IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 3 July 2023, accepted 20 August 2023, date of publication 29 August 2023, date of current version 6 September 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3309663

RESEARCH ARTICLE

Q-Rung Orthopair Fuzzy Petri Nets for Knowledge Representation and Reasoning

KAIYUAN BAI, (Member, IEEE), DAN JIA, WEIYE MENG, AND XINGMIN HE
AI Research Centre, China Telecom Research Institute, Beijing 102209, China
Corresponding author: Kaiyuan Bai (baiky1@chinatelecom.cn)

ABSTRACT This paper investigates a novel fuzzy Petri nets (FPNs) method based on q-rung orthopair fuzzy sets (q-ROFSs) to provide an efficient solution to uncertain knowledge representation and reasoning. It not only improves FPN's flexibility in knowledge parameter representation and reasoning algorithms but also addresses the challenges that most FPNs cannot implement backward reasoning, which is a common reasoning task to infer condition statuses according to consequences reversely. Specifically, we first propose the q-rung orthopair FPNs (q-ROFPNs) by integrating q-ROFSs with FPNs. It achieves an intuitive evaluation of hesitancy information and a flexible adjustment of the knowledge representation ranges. And a reasoning algorithm based on the ordered weighted averaging-weighted average (OWAWA) operator is developed to accomplish the forward reasoning driven by q-ROFPNs, which can balance the proposition weights and its position weights flexibly. Building upon q-ROFPNs, we further propose the q-rung orthopair fuzzy reversed Petri nets (q-ROFRPNs) for backward reasoning task, where a decomposition algorithm for q-ROFRPNs is designed for reducing the inference complexity, and an ordered weighted backward reasoning (OWBR) algorithm is provided to backward reasoning suitable for different fuzzy environments. In addition, to ensure the accuracy and rationality of reasoning results, we propose a knowledge acquisition method by power average (PA) operator to eliminate the negative impact of outliers on knowledge parameter assessments. A simulation experiment on the fault diagnosis of the air conditioning system demonstrates that the proposed method can achieve a more flexible and reliable knowledge representation and reasoning than the state-of-the-art FPNs methods.

INDEX TERMS Expert system, fuzzy petri net, knowledge representation and reasoning, q-rung orthopair fuzzy sets.

I. INTRODUCTION

Fuzzy Petri nets (FPNs) initiate a graphical knowledge representation and reasoning method by integrating fuzzy sets and Petri nets (PNs) theories. They apply fuzzy sets to model the uncertainty of knowledge information and exploit Petri nets to realize the dynamic knowledge reasoning in a graphical manner [1]. Because of their power in uncertainty modeling and graphical inference, FPNs have achieved sustained attention in a wide range of applications, particularly in fault diagnosis of power system [2], reliability evaluation [3], workflow management [4]. Traditional FPNs advocate modeling the uncertain knowledge by Zadeh's fuzzy sets, which characterizes the ambiguous perception by a membership degree (MD). However, a crucial shortcoming of the above method is that Zadeh's fuzzy sets cannot deal with complicated ambiguous or linguistic knowledge information [5]. In addition, traditional FPNs carry out knowledge reasoning tasks only by the min, max, and product operators, thus failing to ensure the precision and reliability of knowledge reasoning [6]. In recent years, improving FPN's knowledge representation and reasoning ability has attracted a growing interest in the research community [7].

To enhance the knowledge representation ability, various advanced fuzzy sets and linguistic models are introduced into FPNs to support complicated uncertainty modeling. In [8], the grey reasoning PNs that integrate grey numbers with FPN have been proposed to handle incomplete knowledge information. In [9] and [10], the linguistic reasoning PNs and cloud reasoning PNs are proposed to realize knowledge

The associate editor coordinating the review of this manuscript and approving it for publication was Chun-Hao Chen.

reasoning under the linguistic environments. To intuitively reveal the hesitancy implied in the knowledge information, two significant FPNs, namely intuitionistic FPNs (IFPNs) and Pythagorean FPNs (PFPNs), have been proposed in [11] and [12]. They assimilate intuitionistic fuzzy sets (IFSs) and Pythagorean fuzzy sets (PFSs) with FPNs and leverage the MD and non-membership degree (NMD) to intuitively represent the hesitancy of knowledge information. However, the knowledge parameters expressed in the form of IFSs and PFSs require that the sum or square sum of MD and NMD is less or equal to one, which limits the flexibility and practicability of FPNs.

On the other hand, existing studies also made many works on enhancing the effectiveness of knowledge reasoning of FPNs. Over the past few years, the aggregation operator, a significant aggregation function for information fusion, has been applied widely in the reasoning algorithm of FPNs to improve its flexibility and reliability. For example, a so-called dynamic adaptive FPN is proposed in [13], which utilizes the weighted averaging (WA) operator to fulfill knowledge inference considering proposition weights. In [6] and [9], the ordered weighted averaging (OWA) operator is used to replace max or min operators of FPNs and enable the knowledge inference considering position weights of propositions. In addition, some FPN methods also introduce the hybrid of WA and OWA to weigh both the propositions themselves and their position information, as achieved in [10] and [14]. Unfortunately, the above methods ignore the situation where the proposition and position weights may be assigned to different importance degrees due to the difference in inference target.

In addition to the grievances in knowledge representation and reasoning, another pivotal shortcoming in existing FPNs is that they only focus on forward reasoning tasks and rarely consider backward reasoning driven by consequences. In some knowledge reasoning tasks, the consequences may be known, and the expert system needs to infer the status of antecedent propositions according to consequent propositions. Reversed FPNs (RFPNs) [15] and reversed dynamic adaptive FPNs (RDAFPNs) [16] were regarded as the solution providers for the above problem. They construct the reversed reasoning mechanism oriented to FPNs to realize the truth evaluation of antecedent propositions. However, RFPNs and RDAFPNs only achieve simple backward reasoning by fuzzy sets and conventional Min and Max operators. The FPN-based backward reasoning suitable for complex and uncertain situations becomes imperative, yet the related achievements are still scarce.

Motivated by the above shortcomings, this paper proposes a new type of FPN method based on q-rung orthopair fuzzy sets (q-ROFSs). It is constituted by q-rung orthopair FPNs (q-ROFPNs) and q-rung orthopair fuzzy reversed Petri nets (q-ROFRPNs), which not only enables a flexible adjustment of the knowledge representation ranges but also realizes forward and backward knowledge reasoning. In addition, a knowledge acquisition method by power average (PA) operator is developed to ensure the accuracy and rationality of knowledge reasoning. The novelty and contributions of this paper are as follows:

- We integrate the q-ROFSs with the FPNs to propose the q-ROFPN model. Compared with IFPNs and PFPNs, q-ROFPNs not only intuitively evaluate the hesitancy implied in knowledge parameters but also avoid the limitation that the sum or square sum of MD and NMD is less or equal to one, which achieves a flexible expansion of knowledge representation ranges. Q-ROFPN can be regarded as the generalization of IFPNs or PFPNs, and it also can reduce to IFPNs or PFPNs by adjusting the model parameter.
- We develop a reasoning algorithm based on the ordered weighted averaging-weighted average (OWAWA) operator to fulfill the forward knowledge reasoning driven by q-ROFPNs. Apart from considering both the proposition weight and the position weight, the proposed algorithm enables a flexible trade-off between them, which fortifies the reliability and flexibility of FPN knowledge reasoning.
- Based on the q-ROFPNs, we accomplish the q-ROFRPNs for backward knowledge reasoning. It comprises a decomposition algorithm that constructs a subnet model of q-ROFRPNs to reduce the inference complexity and an ordered weighted backward reasoning (OWBR) algorithm for executing backward reasoning tasks. Compared with RFPNs and RDAFPNs, q-ROFRPNs realize the backward reasoning suitable for different fuzzy environments, including intuitionistic fuzzy, Pythagorean fuzzy, and q-rung orthopair fuzzy environments.
- To ensure the accuracy of knowledge reasoning, this paper also concerns the knowledge acquisition of model parameters. Traditional knowledge acquisition based on WA or weighted geometric (WG) operators may provide unreasonable aggregation results if there are outliers in the evaluation of knowledge parameters. A knowledge acquisition method based on the power average (PA) operator is to this end presented herein, which can eliminate the negative impact of outlier assessments on knowledge aggregation results.

The rest of this paper is organized as follows. Section II reviews the related works, including q-ROFSs, aggregation operators under q-ROFSs, and FPNs. Sections III and IV elaborate on the knowledge representation and reasoning based on q-ROFPNs and q-ROFRPNs. Section V details the PA-based knowledge parameter acquisition. Section VI employs a case study on fault diagnosis of the air conditioning system to validate the efficacy of the proposed method. The summaries are given in the last section.

II. PRELIMINARIES

A. Q-RUNG ORTHOPAIR FUZZY SETS

The q-ROFSs [17] are the generalization of IFSs and PFSs. It assigns an adjustable parameter q and allows the sum of the q-th power of the MD and the NMD to be less or equal to one, which enables a flexible adjustment for the acceptable fuzzy information space. The details of q-ROFSs is presented as follows [17]:

Definition 1: Let X be a fixed set, then a q-ROFS A in X is:

$$A = \{\langle x \,|\mu_A(x), v_A(x)\rangle \,| x \in X\} \tag{1}$$

where $\mu_A(x) \in [0, 1]$ and $v_A(x) \in [0, 1]$ are respectively the MD and NMD of x in A, and satisfy $0 \leq (\mu_A(x))^q + (v_A(x))^q \leq 1, (q \geq 1)$ for all $x \in X$. For a q-ROFS A, $h_A(x) = \left(1 - \left(\mu_A^q(x) + v_A^q(x)\right)\right)^{1/q}$ is defined as the hesitancy degree of x to A. For convenience, the pair $\langle \mu_A(x), v_A(x)\rangle$ is called as q-rung orthopair fuzzy number (q-ROFN), denoted by $\langle \mu_A, v_A\rangle$.

Definition 2: Given any two q-ROFNs $a = \langle \mu_a, v_a\rangle$ and $b = \langle \mu_b, v_b\rangle$, then the operational laws of q-ROFNs are defined as follows [18], [19]:

1) $a\tilde{\oplus}b = \left\langle \left(\mu_a^q + \mu_b^q - \mu_a^q\mu_b^q\right)^{1/q}, v_a v_b\right\rangle$,
2) $a\tilde{\otimes}b = \left\langle \mu_a\mu_b, \left(v_a^q + v_b^q - v_a^q v_b^q\right)^{1/q}\right\rangle$,
3) $\lambda a = \left\langle \left(1 - (1 - \mu_a^q)^\lambda\right)^{1/q}, v_a^\lambda\right\rangle$,
4) $a^\lambda = \left\langle \mu_a^\lambda, \left(1 - (1 - v_a^q)^\lambda\right)^{1/q}\right\rangle$,
5) $a\tilde{\ominus}b = \left\langle 0 \vee \left(\frac{\mu_a^q - \mu_b^q}{1 - \mu_b^q}\right)^{1/q}, 1 \wedge \frac{v_a}{v_b} \wedge \left(\frac{1 - \mu_a^q}{1 - \mu_b^q}\right)^{1/q}\right\rangle$,
6) $a\tilde{\oslash}b = \left\langle 1 \wedge \frac{\mu_a}{\mu_b} \wedge \left(\frac{1 - v_a^q}{1 - v_b^q}\right)^{1/q}, 0 \vee \left(\frac{v_a^q - v_b^q}{1 - v_b^q}\right)^{1/q}\right\rangle$,

Definition 3: Let $a = \langle \mu_a, v_a\rangle$ be a q-ROFN, then the score function and accuracy function of a are $S(a) = \mu_a^q - v_a^q$ and $H(a) = \mu_a^q + v_a^q$, respectively. Further, a comparison rule for any two q-ROFNs, i.e., $a = \langle \mu_a, v_a\rangle$ and $b = \langle \mu_b, v_b\rangle$, is defined as follows [18]:

1) If $S(a) > S(b)$, then $a \succ b$;
2) If $S(a) = S(b)$, then
   if $H(a) > H(b)$, then $a \succ b$;
   if $H(a) = H(b)$, then$a = b$.

