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RESEARCH ARTICLE

Output Feedback Learning Control of Constrained Nonlinear Systems

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ABSTRACT This paper explores adaptive neural network output feedback control for a group of output-constrained systems using dynamic cooperative learning. The barrier Lyapunov function is employed to constrain the outputs within the required range. For all agents, we use radial basis function neural networks to identify their unknown dynamics. The undirected and connected communication topology is used for exchanging learned knowledge of each agent. Therefore, the approximation domain of learned neural networks is extended. Finally, the stored constant neural weights are used as previous experiences to construct new neural controllers, which not only can be used for the same control tasks, but also can improve dynamic performance and reduce the computational burden. Finally, the approach feasibility is demonstrated by simulation results.

INDEX TERMS Output constraints, dynamic cooperative learning, neural networks, output feedback control.

I. INTRODUCTION

Uncertainties in nonlinear systems adversely affect the control performance. Benefit from the powerful approximation capability of neural networks (NNs) and fuzzy logic systems (FLSs), this problem is effectively solved. Further, adaptive neural/fuzzy control methods are considerably developed by combining the online adjustment capability of adaptive methods and the approximation properties of NNs or FLSs. For example, based on the backstepping technique, the adaptive fault-tolerant control scheme is studied in [1]. Further, an improved adaptive fuzzy controller is designed in [2], which achieves that the system output converges to the desired trajectory within a fixed time. For the nonlinear systems with multiple actuator constraints, an adaptive NN command filter controller is designed to ensure the tracking performance in [3]. However, the above control methods are limited for practical applications due to the lack of consideration of system output or state restrictions.

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In practice, the most important issue is the consideration of various constraints for control performance or safety in control systems. Driven by this practical requirements, significant efforts have been made for output constraints or full state constraints, and the barrier Lyapunov function (BLF)-based approach becomes a convenient solution for solving the constraint problem. Therefore, the BLF-based adaptive controllers are designed to handle output constraints for strict-feedback systems [4], [5], output-feedback systems [6] and nonstrict-feedback systems [7]. Furthermore, an adaptive control approach is studied for full state-constrained pure-feedback systems in [8].

Compared with the single agent, multiple agents can perform tasks in a cooperative manner, their execution efficiency usually exhibits higher. Thus, various research works on multi-agent systems are widely carried out [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. Meanwhile, the output constraint problem of multi-agent control systems is further studied. In [10], two distributed control protocols are designed by using a novel BLF. By combining the backstepping approach with the time-varying BLF, Du *et al*. [11] present an adaptive finite-time decentralized control approach without violating output constraints. In [12], Liu *et al*. construct a distributed controller to ensure that all subsystem outputs are constrained. In addition, other constraints of multi-agent system are explored, such as maximum curvature constraints [13], error constraints [14], state constraints [15], [16] and input saturation [17], [18].

The knowledge acquired from repetitive environments is viewed as experience, and it can be used to improve the efficiency and quality of the tasks. If the control method has such a capability, then it will reduce energy consumption and avoid many invalid behaviors. Although some of the above works on adaptive neural control have achieved the control objective, the learning ability of NNs is not utilized. A deterministic/dynamic learning mechanism is presented in [19] to solve this problem. The authors in [19] focus on the verification that the radial basis function (RBF) NN regression vector satisfies the persistent excitation (PE) condition under the recurrent NN input, which guarantees the convergence of weight estimation. The acquired weight knowledge can be directly used in the controller construction, thereby improving system performance. Benefit from this mechanism, fruitful results are reported, such as affine systems [20], non-affine systems [21] and strict-feedback systems [22], [23], [24]. Inspired by the consensus of multi-agent systems [12], [25], [26], [27], [28], [29], [30], [31] and the cooperative PE [33], a distributed cooperative learning (DCL) mechanism is proposed in [34]. The generalization ability of trained NNs in [34] is larger than that in [19]. In [35], an event-based cooperative controller is designed to overcome the drawback of continuous communication. Since system states may not be available in many practical systems, several cooperative learning output feedback control schemes are explored in [36], [37], and [38]. In recent years, the DCL control is realised under directed communication topology [39], [40]. In addition, the DCL is also applied in practical applications, including unmanned surface vehicles [41], nonholonomic wheeled mobile robots [42] and underwater vehicle formation [43].

According to the above discussion, it is worth noting that output constraints are not considered in the control design of [34], [35], [36], [37], and [38], which implies that these methods have limitations in applications on practical control systems with physical constraints. Motivated by this, this paper investigates dynamic cooperative learning from output feedback control for the output-constrained multi-agent system under undirected communication topologies. The constraint performance of output tracking errors is guaranteed by using BLFs. The cooperative PE condition of RBF NN is satisfied by verifying that all NN input variables along the union orbit are recurrent. After completing cooperative learning, the converged weights are expressed as a group of constants to construct experience-based controllers for the same control tasks. To achieve dynamic cooperative learning from output feedback control under output constraints, two challenges arise in the control design, (i) how to design the cooperative feedback controller such that the closed-loop system remains stable, and the system outputs are restricted when tracking the reference signals; (ii) all neural weights can not converge to small neighborhoods of their common optimal values, resulting in a failure to expand the approximation domain of the NNs. The proposed control scheme effectively addresses the above mentioned challenges, and the main contributions are listed as follows

1) Compared with the existing output feedback-based cooperative learning results [36], [37], [38], the tracking performance of all agents is achieved under the output constraints. Meanwhile, the weights of NNs converge to their optimal value with small errors in the cooperative learning process.
2) The obtained NNs are recalled or recycled to control the same systems. The designed controller ensures the output constraints are never violated. Simultaneously, online computation is reduced.

The organizational structure of this paper is as follows. System statement, basic knowledge on RBF NNs and some lemmas are given in Section II. In Section III, the BLF-based dynamic cooperative learning from output-feedback control is presented. The obtained experience is used to control same tasks in Section IV. In Section V, an example is provided to verify the results that are obtained in Section III and Section IV. In the end, the conclusions and future works are described in Section VI.

Notations: $\otimes$ represents Kronecker product; R denotes the set of real numbers; $\log(\cdot)$ represents natural logarithms; $\|\cdot\|$ denotes the 2-norm; $|\cdot|$ represents the absolute value; I_l denotes the unit matrixs; $\mathcal{L}$ denotes the Laplace matrix under the undirected and connected graph $\mathcal{G}$.

II. SYSTEM STATEMENT AND PRELIMINARIES

A. SYSTEM STATEMENT

Consider the following i-th subsystem in a multi-agent system

$$\begin{cases} \dot{x}_{i,k} = x_{i,k+1}, \quad k = 1, 2, \cdots, m-1, \\ \dot{x}_{i,m} = f(x_i) + u_i \\ y_i = x_{i,1}, \qquad i \in \{1, 2, \cdots, L\}, \end{cases} \tag{1}$$

where m is the order of each subsystem, L is the total number of subsystems (also called agents). $x_i = [x_{i,1}, x_{i,2}, \cdots, x_{i,m}]^T \in R^m$, $u_i \in R$ and $y_i \in R$ are the state, input and output of each agent, respectively. $f(x_i)$ is an unknown smooth function. To be clear, the states of all subsystems are not available except for the output y_i. Note that the structure of all subsystems is identical. The output of (1) is required to remain $|y_i(t)| < k_{y_i}$, $i \in \{1, 2, \cdots, L\}$.

