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TOPICAL REVIEW

Synthesis Identification Analysis for Closed Loop EIV System

WANG JIANHONG AND OUYANG QING

School of Electronic Engineering and Automation, Jiangxi University of Science and Technology, Ganzhou 34100, China

Corresponding author: Wang Jianhong (wangjianhong@nuaa.edu.cn)

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ABSTRACT This new paper combines errors-in-variable (EIV) system with closed loop situation to consider a more complex closed loop EIV system. Consider the identification problem for closed loop EIV system, synthesis analysis is completed from the point of system identification field. Firstly, the reason about why closed loop EIV system is useful in academy and practice is explained in detail. Secondly, to identify that unknown plant, existing in closed loop EIV system with the measurement and process noise simultaneously, nonparametric estimate is proposed to be one plant estimate through our own mathematical derivation. Then statistical analysis above nonparametric estimate is studied to show the unbiased property. Thirdly, for the rational transfer function as the unknown plant, parameterized by one unknown parameter vector, the plant identification is transferred into one parameter estimate, so after reformulating the parameterized input-output relation for closed loop EIV system, least squares solution is derived to be biased. To avoid this explicit biased term, bias compensated estimate and an iterative least squares estimate are proposed to improve the overall identification performance. Finally, one practical example is to show the existence of our considered closed loop EIV system, and the detailed numerical results prove our proposed identification strategies.

INDEX TERMS Closed loop EIV system, synthesis analysis, nonparametric estimate, parametric estimate.

I. INTRODUCTION

System identification applies the data to generate one mathematical model for the considered unknown plant or system, and the data are collected through some physical devices. After collecting some observed data in priori, then the mission of system identification is to extract the useful information for the unknown plant by virtue of some existed methods, such as statistical strategy, machine learning and deep learning etc. These use information can be expressed as some explicit forms, for example, mathematical equation, figure, graph, table and others, but mathematical equation is more widely accepted, due to its convenient tool for modeling and analysis purposes. The reason about why the research on system identification appears so popularly is that the identified mathematical equation, corresponding to the unknown plant, will be used in latter controller design process. It means the controller design process regards the mathematical equation as the model basis, then the final controller is dependent on the mathematical equation. More generally, all natural plants are operating in open loop or closed loop environment with some unavoidable noises, so the existed research on system identification mainly concern on open loop system identification and closed loop system identification. But recent practical results show unstability always exists for open loop system due to disturb, noise and bad mathematical equation for the unknown plant, so now all plants must be operated of controlled within one closed loop system, leading to closed loop system identification and next closed loop controller design.

Roughly speaking, two control structures exist in both industry and academy, i.e. open loop structure and closed loop structure. Before 1960s, open loop structure was used and studied to let the input signal go through the feed forward controller and the considered plant directly, then the collected output signal was determined whether it was satisfied or not.

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And this satisfied requirement or condition was measured by one expected criterion in priori. Latter, thanks to the stability analysis and other interesting analysis, such as tracking property, robust analysis, etc, researchers found there were lots of shortcomings for open loop structure, for example, unstable system and unable to track the desired or expected signal, so closed loop structure was proposed to replace the classical open loop structure. Within the framework of closed loop structure, the output signal is returned back to the input part, then the error, obtained by the subtract operation, is passed to the feed forward controller. Meanwhile, one or more feedback controllers can be added in the feedback path. Due to more controllers exist in closed loop system, i.e. the feed forward controller and feedback controller, so more goals or expected properties are achieved although the complexity and computational load are increased, but they are tolerable in this advanced times.

Consider the problem of closed loop system identification, three kinds of identification strategies are always used, i.e. direct, indirect and their combinations, referring to any book about system identification. As the unknown plant and unknown controller exist in closed loop system simultaneously, so firstly the unknown plant is identified from the theory of system identification field. Then secondly, this identified or estimated plant, corresponding to one explicit mathematical equation, is used to design the controller from the different control goals, i.e. tracking perfectly, robust property, adaptive property, etc, named as model reference tracking control, robust control and adaptive control respectively. Specifically, during the closed loop system, the unknown plant and controller are all deemed as the stochastic case, meaning stochastic noise, and stochastic noise is only imposed on the output signal. It means the input signal is not influences, and the output signal is corrupted by external noise. But this case does not hold in practice, whose input signal and output signal are all corrupted by the measurement and process noise respectively, referring to errors-in-variables system (EIV).

During previous ten years, lots of contributions are proposed to identify EIV system, but due to space limitations, here we only list some not all of them. A generalized instrumental variable method is proposed for EIV identification problems [1], and the detailed statistical analysis of this generalized instrumental variable method is given too. Reference [2] formulates all identification method for EIV system and points our the future perspective. A fast algorithm for EIV filtering is considered in [3], where filtering, estimation and prediction are analyzed together. Nonparametric identification for the unknown plant, existing in linear dynamic EIV system, is derived to estimate one transfer function form [4]. Furthermore, [5] uses a quasi-stationary input to excite EIV system, while identifying it within the frequent domain. To improve the identification accuracy, one additional regularized term is added to guarantee the unbiased estimation [6]. One modified dynamic iterative PCA is proposed to identify EIV ARX models [7], where the unknown plant is modeled as ARX model. Reference [8] analyzes uniform confidence bands for nonparametric EIV regression, and confidence means the identification accuracy is acceptable with one probability level. EIV system exists whatever in academy and economics, meaning some economic phenomenons can be modeled as one EIV system [9]. Other than above references on EIV system identification, identifiability is considered in the behavioral setting for EIV system [10] respectively. The fact about EIV exist is true,and also closed loop structure is more than open loop structure, so this paper combines them together to form a new closed loop structure, whose input and output are all corrupted by the measurement and process noise. Under this circumstance, we call it as closed loop EIV system. To achieve the perfect identification mission, we have some previous contributions on closed loop system identification,then we need to extend our previous contributions to identify this new closed loop EIV system. For example, [11] proposes stealth identification strategy for closed loop system, but the initial estimation is difficult to define, because it is very hard to guarantee the initial estimation approach to its real value [12]. The fact about that closed loop EIV system exists is proved in aircraft flutter model [13], where the aircraft flutter model corresponds to our named closed loop EIV system through many experiments of wind tunnel. Consider the controller design problem for closed loop EIV system, direct data strategy is proposed to satisfy the safety [14] and design one data driven controller from the observed data directly. Furthermore, our previous contributions reply the problem from [15], where the measurement noise must be chosen approximately to satisfy one necessary condition.