B. OWAWA OPERATOR AND PA OPERATOR

The OWAWA operator, as a unification between WA and OWA, not only weights the variables themselves and their ordered positions but also considers the relative degree of importance between them [20]. To deal with the knowledge information expressed by q-ROFNs, we extend it in q-ROFSs and propose a q-rung orthopair OWAWA (q-OWAWA) operator. Let Ω be the set of all q-ROFNs, then q-OWAWA is defined as:

Definition 4: Let a_i $(i = 1, 2, \ldots, n)$ be a collection of q-ROFNs, ρ_i be the weight of a_i $(i = 1, 2, \ldots, n)$, satisfying $\rho_i \in [0, 1]$ and $\sum_{i=1}^n \rho_i = 1$. Meanwhile, ϖ_i be the position weight of a_i $(i = 1, 2, \ldots, n)$, satisfying $\varpi_i \in [0, 1]$ and $\sum_{i=1}^n \varpi_i = 1$, then an q-OWAWA operator of dimension n is a mapping q-OWAWA: $\Omega^n \to \Omega$ such that:

$$\begin{aligned}&\text{q-OWAWA}(a_1, a_2, \ldots, a_n)\\&\quad = w_1 a_{\sigma(1)}\tilde{\oplus} w_2 a_{\sigma(2)}\tilde{\oplus} \cdots \tilde{\oplus} w_n a_{\sigma(n)}\end{aligned} \tag{2}$$

where $a_{\sigma(i)}$ is the i-th largest of the a_i,$(i = 1, 2, \ldots, n)$ and its weight w_i is calculated as follows:

$$w_i = \beta\varpi_i + (1 - \beta)\rho_{\sigma(i)}, \beta \in [0, 1] \tag{3}$$

As pointed out in [20] and [21], q-OWAWA operator satisfies monotonicity, idempotency, and boundary. By adjusting the β, Eq.(3) enables a flexible trade-off between the argument and position weights. Meanwhile, it can reduce q-OWAWA to q-rung orthopair fuzzy OWA (q-OWA) operator or q-rung orthopair fuzzy WA (q-WA) operator by setting $\beta = 1$ or $\beta = 0$, as given by

$$\text{q-OWA}(a_1, a_2, \ldots, a_n) = \mathop{\tilde{\oplus}}_{i=1}^{n} \varpi_i a_{\sigma(i)} \tag{4}$$

or

$$\text{q-WA}(a_1, a_2, \ldots, a_n) = \mathop{\tilde{\oplus}}_{i=1}^{n} \rho_i a_i \tag{5}$$

The PA operator [22] provides a new information aggregating method. By constructing the support degree of variables, PA allows variables to support and reinforce each other to eliminate the negative impact of outliers. Ju et al. [23] further expand the PA operator to q-ROFSs and propose the q-rung orthopair fuzzy power weighted average (q-PWA) operator. Let Ω be the set of all q-ROFNs, the q-PWA operator is defined as follows [23]:

Definition 5: Let $a_i(i = 1, 2, \ldots, n)$ be a collection of q-ROFNs, then q-PWA operator of dimension n is a mapping q-PWA: $\Omega^n \to \Omega$ such that:

$$\text{q-PWA}(a_1, a_2, \ldots, a_n) = \mathop{\tilde{\oplus}}_{j=1}^{n} \left(\frac{\gamma_i(1 + T(a_i))a_i}{\sum_{j=1}^n \gamma_j(1 + T(a_j))}\right) \tag{6}$$

where γ_i is the weight of $a_i(i = 1, 2, \ldots, n)$, satisfying $\gamma_i \in [0, 1]$ and $\sum_{i=1}^n \gamma_i = 1$. $T(a_j) = \sum_{i=1, i\neq j}^n \text{Sup}(a_j, a_i)$, $j = 1, 2, \ldots, n$ and $\text{Sup}(a_j, a_i)$ is the support degree for a_j from a_i, which satisfies following properties:

1) $\text{Sup}(a_j, a_i) \in [0, 1]$.
2) $\text{Sup}(a_j, a_i) = \text{Sup}(a_i, a_j)$.
3) $\text{Sup}(a_j, a_i) \geq \text{Sup}(a_l, a_k)$, if $d(a_j, a_i) \leq d(a_l, a_k)$; $d(a_j, a_i)$ is the distance measure of a_j and a_i.

C. FUZZY PETRI NETS

FPNs, originated by Looney [24], have emerged as a significant knowledge representation and reasoning method, whose key superiority is to assimilate the uncertain modeling of fuzzy sets with the inference mechanism of PNs, thus realizing graphical and visual modeling and inference. A classical FPN model is defined as follows [6], [25]:

Definition 6: An FPN is defined as an 8-tuple:

$$\text{FPN} = (P, T, D, I, O, f, \alpha, \beta) \tag{7}$$

where
1) $P = \{p_1, p_1, \ldots, p_m\}$ denotes a finite nonempty set of places.
2) $T = \{t_1, t_2, \ldots, t_n\}$ denotes a finite nonempty set of transitions, $P \bigcap T = \varnothing$.
3) $D = \{d_1, d_2, \ldots, d_m\}$ denotes a finite nonempty set of propositions, $|P| = |D|$.
4) $I : [P \times T] \rightarrow \{0, 1\}$ is a $m \times n$ input incidence matrix, each element I_{ij} records the relation from place $p_i(i = 1, 2, \ldots, m)$ to transition $t_j(j = 1, 2, \ldots, n)$. $I_{ij} = 1$, if there is a directed arc from p_i to t_j, otherwise, $I_{ij} = 0$.
5) $O : [P \times T]^T \rightarrow \{0, 1\}$ is a $m \times n$ output incidence matrix, each element O_{ij} records the relation from transition $t_j(j = 1, 2, \ldots, n)$ to place $p_i(i = 1, 2, \ldots, m)$. $O_{ij} = 1$, if there is a directed arc from $t_j$top_i, otherwise, $O_{ij} = 0$.
6) $f : P \rightarrow [0, 1]$ is an associated function which defines a mapping from transitions to real values between 0 and 1.
7) $\alpha : T \rightarrow [0, 1]$ is an associated function which defines a mapping from places to real values between 0 and 1.
8) $\beta : P \rightarrow D$ is an associated function which defines a bijective mapping from place p_i to proposition d_i. If $\beta(p_i) = d_i$, p_i is associated with proposition d_i.

Since the introduction of FPNs, a larger number of studies have been proposed to improve FPN's performance, including the extensions of modeling ability for uncertain knowledge [26] and the enhancement of reasoning efficiency [27], [28]. However, existing FPNs still lack flexibility in knowledge parameter representation and reasoning algorithms. Moreover, most FPN methods fail to accomplish the backward knowledge reasoning driven by consequences effectively. That will motivate us to propose a new FPN method based on q-ROFSs, which is composed of q-ROFPNs for forward knowledge reasoning, q-ROFRPNs for backward knowledge reasoning, and PA-based knowledge acquisition. The whole knowledge representation and reasoning process is shown in Fig. 1, and details are elaborated in the following sections.

III. Q-RUNG ORTHOPAIR FUZZY PETRI NETS

This section elaborates on the q-ROFPN and its knowledge representation and reasoning. The q-ROFPN model is firstly established, which inherits the merits of q-ROFSs to flexibly adjust the representation range of knowledge parameters while intuitively revealing the hesitancy implied in knowledge information. Secondly, a forward reasoning algorithm based on q-OWAWA is proposed to attain a flexible trade-off between proposition weights and position weights, thus enhancing the reliability of knowledge reasoning. The above two parts form the knowledge representation and reasoning based on q-ROFPNs.

A. DEFINITION OF Q-ROFPNS

Let Ω be the set of all q-ROFNs, the q-ROFPNs are defined as follows:

Definition 7: A q-ROFPN is a 11-tuple:

$$\text{q-ROFPN} = (P, T, D, I, O, \beta, T_h, U, W_L, W_G, M) \quad (8)$$

where
1) P, T, D, I, O, and β are same as those in Definition 6.
2) $T_h : P \rightarrow \Omega$ is an associated function, which assigns a threshold λ_i represented by q-ROFNs to each place $p_i(i = 1, 2, \ldots, m)$, and $T_h = (\lambda_1, \lambda_2, \ldots, \lambda_m)$.
3) $U : T \rightarrow \Omega$ is an associated function, which assigns a certainty factor u_j represented by q-ROFNs to each transition $t_j(j = 1, 2, \ldots, n)$, and $U = (u_1, u_2, \ldots, u_n)$.
4) $W_L : [P \times T] \rightarrow [0, 1]$ is an input function assigning a local weight to each input arc of the transition. The $W_L = (\mathrm{lw}_{ij})_{m\times n}(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ and element $\mathrm{lw}_{ij} \in [0, 1]$ indicates the impact of places p_i on transition t_j.
5) $W_G : [P \times T]^T \rightarrow [0, 1]$ is an output function assigning a global weight to each output arc of the transition. The $W_G = (\mathrm{gw}_{ij})_{m\times n}$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ and element $\mathrm{gw}_{ij} \in [0, 1]$ indicates the impact of transitions t_j on place p_i.
6) $M = (\alpha(p_1), \alpha(p_2), \ldots, \alpha(p_m))^T$ is the marking vector of the q-ROFPN, where $\alpha(p_i)$ is taken in form of q-ROFNs and indicates the tokens contained in place p_i, which signifies the truth degree of the corresponding proposition d_i $(i = 1, 2, \ldots, m)$. The initial marking vector is denoted as M_0.

B. Q-ROFPN-BASED KNOWLEDGE REPRESENTATIONS

The knowledge representation of q-ROFPN intends to model fuzzy production rules (FPRs) in a graphical way. FPRs are a prevailing knowledge storage method in expert systems, which employs an IF-THEN form to express the domain knowledge. As such, we first propose the novel FPRs, namely q-rung orthopair FPRs (q-ROFPRs), to express the knowledge information.

