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RESEARCH ARTICLE

Aperiodically Intermittent Control Strategy and Adaptive Synchronization of Neural Networks With Inertial and Memristive Terms

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ABSTRACT The problem of adaptive aperiodic intermittent synchronization scheme is studied for inertial memristive delayed neural networks by using a piecewise Lyapunov function in this article. First, an appropriate two-parameter variable replace technique is proposed about the second-order differential equation, and inertial memristive delayed neural systems can be replaced by first-order derivatives of states. Second, an aperiodic intermittent control strategy which can be degenerated periodically intermittent controller or continue feedback controller is designed. Third, by building a piecewise Lyapunov function and using piecewise analytical skills, a number of novel and effective norms to guarantee the exponential synchronization and global exponential synchronization of memristive delayed inertial systems are obtained. Besides, an adaptive control method is proposed to adjust control gains. And asymptotic synchronization and exponential synchronization are ensured by constructing of Lyapunov functional. In the end, simulation results are given to support the significance of the research work.

INDEX TERMS Memristive neural network, aperiodically intermittent control, adaptive control, synchronization, inertial terms.

I. INTRODUCTION

As we all know, the four fundamental variables are charge Q, current I, voltage V and magnetic flux Φ in circuit theory. The three fundamental electric circuit elements are capacitance C, resistance R and inductance L. According to the definition of current and Faraday's law, three circuit elements connect four basic variables: $R = \frac{dV}{dI}$, $C = \frac{dQ}{dV}$, $L = \frac{d\Phi}{dI}$. In 1971, according to the symmetry of variables, American scholars foretold the existence of the fourth fundamental element: memristor. We define the functional relationship between magnetic flux Φ and charge Q, i.e. $\frac{d\Phi}{dQ}$ [1] as memristor. In 2008, Williams of HP Labs and his colleagues produced nanoscale solid-state memristors and pointed out that memristors are memory nonlinear resistors: the resistance is variable and can remember the current flowing through them. Under the periodic voltage, the memristor shows fingerprint characteristics, that is, a twisted hysteresis ring (an oblique figure of eight from the appearance). The resistance of memristor changes continuously with the flow of current, so memristor can be used in analog circuits. It is this characteristic of memristor that makes its application become a research hotspot [2], [3], [4], [5], [6].

Foreign scholars pershin and others verified that the adaptive behavior of cells is similar to the property of memristor through the experiment of single-cell amoeba. Moreover, they successfully verified the associative memory function of dogs by simulating the conditioned reflex behavior of dogs with memristor simulator. This means that the memory resistance is like a neuron's synapse, and the variability of resistance is similar to the variability of synaptic strength. Therefore, we can use memristor to establish an artificial neural network

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with variable weight, which can better simulate the associative memory function of human brain. If the memristor is combined with the powerful high-speed parallel processing ability of neural network, it is expected to form a network with stronger processing capacity, faster running speed and more compact structure. It has great potential application in many fields and will have a huge and far-reaching impact on these industries.

In the past few decades, most scholars usually study first-order ordinary differential equations. By leading inductors into network circuits, the original inertial neural system was put forward by Babcock and Westervelt in 1986, which can be represented by second-order ordinary differential equations [7]. The inertia term is the main element in the application of chaos and bifurcation suppression. Hence, it is very essential for us to research the dynamic action of inertial network. However, owing to the inherent complexity of inertial neural system, it is more challenging to deal with inertial neural system than general neural system. In fact, inertial neural networks have practical biological background. For example, the quasi active membrane behavior of squid axons and neurons can be simulated by circuits involving inductance [8], [9]; Some cell membranes of pigeons can be modeled by artificial circuits containing inducible elements [8].

The word synchronization comes from the Greek root, which means sharing the same time. The Dutch physicist Christian Huygens first studied the phenomenon of synchronization. He found that two weakly connected pendulums can achieve synchronization in phase. Now, synchronization mainly refers to the time consistency of different processes. At present, many different forms of synchronization of various neural systems have been successively proposed, for example exponential synchronization [11], anti-synchronization [12], delay synchronization [13], finite-time synchronization [14], fixed-time synchronization [15], projective synchronization [16], cluster synchronization [17], lag synchronization [18] and so on.

In order to make the network system stable or synchronous, we force the control input to the nodes of the network system. So far, researchers have proposed many control methods, such as feedback control [19], periodic intermittent control [20], adaptive control [22], impulse control [23] and so on. Impulse control method and intermittent control method are discontinuous control strategy, while feedback control method and adaptive control method are continuous control methods. Periodic intermittent control method is more economical and effective than continuous control strategy, and can reduce the number of transmitted signals. As we all know, adaptive control method is provided according to the characteristics of the system under the control goal. The control gains can be automatically adjusted on the basis of the appropriate update law which is the advantage of adaptive control strategy. Therefore, the synchronization of delayed memristive inertial neural systems via aperiodic intermittent adaptive control method is worth studying.

Inspired by the above analysis, conclusions on the synchronization of delayed memristive inertial neural systems are almost via impulsive control or feedback control. But, few researchers concentrate on the asymptotic and exponential synchronization of delayed memristive inertial neural systems under intermittent and adaptive control strategy. This is of great significance to broaden the research of mixture control based on memristive delayed inertial neural network. Owing to the great performance of adaptive intermittent strategy in complex systems, it ought to use in else classes of neural systems, and this paper investigates its application in inertial memristive delayed neural systems. A lots of conclusions are obtained on the question consequently. Two main difficulties will be investigated in the following. (1) How to cut down the complicacy of synchronization analytical for delayed memristive inertial neural networks caused by memristive and delay. (2) How to designed the control parameter of adaptive intermittent control strategy to make delayed memristive inertial neural networks asymptotic synchronization and exponentially synchronization. In the paper, in order to obtain the control gain, some synchronization criteria are obtained. The merit of this article are summarized in the following.

- Compared to lots of synchronization results on the first-order memristive inertial networks, the synchronization of the second-order memristive inertial systems via adaptive aperiodically intermittent controller is first investigated in this article and the previous results are further supplemented.
- To deal with the second-order derivatives, an appropriate two-parameter variable substitution way is presented, and then inertial memristive delayed neural networks may be replaced in the form of first-order differential equation.
- An aperiodic intermittent control strategy which can be degenerated periodically intermittent controller or continue feedback controller is designed.
- By building piecewise Lyapunov functionals and using piecewise analytical skills, some novel and effective criteria to ensure the asymptotic and exponential synchronization of memristive inertial delayed neural systems are obtained.
- Besides, an adaptive controller of memristive neural networks is presented to automatically adjust control parameters. Furthermore, asymptotic synchronization and exponential synchronization are reached under piecewise Lyapunov functional.

The remaining of the article is organized in the following. Model of memristive inertial delayed neural systems and preliminaries are proposed in Section II. Asymptotic synchronization and exponential synchronization of the researched model via aperiodically intermittent adaptive controller are researched in Section III. Some numerical simulations illustrate the validness of the systems methods in Section IV,. Finally, in Section V, some brief conclusions are given.

II. PRELIMINARIES

A class of inertial delayed systems is investigated as follows in the acticle.

$$\ddot{r}_i(t) = -c_i\dot{r}_i(t) - d_ir_i(t) + \sum_{j=1}^{n} a_{ij}(r_i(t))g_j(r_j(t)) + \sum_{j=1}^{n} b_{ij}(r_i(t))g_j(r_j(t-\tau_j)) + I_i, \quad (1)$$

in which $i, j \in \mathscr{I} = \{1, 2, \ldots, n\}$, $r_i(t)$ stands for the state of the ith nerve cell, the second derivative is known as an inertial term of network (1). c_i, $d_i > 0$ represent constant connection weights, $g_j(\cdot)$ corresponds activation functions, τ_j represents the transmission delay, and I_i stands for the external bias on the ith neuron. $a_{ij}(r_i(t))$ and $b_{ij}(r_i(t))$ are memristive connection weights and satisfy the conditions as follows.

$$a_{ij}(r_i(t)) = \frac{\boldsymbol{M}_{ij}}{\boldsymbol{C}_i} \times sign_{ij}, \; b_{ij}(r_i(t)) = \frac{\widetilde{\boldsymbol{M}}_{ij}}{\boldsymbol{C}_i} \times sign_{ij},$$

$$sign_{ij} = \begin{cases} 1, & i \neq j, \\ -1, & i = j, \end{cases}$$

where $\boldsymbol{M}_{ij}$, $\widetilde{\boldsymbol{M}}_{ij}$ stand for the memductances of memristors $\boldsymbol{R}_{ij}$, $\widetilde{\boldsymbol{R}}_{ij}$, respectively. Besides, $\boldsymbol{R}_{ij}$ stands for the memristor between $g_i(r_i(t))$ and $r_i(t)$, $\widetilde{\boldsymbol{R}}_{ij}$ stands for the memristor between $g_i(r_i(t-\tau_i))$ and $r_i(t)$.

Capacitor $\boldsymbol{C}_i$ is invariable and memductances $\boldsymbol{M}_{ij}$, $\widetilde{\boldsymbol{M}}_{ij}$ respond to change in pinched hysteresis loops. Hence, $a_{ij}(r_i(t))$, $b_{ij}(r_i(t))$ will change as pinched hysteresis loops change. Under the current-voltage characteristic and the feature of the memristor, then

$$a_{ij}(r_i(t)) = \begin{cases} \hat{a}_{ij}, & | r_i(t) | < \mathfrak{T}_i, \\ \check{a}_{ij}, & | r_i(t) | \geq \mathfrak{T}_i, \end{cases}$$

$$b_{ij}(r_i(t)) = \begin{cases} \hat{b}_{ij}, & | r_i(t) | < \mathfrak{T}_i, \\ \check{b}_{ij}, & | r_i(t) | \geq \mathfrak{T}_i, \end{cases}$$

in which switching jumps $\mathfrak{T}_i > 0$, $\hat{a}_{ij}$, $\check{a}_{ij}$, $\hat{b}_{ij}$, $\check{b}_{ij}$, $i, j \in \mathscr{I}$ are constants.

