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RESEARCH ARTICLE

Modeling Method for Pose Error of Machine Tool Linear Feed System Based on the Principle of Minimum Potential Energy

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ABSTRACT In the present research, a method to model the pose error of the machine tool feed system is proposed based on the principle of minimum potential energy. Firstly, the force-moment equilibrium equation of the feed system is established by mechanical analysis. Secondly, the force of the single slider is analyzed based on Hertzian contact theory, and the force state of the slider under the action of the guideway error is obtained. The elastic displacements of the single slider in the horizontal and vertical directions are solved based on the principle of the minimal potential energy. Subsequently, the pose error of the feeding system is obtained by using the plane fitting principle. Finally, the correctness of the proposed method is verified through experiments, and the experimental results are in good agreement with the theoretical results.

INDEX TERMS Precision machine tool, feed system, pose error, modeling method, the principle of minimum potential energy.

I. INTRODUCTION

Machine tool is the foundation of equipment manufacturing [1], [2]. The overall performance of machine tools is affected by the linear feed system. The error shapes of guideways are different and coupled to each other, resulting in complex and variable internal forces in the feed system. It is difficult to predict the accuracy and accuracy stability/retention of the feed system. The main reason is that it is difficult to establish a mathematical model between the pose error of the linear feed system and the geometric error of the guideway.

There are three main methods for modeling the pose error of linear feed system, including static balance method, equivalent stiffness modeling method and finite element method. The core idea of the static equilibrium modeling method is to analyze the force state of the linear feed system based on its structural characteristics. And then the equilibrium equations can be obtained based on the directions of force and couple moments. Fan et al. [3] used the static equilibrium equation to solve for the reaction forces generated on the contact surface between the slider and the guide, and the accuracy wear model of the linear axis of the machine tool was established on this basis. Sun et al. [4] established the error model of the repeatability of positioning of linear feed systems by combining material hysteresis with the static equilibrium modeling method, and investigated the mechanism of the effect of assembly error on the repeatability of positioning error. Zha et al. [5] build a static analysis model based on the static equilibrium modeling method and studied the different trends of straightness errors. However, the static balance method can only be used to model the pose error of a linear feed system in an ideal state. In fact, assembly errors inevitably exist in the linear feed system after assembly.

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Therefore, the static balance modeling method is difficult to establish an accurate model between assembly error and pose error.

The equivalent stiffness modeling method is developed on the basis of the Hertzian contact theory and the static equilibrium modeling method. The equivalent stiffness of the guideway slider is considered in the modeling process. Khim et al. [6], [7] used the transfer function theory and the equivalent stiffness modeling method to analyze the pose error of the table. The ratio of the combined force on the slider to the geometric error of the guideway was equated to the stiffness K(0) of the guideway slider, and the pose error of the linear was obtained. And they proposed a method to calculate the five degrees of freedom pose error of the table, and achieved the prediction of the five-term pose error of the linear axis supported by the air-bearing guide. Kim et al. [8] used the equivalent stiffness modeling method to develop a reaction force moment model for a double-spring system, which taking into account the reaction force-moment caused by a pad pitch error, and proposed a new method to measure the straightness and parallelism of the guideway that further improves the prediction accuracy. There is a good versatility in the equivalent stiffness modeling method, and during the modeling process the profile error of the guideway raceway is required. However, due to the great difficulty in measuring the profile error of the guideway raceway, the prediction accuracy of the equivalent stiffness model is limited.

With the development of computer simulation technology, finite element modeling technology is more widely used in the field of machine tools. Majda [9] proposed a method to analyze the effect geometry error of linear guide on the table pose error, equated roller as spring units based on the nonlinear characteristics of the guide geometry error, and simulated guide geometry error by changing the length of spring units. By this method, the finite element model of the dual guideway four-slider system was established and the error analysis was carried out. He et al. [10] presented a method of error measurement and identification by using a stress test plate and built a finite element model to find the critical measurement points of the plates in the method. The disadvantages of the finite element modeling method are that the modeling process is cumbersome, time-consuming, and the model is not universal.

There are some scholars proposed their own modeling approaches in addition to these three approaches. Ekinci and Mayer [11] proposed an error analysis method for machine tools based on error classification to study the relationship between linear axis angular error and straightness error of machine tools. Khan and Chen [12] used the integration method to model the mapping between the straightness error of the guideway and the pose error of table. Ma et al. [13] studied the effect of part manufacturing error and deformation on the pose error of linear axis. He et al. [14] proposed a hierarchical error modeling method based on the static equilibrium method and the equivalent stiffness method. However,

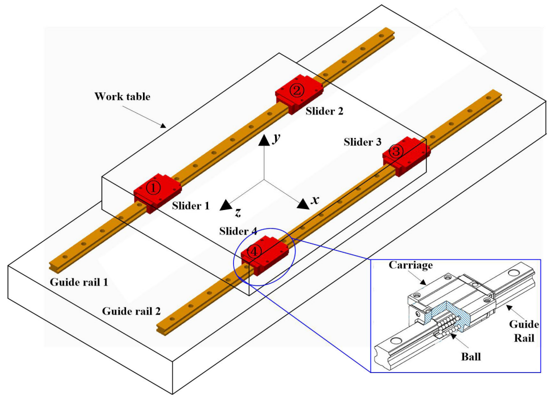

FIGURE 1. Linear feed system of the machine tool.

poor versatility is a disadvantage of these methods, which leads to their applicability only to specific structures.