System identification is widely studied in academy and engineering, for example UAV system identification. More specifically, in UAV system identification, the total number of observations, use to extract the intrinsic principle of the considered system, is the sample size [16]. In case of the number of observations be more exceed this sample size, then the input is persistent excitation, while the identification model satisfies the expected accuracy. From the knowledge of system identification theory, the situation with observed disturbance or noise in the output corresponds to the robust system identification [17], which being also extended to robust optimal control. When using the probabilistic or statistical inference in system identification theory in [18] to measure the asymptotic accuracy about the final identification model. Furthermore in recent years, risk sensitive theory and reinforce learning are all introduced in system theory and advanced control theory [19] and [20], i.e. the risk decision and limitations of policies were considered during the whole process of identification and controller design. Then the final identification system or plant is more realistic then classical theoretical result. From these ongoing subjects about applying risk theory, dynamic programming and probabilistic limitation for system identification and control theory, we are thinking to extend graph theory and topology to system identification. More specifically, the second step-model structure choice is related with graph theory, i.e. the chosen model is constructed as one network system, being formulated as graph theory. System identification theory is not only for our considered aircraft system identification, but also for robot system identification in [21]. As lots of identification processed are transformed into their corresponding constrain optimization problems, so some existed optimization results can be applied directly, for example, convex optimization [22], scenario optimization [23], and scenario robust control [24], etc. Consider the last step for system identification-model validation, some nice properties are satisfied for the final identification model or designed controller, such as controllability, stochastic chance constraints, robustness and nonlinearity [25]. The goal of experiment design is to maximize the information content in the data, subject to practical constraints, for example [26],limits on input or output amplitude to ensure that a linear model structure can be used to estimate parameters from the measured data. Reference [27] considers the identification of output error (OE) model, for the case of constrained output variance, and for the purpose of increasing the practical value of the filter, a heuristic modification is performed. Also, an optimal input is obtained by a minimum variance controller with a Gaussian reference signal [28].

Based on above mentioned references on EIV system identification and our previous contributions on closed loop system identification, this new paper studies synthesis identification analysis for that new structure-closed loop EIV system. Closed loop EIV system combines the dual properties from EIV system and closed loop situation, i.e. the closed loop input and output tare all corrupted or measured by external noises, so our identification mission is to extract one mathematical equation for the unknown plant through these corrupted input-output signal. As the field of system identification includes four steps, optimal input design, model structure, identification algorithm and model validation, so our synthesis identification analysis for closed loop EIV system includes above four steps too. The detailed input signal design and model description can be referred to our previous contributions [13], [14]. Then this new paper concerns on the identification algorithm and model validation for closed loop EIV system, while completing the overall synthesis analysis. More specifically, after describing the called closed loop EIV system with two external noises, the identification algorithms for closed loop EIV system are proposed for nonparametric estimate and parametric estimate respectively. Firstly, the corrupted input-output signal are dealt with to generate one nonparametric estimate, i.e. rough estimate, then through our statistical analysis, an unbiased nonparametric estimate is improved without any parametric form. Secondly, the case of parameterized plant by one unknown parameter vector, we propose the parametric estimate and its improved form to guarantee the unbiased parametric estimate. Consider these two nonparametric estimate and parametric estimate, their statistical analysis are also given to testify whether our obtained estimates are unbiased. Generally, the difficulty for closed loop EIV system identification is how to handle the external noise on the input within closed loop situation.

This paper is organized as follows. In section II, closed loop EIV system is described to give a background for well understanding. Our main work are in latter two sections. Section III proposes nonparametric estimate, and section IV shows the parametric estimate. These two estimates correspond to the identified plant, operating in closed loop situation. Moreover, two statistical analysis, corresponding to these two different estimates, are also given in section III and section IV respectively. Section V uses one practical example to prove the efficiencies of our proposed two estimates. Section VI formulates the main conclusion and points our our next work. Generally, in this paper, we give the detailed mathematical derivation to obtain the plant estimate for one new closed loop EIV system, extending our previous contributions into new fields.

Generally, the improvements of this paper are listed as follows.

(1) Extend the existed EIV system into closed loop EIV system, that is a more general system structure, only appearing in our paper.

(2) Nonparametric estimate and parametric estimate are derived through our own mathematical derivations for that unknown plant.

(3) To get an unbiased estimate, an iterative idea is proposed for further identification mission.

II. CLOSED LOOP EIV SYSTEM AND MOTIVATION

A. PRELIMINARY

For the sake of completeness, before showing or considered closed loop EIV system, the existed EIV system is introduced, plotting in following Figure 1. where in above Figure 1, the considered EIV system is also one open loop system. $P(z)$ is the transfer function of an unknown plant, z is the shift operator. $u_0(t)$ is the external excitation signal, $y_0(t)$ is plant output without any noise. $\{u(t), y(t)\}$ are the measured or corrupted input-output signal, $\{\tilde{u}(t), \tilde{y}(t)\}$ are two kinds of external noises, coming from unavoided factors or disturbances.

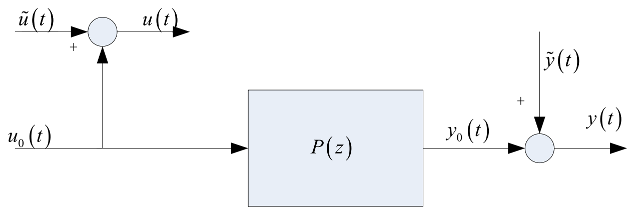


FIGURE 1. EIV system.

Comment: In Figure 1, we always regard $\{u_0(t), y_0(t)\}$ as real or true input-output without noises, and $\{u(t), y(t)\}$ as noisy input-output with the measurement and process

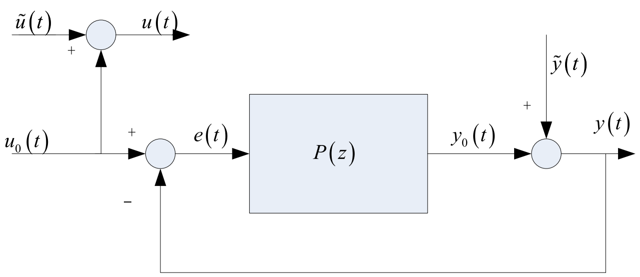


FIGURE 2. Closed EIV system.

noise $\{\tilde{u}(t), \tilde{y}(t)\}$. The classical system identification applies $\{u_0(t), y(t)\}$ to identify that unknown plant $P(z)$ without considering noise $\tilde{u}(t)$. But this noise $\tilde{u}(t)$ exists, leading to bad plant estimate, so to improve the identification accuracy about plant $P(z)$, EIV system identification proposes to use the noisy input-output $\{u(t), y(t)\}$ to estimate the unknown plant $P(z)$, and it is more realistic than before.