Based on the above discussion, the solution for the right-hand sides discontinuous system (1) does not exist in the conventional meaning. However, the dynamical behave of system (1) can be discussed based on Filippov solution. Now, in order to define Filippov solution, the non-autonomous differential equation in the following is considered.

$$\frac{dz}{dt} = g(t, z),$$

in which t represents time, z denotes state vector, $\frac{dz}{dt}$ represents derivative of the time t and $g(t, z)$ is discontinuous about z.

Definition 3 [24]: Assume that the set-valued map $G : R \times R^n \to 2^{R^n}$ satisfies

$$G(t, z) = \bigcap_{\varrho > 0} \bigcap_{\mu(N)=0} \overline{co}[g(t, B(z, \varrho)\backslash N)],$$

where $\overline{co}[\cdot]$ represents the closure of the convex hull, $B(z, \varrho)$ is the ball of center z and radius ϱ and $\mu(N)$ is the Lebesgue measure of set N, intersection is taken over all sets N of measure zero and over all $\varrho > 0$. A solution $z(t)$ of equation $\frac{dz}{dt} = g(t, z)$ in Filippov's sense with initial condition $z(t_0) = z_0$ is an absolutely continuous vector value function which satisfies differential inclusion

$$\frac{dz}{dt} \in G(t, z).$$

Under the above differential inclusion theory and (1),

$$\ddot{r}_i(t) \in -c_i\dot{r}_i(t) - d_ir_i(t) + \sum_{j=1}^{n} co[a_{ij}(r_i(t))]g_j(r_j(t)) + \sum_{j=1}^{n} co[b_{ij}(r_i(t))]g_j(r_j(t-\tau_j)) + I_i, \quad (2)$$

in which

$$co[a_{ij}(r_i(t))] = \begin{cases} \hat{a}_{ij}, & | r_i(t) | < \mathfrak{T}_i, \\ co\{\hat{a}_{ij}, \check{a}_{ij}\}, & | r_i(t) | = \mathfrak{T}_i, \\ \check{a}_{ij}, & | r_i(t) | > \mathfrak{T}_i, \end{cases}$$

$$co[b_{ij}(r_i(t))] = \begin{cases} \hat{b}_{ij}, & | r_i(t) | < \mathfrak{T}_i, \\ co\{\hat{b}_{ij}, \check{b}_{ij}\}, & | r_i(t) | = \mathfrak{T}_i, \\ \check{b}_{ij}, & | r_i(t) | > \mathfrak{T}_i, \end{cases}$$

or equivalently, assume that $a^*_{ij}(r_i(t)) \in co[a_{ij}(r_i(t))]$, $b^*_{ij}(r_i(t)) \in co[b_{ij}(r_i(t))]$, then

$$\ddot{r}_i(t) = -c_i\dot{r}_i(t) - d_ir_i(t) + \sum_{j=1}^{n} a^*_{ij}(r_i(t))g_j(r_j(t)) + \sum_{j=1}^{n} b^*_{ij}(r_i(t))g_j(r_j(t-\tau_j)) + I_i. \quad (3)$$

Throughout the paper, the drive system is given by the model (1) and we discuss the following response network.

$$\ddot{h}_i(t) = -c_i\dot{h}_i(t) - d_ih_i(t) + \sum_{j=1}^{n} a_{ij}(h_i(t))g_j(h_j(t)) + \sum_{j=1}^{n} b_{ij}(h_i(t))g_j(h_j(t-\tau_j)) + I_i + \xi_i(t), \quad (4)$$

in which $\xi_i(t)$ is an suitable controller to be determined.

Similar to the analysis of (3), we obtain

$$\ddot{h}_i(t) \in -c_i\dot{h}_i(t) - d_ih_i(t) + \sum_{j=1}^{n} co[a_{ij}(h_i(t))]g_j(h_j(t)) + \sum_{j=1}^{n} co[b_{ij}(h_i(t))]g_j(h_j(t-\tau_j)) + I_i + \xi_i(t), \quad (5)$$

where

$$co[a_{ij}(h_i(t))] = \begin{cases} \hat{a}_{ij}, & |h_i(t)| < \mathfrak{T}_i, \\ co\{\hat{a}_{ij}, \check{a}_{ij}\}, & |h_i(t)| = \mathfrak{T}_i, \\ \check{a}_{ij}, & |h_i(t)| > \mathfrak{T}_i, \end{cases}$$

$$co[b_{ij}(h_i(t))] = \begin{cases} \hat{b}_{ij}, & |h_i(t)| < \mathfrak{T}_i, \\ co\{\hat{b}_{ij}, \check{b}_{ij}\}, & |h_i(t)| = \mathfrak{T}_i, \\ \check{b}_{ij}, & |h_i(t)| > \mathfrak{T}_i, \end{cases}$$

or equivalently, assume that $a_{ij}^{**}(h_i(t)) \in co[a_{ij}(h_i(t))]$, $b_{ij}^{**}(h_i(t)) \in co[b_{ij}(h_i(t))]$, then

$$\ddot{h}_i(t) = -c_i\dot{h}_i(t) - d_ih_i(t) + \sum_{j=1}^{n} a_{ij}^{**}(h_i(t))g_j(h_j(t)) + \sum_{j=1}^{n} b_{ij}^{**}(h_i(t))g_j(h_j(t-\tau_j)) + I_i + \xi_i(t). \tag{6}$$

$r_i(s) = \varphi_i(s)$, $h_i(s) = \psi_i(s)$, $\dot{r}_i(s) = \chi_i(s)$, $\dot{h}_i(s) = \phi_i(s)$, $i \in \mathscr{I}$ are the initial values of systems (1) and (4), respectively, $s \in [-\tau, 0]$, in which $\tau = \max_{i\in\mathscr{I}}\{\tau_i\}$, $\varphi_i(s)$, $\psi_i(s)$, $\chi_i(s)$, $\phi_i(s) \in C([-\tau, 0], R)$.

Assumption 1: For any $j \in \mathscr{I}$, there exist constants $L_j, M_j > 0$ such that

$$|g_j(r_j) - g_j(h_j)| \le L_j|r_j - h_j|, \quad |g_j(r_j)| \le M_j, \quad r_j, h_j \in R.$$

Assumption 2 [27]: Under the intermittent controller, assume that

$$\begin{cases} \inf_m(\hat{\mathfrak{S}}_m - \hat{\mathfrak{T}}_m) = \theta, \\ \sup_m(\hat{\mathfrak{T}}_{m+1} - \hat{\mathfrak{T}}_m) = \delta, \end{cases} \tag{7}$$

hold for constants $0 < \theta < \delta < +\infty$ and $m \in z^+ = \{0, 1, 2, \ldots\}$.

Lemma 1 [28]: Assume that $\pi(\cdot) : [\hat{\mathfrak{T}}_0 - \tau, +\infty) \to [0, +\infty)$ is a continuous function and $\dot{\pi}(t) \le -a\pi(t) + b\bar{\pi}(t)$ holds for $t \ge \hat{\mathfrak{T}}_0$, in which $\bar{\pi}(t) = \sup_{-\tau\le k\le 0}(\pi(t+k))$. If $a > b > 0$, then

$$\pi(t) \le \bar{\pi}(\hat{\mathfrak{T}}_0)\exp\{-\varepsilon(t - \hat{\mathfrak{T}}_0)\}, \quad t \ge \hat{\mathfrak{T}}_0,$$

in which $\varepsilon > 0$ meets equality $\varepsilon - a + b\exp\{\varepsilon\tau\} = 0$

Lemma 2 [29]: Assume that $\pi(\cdot) : [\hat{\mathfrak{T}}_0 - \tau, +\infty) \to [0, +\infty)$ is a continuous function and $\dot{\pi}(t) \le a(t)\pi(t) + b(t)\bar{\pi}(t)$ holds for $t \ge \hat{\mathfrak{T}}_0$, in which $\bar{\pi}(t) = \sup_{-\tau\le k\le 0}(\pi(t+k))$. If $b(t) > 0$, $a(t) + b(t) \ge m^*$, then

$$\pi(t) \le \bar{\pi}(\hat{\mathfrak{T}}_0)\exp\{m^*(t - \hat{\mathfrak{T}}_0)\}, \quad t \ge \hat{\mathfrak{T}}_0.$$

Especially, when $a(t) = a$ and $b(t) = b$ are constants,

$$\pi(t) \le \bar{\pi}(\hat{\mathfrak{T}}_0)\exp\{(a+b)(t - \hat{\mathfrak{T}}_0)\}, \quad t \ge \hat{\mathfrak{T}}_0.$$

By leading into the variable substitution

$$u_i(t) = \dot{r}_i(t) + \eta_ir_i(t), i \in \mathscr{I},$$

system (3) changes into

$$\begin{cases} \dot{r}_i(t) = -\eta_ir_i(t) + u_i(t), \\ \dot{u}_i(t) = -\rho_iu_i(t) - \sigma_ir_i(t) + \sum_{j=1}^{n} a_{ij}^{*}(r_i(t))g_j(r_j(t)) \\ \qquad + \sum_{j=1}^{n} b_{ij}^{*}(r_i(t))g_j(r_j(t-\tau_j)) + I_i, \end{cases} \tag{8}$$

in which $\rho_i = c_i - \eta_i$, $\sigma_i = d_i - c_i\eta_i + \eta_i^2$.