In summary, the shortcomings of current modeling methods include: (1) poor universality of the model; (2) It is necessary to assume that the error shapes of the two guideways are the same, which does not match the actual assembly requirements.

Therefore, this paper proposed a new and universal modeling method for the feed system. Based on the mechanical balance equation, the principle of minimum elastic potential energy for the system to be in equilibrium under the influence of geometric errors of the guideway is proposed. The elastic deformations of four sliders are calculated, and the pose error of the feed system is solved using the plane fitting method. The proposed method can be used to model the pose error of feed systems with inconsistent shape errors between two guideways.

II. MODELING OF POSE ERROR BASED ON THE PRINCIPLE OF MINIMUM POTENTIAL ENERGY

The structure of the feed system of the machine tool is shown in Fig.1. The worktable is connected to the guideway through four sliders. The rolling ball is in contact with both the guideway and the slider, rolling along the surface of the guideway. Therefore, the pose error of the worktable is significantly affected by the geometric error of the guideway.

A. MECHANICS ANALYSIS OF SINGLE SLIDER

In order to study the influence of geometric errors of the guideway on the motion error of the worktable, it is necessary to conduct force analysis on the slider. The assumptions for analyzing the single slider of the feed system are as follows: 1) The geometric error only exists on the raceway of the guideway. 2) The roller is an elastomer and the guideway is a rigid body. 3) The guideway errors suffered by the rolling elements in the slider are the same.

Fig. 2 shows the curvature centers of the raceway groove, O_{ri} and O_{ci} are the raceway groove curvature centers of rail and carriage. There are four form error vectors for the rail,

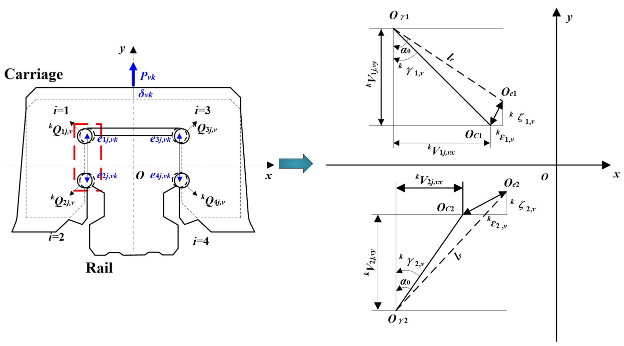

FIGURE 2. Geometry of the ball - raceway contact in the flexible under vertical load.

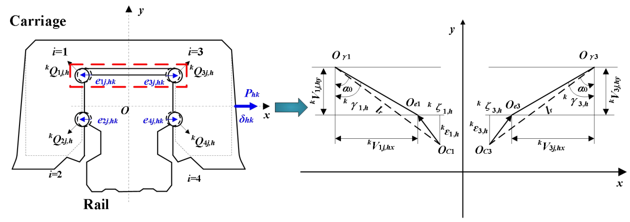

FIGURE 3. Geometry of the ball - raceway contact in the flexible under horizontal load.

assuming that all the four error vector components are equal to the average values.

$$e_{vk} = e_{1j,vk} = e_{2j,vk} = e_{3j,vk} = e_{4j,vk} \tag{1}$$

$$e_{hk} = e_{1j,hk} = e_{2j,hk} = e_{3j,hk} = e_{4j,hk} \tag{2}$$

According to the flexible model established by Ohta and Tanaka [15], when a vertical load is applied to the carriage, the outward elastic deformation will be occurred at the carriage skirt.

Due to the geometric errors and preload of the guideway, a single ball in the raceway is subjected to the normal load Q_{ij}. The elastic deformation of each ball $\xi_{ij,y}$ can be expressed as

$$^k\xi_{ij,v} = \sqrt{^kV_{ij,vx}^2 + {}^kV_{ij,vy}^2} - l_r + \chi_p \tag{3}$$

$$l_r = 2R - D_a \tag{4}$$

$$^kV_{ij,vy} = l_r \cos\alpha_0 + {}^k\zeta_{i,v} \tag{5}$$

$$^kV_{ij,vx} = l_r \sin\alpha_0 - {}^k\varepsilon_{i,v} \tag{6}$$

where χ_p is the ball preload, l_r is the original distance between the centers of curvature O_{ri} and O_{ci}. D_a is the original diameter of the balls, and R is the radius of curvature of the rail and the bearing block grooves. The superscript k represents the kth slider, in this paper, there are four sliders in the feed system, as shown in Fig. 1, so $k = 1,2,3,4$.

$\zeta_{i,v}$ is the equivalent deformation in the vertical direction, $\varepsilon_{i,v}$ is the outward deformation in the horizontal direction [16].