The q-ROFPRs include following five basic forms:

Type 1: A simple q-ROFPR

$$R_1 : \text{IF } a \text{ THEN } c \; (\lambda; \mathrm{lw}; u; \mathrm{gw})$$

Type 2: A composite q-rung orthopair fuzzy weighted conjunctive rule in the antecedent

$$R_2 : \text{IF } a_1 \text{ AND } a_2 \text{ AND} \cdots \text{ AND } a_m \text{ THEN } c \; (\lambda_1, \lambda_2, \ldots, \lambda_m; \mathrm{lw}_1, \mathrm{lw}_2, \ldots, \mathrm{lw}_m; u; \mathrm{gw})$$

Type 3: A composite q-rung orthopair fuzzy weighted conjunctive rule in the consequent

$$R_3 : \text{IF } a \text{ THEN } c_1 \text{ AND } c_2 \text{ AND} \cdots \text{ AND } c_m \; (\lambda; \mathrm{lw}; u; \mathrm{gw}_1, \mathrm{gw}_2, \ldots, \mathrm{gw}_m)$$

Type 4: A composite q-rung orthopair fuzzy weighted disjunctive rule in the antecedent

$$R_4 : \text{IF } a_1 \text{ OR } a_2 \text{ OR} \cdots \text{ OR } a_m \text{ THEN } c \; (\lambda_1, \lambda_2, \ldots, \lambda_m; \mathrm{lw}_1, \mathrm{lw}_2, \ldots, \mathrm{lw}_m; \; u_1, u_2, \ldots, u_m; \mathrm{gw}_1, \mathrm{gw}_2, \ldots, \mathrm{gw}_m)$$

Type 5: A composite q-rung orthopair fuzzy weighted disjunctive rule in the consequent

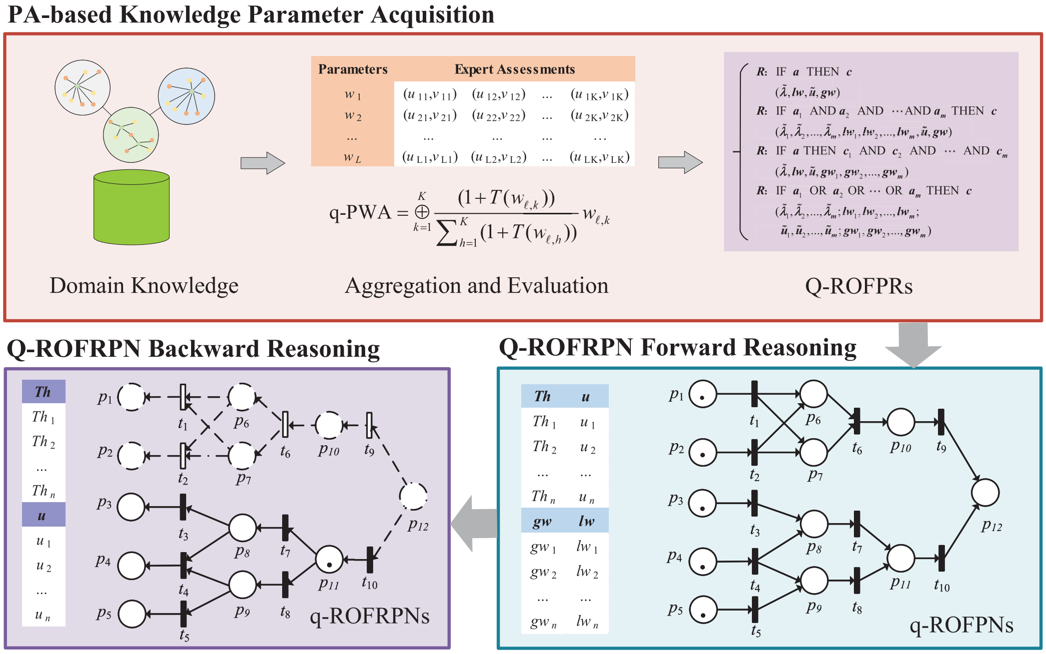

FIGURE 1. Q-ROFPN based knowledge representation and reasoning.

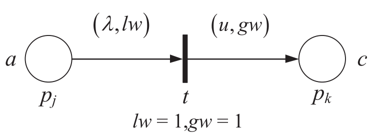

FIGURE 2. q-ROFPN representation of Type 1 rule.

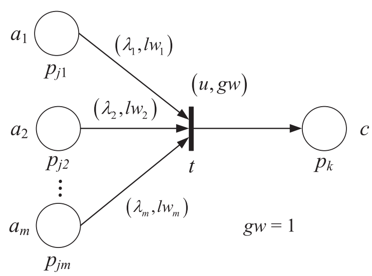

FIGURE 3. q-ROFPN representation of Type 2 rule.

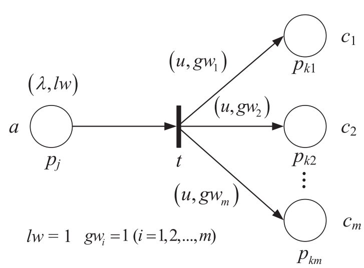

FIGURE 4. q-ROFPN representation of Type 3 rule.

$$R_5 : \text{IF } a \text{ THEN } c_1 \text{ OR } c_2 \text{ OR} \cdots \text{ OR } c_m$$
$$(\lambda; \text{lw}_1, \text{lw}_2, \ldots, \text{lw}_m; u; \text{gw}_1, \text{gw}_2, \ldots, \text{gw}_m)$$

where a and c is the antecedent and consequent propositions and their truth degrees are expressed by q-ROFNs. λ and u, in form of q-ROFNs, represent the threshold of the antecedent proposition and the certainty factors of the rule, respectively. lw $\in [0, 1]$ is the local weight, which indicates the importance degree of antecedent propositions to the outputs of the rule. gw $\in [0, 1]$ is the global weight that indicates the impact of the rule to its consequent propositions.

Next, q-ROFPNs are used to model the above five rule types, as shown in Figs. 2-6, in which propositions are denoted as places, and the firing of rules is regarded as the firing of transitions. Note that Type 4 and Type 5 will not be discussed below because they can be transferred into several rules of Type 1.

C. EXECUTION RULES OF Q-ROFPNS

For any a transition $t \in T$, let $I(t) = \{p_{I1}, p_{I2}, \ldots, p_{Im}\}$ be the input place set with thresholds $\lambda_{I1}, \lambda_{I2}, \ldots, \lambda_{Im}$ and local weights $\text{lw}_{I1}, \text{lw}_{I2}, \ldots, \text{lw}_{Im}$. $O(t) = \{p_{O1}, p_{O2}, \ldots, p_{On}\}$ be the output place set with the global weights $\text{gw}_{O1}, \text{gw}_{O2}, \ldots, \text{gw}_{Om}$. The $u(t)$ indicates the certainty factor of transition t, then the enabling and firing rules of q-ROFPNs are defined as following:

1) Enabling rule: the transition $t \in T$ is enabled, if

$$\forall p_{Ij} \in I(t) : \alpha(p_{Ij}) \geq \lambda_{Ij}, j = 1, 2, \ldots, m; \tag{9}$$

where $\alpha(p_{Ij})$ is the tokens contained in place p_{Ij}.

2) Firing rule: when enabled transition t is fired, the token $\alpha(p_{Ij})$ of its input places $p_{Ij} (j = 1, 2, \ldots, m)$ are copied and

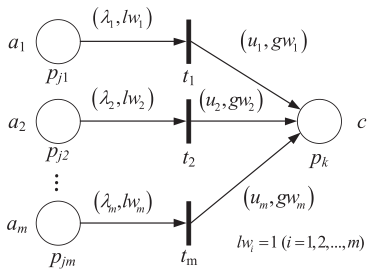

FIGURE 5. q-ROFPN representation of Type 4 rule.

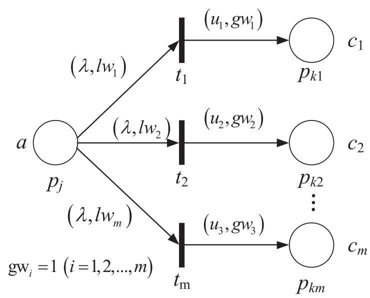

FIGURE 6. q-ROFPN representation of Type 5 rule.

a token with certain degree of fuzzy truth is deposited into its output places p_{Ok} for $k = 1, 2, \ldots, n$. If the output place p_{Ok} has only one input transition t, then the token value of p_{Ok} is calculated by

$$\alpha(p_{Ok}) = \text{q-OWAWA}\left(\mathrm{lw}_{I1}\alpha(p_{I1}), \mathrm{lw}_{I2}\alpha(p_{I2}), \ldots, \mathrm{lw}_{Im}\alpha(p_{Im})\right) \tilde{\otimes} u(t) \tag{10}$$

If the place p_{Ok} has more than one input transitions t_j with global weight gw^i_{Ok} for $i = 1, 2, \ldots, l$, then the values of token of p_{Ok} is calculated by

$$\alpha(p_{Ok}) = \text{q-OWAWA}\left(\mathrm{gw}^1_{Ok}\alpha(p^1_{Ok}), \mathrm{gw}^2_{Ok}\alpha(p^2_{Ok}), \ldots, \mathrm{gw}^l_{Ok}\alpha(p^l_{Ok})\right) \tag{11}$$

where $\tilde{\alpha}(p^i_{Ok})$ $(i = 1, 2, \ldots, l)$ is the token values of output place p_{Ok} determined by k-th input transitions.

D. REASONING ALGORITHM OF Q-ROFPNS

According to the execution mechanism of q-ROFPNs, we propose the forward reasoning algorithm based on q-OWAWA, which intend to infer the truth degree of goal places from the known starting places. Before introducing the algorithm, some basic matrix operations are defined as follows:

1) Operator $\oplus$:

$$\mathbf{A} \oplus \mathbf{B} = \mathbf{D} \tag{12}$$

where $\mathbf{A} = (a_{ij})_{m\times n}$, $\mathbf{B} = (b_{ij})_{m\times n}$ and $\mathbf{D} = (d_{ij})_{m\times n}$, $d_{ij} = \max\{a_{ij}, b_{ij}\}$ for $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$

2) Operator $\odot$:

$$\mathbf{A} \odot \mathbf{B} = \mathbf{D} \tag{13}$$

where $\mathbf{A} = (a_{ij})_{m\times n}$, $\mathbf{B} = (b_{ij})_{m\times n}$ and $\mathbf{D} = (d_{ij})_{m\times n}$, $d_{ij} = a_{ij} \times b_{ij}$ for $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

3) Operator $\otimes$:

$$\mathbf{A} \otimes \mathbf{B} = \mathbf{D} \tag{14}$$

where $\mathbf{A} = (a_{ij})_{m\times l}$, $\mathbf{B} = (b_{ij})_{l\times n}$ and $\mathbf{D} = (d_{ij})_{m\times n}$, $d_{ij} = \max\limits_{1\le k\le l}\{a_{ik} \times b_{kj}\}$ for $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

4) Operator $\triangleright$:

$$\mathbf{A} \triangleright \mathbf{B} = \mathbf{D} \tag{15}$$

where $\mathbf{A} = (a_{ij})_{m\times n}$, $\mathbf{B} = (b_{ij})_{m\times n}$ and $\mathbf{D} = (d_{ij})_{m\times n}$, then $d_{ij} = 1$, if $a_{ij} \ge b_{ij}$; $d_{ij} = 0$, if $a_{ij} < b_{ij}$ for $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

Further, the details of forward reasoning algorithm are given as follows:

Input: I, O, W_L, W_G are $m \times n$ dimensional matrices; U is an n dimensional vector; T_h and M_0 is an m dimensional vectors.

Output: M_k is an m dimensional vector.

Step 1: Let $k = 1$, k is iteration times.

Step 2: Calculate the enabled input place vector $D^{(k)}$, as given by

$$D^{(k)} = (d_i)^{(k)}_{m\times 1} = M_{k-1} \triangleright T_h, i = 1, 2, \ldots, m; \tag{16}$$

Step 3: End the reasoning, if $D^{(k)}$ is zero vector; otherwise, calculate the vector $\Gamma^{(k)}$ of equivalent fuzzy truth values regarding to transitions, as given by

$$\Gamma^{(k)} = \text{q-OWAWA}\left((W_L)^T, \tilde{M}_{k-1}\right), \tag{17}$$

Step 4: Calculate the enabled transition vector $F^{(k)}$, as given by

$$F^{(k)} = (f_i)^{(k)}_{1\times n} = \left(\left(D^{(k)}\right)^T \times I\right) \triangleright (E \times I) \tag{18}$$

where $E = [1, 1, \ldots, 1]_{1\times m}$.