Consider that EIV system in Figure 1, the following equations hold easily.

$$\begin{cases} y(t) = y_0(t) + \tilde{y}(t) = P(z)u_0(t) + \tilde{y}(t) \\ u(t) = u_0(t) + \tilde{u}(t) \end{cases} \tag{1}$$

i.e. the overall noisy input-output is that

$$\begin{aligned} y(t) &= P(z)[u(t) - \tilde{u}(t)] + \tilde{y}(t) \\ &= P(z)u(t) - P(z)\tilde{u}(t) + \tilde{y}(t) \end{aligned} \tag{2}$$

so the problem of identifying that unknown plant $P(z)$ from the noisy input-output $\{u(t), y(t)\}$ corresponds to EIV system identification.

B. CLOSED LOOP EIV SYSTEM

From our many experiments of wing tunnel, we find all operating systems are considered into one closed loop situation, and it is true in academy and practices. It means the open loop system is divergent and replaced by closed loop system, so we put that EIV system in Figure 1 into one closed loop system to get our considered closed loop EIV system, plotting in Figure 2. where in Figure 2, all physical variables are similar to those in Figure 1. By the way, $e(t)$ is the unit feedback error signal or the input for the unknown plant, i.e. $e(t) = u_0(t) - y(t)$. The minimal realization can be obtained by structure decomposition. It is well known that the minimal realization means one system with controllability and observability property, so we need to decompose the obtained system according to controllability or observably decomposition.

Pay attention that the input signal, using to identify the unknown plant, is noisy input $u(t)$, not that real input $u_0(t)$, as here the external noise $\tilde{u}(t)$ is considered. From Figure 2, the noisy input-output relations are follows.

$$\begin{aligned} u(t) &= u_0(t) + \tilde{u}(t) \\ y(t) &= y_0(t) + \tilde{y}(t) \\ &= P(z)e(t) + \tilde{y}(t) \\ &= P(z)[u_0(t) - y(t)] + \tilde{y}(t) \\ &= P(z)[u(t) - \tilde{u}(t) - y(t)] + \tilde{y}(t) \\ &= P(z)u(t) - P(z)\tilde{u}(t) + \tilde{y}(t) - P(z)y(t) \end{aligned} \tag{3}$$

i.e. the direct computation to get

$$y(t) = \frac{P(z)}{1+P(z)}u(t) + \frac{\tilde{y}(t) - P(z)\tilde{u}(t)}{1+P(z)} \tag{4}$$

The difficulty in identifying closed loop EIV system is to handle the above second noisy combination term $\frac{\tilde{y}(t)-P(z)\tilde{u}(t)}{1+P(z)}$, being studied in latter sections.

III. NONPARAMETRIC ESTIMATE

As model can be expressed into two forms, one is parametric form and the other is nonparametric form. This paper spends more time and content on parametric form. The goal of considering that nonparametric form is for a complete analysis.

To get one plant estimate from the noisy input-output $\{u(t), y(t)\}$ in equation (4), we directly observe equation (4) and generate one direct estimation for that unknown plant $P(z)$, i.e. rough estimate.

A. ROUGH ESTIMATE

Observing equation (4), its one step ahead prediction output $\hat{y}(t)$ is defined as

$$\begin{aligned} \hat{y}(t) &= [1 + P(z)]\frac{P(z)}{1+P(z)}u(t) + [1 - 1 - P(z)]y(t) \\ &= P(z)u(t) - P(z)y(t) \end{aligned} \tag{5}$$

where one step ahead prediction output $\hat{y}(t)$ is dependent on the noisy input-output $\{u(t), y(t)\}$. So the prediction output error $\xi(t)$ is defined as follows.

$$\begin{aligned} \xi(t) &= y(t) - \hat{y}(t) \\ &= y(t) - P(z)u(t) + P(z)y(t) \\ &= [1 + P(z)]y(t) - P(z)u(t) \end{aligned} \tag{6}$$

Making use of noisy input-output sequence $\{u(t), y(t)\}_{t=1}^{N}$, where N is the total number of data, the unknown plant is generated from the following numerical optimization problem, i.e. solving an optimization problem to get an optimal decision variable.

$$\begin{aligned} \hat{P}(z) &= \arg\min_{P(z)} J(P(z)) \\ &= \arg\min_{P(z)} \frac{1}{N}\sum_{t=1}^{N} \xi^2(t) \end{aligned}$$

$$\frac{1}{N}\sum_{t=1}^{N} \xi^2(t) = \frac{1}{N}\sum_{t=1}^{N} ([1 + P(z)]y(t) - P(z)u(t))^2 \tag{7}$$

Cost function in equation (7) is the commonly used Euclidean norm, being used to measure the derivation between prediction output and noisy output, due to its simple form for equation (7). Other cost functions or performance index can be applied in different simulations, for example, min-max, robust norm etc.

Comment: Observing above optimization problem (7), whose decision variable is that unknown plant $P(z)$, and noisy input-output sequence $\{u(t), y(t)\}_{t=1}^{N}$ are collected in priori.

Making use of the optimality necessary condition to differentiate with respect to $P(z)$ and set the derivative equal to zero, we have

$$\frac{\partial J(P(z))}{\partial P(z)} = \frac{2}{N}([1+P(z)]y(t) - P(z)u(t))^T \times (y(t)-u(t)) = 0 \quad (8)$$

i.e.

$$\frac{2}{N}\sum_{t=1}^{N}(y(t)+P(z)(y(t)-u(t)))^T(y(t)-u(t)) = 0$$
$$\sum_{t=1}^{N} y(t)^T(y(t)-u(t)) = P(z)\sum_{t=1}^{N}(y(t)-u(t))^2 \quad (9)$$

Using the knowledge of power spectral theory to get one rough estimate $\hat{P}(z)$.

$$\hat{P}(z) = \frac{\sum_{t=1}^{N} y(t)^T(y(t)-u(t))}{\sum_{t=1}^{N}(y(t)-u(t))^2} = \frac{\phi_y(w)-\phi_{yu}(w)}{\phi_y(w)-2\phi_{yu}(w)+\phi_u(w)} \quad (10)$$

where $\phi_y(w), \phi_u(w), \phi_{yu}(w)$ are auto spectrum and cross spectrum between noisy input-output $\{u(t), y(t)\}_{t=1}^{N}$, i.e.

$$\phi_y(w) = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N} y^T(t)y(t)$$
$$\phi_u(w) = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N} u^T(t)y(t)$$
$$\phi_{yu}(w) = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N} y^T(t)u(t) \quad (11)$$

From that rough estimate $\hat{P}(z)$, after collecting the noisy $\{u(t), y(t)\}_{t=1}^{N}$, three kinds of spectrums

$\{\phi_y(w), \phi_u(w), \phi_{yu}(w)\}$ are computed. Then we substitute three spectrums into equation (10) to obtain one rough estimate $\hat{P}(z)$ for that unknown plant $P(z)$.