When $i = 2$ or 4

$$^k\zeta_{i,v} = e_{vk} + \delta_{vk} \tag{7}$$

$$\begin{aligned} ^k\varepsilon_{i,v} = {} & 3.28 \times 10^{-6} \cdot {}^kQ_{2j,v} \sin {}^k\gamma_{2,v} \\ & + 8.86 \times 10^{-6} \cdot {}^kQ_{1j,v} \sin {}^k\gamma_{1,v} \end{aligned} \tag{8}$$

When $i = 1$ or 3

$$^k\zeta_{1,v} = e_{vk} - \delta_{vk} \tag{9}$$

$$\begin{aligned} ^k\varepsilon_{i,v} = {} & 8.86 \times 10^{-6} \cdot {}^kQ_{2j,v} \sin {}^k\gamma_{2,v} \\ & + 1.59 \times 10^{-5} \cdot {}^kQ_{1j,v} \sin {}^k\gamma_{1,v} \end{aligned} \tag{10}$$

$\gamma_{i,v}$ is the actual contact angle under vertical load, can be calculated as:

$$\sin {}^k\gamma_{i,v} = \frac{^kV_{ij,vx}}{\sqrt{^kV_{ij,vx}^2 + {}^kV_{ij,vy}^2}} \tag{11}$$

By the Hertz contact theory, the bearing force $Q_{ij,v}$ due to the rail form errors can be obtained.

$$^kQ_{ij,v} = (2C)^{-\frac{3}{2}} \sqrt{D_a} \left({}^k\xi_{ij,v}\right)^{\frac{3}{2}} \tag{12}$$

According to the static balance of the carriage in the vertical direction, the balance equation can be expressed as (13), and P_{vk} can be obtained.

$$\begin{aligned} & P_{vk} + \sum_{j=1}^{p} \left({}^kQ_{1j,v} \cdot \cos {}^k\gamma_{1,v} + {}^kQ_{3j,v} \cdot \cos {}^k\gamma_{3,v}\right) \\ & = \sum_{j=1}^{p} \left({}^kQ_{2j,v} \cdot \cos {}^k\gamma_{2,v} + {}^kQ_{4j,v} \cdot \cos {}^k\gamma_{4,v}\right) \end{aligned} \tag{13}$$

where, p is the number of roller.

Similarly, the horizontal force of the slider P_{hk} can be obtained according to Fig.3.

$$^k\xi_{ij,h} = \sqrt{^kV_{ij,hx}^2 + {}^kV_{ij,hy}^2} - l_r + \chi_p \tag{14}$$

$$^kV_{ij,hy} = l_r \cos\alpha_0 - {}^k\varepsilon_{i,h} \tag{15}$$

$$^kV_{ij,hx} = l_r \sin\alpha_0 + {}^k\zeta_{i,h} \tag{16}$$

ζ_{ih} is the equivalent deformation in the horizontal direction, ε_{ih} is the outward deformation in the vertical direction.

When $i = 1$ or 2

$$\begin{aligned} ^k\varepsilon_{i,h} = {} & 3.28 \times 10^{-6} \cdot {}^kQ_{1j,h} \sin {}^k\gamma_{1,h} \\ & + 8.86 \times 10^{-6} \cdot {}^kQ_{3j,h} \sin {}^k\gamma_{3,h} \end{aligned} \tag{17}$$

$$^k\zeta_{i,h} = e_{hk} + \delta_{hk} \tag{18}$$

When $i = 3$ or 4

$$\begin{aligned} ^k\varepsilon_{i,h} = {} & 8.86 \times 10^{-6} \cdot {}^kQ_{1j,h} \sin {}^k\gamma_{1,h} \\ & + 1.59 \times 10^{-5} \cdot {}^kQ_{3j,h} \sin {}^k\gamma_{3,h} \end{aligned} \tag{19}$$

$$^k\zeta_{i,h} = e_{hk} - \delta_{hk} \tag{20}$$

$$\sin {}^k\gamma_{i,h} = \frac{^kV_{ij,hx}}{\sqrt{^kV_{ij,hx}^2 + {}^kV_{ij,hy}^2}} \tag{21}$$

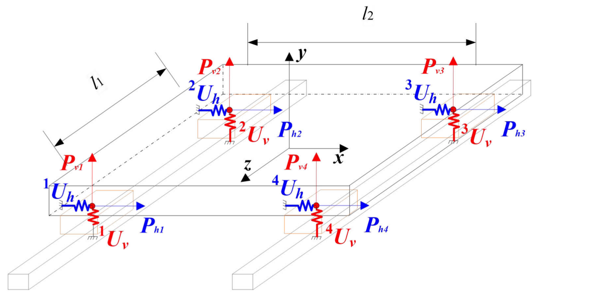

FIGURE 4. Mechanical and potential energy analysis of worktable.

