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SURVEY

Disturbance Observer-Based Data Driven Model Predictive Tracking Control of Linear Systems

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ABSTRACT This paper introduces a new data-driven MPC structure based on two offline and online parts to achieve the robust and constrained performance in an optimal scheme. In the first step, according to the model matching condition, an offline data-driven controller is designed to reach the tracking performance. In addition, to reduce the effects of the external disturbance, a data-driven-based disturbance observer is presented to estimate the external disturbance. Therefore, the robustness against the external disturbances is achieved in an offline procedure. Then, a data-driven model predictive control (MPC) is structured based on a data-driven-based model of a stabilized system. In other words, the overall controller is configured such that the limitations of the system states and control input are considered in the control design process. Moreover, by employing the move blocking strategy, the online computational burden of the suggested controller is greatly reduced. To further improve of the feasibility problem, an ellipsoidal terminal (ET) constraint is considered. The rows number of the blocking matrix influences on the ET set which leads to feasibility enhancement. So, the main contributions of the presented data-driven controller are feasibility improvement and reducing online computational burden in an optimal and constrained scheme which are illustrated in the simulation section.

INDEX TERMS Data-driven method, model predictive control, model matching condition, disturbance observer, move blocking strategy.

I. INTRODUCTION

Control of practical and industrial systems has always been an inevitable part of studies [1], [2], [3]. Recently, several control methods have been investigated based on the measured data with lack of the dynamic model [4], [5]. The main approaches to design the data-driven-based controller are usually called *indirect data-driven* and *direct data-driven* methods [6]. In the *indirect data-driven* method, a model is first estimated (or identified) based on the measured data. Then, the control input is developed using the estimated dynamic model. Hence, the measured data is not employed in the designing process of the control law [6]. To decrease the identifying complexity of the *indirect data-driven* method, the *direct data-driven*-based controllers have been significantly considered. The controller is directly designed based on the data collected from the real system [6].

In [7], it is shown that persistently exciting (PE) data can parametrize all trajectories of a linear time invariant (LTI) system. According to this result, various studies have been presented to design the direct data-driven controller. For example, several methods are developed for system analysis in [8], [9], [10], and [11], designing the output/state feedback control in [4], [12], [13], [14], and [15], optimal controller design in [16], internal model control and model reduction in [17] and [18]. In addition, the advantages of model predictive control (MPC) have led to many studies in the data-driven-based MPC field [19], [20], [21], [22], [23].

In [22], a data-driven MPC is developed to control an unknown LTI systems based on the noisy data. The recursive

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feasibility and closed loop stability of the system is guaranteed with constraint satisfaction. By considering a constraint tightening which ensures the constrained output tracking performance, the output constraint is considered in the control design procedure. A data-driven MPC with the satisfaction closed-loop stability is presented for controlling unknown LTI systems in [24]. In addition, it can be used to control the nonlinear systems by repeatedly updating data and utilizing the local linear approximations. The simulation and experimental results of the nonlinear four-tank system show the advantages of the proposed data-driven MPC. Moreover, a data-driven MPC scheme with terminal equality constraint is suggested in [21] to control the LTI systems. First, this approach is developed for the nominal data with no measurement noise. Then, by considering the measurement noise of data, a robust data-driven MPC is presented with guaranteeing the closed-loop stability, recursive feasibility, and constraint satisfaction. However, due to the prediction process during the online optimization problem, the number of decision variables of the proposed approaches ([21], [22], [24]) is increased. As a result, the online computation burden of [21], [22], and [24] is high which can be a major challenge in real implementation. Moreover, the robustness against the external disturbances is not considered in [21], [22], and [24].

Some other data-driven controllers have been provided in [13], [25], [26], and [27] to reach the robust performance. In [13], a robust state-feedback data-driven controller is designed for discrete-time LTI systems. First, an exact parametrization of the closed-loop LTI system with a fixed state-feedback is proposed. Then, by utilizing this exact parametrization, an H_∞ robust state-feedback controller is developed. However, the optimality and constraint satisfaction are not considered during the designing the controller. A data-driven robust controller is designed in [25] for discrete- and continuous-time LTI systems in the presence of multi-model uncertainty and measurement noise. The controller is provided based on a convex–concave optimization problem. Then, the concave part is convexified by linearization around the initial controller. Although, the iterative solving increases the computational complexity.

