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RESEARCH ARTICLE

Asymptotical Control Strategy for a Class of High-Order Nonlinear Systems With Multiple Uncertainties

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ABSTRACT This study explores the issue of adaptive stabilization for a class of high-order uncertain nonlinear systems with asymmetric output constraint and zero dynamics. By combining skillful Barrier Lyapunov Function (BLF) with the technique of continuous feedback domination equipped with a group of integral functions including nested sign functions, a continuous state feedback stabilizer is established, which guarantees that the closed-loop system's states converge to zero asymptotically while keeping the asymmetric output restriction inviolate. Superior to the existing methods, the developed strategy can not only simultaneously cope with the output asymmetric constraints and dynamic uncertainties, but also unifies the construction and theory analysis for the limited and unlimited output. At last, a numerical simulation is offered to verify the efficiency of the developed method.

INDEX TERMS High-order nonlinear systems, dynamic uncertainty, asymmetric output constraint, zero dynamics.

I. INTRODUCTION

It is well known that the issue of nonlinear systems' stabilization in the presence of parametric uncertainties is capable of being efficiently addressed via adaptive techniques [1], [2]. Although several approaches including the backstepping technique and feedback linearization have the potential to be employed to nonlinear systems' adaptive construction, they are not applicable to p-normal form systems owing to the intrinsic nonlinearities triggered by the Jacobian linearization's uncontrollability. Luckily, the idea of adding a power integrator originally developed by [3] and [4] created an important leap forward and further sparked numerous studies on high-order uncertain nonlinear systems's adaptive control, see, e.g., [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], and [15]. What matters is that intriguing solutions incorporating neural networks, fuzzy tools, homogeneous domination approach and filters [16], [17], [18], [19], are presented to handle more severe nonlinearities.

In addition to adaptive mechanisms for addressing uncertainties, the topic of zero dynamic (i.e., dynamic uncertainty) has also drawn a lot of focus [20], [21], [22], [23], [24]. As a matter of fact, control systems in real-world applications unavoidably consist of zero dynamics because of the restrictions of measurement equipment and techniques. With extra constraints placed on dynamic uncertainty, the small-gain theorem [20] and modifying supply functions [21] were capable of effectively overcoming this challenge. Particularly, the research [22] tackled a particular kind of tight feedback cascade systems' asymptotical stability working with the small-gain theorem and parameter division strategy. The same methodologies were put to use in [23] to explore a category of high-order nonlinear systems' finite-time stabilization with dynamic uncertainty. Furthermore,

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backstepping technique was applied in [24] to cope with the creation of adaptive robust controller for certain kinds of nonlinear systems with zero dynamic. Hence, it is regarded as critical to build a strategy to the stabilization issue for a category of high-order uncertain nonlinear systems equipped with both parameter uncertainties and dynamic uncertainty (i.e., zero dynamics), which constitute one of the motivations of this paper.

Remarkably, paying attention to transient actions of system states, especially, the system output throughout the stabilizing mission, also constitutes an essential and noteworthy topic [25], [26], [27], [28], [29], [30], [31], [32], since any break of the restrictions on system output may produce an undesirable inclination to degrade system efficiency or may be even to cause destabilization. For instance, the location of a maritime vessel is supposed to be confined by its maximum distance of trip [31]; the output of a adaptable crane system is required to be faced with an output limitation for guaranteeing security and reliability [32]. As a consequence, during the stabilizing process, the system output meeting come predetermined restrictions is frequently desired; such a requirement may also be inferred from several beneficial approaches emerged to deal with diverse constraints, see, e.g., [33], [34], [35], [36], [37], [38], and [39]. So far, several approaches for tacking such a problem have been put forward, including invariability control, model-predictable control and reference leaders [40], [41]. In general, the BLF presented in [26] and [27] has become a practical technique for coping with output constraints, where a BLF being log-type was offered for a kind of strict-feedback nonlinear systems with asymmetric or symmetric output restriction. Subsequently, quite a number of innovative ideas on how to deal with the various constraints have appeared, see, e.g., [33], [34], [35], [36], [37], [38], and [39]. However, [26] can not be suitable for a system with an especially precise control requirements since the time derivative of the BLF is merely less tham or equivalent to zero. Moreover, control schemes possibly overly manage the restriction in instances where there possesses no any constraints. This observation inspires this study to dedicate to establishing one specific stability criterion to ensure the viability of general control mechanism for the constrained and unconstrained output.

As a consequence, a fascinating query is presented concurrently: *Is there a way to find a new solution to cope with the challenge of adaptive stabilization for a group of high-order uncertain nonlinear systems equipped with both the parameter uncertainty and zero dynamics, which is suitable for both the restricted and unrestricted output?* We are going to address the aforementioned question and offer a satisfactory response according to our research and materials mentioned above. Indeed, this procedure is indeed challenging due to the absence of specific theoretical backing and rigorous guidance. In this study, we develop a scheme by entailing both a skillfull BLF and the technique of continuous feedback domination equipped with a serial of integral functions. The primary achievements/innovations of this study are classified into three categories: (i) This study provides an entirely novel criteria for constructing the adaptive stability for a class of high-order uncertain nonlinear systems when the parameter uncertainties, zero dynamic and output constraint both exist. In other words, this paper conceivably offers a new insight into constructing a stabilizer by an adaptive fashion, capable of dominating complex dynamics and parameter uncertainties. (ii) Without modifying the controller's construction, the development and analysis procedures for constrained and unconstrained output are unified. That is, the case that the control strategies dominate the constraint excessively when there is no constraint can be avoided. (iii) In order to prevent zero division and streamline stability analysis, new algebraic techniques are employed, including the barrier function and several creative transformation algorithms.

Notations: Throughout this paper, we are going to employ the following notations. $\mathbb{R}$ represents the sequence of real numbers, $\mathbb{R}^+$ represents the sequence of all non-negative real numbers, and $\mathbb{R}^n$ represents Euclidean space with dimension n, and $\mathbb{R}^{>i}_{\text{odd}} \triangleq \{q_1/q_2 > i \,|\, q_1$ and q_2 are positive odd integers$\}$ with $i = 0, 1$. Given a real vector $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ and three real positive numbers s_1, s_2, s_3, for $i = 1, \ldots, n$, $\bar{x}_i \triangleq [x_1, \ldots, x_i]^T \in \mathbb{R}^i$, $\mathbb{S}_i(s_1, s_2) \triangleq \{\bar{x}_i | \bar{x}_i \in \mathbb{R}^i$ with $-s_1 < x_1 < s_2\} \subset \mathbb{R}^i$, and $\partial\mathbb{S}_i(s_1, s_2)$ denotes the boundary of $\mathbb{S}_i(s_1, s_2)$; $\lceil s \rfloor^{s_3} = |s|^{s_3} sign(s)$ for all $s \in \mathbb{R}$ with $sign(\cdot)$ being the sign function which satisfies $sign(s) = -1$ if $s < 0$, $sign(s) = 1$ if $s > 0$, and $sign(s) = 0$ if $s = 0$. $\|A\| = \sqrt{\lambda_{max}(A^T A)}$ denotes the norm of $A \in \mathbb{R}^n$, where $\lambda_{max}(A^T A)$ denotes maximum eigenvalue of square matrix $A^T A$. If a continuous function $\varsigma : [0, s) \to [0, \infty)$ is rigorously increasing along with $\varsigma(0) = 0$, it can be considered to be class $\mathcal{K}_\infty$; and if $s = \infty$ and $\varsigma(r) = \infty$ while $r \to \infty$, it can be declared to be a member of $\mathcal{K}_\infty$. Regarding a continuously differentiable function $W : \mathbb{R}^n \to \mathbb{R}^+$, if $W(x) \geq 0$ along with $W(x) = 0$, it can be regarded as positive definite; moreover, if $W(x) \to \infty$ as $\|x\| \to \infty$, it further can be viewed as radially unbounded. In certain circumstances, functions' arguments may be abbreviated, for instance, a function $g(x(t))$ is denoted by $g(x)$ or g and $\|x\|_2$ can be represented by $\|x\|$.

II. PRELIMINARIES AND PROBLEM FORMULATION

Take the following high-order uncertain nonlinear systems into account:

$$\begin{cases} \dot{z}(t) = f_0(z(t), y(t), d_0(t)), \\ \dot{x}_i(t) = \beta_i(x_i(t)) x_{i+1}^{p_i}(t) + f_i(z(t), x(t), d_i(t)), \\ \qquad i = 1, \ldots, n-1, \\ \dot{x}_n(t) = \beta_n(x(t)) u^{p_n}(t) + f_n(z(t), x(t), d_n(t)), \\ y(t) = x_1(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$ and $z(t) \in \mathbb{R}^m$ denote system state, control input and unmeasured state, respectively. For each $i = 1, \cdots, n$, $d_i(t) \in \mathbb{R}^{n_{d_i}}$ denotes the unknown bounded time-varying vector; $p_i \in \mathbb{R}^{\geq 1}_{odd}$ is called systems' high-order, $f_i(\cdot), f_n(\cdot)$ and $\beta_i(\cdot)$ are continuous functions. The initial condition is $x(0) = 0$, $z(0) = 0$. $y(t) \in \mathbb{R}$ represents the output that is constrained by $-a < y(t) < b, \forall t \geq 0$ with a, b being two positive predetermined constants.

The purpose of this study is to build up an adaptive continuous feedback stabilizer for system (1) such that (i) all the closed-loop system's states are bounded and the system output meets the asymmetric constraint $-a < y(t) < b, \forall t \geq 0$. (ii) $x(t)$ converges to zero.

In order to accomplish the control purpose, the following assumptions are required.