So, by leading into variable substitution

$$v_i(t) = \dot{h}_i(t) + \eta_ih_i(t), i \in \mathscr{I},$$

system (4) changes into

$$\begin{cases} \dot{h}_i(t) = -\eta_ih_i(t) + v_i(t) + \xi_{1i}, \\ \dot{v}_i(t) = -\rho_iv_i(t) - \sigma_ih_i(t) + \sum_{j=1}^{n} a_{ij}^{**}(h_i(t))g_j(h_j(t)) \\ \qquad + \sum_{j=1}^{n} b_{ij}^{**}(h_i(t))g_j(h_j(t-\tau_j)) + I_i + \xi_{2i}. \end{cases} \tag{9}$$

III. SYNCHRONIZATION AND EXPONENTIAL SYNCHRONIZATION

First, assume the synchronization error is $\omega_{1i}(t) = h_i(t) - r_i(t)$, $\omega_{2i}(t) = v_i(t) - u_i(t)$. From (8) and (9), then

$$\begin{cases} \dot{\omega}_{1i}(t) = -\eta_i\omega_{1i}(t) + \omega_{2i}(t) + \xi_{1i}(t), \\ \dot{\omega}_{2i}(t) = -\rho_i\omega_{2i}(t) - \sigma_i\omega_{1i}(t) + \sum_{j=1}^{n}\Big(a_{ij}^{**}(h_i(t))g_j(h_j(t)) \\ -a_{ij}^{*}(r_i(t))g_j(r_j(t))\Big) + \sum_{j=1}^{n}\Big(b_{ij}^{**}(h_i(t))g_j(h_j(t \\ -\tau_j)) - b_{ij}^{*}(r_i(t))g_j(r_j(t-\tau_j))\Big) + \xi_{2i}(t). \end{cases} \tag{10}$$

Now, the adaptive intermittent control strategies $\xi_{1i}(t)$, $\xi_{2i}(t)$ are introduced as follows:

$$\xi_{1i}(t) = \begin{cases} -\alpha_{1i}(t)\omega_{1i}(t) - \mu\beta_1\Big(\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\omega_{1i}^2(s)ds\Big) \\ \dfrac{\omega_{1i}(t)}{||\omega_1(t)||^2}, \quad \hat{\mathfrak{T}}_m \le t \le \hat{\mathfrak{S}}_m, \\ 0, \quad \hat{\mathfrak{S}}_m < t < \hat{\mathfrak{T}}_{m+1}, \end{cases} \tag{11}$$

$$\alpha_{1i}(t) = \begin{cases} \alpha_{1i}(\hat{\mathfrak{T}}_0), & t = \hat{\mathfrak{T}}_0, \\ \alpha_{1i}(\hat{\mathfrak{S}}_m), & t = \hat{\mathfrak{T}}_{m+1}, \\ 0, & \hat{\mathfrak{S}}_m < t < \hat{\mathfrak{T}}_{m+1}, \end{cases} \tag{12}$$

and

$$\dot{\alpha}_{1i}(t) = \zeta_{1i}\omega_{1i}^2(t), \quad \hat{\mathfrak{T}}_m \le t \le \hat{\mathfrak{S}}_m, \tag{13}$$

$$\xi_{2i}(t)=\begin{cases}-\alpha_{2i}(t)\omega_{2i}(t)-\beta_2\mathrm{sign}(\omega_{2i}(t)), & \hat{\mathfrak{T}}_m\le t\le\hat{\mathfrak{S}}_m,\\ -\beta_2\mathrm{sign}(\omega_{2i}(t)), & \hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1},\end{cases} \tag{14}$$

$$\alpha_{2i}(t)=\begin{cases}\alpha_{2i}(\hat{\mathfrak{T}}_0), & t=\hat{\mathfrak{T}}_0,\\ \alpha_{2i}(\hat{\mathfrak{S}}_m), & t=\hat{\mathfrak{T}}_{m+1},\\ 0, & \hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1},\end{cases} \tag{15}$$

and

$$\dot{\alpha}_{2i}(t)=\zeta_{2i}\omega_{2i}^2(t),\quad \hat{\mathfrak{T}}_m\le t\le\hat{\mathfrak{S}}_m, \tag{16}$$

here $m\in Z^+$, μ, β_1, β_2, ζ_{1i}, $\zeta_{2i}>0$ are the control strengths.

Theorem 1: Under Assumptions 1-2, for β_1, β_2, $\lambda>0$, assume that

(i): $\beta_1\ge\frac{1}{2}\sum_{j=1}^{n}\bar{b}_{ji}L_i$,

(ii): $\sum_{j=1}^{n}\big(|\hat{a}_{ij}-\check{a}_{ij}|+|\hat{b}_{ij}-\check{b}_{ij}|\big)M_j-\beta_2\le 0$,

(iii): $-\rho_i+\frac{1}{2}\sum_{j=1}^{n}(\bar{a}_{ij}+\bar{b}_{ij})L_j+\frac{|1-\sigma_i|}{2}-\frac{\lambda}{2}\le 0$,

(v): $-\eta_i+\beta_1+\frac{1}{2}\sum_{j=1}^{n}\bar{a}_{ji}L_i+\frac{|1-\sigma_i|}{2}-\frac{\lambda}{2}\le 0$, hold, then the inertial networks (8) and (9) are global synchronized via control scheme (11)-(16).

Proof: Under assumption 1, then

$$\begin{aligned}&\big|a_{ij}^{**}(h_i(t))g_j(h_j(t))-a_{ij}^{*}(r_i(t))g_j(r_j(t))\big|\\ &\le\big|a_{ij}^{**}(h_i(t))g_j(h_j(t))-a_{ij}^{**}(h_i(t))g_j(r_j(t))\big|\\ &\quad+\big|a_{ij}^{**}(h_i(t))g_j(r_j(t))-a_{ij}^{*}(r_i(t))g_j(r_j(t))\big|\\ &\le\bar{a}_{ij}L_j\big|\omega_{1j}(t)\big|+\big|\hat{a}_{ij}-\check{a}_{ij}\big|M_j,\end{aligned} \tag{17}$$

where $\bar{a}_{ij}=\max\{|\hat{a}_{ij}|,|\check{a}_{ij}|\}$.

In a similar way, we can get

$$\begin{aligned}&\big|b_{ij}^{**}(h_i(t))g_j(h_j(t-\tau_j))-b_{ij}^{*}(r_i(t))g_j(r_j(t-\tau_j))\big|\\ &\qquad\le\bar{b}_{ij}L_j\big|\omega_{1j}(t-\tau_j)\big|+\big|\hat{b}_{ij}-\check{b}_{ij}\big|M_j,\end{aligned} \tag{18}$$

where $\bar{b}_{ij}=\max\{|\hat{b}_{ij}|,|\check{b}_{ij}|\}$.

Construct the following piecewise function

$$Q(t)=\begin{cases}\frac{1}{2}\exp\{-\mu(t-\hat{\mathfrak{T}}_m)\}\sum_{i=1}^{n}\Big[\frac{1}{\zeta_{1i}}(\alpha_{1i}^{*}-\alpha_{1i}(t))^2\\ +\frac{1}{\zeta_{2i}}(\alpha_{2i}^{*}-\alpha_{2i}(t))^2\Big],\hat{\mathfrak{T}}_m\le t\le\hat{\mathfrak{S}}_m,\\ \frac{1}{2}\exp\{\lambda(t-\hat{\mathfrak{S}}_m)-\mu(\hat{\mathfrak{S}}_m-\hat{\mathfrak{T}}_m)\}\sum_{i=1}^{n}\Big[\frac{1}{\zeta_{1i}}(\alpha_{1i}^{*}\\ -\alpha_{1i}(\hat{\mathfrak{S}}_m))^2+\frac{1}{\zeta_{2i}}(\alpha_{2i}^{*}-\alpha_{2i}(\hat{\mathfrak{S}}_m))^2\Big],\\ \hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1},\end{cases} \tag{19}$$

where α_{1i}^{*}, α_{2i}^{*} are positive constants. From (19), $Q(t)$ is continuous apart from $t=\hat{\mathfrak{T}}_{m+1}$ with $m\in Z^+$ and

$$\begin{aligned}&Q_+(\hat{\mathfrak{T}}_{m+1})\\ &=\exp\{\mu(\hat{\mathfrak{S}}_m-\hat{\mathfrak{T}}_m)-\lambda(\hat{\mathfrak{T}}_{m+1}-\hat{\mathfrak{S}}_m)\}Q_-(\hat{\mathfrak{T}}_{m+1}),\end{aligned} \tag{20}$$

in which $Q_-(\hat{\mathfrak{T}}_{m+1})$ and $Q_+(\hat{\mathfrak{T}}_{m+1})$ represent respectively the left limit and the right limit of $Q(t)$ at time $\hat{\mathfrak{T}}_{m+1}$.

Assume that

$$H(t)=W(t)+Q(t), \tag{21}$$

in which

$$W(t)=\frac{1}{2}\sum_{i=1}^{n}\omega_{1i}^2(t)+\beta_1\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\omega_{1i}^2(s)ds+\frac{1}{2}\sum_{i=1}^{n}\omega_{2i}^2(t). \tag{22}$$

Obviously, $W(t)$ is continuous at every $t\ge\hat{\mathfrak{T}}_0$. $Q(t)$ is continuous apart from $t=\hat{\mathfrak{T}}_{m+1}$ with $m\in Z^+$. But, $Q(t)$ is right continuous for $t=\hat{\mathfrak{T}}_{m+1}$.