Consider the following reference system

$$\begin{cases} \dot{x}_{r_{i,k}} = x_{r_{i,k+1}} \quad k = 1, 2, \cdots, m-1, \\ \dot{x}_{r_{i,m}} = g_{r_i}(x_{r_i}, t) \\ y_{r_i} = x_{r_{i,1}} \qquad i \in \{1, 2, \cdots, L\}, \end{cases} \tag{2}$$

where $x_{r_i} = [x_{r_{i,1}}, x_{r_{i,2}}, \cdots, x_{r_{i,m}}]^T \in R^m$ is the state, $y_{r_i} \in R$ is the output and $g_{r_i}(x_{r_i}, t)$ is a known smooth function. Let φ_{r_i} be the ith reference signal starting from the initial condition $y_{r_i}(0)$.

Assumption 1: For any k_{y_i}, there exist positive constants $\underline{D}_{0_i}, \overline{D}_{0_i}, D_{0_i}$ satisfying $\max\{\underline{D}_{0_i}, \overline{D}_{0_i}\} \le D_{0_i} \le k_{y_i}$ such that the reference signal $y_{r_i}(t)$ satisfy $\underline{D}_{0_i} \le y_{r_i}(t) \le \overline{D}_{0_i}$.

B. RBF NEURAL NETWORKS AND USEFUL LEMMAS

This paper uses the RBF NN to approximate the unknown function $f(Z)$ over a compact set

$$f(Z) = W^T S(Z) + \varepsilon(Z), \quad \forall Z \in \Omega_Z, \tag{3}$$

where $W = [w_1, \cdots, w_n]^T$ is the weight vector, $n > 1$ is the neural number. $Z \subset R^q$ is the input vector. $S(Z) = [s_1(Z), \cdots, s_n(Z)]^T$ is the regression vector, $\varepsilon(Z)$ represents the approximation error. According to [44], we select the Gaussian function as the activation function, that is, $s_k(Z)=\exp[-\frac{\|Z-\beta_k\|^2}{\pi^2}]$, $k = 1, \cdots, n$ with β_k and π being the center and the width of $s_k(Z)$.

For the multi-agent system, the authors in [34] summarize the local cooperative PE condition of RBF NNs, which plays a key role in demonstrating the convergence of neural weights.

Lemma 1 ([34]): Suppose that the union orbit $\varphi(t) = \cup_1^L \varphi_i(t)$ are periodic. Denote $\mathcal{I} = [t_0, t_0+T_0]$ be the bounded μ-measurable subsets of $[0, \infty)$, where T_0 is the period of $\varphi(t)$. Then $S_\varsigma(\varphi_i(t))$ $(i = 1, \cdots, L)$ satisfy cooperative PE condition, where $(\cdot)_\varsigma$ denotes close to the union orbit $\varphi(t)$.

Lemma 2 ([4]): There exist any positive constant k_e, let $E_1 := \{\hat{e}_1 \in R : |\hat{e}_1| < k_e\}$ and $\mathcal{N} := R^l \times E_1 \subset R^{l+1}$ be open sets. Consider the following system

$$\dot{\theta} = h(t, \theta), \tag{4}$$

where $\theta := [w, \hat{e}_1]^T \in \mathcal{N}$ is the state, and $h : R_+ \times \mathcal{N} \to R^{l+1}$ is piecewise continuous in t and satisfies locally Lipschitz in $\hat{e}_1$, uniformly in t, on $R_+ \times \mathcal{N}$. Suppose that there exist positive definite functions $U : R^l \to R_+$ and $V_1 : E_1 \to R_+$ that satisfy continuous differentiability in their respective domains, such that

$$V_1(\hat{e}_1) \to \infty \;\; as \;\; |\hat{e}_1| \to k_{b_1}, \tag{5}$$
$$\eta_1(\|w\|) \le U(w) \le \eta_2(\|w\|), \tag{6}$$

where η_1 and η_2 are class K_∞ functions. Let $V(\theta) := V_1(\hat{e}_1) + U(w)$, and $\hat{e}_1(0) \in E_1$. If the inequality holds

$$\dot{V} = \frac{\partial V}{\partial \theta} \le -\nu_1' V + \nu_2' \le 0, \tag{7}$$

where ν_1' and ν_2' are both positive constants, and $\hat{e}_1(t)$ remains in the open set $E_1, \forall t \ge 0$.

Lemma 3 ([6]): If $\hat{e}_1 \in (-k_e, k_e)$, then the inequality holds as follows

$$\log \frac{k_e^2}{k_e^2 - \hat{e}_1^2} < \frac{\hat{e}_1^2}{k_e^2 - \hat{e}_1^2}. \tag{8}$$

III. DYNAMIC COOPERATIVE LEARNING FROM OUTPUT FEEDBACK CONTROL

The controller construction requires full states. However, only the output of (1) is available for measurement. Thus we need to use this known information to estimate other states. Therefore, a high-gain observer (HGO) is used to estimate the states $x_{i,2}, \cdots, x_{i,m}$ of (1).

Lemma 4 ([45]): Consider the following linear systems:

$$\begin{cases} \epsilon_i \dot{z}_{i,k} = z_{i,k+1}, \quad k = 1, \cdots, m-1, \\ \epsilon_i \dot{z}_{i,m} = -\lambda_{i,1} z_{i,m} - \lambda_{i,2} z_{i,m-1} - \cdots - \lambda_{i,m-1} z_{i,2} \\ -z_{i,1} + y_i(t), \quad i = 1, \cdots, L, \end{cases} \tag{9}$$

where ϵ_i is any small positive constant and $z_{i,k}, k = 1, \cdots, m$ is the state. $y_i(t)$ and its first m derivatives are assumed to be bounded. Choose the parameters $\lambda_{i,1}$ to $\lambda_{i,m-1}$ such that the polynomial $s^m + \lambda_{i,1} s^{m-1} + \cdots + \lambda_{i,m-1} s + 1$ is Hurwitz. Then, there exist positive constants $h_{l+1}, l = 1, \cdots, m-1$, and t^* such that for all $t > t^*$,

(1) $\frac{z_{i,k+1}}{\epsilon_i^l} - y_i^{(l)} = -\epsilon_i \psi_i^{(l+1)} \quad l = 1, \cdots, m-1,$

(2) $|\frac{z_{i,k+1}}{\epsilon_i^l} - y_i^{(l)}| \le \epsilon_i h_{l+1} \quad l = 1, \cdots, m-1,$

where $\psi_i = z_{i,m} + \lambda_{i,1} z_{i,m-1} + \cdots + \lambda_{i,m-1} z_{i,1}$. $\psi_i^{(l)}$ is the lth derivative of ψ_i, and $|\psi_i^{(l)}| \le h_{l+1}$.

According to the property of the observer (9), we can obtain accurate estimation of system state by setting ϵ_i to a smaller value. Therefore, this observer is suitable for estimating unavailable states of the system (1). To clarify further, define the state estimation as

$$\hat{x}_i = [\hat{x}_{i,1}, \cdots, \hat{x}_{i,m}]^T = [x_{i,1}, \frac{z_{i,2}}{\epsilon_i}, \frac{z_{i,3}}{\epsilon_i^2}, \cdots, \frac{z_{i,m}}{\epsilon_i^{m-1}}]^T. \tag{10}$$

The unknown function $f(x_i)$ can be approximated by the RBF NN, that is,

$$f(x_i) = W^T S(\hat{x}_i) + \varepsilon_i, \tag{11}$$

where W is the optimal weight for all agents and $|\varepsilon_i| < \varepsilon$ is NN approximation error. Since the structure of all agents is identical, the final optimal constant weights obtained are also the same but they are unknown before dynamic cooperative learning process.