Assumption 1: Given $i = 1, \ldots, n$, there is an unknown constant $\bar{\theta} > 0$ and nonnegative continuous functions $\bar{f}_o(\cdot), \bar{f}_i(\cdot)$ and $\bar{f}_i(0) = 0$ such that

$$|f_i(\cdot)| \leq \bar{f}_0(\|z\|) + \bar{\theta} \sum_{j=1}^{i} |x_j|^{\delta_j + \mu_{ij}} \bar{f}_i(\bar{x}_i), \tag{2}$$

where $\mu_{ij} \geq 0$, $\delta_j = \frac{h_i+\eta}{h_j}$, h_i are specified recursively by $h_1 = 1, h_j = \frac{h_{j-1}+\eta}{p_{j-1}}, j = 2, \ldots, n+1$, and η satisfies $\eta \in (-\frac{1}{\sum_{i=1}^{n} p_0 \cdots p_{i-1}}, 0)$. Notably, (2) takes the following form:

$$|f_i(\cdot)| \leq \bar{f}_0(\|z\|) + \bar{\theta} \sum_{j=1}^{i} |x_j|^{\delta_j} l_i(\bar{x}_i(t)), \tag{3}$$

where $l_i(\bar{x}_i(t)) \triangleq \sum_{j=1}^{i} |x_j|^{\mu_{ij}} \bar{f}_i(\bar{x}_i)$ is continuous nonnegative differential function and $l_i(0) = 0$.

Assumption 2: There exists a positive definite continuous differentiable function $U_0(z)$ satisfies:

$$\begin{cases} \underline{\pi}(\|z\|) \leq U_0(z) \leq \overline{\pi}(\|z\|), \\ \dfrac{\partial U_0(z)}{\partial z} f_0(x_1, z, d_0(t)) \leq -\pi(\|z\|) + \sigma\tau(|x_1|), \end{cases} \tag{4}$$

where $\overline{\pi}(\cdot)$, $\underline{\pi}(\cdot)$, $\pi(\cdot)$, $\tau(\cdot) \in K_{\infty}$,, $k_0, \alpha < 1$ are positive constants, and $\sigma > 0$ is an unknown constant.

Remark 1: By utilizing the following inequality, it is not hard for designer to transform the asymmetric constraints into intelligibly symmetric constraints.

$$-\frac{a+b}{2} < y(t) - \frac{b-a}{2} < \frac{a+b}{2}, \quad \forall t \geq 0$$

Let $\tilde{y}(t) = y(t) - \frac{b-a}{2}$ *and* $\varepsilon = \frac{a+b}{2}$, *then for all* $t \geq 0$, $-\varepsilon < \tilde{y}(t) < \varepsilon$ *holds.*

Several lemmas that are significant to the main results'proof are presented below.

Lemma 1 [4]: For a continuous function $f(x, y)$ with $x \in \mathbb{R}^m$ and $y \in \mathbb{R}^n$, there exist smooth functions $a(x) \geq 0, b(y) \geq 0, c(x) \geq 1$ and $d(y) \geq 1$ such that $|f(x, y)| \leq a(x) + b(y)$, $|f(x, y)| \leq c(x)d(y)$.

Lemma 2 [4]: For given positive constants m, n, there is a function $a(x, y)$ such that the inequality $|a(x, y)x^m y^n| \leq c(x, y)|x|^{m+n} + \frac{n}{m+n}|a(x, y)|^{\frac{m+n}{n}} \left(\frac{m}{(m+n)c(x,y)}\right)^{\frac{m}{n}} |y|^{m+n}$ holds for any $x \in \mathbb{R}$ and any $y \in \mathbb{R}$, where $c(x, y) > 0$.

Lemma 3 [4]: For any $x \in \mathbb{R}$ and any $y \in \mathbb{R}$, the inequalities $|x+y|^p \leq 2^{p-1}(x^p + y^p)$, $|x-y|^p \leq 2^{p-1}(x^p - y^p)$, $(|x| + |y|)^{\frac{1}{p}} \leq |x|^{\frac{1}{p}} + |y|^{\frac{1}{p}} \leq 2^{\frac{p-1}{p}} ||x| + |y||^{\frac{1}{p}}$, $|x^{\frac{1}{p}} - y^{\frac{1}{p}}| \leq 2^{\frac{p-1}{p}} |x - y|^{\frac{1}{p}}$ hold for given $p \in \mathbb{R}^{\geq 1}_{\text{odd}}$, and $(x_1 + \cdots + x_n)^p \leq \max(n^{p-1}, 1)(x_1^p + \cdots + x_n^p)$ hold for given $p \in \mathbb{R}^{>0}_{\text{odd}}$ and any $x_1, \ldots, x_n \in \mathbb{R}$.

Lemma 4 [9]: For any continuous functions $x(t)$ and $\varepsilon(t)$ defined on $[0, \infty)$ satisfying $\lim_{t\to\infty} \varepsilon(t) = 0$ and $\varepsilon(t) > 0$, there is $|x(t)| < \varepsilon(t) + \frac{x^2(t)}{\sqrt{x^2(t)+\varepsilon^2(t)}}, \forall t \geq 0$.

III. CONTROL DESIGN PROCEDURE

To begin with, the designer establish the following Proposition to eliminate the effect of zero dynamics.

Proposition 1: With an identified continuous and monotone non-diminishing function $K : \mathbb{R}^+ \to [1, \infty)$ and the function $V_0(z) = \int_0^{U_0(z)} K(s)ds$ with $U_0(z)$ being presented by Assumption 2 is positive, continuously differential and radially unbounded, there arises a nondecreasing smooth function $\bar{\tau}(\cdot)$, a positive constant $\epsilon \in (0, 1)$ and an unknown constant $\bar{\sigma}$ such that

$$\frac{\partial V_0}{\partial z} f_0 \leq -(1 - \epsilon)K(s) \circ \underline{\pi}(\|z\|)\pi(\|z\|) + \bar{\sigma} x_1^2 \bar{\tau}(x_1), \tag{5}$$

Proof: Check Appendix A. □

The following coordinate transformations are then offered:

$$\begin{cases} \xi_i(t) = \lceil x_i(t) \rceil^{\frac{1}{h_i}} - \lceil \alpha_{i-1}(\bar{x}_{i-1}(t), \hat{\theta}(t)) \rceil^{\frac{1}{h_i}}, \\ u(t) = \alpha_n(x(t), \hat{\theta}(t)), \\ \alpha_i(\bar{x}_i(t), \hat{\theta}(t)) = -g_i^{h_{i+1}}(\bar{x}_i(t), \hat{\theta}(t)) \lceil \xi_i(t) \rceil^{h_{i+1}}, \end{cases} \tag{6}$$

where $i = 1, \ldots, n$, $\hat{\theta}$ actually denotes the estimate of $\theta \triangleq \max\{\bar{\sigma}, \bar{\theta}, \bar{\theta}^{\frac{2}{1-\eta}}\}$. $g_1(\cdot), \cdots, g_n(\cdot)$ are smooth positive functions to be determined subsequently. For convenience, let $g_0 = \bar{x}_0 = \alpha_0 = 0$. According to Assumption 1, there holds

$$\frac{1}{h_i} \geq 1, 2 - h_i + h_{i+1}p_i = 2 + \eta,$$
$$0 < h_{i+1}p_i < 1, \ i = 1, \cdots, n, \tag{7}$$

on the basis of (7), we represent an integral function with nested sign functions $W_k : \mathbb{R}^i \times \mathbb{R} \to \mathbb{R}, i = 2, \cdots, n$ as

$$W_i(\cdot) = \int_{\alpha_{i-1}}^{x_i} \left\lceil \lceil s \rceil^{\frac{1}{h_i}} - \lceil \alpha_{i-1} \rceil^{\frac{1}{h_i}} \right\rceil^{2-h_{i+1}p_i} ds, \tag{8}$$

Repeat the procedures in [5], the continuity of $W_k(\cdot)$ can be obtained and satisfies

$$\begin{cases} \dfrac{\partial W_i}{\partial x_i} = \lceil \xi_i \rceil^{2-h_i-\eta}, \\ \dfrac{\partial W_i}{\partial \chi_k} = -\displaystyle\int_{\alpha_{i-1}}^{x_i} \left| \lceil s \rceil^{\frac{1}{h_i}} - \lceil \alpha_{i-1} \rceil^{\frac{1}{h_i}} \right|^{1-h_i-\eta} ds \,(2 - h_{i+1}p_i) \\ \quad \cdot \dfrac{\partial}{\partial \chi_k} \left(\left\lceil \alpha_{i-1} \right|^{\frac{1}{h_i}} \right), \\ c_{i1} \mid x_i - \alpha_{i-1} \mid^{\frac{2-\eta}{h_i}} \leq W_i \leq c_{i2} \, |\xi_i|^{2-\eta}, \end{cases} \tag{9}$$

where $\chi_k = x_k$ for $k = 2, \cdots, i-1$, $\chi_i = \hat{\theta}$, $c_{i1} = \frac{h_i}{2-\eta} 2^{(2-h_{i+1}p_i)(h_i-1)/h_i}$ and $c_{i2} = 2^{1-h_i}$. According to (6), one has

$$u = \alpha_n = -\left\lceil \sum_{l=1}^{n} \left(\prod_{j=l}^{n} g_j(\bar{x}_j, \hat{\theta}) \right) \lceil x_l \rceil^{\frac{1}{h_l}} \right\rceil^{h_{n+1}}. \tag{10}$$

Subsequently, the mission is to determine the exact form of g_i recursively.