Step 5: End the reasoning, if $F^{(k)}$ is zero vector; otherwise, calculate output truth degree vector $\tilde{\Psi}^{(k)}$ of the transition, as given by

$$\tilde{\Psi}^{(k)} = \left(F^{(k)} \odot \Gamma^{(k)}\right) \tilde{\otimes} \tilde{U} \tag{19}$$

Step 6: Calculate the new marking vector $\tilde{M}_k$, as given by

$$\tilde{M}_k = \tilde{M}_{k-1} \oplus \text{q-OWAWA}\left(W_G, \tilde{\Psi}^{(k)}\right) \tag{20}$$

Step 7: End reasoning, if $\tilde{M}_k = \tilde{M}_{k-1}$; otherwise, let $k = k + 1$ and go back to Step 2.

Remark 1: Q-ROFPNs accomplish a flexible and reliable knowledge representation and reasoning. On the one hand, q-ROFPNs can change the parameter q to release the restriction of IFPNs and PFPNs that the sum or square sum of the MD and the NMD is less or equal to one. On the other hand, the adjustment of parameter β in q-OWAWA enables the knowledge reasoning capable of flexibly balancing the importance degrees between proposition weights and position weights. In addition, the forward reasoning algorithm based on OWA or WA can also be realized by setting β as 0 or 1.

IV. Q-RUNG ORTHOPAIR FUZZY REVERSED PETRI NETS

In many knowledge reasoning tasks, the consequences are often known, and the expert system needs to infer the status of antecedent propositions according to consequent propositions. In this section, we investigate the backward knowledge reasoning driven by q-ROFRPNs, where the principal constituents include: 1) Constructing the q-ROFRPN model under the framework of q-ROFPN; 2) Developing the decomposition algorithm of q-ROFRPNs, which intends to eliminate the propositions and rules unrelated to the inference task and enhance the inference efficiency. 3) Developing the OWBR algorithm for reliable and flexible backward reasoning.

A. DEFINITION OF Q-ROFRPNS

Based on the framework of q-ROFPN, the q-ROFRPNs are defined as follows:

Definition 8: Let q-ROFPN $= (P, T, I, O, D, \beta, T_h, U, W_L, W_G, M)$ be the original q-ROFPN model, then, a q-ROFRPN is an 9-tuple:

$$\text{q-ROFRPN} = (\tilde{P}, \tilde{T}, \tilde{I}, \tilde{O}, \tilde{D}, \tilde{\beta}, \tilde{T}_h, \tilde{U}, \tilde{M}) \quad (21)$$

where $\tilde{P} = P$, $\tilde{T} = T$, $\tilde{I} = O$, $\tilde{O} = I$, $\tilde{D} = D$, $\tilde{\beta} = \beta$, $\tilde{T}_h = T_h$, $\tilde{U} = U$ and $\tilde{M} = M$.

Q-ROFRPN is obtained by reversing the input-output relationship between the places and transactions of the original q-ROFPN. More specifically, the output matrix of the original model is changed into the input matrix of the reverse model, and its input matrix is changed into the output matrix of the reverse model. The places, transitions, proposition, etc., are consistent with the original model. Note that W_L and W_G of the original model don't include in the reverse model due to the change in input and output relationships.

B. SUBNET MODELS OF Q-ROFRPNS

Compared with the forward reasoning task, backward reasoning is a process of inferring antecedent propositions according to consequent propositions. Therefore, we can search these propositions and rules relevant only to the consequence and then implement the backward reasoning task. Such a strategy avoids the interference of irrelevant propositions on the reasoning task and enhances reasoning efficiency.

To achieve the above purpose, we propose a decomposition algorithm to construct the subnet model of q-ROFRPNs (S-q-ROFRPNs), which only involves the places and transitions related to the appointed goal place. Before constructing the S-q-ROFRPNs, two vectors are introduced as follows:

1) *Place vector*: Let $X = (x_1, x_2, \ldots, x_m)^T$ be an incident place vector, the element x_i is related to place p_i for $i = 1, 2, \ldots, m$. If p_i is an appointed goal place or a place related to appointed goal place, $x_i = 1$. Otherwise, $x_i = 0$.
2) *Transition vector*: Let $Y = (y_1, y_2, \ldots, y_n)^T$ be an incident transition vector, the each element y_j is related to transition t_j for $j = 1, 2, \ldots, n$. If t_j is a transition related to appointed goal place, $y_i = 1$. Otherwise, $y_i = 0$.

With the place and transition vectors, we propose the decomposition algorithm to identify all places and transitions related to the goal places. Let q-ROFPN $= (P, T, D, I, O, \beta, T_h, U, W_L, W_G, M)$ be the original q-ROFPN model, the procedure of algorithm are presented as follows:

Step 1: Initialize incident place and transition vectors $X^{(0)} = (x_1, x_2, \ldots, x_m)^T$ and $Y^{(0)} = y_1, y_2, \ldots, y_n)^T$, where $x_i = 1$, if p_i is the appointed output place, otherwise $x_i = 0$, and $y_j = 0$ for $j = 1, 2, \ldots, n$. Meanwhile, let be $k = 1$, k denotes the iteration time.

Step 2: Calculate the incident transition vector $Y^{(k)}$ by

$$Y^{(k)} = O^T \otimes X^{(k-1)} \quad (22)$$

Step 3: Calculate the incident place vector $X^{(k)}$ by

$$X^{(k)} = \left(I \otimes Y^{(k)}\right) \oplus X^{(k-1)} \quad (23)$$

Step 4: Let $k = k + 1$ and repeat Step 3 and Step 4, until $X^{(k)} = X^{(k-1)}$ and $Y^{(k)} = Y^{(k-1)}$ are satisfied.

After obtaining vectors $X^{(k)}$ and $Y^{(k)}$, the S-q-ROFRPN is generated from the original q-ROFPN model. Let Ω be the set of all q-ROFNs

Definition 9: Definition 5. A S-q-ROFRPNs is an 9-tuple:

$$\text{S-q-ROFPN} = (\tilde{P}', \tilde{T}', \tilde{I}', \tilde{O}', \tilde{D}', \tilde{\beta}', \tilde{T}_h', \tilde{U}', \tilde{M}') \quad (24)$$

where

1) $\tilde{P}' \subseteq P$, the element $\tilde{p'}$ in $\tilde{P}'$ corresponds to the nonzero element $x_i^{(l)}$ in $X^{(l)}$, and $\tilde{p'} \in P$.
2) $\tilde{T}' \subseteq T$, the element $\tilde{t}'$ in $\tilde{T}'$ corresponds to the nonzero element $y_j^{(l)}$ in $Y^{(l)}$, and $\tilde{t}' \in T$.
3) $\tilde{D}' \subseteq D$ is a finite nonempty set of propositions, $\tilde{P}' \bigcap \tilde{T}' \bigcap \tilde{D}' = \varnothing$, $|\tilde{P}'|$=$|\tilde{D}'|$.
4) $\tilde{I}' = O \bigcap (\tilde{P}' \times \tilde{T}')^T$
5) $\tilde{O}' = I \bigcap (\tilde{P}' \times \tilde{T}')$.
6) $\tilde{\beta}' : \tilde{P}' \to \tilde{D}'$ is an associated function defining a bijective mapping between places and propositions, such that $\beta'(\tilde{p}') = \beta(\tilde{p}')$, $\forall \tilde{p}' \in \tilde{P}'$.
7) $\tilde{T}_h'$: $\tilde{P}' \to \Omega$ is an associated function defining a threshold $\tilde{\lambda}'$ to place $\tilde{p}'$, $\tilde{T}_h'(\tilde{p}') = T_h(\tilde{p}')$.
8) $\tilde{U}' : \tilde{T}' \to \Omega$ is an associated function defining a certainty factor $\tilde{u}'$ to transition $\tilde{t}'$, $\tilde{U}'(\tilde{t}') = U(\tilde{t}')$.
9) $\tilde{M}' \subseteq M$ is the marked vector of S-q-ROFPN, such that $\tilde{M}'(\tilde{p}') = M(\tilde{p}')$, $\forall \tilde{p}' \in \tilde{P}'$.

It can be seen that the S-q-ROFRPN model is generated based on the original q-ROFPN model. It only preserves the places, transitions, thresholds, certainty factors, and their mapped relationship relevant to the goal place. The above strategy simplifies the model structure and avoids unnecessary inference computation.

C. REASONING ALGORITHM OF Q-ROFRPNS

Next, we investigate the backward reasoning driven by q-ROFRPN. To address the limitation in the reliability and flexibility of backward reasoning caused by Min and Max operators, we develop an OWBR algorithm that embeds the q-OWA into backward reasoning to adapt to various inference situations.

Input: $\tilde{I}'$ and $\tilde{O}'$ are $m \times n$ dimensional matrices, $\tilde{U}'$ is an n-dimensional vector; $\tilde{T}'_h$ and $\tilde{M}'_0$ is an m-dimensional vector.

Output: $\tilde{M}'_k$ is an m-dimensional vector.

Step 1: Let $k = 1$, k is iteration times.

Step 2: Calculate the enabled input places vector $\tilde{D}'_{(k)}$, as given by

$$\tilde{D}'_{(k)} = (\tilde{d}'_i)^{(k)}_{m\times 1} = \tilde{M}'_{k-1} \triangleright \tilde{T}'_h, i = 1, 2, \ldots, m; \tag{25}$$

Step 3: End the reasoning, if $\tilde{D}'_{(k)}$ is zero vector; Otherwise, calculate the vector $\tilde{\Gamma}'_{(k)}$ of equivalent fuzzy truth values regarding to transitions, as given by

$$\tilde{\Gamma}'_{(k)} = \text{q-OWA}\left(\tilde{I}', \tilde{M}'_{k-1}\right) \tag{26}$$

Step 4: Calculate the enabled transitions vector $\tilde{F}'_{(k)}$, as given by

$$\tilde{F}'_{(k)} = (\tilde{f}'_i)^{(k)}_{1\times n} = \left(\left(\tilde{D}'_{(k)}\right)^T \times \tilde{I}'\right) \triangleright (E \times \tilde{I}') \tag{27}$$

where $E = [1, 1, \ldots, 1]_{1\times m}$.

Step 5: End the reasoning, if $\tilde{F}'_{(k)}$ is zero vector; Otherwise, calculate output truth degree vector $\tilde{\Psi}'_{(k)}$ of the transition, as given by

$$\tilde{\Psi}'_{(k)} = \left(\tilde{F}'_{(k)} \odot \tilde{\Gamma}'_{(k)}\right) \tilde{\oslash} \tilde{U}' \tag{28}$$

Step 6: Calculate the new marking vector $\tilde{M}_k$, as given by

$$\tilde{M}'_k = \tilde{M}'_{k-1} \oplus \text{q-OWA}\left(\tilde{O}', \tilde{\Psi}'_{(k)}\right) \tag{29}$$

Step 7: End reasoning, if $\tilde{M}_k = \tilde{M}_{k-1}$; Otherwise, let $k = k + 1$ and go back to Step 2.

Remark 2: Q-ROFRPNs has the ability to fulfill backward reasoning task in various complex situations. Like q-ROFPNs, q-ROFRPNs can execute the backward reasoning task in different fuzzy environments, including IFSs, PFSs, and q-ROFSs. By constructing the OWBR algorithm, q-ROFRPNs enable backward inference considering position weights, thus avoiding inflexible and imprecise inference due to min and max operators. It should be noted that the OWBR algorithm can also degrade to the Max-Min inference mechanism by adjusting the position weight vector.

TABLE 1. Linguistic term set to corresponding Q-ROFNs.