Motivated by the existing restrictions and challenges, in this article, a new disturbance observer-based move blocking data driven model predictive controller is developed for discrete-time LTI systems. The proposed controller includes two online and offline parts. In the first part, the controller is designed based on the model matching condition in an offline procedure. Moreover, a data-driven-based disturbance observer is provided to estimate the external disturbance which is employed to reduce the effects of the disturbance. In other words, to reach the robust tracking performance against the external disturbance, an offline data-driven controller is designed based on the model matching condition and the estimation of the disturbance. A data-driven MPC is configured in second part to satisfy the states and control input constraints and guarantee the optimal performance. To do this, the data-driven MPC is structured based on the data of a stabilized system. Moreover, to reduce the number of decision variables, a move blocking method is utilized and consequently, the computational complexity of solving online optimization problem is greatly reduced. The number of rows of the blocking matrix influences on the considered ellipsoidal terminal (ET) constraint which leads to feasibility enhancement. In summary, the contributions of the proposed data-driven controller in comparison with the exiting methods are as follows:

- Developing a new configuration of data-driven MPC based on two offline and online parts to reach the robust and optimal tracking performance and constraints satisfaction.
- Compared with [25] and [27], the robustness against the external disturbance is obtained in an offline process which leads to reducing the conservatism of the online optimization problem.
- Compared with the terminal equality-based MPC [21], by employing the move blocking strategy and the ET constraint, the feasibility of the online optimization problem is improved.
- Structuring the MPC problem based on a stabilized systems is also decreased the computational complexity of the online optimization problem against the [21], [22], and [24].

The reminder of the paper is: The problem formulation is presented in Section II. The main results of the proposed data-driven controller, Simulation examples, and Conclusion and Future works are illustrated in Sections III, IV, and V, respectively.

Notation: $\|\cdot\|$ and $(\Omega)^\dagger$ are the Euclidean norm, the pseudo-inverse of Ω which satisfies $\Omega\Omega^\dagger\Omega = \Omega$. I_m is a $(m \times m)$ identity matrix and $\mathbf{0}_n$ is a $(n \times 1)$ zeros vector. $x(k)$ is the real state and $x_t = x(t|k)$ is the predicted state at step t at time step k. $(\star)$ shows the transpose element of the symmetric matrix.

II. PROBLEM FORMULATION

In this section, problem formulation and preliminary lemmas are explained. First, a model-based tracking controller and disturbance observer are introduced for a discrete-time LTI system. Then, the data driven-based representation of LTI systems is presented according to [12]. To do this, by considering Rouche-Capelli theorem, the controller gains is expressed to reach the tracking performance based on model matching condition. The nomenclature of the symbols is shown in Table 1.

A. MODEL-BASED NOMINAL TRACKING CONTROLLER

Consider the following nominal observable and controllable discrete-time LTI system:

$$x(k+1) = Ax(k) + Bu(k), \tag{1}$$

where $x(k) \in \mathcal{R}^n$, $u(k) \in \mathcal{R}^m$, $A \in \mathcal{R}^{n\times n}$, and $B \in \mathcal{R}^{n\times m}$ are system states, control input, known constant matrix, and

TABLE 1. The nomenclature of the symbols.

$x(k)$	System states
$\bar{x}(k)$	Reference states
$\omega(k)$	External disturbance
$\widehat{\omega}(k)$	Estimation of external disturbance
$\boldsymbol{u}(k)$	Overall control law
$\mathcal{L}(k)$	Blocking matrix
K_ω	Disturbance observer gain
K_1,K_2	Controller gains

a full ranked input matrix, respectively. The design purpose is that the stabilizing controller that conforms to a pre-defined reference model, dictates the states of the closed-loop system to a prescribed adjustable set point. The pre-defined reference model is as follows:

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}r(k), \tag{2}$$

where $\bar{x}(k) \in \mathcal{R}^n$ is the desired state of the pre-defined reference model at instant $k \in \mathcal{N}$ and $r(k) \in \mathcal{R}^n$ is the prescribed adjustable set point. The dynamic model (2) is supposed to be stable, thus the matrices $\bar{A}$ (which is Hurwitz) and $\bar{B}$ are known and constant.

Moreover, to reach the tracking performance, the control signal is structured as follows:

$$u(k) = K_1 x(k) + K_2 r(k), \tag{3}$$

where $K_1 \in \mathcal{R}^{m\times n}$ obligates the closed-loop stability and $K_2 \in \mathcal{R}^{m\times n}$ is a feed-forward term which guarantees the tracking performance. The control law (3) is employed to guarantee the matching problem as in the following lemma.

Lemma 1 (Matching Condition [28]): Consider the LTI system (1), the pre-defined reference model (2) and the control law (3). If there exist matrices K_1 and K_2 such that the following conditions are hold,

$$A + BK_1 = \bar{A}, \tag{4a}$$
$$BK_2 = \bar{B}, \tag{4b}$$

then, one can say the matching problem is satisfied. ■

To design the control signal based on Lemma 1, the matrices of the system must be known. In the next subsection the design process of the disturbance observer based on the system model (1) is introduced.

B. MODEL-BASED LINEAR DISTURBANCE OBSERVER

Consider the LTI system (1) in presence of the external disturbance $\omega(k) \in \mathcal{R}^m$ as follows:

$$x(k+1) = Ax(k) + B(u(k) + \omega(k)), \tag{5}$$

where the external disturbance $\omega(k)$ is supposed to be unknown and slowly time-varying in the following.