Denote $\hat{W}_i$ as the estimate of W for the agent i and let $\tilde{W}_i = \hat{W}_i - W$. Then, we design the feedback controller for the agent i as

$$u_i = -\hat{e}_{i,m-1} - c_{i,m}\hat{e}_{i,m} - \hat{W}_i^T S(\hat{x}_i) + \dot{\alpha}_{i,m-1}, \tag{12}$$

where

$$\hat{e}_{i,1} = \hat{x}_{i,1} - y_{r_i}, \tag{13}$$
$$\hat{e}_{i,j} = \hat{x}_{i,j} - \alpha_{i,j-1}, \quad j = 2, \cdots, m, \tag{14}$$
$$\alpha_{i,1} = y_{i,2} - c_{i,1}\hat{e}_{i,1}, \tag{15}$$
$$\alpha_{i,2} = \dot{\alpha}_{i,1} - c_{i,2}\hat{e}_{i,2} - \frac{\hat{e}_{i,1}}{k_{e_1}^2 - \hat{e}_{i,1}^2}, \tag{16}$$
$$\alpha_{i,k} = \dot{\alpha}_{i,k-1} - \hat{e}_{i,k-1} - c_{i,k}\hat{e}_{i,k}, \quad k = 3, \cdots, m, \tag{17}$$

with $c_{i,1}, c_{i,2}, \cdots, c_{i,m} > 0$ being control gains. From [46], $\dot{\alpha}_{i,1}, \dot{\alpha}_{i,2}, \dot{\alpha}_{i,k}$ are given by

$$\dot{\alpha}_{i,1} = -c_{i,1}(-c_{i,1}\hat{e}_{i,1} + \hat{e}_{i,2}) + x_{r_{i,3}}, \tag{18}$$

$$\dot{\alpha}_{i,2} = -\phi(\hat{e}_{i,1})^{(1)} - c_{i,2}(-\phi(\hat{e}_{i,1}) - c_{i,2}\hat{e}_{i,2} + \hat{e}_{i,3}) + \alpha_{i,1}^{(2)}, \tag{19}$$

$$\dot{\alpha}_{i,k} = -\phi(\hat{e}_{i,1})^{(k-1)} - \sum_{a=2}^{k-1} \hat{e}_{i,a}^{(k-a)} - \sum_{b=1}^{k-1} c_{i,b+1}\hat{e}_{i,b+1}^{(k-b)} + \alpha_{i,1}^{(k)}, \quad k = 3, \cdots, m-1, \tag{20}$$

where $\phi(\hat{e}_{i,1}) = \frac{\hat{e}_{i,1}}{k_{e_1}^2 - \hat{e}_{i,1}^2}$ and $(\cdot)^{(n)}$ denotes the nth derivative of $(\cdot)$.

In order to exchange information among the agents, we construct the cooperative NN weight update law as

$$\dot{\hat{W}}_i = \gamma[S(\hat{x}_i)\hat{e}_{i,m} - \sigma_i \hat{W}_i] - \mu \sum_{s\in\mathcal{N}_i} a_{is}(\hat{W}_i - \hat{W}_s), \tag{21}$$

where $i \in \{1, 2, \cdots, L\}$, and $\gamma, \mu, \sigma_i > 0$ are design parameters and $\mathcal{N}_i$ is the set of neighbouring agents of the agent i. Further, we have

$$\mu(\mathcal{L} \otimes I_l)\tilde{W} = \begin{bmatrix} \mu \sum_{j\in\mathcal{N}_1} a_{1s}(\tilde{W}_1 - \tilde{W}_s) \\ \vdots \\ \mu \sum_{j\in\mathcal{N}_L} a_{Ls}(\tilde{W}_L - \tilde{W}_s) \end{bmatrix}, \tag{22}$$

where $\tilde{W} = [\tilde{W}_1^T, \cdots, \tilde{W}_L^T]^T$.

Assumption 2: All the states in the reference model (2) are bounded.

Assumption 3: The communication topologies among all agents is undirected and connected.

Remark 1: Under Assumption 3, each agent can share the learned knowledge with neighborhood agents due to the consensus term $\mu \sum_{s\in\mathcal{N}_i} a_{is}(\hat{W}_i - \hat{W}_s)$ introduced in the weight update law (21). Note that when $a_{is} > 0$, the agents are able to obtain the NN weights of their neighborhood agents, which is called DCL. But when $a_{is} = 0$, it means that each agent learns independently, rather than learns cooperatively, which is called decentralized learning (DL). Thus, compared with the DL control approach, using the DCL control approach such that RBF NNs have better generalization capabilities after completing the cooperative learning.

Theorem 1: Under Assumptions 2 and 3, consider a multi-agent system consisting of L subsystems (1), the reference models (2), the HGOs (9), the feedback controllers (12) and cooperative weight update laws (21). For any initial condition $\hat{e}_{i1}(t)(0) \in \Omega_{\hat{e}}^0$ (where $\Omega_{\hat{e}}^0$ is a compact set) and $\hat{W}_i(0) = 0$, if all reference orbits are periodic, then we have

(i) all the signals in the closed-loop system are bounded;
(ii) the output constraints are never violated;
(iii) the outputs track their reference signals with the errors by the proper selection of design parameters;
(iiii) along the union orbit $\varphi_r = \cup_{i=1}^{L}\varphi_{r_i}$, the estimated weights $\hat{W}_i$, $i = 1, \cdots, L$, converge to small neighborhoods of their optimal weight W, and L approximations of $f(x_i)$ are obtained by $\overline{W}_i^T S(\hat{x}_i)$, where $\overline{W}_i = mean_{t\in[t_a,t_b]}\hat{W}_i$ represents the average value on a time segment and $[t_a, t_b](t_b > t_a > T)$ stands for a time segment after the transient process.

Proof :(i) Based on (12)–(21), the closed-loop error signals are written as

$$\begin{cases} \dot{\hat{e}}_{i,1} = -c_{i,1}\hat{e}_{i,1} + \hat{e}_{i,2}, \\ \dot{\hat{e}}_{i,2} = -\dfrac{\hat{e}_{i,1}}{k_{e_i}^2 - \hat{e}_{i,1}^2} - c_{i,2}\hat{e}_{i,2} + \hat{e}_{i,3}, \\ \dot{\hat{e}}_{i,k} = -\hat{e}_{i,k-1} - c_{i,k}\hat{e}_{i,k} + \hat{e}_{i,k+1}, \quad k = 3, \cdots, m-1, \\ \dot{\hat{e}}_{i,m} = -\hat{e}_{i,m-1} - c_{i,m}\hat{e}_{i,m} - \tilde{W}_i^T S(\hat{x}_i) + \epsilon_i, \\ \dot{\hat{W}}_i = \gamma[S(\hat{x}_i)\hat{e}_{i,m} - \sigma_i\hat{W}_i] - \mu \sum_{s\in\mathcal{N}_i} a_{is}(\hat{W}_i - \hat{W}_s). \end{cases} \tag{23}$$

Consider the following BLF candidate

$$V = \frac{1}{2}\sum_{i=1}^{L} \log \frac{k_{e_i}^2}{k_{e_i}^2 - \hat{e}_{i,1}^2} + \frac{1}{2}\sum_{i=1}^{L}\sum_{k=2}^{m} \hat{e}_{i,k}^2 + \frac{1}{2\gamma}\sum_{i=1}^{L} \tilde{W}_i^T \tilde{W}_i, \tag{24}$$

where k_{e_i} denotes the constraint on $\hat{e}_{i,1}$, i.e., $|\hat{e}_{i,1}| < k_{e_i}$. Then, its derivative along (23) is