Linguistic Terms	Corresponding q-ROFNs
Extremely High (EH)	(0.95,0.05)
High (H)	(0.85,0.10)
Moderately High (MH)	(0.70,0.20)
Medium (M)	(0.50,0.35)
Moderately Low (ML)	(0.35,0.55)
Low (L)	(0.25,0.70)
Extremely Low (EL)	(0.10,0.90)

V. ACQUISITION OF KNOWLEDGE PARAMETERS

After constructing q-ROFPN and q-ROFRPN models, determining knowledge parameter values becomes an imperative mission. In evaluating knowledge parameters, unreasonable assessments may be obtained if experts provide some outliers. In this section, a novel knowledge parameter acquisition method based on the q-PWA operator is proposed to derive the truth values of thresholds, certainty factors, and weights of propositions. The acquisition process includes 1) Evaluating knowledge parameters through interacting with experts; 2) Aggregating individual assessments; and 3) Defuzzifizing and normalizing the aggregation results if necessary.

A. ASSESSMENT OF KNOWLEDGE REPRESENTATION PARAMETERS

Suppose that there are N fuzzy rules $\{R_1, R_2, \ldots, R_N\}$ with L antecedent or consequent propositions $\{P_1, P_2, \ldots, P_L\}$. For a given rule R_i $(i = 1, 2, \ldots, N)$, the thresholds and weights (e.g., local weights and global weights) are associated with propositions, and the certainty factors are associated with fuzzy rules. In what follows, we select the weights of propositions as an example to illustrate the acquisition of knowledge parameters.

We assume that there are K domain experts $\{E_1, E_2, \ldots, E_K\}$ responsible for evaluating the proposition weights. In view of the complexity and diversity of expert systems, the q-ROFNs and linguistic terms are employed for evaluating the knowledge parameters. A linguistic term set with seven cardinalities and its corresponding q-ROFNs is listed in Table 1 [29]. As such, an evaluation matrix for the proposition weights by experts is given by:

$$W_i = \begin{matrix} & \begin{matrix} E_1 & E_2 & \cdots & E_K \end{matrix} \\ \begin{matrix} P_1 \\ P_2 \\ \vdots \\ p_L \end{matrix} & \begin{pmatrix} w^i_{1,1} & w^i_{1,2} & \cdots & w^i_{1,K} \\ w^i_{2,1} & w^i_{2,2} & \cdots & w^i_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ w^i_{L,1} & w^i_{L,2} & \cdots & w^i_{L,K} \end{pmatrix} \end{matrix} \tag{30}$$

where $w^i_{jk} = (\mu^i_{j,k}, v^i_{j.k})$ represents the weight assessments associated with proposition P_j $(j = 1, 2, \ldots, L)$ given by expert $E_k(k = 1, 2, \ldots, K)$. Note that if there is only one antecedent or consequent proposition in a rule, the local weight or the global weight is 1 and does not need to be evaluated.

B. AGGREGATION OF KNOWLEDGE REPRESENTATION PARAMETERS

Based on the obtained evaluation matrix $W_i = [w^i_{jk}]_{L\times K}$, we implement the aggregation of proposition weights. Assume that the expert weight vector is $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_K)$, satisfying $\gamma_k \in [0, 1]$ and $\sum_{k=1}^{K} \gamma_k = 1$. Then, the aggregation process of proposition weights is presented as follows:

Step 1: Calculate the support degree $\mathrm{Sup}(w^i_{jk}, w^i_{jl})$ between w^i_{jk} and w^i_{jl} $(k, l = 1, 2, \ldots, K)$, as given by

$$\mathrm{Sup}(w^i_{jk}, w^i_{jl}) = 1 - d(w^i_{jk}, w^i_{jl}) \quad (i = 1, 2, \ldots, N; j = 1, 2, \ldots, L); \tag{31}$$

where $d(w^i_{jk}, w^i_{jl})$ is the normalized Hamming distance between of q-ROFNs w^i_{jk} and w^i_{jl} [30].

Step 2: Calculate the power weight θ^i_{jk} by

$$\theta^i_{jk} = \gamma_k \left(1 + T(w^i_{jk})\right) / \sum_{k=1}^{K} \gamma_l \left(1 + T(w^i_{jk})\right) \quad (i=1, 2, \ldots, N; j = 1, 2, \ldots, L; k = 1, 2, \ldots, K); \tag{32}$$

and

$$T\left(w^i_{jk}\right) = \sum_{l=1, l\neq k}^{K} \mathrm{Sup}\left(w^i_{jk}, w^i_{jl}\right), \quad (i = 1, 2, \ldots, N; j = 1, 2, \ldots, L; k = 1, 2, \ldots, K) \tag{33}$$

Step 3: Aggregate the individual assessments $w^i_{jk} (k = 1, 2, \ldots, K)$ into a collective assessment by

$$\text{q-PWA}(w^i_{j1}, w^i_{j2}, \ldots, w^i_{jK}) = \tilde{\oplus}_{k=1}^{K} \left(\theta^i_{jk} \tilde{\otimes} w^i_{jk}\right) \tag{34}$$

According to the above aggregation steps, the overall assessment vector of the proposition's weight is obtained as $W_i = (w^i_1, w^i_2, \ldots, w^i_L)$. Similarly, the overall assessment of threshold values and certainty factors can be acquired in the same way.

By assigning a power weight θ^i_{jk} to each assessment w^i_{jk}, q-PWA enables the parameter assessments to support and reinforce each other, which eliminates the influence of unreasonable outliers on final knowledge parameter values.

C. DEFUZZIFICATION AND NORMALIZATION

The threshold values and the certainty factors can be directly determined based on the above overall assessments. However, the assessments of local weights and global weights need to be defuzzified and normalized. Yager develops a defuzzification method [31], which assigns a crisp value to each intuitionistic fuzzy number. Since IFSs is a special case of q-ROFNs [17], we extend the above defuzzification method into q-ROFSs to convert each knowledge parameter denoted by q-ROFN into a crisp value.

For a q-ROFN $(\mu(x), v(x))$, its defuzzification result is calculated as follows:

$$V(x) = (\mu(x))^q + 0.5(h(x))^q, \; q \geq 1 \tag{35}$$

where $h_A(x)$ is the hesitancy degree of x.

As such, the overall assessments of the proposition weight are defuzzified by using (35) and a crisp evaluation vector be formed as $V_i = (V^i_1, V^i_2, \ldots, V^i_L)$. Next, the crisp evaluation vector are normalized by

$$\tilde{w}^i_j = V^i_j / \sum_{j=1}^{L} V^i_j \; i = 1, 2, \ldots N; j = 1, 2, \ldots, L \tag{36}$$

where $\tilde{w}^i_j$ is the local weight or global weight corresponding to proposition P_j of rule R_i.

VI. ILLUSTRATIVE EXAMPLE

In this section, a practical case regarding fault diagnosis of a variable refrigerant flow (VRF) air-conditioning system [32] is employed to demonstrate the effectiveness and advantages of the proposed method. In recent years, energy management of heating, ventilation, and air conditioning (HVAC) systems has become an important research issue in energy-saving. VRF is a newly emerged air-conditioning system with advanced control and energy efficiency. Unfortunately, equipment faults will result in much energy waste and the service life decrease of the VRF system. Therefore, fault diagnosis is significant for the energy-saving operation of VRF systems. Due to the specialization of domain knowledge of HVAC, the expert system is still a prevailing method in HVAC fault diagnosis. We utilize the proposed q-ROFPNs method to solve the diagnosis problems of the VRF system, and the experimental details are presented in the following sections.

A. KNOWLEDGE ACQUISITION OF Q-ROFPRS

We first conduct the acquirement of q-ROFPRs. As pointed out in [32], the fault of VRF mainly consists of sensor faults and indoor unit faults, where each type of fault involves a series of equipment faults. According to the domain knowledge provided by [32], q-ROFPRs are used to model the fault rule of the VRF system.

Let d_j $(j = 1, 2, \ldots, 14)$ be fourteen propositions, and the q-ROFPRs for VRF system faults are identified as follows:

R_1: IF d_1 THEN d_8 AND d_9;
$(\lambda_1, \lambda_8, \lambda_9; \mathrm{lw}_{1,1}, u_1; \mathrm{gw}_{1,8}, \mathrm{gw}_{1,9})$
R_2: IF d_2 THEN d_8 AND d_9;
$(\lambda_2, \lambda_8, \lambda_9; \mathrm{lw}_{2,2}, u_2; \mathrm{gw}_{2,8}, \mathrm{gw}_{2,9})$
R_3: IF d_3 AND d_4 THEN d_{10};
$(\lambda_3, \lambda_4, \lambda_{10}; \mathrm{lw}_{3,3}, \mathrm{lw}_{4,3}, u_3; \mathrm{gw}_{3,10})$
R_4: IF d_5 THEN d_{10} AND d_{11};
$(\lambda_5, \lambda_{10}, \lambda_{11}; \mathrm{lw}_{5,4}; u_4; \mathrm{gw}_{4,10}, \mathrm{gw}_{4,11})$
R_5: IF d_6 THEN d_{11}; $(\lambda_6, \lambda_{11}; \mathrm{lw}_{6,5}; u_5; \mathrm{gw}_{5,11})$
R_6: IF d_7 THEN d_{11}; $(\lambda_7, \lambda_{11}; \mathrm{lw}_{7,6}; u_6; \mathrm{gw}_{6,11})$
R_7: IF d_8 AND d_9 THEN d_{12};

TABLE 2. Places in the Q-ROFPN model and their propositions.

Place p_i	Proposition d_i
p_1	The temperature difference between the outlet and inlet refrigerant temperatures of the running IDU have a significantly difference.
p_2	The inlet refrigerant temperature of target IDU will be significantly different than that of other IDUs
p_3	The compressor discharge temperature will be approximately equal to the outdoor temperature
p_4	The vapor-liquid separator inlet temperature will be approximately equal to the outdoor temperature
p_5	The relative error between the measured and predicted sensor value will has a significant difference
p_6	The vapor-liquid separator inlet temperature will not be approximately equal to the outdoor temperature
p_7	The compressor discharge temperature will not be approximately equal to the outdoor temperature
p_8	Electronic expansion valve (EXV) stuck closed fault
p_9	EXV stuck at intermediate position fault
p_{10}	Sensor detached
p_{11}	Sensor bias
p_{12}	Indoor unit faults
p_{13}	Sensor faults
p_{14}	VRF system faults

TABLE 3. Assessment information of thresholds by the expert panel.