$$^{k}Q_{ij,h} = (2C)^{-\frac{3}{2}}\sqrt{D_a}\left(^{k}\xi_{ij,h}\right)^{\frac{3}{2}} \tag{22}$$

$$P_{hk} + \sum_{j=1}^{p}\left(^{k}Q_{3j,h}\cdot\cos{}^{k}\gamma_{3,h} + {}^{k}Q_{4j,h}\cdot\cos{}^{k}\gamma_{4,h}\right) = \sum_{j=1}^{p}\left(^{k}Q_{1j,h}\cdot\cos{}^{k}\gamma_{1,h} + {}^{k}Q_{2j,h}\cdot\cos{}^{k}\gamma_{2,h}\right) \tag{23}$$

B. MECHANICS ANALYSIS OF FEED SYSTEM

The worktable moves in a reciprocating linear motion along the z direction, as shown in Fig.4. The worktable is subjected to four forces (P_{h1}, P_{h2}, P_{h3}, and P_{h4}) in the horizontal direction and four forces (P_{v1}, P_{v2}, P_{v3}, and P_{v4}) in the vertical direction transmitted from the slider.

The force equilibrium equation and the moment equilibrium equation are expressed as:

$$\begin{cases} P_{v1}+P_{v2}+P_{v3}+P_{v4}+F_y=0 \\ (P_{v1}+P_{v2}-P_{v3}-P_{v4})\,l_2/2+M_z=0 \\ (-P_{v1}+P_{v2}+P_{v3}-P_{v4})l_1/2+M_x=0 \\ P_{h1}+P_{h2}+P_{h3}+P_{h4}+F_x=0 \\ (P_{h1}-P_{h2}-P_{h3}+P_{h4})l_1/2+M_y=0 \end{cases} \tag{24}$$

In order to solve the equations, it is necessary to analyze the conditions of the feed system in a mechanical equilibrium state. According to the principle of minimum potential energy, a system would be in an equilibrium state when the potential energy of the system is minimal. Therefore, it is necessary to analyze the elastic potential energy of the feed system.

C. ANALYSIS OF ELASTIC POTENTIAL ENERGY

There are two reasons for providing minimum potential energy. On the one hand, the rolling element in the guideway slider is in a pre-tightened state, which generates elastic potential energy; On the other hand, the geometric error between the two guideways supporting the worktable results in different stress states of the four sliders, which in turn leads to different elastic deformations of the rolling elements in the four sliders, resulting in the generation of elastic potential energy.

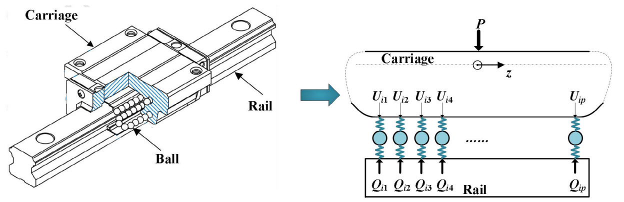

FIGURE 5. The linear rolling guide and elastic potential energy.

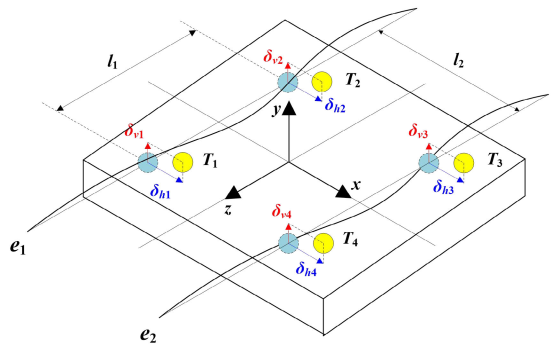

FIGURE 6. The coordinates of the four sliders.

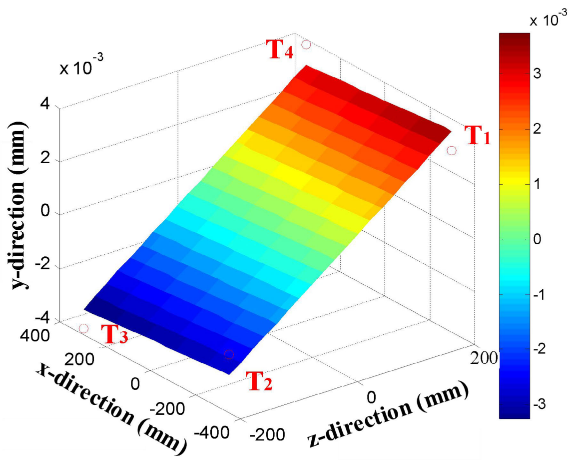

FIGURE 7. Surface plane fitting of worktable.

The linear rolling guide and elastic potential energy are shown in Fig. 5. Elastic potential energy of a ball in the vertical direction and horizontal direction can be expressed as:

$$^{k}U_{ij,v} = \frac{1}{2}K\cdot{}^{k}\xi_{ij,v}^{2} \tag{25}$$

$$^{k}U_{ij,h} = \frac{1}{2}K\cdot{}^{k}\xi_{ij,h}^{2} \tag{26}$$

where K is the stiffness coefficient.