step 1 As a matter of fact, symmetric constraints are a special case of asymmetric constraints. In order to provide flexibility and versatility in the control design process, the designer attempts to explore a more extensive and flexible BLF. That is to say, the BLF should be set to utilize the system's nonlinear properties to their greatest potential and be able to handle both symmetric and asymmetric cases. So we construct the following BLF:

$$V_{blf} = \frac{a^{2-\eta}b^{2-\eta}|x_1|^{2-\eta}}{(2-\eta)(b-x_1)^{2-\eta}(a+x_1)^{2-\eta}}, \tag{11}$$

if $x_1 \to \partial\mathbb{S}_1(a,b)$, then $V_{blf} \to \infty$ holds. For any initial condition $-a < x_1(0) = y(0) < b$, as long as $-a < y(0) < b$ and $y(t)$ is bounded, whether $y(t) \to -a$ or $y(t) \to b$ means $V_{blf} \to \infty$; that is, if V_{blf} and $y(t)$ are bounded, then the output constraint $-a < y(t) < b$ is not violated. Moreover, it follows from (11) that

$$\begin{aligned}\frac{\partial V_{blf}(x_1)}{\partial x_1} &= \frac{a^{2-\eta}b^{2-\eta}(x_1{}^2+ab)}{(b-x_1)^{3-\eta}(a+x_1)^{3-\eta}}\lceil x_1\rceil^{1-\eta}\\ &\triangleq \rho(x_1)\lceil x_1\rceil^{1-\eta}, \quad -a<x_1<b,\end{aligned} \tag{12}$$

where $\rho(x_1)$ is a smooth positive function. For obtaining the control objective of (1) with output constraint, define

$$V_1 = V_{blf} + \frac{1}{2}\tilde{\theta}^2 + V_0, \tag{13}$$

where $\tilde{\theta} \triangleq \theta - \hat{\theta}$. It should be noted that the positive definiteness of V_1 can be guaranteed. To be specific, with V_1 being a function of z, x_1 and $\tilde{\theta}$ in mind, we conclude that $U_0(0) = 0, V_{blf}(0) = 0, \frac{1}{2}\tilde{\theta}^2 = 0$ holds if $z = 0, x_1 = 0, \tilde{\theta} = 0$, thus $V_1(0) = 0$. On the other hand, $V_1(t) > 0$ holds whenever $z(t) \neq 0$ or $x_1(t) \neq 0$ or $\tilde{\theta}(t) \neq 0$. The time derivative of V_1 along with (6) is as follows

$$\begin{aligned}\dot{V}_1 &= \dot{V}_{blf} - \tilde{\theta}\dot{\hat{\theta}} + \dot{V}_0\\ &= \rho\lceil x_1\rceil^{1-\eta}(\beta_1 x_2^{p_1} + f_1) - \tilde{\theta}\dot{\hat{\theta}} - (1-\epsilon)K\circ\underline{\pi}\pi + \bar{\sigma}x_1^2\bar{\tau}\\ &= \rho\lceil x_1\rceil^{1-\eta}\beta_1(x_2^{p_1}-\alpha_1^{p_1}) + \rho\beta_1\lceil x_1\rceil^{1-\eta}\alpha_1^{p_1}\\ &\quad - \tilde{\theta}\dot{\hat{\theta}} - (1-\epsilon)K\\ &\quad \circ\underline{\pi}\pi + \bar{\sigma}x_1^2\bar{\tau} + \rho\lceil x_1\rceil^{1-\eta}f_1,\end{aligned} \tag{14}$$

the next task is calculate the final two terms in (14). On the basis of (3), (7) and Lemma 2, we have

$$\begin{aligned}\rho\lceil x_1\rceil^{1-\eta}f_1 &\le \rho\,|x_1|^{1-\eta}\left(\bar{f}_0 + \tilde{\theta}\,|x_1|^{p_1h_2}\,l_1 + \hat{\theta}\,|x_1|^{p_1h_2}\,\bar{l}_1\right)\\ &\le \phi_1\xi_1^2 + \bar{f}_0^{\frac{2}{\eta+1}} + \bar{f}_0^{\frac{2}{p_1h_2}} + \tilde{\theta}\rho l_1\xi_1^2,\end{aligned} \tag{15}$$

where $\phi_1 = \rho\bar{l}\hat{\theta} + \frac{1-\eta}{2}(\frac{4}{1+\eta})^{\frac{1+\eta}{\eta-1}}\rho^{\frac{2}{1-\eta}}$, ϕ_1 and $\bar{l}_1$ are positive smooth functions. Additionally, there holds

$$\bar{\sigma}x_1^2\bar{\tau} \le \tilde{\theta}\xi_1^2\bar{\tau} + \hat{\theta}\xi_1^2\bar{\tau}, \tag{16}$$

substituting (15) and (16) into (14), it can be observed from $1-\eta+p_1h_2 = 2$ and $h_1 = 1$ that $\lceil x_1\rceil^{1-\eta}\alpha_1^{p_1} = -g_1^{p_1h_2}\xi_1^2$. Then, (14) can be simplified as

$$\begin{aligned}\dot{V}_1 &\le -(n+1)\xi_1^2 + \rho\beta_1\lceil x_1\rceil^{1-\eta}\left(x_2^{p_1}-\alpha_1^{p_1}\right) + \bar{f}_0^{\frac{2}{1+\eta}} + \bar{f}_0^{\frac{2h_2}{p_1}}\\ &\quad - (1-\epsilon)K\circ\underline{\pi}\pi - \tilde{\theta}\dot{\hat{\theta}} + \tilde{\theta}\bar{\tau}\xi_1^2 + \tilde{\theta}\rho l_1\xi_1^2\\ &\quad + \xi_1^2\left(\phi_1 + n + 1 + \hat{\theta}\bar{\tau} - \rho\beta_1 g_1^{p_1h_2}\right).\end{aligned} \tag{17}$$

So far, one can choose

$$g_1 = \left(\frac{\phi_1 + n + 1 + \hat{\theta}\bar{\tau}}{\rho\beta_1}\right)^{\frac{1}{p_1h_2}}, \tag{18}$$

(17) can be transformed into

$$\begin{aligned}\dot{V}_1 &\le -(n+1)\xi_1^2 + \rho\beta_1\lceil x_1\rceil^{1-\eta}\left(x_2^{p_1}-\alpha_1^{p_1}\right) + \bar{f}_0^{\frac{2}{1+\eta}} + \bar{f}_0^{\frac{2}{p_1h_2}}\\ &\quad - (1-\epsilon)K\circ\underline{\pi}\pi + \tilde{\theta}\left(\varpi_1\xi_1^2 - \dot{\hat{\theta}}\right),\end{aligned} \tag{19}$$

where $\varpi_1(x_1) = \rho l_1 + \bar{\tau}(x_1)$.

step 2 In this step, we will calculate the explicit expression of g_2. Consider $V_2 = V_1 + W_2$. Considering (9) and (19), one has

$$\begin{aligned}\dot{V}_2 &\le -(n+1)\xi_1^2 + \rho\beta_1\lceil x_1\rceil^{1-\eta}\left(x_2^{p_1}-\alpha_1^{p_1}\right)\\ &\quad + \bar{f}_0^{\frac{2}{1+\eta}} + \bar{f}_0^{\frac{2}{p_1h_2}} - W_2\\ &\quad + c_{22}\left(1+\xi_2^2\right)^{\frac{-\eta}{2}}\xi_2^2 + \beta_2\lceil\xi_2\rceil^{2-\eta-h_2}\alpha_2^{p_2} + \frac{\partial W_2}{\partial\hat{\theta}}\dot{\hat{\theta}}\\ &\quad - (1-\epsilon)K\circ\underline{\pi}\pi + \tilde{\theta}\left(\varpi_1\xi_1^2 - \dot{\hat{\theta}}\right) + \frac{\partial W_2}{\partial x_1}\dot{x}_1\\ &\quad + \beta_2\lceil\xi_2\rceil^{2-\eta-h_2}\left(x_3^{p_2}-\alpha_2^{p_2}\right) + \lceil\xi_2\rceil^{2-\eta-h_2}f_2,\end{aligned} \tag{20}$$

we next simplify the indefinite terms of (20). According to Lemma 2, (6) and (7), one can get

$$\begin{aligned}&\rho\beta_1\lceil x_1\rceil^{1-\eta}\left(x_2^{p_1}-\alpha_1^{p_1}\right)\\ &\le \rho\beta_1\,|x_1|^{1-\eta}\cdot 2^{1-p_1h_2}\left|\lceil x_2\rceil^{\frac{1}{h_2}} - \lceil\alpha_1\rceil^{\frac{1}{h_2}}\right|^{p_2h_2}\\ &\le 2^{-\eta}\rho\beta_1\,|\xi_1|^{1-\eta}\,|\xi_2|^{1+\eta}\\ &\le \frac{1}{4}\xi_1^2 + \phi_{21}\xi_2^2,\end{aligned} \tag{21}$$

where ϕ_{21} is positive smooth function. It follows (6) and (7) that $|x_i|^{\frac{h_3p_2}{h_i}} \le |\xi_i|^{p_2h_3} + |g_{i-1}|^{p_2h_3}\,|\xi_{i-1}|^{p_2h_3}$, $i = 1, 2$. Then, according to (3) and Lemma 2, one has

$$\begin{aligned}\lceil\xi_2\rceil^{2-\eta-h_2}f_2 &\le |\xi_2|^{2-\eta-h_2}\left(\bar{f}_0 + \bar{\theta}\sum_{i=1}^{2}|x_1|^{\frac{p_2h_3}{h_i}}\,l_2\right)\\ &\le |\xi_2|^{2-\eta-h_2}\bar{f}_0 + \bar{\theta}l_2\bar{g}_1\,|\xi_2|^{2-\eta-h_2}\sum_{i=1}^{2}|\xi_i|^{p_2h_3}\\ &\le \phi_{22}\xi_2^2 + \bar{f}_0^{\frac{2}{p_2h_3}} + \frac{1}{4}\xi_1^2 + \tilde{\theta}\varpi_{21}\xi_2^2,\end{aligned} \tag{22}$$