λ_i	TM1	TM2	TM3	TM4	TM5	q-PWA	q-WA
λ_1	(0.3,0.6)	(0.4,0.6)	(0.3,0.6)	(0.35,0.55)	(0.25,0.75)	(0.302,0.619)	(0.302,0.619)
λ_2	(0.2,0.6)	(0.25,0.6)	(0.25,0.7)	(0.6,0.4)	(0.15,0.8)	(0.278,0.623)	(0.289,0.617)
λ_3	(0.2,0.7)	(0.1,0.8)	(0.2,0.7)	(0.25,0.6)	(0.25,0.7)	(0.2,0.702)	(0.2,0.703)
λ_4	(0.35,0.55)	(0.4,0.6)	(0.3,0.65)	(0.2,0.8)	(0.1,0.9)	(0.281,0.673)	(0.277,0.676)
λ_5	(0.25,0.6)	(0.4,0.6)	(0.25,0.5)	(0.1,0.9)	(0.25,0.6)	(0.265,0.582)	(0.263,0.587)
λ_6	(0.2,0.8)	(0.15,0.8)	(0.2,0.8)	(0.4,0.5)	(0.2,0.6)	(0.221,0.71)	(0.224,0.704)
λ_7	(0.2,0.7)	(0.3,0.5)	(0.15,0.8)	(0.1,0.9)	(0.2,0.7)	(0.196,0.702)	(0.198,0.698)
λ_8	(0.2,0.8)	(0.2,0.8)	(0.2,0.7)	(0.4,0.5)	(0.25,0.7)	(0.241,0.711)	(0.244,0.707)
λ_9	(0.2,0.8)	(0.2,0.8)	(0.2,0.7)	(0.4,0.5)	(0.25,0.7)	(0.241,0.711)	(0.244,0.707)
λ_{10}	(0.35,0.6)	(0.15,0.6)	(0.25,0.7)	(0.5,0.4)	(0.25,0.7)	(0.3,0.605)	(0.302,0.6)
λ_{11}	(0.3,0.5)	(0.35,0.6)	(0.3,0.6)	(0.1,0.9)	(0.1,0.8)	(0.249,0.644)	(0.247,0.645)
λ_{12}	(0.4,0.5)	(0.3,0.6)	(0.6,0.2)	(0.3,0.5)	(0.2,0.75)	(0.374,0.479)	(0.382,0.468)
λ_{13}	(0.3,0.6)	(0.3,0.5)	(0.4,0.6)	(0.25,0.5)	(0.3,0.5)	(0.313,0.542)	(0.274,0.543)
λ_{14}	(0.4,0.5)	(0.5,0.4)	(0.3,0.6)	(0.3,0.55)	(0.4,0.6)	(0.388,0.523)	(0.389,0.522)

$(\lambda_8, \lambda_9, \lambda_{12}; \mathrm{lw}_{8,7}, \mathrm{lw}_{9,7}; u_7; \mathrm{gw}_{7,12})$

R_8: IF d_{10} THEN d_{13}; $(\lambda_{10}, \lambda_{13}; \mathrm{lw}_{10,8}; u_8; \mathrm{gw}_{8,13})$

R_9: IF d_{11} THEN d_{13}; $(\lambda_{11}, \lambda_{13}; \mathrm{lw}_{11,9}; u_9; \mathrm{gw}_{9,13})$

R_{10}: IF d_{12} THEN d_{14}; $(\lambda_{12}, \lambda_{14}; \mathrm{lw}_{12,10}; u_{10}; \mathrm{gw}_{10,14})$

R_{11}: IF d_{13} THEN d_{14}; $(\lambda_{13}, \lambda_{14}; \mathrm{lw}_{13,11}; u_{11}; \mathrm{gw}_{11,14})$

The fourteen q-ROFPRs mentioned above are mapped into a q-ROFPN, as shown in Fig 7. Note that the labels are omitted, if $\mathrm{lw}_{ij} = 1$ or $\mathrm{gw}_{ij} = 1$, and the corresponding propositions of places are presented in Table 2.

Next, we apply the knowledge acquisition based on q-PWA to determine the knowledge parameters of rules. An expert panel consisting of five team members uses the q-ROFNs and linguistic terms (in Table 1) to evaluate the knowledge parameters of the q-ROFPN, including thresholds, certainty factors, local weights and global weights. Tables 3-5 present the assessment results of parameters, where the local and global weights equaling 1 are not evaluated. Suppose the expert weights are $\gamma = (0.25, 0.2, 0.2, 0.15, 0.2)$, then the q-PWA operator (setting $q = 1$) is used to aggregate individual assessments into a collective assessment. In addition, we also apply the traditional WA operator to aggregate individual assessments, thus validating the effectiveness of the PA-based knowledge aggregation. The aggregation results by q-PWA and q-WA are shown in the last two columns of Tables 3-5.

TABLE 4. Assessment information of certain factors by the expert panel.

u_j	TM1	TM2	TM3	TM4	TM5	q-PWA	q-WA
u_1	(0.8,0.2)	(0.9,0.1)	(0.85,0.05)	(0.8,0.1)	(0.85,0.15)	(0.845,0.112)	(0.845,0.112)
u_2	(0.85,0.1)	(0.85,0.1)	(0.9,0.1)	(0.85,0.1)	(0.8,0.1)	(0.853,0.1)	(0.854,0.1)
u_3	(0.9,0)	(0.9,0)	(0.9,0)	(0.85,0.1)	(0.95,0.05)	(0.907,0)	(0.908,0)
u_4	(0.95,0.05)	(0.9,0)	(0.9,0.05)	(0.9,0)	(0.6,0.4)	(0.895,0)	(0.889,0)
u_5	(0.9,0)	(0.9,0)	(0.87,0.05)	(0.9,0)	(0.95,0)	(0.908,0)	(0.908,0)
u_6	(0.9,0)	(0.95,0.05)	(0.7,0.2)	(0.85,0.1)	(0.95,0.05)	(0.902,0)	(0.9,0)
u_7	(0.95,0.05)	(0.95,0.05)	(0.9,0.1)	(0.9,0.1)	(0.85,0.1)	(0.921,0.073)	(0.921,0.073)
u_8	(0.95,0)	(0.9,0.1)	(0.7,0.2)	(0.9,0.1)	(0.95,0.05)	(0.911,0)	(0.909,0)
u_9	(0.9,0.1)	(0.9,0.05)	(0.95,0.05)	(0.85,0.15)	(0.9,0.1)	(0.908,0.081)	(0.908,0.081)
u_{10}	(0.95,0.05)	(0.9,0.1)	(0.95,0.05)	(0.9,0)	(0.95,0.05)	(0.937,0)	(0.936,0)
u_{11}	(0.95,0.05)	(0.9,0.1)	(0.95,0.05)	(0.85,0.15)	(0.95,0.05)	(0.933,0.067)	(0.932,0.068)

TABLE 5. Assessment information of weights by the expert panel.

$\mathrm{lw}_{i,j}/\mathrm{gw}_{i,j}$	TM1	TM2	TM3	TM4	TM5	q-PWA	Defuzzy	q-WA
$\mathrm{gw}_{1,8}$	H	EH	MH	H	H	(0.862,0.1)	0.584	(0.862,0.1)
$\mathrm{gw}_{1,9}$	H	EH	MH	H	H	(0.862,0.1)	0.584	(0.862,0.1)
$\mathrm{gw}_{2,8}$	M	L	MH	M	MH	(0.566,0.312)	0.416	(0.558,0.321)
$\mathrm{gw}_{2,9}$	M	L	MH	M	MH	(0.566,0.312)	0.416	(0.558,0.321)
$\mathrm{lw}_{3,3}$	H	H	MH	MH	M	(0.76,0.162)	0.55	(0.757,0.164)
$\mathrm{lw}_{4,3}$	M	MH	M	M	MH	(0.59,0.282)	0.45	(0.592,0.28)
$\mathrm{gw}_{3,10}$	H	MH	MH	M	M	(0.695,0.207)	0.576	(0.698,0.205)
$\mathrm{gw}_{4,10}$	M	M	ML	M	M	(0.476,0.38)	0.424	(0.473,0.383)
$\mathrm{gw}_{4,11}$	M	M	H	ML	M	(0.575,0.303)	0.299	(0.591,0.292)
$\mathrm{gw}_{5,11}$	H	M	H	H	MH	(0.787,0.143)	0.386	(0.781,0.148)
$\mathrm{gw}_{6,11}$	H	M	M	M	M	(0.609,0.271)	0.315	(0.63,0.256)
$\mathrm{lw}_{8,7}$	M	MH	M	MH	M	(0.58,0.289)	0.54	(0.582, 0.288)
$\mathrm{lw}_{9,7}$	M	ML	M	M	M	(0.478,0.38)	0.46	(0.473,0.383)
$\mathrm{gw}_{8,13}$	M	H	MH	ML	H	(0.716,0.198)	0.474	(0.71,0.203)
$\mathrm{gw}_{9,13}$	H	H	MH	MH	H	(0.81,0.127)	0.526	(0.809,0.128)
$\mathrm{gw}_{10,14}$	EH	EH	H	MH	MH	(0.884,0.093)	0.504	(0.883,0.093)
$\mathrm{gw}_{11,14}$	H	EH	H	H	MH	(0.862,0.1)	0.496	(0.862,0.1)

From Tables 3-5, we can observe that the overall assessments given by the q-PWA are different from those by the q-WA when individual assessments by expert members vary greatly. For example, TM4 assesses the threshold λ_2 as (0.6,0.4), which is much larger than those by the other four experts. The overall assessment λ_2 given by the q-PWA is (0.278, 0.623), which is smaller than the result (0.289,0.617) by the q-WA. That is because the q-PWA assigns a power weight to each assessment, which reduces the impact of the outlier (0.6,0.4) on aggregation results. Further, we also observe that the results by the q-PWA are the same as that by the q-WA, when the differences are small among expert assessments. For example, the assessments of λ_1 by five experts are slightly different, and the overall assessment λ_1 by q-PWA is (0.302,0.619), which is same as the ones by q-WA operator. It is proven that q-PWA based knowledge acquisition method can eliminate the negative effects of outliers on aggregation results, thus guaranteeing the veracity of knowledge parameters.

B. FORWARD REASONING BASED ON Q-ROFPNs

In the following, the forward reasoning is conducted to estimate the fault probability of air condition system. We assume

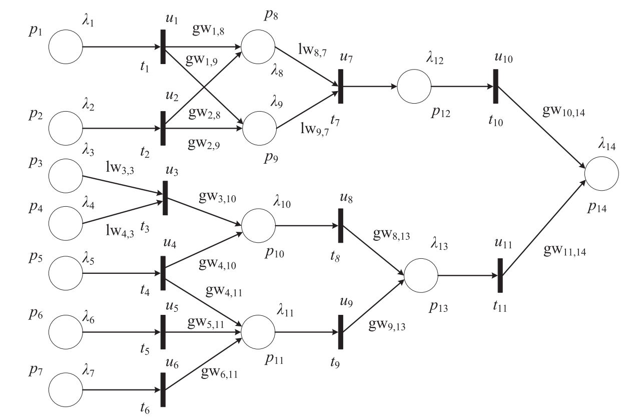

FIGURE 7. q-ROFPN of example.

the initial marking vector of q-ROFPN is

$$M_0 = [\ (0.77,0.1)(0.68,\ 0.25)(0.84,\ 0.05)\\ (0.79,\ 0.15)(0.81,\ 0.1)(0.8,\ 0.15)(0.75,\ 0.1)\\ (0,\ 1)(0,\ 1)(0,\ 1)(0,\ 1)(0,\ 1)\ (0,\ 1)(0,\ 1)]^T$$

According to the aggregation results of Tables 3-5 and Fig 7, the fault reasoning of VRF is carried out by the q-OWAWA reasoning algorithm, and the inference processes are the same as those in Steps 1-7 in Section III-D. The parameter $q = 1$, $\beta = 0.5$, and $\omega = (0.5, 0.5)$, when $n = 2$; $\omega = (0.2429, 0.5142, 0.2429)$, when $n = 3$ [33]. After four iterations, the final marking vector is obtained as follows:

$$M_4 = [(0.77,\ 0.1)(0.68,\ 0.25)(0.84,\ 0.05)(0.79,\ 0.15)\\ (0.81,\ 0.1)(0.8,\ 0.15)(0.75,\ 0.1)(0.62,\ 0.25)\\ (0.62,\ 0.25)(0.734,\ 0.091)(0.706,\ 0.114)\\ (0.571,\ 0.305)(0.655,\ 0.131)(0.575,\ 0.241)]^T,$$

which shows the final truth values of all propositions in the considered situation. For the fault diagnosis problem, we can obtain that the VRF system may have the fault of d_{14} with q-rung orthoapir fuzzy truth degree (0.575, 0.241), in which the 0.575 and 0.241 indicate degree of supporting for and against the proposition "the VRF system have faults", respectively.

Next, we investigate the influence of parameter β on the diagnosis results. We conduct the simulations of the VRF system fault diagnosis under different β. Table 6 delineates the truth values of non-initial places. From 6, we can observe that the truth values of the places become smaller and smaller as the value of parameter β increases. The parameter β indicates the relative degree of importance between proposition weights and the position weights. The greater the parameter β, the more importance is assigned to the position weights. Conversely, the smaller the parameter β, the more importance is assigned to the proposition weights. Note that the q-OWAWA can reduce to the q-WA (when $\beta = 0$), which only weighs the proposition itself. And the q-OWAWA also reduces to the q-OWA (when $\beta = 1$), which only considers the ordered positions of propositions. In practical reasoning, the parameter β can take any values between 0 and 1, thus assigning different degrees of importance to the place and position weights depending on the analyzed problem.