B. STATISTICAL ANALYSIS

To testify whether that rough estimate $\hat{P}(z)$ in equation (10) is biased or unbiased, its expectation operation $E[\hat{P}(z)]$ is computed. As three spectrums $\{\phi_y(w), \phi_u(w), \phi_{yu}(w)\}$ appear in that right hand of rough estimate, so firstly, we compute them in detail.

After simple but tedious calculation, we have

$$\phi_u(w) = \phi_{u_0}(w) + \sigma_u^2$$
$$\phi_y(w) = \frac{|P(z)|^2\phi_{u_0}(w)+\sigma_y^2}{|1+P(z)|^2}$$
$$\phi_{yu}(w) = \frac{P(z)\phi_u(w) - P(z)\sigma_u^2}{1+P(z)} \quad (12)$$

where $\{\sigma_u^2, \sigma_y^2\}$ are variances for those two external noises $\{\tilde{u}(t), \tilde{y}(t)\}$ respectively, being assumed to be white noises and variances $\{\sigma_u^2, \sigma_y^2\}$.

Substituting equation (12) into rough estimate to get the denominator and numerator, i.e.

$$\phi_y(w) - \phi_{yu}(w) = \frac{\sigma_y^2 - P(z)\phi_{u_0}(w)}{|1+P(z)|^2}$$
$$\phi_y(w) - 2\phi_{yu}(w) + \phi_u(w) = \frac{|P(z)|^2\phi_{u_0}(w)+\sigma_y^2}{|1+P(z)|^2} - \frac{2P(z)\phi_{u_0}(w)}{1+P(z)} + \frac{\phi_{u_0}(w)+\sigma_y^2+P(z)\phi_{u_0}(w)+P(z)\sigma_u^2}{1+P(z)} = \frac{\phi_{u_0}(w)+\sigma_y^2+(1+P(z))^2\sigma_u^2}{|1+P(z)|^2} \quad (13)$$

dividing each other to get the expectation operation of that rough estimate $\hat{P}(z)$, i.e.

$$E[\hat{P}(z)] = \frac{\sigma_y^2 - P(z)\phi_{u_0}(w)}{|1+P(z)|^2} \times \frac{|1+P(z)|^2}{\phi_{u_0}(w)+\sigma_y^2+(1+P(z))^2\sigma_u^2} = \frac{\sigma_y^2 - P(z)\phi_{u_0}(w)}{\phi_{u_0}(w)+\sigma_y^2+(1+P(z))^2\sigma_u^2} \quad (14)$$

From above equation (14), we see that rough estimate $\hat{P}(z)$ is an biased term, being dependent of $\{\phi_{u_0}(w), \sigma_y^2, \sigma_u^2\}$. But in practice, an ideal case is used to replaces that rough estimate $\hat{P}(z)$, such as

$$\hat{P}(z) = -\frac{\frac{1}{N}\sum_{t=1}^{N} y(t)}{\frac{1}{N}\sum_{t=1}^{N}[y(t)-u(t)]} = \frac{Ey(t)}{E[u(t)-y(t)]} \quad (15)$$

Due to the following equity holds.

$$\hat{P}(z) = \frac{Ey(t)}{E[u(t)-y(t)]} = \frac{\frac{P(z)}{1+P(z)}Eu(t)}{\frac{1}{1+P(z)}Eu(t)} = \frac{P(z)}{1+P(z)}\frac{1+P(z)}{1} = P(z) \quad (16)$$

It means above estimate $\hat{P}(z)$ is an unbiased estimate. In practice, in case of relaxed condition, above estimate (15) is used to save lots of computations.

Comment: Whatever plant estimate (10) and (15), we do not parameterize that unknown plant $P(z)$, and only use the noisy input-output sequence $\{u(t), y(t)\}_{t=1}^{N}$ to identify the unknown plant by virtue of power spectrum theory.

IV. PARAMETRIC ESTIMATE

From an applied point of view, the unknown plant $P(z)$ is a transfer function form, appearing widely in some engineering problems.

A. ESTIMATE

Let

$$P(z) = \frac{B(z)}{A(z)};$$
$$A(z) = 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a};$$
$$B(z) = b_1 z^{-1} + \cdots + b_{n_b} z^{-n_b}; \tag{17}$$

where n_a and n_b are two known orders for two different polynomials $A(z)$ and $B(z)$. Unknown parameters $\{a_i\}_{i=1}^{n_a}$ and $\{b_i\}_{i=1}^{n_b}$ are needed to identify.

Based on above transfer function form, the identification problem for that unknown plant $P(z)$ is changed to identify these unknown parameters. Substituting above parameterized transfer function into equation (4), it holds that.

$$y(t) = \frac{B(z)}{A(z)+B(z)}u(t) + \frac{A(z)\tilde{y}(t) - B(z)\tilde{u}(t)}{A(z)+B(z)} \tag{18}$$

rewriting above as that

$$[A(z)+B(z)]y(t) = B(z)u(t) + [A(z)\tilde{y}(t) - B(z)\tilde{u}(t)] \tag{19}$$

and

$$\begin{aligned} A(z) &= 1 + a_1 z^{(-1)} + \cdots + a_{n_a} z^{-n_a} \\ &= 1 + \begin{bmatrix} z^{-1} & z^{-2} & \cdots & z^{-n_a} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n_a} \end{bmatrix} \\ &= 1 + \alpha(z)a \\ B(z) &= b_1 z^{-1} + \cdots + b_{n_b} z^{-n_b} \\ &= \begin{bmatrix} z^{-1} & z^{-2} & \cdots & z^{-n_b} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n_b} \end{bmatrix} \\ &= \beta(z)b \end{aligned} \tag{20}$$

where

$$\alpha(z) = \begin{bmatrix} z^{-1} & z^{-2} & \cdots & z^{-n_a} \end{bmatrix};$$
$$\beta(z) = \begin{bmatrix} z^{-1} & z^{-2} & \cdots & z^{-n_b} \end{bmatrix};$$
$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n_a} \end{bmatrix}; b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n_b} \end{bmatrix} \tag{21}$$