where ϕ_{22}, ϖ_{21} and $\bar{g_1} \geq 1 + g_1^{p_2 h_3}$ are smooth positive functions. By using (3), (6) and Lemma 1, the designer ia capable of calculating the following estimate:

$$\left|\frac{\partial \lceil\alpha_1\rceil^{\frac{1}{h_2}}}{\partial x_1} f_1\right| \leq \left(g_1 + \left|\frac{\partial g_1}{\partial \xi_1}\right| |\xi_1|\right)\left(\bar{f_0} + \bar{\theta}\,|\xi_1|^{p_1 h_2}\, l_1\right) \leq \gamma_{21}(\bar{f_0} + \bar{\theta}\,|\xi_1|^{1+\eta}), \tag{23}$$

where $\gamma_{21}(x_1) \geq \left(g_1 + |\frac{\partial g_1}{\partial \xi_1}\xi_1|\right)(1 + l_1)$ is a smooth positive function. Further, since $\left|x_2^{p_1}\right| \leq (|\xi_2| + g_1\,|\xi_1|)^{\eta+1} \leq (1 + g_1^{\eta+1})\left(|\xi_2|^{\eta+1} + |\xi_1|^{\eta+1}\right)$, we have

$$\left|\frac{\partial \lceil\alpha_1\rceil^{\frac{1}{h_2}}}{\partial x_1}\beta_1 x_2^{p_1}\right| \leq \varrho_{21}(x_1)\sum_{j=1}^{2}\left|\xi_j\right|^{\eta+1}, \tag{24}$$

where $\varrho_{21} \geq |g_1 + |\frac{\partial g_1}{\partial \xi_1}\xi_1||\beta_1(x_1,t)\cdot(1 + g_1^{\eta+1})$ is a smooth positive function. Besides, Combining Lemmas 3 with 4 leads to

$$\begin{aligned}
&-(2-\eta-h_2)\int_{\alpha_1}^{x_2}\left|\lceil s\rceil^{\frac{1}{h_2}} - \lceil\alpha_1(x_1)\rceil^{\frac{1}{h_2}}\right|^{1-h_2-\eta} ds \\
&\quad\leq (2-\eta-h_2)\,|x_2-\alpha_1|\cdot|\xi_2|^{1-h_2-\eta} \\
&\quad\leq 2^{1-h_2}(2-\eta-h_2)\,|\xi_2|^{h_2}\,|\xi_2|^{1-h_2-\eta} \\
&\quad\leq \tilde{c}_2\,|\xi_2|^{1-\eta},
\end{aligned} \tag{25}$$

where $\tilde{c}_2 = 2^{1-h_2}(2-\eta-h_2) > 0$ is a constant. In the subsequence, on the basis of (3), (7), (23)-(25) and Lemma 2, we have

$$\begin{aligned}
&\frac{\partial W_2}{\partial x_1}\dot{x}_1 \\
&= -(2-\eta-h_2)\int_{\alpha_1}^{x_2}|\lceil s\rceil^{\frac{1}{h_2}} - \lceil\alpha_1\rceil^{\frac{1}{h_2}}|^{1-\eta-h_2} ds \\
&\quad\cdot\frac{\partial}{\partial x_1}\left(\lceil\alpha_1\rceil^{\frac{1}{h_2}}\right)\left(\beta_1 x_2^{p_1} + f_1\right) \\
&\leq \tilde{c}_2(\varrho_{21}+\gamma_{21})\,|\xi_2|^{1-\eta}\left(\bar{f_0} + \sum_{j=1}^{2}\left|\xi_j\right|^{\eta+1} + \bar{\theta}\,|\xi_1|^{\eta+1}\right) \\
&\leq \frac{1}{4}\xi_1^2 + \phi_{23}\xi_2^2 + f_0^{\frac{2}{\eta+1}} + \tilde{\theta}\varpi_{22}\xi_2^2,
\end{aligned} \tag{26}$$

where ϖ_{22} and ϕ_{23} are positive smooth functions. On the other hand, it should be noted that

$$\frac{\partial W_2}{\partial\hat{\theta}}\left(\varpi_1\xi_1^2 + \varpi_2\xi_2^2\right) \leq \frac{1}{4}\xi_1^2 + \phi_{24}\xi_2^2, \tag{27}$$

where ϕ_{24} is a positive function, and $\varpi_{21} + \varpi_{22} = \varpi_2$. Let $\phi_2(\bar{x}_2) = \phi_{21} + \phi_{22} + \phi_{23} + \phi_{24}$. Substituting (21)-(27) into (20) and taking $\lceil\xi_2\rceil^{2-\eta-h_2}\alpha_2^{p_2} = -g_2^{p_2 h_3}\xi_2^2$ into consideration, then (20) takes the form

$$\begin{aligned}
\dot{V}_2 \leq& -(n-1)(\xi_1^2+\xi_2^2) - \xi_1^2 + 2\bar{f_0}^{\frac{2}{1+\eta}} + \sum_{i=1}^{2}\bar{f_0}^{\frac{2}{p_i h_{i+1}}} - W_2 \\
&- (1-\epsilon)K\circ\underline{\pi}\pi + \beta_2\lceil\xi_2\rceil^{2-\eta-h_2}\left(x_3^{p_2}-\alpha_2^{p_2}\right) \\
&+ \xi_2^2\left(\phi_2 + n - 1 - \beta_2 g_2^{p_2 h_3} + c_{22}(1+\xi_1^2)^{-\frac{2}{\eta}}\right) \\
&+ \left(\tilde{\theta} - \frac{\partial W_2}{\partial\hat{\theta}}\right)\left(\sum_{i=1}^{2}\varpi_i\xi_i^2 - \dot{\hat{\theta}}\right).
\end{aligned} \tag{28}$$

Choose

$$g_2 = \left(\frac{\phi_2 + n - 1 + c_{22}\left(1+\xi_2^2\right)^{-\frac{\eta}{2}}}{\beta_2}\right)^{\frac{1}{p_2 h_3}}, \tag{29}$$

At last, (28) can be rewritten as

$$\begin{aligned}
\dot{V}_2 \leq& -(n-1)(\xi_1^2+\xi_2^2) - \xi_1^2 + 2\bar{f_0}^{\frac{2}{1+\eta}} + \sum_{i=1}^{2}\bar{f_0}^{\frac{2}{p_i h_{i+1}}} \\
&- W_2 - (1-\epsilon)K\circ\underline{\pi}\pi + \beta_2\lceil\xi_2\rceil^{2-\eta-h_2}\left(x_3^{p_2}-\alpha_2^{p_2}\right) \\
&+ \left(\tilde{\theta} - \frac{\partial W_2}{\partial\hat{\theta}}\right)\left(\sum_{i=1}^{2}\varpi_i\xi_i^2 - \dot{\hat{\theta}}\right).
\end{aligned} \tag{30}$$

step i ($i = 3,\ldots,n$) Assuming at step $i-1$, one has created a suitable continuously differential function $V_{i-1}(\bar{x}_{i-1})$ and smooth positive functions $g_1,\cdots,g_{i-1}$ such that

$$\begin{aligned}
&\dot{V}_{i-1} \\
&\leq -(n-i+2)\sum_{k=1}^{i-1}\xi_k^2 - \xi_1^2 \\
&\quad+ \left(\tilde{\theta} - \sum_{k=2}^{i-1}\frac{\partial W_k}{\partial\hat{\theta}}\right)\left(\sum_{k=1}^{i-1}\varpi_k\xi_k^2 - \dot{\hat{\theta}}\right) \\
&\quad- \sum_{k=2}^{i-1}W_k - (1-\epsilon)K\circ\underline{\pi}\pi + \sum_{k=1}^{i-1}\bar{f_0}^{\frac{2}{p_k h_{k+1}}} + (i-1)\bar{f_0}^{\frac{2}{\eta+1}} \\
&\quad+ \beta_{i-1}\lceil\xi_{i-1}\rceil^{2-\eta-h_{i-1}}\left(x_i^{p_{i-1}} - \alpha_{i-1}^{p_{i-1}}\right),
\end{aligned} \tag{31}$$

where $\varpi_k(\bar{x}_k,\hat{\theta})$ is obvious a nonnegative continuous function with $\varpi_k(0,\hat{\theta}) = 0$. What comes next, we need to verify that (29) still remains valid in step i. Thus consider $V_i = V_{i-1} + W_i$. Utilizing (9), (29) can be changed to

$$\begin{aligned}
&\dot{V}_i \\
&\leq -(n-i+2)\sum_{k=1}^{i-1}\xi_k^2 - \xi_1^2 + (i-1)\bar{f_0}^{\frac{2}{\eta+1}} + \sum_{k=1}^{i-1}\bar{f_0}^{\frac{2}{p_k h_{k+1}}} \\
&\quad+ \beta_i\lceil\xi_i\rceil^{2-\eta-h_i}\alpha_i^{p_i} + \beta_i\lceil\xi_i\rceil^{2-\eta-h_i}\left(x_{i+1}^{p_i} - \alpha_i^{p_i}\right) - \sum_{k=2}^{i}W_k \\
&\quad+ W_i - (1-\epsilon)K\circ\underline{\pi}\pi \\
&\quad+ \left(\tilde{\theta} - \sum_{k=2}^{i-1}\frac{\partial W_k}{\partial\hat{\theta}}\right)\left(\sum_{k=1}^{i-1}\varpi_k\xi_k^2 - \dot{\hat{\theta}}\right) \\
&\quad+ \lceil\xi_i\rceil^{2-\eta-h_i}f_i + \beta_{i-1}\lceil\xi_{i-1}\rceil^{2-\eta-h_i}\left(x_i^{p_{i-1}} - \alpha_{i-1}^{p_{i-1}}\right) \\
&\quad+ \frac{\partial W_i}{\partial\hat{\theta}}\dot{\hat{\theta}} + \sum_{k=1}^{i-1}\frac{\partial W_i}{\partial x_k}\dot{x}_k,
\end{aligned} \tag{32}$$