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{L} \frac{\hat{e}_{i,1}\dot{\hat{e}}_{i,1}}{k_{e_i}^2 - \hat{e}_{i,1}^2} + \sum_{i=1}^{L}\sum_{k=2}^{m} \hat{e}_{i,k}\dot{\hat{e}}_{i,k} + \frac{1}{\gamma}\sum_{i=1}^{L} \tilde{W}_i^T \dot{\hat{W}}_i \\ &= -\sum_{i=1}^{L}\left(\frac{c_{i1}\hat{e}_{i,1}^2}{k_{e_i}^2 - \hat{e}_{i,1}^2} - \hat{e}_{i,m}\varepsilon_i\right) - \sum_{i=1}^{L}\sum_{k=2}^{m} c_{i,k}\hat{e}_{i,k}^2 \\ &\quad - \sum_{i=1}^{L} \sigma_i \tilde{W}_i^T \hat{W}_i - \frac{\mu}{\gamma}\tilde{W}^T(\mathcal{L} \otimes I_l)\tilde{W}. \end{aligned} \tag{25}$$

Since the following appropriate inequalities hold

$$\begin{cases} \sum_{i=1}^{L} \hat{e}_{i,m}\varepsilon_i \leq \sum_{i=1}^{L} \dfrac{c_{i,m}}{2}\hat{e}_{i,m}^2 + \sum_{i=1}^{L} \dfrac{1}{2c_{i,m}}|\varepsilon|^2, \\ -\sum_{i=1}^{L} \sigma_i \tilde{W}_i^T \hat{W}_i \leq -\sum_{i=1}^{L} \dfrac{\sigma}{2}\tilde{W}_i^T \tilde{W}_i + \sum_{i=1}^{L} \dfrac{\sigma_i}{2}\|W\|^2, \end{cases} \tag{26}$$

where $\sigma = \min\{\sigma_1, \cdots, \sigma_L\}$ and the term $\frac{\mu}{\gamma}\tilde{W}^T(\mathcal{L} \otimes I_l)\tilde{W}$ of (25) always remains positive under Assumption 3.

Then, by using (26) and Lemma 3, (25) becomes

$$\dot{V} \leq -\sum_{i=1}^{L} c_{i,1} \log \frac{k_{e_1}^2}{k_{e_1}^2 - \hat{e}_{i,1}^2} - \sum_{i=1}^{L} c_{i,k} \sum_{k=2}^{m-1} \hat{e}_{i,k}^2 - \sum_{i=1}^{L} \frac{c_{i,m}}{2} \hat{e}_{i,m}^2 - \sum_{i=1}^{L} \frac{\sigma}{2} \tilde{W}_i^T \tilde{W}_i + \sum_{i=1}^{L} \frac{1}{2c_{i,m}} |\varepsilon|^2 + \sum_{i=1}^{L} \frac{\sigma_i}{2} \|W\|^2. \quad (27)$$

Choose $\nu_1' = \min\{2c_{i,k}, c_{i,m}, \gamma\sigma\}$, $i = 1, \cdots, L$, $k = 1, \cdots, m-1$, (27) is rewritten as

$$\dot{V} \leq -\nu_1' V + \nu_2' \quad (28)$$

where $\nu_2' = \sum_{i=1}^{L}(\frac{1}{2c_{i,m}}|\varepsilon|^2 + \frac{\sigma_i}{2}\|W\|^2)$. Furthermore, one has

$$V(t) \leq V(0)e^{-\nu_1' t} + \frac{\nu_2'}{\nu_1'}. \quad (29)$$

According to (29), we have that $\hat{e}_{i,k}$, $\tilde{W}_i$, $i = 1, \cdots, L$, $k = 1, \cdots, m$ are bounded. Recursively, $\hat{W}_i$, $i = 1, \cdots, L$, are also bounded due to the boundedness of $\tilde{W}_i$, $i = 1, \cdots, L$. Further, $\hat{x}_{i,k}, x_{i,k}, i = 1, \cdots, L, k = 1, \cdots, m$, always remain bounded. Therefore, all the closed-loop signals are bounded.

(ii)According to Lemma 2 and (29), it follows that $|\hat{e}_{i,1}(t)| < k_{e_i}$, $\forall t > 0$. From $y_i(t) = \hat{e}_{i,1}(t) + y_{r_i}(t)$ and $|y_{r_i}(t)| < D_{0_i}$, it is straightforward to obtain that

$$|y_i(t)| < k_{e_i} + D_{0_i} = k_{y_i}. \quad (30)$$

As such, for all $t \in [0, \infty)$, we infer that the outputs are restricted.

(iii)Consider the following BLF candidate

$$V_1 = \frac{1}{2} \sum_{i=1}^{L} \log \frac{k_{e_i}^2}{k_{e_i}^2 - \hat{e}_{i,1}^2} + \frac{1}{2} \sum_{i=1}^{L} \sum_{k=2}^{m} e_{i,k}^2. \quad (31)$$

The derivative of V_1 is given by

$$\dot{V}_1 = \sum_{i=1}^{L} \frac{\hat{e}_{i,1}\dot{\hat{e}}_{i,1}}{k_{e_i}^2 - \hat{e}_{i,1}^2} + \sum_{i=1}^{L} \sum_{k=2}^{m} \hat{e}_{i,k}\dot{\hat{e}}_{i,k} = -\sum_{i=1}^{L} \frac{c_{i1}\hat{e}_{i,1}^2}{k_{e_i}^2 - \hat{e}_{i,1}^2} - \sum_{i=1}^{L} \sum_{k=2}^{m} c_{i,k}\hat{e}_{i,k}^2 + \sum_{i=1}^{L} \hat{e}_{i,m}\varepsilon_i. \quad (32)$$

According to (26) and Lemma 3, (32) becomes

$$\dot{V}_1 \leq -\sum_{i=1}^{L} c_{i,1} \log \frac{k_{e_1}^2}{k_{e_1}^2 - \hat{e}_{i,1}^2} - \sum_{i=1}^{L} c_{i,k} \sum_{k=2}^{m-1} \hat{e}_{i,k}^2 - \sum_{i=1}^{L} \frac{c_{i,m}}{2} \hat{e}_{i,m}^2 + \sum_{i=1}^{L} \frac{1}{2c_{i,m}} |\varepsilon|^2 \leq -\beta_1 V_1 + \beta_2, \quad (33)$$

where $\beta_1 = \min\{2c_{i,k}, c_{i,m}\}$, $i = 1, \cdots, L$, $k = 1, \cdots, m-1$ and $\beta_2 = \sum_{i=1}^{L} \frac{1}{2c_{i,m}} |\varepsilon|^2$. Then (31) satisfies

$$V_1(t) \leq V_1(0)e^{-\beta_1 t} + \frac{\beta_2}{\beta_1}. \quad (34)$$

In view of (34), if choose large $c_{i,m}$, the term $\frac{\beta_2}{\beta_1}$ can be made small enough. Thus the output tracking errors $\hat{e}_{i,1}$, $i = 1, \cdots, L$, will asymptotically converge to small neighborhoods of zero. This indicates that $\hat{x}_{i,k}$, $i = 1, \cdots, L$, $k = 1, \cdots, m$, can track the reference signals with very small errors.