The total elastic potential energy of the k-th slider in the vertical and horizontal directions can be obtained.

$$^{k}U_{v} = \sum_{i}^{4}\sum_{j}^{p}{}^{k}U_{ij,v} \tag{27}$$

$$^{k}U_{h} = \sum_{i}^{4}\sum_{j}^{p}{}^{k}U_{ij,h} \tag{28}$$

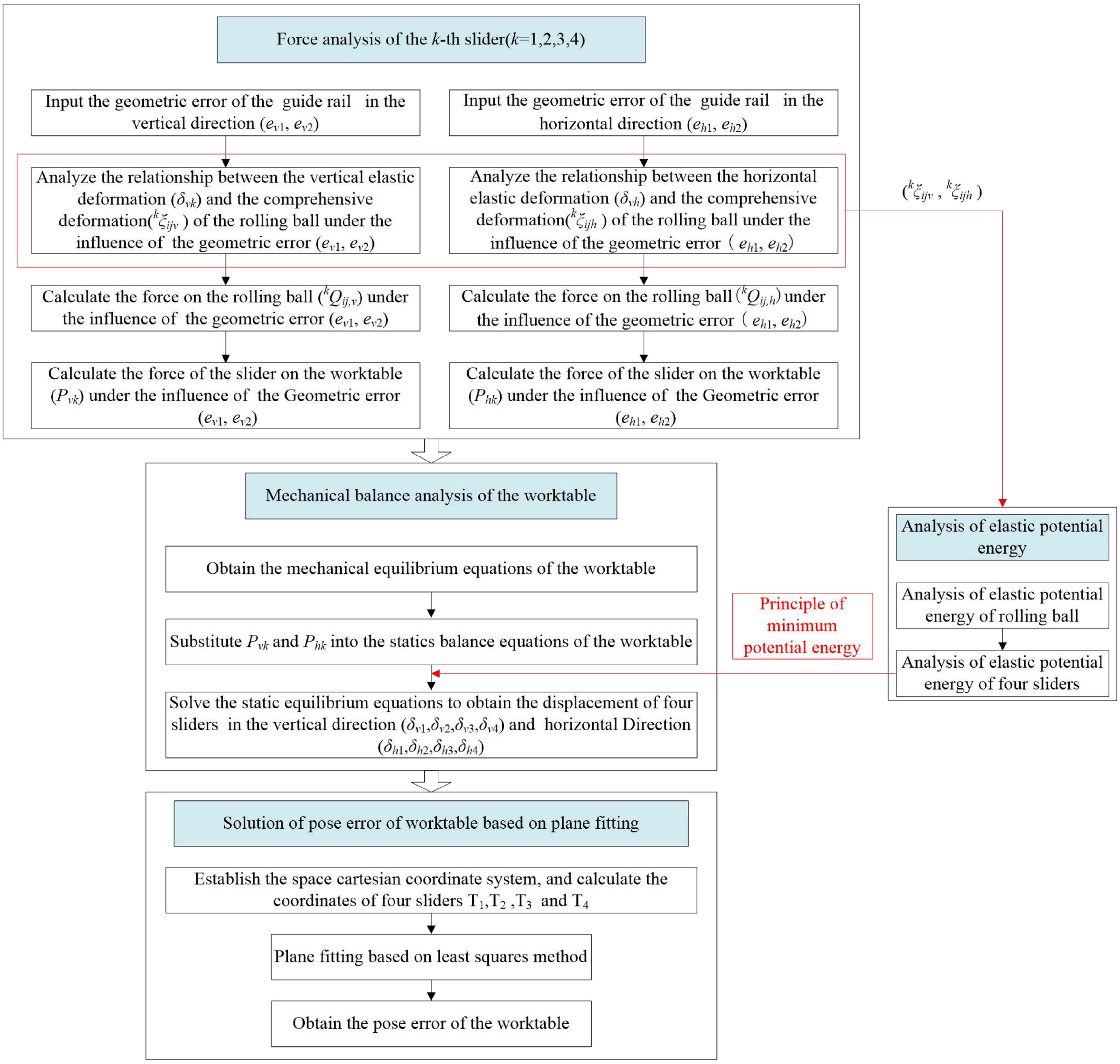

FIGURE 8. Flowchart of pose error based on the principle of minimum potential energy calculation.

The equations for the principle of minimum potential energy are shown as

$$U_{V,\min} = {}^1U_v + {}^2U_v + {}^3U_v + {}^4U_v \tag{29}$$

$$U_{H1,\min} = {}^1U_h + {}^2U_h + {}^3U_h + {}^4U_h \tag{30}$$

The system would be in an equilibrium state of when the difference in potential energy between the two guideways is minimal. Additional equations are obtained as follows:

$$U_{H2,\min} = {}^1U_h + {}^2U_h - {}^3U_h - {}^4U_h \tag{31}$$

By combining (24) to (31), δ_{v1}, δ_{v2}, δ_{v3}, δ_{v4}, δ_{h1}, δ_{h2}, δ_{h3} and δ_{h4} can be obtained.