For $\hat{\mathfrak{T}}_m\le t\le\hat{\mathfrak{S}}_m,\ m\in Z^+$

$$\begin{aligned}\dot{H}(t)\le&-\sum_{i=1}^{n}\eta_i\omega_{1i}^2(t)+\sum_{i=1}^{n}\omega_{1i}(t)\omega_{2i}(t)-\sum_{i=1}^{n}\alpha_{1i}(t)\\ &\times\omega_{1i}^2(t)+\beta_1\sum_{i=1}^{n}\omega_{1i}^2(t)-\beta_1\sum_{i=1}^{n}\omega_{1i}^2(t-\tau_i)\\ &-\sum_{i=1}^{n}\rho_i\omega_{2i}^2(t)-\sum_{i=1}^{n}\sigma_i\omega_{1i}(t)\omega_{2i}(t)+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\\ &\Big(\bar{a}_{ij}+\bar{b}_{ij}\Big)L_j\omega_{2i}^2(t)+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\bar{a}_{ji}L_i\omega_{1i}^2(t)\\ &+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\bar{b}_{ji}L_i\omega_{1i}^2(t-\tau_i)+\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(|\hat{a}_{ij}-\check{a}_{ij}|\\ &+|\hat{b}_{ij}-\check{b}_{ij}|\Big)M_j|\omega_{2i}(t)|-\sum_{i=1}^{n}\alpha_{2i}(t)\omega_{2i}^2(t)\\ &-\beta_2\sum_{i=1}^{n}|\omega_{2i}(t)|-\mu\beta_1\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\omega_{1i}^2(s)ds-\mu Q(t)\\ &-\exp\{-\mu(t-\hat{\mathfrak{T}}_m)\}\sum_{i=1}^{n}\Big[(\alpha_{1i}^{*}-\alpha_{1i}(t))\omega_{1i}^2\\ &+(\alpha_{2i}^{*}-\alpha_{2i}(t))\omega_{2i}^2\Big]\\ \le&\sum_{i=1}^{n}\Big(-\eta_i+\beta_1+\frac{1}{2}\sum_{j=1}^{n}\bar{a}_{ji}L_i\Big)\omega_{1i}^2(t)+\sum_{i=1}^{n}(1\\ &-\sigma_i)\omega_{1i}(t)\omega_{2i}(t)+\sum_{i=1}^{n}\Big(-\beta_1+\frac{1}{2}\sum_{j=1}^{n}\bar{b}_{ji}L_i\Big)\\ &\omega_{1i}^2(t-\tau_i)+\sum_{i=1}^{n}\Big(-\rho_i+\frac{1}{2}\sum_{j=1}^{n}(\bar{a}_{ij}+\bar{b}_{ij})L_j\Big)\end{aligned}$$

$$\omega_{2i}^2(t)+\sum_{i=1}^{n}\Big(\sum_{j=1}^{n}(|\hat{a}_{ij}-\check{a}_{ij}|+|\hat{b}_{ij}-\check{b}_{ij}|)M_j$$
$$-\beta_2\Big)|\omega_{2i}(t)|-\mu\beta_1\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\omega_{1i}^2(s)ds-\mu Q(t)$$
$$-\exp\{-\mu(t-\hat{\mathfrak{T}}_m)\}\sum_{i=1}^{n}(\alpha_{1i}^*\omega_{1i}^2+\alpha_{2i}^*\omega_{2i}^2)$$
$$\leq\sum_{i=1}^{n}\Big(-\eta_i+\beta_1+\frac{1}{2}\sum_{j=1}^{n}\overline{a}_{ji}L_i-\exp\{-\mu\delta\}\alpha_{1i}^*$$
$$+\frac{|1-\sigma_i|}{2}+\frac{\mu}{2}\Big)\omega_{1i}^2(t)-\frac{\mu}{2}\sum_{i=1}^{n}\omega_{1i}^2(t)+\sum_{i=1}^{n}\Big($$
$$-\rho_i+\frac{1}{2}\sum_{j=1}^{n}(\overline{a}_{ij}+\overline{b}_{ij})L_j-\exp\{-\mu\delta\}\alpha_{2i}^*$$
$$+\frac{|1-\sigma_i|}{2}+\frac{\mu}{2}\Big)\omega_{2i}^2(t)-\frac{\mu}{2}\sum_{i=1}^{n}\omega_{2i}^2(t)$$
$$-\mu\beta_1\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\omega_{1i}^2(s)ds-\mu Q(t). \tag{23}$$

It is easy to see that some suitable $\alpha_{1i}^*,\ \alpha_{2i}^*$ can be chosen such that

$$-\eta_i+\beta_1+\frac{1}{2}\sum_{j=1}^{n}\overline{a}_{ji}L_i-\exp\{-\mu\delta\}\alpha_{1i}^*+\frac{|1-\sigma_i|}{2}+\frac{\mu}{2}\leq 0,$$

and

$$-\rho_i+\frac{1}{2}\sum_{j=1}^{n}(\overline{a}_{ij}+\overline{b}_{ij})L_j-\exp\{-\mu\delta\}\alpha_{2i}^*+\frac{|1-\sigma_i|}{2}+\frac{\mu}{2}\leq 0,$$

which leads that

$$\dot{H}(t)\leq-\mu H(t). \tag{24}$$

So, for $\hat{\mathfrak{T}}_m\leq t\leq\hat{\mathfrak{S}}_m$,

$$H(t)\leq\exp\{-\mu(t-\hat{\mathfrak{T}}_m)\}H_+(\hat{\mathfrak{T}}_m). \tag{25}$$

When $\hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1}$,

$$\dot{H}(t)\leq\sum_{i=1}^{n}\Big(-\eta_i+\beta_1+\frac{1}{2}\sum_{j=1}^{n}\overline{a}_{ji}L_i\Big)\omega_{1i}^2(t)$$
$$+\sum_{i=1}^{n}(1-\sigma_i)\omega_{1i}(t)\omega_{2i}(t)+\sum_{i=1}^{n}\Big(-\beta_1$$
$$+\frac{1}{2}\sum_{j=1}^{n}\overline{b}_{ji}L_i\Big)\omega_{1i}^2(t-\tau_i)+\sum_{i=1}^{n}\Big(-\rho_i$$
$$+\frac{1}{2}\sum_{j=1}^{n}(\overline{a}_{ij}+\overline{b}_{ij})L_j\Big)\omega_{2i}^2(t)$$
$$+\sum_{i=1}^{n}\Big(\sum_{j=1}^{n}(|\hat{a}_{ij}-\check{a}_{ij}|+|\hat{b}_{ij}-\check{b}_{ij}|)M_j$$
$$-\beta_2\Big)|\omega_{2i}(t)|+\lambda Q(t)$$
$$\leq\sum_{i=1}^{n}\Big(-\eta_i+\beta_1+\frac{1}{2}\sum_{j=1}^{n}\overline{a}_{ji}L_i+\frac{|1-\sigma_i|}{2}$$
$$-\frac{\lambda}{2}\Big)\omega_{1i}^2(t)+\frac{\lambda}{2}\sum_{i=1}^{n}\omega_{1i}^2(t)+\sum_{i=1}^{n}\Big(-\rho_i$$
$$+\frac{1}{2}\sum_{j=1}^{n}(\overline{a}_{ij}+\overline{b}_{ij})L_j+\frac{|1-\sigma_i|}{2}-\frac{\lambda}{2}\Big)\omega_{2i}^2(t)$$
$$+\frac{\lambda}{2}\sum_{i=1}^{n}\omega_{2i}^2(t)+\lambda Q(t)$$
$$\leq\lambda H(t), \tag{26}$$

so,

$$H(t)\leq\exp\{\lambda(t-\hat{\mathfrak{S}}_m)\}H(\hat{\mathfrak{S}}_m). \tag{27}$$

Via (17), (22) and (25), then

$$H_-(\hat{\mathfrak{T}}_{m+1})\leq\exp\{\lambda(\hat{\mathfrak{T}}_{m+1}-\hat{\mathfrak{S}}_m)\}\exp\{-\mu(\hat{\mathfrak{S}}_m-\hat{\mathfrak{T}}_m)\}H_+(\hat{\mathfrak{T}}_m)$$
$$\leq\exp\{-\alpha\}H_-(\hat{\mathfrak{T}}_m)+(1-\exp\{-\alpha\})Q_-(\hat{\mathfrak{T}}_m),$$

where $\alpha=\mu\theta-\lambda(\delta-\theta)>0$ and

$$H_-(\hat{\mathfrak{T}}_{m+1})-H_-(\hat{\mathfrak{T}}_m)\leq(\exp\{-\alpha\}-1)H_-(\hat{\mathfrak{T}}_m)$$
$$+(1-\exp\{-\alpha\})Q_-(\hat{\mathfrak{T}}_m)$$
$$=(\exp\{-\alpha\}-1)W(\hat{\mathfrak{T}}_m),$$

and then

$$H_-(\hat{\mathfrak{T}}_{m+1})-H(\hat{\mathfrak{T}}_0)\leq(\exp\{-\alpha\}-1)\sum_{i=0}^{m}W(\hat{\mathfrak{T}}_i),$$

it leads that

$$\sum_{i=0}^{\infty}W(\hat{\mathfrak{T}}_i)\leq\frac{H(\hat{\mathfrak{T}}_0)}{1-\exp\{-\alpha\}}, \tag{28}$$

hence,

$$\lim_{i\to+\infty}W(\hat{\mathfrak{T}}_i)=0.$$

Besides, when $\hat{\mathfrak{T}}_m<t<\hat{\mathfrak{T}}_{m+1}$, owing to the nonnegativeness of $\alpha_{1i}(t)$, $\alpha_{2i}(t)$ and $\mu\beta_1$, it's easy to get

$$\dot{H}(t)\leq\sum_{i=1}^{n}\Big(-\eta_i+\beta_1+\frac{1}{2}\sum_{j=1}^{n}\overline{a}_{ji}L_i\Big)\omega_{1i}^2(t)$$
$$+\sum_{i=1}^{n}(1-\sigma_i)\omega_{1i}(t)\omega_{2i}(t)+\sum_{i=1}^{n}\Big(-\beta_1$$
$$+\frac{1}{2}\sum_{j=1}^{n}\overline{b}_{ji}L_i\Big)\omega_{1i}^2(t-\tau_i)+\sum_{i=1}^{n}\Big(-\rho_i$$

$$+\frac{1}{2}\sum_{j=1}^{n}(\overline{a}_{ij}+\overline{b}_{ij})L_j\Big)\omega_{1i}^2(t)+\sum_{i=1}^{n}\Big(\sum_{j=1}^{n}$$
$$\big(|\hat{a}_{ij}-\check{a}_{ij}|+|\hat{b}_{ij}-\check{b}_{ij}|\big)M_j-\beta_2\Big)|\omega_{2i}(t)|$$
$$\leq\sum_{i=1}^{n}\Big(-\eta_i+\beta_1+\frac{1}{2}\sum_{j=1}^{n}\overline{a}_{ji}L_i+\frac{|1-\sigma_i|}{2}\Big)$$
$$\times\omega_{1i}^2(t)+\sum_{i=1}^{n}\Big(-\rho_i+\frac{1}{2}\sum_{j=1}^{n}(\overline{a}_{ij}+\overline{b}_{ij})L_j$$
$$\leq+\frac{|1-\sigma_i|}{2}\Big)\omega_{2i}^2(t)\hat{\lambda}H(t), \tag{29}$$

where

$$\hat{\lambda}=\max_{i\in\mathscr{I}}\{-\eta_i+\beta_1+\frac{1}{2}\sum_{j=1}^{n}\overline{a}_{ji}L_i+\frac{|1-\sigma_i|}{2},$$
$$-\rho_i+\frac{1}{2}\sum_{j=1}^{n}(\overline{a}_{ij}+\overline{b}_{ij})L_j+\frac{|1-\sigma_i|}{2}\}.$$