TABLE 6. Truth values of intermediate and terminating places with different β.

p_i	$\beta=0$	$\beta=0.2$	$\beta=0.4$	$\beta=0.6$	$\beta=0.8$	$\beta=1$
p_8	(0.623,0.245)	(0.622,0.247)	(0.621,0.249)	(0.619,0.251)	(0.618,0.253)	(0.617,0.256)
p_9	(0.623,0.245)	(0.622,0.247)	(0.621,0.249)	(0.619,0.251)	(0.618,0.253)	(0.617,0.256)
p_{10}	(0.736,0.089)	(0.735,0.09)	(0.734,0.091)	(0.734,0.092)	(0.733,0.092)	(0.733,0.093)
p_{11}	(0.711,0.117)	(0.709,0.116)	(0.707,0.114)	(0.705,0.113)	(0.703,0.112)	(0.701,0.11)
p_{12}	(0.574,0.3)	(0.573,0.302)	(0.572,0.304)	(0.57,0.306)	(0.569,0.308)	(0.568,0.31)
p_{13}	(0.657,0.132)	(0.656,0.132)	(0.655,0.131)	(0.655,0.131)	(0.654,0.131)	(0.653,0.13)
p_{14}	(0.577,0.24)	(0.576,0.24)	(0.575,0.24)	(0.574,0.241)	(0.573,0.241)	(0.572,0.242)

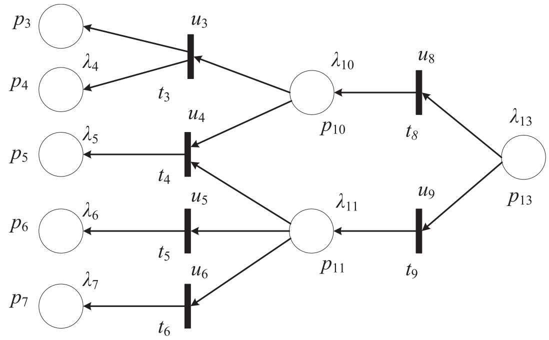

FIGURE 8. q-ROFRPN of example.

C. BACKWARD REASONING BASED ON q-ROFRPNs

In fault diagnosis, the fault status may be known, and the expert system needs to identify the fault's causes and their probability based on known outcomes. For the VRF system fault diagnosis problem, we assume the indoor unit p_{13} has faults with a truth degree of (0,655,0.131), and we use q-ROFRPN to identify its fault causes and truth degrees.

With the decomposition algorithm, the S-q-ROFRPN associated with the appointed goal place p_{13} is first constructed from the original q-ROFPN model. The detailed steps are shown in Steps 1-4 in Section IV-B, and the obtained S-q-ROFRPN is presented in Fig. 7. Next, the OWBR algorithm is performed according to Steps 1-7 in Section IV-C, which determines the truth values of the S-q-ROFRPN model. We set $q = 1$, and the final marking vector is obtained as follows:

$$\tilde{M}'_3 = [(0.793,\ 0.131)(0.793,\ 0.131)(0.805,\ 0.085)\\ (0.795,\ 0.055)(0.8,\ 0.055)(0.719,\ 0.131)\\ (0.721,\ 0.055)(0.655,\ 0.131\ )]^T$$

We can observe that the faults causes of p_{13} are p_3, p_4, p_5, p_6, and p_7 with truth degree (0.793,0.131), (0.793,0.131), (0.805,0.085), (0.795,0.055), and (0.8,0.055), respectively. The MD and NMD indicate degree of supporting for and against the cause propositions.

TABLE 7. Reasoning results with different reasoning operators.

Input-Output operator	Final marker vector: $[p_3, p_4, p_5, p_6, p_7, p_{10}, p_{11}, p_{13}]$
Min-Max	[(0.793,0.131)(0.793,0.131)(0.803,0.131) (0.795,0.055) (0.8,0.055) (0.719,0.131) (0.721,0.055) (0.655,0.131)]
Max-Max	[(0.793,0.131)(0.793,0.131)(0.806,0.055) (0.795,0.055) (0.8,0.055) (0.719,0.131) (0.721,0.055) (0.655,0.131)]
OWA-OWA	[(0.793,0.102)(0.793,0.102)(0.805,0.085) (0.795,0.055) (0.8,0.055) (0.719,0.131) (0.721,0.055) (0.655,0.131)]

TABLE 8. The truth values of intermediate and goal places by different FPN models.

place	IFPN	PFPN	q-ROFLRPN (q = 1)	q-ROFPN ($\beta = 1, q = 1$)	q-ROFPN ($\beta = 0, q = 2$)	q-ROFPN ($\beta = 0.5, q = 1$)	q-ROFPN ($\beta = 0.5, q = 2$)
p_8	(0.62,0.256)	(0.651,0.15)	$\langle s_8, 0.623, 0.191\rangle$	(0.62,0.256)	(0.651,0.15)	(0.62,0.256)	(0.621,0.196)
p_9	(0.62,0.256)	(0.651,0.15)	$\langle s_8, 0.623, 0.191\rangle$	(0.62,0.256)	(0.651,0.15)	(0.62,0.256)	(0.621,0.196)
p_{10}	(0.733,0.093)	(0.743,0.082)	$\langle s_8, 0.741, 0.085\rangle$	(0.733,0.093)	(0.743,0.082)	(0.734,0.091)	(0.734,0.091)
p_{11}	(0.701,0.11)	(0.725,0.1)	$\langle s_8, 0.716, 0.12\rangle$	(0.701,0.11)	(0.725,0.1)	(0.706,0.114)	(0.713,0.12)
p_{12}	(0.568,0.31)	(0.599,0.166)	$\langle s_8, 0.574, 0.204\rangle$	(0.568,0.31)	(0.599,0.166)	(0.571,0.305)	(0.572,0.208)
p_{13}	(0.653,0.13)	(0.677,0.082)	$\langle s_8, 0.664, 0.1\rangle$	(0.653,0.13)	(0.677,0.082)	(0.655,0.132)	(0.668,0.116)
p_{14}	(0.572,0.242)	(0.632,0.106)	$\langle s_8, 0.572, 0.157\rangle$	(0.572,0.242)	(0.632,0.106)	(0.575,0.241)	(0.577,0.167)

TABLE 9. The final marking vectors by different RFPN models.

Models	Final marking vector
RFPNs	[0, 0, 0.793, 0.793, 0.803, 0.795, 0.8, 0, 0, 0.719, 0.721, 0, 0.655, 0]
DRAFPNs	[0, 0, 0.793, 0.793, 0.806, 0.795, 0.8, 0, 0, 0.719, 0.721, 0, 0.655, 0]
q-ROFRPNs (Min-Max)	[(0.793,0.131)(0.793,0.131)(0.803,0.131) (0.795,0.055) (0.8,0.055) (0.719,0.131) (0.721,0.055) (0.655,0.131)]
q-ROFRPNs (Max-Max)	[(0.793,0.131)(0.793,0.131)(0.806,0.055) (0.795,0.055) (0.8,0.055) (0.719,0.131) (0.721,0.055) (0.655,0.131)]

We further validate the flexibility of the OWBR algorithm in backward inference. The identification of fault causes of the indoor unit p_{13} is carried out by two particular cases of the OWBR algorithm (e.g. minimum and maximum). Table 7 presents the final marking vector obtained by the Min, Max and q-OWA reasoning operators. It can be known that the obtained truth value of p_5 by q-OWA is different from that given by the other two operators. That is because the q-OWA considers the ordered position weights of places when dealing with the conjunctive rule types. This also indicates that the proposed OWBR algorithm can execute the Max and Min inference mechanism by adjusting the position weight vector.

D. COMPARISON AND DISCUSSION

In this section, we demonstrate the efficacy of the proposed method by comparing them with five representative FPNs, i.e., IFPNs [6], PFPNs [12], the q-rung orthopair fuzzy linguistic Petri nets (q-ROFLPNs) [14], the RFPNs [15], and the RDAFPNs [16]. We employ the above five FPN models to solve the VRF system's fault diagnosis problem and conduct an in-depth look at the experiment results.

1) KNOWLEDGE REASONING VIA IFPN, PFPN AND Q-ROFLPNs

To confirm the efficacy of the proposed method in forward reasoning, we will compare the knowledge representation and reasoning ability of q-ROFPNs with IFPNs, PFPNs, and q-ROFLPNs. The fault diagnosis case in part B of Section VI is used to evaluate their performances. To guarantee the comparability of results, the knowledge parameters and the initial marking vectors of the IFPN and PFPN models are the same as the data in part B of Section VI. For the q-ROFLPN model, we assume the linguistic variables of all knowledge parameters are s_8, and their membership and non-membership degrees are the same as the data in part B of Section VI. The inference results of all non-initial places are presented in Table 8.

From Table 8, we can observe that the place truths by the q-ROFPN model ($\beta = 1$ and $q = 1$) are equal to those by the IFPN model, and the results of the q-ROFPN model ($\beta = 0$ and $q = 2$) are the same as those by the PFPN model. The above results demonstrate the validity of q-ROFPNs and further prove that q-ROFPNs can reduce to IFPNs or PFPNs by setting $q = 1$ or $q = 2$. On the other hand, unlike IFPNs and the PFPNs, q-ROFPNs also can set the parameter q to any values greater than or equal to one, which enables q-ROFPNs to eliminate the limitation to MD and NMD and realizes the flexible adjustment of the representation range of knowledge parameters.

In addition, we also find that the inference results given by the IFPN and PFPN models are different from that given by the q-ROFPN model ($\beta = 0.5$, and $q = 1$ or $q = 2$). This is because IFPNs and PFPNs employ the OWA and WA as reasoning operators, which are incapable of both considering the place weights and their position weights. Instead, the q-ROFPNs can both weigh proposition themselves and their ordered positions when parameter $\beta = 0.5$.

Comparing q-ROFLPNs with q-ROFPNs, we can observe that the equivalent fuzzy values of places given by the q-ROFLPN are different with that inference results by the q-ROFPN ($\beta = 0.5$ or $q = 1$). Although they both consider the weights of the proposition themselves and their position weights, the internal mechanism of the reasoning algorithm are greatly different. In the former, the WOWA operator is embedded into the reasoning algorithm, which unifies the OWA and WA by interpolation functions, whereas the latter, which integrates the OWAWA operator, can be regarded as a convex combination of the WA and the OWA. However, the reasoning algorithm based on q-OWAWA can assign different degrees of importance to the proposition weights and the position weights, thus fortifying the reliability and flexibility of knowledge reasoning.

2) KNOWLEDGE REASONING VIA RFPN AND RDAFPN

To demonstrate the efficacy of the proposed method in backward reasoning, the comparison among q-ROFRPNs, RFPNs and RDAFPNs is undertaken through the fault diagnosis case in part C of Section VI. To guarantee the comparability of results, the Min-Max and Max-Max are selected as the reasoning operators in the q-ROFRPN model, respectively. We assume the truth degree of certain factors of the RFPN and RDAFPN models are equal to the membership degree values of the data in Table 4, and their initial marking vector are $\Theta_0 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.655, 0]^T$.