D. ANALYSIS OF POSE ERROR

The coordinate system is established with the center of the workbench as the origin, as shown in Fig.6. The coordinates of the four sliders are $T_1(-l_2/2+\delta_{h1}, \delta_{v1}, l_1/2)$, T_2 $(-l_2/2+\delta_{h2}, \delta_{v2},-l_1/2)$, T_3 $(l_2/2+\delta_{h3}, \delta_{v3},-l_1/2)$ and T_4 $(l_2/2+\delta_{h4}, \delta_{v4}, l_1/2)$, respectively. Curved surface of worktable is fitted to a plane using the least squares method with the T_1 to T_4.

The general expression for the plane equation can be expressed as:

$$Ax + By + Cz + D = 0 \tag{32}$$

Equation (32) is equivalently transformed to obtain the following equation.

$$z = a_0x + a_1y + a_2 \tag{33}$$

where $a_0 = -\frac{A}{C}, a_1 = -\frac{B}{C}, a_2 = -\frac{D}{C}$

The coordinates of these four points (T_1, T_2, T_3, T_4) in the spatial coordinate system are represented as:

$$(x_i, y_i, z_i), i = 0, 1, \cdots 4 \tag{34}$$

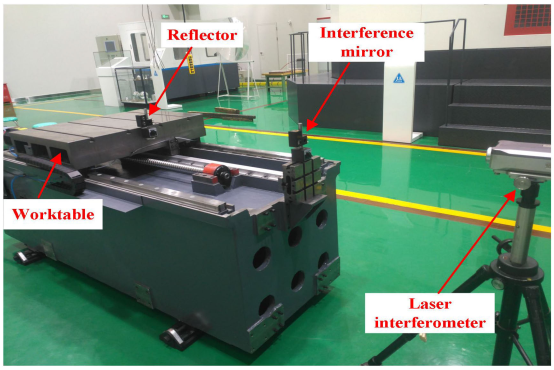

FIGURE 9. Experimental setup.

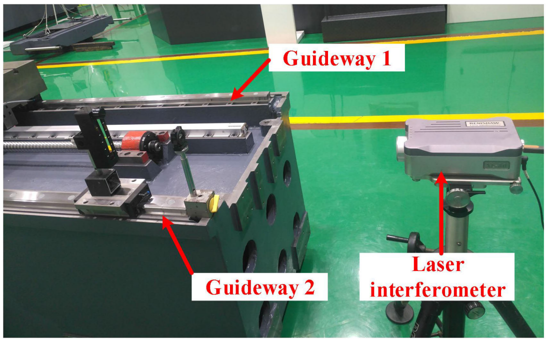

FIGURE 10. Measurement of guideway straightness error.

Plane fitting is performed by using the least squares method to minimize the sum of squares of the distances from these four points to the plane.

$$\min\left\{S=\sum_{i=0}^{4}\left(a_0x+a_1y+a_2-z\right)^2\right\} \tag{35}$$

To minimize S, $\frac{\partial S}{\partial a_k}=0, k=0,1,2$ should be satisfied, which means (36) needs to be satisfied.

$$\begin{cases}\sum_{i=0}^{4}2\left(a_0x_i+a_1y_i+a_2-z_i\right)x_i=0\\ \sum_{i=0}^{4}2\left(a_0x_i+a_1y_i+a_2-z_i\right)y_i=0\\ \sum_{i=0}^{4}2\left(a_0x_i+a_1y_i+a_2-z_i\right)=0\end{cases} \tag{36}$$

Equation (36) can be equivalently transformed into matrix (37).

$$\begin{bmatrix}\sum_{i=0}^{4}x_i^2 & \sum_{i=0}^{4}x_iy_i & \sum_{i=0}^{4}x_i\\ \sum_{i=0}^{4}x_iy_i & \sum_{i=0}^{4}y_i^2 & \sum_{i=0}^{4}y_i\\ \sum_{i=0}^{4}x_i & \sum_{i=0}^{4}y_i & 4\end{bmatrix}\begin{bmatrix}a_0\\a_1\\a_2\end{bmatrix}=\begin{bmatrix}\sum_{i=0}^{4}x_iz_i\\ \sum_{i=0}^{4}y_iz_i\\ \sum_{i=0}^{4}z_i\end{bmatrix} \tag{37}$$