Substituting above simplified forms into equation (19), we have

$$\begin{aligned} &[1 + \alpha(z)a + \beta(z)b]y(t) \\ &\quad = \beta(z)bu(t) \\ &\quad\quad + [(1 + \alpha(z)a)\tilde{y}(t) - \beta(z)b\tilde{u}(t)]; \\ &y(t) + \begin{bmatrix} \alpha(z)y(t)\beta(z)y(t) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \\ &\quad = \beta(z)u(t)b + \tilde{y}(t) \\ &\quad\quad + \alpha(z)\tilde{y}(t)a - \beta(z)\tilde{u}(t)b \end{aligned} \tag{22}$$

i.e. it is equivalent to that

$$\begin{aligned} y(t) &= \tilde{y}(t) - \begin{bmatrix} \alpha(z)y(t)\beta(z)y(t) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \\ &\quad + \begin{bmatrix} 0\beta(z)u(t) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \\ &\quad + \begin{bmatrix} \alpha(z)\tilde{y}(t) & -\beta(z)\tilde{u}(t) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \\ &= \begin{bmatrix} -\alpha(z)(y(t) - \tilde{y}(t)) & -\beta(z)(y(t) - u(t) + \tilde{u}(t)) \end{bmatrix} \\ &\quad \times \begin{bmatrix} a \\ b \end{bmatrix} + \tilde{y}(t) \\ &= \varphi_1^T(t)\theta + \tilde{y}(t) \end{aligned} \tag{23}$$

where regressor variable $\varphi_1(t)$ and unknown parameter vector θ are defined as follows

$$\varphi_1(t) = [-\alpha(z)(y(t) - \tilde{y}(t)) \ -\beta(z)(y(t) - u(t) + \tilde{u}(t))];$$
$$\theta = \begin{bmatrix} a \\ b \end{bmatrix} \tag{24}$$

Thanks for above least regressor form, then the unknown parameter vector θ is very easily solved to get one classical least squares solution $\hat{\theta}$, i.e.

$$\hat{\theta} = \arg\min_\theta \frac{1}{N} \sum_{t=1}^{N} [y(t) - \varphi_1^T(t)\theta + \tilde{y}(t)]^2 \tag{25}$$

then

$$\hat{\theta} = [\sum_{t=1}^{N} \varphi_1^T(t)\varphi_1(t)]^{-1} [\sum_{t=1}^{N} \varphi_1^T(t)y(t)] \tag{26}$$

When to check the identification accuracy of above least squares solution $\hat{\theta}$, we take the expectation operation on both sides of equation (26).

$$\begin{aligned} E\hat{\theta} &= [\sum_{t=1}^{N} \varphi_1^T(t)\varphi_1(t)]^{-1} [\sum_{t=1}^{N} \varphi_1^T(t)\varphi_1(t)\theta + \sum_{t=1}^{N} \varphi_1^T(t)\tilde{y}(t)] \\ &= \theta + [\sum_{t=1}^{N} \varphi_1^T(t)\varphi_1(t)]^{-1} [\sum_{t=1}^{N} E\varphi_1^T(t)\tilde{y}(t)] \end{aligned} \tag{27}$$

Making use of the following results to continuous computations.

$$\sum_{t=1}^{N} \varphi_1^T(t)\tilde{y}(t) = \begin{bmatrix} \alpha(z)y_0(t) & -\beta(z)y(t) + u_0(t) \end{bmatrix} \tilde{y}(t)$$

$$\begin{aligned} y_0(t) &= y(t) - \tilde{y}(t) \\ &= \frac{P(z)}{1+P(z)}u(t) + \frac{\tilde{y}(t) - P(z)\tilde{u}(t)}{1+P(z)} - \tilde{y}(t) \\ &= \frac{P(z)}{1+P(z)}u(t) - \frac{P(z)(\tilde{u}(t) - \tilde{y}(t))}{1+P(z)} \\ Ey_0^T(t)\tilde{y}(t) &= \frac{P(z)}{1+P(z)}\sigma_y^2 \end{aligned} \tag{28}$$

$$-\beta(z)[y(t)-u_0(t)] = -\beta(z)[\frac{P(z)}{1+P(z)}u(t) + \frac{\tilde{y}(t)-P(z)\tilde{u}(t)}{1+P(z)} - u_0(t)] = -\beta(z)\frac{1}{1+P(z)}\sigma_y^2 \tag{29}$$

The above detailed computational process are omitted due to space limitations.

Substituting above derived results into equation (27) to obtain

$$E\hat{\theta} = \theta - [\sum_{t=1}^{N}\varphi_1^T(t)\varphi_1(t)]^{-1}\begin{bmatrix}\alpha(z) & 0\end{bmatrix}\sigma_y^2 \tag{30}$$

From equation (30), we see that least squares solution $\hat{\theta}$ is a biased estimate, and the biased error is $[\sum_{t=1}^{N}\varphi_1^T(t)\varphi_1(t)]^{-1}[\alpha(z)\ \ 0]\sigma_y^2$.

Comment: Although above least squares estimate $\hat{\theta}$ is biased, but we can compensate it to be unbiased, i.e. the bias compensated estimate.

$$\hat{\theta} = [\sum_{t=1}^{N}\varphi_1^T(t)\varphi_1(t)]^{-1}[\sum_{t=1}^{N}\varphi_1^T(t)y(t)] + [\sum_{t=1}^{N}\varphi_1^T(t)\varphi_1(t)]^{-1}\begin{bmatrix}\alpha(z) & 0\end{bmatrix}\sigma_y^2 \tag{31}$$

where our derived biased error is summed in that least squares estimate.

As one inverse operation exists in both equation (26) and (31), so these two equations hold on the condition that regressor matrix $\sum_{t=1}^{N}\varphi_1^T(t)\varphi_1(t)$ is inverse. This condition corresponds to the persistent excitation, being described in following Assumption 1.

Assumption 1: To guarantee our identified parameter estimation feasible, i.e.equation (26) and (31) hold, the persistent excitation about input signal is needed here, meaning there exists one positive value M, such that.

$$\sum_{t=1}^{N}\varphi_1^T(t)\varphi_1(t) \geq MI \tag{32}$$

where I is one identity matrix.

B. IMPROVED ESTIMATE

To avoid the biased error and get the unbiased estimate, this section proposes an iterative method to improve that least squares estimate.