Hence,

$$W(t)\leq\exp\{\hat{\lambda}(t-\hat{\mathfrak{T}}_m)\}W(\hat{\mathfrak{T}}_m)$$
$$\leq\exp\{\hat{\lambda}\delta\}W(\hat{\mathfrak{T}}_m),\quad\hat{\mathfrak{T}}_m\leq t\leq\hat{\mathfrak{T}}_{m+1}. \tag{30}$$

Clearly, $m\to\infty$ for $t\to\infty$, so

$$\lim_{t\to+\infty}W(t)=0.$$

Hence, the proof of Theorem 1 is achieved. □

Now, the adaptive control strategy $\xi_{1i}(t),\ \xi_{2i}(t)$ is introduced in the following.

$$\xi_{1i}(t)=\begin{cases}-\alpha_{1i}(t)\omega_{1i}(t), & \hat{\mathfrak{T}}_m\leq t\leq\hat{\mathfrak{S}}_m,\\ 0, & \hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1},\end{cases} \tag{31}$$

$$\alpha_{1i}(t)=\begin{cases}\alpha_{1i}(t_0), & t=\hat{\mathfrak{T}}_0,\\ \alpha_{1i}(\hat{\mathfrak{S}}_m), & t=\hat{\mathfrak{T}}_{m+1},\\ 0, & \hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1},\end{cases} \tag{32}$$

and

$$\dot{\alpha}_{1i}(t)=\zeta_{1i}\exp\{p_1t\}\omega_{1i}^2(t),\quad\hat{\mathfrak{T}}_m\leq t\leq\hat{\mathfrak{S}}_m, \tag{33}$$

$$\xi_{2i}(t)=\begin{cases}-\alpha_{2i}(t)\omega_{2i}(t)-\beta_2\mathrm{sign}(\omega_{2i}(t)), & \hat{\mathfrak{T}}_m\leq t\leq\hat{\mathfrak{S}}_m,\\ -\beta_2\mathrm{sign}(\omega_{2i}(t)), & \hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1},\end{cases} \tag{34}$$

$$\alpha_{2i}(t)=\begin{cases}\alpha_{2i}(t_0), & t=\hat{\mathfrak{T}}_0,\\ \alpha_{2i}(\hat{\mathfrak{S}}_m), & t=\hat{\mathfrak{T}}_{m+1},\\ 0, & \hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1},\end{cases} \tag{35}$$

and

$$\dot{\alpha}_{2i}(t)=\zeta_{2i}\exp\{p_1t\}\omega_{2i}^2(t),\quad\hat{\mathfrak{T}}_m\leq t\leq\hat{\mathfrak{S}}_m, \tag{36}$$

in which $m\in Z^+,\ p_1,\ \beta_2,\ \zeta_{1i},\ \zeta_{2i}>0$ are the control strengths.

Theorem 2: Under Assumptions 1-2, if there exist constants $\beta_2,\ q,\ p_1,\ p_2>0$ such that

(*i*): $q-p_1<0$,

(*ii*): $\sum_{j=1}^{n}\big(|\hat{a}_{ij}-\check{a}_{ij}|+|\hat{b}_{ij}-\check{b}_{ij}|\big)M_j-\beta_2\leq0$,

(*iii*): $-\rho_i+\frac{1}{2}\sum_{j=1}^{n}(\overline{a}_{ij}+\overline{b}_{ij})L_j+\frac{|1-\sigma_i|}{2}-\frac{p_2-p_1}{2}\leq0$,

(*iv*): $-\eta_i+\frac{1}{2}\sum_{j=1}^{n}\overline{a}_{ji}L_i+\frac{|1-\sigma_i|}{2}-\frac{p_2-p_1}{2}\leq0$,

(*v*): $\tau<\inf_m\{\hat{\mathfrak{S}}_m-\hat{\mathfrak{T}}_m\}$,

(*vi*): $\tilde{\mu}=\varepsilon(\theta-\tau)-\gamma(\delta-\theta)$,

then the inertial networks (8) and (9) are global synchronized via control scheme (31)-(36).

Proof: Assume that

$$Q(t)=\begin{cases}\frac{1}{2}\exp\{-p_1t\}\sum_{i=1}^{n}\Big[\frac{1}{\zeta_{1i}}(\alpha_{1i}^*-\alpha_{1i}(t))^2\\ +\frac{1}{\zeta_{2i}}(\alpha_{2i}^*-\alpha_{2i}(t))^2\Big],\ \hat{\mathfrak{T}}_m\leq t\leq\hat{\mathfrak{S}}_m,\\ \frac{1}{2}\exp\{-p_1t\}\sum_{i=1}^{n}\Big[\frac{1}{\zeta_{1i}^*}(\alpha_{1i}^*-\alpha_{1i}(\hat{\mathfrak{S}}_m))^2\\ +\frac{1}{\zeta_{2i}^*}(\alpha_{2i}^*-\alpha_{2i}(\hat{\mathfrak{S}}_m))^2\Big],\quad\hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1},\end{cases} \tag{37}$$

here $\alpha_{1i}^*,\ \alpha_{2i}^*>0$ are positive constants.

Give the following Lyapunov function

$$H(t)=W(t)+Q(t), \tag{38}$$

in which

$$W(t)=\frac{1}{2}\sum_{i=1}^{n}\omega_{1i}^2(t)+\frac{1}{2}\sum_{i=1}^{n}\omega_{2i}^2(t). \tag{39}$$

Obviously, $W(t)$ and $H(t)$ are continuous.

When $\hat{\mathfrak{T}}_m\leq t\leq\hat{\mathfrak{S}}_m,\ m\in Z^+$

$$\dot{H}(t)\leq-\sum_{i=1}^{n}\eta_i\omega_{1i}^2(t)+\sum_{i=1}^{n}\omega_{1i}(t)\omega_{2i}(t)$$
$$-\sum_{i=1}^{n}\alpha_{1i}(t)\omega_{1i}^2(t)-\sum_{i=1}^{n}\rho_i\omega_{2i}^2(t)-\sum_{i=1}^{n}\sigma_i$$
$$\omega_{1i}(t)\omega_{2i}(t)+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(\overline{a}_{ij}+\overline{b}_{ij}\Big)L_j\omega_{2i}^2(t)$$
$$+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\overline{a}_{ji}L_i\omega_{1i}^2(t)+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\overline{b}_{ji}L_i\omega_{1i}^2(t$$
$$-\tau_i)+\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(|\hat{a}_{ij}-\check{a}_{ij}|+|\hat{b}_{ij}-\check{b}_{ij}|\Big)$$
$$M_j|\omega_{2i}(t)|-\sum_{i=1}^{n}\alpha_{2i}(t)\omega_{2i}^2(t)-\beta_2\sum_{i=1}^{n}|\omega_{2i}(t)|$$
$$-p_1Q(t)-\sum_{i=1}^{n}\Big[(\alpha_{1i}^*-\alpha_{1i}(t))\omega_{1i}^2+(\alpha_{2i}^*$$

$$
\begin{aligned}
&-\alpha_{2i}(t))\omega_{2i}^2\Big]\\
&\le \sum_{i=1}^n\Big(-\eta_i+\frac{1}{2}\sum_{j=1}^n \bar{a}_{ji}L_i\Big)\omega_{1i}^2(t)+\sum_{i=1}^n(1\\
&-\sigma_i)\omega_{1i}(t)\omega_{2i}(t)+\sum_{i=1}^n\Big(\frac{1}{2}\sum_{j=1}^n \bar{b}_{ji}L_i\Big)\\
&\omega_{1i}^2(t-\tau_i)+\sum_{i=1}^n\Big(-\rho_i+\frac{1}{2}\sum_{j=1}^n(\bar{a}_{ij}+\bar{b}_{ij})L_j\Big)\\
&\omega_{2i}^2(t)+\sum_{i=1}^n\Big(\sum_{j=1}^n(|\hat{a}_{ij}-\check{a}_{ij}|+|\hat{b}_{ij}-\check{b}_{ij}|)M_j\\
&-\beta_2\Big)|\omega_{2i}(t)|-p_1Q(t)-\sum_{i=1}^n(\alpha_{1i}^*\omega_{1i}^2+\alpha_{2i}^*\omega_{2i}^2)\\
&\le \sum_{i=1}^n\Big(-\eta_i+\frac{1}{2}\sum_{j=1}^n \bar{a}_{ji}L_i-\alpha_{1i}^*+\frac{|1-\sigma_i|}{2}\\
&+\frac{p_1}{2}\Big)\omega_{1i}^2(t)-\frac{p_1}{2}\sum_{i=1}^n\omega_{1i}^2(t)+\sum_{i=1}^n\Big(-\rho_i\\
&+\frac{1}{2}\sum_{j=1}^n(\bar{a}_{ij}+\bar{b}_{ij})L_j-\alpha_{2i}^*+\frac{|1-\sigma_i|}{2}\\
&+\frac{p_1}{2}\Big)\omega_{2i}^2(t)-\frac{p_1}{2}\sum_{i=1}^n\omega_{2i}^2(t)-p_1Q(t)\\
&+q\sup_{t-\tau\le s\le t}H(s),
\end{aligned}
\tag{40}
$$

where $q=\frac{1}{2}\sum_{j=1}^n \bar{b}_{ji}L_i$.