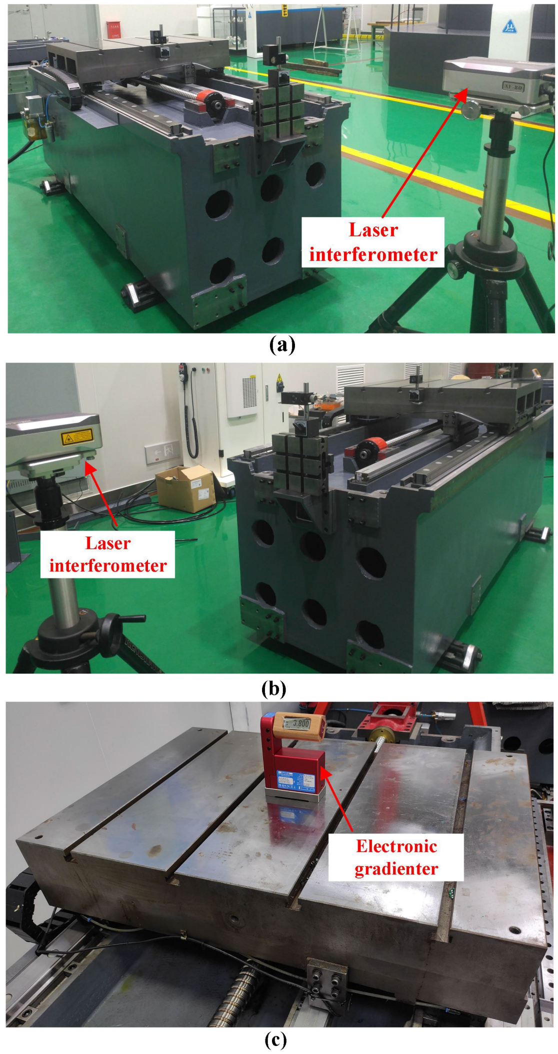

FIGURE 11. Measurement of angle error. (a) Measurement of pitch error. (b) Measurement of yaw error. (c) Measurement of roll error.

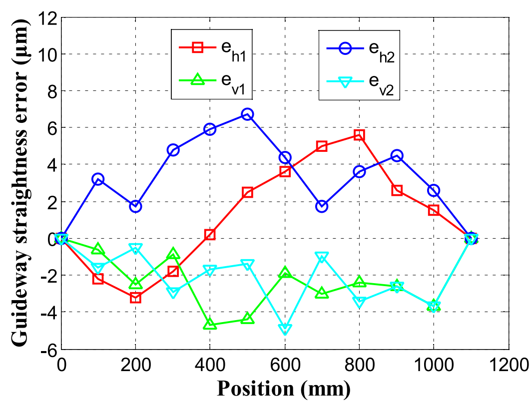

FIGURE 12. Measurement results of guideway geometric errors.

By using MATLAB software to solve the three parameters of matrix (37). a_0, a_1, and a_2, which are solved and brought into (33), the fitting plane equation can be obtained, as shown in