Rewriting equation (23) as the following two parts.

$$\begin{aligned} y(t) &= \tilde{y}(t) - \begin{bmatrix}\alpha(z)y(t)\beta(z)y(t)\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} + \begin{bmatrix}0 \ \beta(z)u(t)\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} + \begin{bmatrix}\alpha(z)\tilde{y}(t)-\beta(z)\tilde{u}(t)\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} \\ &= \begin{bmatrix}-\alpha(z)y(t)\beta(z)(u(t)-y(t))\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} + w(t) \\ &= \varphi_2^T(t)\theta + w(t) \end{aligned} \tag{33}$$

where

$$\begin{aligned} \varphi_2(t) &= \begin{bmatrix}-\alpha(z)y(t) & \beta(z)(u(t)-y(t))\end{bmatrix}; \\ w(t) &= \begin{bmatrix}\alpha(z)\tilde{y}(t) & -\beta(z)\tilde{u}(t)\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} + \tilde{y}(t) \\ &= \varphi_3^T(t)\theta + \tilde{y}(t); \\ \varphi_3(t) &= \begin{bmatrix}\alpha(z)\tilde{y}(t) & -\beta(z)\tilde{u}(t)\end{bmatrix} \end{aligned} \tag{34}$$

Observing equation (32) and (33) together and rewriting two main equations as

$$\begin{aligned} y(t) &= \varphi_2^T(t)\theta + w(t); \\ w(t) &= \varphi_3^T(t)\theta + \tilde{y}(t) \end{aligned} \tag{35}$$

where $\varphi_2(t)$ and $\varphi_3(t)$ are two regressor vectors. They are constituted from noisy input-output sequence $\{u(t), y(t)\}_{t=1}^{N}$ and two noises $\{\tilde{u}(t), \tilde{y}(t)\}_{t=1}^{N}$ respectively. After collecting the noisy input-output sequence and white noises,two regressor vectors are obtained only through shift operative.

As that unknown parameter vector θ exists in $y(t)$ and $w(t)$ simultaneously, iterative least squares method is well applied to improve the identification performance. The main steps of iterative least squares method are formulated as follows.

> Step 1: Collect noisy input-output $\{u(t), y(t)\}_{t=1}^{N}$ and sample one white noise twice to form $\{\tilde{u}(t), \tilde{y}(t)\}_{t=1}^{N}$, where N is the total number, for example, $N = 200$.
> Step 2: Construct two regressor vectors $\varphi_2(t)$ and $\varphi_3(t)$.
> Step 3: Given one initial parameter vector θ_0.
> Step 4: Substitute θ_0 into $\varphi_3^T(t)\theta + \tilde{y}(t)$ to generate $w_0(t)$.
> Step 5: Substitute $w_0(t)$ into $y(t) = \varphi_2^T(t)\theta + w(t)$ and the least squares estimate is solved, i.e.
> $\hat{\theta}_1 = [\sum_{t=1}^{N}\varphi_2^T(t)\varphi_2(t)]^{-1}[\sum_{t=1}^{N}\varphi_2^T(t)y(t)]$.
> Step 6: Substitute $\hat{\theta}_1$ into $\varphi_3^T(t)\hat{\theta}_1 + \tilde{y}(t)$ to generate $w_1(t)$, i.e.
> $w_1(t) = \varphi_3^T(t)\hat{\theta}_1 + \tilde{y}(t)$.
> iteratively run step 5 and step 6
> $\vdots$
> generate a sequence of parameter estimate as
> $\theta_0, \hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_i, \hat{\theta}_{i+1}$
> Check whether $|\hat{\theta}_{i+1} - \hat{\theta}_i| \leq 0.05$, if it holds, then terminate the above iterative processes, or go to step 5 again, until it holds
> When to start above given iterative least squares method, the initial parameter vector θ_0 is chosen as $\theta_0 = 0.5I$, where I is one identity vector.

As the least squares estimate and its improved one shows the parameter estimate for that unknown parameter vector θ, so we call it the parametric estimate, being different from that nonparametric estimated in section III. Generally, section III and section IV propose nonparametric estimate and parametric estimate for that unknown plant.

Comment: Due to the simple form for iterative least squares method and many existed softwares about it can be downloaded directly, so the application of our improved iterative least squares method is feasible and implemented. Our proposed identification algorithm is iterative least squares algorithm, belonging to the kind of common least squares algorithm. As research on least squares algorithm can be referred to each book about system identification, so the convergence stability and other statistical properties are mature from each book.

The reason about why we consider nonparametric estimation and parametric estimation is that these two difference forms exist actually within different situations for nonparametric or parametric forms. Nonparametric form and parametric form are determined by designer, then their own nonparametric estimation or parametric estimation are chosen freely.

V. NUMERICAL EXAMPLE

Here we start to prove our identification strategy, describing in above steps, through one simulation example. More specifically, the unknown's plant real or true transfer function form is given as follows.

$$
\begin{aligned}
y_0(t) &= \frac{B(z)}{A(z)} e(t); \\
A(z) &= 1 - 1.2z^{-1} + 0.8z^{-2} - 0.27z^{-3}; \\
B(z) &= 0.88 + 0.16z^{-1} + 0.8z^{-2} - 0.38z^{-3}
\end{aligned}
$$

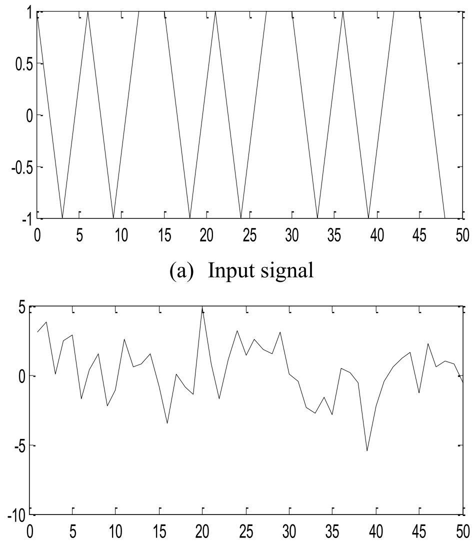
and two variance values for those two white noises are $\sigma_u^2 = 0.25; \sigma_y^2 = 0.64$.

(1) Identification results

Let above plant operate within one unit feedback situation, then one input signal $u_0(t)$, plotting in Figure 3, is chosen to excite the whole closed loop system whose the measurement and process noises are white noises. Figure 3 shows the input-output signal pair $\{u_0(t), y_0(t)\}$, i.e. the real input-output signal without noises.To introduce the measurement and process noise together and construct our considered closed loop EIV system, those two real input-output signals are corrupted with two white noises,i.e. the corrupted input-output, plotting in Figure 4, so our mission is to get some information about $\frac{B(z)}{A(z)}$ from these corrupted input-output.

As two polynomials $A(z)$ and $B(z)$ are all parameterized by two kinds of unknown parameters, i.e.

$$
\begin{aligned}
a &= \left[\, a_1 \; a_2 \; a_3 \,\right] \\
&= \left[\, -1.2 \; 0.8 \; -0.27 \,\right]; \\
b &= \left[\, b_0 \; b_1 \; b_2 \; b_3 \,\right] \\
&= \left[\, 0.88 \; 0.16 \; 0.8 \; -0.38 \,\right]; \\
\theta &= \left[\, a \; b \,\right]
\end{aligned}
$$



(a) Input signal

(b) Output signal

FIGURE 3. The real input-output signal without noise.