To eliminate tedious calculations, we put the estimation of the final five terms of (32) in Appendix B. In other words, after laborious calculations, we are left with the following inequality:

$$\beta_{i-1}\lceil\xi_{i-1}\rceil^{2-\eta-h_i}\left(x_i^{p_{i-1}} - \alpha_{i-1}^{p_{i-1}}\right) + \lceil\xi_i\rceil^{2-\eta-h_i}f_i$$

$$+\sum_{k=1}^{i-1}\frac{\partial W_i}{\partial x_k}\dot{x}_k+\frac{\partial W_i}{\partial\hat{\theta}}\sum_{k=1}^{i-1}\varpi_k\xi_k^2+\sum_{k=2}^{i}\varpi_i\xi_i^2$$
$$\leq \xi_{i-1}^2+\phi_i\xi_i^2+\sum_{k=1}^{i-2}\xi_k^2+\tilde{\theta}\varpi_i\xi_i^2+\bar{f}_0^{\frac{2}{p_ih_{i+1}}}+\bar{f}_0^{\frac{2}{\eta+1}}, \quad (33)$$

then, one can choose the positive smooth function g_i as follows:

$$g_i=\Big(\frac{\phi_i+n-i+1+c_{i2}\left(1+\xi_i^2\right)^{\frac{-\eta}{2}}}{\beta_i}\Big)^{\frac{1}{p_ih_{i+1}}},\ i\geq 2, \quad (34)$$

substituting (33) and (34) into (32) produces

$$\dot{V}_i \leq -(n-i+1)\sum_{k=1}^{i}\xi_k^2-\xi_1^2+i\bar{f}_0^{\frac{2}{\eta+1}}+\sum_{k=1}^{i}\bar{f}_0^{\frac{2}{p_kh_{k+1}}}-\sum_{k=2}^{i}W_k$$
$$+\beta_i\lceil\xi_i\rfloor^{2-\eta-h_i}\left(x_{i+1}^{p_i}-\alpha_i^{p_i}\right)-(1-\epsilon)K\circ\underline{\pi}\pi$$
$$+\Big(\tilde{\theta}-\sum_{k=2}^{i}\frac{\partial W_k}{\partial\hat{\theta}}\Big)\Big(\sum_{k=1}^{i}\varpi_k\xi_k^2-\dot{\hat{\theta}}\Big). \quad (35)$$

It should be noticed that (35) is also holds for $i=n$ with $\xi_{n+1}=0$. Hence, we are able to design an adaptive feedback stabilizer as follows:

$$\dot{\hat{\theta}}=\sum_{k=1}^{n}\varpi_k\xi_k^2,\ \hat{\theta}(0)=\hat{\theta}_0, \quad (36)$$
$$u=-\Big\lceil\sum_{l=1}^{n}\Big(\prod_{j=l}^{n}g_j(\bar{x}_j,\hat{\theta})\Big)\lceil x_l\rfloor^{\frac{1}{h_l}}\Big\rfloor^{h_{n+1}}. \quad (37)$$

Finally, using (35) and letting $i=n$, one achieves

$$\dot{V}_n\leq-\sum_{k=1}^{n}\xi_k^2-\xi_1^2+n\bar{f}_0^{\frac{2}{\eta+1}}+\sum_{k=1}^{n}\bar{f}_0^{\frac{2}{p_kh_{k+1}}}-\sum_{k=2}^{n}W_k$$
$$-(1-\epsilon)K\circ\underline{\pi}\pi, \quad (38)$$

where $V_n=V_1+\Sigma_{k=2}^{n}W_k$. So far, the whole design procedure is completed.

Although lack of uniqueness, the continuity of the closed-loop system guarantees the existence of its solution, which does not decrease the significance of this paper. Last but not the least, we draw attention to the distinctive characteristics of V_{blf} from two aspects.

Remark 2: (i) Control design procedure can be unified by V_{blf} when coping with both constrained and unconstrained systems. In fact, we set $b=a\to\infty$ and obtain

$$\lim_{a\to\infty}V_{blf}(x_1)=\lim_{a\to\infty}\frac{a^{4-2\eta}|x_1|^{2\eta}}{(2-\eta)(a-x_1)^{2-\eta}(a+x_1)^{2-\eta}}$$
$$=\frac{|x_1|^{2-\eta}}{2-\eta}.$$

In contrast, the current findings in [5], [6], and [7] likewise utilize the following Lyapunov function:

$$W_1(x_1)=\int_0^{x_1}\lceil s\rfloor^{2-r_2p_1}ds=\int_0^{x_1}\lceil s\rfloor^{1-\eta}ds=\frac{|x_1|^{2-\eta}}{2-\eta}.$$

As a result, the scenario where x_1 has no constraint is identical to $a=b\to\infty$. Hence, the barrier function turns into the same as well, and the remainder design and analysis apply the same techniques as that in [5] and [7].

(ii) V_{blf} is constructed by effectively leveraging the characteristics of nonlinearities. It can be indicated that η moves into the powers of V_{blf}, which effectively depicts nonlinear features of functions f_i's. On the other hand, this knowledge fails to be taken into account while developing barrier functions in [26], [27], and [33]. It further clarifies why it is impossible to employ logarithm or tangent functions to control design from a different perspective.

IV. MAIN RESULTS

The main result of this paper is summarized as follows.

Theorem 1: For the high-order uncertain nonlinear system (1) under Assumptions 1 and 2, if (1) satisfies:

$$\lim_{s\to 0^+}\sup\frac{\bar{\tau}(s)}{s^2}<+\infty,\ \lim_{s\to 0^+}\sup\frac{\bar{f}_0^2(s)}{\underline{\pi}(s)}<+\infty, \quad (39)$$

then there exists a continuous controller guarantees that the states of system (1) converge to the origin as well as keeping the asymmetric output constraint inviolate.

Proof: The entire proof can be split into two halves.

(a) Verification of asymptotic stability. To begin with, we need to prove $\dot{V}_n\leq 0$. Owing to $\frac{2}{\eta+1}>2$ and $\frac{2}{p_ih_{i+1}}>2$, (39), $\bar{f}_0^{\frac{2}{\eta+1}}$ and $\bar{f}_0^{\frac{2}{p_ih_{i+1}}}$, it can be concluded from the boundedness near the origin that $\lim_{s\to 0^+}\sup\frac{\hat{j}_1(s)}{\underline{\pi}(s)}<\infty$, and

$$\hat{j}_1(\|z\|)=n\bar{f}_0^{\frac{2}{\eta+1}}(\|z\|)+\sum_{i=1}^{n}\bar{f}_0^{\frac{2}{p_ih_{i+1}}}(\|z\|), \quad (40)$$

define:

$$K(s)=\begin{cases}\dfrac{2}{(1-e)(1-\epsilon)}\displaystyle\limsup_{s\to 0+}\frac{\hat{j}_1(s)}{\underline{\pi}(s)}+1, & s=0,\\ \dfrac{2}{(1-e)(1-\epsilon)}\displaystyle\sup_{0<s'\leq s}\frac{\hat{j}_1(s')}{\underline{\pi}(s')}+1, & s>0,\end{cases} \quad (41)$$

where $0<e<1$ is a specified constant. $K(s)$ is nondecreasing, positive and continuous on $[0,\infty)$. By means of (39), one has

$$-\frac{(1-e)(1-\epsilon)}{2}K(s)\circ\underline{\pi}(\|z\|)\pi(\|z\|)+\hat{j}_1(\|z\|)\leq 0, \quad (42)$$

which combines with $-(1-\epsilon)K(s)\circ\underline{\pi}(\|z\|)\pi(\|z\|)\leq 0$ leads to

$$\hat{j}_1(\|z\|)-(1-\epsilon)K(s)\circ\underline{\pi}(\|z\|)\pi(\|z\|)$$
$$\leq-\frac{(1-e)(1-\epsilon)}{2}K(s)\circ\underline{\pi}(\|z\|)\pi(\|z\|). \quad (43)$$

Substituting (43) into (38), there holds

$$\dot{V}_n\leq-\frac{(1-e)(1-\epsilon)}{2}K(s)\circ\underline{\pi}(\|z\|)\pi(\|z\|)-\sum_{i=1}^{n}\xi_i^2-\xi_1^2$$
$$-\sum_{i=2}^{n}W_i\triangleq-V_n^*(x,z)\leq 0, \quad (44)$$

therefore, $V(x, z)$ is positive and continuous. Furthermore, it can be deduced from Lyapunov stability theory that system (1) is uniformly asymptotically stable.

(b)Validation of output constraints. The following is to verify that there is $a < |y(t)| < b$, for all $t \geq 0$. At first, here we define the preliminary state $x(0) \in \mathbb{S}_n^{\lambda}$. By means of (44), we can draw a conclusion that $0 \leq V_n(x(t)) \leq V_n(x(0))$ for all $t \geq 0$, which shows that

$$\frac{a^{2-\eta}b^{2-\eta}|x_1|^{2-\eta}}{(2-\eta)(b-x_1)^{2-\eta}(a+x_1)^{2-\eta}} \leq V_n(x(0)). \tag{45}$$

According to $V(0) = 0$ and the continuity of V, we know that there is a constant $\lambda_2 > 0$ so that$V < 1$ holds, for any given $\|y(t)\| < \lambda_2$. Besides, there is also a constant $\lambda_3 = \min\{1, (a+b+ab-ab(2-\eta))^{\frac{1}{\eta-2}})^{\frac{1}{2}}\} > 0$ such that for any $|x_1| < \lambda_3$, the following holds

$$\frac{a^{2-\eta}b^{2-\eta}|x_1|^{2-\eta}}{(2-\eta)(b-x_1)^{2-\eta}(a+x_1)^{2-\eta}} < 1. \tag{46}$$

Let $\lambda = \min\{\lambda_1, \lambda_2, \lambda_3, a, b\}$, $|y| < \lambda$ holds. In other words, for any $t \geq 0$, $|y(t)| = |x_1(t)| < \lambda$ holds, which means $\mathbb{S}_n^{\lambda} \subset \mathbb{R}^n$ is an estimate of attractive domain. □

Remark 3: It is worthy pointing out that the difficulties encountered and novelties made from two aspects.