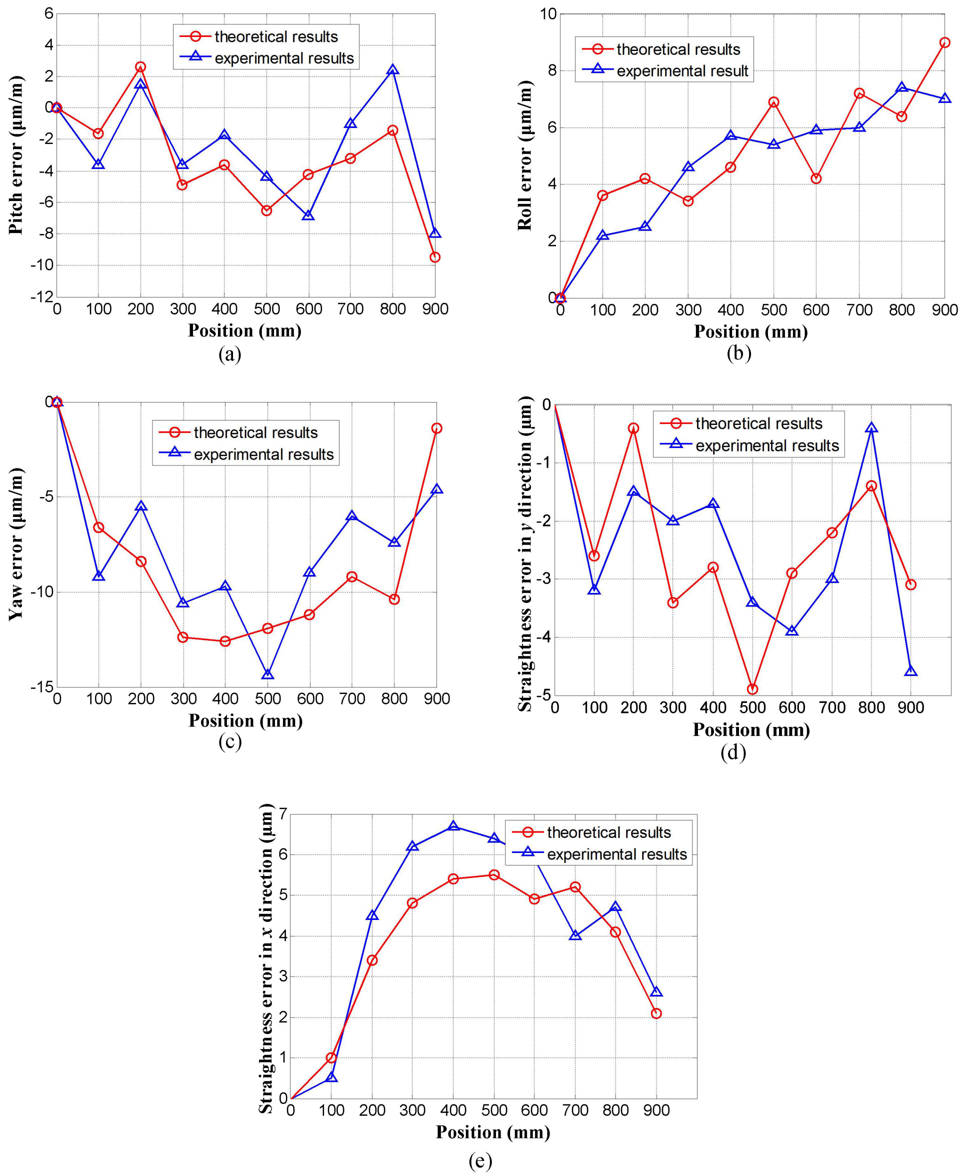

FIGURE 13. **Comparison of theoretical and experimental results. (a) Pitch error. (b) Roll error. (c) Yaw error. (d) Straightness error in y direction. (e) Straightness error in x direction.**

Fig. 7. The fitted plane is used to define the pose error of the table.

The modeling and solving process of the pose error of worktable based on the principle of minimum potential energy is shown in Fig. 8.

III. EXPERIMENTAL VERIFICATION

A. EXPERIMENTAL SETUP

Experimental validation is carried out to verify the correctness of the proposed model. The test table is consisted of a bed, two guideways, four slides, ball screw, worktable, scale, motor, and CNC system, as shown in Fig. 9. The worktable is supported by two rolling linear guideways and performs reciprocating linear motion driven by a ball screw. The test table is equipped with a linear encoder and the motion accuracy can be closed-loop controlled. Table 1 provides the specifications for the test table used in the experiment.

B. EXPERIMENTAL RESULTS

The experiment is carried out in three steps. Firstly, the geometric errors of the two guideways in the vertical direction and horizontal direction are measured using a laser interferometer, as shown in Fig. 10. Secondly, the pose errors include pitch error, yaw error, and two straightness error are measured with a interferometer, and the roll error is measured using an electronic gradienter, as shown in Fig. 11. Finally, the measured geometric error of the guideway is brought into the theoretical model in Section II to obtain the theoretical pose

TABLE 1. Specific parameters of the experimental setup.

Parameters	Specifications
Numerical control system	Fanuc 0iF(1)
Motor	Fanuc α iF 22/3000-B
Linear encoder	HEIDENHAIN,LC195F-ML1240
Guideway	THK:SHS45LC2SSC1+2100LP-II
Ball Screw	THK:SBN4016-5RRG0+1930LC2
Distance between two guideways	650 mm
Slider spacing	380 mm
Experimental table travel	1100 mm

error of the worktable, and compared with the experimental results.

The experiment is conducted five times on a full closed loop control machine tool in a constant temperature environment, and the results of the five experiments are averaged. The measurement results of guideway geometric errors are shown in Fig. 12. The red and green lines represent the straightness error of guideway 1 in the horizontal and vertical directions. And the blue and cyan lines represent the straightness error of guideway 2 in the horizontal and vertical directions. It can be seen that the straightness error of guideway 1 in the vertical direction e_{v1} is -4.7μm, and in the horizontal direction e_{h1} is 5.6μm. For guideway 2, the straightness error e_{v2} is -4.9μm in the vertical direction, and e_{h2} is 6.7μm in the horizontal direction. The obtained e_{v1}, e_{h1}, e_{v2} and e_{h2} are introduced into the theoretical model to solve for the pose error.

Results of the comparison between the estimated and experimental data are presented in Fig. 13. It can be seen that the estimated motion error is very close to the experimental result. The results show that the theoretical results were in good agreement with the experiment. The difference between the predicted and experimental results is caused primarily by the thermal error, and random error in the movement process of the worktable. The measurement results prove the validity of the presented model.

IV. CONCLUSION

In this paper, a new method for modeling the pose error of linear feed system based on the principle of minimum potential energy is proposed and verified by experiments. The following conclusions are obtained:

1) In this method, the elastic potential energy of the feed system under the action of guideway error is analyzed and the minimum sum of elastic potential energy is taken as the condition for the system to reach equilibrium state. This method provides a new method for modeling the pose error of the feed system.

2) The advantage of this method is that it provides a new and more universal modeling method for the linear feed system, which does not assume that the error forms of two guideways are the same and can be applied to various types of errors of the guideways. Compared to traditional methods, the proposed method has stronger universality and higher prediction accuracy of the model.

3) The method not only provides a more accurate model of the pose error, but also provides a reference for the accuracy modeling of other machines with similar structures.

4) The proposed method established a more accurate mathematical model between the geometric error of the guideway and the pose error of the worktable, which can provide guidance for designers and engineers to control the error of the guideway during the assembly process of the machine tool. It can effectively improve the assembly efficiency of machine tools and reduce costs.

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