Some appropriate α_{1i}^*, α_{2i}^* can be selected such that

$$
-\eta_i+\frac{1}{2}\sum_{j=1}^n \bar{a}_{ji}L_i-\alpha_{1i}^*+\frac{|1-\sigma_i|}{2}+\frac{p_1}{2}\le 0,
$$

and

$$
-\rho_i+\frac{1}{2}\sum_{j=1}^n(\bar{a}_{ij}+\bar{b}_{ij})L_j-\alpha_{2i}^*+\frac{|1-\sigma_i|}{2}+\frac{p_1}{2}\le 0,
$$

which lead to

$$
\dot{H}(t)\le -p_1H(t)+q\sup_{t-\tau\le s\le t}H(s),\quad \hat{\mathfrak{T}}_m\le t\le\hat{\mathfrak{S}}_m. \tag{41}
$$

Similarly, when $\hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1}$,

$$
\begin{aligned}
\dot{H}(t)&\le \sum_{i=1}^n\Big(-\eta_i+\frac{1}{2}\sum_{j=1}^n \bar{a}_{ji}L_i\Big)\omega_{1i}^2(t)+\sum_{i=1}^n(1\\
&-\sigma_i)\omega_{1i}(t)\omega_{2i}(t)+\sum_{i=1}^n\frac{1}{2}\sum_{j=1}^n \bar{b}_{ji}L_i\omega_{1i}^2(t\\
&-\tau_i)+\sum_{i=1}^n\Big(-\rho_i+\frac{1}{2}\sum_{j=1}^n(\bar{a}_{ij}+\bar{b}_{ij})L_j\Big)\omega_{2i}^2(t)\\
&+\sum_{i=1}^n\Big(\sum_{j=1}^n(|\hat{a}_{ij}-\check{a}_{ij}|+|\hat{b}_{ij}-\check{b}_{ij}|)M_j\\
&-\beta_2\Big)|\omega_{2i}(t)|-p_1Q(t)\\
&\le \sum_{i=1}^n\Big(-\eta_i+\frac{1}{2}\sum_{j=1}^n \bar{a}_{ji}L_i+\frac{|1-\sigma_i|}{2}-\frac{p_2-p_1}{2}\Big)\\
&\omega_{1i}^2(t)+\frac{p_2-p_1}{2}\sum_{i=1}^n\omega_{1i}^2(t)+\sum_{i=1}^n\Big(-\rho_i\\
&+\frac{1}{2}\sum_{j=1}^n(\bar{a}_{ij}+\bar{b}_{ij})L_j+\frac{|1-\sigma_i|}{2}\\
&-\frac{p_2-p_1}{2}\Big)\omega_{2i}^2(t)+\frac{p_2-p_1}{2}\sum_{i=1}^n\omega_{2i}^2(t)\\
&-p_1Q(t)+q\sup_{t-\tau\le s\le t}H(s)\\
&\le (p_2-p_1)H(t)+q\sup_{t-\tau\le s\le t}H(s).
\end{aligned}
\tag{42}
$$

So

$$
\begin{cases}
\dot{H}(t)\le -p_1H(t)+q\big(\sup\limits_{t-\tau\le s\le t}H(s)\big),\ \hat{\mathfrak{T}}_m\le t\le\hat{\mathfrak{S}}_m,\\
\dot{H}(t)\le (p_2-p_1)H(t)+q\big(\sup\limits_{t-\tau\le s\le t}H(s)\big),\\
\hat{\mathfrak{S}}_m<t<\hat{\mathfrak{T}}_{m+1}.
\end{cases}
$$

Now, $H(t)$ will be estimated via above equations.

When $0=\hat{\mathfrak{T}}_0\le t\le\hat{\mathfrak{S}}_0$, based on Lemma 1

$$
H(t)\le \max_{-\tau\le s\le 0}H(s)\exp\{-\varepsilon t\},
$$

in which $\varepsilon>0$ is the only positive solution of $\varepsilon-p_1+q\exp\{\varepsilon\tau\}=0$.

When $\hat{\mathfrak{S}}_0<t<\hat{\mathfrak{T}}_1$, based on Lemma 2

$$
\begin{aligned}
H(t)\le \max_{\hat{\mathfrak{S}}_0-\tau\le s\le\hat{\mathfrak{S}}_0}H(s)\exp\big\{\gamma(t-\hat{\mathfrak{S}}_0)\big\}\le \max_{-\tau\le s\le 0}H(s)\\
\exp\big\{\gamma(t-\hat{\mathfrak{S}}_0)-\varepsilon(\hat{\mathfrak{S}}_0-\tau)\big\},
\end{aligned}
$$

where $\gamma=p_2-p_1+q$.

Hence, via Assumption 2, then

$$
\begin{aligned}
H(\hat{\mathfrak{T}}_1)&\le \max_{-\tau\le s\le 0}H(s)\exp\big\{\gamma(\hat{\mathfrak{T}}_1-\hat{\mathfrak{S}}_0)-\varepsilon(\hat{\mathfrak{S}}_0-\tau)\big\}\\
&\le \max_{-\tau\le s\le 0}H(s)\exp\big\{\gamma(\delta-\theta)-\varepsilon(\theta-\tau)\big\}\\
&=\max_{-\tau\le s\le 0}H(s)\exp\{-\tilde{\mu}\},
\end{aligned}
$$

in which $\tilde{\mu}=\varepsilon(\theta-\tau)-\gamma(\delta-\theta)$. evidently,

$$
\max_{\hat{\mathfrak{T}}_1-\tau\le s\le\hat{\mathfrak{T}}_1}H(s)\le \max_{-\tau\le s\le 0}H(s)\exp\{-\tilde{\mu}\}.
$$

When $\hat{\mathfrak{T}}_1\le t\le\hat{\mathfrak{S}}_1$, via Lemma 1

$$
\begin{aligned}
H(t)&\le \max_{\hat{\mathfrak{T}}_1-\tau\le s\le\hat{\mathfrak{T}}_1}H(s)\exp\big\{-\varepsilon(t-\hat{\mathfrak{T}}_1)\big\}\\
&\le \max_{-\tau\le s\le 0}H(s)\exp\big\{-\tilde{\mu}-\varepsilon(t-\hat{\mathfrak{T}}_1)\big\}.
\end{aligned}
$$

Therefore

$$\max_{\hat{\mathfrak{S}}_1-\tau\le s\le\hat{\mathfrak{S}}_1} H(s) \le \max_{-\tau\le s\le 0} H(s)\exp\big\{-\tilde{\mu}-\varepsilon(\hat{\mathfrak{S}}_1-\hat{\mathfrak{T}}_1-\tau)\big\}.$$

When $\hat{\mathfrak{S}}_1 < t < \hat{\mathfrak{T}}_2$, via Lemma 2

$$\begin{aligned} H(t) &\le \max_{\hat{\mathfrak{S}}_1-\tau\le s\le\hat{\mathfrak{S}}_1} H(s)\exp\big\{\gamma(t-\hat{\mathfrak{S}}_1)\big\} \\ &\le \max_{-\tau\le s\le 0} H(s)\exp\big\{-\tilde{\mu}+\gamma(t-\hat{\mathfrak{S}}_1) \\ &\quad -\varepsilon(\hat{\mathfrak{S}}_1-\hat{\mathfrak{T}}_1-\tau)\big\}. \end{aligned}$$

Thus

$$\begin{aligned} &\max_{\hat{\mathfrak{T}}_2-\tau\le s\le\hat{\mathfrak{T}}_2} H(s) \\ &\le \max_{-\tau\le s\le 0} H(s)\exp\big\{-\tilde{\mu}+\gamma(\delta-\theta) \\ &\quad -\varepsilon(\theta-\tau)\big\} \\ &= \max_{-\tau\le s\le 0} H(s)\exp\{-2\tilde{\mu}\}. \end{aligned}$$

Repeat the above process, it can be obtained

$$\max_{\hat{\mathfrak{T}}_m-\tau\le s\le\hat{\mathfrak{T}}_m} H(s) \le \max_{-\tau\le s\le 0} H(s)\exp\{-m\tilde{\mu}\}.$$

For any $t > 0$, $\hat{\mathfrak{T}}_{m^*} \le t < \hat{\mathfrak{T}}_{m^*+1}$ for m^*. If $\hat{\mathfrak{T}}_{m^*} \le t \le \hat{\mathfrak{S}}_{m^*}$, then

$$\begin{aligned} H(t) &\le \max_{\hat{\mathfrak{T}}_{m^*}-\tau\le s\le\hat{\mathfrak{T}}_{m^*}} H(s)\exp\big\{-\varepsilon(t-\hat{\mathfrak{T}}_{m^*})\big\} \\ &\le \max_{-\tau\le s\le 0} H(s)\exp\big\{-m^*\tilde{\mu}-\varepsilon(t-\hat{\mathfrak{T}}_{m^*})\big\} \\ &\le \max_{-\tau\le s\le 0} H(s)\exp\{-m^*\tilde{\mu}\}. \end{aligned} \tag{43}$$

Otherwise, if $\hat{\mathfrak{S}}_{m^*} < t < \hat{\mathfrak{T}}_{m^*+1}$, then $\hat{\mathfrak{T}}_{m^*} \le t < \hat{\mathfrak{T}}_{m^*+1}$, it can be gotten $-m^*\tilde{\mu} \le (-\frac{t}{\delta}+1)\tilde{\mu}$. Hence

$$\begin{aligned} H(t) &\le \max_{\hat{\mathfrak{S}}_{m^*}-\tau\le s\le\hat{\mathfrak{S}}_{m^*}} H(s)\exp\big\{\gamma(t-\hat{\mathfrak{S}}_{m^*})\big\} \\ &\le \max_{-\tau\le s\le 0} H(s)\exp\big\{-m^*\tilde{\mu}+\gamma(t-\hat{\mathfrak{S}}_{m^*})\big\} \\ &\le \max_{-\tau\le s\le 0} H(s)\exp\big\{-m^*\tilde{\mu}+\gamma\delta\big\} \\ &\le \max_{-\tau\le s\le 0} H(s)\exp\big\{\tilde{\mu}+\gamma\delta\big\}\exp\big\{-\frac{\tilde{\mu}}{\delta}t\big\}. \end{aligned} \tag{44}$$

Based on (43) and (44), it can be easily gotten

$$\max_{-\tau\le s\le 0} H(s)\exp\{-m^*\tilde{\mu}\} \le \max_{-\tau\le s\le 0} H(s)\exp\big\{\tilde{\mu}+\gamma\delta\big\} \exp\big\{-\frac{\tilde{\mu}}{\delta}t\big\}.$$

So, for any $t > 0$, then

$$H(t) \le M \max_{-\tau\le s\le 0} H(s)\exp\big\{-\frac{\tilde{\mu}}{\delta}t\big\},$$

where $M = \exp\big\{\tilde{\mu}+\gamma\delta\big\}$. So, the proof is achieved. □

In system (8), if the constants $b^*_{ij}(r_i(t)) = 0$ and $b^{**}_{ij}(r_i(t)) = 0$, then network (8) become the following form

$$\begin{cases} \dot{r}_i(t) = -\eta_i r_i(t) + u_i(t), \\ \dot{u}_i(t) = -\rho_i u_i(t) - \sigma_i r_i(t) + \sum_{j=1}^{n} a^*_{ij}(r_i(t)) \\ \quad \times g_j(r_j(t)) + I_i, \end{cases} \tag{45}$$

in which $\rho_i = c_i - \eta_i,\ \sigma_i = d_i - c_i\eta_i + \eta_i^2$.