(i) We additionally consider the asymmetric constraint which is a challenging in both practical applications and control theories in this study. Based on this, how to develop a novel barrier Lyapunov function to unify the control design in dealing with both constrained and unconstrained systems without changing the structure of the controller can be regarded as the first difficulty of this paper. By efficiently exploiting the characteristics of nonlinearities, an innovative barrier Lyapunov function is skillfully constructed to keep the asymmetric output constraint inviolate. More specific clues can be found in Remark 2.

(ii) The high-orders p_i's renders the recursive design ineffective in dominating terms which are associated with θ and have unmatched powers with other destabilized nonlinear terms. On the other hand, $p_i > 1$ inevitably leads to different homogeneous degrees in each equation of system (1). By improving the technique of continuous feedback domination equipped with a serial of integral functions including nested sign functions, and developing subtle state transformations, a controller producing effective actions is constructed accordingly to compensate for the effects of inherent nonlinearities while manipulating high-orders p_i's.

V. SIMULATION EXAMPLE

This part provides a numerical simulation to evaluate the efficiency of the design strategy, consider the following example:

$$\begin{cases} \dot{z} = -2z^{\frac{3}{5}} + \dfrac{1}{4}\theta x_1^{\frac{3}{5}}, \\ \dot{x}_1 = x_2^{\frac{5}{3}} + \theta x_1 + z^2, \\ \dot{x}_2 = u + \theta x_2^{\frac{4}{3}} + z^2. \end{cases}$$

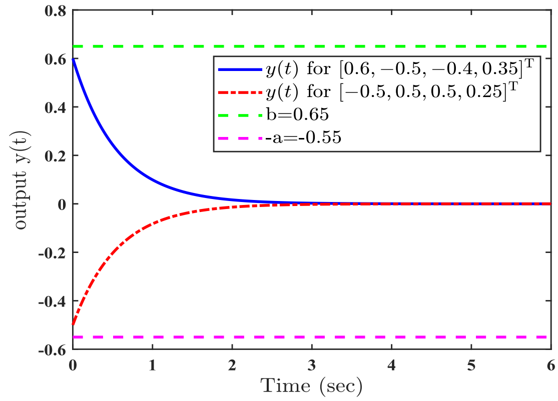

FIGURE 1. Trajectories of y.

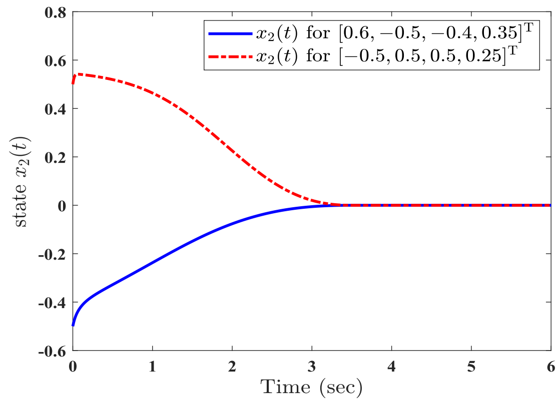

FIGURE 2. Trajectories of x_2.

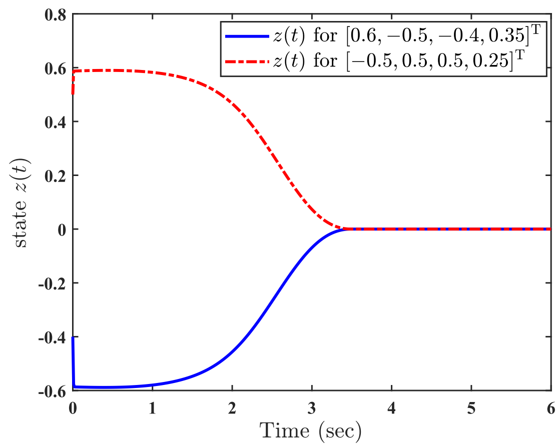

FIGURE 3. Trajectories of z.

where $y = x_1$. Choose $\beta_1 = \beta_2 = 1, a = 0.55, b = 0.65, d_0 = 0.3$. Besides, $p_1 = \frac{3}{5}, p_2 = 1, \eta \in (-\frac{1}{3}, 0), h_1 = 1, h_2 = \frac{h_1+\eta}{p_1} = \frac{8}{25}, h_3 = \frac{h_2+\eta}{p_2} = \frac{7}{25}$. By (2), one has

$$|f_1| = \left|\theta x_1 + z^2\right| \leq |\theta|\,|x_1|^{\frac{4}{5}}\,|x_1|^{\frac{1}{5}} + z^2,$$

$$|f_2| = \left|\theta x_2^{\frac{4}{3}} + z^2\right| \leq |\theta|\left(|x_1|^{\frac{7}{25}} + |x_2|^{\frac{7}{12}}\right)|x_2|^{\frac{3}{4}} + z^2,$$

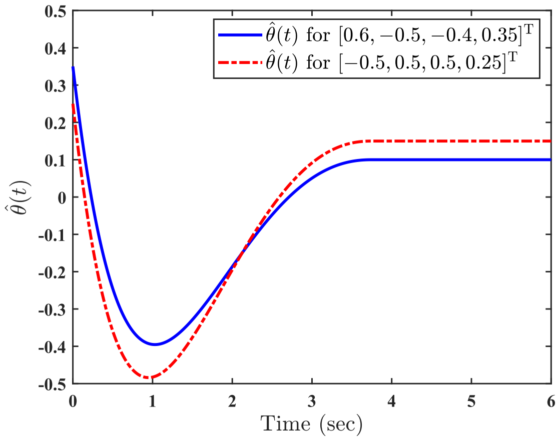

FIGURE 4. Trajectories of $\hat{\theta}$.

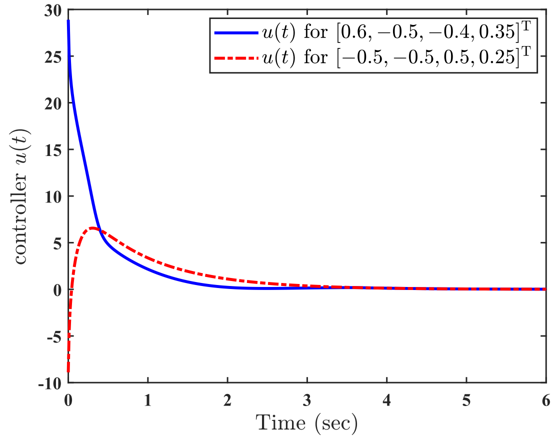

FIGURE 5. Trajectories of u.

some calculation illustrate that $\bar{f}_1 = |x_1|^{\frac{1}{5}}, \bar{f}_2 = |x_2|^{\frac{3}{4}}, \bar{f}_0 = z^2$ meets Assumption 1. Here we select $U_0 = z^4$, thus $\frac{\partial U_0(z)}{\partial z} f_0(x_1, z, d_0) = 4z^3 \left(-2z^{\frac{3}{5}} + \frac{1}{4} d_0 x_1^{\frac{3}{5}}\right) \leq -\frac{1}{4}\left(z^4\right)^{\frac{9}{10}} + d_0^6 |x_1|^{\frac{18}{5}}$ satisfies Assumption 2, and $\alpha = \frac{9}{10}, 1-\epsilon = \frac{1}{4}, \epsilon = \frac{3}{4}, \sigma = 1, \tau(|x_1|) = d_0^6 |x_1|^{\frac{18}{5}}$. After complicated calculation, the controller can be constructed as $u = -g_2(g_1 x_1 + |x_2|^{\frac{25}{12}})^{\frac{7}{25}}$, where $g_1 = (\phi_1 + 3 + \hat{\theta}\tau_1(x_1))^{\frac{5}{4}}$, $g_2 = (\phi_2 + 1 + 1.4(1 + \xi_2^2)^{\frac{1}{10}})^{\frac{25}{7}}$, $\phi_1 = \hat{\theta}\rho(1 + x_1^2)^{\frac{1}{10}} + 0.2\rho^{\frac{5}{3}}$, $\phi_2 = \hat{\theta}(1 + x_2^2)^{\frac{3}{8}} + 0.63\hat{\theta}(1 + g_1^{\frac{7}{25}})^{\frac{50}{43}}(1 + x_2^2)^{\frac{75}{172}} + 2.1\rho^{\frac{5}{2}} + 2.47(\varrho_{21} + \gamma_{21}) + (6 + 10.6)\hat{\theta}(\varrho_{21} + \gamma_{21})^{\frac{5}{3}} + 2.7((g_1 + (1 + (\frac{\partial g_1}{\partial x_1})^2 x_1^2)^{\frac{1}{2}}(\varpi_1 + \varpi_2)(\xi_1^{\frac{6}{5}} + \xi_2^{\frac{6}{5}}))^{\frac{4}{3}} + 2.47(g_1 + (1 + \frac{\partial g_1}{\partial x_1} x_1^2)^{\frac{1}{2}}(\varpi_1 + \varpi_2)(\xi_1^{\frac{6}{5}} + \xi_2^{\frac{6}{5}})$.