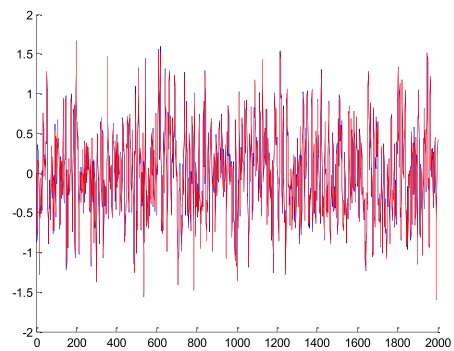


FIGURE 4. The corrupted input-output signal.

TABLE 1. The parameter estimate for polynomial A(z).

t	a_1	a_2	a_3
100	-1.62200	0.80321	-0.69354
150	-1.56115	0.61979	-0.52254
200	-1.47441	0.52765	-0.32766
250	-1.39615	0.61037	-0.28292
300	-1.19751	0.79660	-0.27881
True value	-1.2	0.8000	-0.27

so our goal is to identify above unknown parameter vector θ through using the corrupted input-output signa, plotting in Figure 4.

Before to start our proposed iterative least squares method, the noisy input-output $\{u(t), y(t)\}_{t=1}^{N}$ are sampled from Figure 4, and the initial parameter vector θ_0 is set as $\theta_0 = [0.5, \cdots, 0.5]$. After 300 iterations, the final parameter estimates are formulated in the following Table 1 and Table 2, showing the parameter estimates with time increase.

TABLE 2. The parameter estimate for polynomial B(z).

t	b_0	b_1	b_2	b_3
100	1.234	0.743	0.80321	-0.15739
150	1.023	0.701	0.81979	-0.22146
200	0..987	0.682	0.82765	-0.2881
250	0.942	0.432	0.81037	-0.34881
300	0.901	0.210	0.80660	-0.37915
True value	0.88	0.16	0.8	-0.38

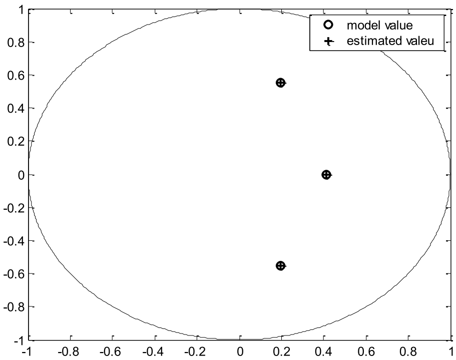


FIGURE 5. Three roots for different polynomial A(z).

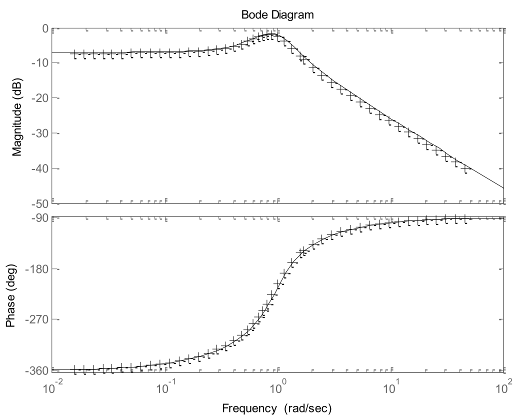


FIGURE 6. Bode response.

From above two Tables, we see after 300 iterations, all parameter estimates converge to their own true values. To understand well, we substitute these parameter estimates into polynomial $A(z)$ to yield one estimated or identified polynomial. Figure 5 shows three roots corresponding to the estimated polynomial and its real polynomial, furthermore, Figure 6 plots two Bode response curves for the estimated polynomial and real polynomial. From Figure 5 and Figure 6, the real roots and estimated roots are all the same with other, also this fact holds for Bode response curves.

(2) Model validation

After the parameter vector θ is identified by above iterative least squares method and substitute in the two polynomials to form that transfer function $\frac{B(z)}{A(z)}$. Then another problem

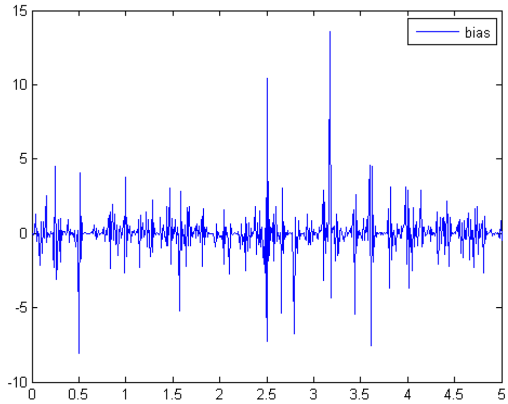


FIGURE 7. Biased error curve.

appears on how to check the accuracy of the identified plant,i.e. guaranteeing the identified plant converge to its real or true plant.

$$\frac{B(z,\hat{\theta})}{A(z,\hat{\theta})} \rightarrow \frac{B(z)}{A(z)} = \frac{B(z,\theta_0)}{A(z,\theta_0)};$$
$$\hat{\theta} \rightarrow \theta_0$$

where $\hat{\theta}$ is the identified parameter vector, and θ_0 is the true parameter vector.

To guarantee the above limit equity hold, it is similar to let that biased error $[\sum_{t=1}^{N} \varphi_1^T(t)\varphi_1(t)]^{-1}[\alpha(z)\ \ 0]\sigma_y^2$ be zero or as small as possible. During the simulation, we choose $N = 300$ and use above identified parameter estimates in Table 1 and Table 2 to compute the corresponding biased error, which curve is plotting in latter Figure 7. Figure 7 tells us that biased error is around zero with some fluctuations,which means the noisy input-output include other unavoided external noise. And these external noise can not described by white noise.Furthermore, we think the white noise is one ideal case in academy, but in practice more widely used noise is the called unknown but bounded noise. During this whole numerical example, two parts corresponds to two different data sets, i.e. identification and validation. Specifically, one data set is for identification part, and the second data set for validation part. Before to deal with each data set, one filter is applied to filter the unavoided noise, as noise always causes the bad effect for identification result.

(3) To combine our theoretical result and practical application, we give the second simulation example to verify our identification strategies and optimal input signal within closed loop identification for aircraft flutter model parameters. A necessary test for flutter model parameters identification is the flutter wind tunnel test, that includes pre-test preparation, excitation section, test operation, post-test model check and data processing. In the flutter wind tunnel test, plotting in Figure 8, to correctly simulate the flight motion and support conditions of the flutter model, a support system, that meets the test requirements needs to be specially designed. When the aircraft is in the low speed test, the component model is usually supported on the rigid frame.