Accordingly, system (9) become the following form

$$\begin{cases} \dot{h}_i(t) = -\eta_i h_i(t) + v_i(t) + \xi_{1i}, \\ \dot{v}_i(t) = -\rho_i v_i(t) - \sigma_i h_i(t) + \sum_{j=1}^{n} a^{**}_{ij}(h_i(t)) \\ \quad \times g_j(h_j(t)) + I_i + \xi_{2i}. \end{cases} \tag{46}$$

Here, $\xi_{1i}(t)$, $\xi_{2i}(t)$ are similar with the equations (11)-(16) of $\beta_1 = 0$.

Corollary 1: Under Assumptions 1-2, assume that $\beta_2, \lambda > 0$,

(i): $\sum_{j=1}^{n} \big(|\hat{a}_{ij} - \check{a}_{ij}| + |\hat{b}_{ij} - \check{b}_{ij}|\big)M_j - \beta_2 \le 0$,

(ii): $-\rho_i + \frac{1}{2}\sum_{j=1}^{n} \big(\bar{a}_{ij} + \bar{b}_{ij}\big)L_j + \frac{|1-\sigma_i|}{2} - \frac{\lambda}{2} \le 0$,

(iii): $-\eta_i + \frac{1}{2}\sum_{j=1}^{n} \bar{a}_{ji}L_i + \frac{|1-\sigma_i|}{2} - \frac{\lambda}{2} \le 0$, then the inertial networks (15) and (16) are global synchronized.

Proof: Give Lyapunov function

$$H(t) = W(t) + Q(t), \tag{47}$$

in which

$$W(t) = \frac{1}{2}\sum_{i=1}^{n} \omega^2_{1i}(t) + \frac{1}{2}\sum_{i=1}^{n} \omega^2_{2i}(t). \tag{48}$$

By the Theorem 1, Corollary 1 is achieved. □

Remark 1: In [10], [11], and [21], the problem of synchronization for inertial delayed neural system via intermittent control strategy was researched. However, the problem of adaptive aperiodic intermittent synchronization is researched of memristive delayed inertial neural systems in this article. If $\hat{\mathfrak{T}}_{m+1} - \hat{\mathfrak{T}}_m \equiv T$, $\hat{\mathfrak{S}}_m - \hat{\mathfrak{T}}_m \equiv \iota$, the intermittent adaptive control strategy develops into the periodic form, where $T,\ \iota > 0$ are constants, $\hat{\mathfrak{T}}_0 = 0$, $m \in Z^+$. In other words, the conclusions given in [10], [11], and [21] are the distinct form of the conclusions in the article.

Remark 2: It is well known that the control gains can automatically regulate themselves on the basis of some appropriate updating rules which is the advantage of adaptive control strategy. Hence, an adaptive control strategy of inertial memristive neural networks is applied to automatically regulate control gains. Synchronization is obtained by the construction of piecewise Lyapunov functional. From Theorem 1, we know that each term in (11)-(16) has various contribution in order to reach synchronization. The influence of $-\alpha_{1i}(t)\omega_{1i}(t)$, $-\alpha_{2i}(t)\omega_{2i}(t)$ is to guarantee the synchronization of memristive inertial delayed neural systems. $-\beta_2 sign(\omega_{2i}(t))$ can remove the influence of memristive error. Ultimately, the integral term $-\mu\beta_1\big(\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\omega_{1i}^2(s)ds\big)\frac{\omega_{1i}(t)}{||\omega_1(t)||^2}$ can remove the influence of delayed error nonlinearity about the states of inertial memristive delayed neural networks.

Remark 3: Although some results on exponential or asymptotical synchronization for memristor-based inertial neural systems contain time delays [12], [14], [19], [24] are obtained, there are little works concerning asymptotical or exponential synchronization for memristive inertial neural systems contain delays by aperiodic intermittent adaptive control. The major difficulties in researching the asymptotical or exponential synchronization of memristor inertial neural systems contain delays, memristive connection terms, aperiodic intermittent adaptive controller and the complicate structures of inertial term. In the light of the existing analysis technology, these difficulties should be solved in time. Nevertheless, disposing of these difficulties is not easily work. Some innovative analysis techniques to deal with the difficulties are adopted in this paper.

Remark 4: By employing the variable substitution $u_i(t) = \dot{r}_i(t) + \eta_i r_i(t)$ and $v_i(t) = \dot{h}_i(t) + \eta_i h_i(t)$, we can transform the driven network (1) and response network (3) into the first-order differential equality (8) and (9). What we need to note here is that we introduce the free weights η_i in (8) and (9). By changing different values of η_i according to various cases, the different variable substitutions can be reached. Hence, the research results can be popularized in application.

Remark 5: Obviously, from intermittent adaptive control strategy (11)-(16), the adaptive control gains $\alpha_{1i}(t)$, $\alpha_{2i}(t)$ in $[\hat{\mathfrak{S}}_m, \hat{\mathfrak{T}}_{m+1}]$ are equal to zero and increasing in $[\hat{\mathfrak{T}}_m, \hat{\mathfrak{S}}_m]$ on the basis of the update law (13), (16). The values of $\alpha_{1i}(t)$, $\alpha_{2i}(t)$ are gather in certain positive constants in $[\hat{\mathfrak{T}}_m, \hat{\mathfrak{S}}_m]$ when the synchronization is reached. In the four section, the adaptive control gains will be reflected.

Remark 6: In [2],

$$\sum_{i=1}^{n} e_{1i}(t)e_{2i}(t) - \beta_i \sum_{i=1}^{n} e_{1i}(t)e_{2i}(t) \leq \sum_{i=1}^{n} \frac{1+|\beta_i|}{2}(e_{1i}^2(t) + e_{2i}^2(t)).$$

In this paper,

$$\sum_{i=1}^{n} \omega_{1i}(t)\omega_{2i}(t) - \beta_i \sum_{i=1}^{n} \omega_{1i}(t)\omega_{2i}(t) \leq \sum_{i=1}^{n} \frac{|1-\beta_i|}{2}(\omega_{1i}^2(t) + \omega_{2i}^2(t)).$$

Obviously,

$$|1-\beta_i| \leq 1 + |\beta_i|.$$

From this point of view, our conclusion is less conservative.

IV. NUMERICAL SIMULATIONS

This part presents a chaotic system to support the validness of the methods and results.

Example: Investigate a inertial network model as below.

$$\begin{cases} \dot{r}_i(t) = -\eta_i r_i(t) + u_i(t), \\ \dot{u}_i(t) = -\rho_i u_i(t) - \sigma_i r_i(t) + \sum_{j=1}^{3} a_{ij}(r_i(t))g_j(r_j(t)) \\ \quad + \sum_{j=1}^{n} b_{ij}(r_i(t))g_j(r_j(t-\tau_j)) + I_i, \end{cases} \tag{49}$$

in which $\rho_i = c_i - \eta_i$, $\sigma_i = d_i - c_i\eta_i + \eta_i^2$, $i = 1, 2, 3$.

$$a_{11}(r_1) = \begin{cases} 1.25, & |r_1(t)| < 0.8, \\ 1, & |r_1(t)| \geq 0.8, \end{cases}$$
$$a_{12}(r_1) = \begin{cases} -0.32, & |r_1(t)| < 0.8, \\ -0.4, & |r_1(t)| \geq 0.8, \end{cases}$$
$$a_{13}(r_1) = \begin{cases} -0.32, & |r_1(t)| < 0.8, \\ -0.22, & |r_1(t)| \geq 0.8, \end{cases}$$
$$a_{21}(r_2) = \begin{cases} -0.32, & |r_2(t)| < 0.8, \\ -0.22, & |r_2(t)| \geq 0.8, \end{cases}$$
$$a_{22}(r_2) = \begin{cases} 1.1, & |r_2(t)| < 0.8, \\ 1.5, & |r_2(t)| \geq 0.8, \end{cases}$$
$$a_{23}(r_2) = \begin{cases} -0.44, & |r_2(t)| < 0.8, \\ -0.4, & |r_2(t)| \geq 0.8, \end{cases}$$
$$a_{31}(r_3) = \begin{cases} -0.32, & |r_3(t)| < 0.8, \\ -0.3, & |r_3(t)| \geq 0.8, \end{cases}$$
$$a_{32}(r_3) = \begin{cases} 0.44, & |r_3(t)| < 0.8, \\ 0.5, & |r_3(t)| \geq 0.8, \end{cases}$$
$$a_{33}(r_3) = \begin{cases} 1, & |r_3(t)| < 0.8, \\ 1.2, & |r_3(t)| \geq 0.8, \end{cases}$$
$$b_{11}(r_1) = \begin{cases} -1.5, & |r_1(t)| < 0.8, \\ -1, & |r_1(t)| \geq 0.8, \end{cases}$$
$$b_{12}(r_1) = \begin{cases} -0.1, & |r_1(t)| < 0.8, \\ -0.2, & |r_1(t)| \geq 0.8, \end{cases}$$
$$b_{13}(r_1) = \begin{cases} -0.1, & |r_1(t)| < 0.8, \\ -0.3, & |r_1(t)| \geq 0.8, \end{cases}$$
$$b_{21}(r_2) = \begin{cases} -0.1, & |r_2(t)| < 0.8, \\ -0.5, & |r_2(t)| \geq 0.8, \end{cases}$$
$$b_{22}(r_2) = \begin{cases} -1.5, & |r_2(t)| < 0.8, \\ -1, & |r_2(t)| \geq 0.8, \end{cases}$$