To perform the simulation, we assign $\theta = 1$ and select the initial values as $[x_1(0), x_2(0), z(0), \hat{\theta}(0)]^T = [0.6, -0.5, -0.4, 0.35]^{\mathrm{T}}$ and $[x_1(0), x_2(0), z(0), \hat{\theta}(0)]^T = [-0.5, 0.5, 0.5, 0.25]^{\mathrm{T}}$. As demonstrated in Figs.1-3, the states of the closed-loop system can converge to the origin and the output constraint $-0.55 < y(t) < 0.65$ can not be broken.

VI. CONCLUSION

Considering asymmetric output constraints and zero dynamics, the challenge of adaptive stabilization for a category of high-order uncertain nonlinear systems is deal with in this study. The establishment of the continuous feedback stabilizer is on the basis of a novel Barrier Lyapunov Function (BLF) with the methodology of continuous feedback domination equipped with a serial of integral functions including nested sign functions. There are still several issues to be explored in the future. (i) It's not certain if strategy can be employed to cope with the prescribed-time stabilization for nonlinear systems with asymmetric time-varying output constraints. (ii) Whether or not this approach could be applied to cope with the stabilization of stochastic nonlinear systems.

APPENDIX

A. PROOF OF PROPOSITION 1

This part offers the specific proof of Proposition 1. At first, since $U_0(z)$ is continuously differential, positive and radially unbounded, it may be inferred from Assumption 2 that

$$
\begin{aligned}
U_0(z) &= K(U_0(z))\dot{U}_0(z) \\
&\leq -\pi(\|z\|)K(U_0(z)) + \sigma\tau(|x_1|)K(U_0(z)),
\end{aligned}
$$

There are two cases to be discussed here:

Case 1: when $\sigma\tau(|x_1|) < \epsilon\pi(\|z\|)$, one has

$$
\begin{aligned}
&-\pi(\|z\|)K(U_0(z)) + \sigma\tau(|x_1|)K(U_0(z)) \\
&\quad \leq -\pi(\|z\|)K(U_0(z)) + \epsilon\pi(\|z\|)K(U_0(z)) \\
&\quad = -(1-\epsilon)\pi(\|z\|)K(U_0(z)).
\end{aligned}
$$

Case 2: when $\sigma\tau(|x_1|) \geq \epsilon\pi(\|z\|)$, if $\pi(\|z\|) \leq \frac{\sigma}{\epsilon}\tau(|x_1|)$ and $|z| \leq \pi^{-1} \circ (\frac{\sigma}{\epsilon}\tau(|x_1|))$. Considering $U_0(z) \leq \bar{\pi}(\|z\|)$ as well as K being nondecreasing, then the subsequent inequality is true:

$$
K(U_0(z)) \leq K \circ \bar{\pi} \circ \pi^{-1} \circ (\frac{\sigma}{\epsilon}\tau(|x_1|)),
$$

further,

$$
\begin{aligned}
&-\pi(\|z\|)K(U_0(z)) + \sigma\tau(|x_1|)K(U_0(z)) \\
&\leq -\pi(\|z\|)K(U_0(z)) + \sigma\tau(|x_1|)K \circ \bar{\pi} \circ \pi^{-1} \circ (\frac{\sigma}{\epsilon}\tau(|x_1|)) \\
&\leq -(1-\epsilon)\pi(\|z\|)K(U_0(z)) + \sigma\tau(|x_1|)K \circ \bar{\pi} \\
&\quad \circ \frac{1}{\pi} \circ (\frac{\sigma}{\epsilon}\tau(|x_1|)).
\end{aligned}
$$

Taken $U_0(z) \geq \underline{\pi}(\|z\|)$ into account, one has

$$
K(U_0(z)) \geq K \circ \underline{\pi}(\|z\|),
$$

at last,

$$
\begin{aligned}
&-\pi(\|z\|)K(U_0(z)) + \sigma\tau(|x_1|)K(U_0(z)) \\
&\leq -(1-\epsilon) \circ \underline{\pi}(\|z\|)\pi(\|z\|) + \sigma\tau(|x_1|)K \circ \bar{\pi} \circ \frac{1}{\pi} \\
&\quad \circ (\frac{\sigma}{\epsilon}\tau(|x_1|)).
\end{aligned}
$$

It can be deduced from Lemma 2.5 in [42] that there is a constant $c(\sigma)$ depending on σ and positive smooth function $\hat{\tau}(|x_1|) \geq 1$ such that

$$K \circ \bar{\pi} \circ \pi^{-1} \circ (\frac{\sigma}{\epsilon}\tau(|x_1|)) \leq c(\sigma)\hat{\tau}(|x_1|).$$

Define $\bar{\sigma} = \sigma c(\sigma)$ and $\tilde{\tau}(s) = \tau(s)\hat{\tau}(s)$, we have

$$\frac{\partial U_0(z)}{\partial z}f_0 \leq -(1-\epsilon)K(s) \circ \underline{\pi}(\|z\|)\pi(\|z\|) + \bar{\sigma}\tilde{\tau}(|x_1|).$$

Given that $\hat{\tau}(|x_1|) \geq 1$ and $\tilde{\tau}(s) = \tau(s)\hat{\tau}(s)$ with $\tau(s) = O(s^2)$. To sum up, there is a smooth nondecreasing function $\bar{\tau}$ satisfies

$$\tilde{\tau}(|x_1|) = x_1^2\bar{\tau}(|x_1|).$$

Finally, there holds

$$\frac{\partial V_0(z)}{\partial z}f_0 \leq -(1-\epsilon)K(s) \circ \underline{\pi}(\|z\|)\pi(\|z\|) + \bar{\sigma}x_1^2\bar{\tau}(x_1).$$

This completes the whole proof. □

B. PROOF OF (33)

This section provides the detailed proof of (33). Firstly, according to Lemmas 2 and 3, there holds

$$\begin{aligned}&\beta_{i-1}(\bar{x}_{i-1},t)\lceil\xi_{i-1}\rceil^{2-\eta-h_i}\left(x_i^{p_{i-1}} - \alpha_{i-1}^{p_{i-1}}\right)\\ &\leq \beta_{i-1}(\bar{x}_{i-1},t)\,|\xi_{i-1}|^{2-\eta-h_i}\left|x_i^{p_{i-1}} - \alpha_{i-1}^{p_{i-1}}\right|\\ &\leq \beta_{i-1}(\bar{x}_{i-1},t)2^{1-p_{i-1}h_i}\,|\xi_{i-1}|^{2-\eta-h_i}\,|\xi_i|^{p_{i-1}h_i}\\ &\leq \frac{1}{4}\xi_{i-1}^2 + \phi_{i1}\xi_i^2,\end{aligned} \tag{47}$$

where ϕ_{i1} is a constant. Then, based on $|x_k|^{\frac{h_{i+1}p_i}{h_k}} \leq |\xi_k|^{p_ih_{i+1}} + |g_{k-1}|^{p_ih_{i+1}}\,|\xi_{k-1}|^{p_ih_{i+1}}$, Lemma 2, (3) and (7), one has

$$\begin{aligned}&\lceil\xi_i\rceil^{2-\eta-h_i}f_i\\ &\leq |\xi_i|^{2-\eta-h_i}\bar{f}_0 + |\xi_i|^{2-\eta-h_i}\,\bar{\theta}\sum_{k=1}^{i}|x_k|^{\frac{p_ih_{i+1}}{h_k}}\,l_i\\ &\leq |\xi_i|^{2-\eta-h_i}\bar{f}_0 + |\xi_i|^{2-\eta-h_i}\,\bar{g}_{i-1}l_i\bar{\theta}\sum_{k=1}^{i}|\xi_k|^{p_ih_{i+1}}\\ &\leq \phi_{i2}\xi_i^2 + \bar{f}_0^{\frac{2}{p_ih_{i+1}}} + \frac{1}{4}\xi_{i-1}^2 + \frac{1}{3}\sum_{k=1}^{i-2}\xi_k^2 + \tilde{\theta}\varpi_{i1}\xi_i^2,\end{aligned} \tag{48}$$

where ϕ_{i2}, ϖ_i1 and $\bar{g}_{i-1} \geq 1 + \sum_{k=1}^{i-1} g_k^{p_ih_{i+1}}$ are positive smooth functions. Next, it can be deduced from (9) that

$$\begin{aligned}&\sum_{k=1}^{i-1}\frac{\partial W_i}{\partial x_k}\dot{x}_k\\ &\leq -(2-\eta-h_i)\int_{\alpha_{i-1}}^{x_i}|\lceil s\rceil^{\frac{1}{h_i}} - \lceil\alpha_{i-1}(\bar{x}_{i-1})\rceil^{\frac{1}{h_i}}|^{1-\eta-h_i}ds\\ &\quad\cdot\sum_{k=1}^{i-1}\frac{\partial\lceil\alpha_{i-1}\rceil^{\frac{1}{h_i}}}{\partial x_k}(\beta_k(\bar{x}_k,t)x_{k+1}^{p_k} + f_k).\end{aligned} \tag{49}$$