Table 9 presents the final reasoning results of different models. The reasoning results by the RPFN model exactly match the memberships of results by the q-ROFRPN model with Mix-Max operators. Similarly, the results by the DRAFPN model exactly match the memberships of the results obtained by the q-ROFRPN model with Max-Max operators. Therefore, the validity of the q-ROFRPN model is proved. However, we also know that RFPNs and DRAFPNs employ traditional fuzzy values between 0 and 1 to represent uncertain information, which ignores the non-membership and hesitancy in knowledge information.

In addition, the backward reasoning by RFPNs and DRAFPNs is oriented to the whole original network, which will lead to more computing loss with the increase of the scale of the model. In contrast, the backward reasoning by q-ROFRPNs simplifies the model structure by the decomposition algorithm. It is worth stressing that the reasoning results produced by q-ROFRPNs are different from those by the RFPNs and DRAFPNs if other cases of reasoning operators are applied. This further proves the flexibility of q-ROFRPN in backward reasoning.

VII. CONCLUSION

This paper proposed a novel FPN method based on q-ROFSs to achieve a flexible and reliable knowledge representation and forward and backward knowledge reasoning. A prominent aspect of the proposed method was constructing the q-ROFPNs and the corresponding reasoning algorithm based on q-OWAWA for knowledge representation and forward knowledge reasoning. Building upon q-ROFPNs, we proposed q-ROFRPNs, which involved a decomposition algorithm of the model and an OWBR algorithm to accomplish reliable and efficient backward knowledge reasoning. In addition, a PA-based knowledge acquisition method was introduced to guarantee the accuracy of model parameter acquisition.

By a fault diagnosis case of the VRF system, the results demonstrated that the q-ROFPNs and q-ROFRPNs could intuitively characterize the hesitancy of knowledge information and allow for a flexible adjustment of the representation range of knowledge parameters. By developing the q-OWAWA-based reasoning algorithm, q-ROFPNs accomplished the forward reasoning capable of flexibly balancing the proposition and position weights. Experiment results also showed the decomposition algorithm of q-ROFRPNs can avoid the interference of unrelated propositions, and its OWBR algorithm enabled more reliable and flexible backward reasoning than existing RFPN methods.

In knowledge representation and reasoning of the q-ROFPN method, the reasonable determination of rule knowledge and its parameters is a key challenge. In future works, we will focus on the adaptive learning of q-ROFPNs, which intend to train model structure and parameters in a data-driven way and thus automatically accomplish knowledge reasoning tasks. In addition, the proposed method can also be used to solve other domain problems, such as reliability assessment, failure mode and effects analysis, etc.

REFERENCES

[1] H.-C. Liu, J.-X. You, Z. Li, and G. Tian, "Fuzzy Petri nets for knowledge representation and reasoning: A literature review," *Eng. Appl. Artif. Intell.*, vol. 60, pp. 45–56, Apr. 2017.
[2] B. Xu, X. Yin, X. Yin, Y. Wang, and S. Pang, "Fault diagnosis of power systems based on temporal constrained fuzzy Petri nets," *IEEE Access*, vol. 7, pp. 101895–101904, 2019.
[3] J. Wu, S. Yan, L. Xie, and P. Gao, "Reliability apportionment approach for spacecraft solar array using fuzzy reasoning Petri net and fuzzy comprehensive evaluation," *Acta Astronautica*, vol. 76, pp. 136–144, Jul. 2012.
[4] Y. Ye, Z. Jiang, X. Diao, and G. Du, "Extended event-condition-action rules and fuzzy Petri nets based exception handling for workflow management," *Exp. Syst. Appl.*, vol. 38, no. 9, pp. 10847–10861, Sep. 2011.
[5] G. Deschrijver and E. E. Kerre, "On the relationship between some extensions of fuzzy set theory," *Fuzzy Sets Syst.*, vol. 133, no. 2, pp. 227–235, Jan. 2003.
[6] H.-C. Liu, J.-X. You, X.-Y. You, and Q. Su, "Fuzzy Petri nets using intuitionistic fuzzy sets and ordered weighted averaging operators," *IEEE Trans. Cybern.*, vol. 46, no. 8, pp. 1839–1850, Aug. 2016.
[7] W. Jiang, K.-Q. Zhou, A. Sarkheyli-Hägele, and A. M. Zain, "Modeling, reasoning, and application of fuzzy Petri net model: A survey," *Artif. Intell. Rev.*, vol. 55, no. 8, pp. 6567–6605, Dec. 2022.
[8] H.-C. Liu, X. Luan, W. Lin, and Y. Xiong, "Grey reasoning Petri nets for large group knowledge representation and reasoning," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 12, pp. 3315–3329, Dec. 2020.
[9] H.-C. Liu, J.-X. You, X.-Y. You, and Q. Su, "Linguistic reasoning Petri nets for knowledge representation and reasoning," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 4, pp. 499–511, Apr. 2016.
[10] H.-C. Liu, X. Luan, Z. Li, and J. Wu, "Linguistic Petri nets based on cloud model theory for knowledge representation and reasoning," *IEEE Trans. Knowl. Data Eng.*, vol. 30, no. 4, pp. 717–728, Apr. 2018.
[11] M. Fei-Xiang, L. Ying-Jie, Z. Bo, S. Xiao-Yong, and Z. Jing-Yu, "Intuitionistic fuzzy Petri nets for knowledge representation and reasoning," *J. Digit. Inf. Manag.*, vol. 14, no. 2, pp. 1–10, 2016.
[12] H.-C. Liu, D.-H. Xu, C.-Y. Duan, and Y. Xiong, "Pythagorean fuzzy Petri nets for knowledge representation and reasoning in large group context," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 8, pp. 5261–5271, Aug. 2021.

[13] H.-C. Liu, Q.-L. Lin, L.-X. Mao, and Z.-Y. Zhang, "Dynamic adaptive fuzzy Petri nets for knowledge representation and reasoning," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 43, no. 6, pp. 1399–1410, Nov. 2013.
[14] K. Bai, X. Zhu, R. Zhang, and J. Gao, "Linguistic reasoning Petri nets using q-rung orthopair fuzzy linguistic sets and weighted ordered weighted averaging operators," *IEEE Access*, vol. 7, pp. 103167–103183, 2019.
[15] H. Hu, Z. Li, and A. Al-Ahmari, "Reversed fuzzy Petri nets and their application for fault diagnosis," *Comput. Ind. Eng.*, vol. 60, no. 4, pp. 505–510, May 2011.
[16] H.-C. Liu, Q.-L. Lin, and M.-L. Ren, "Fault diagnosis and cause analysis using fuzzy evidential reasoning approach and dynamic adaptive fuzzy Petri nets," *Comput. Ind. Eng.*, vol. 66, no. 4, pp. 899–908, Dec. 2013.
[17] R. R. Yager, "Generalized orthopair fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 5, pp. 1222–1230, Oct. 2017.
[18] P. Liu and P. Wang, "Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making," *Int. J. Intell. Syst.*, vol. 33, no. 2, pp. 259–280, Feb. 2018.
[19] W. S. Du, "Research on arithmetic operations over generalized orthopair fuzzy sets," *Int. J. Intell. Syst.*, vol. 34, no. 5, pp. 709–732, 2019.
[20] J. M. Merigó, "A unified model between the weighted average and the induced OWA operator," *Exp. Syst. Appl.*, vol. 38, no. 9, pp. 11560–11572, Sep. 2011.
[21] B. Llamazares, "An analysis of some functions that generalizes weighted means and OWA operators," *Int. J. Intell. Syst.*, vol. 28, no. 4, pp. 380–393, Apr. 2013.
[22] R. R. Yager, "The power average operator," *IEEE Trans. Syst., Man, Cybern. A, Syst. Humans*, vol. 31, no. 6, pp. 724–731, Nov. 2001.
[23] Y. Ju, C. Luo, J. Ma, and A. Wang, "A novel multiple-attribute group decision-making method based on q-rung orthopair fuzzy generalized power weighted aggregation operators," *Int. J. Intell. Syst.*, vol. 34, no. 9, pp. 2077–2103, Sep. 2019.
[24] C. G. Looney, "Fuzzy Petri nets for rule-based decisionmaking," *IEEE Trans. Syst., Man, Cybern.*, vol. 18, no. 1, pp. 178–183, Feb. 1988.
[25] S.-M. Chen, J.-S. Ke, and J.-F. Chang, "Knowledge representation using fuzzy Petri nets," *IEEE Trans. Knowl. Data Eng.*, vol. 2, no. 3, pp. 311–319, Sep. 1990.
[26] M.-H. Ha, Y. Li, and X.-F. Wang, "Fuzzy knowledge representation and reasoning using a generalized fuzzy Petri net and a similarity measure," *Soft Comput.*, vol. 11, no. 4, pp. 323–327, Nov. 2006.
[27] B. Fryc, K. Pancerz, J. F. Peters, and Z. Suraj, "On fuzzy reasoning using matrix representation of extended fuzzy Petri nets," *Fundam. Informat.*, vol. 60, pp. 143–157, Jan. 2004.
[28] K.-Q. Zhou, A. M. Zain, and L.-P. Mo, "A decomposition algorithm of fuzzy Petri net using an index function and incidence matrix," *Exp. Syst. Appl.*, vol. 42, no. 8, pp. 3980–3990, May 2015.
[29] R. Krishankumar, K. S. Ravichandran, and A. B. Saeid, "A new extension to PROMETHEE under intuitionistic fuzzy environment for solving supplier selection problem with linguistic preferences," *Appl. Soft Comput.*, vol. 60, pp. 564–576, Nov. 2017.
[30] P. Liu, S.-M. Chen, and P. Wang, "Multiple-attribute group decision-making based on q-rung orthopair fuzzy power Maclaurin symmetric mean operators," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 10, pp. 3741–3756, Oct. 2020.
[31] R. R. Yager, "Some aspects of intuitionistic fuzzy sets," *Fuzzy Optim. Decis. Making*, vol. 8, no. 1, pp. 67–90, Mar. 2009.
[32] Y. Guo, J. Wang, H. Chen, G. Li, R. Huang, Y. Yuan, T. Ahmad, and S. Sun, "An expert rule-based fault diagnosis strategy for variable refrigerant flow air conditioning systems," *Appl. Thermal Eng.*, vol. 149, pp. 1223–1235, Feb. 2019.
[33] Z. Xu, "An overview of methods for determining OWA weights," *Int. J. Intell. Syst.*, vol. 20, no. 8, pp. 843–865, 2005.

KAIYUAN BAI (Member, IEEE) received the B.S. degree in industrial engineering from the Lanzhou University of Technology, Lanzhou, Gansu, China, in 2016, and the Ph.D. degree in mechanical engineering from Beijing Jiaotong University, Beijing, China, in 2022. He is currently with the AI Research Centre, China Telecom Research Institute, Beijing. His research interests include fuzzy neural networks, machine learning, and expert systems.

DAN JIA received the B.S. degree in computer science from China Women's University, Beijing, China, in 2014, and the M.Sc. degree in computer science and engineering from Beihang University, Beijing, in 2017. She is currently with the AI Research Centre, China Telecom Research Institute, Beijing. Her current research interests include machine learning, deep learning, and 3D reconstruction based on learning.

WEIYE MENG received the B.S. degree in automatic control and the M.Sc. degree in software engineering from the Beijing Institute of Technology, Beijing, China, in 2003 and 2005, respectively. He is currently a Senior Engineer with the AI Research Centre, China Telecom Research Institute, Beijing. His research interests include software architecture, software system development, and smart energy saving.

XINGMIN HE received the B.S. degree in computer science and technology from the North China Institute of Aerospace Engineering University, Hebei, China, in 2011, and the M.Sc. degree in computer science and technology from the Harbin Institute of Technology, Heilongjiang, China, in 2014. She is currently with the AI Research Centre, China Telecom Research Institute, Beijing, China. Her research interests include cloud computing, edge computing, and micro-services architecture.