(iiii)Based on the local property of RBF NNs [19], along the union orbit $\varphi_r = \cup_{i=1}^{L} \varphi_{r_i}$, (23) is rewritten as

$$\begin{cases} \dot{\hat{e}}_{i,m} = -\hat{e}_{i,m-1} - c_{i,m}\hat{e}_{i,m} - \tilde{W}_{i_\varsigma}^T S_\varsigma(\hat{x}_i) + \varepsilon_{i_\varsigma}, \\ \dot{\hat{W}}_{i_\varsigma} = \gamma[S_\varsigma(\hat{x}_i)\hat{e}_{im} - \sigma_i \hat{W}_{i_\varsigma}] - \mu \sum_{s\in\mathcal{N}_i} a_{is}(\hat{W}_{i_\varsigma} - \hat{W}_{s_\varsigma}), \end{cases} \quad (35)$$

where $\varepsilon_{i_\varsigma} = \varepsilon_i - \tilde{W}_{i_{\bar{\varsigma}}}^T S_{\bar{\varsigma}}(\hat{x}_i)$, $(\cdot)_\varsigma$ and $(\cdot)_{\bar{\varsigma}}$ denote the parts that are close to and far away from the union orbit φ_r, respectively. For neurons with their centers far away from the orbit φ_r, we have

$$\dot{\hat{W}}_{i_{\bar{\varsigma}}} = \gamma[S_{\bar{\varsigma}}(\hat{x}_i)\hat{e}_{i,m} - \sigma_i \hat{W}_{i_{\bar{\varsigma}}}] - \mu \sum_{s\in\mathcal{N}_i} a_{is}(\hat{W}_{i_{\bar{\varsigma}}} - \hat{W}_{s_{\bar{\varsigma}}}). \quad (36)$$

Therefore, $S_{\bar{\varsigma}}(\hat{x}_i)$ is very small. $\hat{W}_{i_{\bar{\varsigma}}}$ is marginally updated due to $\hat{W}_i(0) = 0$, and remains very small. Note that $\tilde{W}_{i_{\bar{\varsigma}}} = \hat{W}_{i_{\bar{\varsigma}}} - W_{\bar{\varsigma}}$, which means that $\tilde{W}_{i_{\bar{\varsigma}}}^T S_{\bar{\varsigma}}(\hat{x}_i)$ is also very small such that $\varepsilon_{i_\varsigma} = O(\epsilon_i)$.

In view of (35), the overall closed-loop systems can be simplified as follows

$$\begin{bmatrix} \dot{\hat{e}}_m \\ \dot{\tilde{W}}_\varsigma \end{bmatrix} = \begin{bmatrix} A & -\Psi(\hat{x})^T \\ \gamma\Psi(\hat{x}) & -\mu(\mathcal{L}\otimes I_{l_\varsigma}) \end{bmatrix} \begin{bmatrix} \hat{e}_m \\ \tilde{W}_\varsigma \end{bmatrix} + \begin{bmatrix} \bar{\varepsilon}_\varsigma - \hat{e}_{m-1} \\ -\lambda\hat{W}_\varsigma \end{bmatrix}, \quad (37)$$

where $\hat{e}_m = [\hat{e}_{1,m}, \cdots, \hat{e}_{L,m}]^T$, $\tilde{W}_\varsigma = [\tilde{W}_{1_\varsigma}^T, \cdots, \tilde{W}_{N_\varsigma}^T]^T$, $\bar{\varepsilon}_\varsigma = [\varepsilon_{1_\varsigma}, \cdots, \varepsilon_{L_\varsigma}]^T$, $A = diag\{-c_{1,m}, \cdots, -c_{L,m}\}$, $\Psi(\hat{x}) = diag\{S_\varsigma(\hat{x}_1), \cdots, S_\varsigma(\hat{x}_L)\}$, $\lambda = diag\{\gamma\sigma_1 I_{l_\varsigma}, \cdots, \gamma\sigma_N I_{l_\varsigma}\}$, $\hat{e}_{m-1} = [\hat{e}_{1,m-1}, \cdots, \hat{e}_{L,m-1}]^T$. Further, (37) is considered as a perturbation system [47].

According to the result in (iii), and noting that the trajectory of the reference model is periodic, it is directly concluded that $\hat{x}_{i,k}$, $i = 1, \cdots, L$, $k = 1, \cdots, m$, also become recurrent signals. Besides, based on Lemma 1, $S_\varsigma(\hat{x}_i)$, $i = 1, \cdots, L$ satisfy the cooperative PE condition. Since $\gamma A + A^T\gamma = diag\{-2\gamma c_{1,m}, \cdots, -2\gamma c_{L,m}\}$ is negative definite, then there exists a symmetric positive definite matrix $Q(t)$ that makes

$$\dot{P}(t) + P(t)A(t) + A^T(t)P(t) = -Q(t),$$

where $Q(t) = -\gamma[A(t) + A^T(t)]$.

The convergence of $\hat{e}_{i,m-1}$, $i = 1, \cdots, L$, guarantee that $\bar{\varepsilon}_\varsigma - \hat{e}_{m-1}$ is very small. Since σ_i can be designed small enough and $\hat{W}_{i_\varsigma}$ is boundedness, then $\lambda\hat{W}_\varsigma$ is also very small. Based on Lemma 9.2 of [47], the solution of (37) converges to a small neighborhood of origin. This means that $\hat{W}_i$, $i = 1, \cdots, L$, converge to small neighborhoods of their optimal weight W, that is, $\overline{W}_1 \cong \cdots \cong \overline{W}_L$. In addition, along the union orbit φ_r, $f(\cdot)$ can be approximated by RBF NNs as follows

$$\begin{aligned} f(\varphi_r) &= W_\varsigma^T S_\varsigma(\varphi_r) + \varepsilon_\varsigma \\ &= \overline{W}_{i_\varsigma}^T S_\varsigma(\varphi_r) + \varepsilon_{i_{\varsigma 1}} \\ &= \overline{W}_{i_\varsigma}^T S_\varsigma(\varphi_r) + \overline{W}_{i_{\bar{\varsigma}}}^T S_{\bar{\varsigma}}(\varphi_r) + \varepsilon_{i_{\varsigma 1}} - \overline{W}_{i_{\bar{\varsigma}}}^T S_{\bar{\varsigma}}(\varphi_r) \\ &= \overline{W}_i^T S(\varphi_r) + \varepsilon_{i_2}, \end{aligned} \tag{38}$$

where $\varepsilon_{i_{\varsigma 1}} = \varepsilon_\varsigma - \tilde{W}_\varsigma^T S_\varsigma(\varphi_r)$ and $\varepsilon_{i_2} = \varepsilon_{i_{\varsigma 1}} - \overline{W}_{i_{\bar{\varsigma}}}^T S_{\bar{\varsigma}}(\varphi_r)$ are NN approximation errors. Thus, L approximations of $f(\cdot)$ are obtained. The proof is completed. ■

Remark 2: According to (34), we know that the convergence rate and the tracking performance depend on $\beta_1 = \min\{2c_{i,k}, c_{i,m}\}$, $i = 1, \cdots, L$, $k = 1, \cdots, m-1$ and $\beta_2 = \sum_{i=1}^{L} \frac{1}{2c_{i,m}} |\varepsilon|^2$. Thus, we can appropriately choose large $c_{i,k}$ and $c_{i,m}$ to increase the convergence rate and reduce the tracking errors. σ_i is a modification coefficient, which is introduced to prevent the divergence of the neural weights. Based on (37), σ_i should be chosen small to guarantee the convergence of neural weights. For the other parameters γ and μ, according to Theorem 1, we can guarantee control performance and learning performance as long as $\gamma > 0$ and $\mu > 0$.