By performing the similar procedures in [5], it is not hard to get that

$$|\frac{\partial}{\partial x_k}(\lceil\alpha_{i-1}\rceil^{\frac{1}{h_i}})f_k| \leq \gamma_{ik}(\bar{x}_{i-1})(\bar{f}_0 + \bar{\theta}\sum_{j=1}^{i-1}|\xi_j|^{\eta+1}), \tag{50}$$

where $\gamma_{ik} > 0$ is a smooth function. Obviously, (23) is the case that $i = 2$. Suppose when $i = m-1$, (50) holds, then when $i = m, k = 1, \cdots, m-2$, one has

$$\begin{aligned}&|\frac{\partial\lceil\alpha_{m-1}\rceil^{\frac{1}{h_m}}}{\partial x_k}f_k|\\ &\leq |\frac{\partial\lceil\alpha_{m-2}\rceil^{\frac{1}{h_{m-1}}}}{\partial x_k}g_{m-1}f_k| + |\frac{\partial g_{m-1}}{\partial x_k}\xi_{m-1}f_k|\\ &\leq g_{m-1}\gamma_{m-1,k}(\bar{f}_0 + \bar{\theta}\sum_{j=1}^{m-2}|\xi_j|^{\eta+1}) + |\xi_{m-1}|\cdot|\frac{\partial g_{m-1}}{\partial x_k}|\\ &\quad\cdot(\bar{f}_0 + \bar{\theta}\sum_{j=1}^{k}|x_j|^{\frac{p_kh_{k+1}}{h_j}}\bar{f}_k)\\ &\leq \bar{\theta}\sum_{j=1}^{m-1}|\xi_j|^{\eta+1}\Big(g_{m-1}\gamma_{m-1,k} + |\xi_{m-1}|^{-\eta}|\frac{\partial g_{m-1}}{\partial x_k}|\\ &\quad\cdot\sum_{j=1}^{k}|x_j|^{\frac{p_kh_{k+1}}{h_j}}\bar{f}_k\Big) + \bar{f}_0\Big(g_{m-1}\gamma_{m-1,k} + |\xi_{m-1}||\frac{\partial g_{m-1}}{\partial x_k}|\Big)\\ &\leq \gamma_{mk}(\bar{x}_{m-1})\Big(\bar{f}_0 + \bar{\theta}\sum_{j=1}^{m-1}|\xi_j|^{\eta+1}\Big),\end{aligned} \tag{51}$$

where $\gamma_{mk} \geq g_{m-1}\gamma_{m-1,k} + \bar{\theta}|\xi_{m-1}|^{-\eta}\cdot|\frac{\partial g_{m-1}}{\partial x_k}|\cdot\sum_{j=1}^{k}|x_j|^{\frac{p_kh_{k+1}}{h_j}}\bar{f}_k + |\xi_{m-1}|\cdot|\frac{\partial g_{m-1}}{\partial x_k}|$ is a positive smooth function. if $k = m-1$, there holds

$$\begin{aligned}&|\frac{\partial\lceil\alpha_{m-1}\rceil^{\frac{1}{h_m}}}{\partial x_{m-1}}f_{m-1}|\\ &\leq |f_{m-1}|(|\xi_{m-1}\frac{\partial g_{m-1}}{\partial x_{m-1}}| + \frac{g_{m-1}}{h_{m-1}}|\lceil x_{m-1}\rceil^{\frac{1}{h_{m-1}}-1}|)\\ &\leq (|\xi_{m-1}\frac{\partial g_{m-1}}{\partial x_{m-1}}| + \frac{g_{m-1}}{h_{m-1}}|\lceil x_{m-1}\rceil^{\frac{1}{h_{m-1}}-1}|)\\ &\quad\cdot(\bar{f}_0 + \bar{\theta}\sum_{j=1}^{m-1}|x_j|^{\frac{p_{m-1}h_m}{h_j}}\bar{f}_{m-1}).\end{aligned} \tag{52}$$

Applying Lemma 2, there has

$$\begin{aligned}&\bar{\theta}\sum_{j=1}^{m-1}|x_j|^{\frac{p_{m-1}h_m}{h_j}}\cdot|\lceil x_{m-1}\rceil^{\frac{1}{h_{m-1}}-1}|\\ &\leq (|\xi_{m-1}|^{1-h_{m-1}} + g_{m-2}^{1-h_{m-1}}|\xi_{m-2}|^{1-h_{m-1}})\\ &\quad\cdot(\bar{\theta}\sum_{j=1}^{m-1}(|\xi_j|^{p_{m-1}h_m} + |g_{j-1}\xi_{j-1}|^{p_{m-1}h_m}))\\ &\leq \tilde{\gamma}_{m,m-1}(\bar{x}_{m-1})(|\xi_{m-1}|^{1-h_{m-1}} + |\xi_{m-2}|^{1-h_{m-1}})\end{aligned}$$

$$\cdot\bar{\theta}\sum_{j=1}^{m-1}(|\xi_j|^{p_{m-1}h_m}+|\xi_{j-1}|^{p_{m-1}h_m})$$
$$\leq \bar{\gamma}_{m,m-1}(\bar{x}_{m-1})\cdot\bar{\theta}\sum_{j=1}^{m-1}|\xi_j|^{\eta+1}, \tag{53}$$

where $\bar{\gamma}_{m,m-1}=(2m-3)[\frac{p_{m-1}h_m}{2+\eta}\cdot(\frac{\frac{1}{h_{m-1}}-1}{(2+\eta)\frac{1}{h_{m-1}}})^{\frac{(\frac{1}{h_{m-1}}-1)h_{m-1}}{h_m p_{m-1}}}+1]\tilde{\gamma}_{m,m-1}$, $\tilde{\gamma}_{m,m-1}=(1+g_{m-1}^{1-h_{m-1}})\bar{\theta}\sum_{j=1}^{m-1}(1+g_{j-1}^{p_{m-1}h_m})$ are all positive smooth functions. Considering (52) and (53), it is not hard to get

$$|\frac{\partial\lceil\alpha_{m-1}\rceil^{\frac{1}{h_m}}}{\partial x_{m-1}}f_{m-1}|$$
$$\leq\bar{f}_0(|\xi_{m-1}|\cdot|\frac{\partial g_{m-1}}{\partial x_{m-1}}|+\frac{g_{m-1}}{h_{m-1}}|\lceil x_{m-1}\rceil^{\frac{1}{h_{m-1}}-1}|)$$
$$+|\xi_{m-1}|\cdot|\frac{\partial g_{m-1}}{\partial x_{m-1}}|^{-\eta}\cdot\bar{\theta}\sum_{j=1}^{m-1}|x_j|^{\frac{p_{m-1}h_m}{h_j}}\bar{f}_{m-1}\cdot|\xi_{m-1}|^{\eta+1}$$
$$+\frac{g_{m-1}}{h_{m-1}}\bar{f}_{m-1}\cdot\bar{\gamma}_{m,m-1}\sum_{j=1}^{m-1}|\xi_j|^{\eta+1}$$
$$\leq\gamma_{m,m-1}(\bar{x}_{m-1})(\bar{f}_0+\bar{\theta}\sum_{j=1}^{m-1}|\xi_j|^{\eta+1}), \tag{54}$$

where $\gamma_{m,m-1}\geq|\frac{\partial g_{m-1}}{\partial x_{m-1}}|(|\xi_{m-1}|+|\xi_{m-1}|^{-\eta}\cdot\bar{\theta}\sum_{j=1}^{m-1}|x_j|^{\frac{p_{m-1}h_m}{h_j}}\bar{f}_{m-1})+\frac{g_{m-1}}{h_{m-1}}(|x_{m-1}|^{\frac{1}{h_{m-1}}-1}+\bar{f}_{m-1}\bar{\gamma}_{m,m-1})$. On the basis of (51) and (54), (50) holds. In addition, following the same process in [5], one can conclude that there is a positive smooth function $\varrho_{ik}(\bar{x}_i)$ such that:

$$|\frac{\partial(\lceil\alpha_{i-1}^{\frac{1}{h_i}}\rceil)}{\partial x_k}x_{k+1}^{p_k}\beta_k(\bar{x}_{k,t})|\leq\varrho_{ik}(\bar{x}_i)\sum_{j=1}^{i}|\xi_j|^{\eta+1}, \tag{55}$$

where $i=2,\cdots,n$. Similar to (25), one has

$$-(2-\eta-h_i)\int_{\alpha_{i-1}}^{x_i}\left|\lceil s\rceil^{\frac{1}{h_i}}-\lceil\alpha_{i-1}(x_{i-1})\rceil^{\frac{1}{h_i}}\right|^{1-h_i-\eta}ds$$
$$\leq\tilde{c}_i|\xi_i|^{1-\eta}, \tag{56}$$

where $\tilde{c}_i=(2-\eta-h_i)2^{1-h_i}$ is a constant. To sum up, (50) can be rewritten as

$$\sum_{k=1}^{i-1}\frac{\partial W_i}{\partial x_k}\dot{x}_k$$
$$\leq\tilde{c}_i|\xi_i|^{1-\eta}\sum_{k=1}^{i-1}\left(\gamma_{k_k}+\varrho_{i_k}\right)$$
$$\times\left(\bar{f}_0+\sum_{j=1}^{i}|\xi_j|^{\eta+1}+\bar{\theta}\sum_{j=1}^{i}|\xi_j|^{\eta+1}\right)$$
$$\leq\phi_{i_3}\xi_i^2+\bar{f}_0^{\frac{2}{\eta+1}}+\frac{1}{3}\sum_{k=1}^{i-2}\xi_k^2+\frac{1}{4}\xi_{i-1}^2+\tilde{\theta}\varpi_{i2}\xi_i^2, \tag{57}$$

where ϕ_{i3} and ϖ_{i2} are positive smooth functions. On the other hand, let $\varpi_i=\varpi_{i1}+\varpi_{i2}$, and taken Lemma 3 and (9) into consideration, we have

$$\frac{\partial W_i}{\partial\hat{\theta}}\sum_{k=1}^{i-1}\varpi_k\xi_k^2+\sum_{k=2}^{i}\varpi_i\xi_i^2\leq\frac{1}{3}\sum_{k=1}^{i-2}\xi_k^2+\frac{1}{4}\xi_{i-1}^2+\phi_{i4}\xi_i^2, \tag{58}$$

where ϕ_{i4} is a positive function. Finally, defining $\phi_i=\phi_{i1}+\phi_{i2}+\phi_{i3}+\phi_{i4}$ and performing simple substitution operation, it is directly deduced from (47), (48), (57), and (58) that the inequality (33) holds. □

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