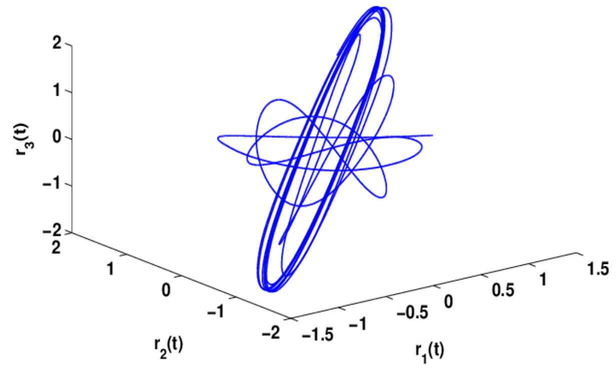

FIGURE 1. The phase trajectory of neural networks (49).

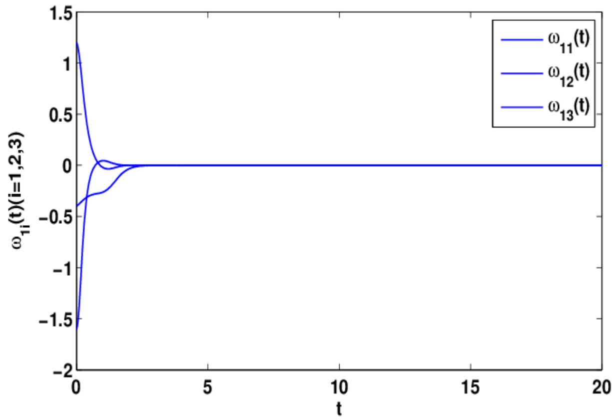

FIGURE 2. Synchronization error between $r_i(t)$ and $h_i(t)$, i = 1,2,3.

$$b_{23}(r_2) = \begin{cases} -0.1, & \mid r_2(t) \mid < 0.8, \\ -0.5, & \mid r_2(t) \mid \geq 0.8. \end{cases}$$

$$b_{31}(r_3) = \begin{cases} -0.1, & \mid r_3(t) \mid < 0.8, \\ -0.3, & \mid r_3(t) \mid \geq 0.8. \end{cases}$$

$$b_{32}(r_3) = \begin{cases} -1.5, & \mid r_3(t) \mid < 0.8, \\ -1, & \mid r_3(t) \mid \geq 0.8. \end{cases}$$

$$b_{33}(r_3) = \begin{cases} -1.1, & \mid r_3(t) \mid < 0.8, \\ -1.5, & \mid r_3(t) \mid \geq 0.8. \end{cases}$$

Let $c_1 = c_2 = c_3 = \eta_1 = \eta_2 = \eta_3 = d_1 = d_2 = d_3 = 1$.

In the following, the controlled system (50) is described by

$$\begin{cases} \dot{h}_i(t) = -\eta_i h_i(t) + u_i(t), \\ \dot{u}_i(t) = -\rho_i u_i(t) - \sigma_i h_i(t) + \sum_{j=1}^{3} a_{ij}(h_i(t)) g_j(h_j(t)) \\ \quad + \sum_{j=1}^{n} b_{ij}(h_i(t)) g_j(h_j(t-\tau_j)) + I_i, \end{cases} \tag{50}$$

in which $\rho_i = c_i - \eta_i,\ \sigma_i = d_i - c_i\eta_i + \eta_i^2, i = 1, 2, 3.$

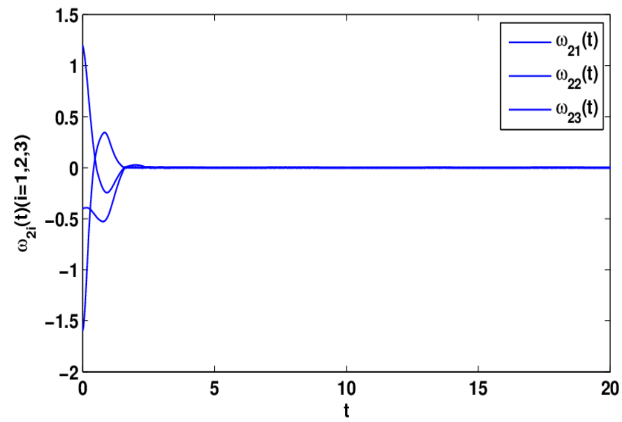

FIGURE 3. Synchronization error between $u_i(t)$ and $v_i(t)$, i = 1,2,3.

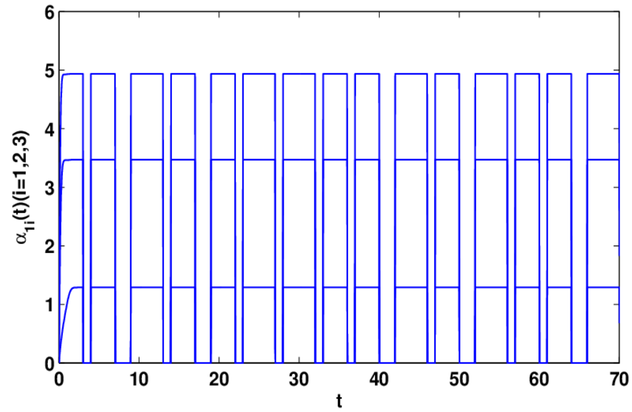

FIGURE 4. Adaptive intermittent control gain $\alpha_{1i}(t), i = 1, 2, 3.$

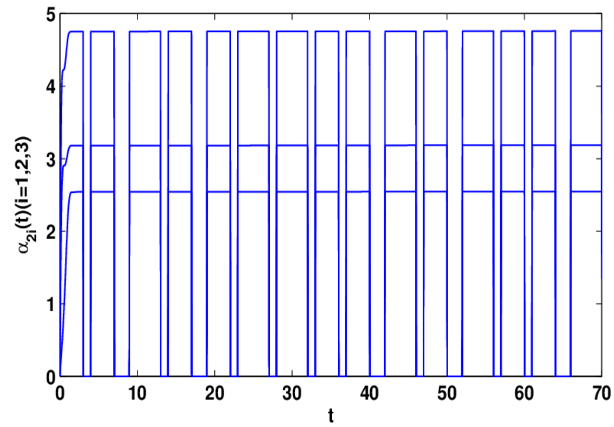

FIGURE 5. Adaptive intermittent control gain $\alpha_{2i}(t), i = 1, 2, 3.$

Take $g_1(\cdot) = g_2(\cdot) = g_3(\cdot) = \tanh(\cdot)$, $I_1 = I_2 = I_3 = 0$, $\tau_1 = \tau_2 = \tau_3 = 0.7$. The dynamic behavior of (49) with the initial conditions $r_1(s) = 0.8,\ r_2(s) = -0.6,\ r_3(s) = 0.2,\ \forall s \in [-0.8, 0)$ is given in Fig. 1. System of (50) has the initial conditions $h_1(s) = -0.8,\ h_2(s) = 0.6,\ h_3(s) = -0.2,\ \forall s \in [-0.8, 0)$. The controller (11)-(16) exists on the following time period

$$[0,3]\cup[4,7]\cup[9,13]\cup[14,17]\cup[19,22]\cup[23,27]\cup[28,32]\cup[33,36]\cup[37,40]\cup[42,46]\cup[47,50]\cup[52,56]\cup[19,22]\cup[57,60]\cup[61,64]\cup[66,70]\cup\cdots.$$

Hence, $\theta = 3$, $\delta = 5$. By simple computing, we obtain $L_1 = L_2 = L_3 = 1$, $M_1 = M_2 = M_3 = 1$. Now, let $\beta_1 = 1.8$, $\beta_2 = 2$, $\lambda = 4.5$, $\mu = 3.5$. Now, all the conditions of Theorem 1 are contented. Under Theorem 1, (49) and (50) are synchronization via the controller method (11)-(16). Figs. 2 and 3 describe the synchronization error of $\omega_{11}(t)$, $\omega_{12}(t)$, $\omega_{13}(t)$, $\omega_{21}(t)$, $\omega_{22}(t)$, $\omega_{23}(t)$ between systems (49) and (50). Time evolution of $\alpha_{1i}(t)$, $\alpha_{2i}(t)$ with $\alpha_{1i}(0) = \alpha_{2i}(0) = 0$ and $\zeta_{1i} = \zeta_{2i} = 10$ for $i = 1, 2, 3$ are given in Figs. 4-5.

V. CONCLUSION

By building aperiodically intermittent control scheme and applying the Lyapunov functional method, we study the asymptotic and exponential synchronization of delayed memristive inertial neural systems in the article. First, a class of aperiodically intermittent adaptive controller including time delayed term and integral term are proposed. Besides, compared with other classes of neural systems, adding inertia term and memristive term to electronic neural system may lead to very complex behavior. In addition, by constructing a common Lyapunov functional and applying the inequality skill, we obtain lots of less conservation conditions to guarantee the synchronization for delayed inertial memristive neural systems. The conclusions of the article can be regarded as an improvement and generalization of the earlier asymptotic synchronization and exponential synchronization for memristive delayed inertial neural systems. In the end, a numerical example make clear the validness of the conclusions and the proposed way.

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