IV. CONTROL WITH EXPERIENCE

In the previous section, with the help of weight exchange among agents, the RBF NN approximations are obtained along the union orbit. For the same control systems, the obtained NNs will be directly used to design the controllers without recalculating the neural weights. Thus, the system performance can be improved. To show this, consider the same system as (1)

$$\begin{cases} \dot{x}_k = x_{k+1}, \quad k = 1, 2, \cdots, m-1, \\ \dot{x}_m = f(x) + u \\ y = x_1, \end{cases} \tag{39}$$

where $x = [x_1, \cdots, x_m]^T$ and u are the state and system input, respectively. y is the output, which is measurable. $x_2, \cdots, x_m$ are also not available.

Consider the reference model as follows

$$\begin{cases} \dot{x}_{r_k} = x_{r_{k+1}} \quad k = 1, 2, \cdots, m-1, \\ \dot{x}_{r_m} = g_r(x_r, t) \\ y_r = x_{r_1}, \end{cases} \tag{40}$$

where $x_{r_k} = [x_{r_1}, \cdots, x_{r_m}]^T$ is model bounded state. $g_r(x_r, t)$ is a known function. Let Υ be the orbit of (40). Then, a HGO (9) is employed to estimate the states x_k, $k = 2, \cdots, m$ of (39). Denote the state variables as $\hat{\mathcal{X}} = [x_1, \hat{x}_2, \cdots, \hat{x}_m]^T$.

According to (13)–(20), and using the obtained NNs, the experience-based controller is given by

$$u = -\hat{e}_{m-1} - c_m \hat{e}_m - \overline{W}_i^T S(\hat{\mathcal{X}}) + \dot{\alpha}_{m-1}, \tag{41}$$

where $\overline{W}_i^T S(\hat{\mathcal{X}})$ is the obtained NN in Theorem 1.

Theorem 2: Consider a control system consisting of the plant (39), the reference model (40), the HGO (9) and the experience-based controller (41). For a reference orbit Υ that is contained in the union orbit $\varphi_r = \cup_{i=1}^{L} \varphi_{r_i}$, and initial condition $\hat{e}_1(t)(0) \in \Omega_{\hat{e}}^0$ (where $\Omega_{\hat{e}}^0$ is a compact set), then we have

(i) all the signals in the closed-loop system are bounded;
(ii) the output constraint is never violated;
(iii) the output y track its reference signal with a very small error.

Proof :The main line is similar to the proof in Theorem 1.

(i) From (23), we deduce the derivatives of errors $\hat{e}_1, \cdots, \hat{e}_m$ are that

$$\begin{cases} \dot{\hat{e}}_1 = -c_1 \hat{e}_1 + \hat{e}_2, \\ \dot{\hat{e}}_2 = -\dfrac{\hat{e}_1}{k_e^2 - \hat{e}_1^2} - c_2 \hat{e}_2 + \hat{e}_3, \\ \dot{\hat{e}}_k = -\hat{e}_{k-1} - c_k \hat{e}_k + \hat{e}_{k+1}, \quad k = 3, \cdots, m-1, \\ \dot{\hat{e}}_m = -\hat{e}_{m-1} - c_m \hat{e}_m - \overline{W}_i^T S(\hat{\mathcal{X}}) + f(x). \end{cases} \tag{42}$$

Consider the following BLF candidate

$$V_1 = \frac{1}{2} \log \frac{k_e^2}{k_e^2 - \hat{e}_1^2} + \frac{1}{2} \sum_{k=2}^{m} \hat{e}_k^2. \tag{43}$$

Then, its derivative along (42) is

$$\dot{V}_1 = -\frac{c_1 \hat{e}_1^2}{k_e^2 - \hat{e}_1^2} - \sum_{k=2}^{m} c_k \hat{e}_k^2 - \hat{e}_m (\overline{W}_i^T S(\hat{\mathcal{X}}) - f(x)). \tag{44}$$

Using the following inequality

$$-\frac{1}{2} c_m \hat{e}_m^2 - \hat{e}_m (\overline{W}_i^T S(\hat{\mathcal{X}}) - f(x)) \leq \frac{|\overline{W}_i^T S(\hat{\mathcal{X}}) - f(x)|^2}{2c_m}, \tag{45}$$

we have

$$\begin{aligned} \dot{V}_1 \leq & -\frac{c_1 \hat{e}_1^2}{k_e^2 - \hat{e}_1^2} - \sum_{k=2}^{m-1} c_k \hat{e}_k^2 - \frac{1}{2} c_m \hat{e}_m^2 \\ & + \frac{|\overline{W}_i^T S(\hat{\mathcal{X}}) - f(x)|^2}{2c_m}. \end{aligned} \tag{46}$$

For $|e| < d_e^*$, where $e = [e_1, \cdots, e_m]^T$, along the orbit Υ, we arrive at

$$|\overline{W}_i^T S(\hat{\mathcal{X}}) - f(x)| \leq \varepsilon_m^*. \tag{47}$$

Combined with (46), (47) and Lemma 3, we have

$$\dot{V}_1 \leq -c_1 \log \frac{k_e^2}{k_e^2 - \hat{e}_1^2} - \sum_{k=2}^{m-1} c_k \hat{e}_k^2 - \frac{1}{2} c_m \hat{e}_m^2 + \frac{\varepsilon_m^{*2}}{2c_m}. \tag{48}$$

Let $c^* \leq \frac{1}{2}c_m$, and then (43) becomes

$$V_1(t) \leq \delta' + V_1(0)e^{-2c^*t}, \tag{49}$$

where $\delta' = \frac{\epsilon_m^{*2}}{4c_k c_m}$. Since

$$\hat{e}(0) \in \Omega_{\hat{e}}^0 = \{\hat{e} | V_1 \leq \frac{1}{2}d_e^{*2} - \delta'\}, \tag{50}$$

then we obtain

$$V_1(t) \leq \frac{1}{2}d_e^{*2}. \tag{51}$$

Thus, all the closed-loop signals are bounded.

(ii)Based on the above analysis and Lemma 1, we directly obtain that the output is constrained.

(iii)From (49), the errors $\hat{e}_1, \cdots, \hat{e}_m$ exponentially converge to small neighborhoods of zero by setting c_m large enough. This implies that the output y of (39) tracks its reference trajectory with a very small error. The proof is completed. ∎

Remark 3: Up to now, numerous works have been reported on adaptive NN control methods under various constraints, including output constraints [7], full-state constraints [16], input saturation [17], just to name a few. However, these methods [7], [16], [17] focus on the universal approximation property of NNs. Compared with these existing results [7], [16], [17], this paper further considers the learning capability of NNs. The proposed BLF-based dynamic cooperative learning control scheme achieves that the neural weights of all subsystems with output constraints converge to their common optimal values, and the obtained weights $\overline{W}_i, i = 1, \cdots, L$ can be recycled for controller design (41) of the same control system to improve control performance and reduce computation. Meanwhile, the output constraints are also guaranteed. In addition, since $\overline{W}_i, i = 1, \cdots, L$ are obtained along the union orbit $\varphi_r = \cup_{i=1}^{L} \varphi_{r_i}$, the approximation domain of NNs is extended.

Remark 4: In this paper, we first design the controller and the neural weight update law for the multiagent system. Then, we give the detailed proof for all conclusions in Theorem 1 and Theorem 2. This means that all conclusions are feasible as long as the selection of parameters follows the instructions in this paper. Thus, the proposed scheme in this paper is conservative and rational although the different selections of parameters affect control and learning performance. In Remark 2, we provide appropriate advises to choose parameters.