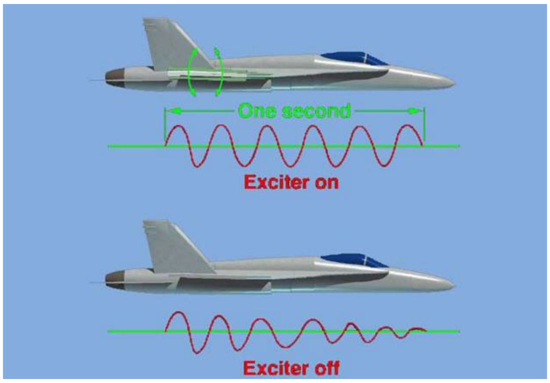


FIGURE 8. Flutter excitation.

Comparing with the model, the rigid frame has a much larger stiffness and mass, and within the relevant flutter frequency range, involved in the test, the inherent rigid frame can not appear to be its natural frequency. When doing a full model test, generally support system can support the model and maintain the model with at least three rigid body motions, such as lifting, pitching and rolling. On the other hand in the high speed model test, the component model can be fixed on the side wall of the wind tunnel. To reduce the influence of the surface layer or the cave wall, then width of the fuselage can be appropriately increased. More information about aircraft flutter model parameter identification can be seen our newly published book [29].

One real aircraft, produced by our lab in Figure 9, is used for flutter experiment, and two excitation motors are installed on the left wing. One excitation motor is installed on the left front beam, and the other one is on the rear beam, so all flutter models are excited. During the whole flutter experiment, one important closed loop structure is the current loop. The function of the current loop is to control the current of the motor not exceed the maximum locked-rotor current of the motor. At the same time, it is also necessary to make the armature current strictly follow the change of the control voltage command, so that we can accurately control the torque output by the motor to eliminate the effect of back-EMF on the output torque.

Here we apply our derived results on one single input and single output system, controlled by one feedback controller. The true data generating system is given as follows.

$$G_0(z) = \frac{0.25z^{-1} + 0.12z^{-2}}{1 - 1.6z^{-1} + 0.8z^{-2} - 0.64z^{-3} + 0.65z^{-4}} = \frac{0.25z^3 + 0.12z^2}{z^4 - 1.6z^3 + 0.8z^2 - 0.64z + 0.65}$$

Their corresponding parametrized forms are denoted as follows.

$$G(z,\theta) = \frac{a_z^{-1} + a_6 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}} = \frac{a_5 z^3 + a_6 z^2}{z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4}$$

FIGURE 9. Aircraft used for flutter experiment.

TABLE 3. Comparison of two identification results.

t	a_1	a_2	a_3	a_4	a_5	a_6
Iterative least squares method						
100	-1.66010	0.71224	-0.36960	0.76926	0.15739	0.11412
500	-1.61858	0.71936	-0.57727	0.75419	0.22146	0.12554
1000	-1.63117	0.71189	-0.56187	0.68829	0.24662	0.11467
1500	-1.63560	0.70161	-0.58364	0.68297	0.24484	0.11423
2000	-1.63341	0.70037	-0.60951	0.65304	0.24476	0.11680
2500	-1.64755	0.70421	-0.62808	0.65519	0.24388	0.11213
3000	-1.64664	0.73523	-0.64690	0.64788	0.24975	0.11534
True	−1.6	0.8	−0.64	0.65	0.25	0.12
Classical least squares method						
100	-1.68	0.72	-0.45	0.77	0.2	0.12
500	-1.62	0.72	-0.57	0.769	0.22	0.13
1000	-1.63	0.719	-0.571	0.78	0.25	0.121
1500	-1.64	0.71	-0.58	0.76	0.254	0.125
2000	-1.64	0.72	-0.61	0.77	0.256	0.131
2500	-1.65	0.71	-0.62	0.70	0.25	0.132
3000	-1.64	0.73	-0.65	0.67	0.256	0.117
True	−1.6	0.8	−0.64	0.65	0.25	0.12

Two Gaussian white noises $\{\tilde{u}(t), \tilde{y}(t)\}$ with unit variance are added, the sampled time is $T_s = 1$ second, the true parameter vector θ_0 is defined as follows.

$$\theta_0 = \left[-1.6 \ 0.8 \ -0.64 \ 0.65 \ 0.25 \ 0.12 \right]^T$$

The data generating system is operated in one closed loop system with one unit feedback controller. In solving that numerical optimization problem to identify the unknown parameter vector, the initial value for unknown parameter vector θ_{int} is chosen as.

$$\theta_{int} = \left[-1.7 \ 0.7 \ -0.4 \ 0.8 \ 0.15 \ 0.1 \right]^T$$

The iterative least squares identification algorithm is applied to estimate those unknown parameters in the polynomials. The identification results are shown in Table 3, which gives the detailed iterative parameter estimations with the number of data increases. From Table 3, as the number of observed data increases, the parameter estimations will converge to their own true values respectively. Furthermore, identification result for classical least squares algorithm is also given in Table 3, where we can see the accuracy of iterative least squares algorithm is more better than classical least squares algorithm.

VI. CONCLUSION

This paper studied the identification analysis for closed loop EIV system, which keeps the dual properties for EIV system and closed loop system. More specifically, as one unknown plant, existing in closed loop system, is needed to identify, we propose nonparametric estimate and parametric estimate to replace that unknown plant for two cases respectively. Furthermore, after giving statistical analysis for these two different estimate, their corresponding improved forms are generated to get more excellent identification performance. To the best of our knowledge, identification is for control, so our future work concerns on direct data driven for closed loop EIV system, being describing by nonlinear form..

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WANG JIANHONG received the Diploma degree in engineering cybernetics from Yunnan University, China, in 2007, and the Dr.Sc. degree from the College of Automation Engineer, Nanjing University of Aeronautics and Astronautics, China, in 2011. From 2013 to 2015, he was a Postdoctoral Fellow of informazione with Politecnico di Milano. From 2016 to 2018, he was a Professor with the University of Seville. Since 2019, he has been a full time Professor at the Tecnologico de Monterrey, and also a part time Professor at the Jiangxi University of Science and Technology. His research interests include real-time and distributed control and optimization and system identification.

OUYANG QING received the bachelor's degree in control engineering from Dalian Jiaotong University, Dalian, Liaoning, China, in 2017. She is currently pursuing the master's degree with the Jiangxi University of Science and Technology, Ganzhou. Her research interest includes advanced control strategy.