Remark 5: According to (37) and (38), we obtain the weights of the neural networks, and store them as constants in the RBF NNs, that is, $\overline{W}_i^T S(\hat{\mathcal{X}})$. Using this learned knowledge, we design the experience-based controller (41) without recalculating the neural weights for the same control tasks. Thus, the online computation is reduced.

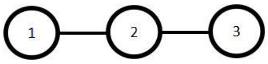

FIGURE 1. Communication topology of the three agents.

V. SIMULATION

Consider the following three systems [36]

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \dot{x}_{i,2} = x_{i,1}x_{i,2}e^{-x_{i,1}^2} + u_i \\ y_i = x_{i,1}, \quad i = 1, 2, 3, \end{cases} \tag{52}$$

where $x_{i,1}$ and $x_{i,2}$ are the system states, and y_i is the output. Assume that $x_{i,2}$, $i = 1, 2, 3$, and $f(x_i) = x_{i,1}x_{i,2}e^{-x_{i,1}^2}$ are totally unknown, and only $y_i, i = 1, 2, 3$, are measurable. The initial state values of (52) are $x_1(0) = [0.77, 0.7]^T$, $x_2(0) = [0.87, 0.8]^T$ and $x_3(0) = [0.97, 0.9]^T$.

Since only $y_i, i = 1, 2, 3$, are measurable, then we use three HGOs to estimate other unmeasurable states of (52)

$$\begin{cases} \epsilon_i \dot{z}_{i,1} = z_{i,2} \\ \epsilon_i \dot{z}_{i,2} = -\lambda_{i,1}z_{i,2} - z_{i,1} + y_i(t), \end{cases} \tag{53}$$

where $\epsilon_1 = \epsilon_2 = \epsilon_3 = 0.002$, $\lambda_{1,1} = \lambda_{2,1} = \lambda_{3,1} = 1$. The communication topology among three systems is shown in Fig. 1.

Three Duffing oscillators [19] are taken as the reference models

$$\begin{cases} \dot{x}_{r_{i,1}} = x_{r_{i,2}} \\ \dot{x}_{r_{i,2}} = -p_{i,1}x_{r_{i,1}} - p_{i,2}x_{r_{i,1}}^3 - p_{i,3}x_{r_{i,2}} + q_i\cos(\omega_i t) \\ y_{r_i} = x_{r_{i,1}}, \quad i = 1, 2, 3, \end{cases} \tag{54}$$

where $x_{r_{i,1}}$ and $x_{r_{i,2}}$ are the states, y_{r_i} is the output, $[p_{1,1}, p_{2,1}, p_{3,1}] = [1.3, 1, 0.8]$, $p_{1,2} = p_{2,2} = p_{3,2} = 0.7$, $[p_{1,3}, p_{2,3}, p_{3,3}] = [0.6, 0.5, 0.4]$, $[q_1, q_2, q_3] = [0.93, 0.75, 0.6]$, and $\omega_1 = \omega_2 = \omega_3 = 1.8$. The initial state values of (54) are $x_{r_1}(0) = [0.5, 0.2]^T$, $x_{r_2}(0) = [0.6, 0.4]^T$, $x_{r_3}(0) = [0.7, 0.6]^T$.

In this simulation, we construct each RBF NN $\hat{W}_i^T S(Z)$ with 441 neurons, $i \in \{1, 2, 3\}$. Their centers are evenly spaced on $[-1.2, 1.2]\times[-1.2, 1.2]$ and width is $\pi = 0.2$. The design parameters are $\gamma = 40$, $\mu = 1$, $k_{e_1} = k_{e_2} = k_{e_3} = 0.3$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.00001$, $c_{1,1} = c_{2,1} = c_{3,1} = 1$, and $c_{1,2} = c_{2,2} = c_{3,2} = 5$.

Simulation results for the adaptive cooperative control with output constraints are given in Fig. 2–5. Fig. 2 shows the output tracking performance of three agents. Fig. 3 shows $f(\hat{x}_i)$, $1 = 1, 2, 3$ are accurately identified by RBF NNs with small errors. In Fig. 4, the output errors $\hat{e}_{i,1}, i = 1, 2, 3$, never violate the constraints $k_{e_i} = 0.3$, $i = 1, 2, 3$. The convergence of NN weights is shown in Fig. 5. For the purpose of comparison, we use the control method in [36], [37], and [38], in which the quadratic Lyapunov candidate function is used in design process, and other parameters are the same. It is clear to see that the output errors $\hat{e}_{i,1}, i = 1, 2, 3$ are not constrained

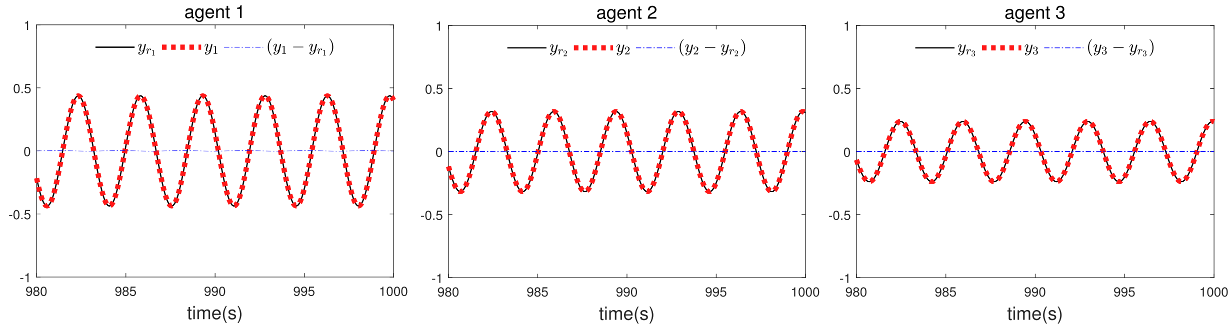

FIGURE 2. Output tracking performance under dynamic cooperative learning control.

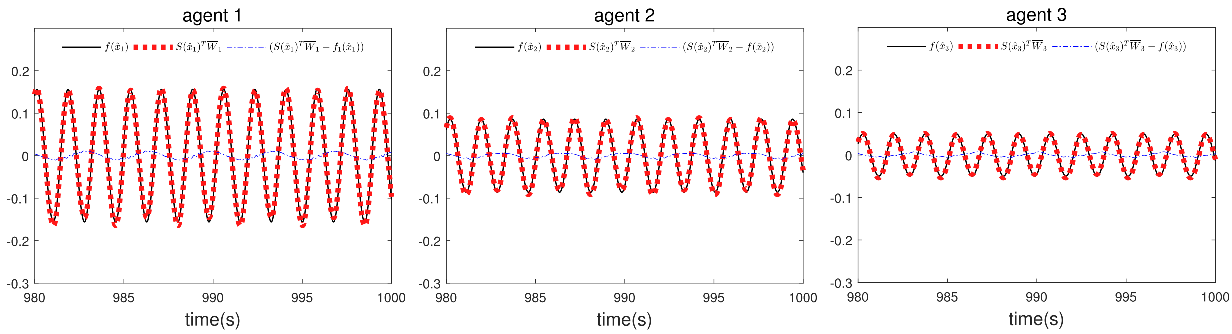

FIGURE 3. Approximation performance of RBF NNs under dynamic cooperative learning control.

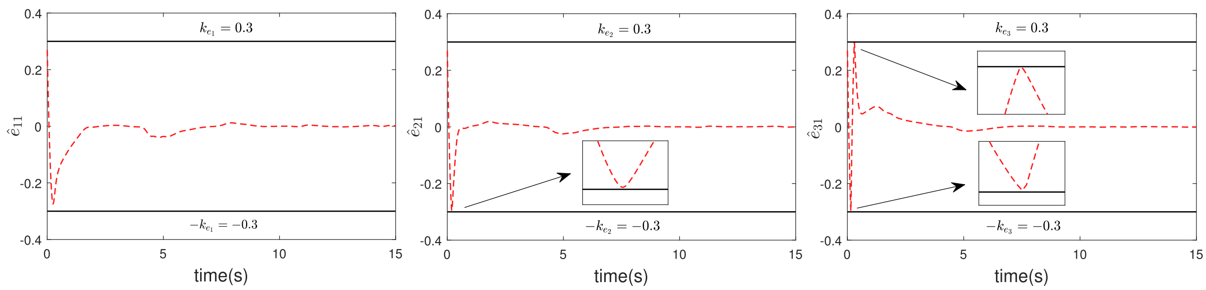

FIGURE 4. Constraint performance of output tracking errors $\hat{e}_{i,1}$ ($i = 1, 2, 3$) using the proposed approach (-).

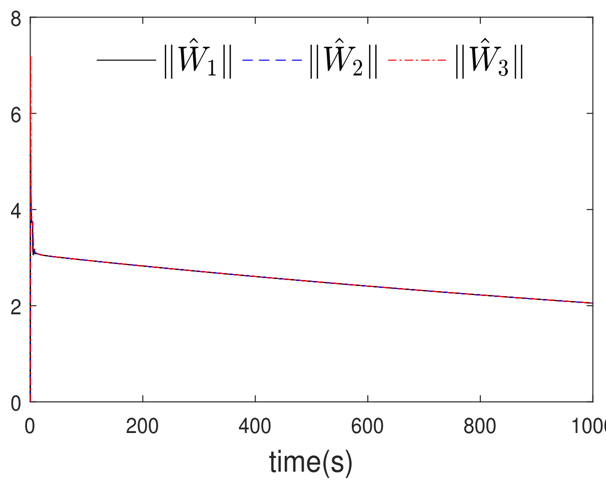

FIGURE 5. The convergence of NN weights.

in Fig. 6. As a result, the proposed approach guarantees that the output constraints are not violated.

Further, after completing the previous cooperative learning process, we employ obtained $\overline{W}_1^T S(Z)$ to design the experience-based controller for the same system to enhance its performance. we use the second of three systems (54) as the reference model

$$\begin{cases} \dot{x}_{r_1} = x_{r_2} \\ \dot{x}_{r_2} = -x_{r_1} - 0.7x_{r_1}^3 - 0.5x_{r_2} + 0.75\cos(1.8t) \\ y_r = x_{r_1}, \end{cases} \quad (55)$$

where $x_r(0) = [0.4, 0.4]^T$. In addition, we add the control performance using the methods in [36], [37], and [38], in which TQLFs are used in design process. The tracking error $\hat{e}_1$ is shown in Fig. 7. It indicates that our control performance is superior to the existing works [36], [37], [38].

Remark 6: Compared with traditional adaptive neural control schemes, the proposed approach not only achieves good tracking and constraint performance, as shown in Figs. 2 and 4, but also realizes the accurate approximation

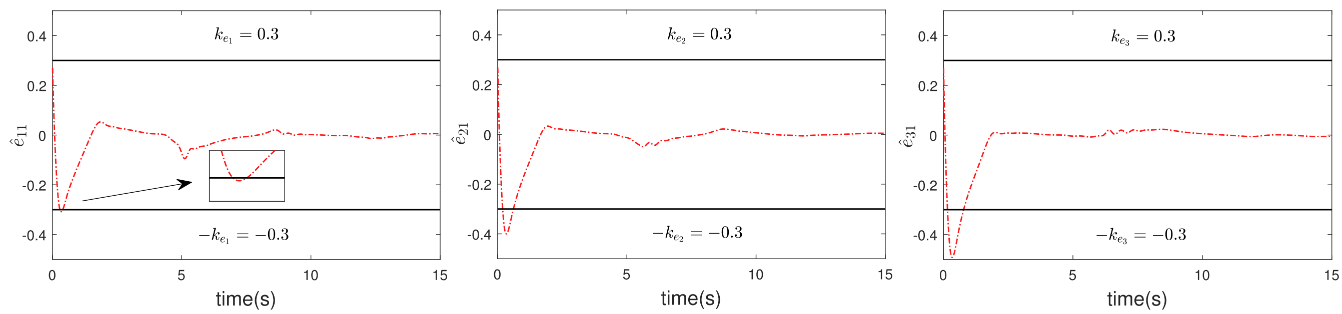

FIGURE 6. **Constraint performance of output tracking errors $\hat{e}_{i,1}$ ($i = 1, 2, 3$) using the approach in [36], [37], and [38] (-).**

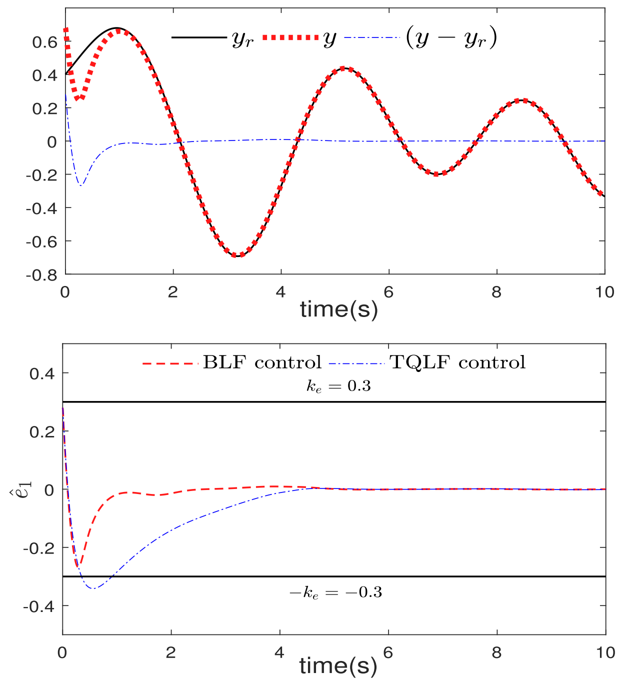

FIGURE 7. **Control performance of output tracking error $\hat{e}_1$ in this paper (- -) and (-.) [36], [37], [38] under the experience-based control approach.**

of the unknown function of all subsystems in the closed-loop control process, as shown in Fig. 3. From the results of dynamic cooperative learning, the control method proposed in this paper improves the system control performance, and maintains the constraint ability, as shown in Fig. 7. In Remark 3, we provide the advantages of the approach proposed in this paper compared with the existing constraint methods.

VI. CONCLUSION

This paper investigates the dynamic cooperative learning-based output feedback control for a group of output-constrained systems. For unmeasurable states, they are estimated by employing high-gain observers. The use of the barrier Lyapunov function in the controller design ensures that the output error is restricted within a required range. Thus, the output constraint is implemented. The proposed control approach obtains the neural approximation over the union orbit. Finally, the obtained neural networks are used in controller design for the same control tasks to enhance system performance. Meanwhile, the output constraints are also guaranteed. However, the communication of neural weights between agents is continuous in this paper, which causes the computation burden. In the future, we will explore the effective communication method to address this drawback.

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