Assumption 1 [29]: The external disturbance $\omega(k)$ is considered as a slowly time-varying so that, for positive constant α,

$$|\Delta\omega(k)| \le \alpha,$$

where $\Delta\omega(k) = \omega(k) - \omega(k-1)$. The design process of the model-based disturbance observer is presented in Lemma 2.

Lemma 2 [29]: For the LTI system (5), the estimation of the external disturbance $\omega(k)$ can be obtained as follows:

$$\widehat{\omega}(k) = K_\omega x(k) - \mathfrak{s}(k), \tag{6a}$$
$$\mathfrak{s}(k+1) = \mathfrak{s}(k) + K_\omega (Ax(k) + B(u(k) + \widehat{\omega}(k)) - x(k)), \tag{6b}$$

where $\widehat{\omega}(k) \in \mathcal{R}^m$ and $\mathfrak{s}(k) \in \mathcal{R}^m$ are the disturbance estimate and the variable state. The stability of the disturbance observer has been proved in [29]. ■

According to Lemma 1 of [29],the matrix $K_\omega \in \mathcal{R}^{m\times n}$ is chosen as follows:

$$K_\omega = (I_m - \mathfrak{D})(MB)^\dagger M, \tag{7}$$

where $\mathfrak{D} = \text{diag}(\delta_1, \delta_2, \ldots, \delta_m)$ with $|\delta_i| < 1$ for $i = 1, 2, \ldots, m$ and matrix M satisfying $(MB)^\dagger MB = I_m$. By selecting the matrix K_ω as in (7), the exponential stability of the estimation error $e_\omega(k) = \omega(k) - \widehat{\omega}(k)$ is achieved [30], [31].

Lemmas 1 and 2 have been developed based on the dynamic model of the system and require the system matrices. Since, the proposed control method is developed based on the available data of the system, so, the data driven-based representation of LTI systems is introduced in the next subsection.

C. DATA DRIVEN-BASED REPRESENTATION OF LINEAR SYSTEMS

Consider the following available data of the LTI system (1) with length T:

$$\mathbb{U}_{[0\ \ T-1]} = \begin{bmatrix} u(0) & u(1) & \ldots & u(T-1) \end{bmatrix}, \tag{8a}$$
$$\mathbb{X}_{[0\ \ T-1]} = \begin{bmatrix} x(0) & x(1) & \ldots & x(T-1) \end{bmatrix}, \tag{8b}$$
$$\mathbb{U}_{[1\ \ T]} = \begin{bmatrix} u(1) & u(2) & \ldots & u(T) \end{bmatrix}, \tag{8c}$$
$$\mathbb{X}_{[1\ \ T]} = \begin{bmatrix} x(1) & x(2) & \ldots & x(T) \end{bmatrix}, \tag{8d}$$

The following assumption is necessary in the design process of the data-based control methods.

Assumption 2 ([7], [12]): Consider the PE input data sequence (8a), the following rank condition must be satisfied.

$$\text{rank}\begin{bmatrix} \mathbb{U}_{[0\ \ T-1]} \\ \mathbb{X}_{[0\ \ T-1]} \end{bmatrix} = m + n. \tag{9}$$

In other words, by satisfying condition (9), it is ensured that the data matrices (8) include all the information of the LTI system (1) with the controller (3). The only necessary condition for T is $T \ge (m+1)n + m$. For more details, one can see [7] and [12].

Proposition 1 [12]: By validating Assumption 2, the LTI system (1) with control law (3) can be equivalently represented in terms of data matrices as follows:

$$x(k+1) = \mathcal{A}x(k) + \mathcal{B}r(k), \tag{10}$$

where $\mathcal{A} = \mathbb{X}_{[1\ T]}G_1$ and $\mathcal{B} = \mathbb{X}_{[1\ T]}G_2$. The matrices $G_1 \in \mathcal{R}^{T\times n}$ and $G_2 \in \mathcal{R}^{T\times n}$ must satisfy the following conditions:

$$\begin{bmatrix} K_1 \\ I_n \end{bmatrix} = \begin{bmatrix} \mathbb{U}_{[0\ \ T-1]} \\ \mathbb{X}_{[0\ \ T-1]} \end{bmatrix} G_1, \tag{11a}$$

$$\begin{bmatrix} K_2 \\ \mathbf{0}_n \end{bmatrix} = \begin{bmatrix} \mathbb{U}_{[0\ \ T-1]} \\ \mathbb{X}_{[0\ \ T-1]} \end{bmatrix} G_2. \tag{11b}$$

Proof: By substituting (3) in (1), the closed-loop system is obtained as follows:

$$\begin{aligned} x(k+1) &= (A + BK_1)\, x(k) + BK_2 r(k) \\ &= [B\ A] \left(\begin{bmatrix} K_1 \\ I_n \end{bmatrix} x(k) + \begin{bmatrix} K_2 \\ \mathbf{0}_n \end{bmatrix} r(k) \right). \end{aligned} \tag{12}$$

According to Rouche-Capelli theorem, there exist two matrices G_1 and G_2 so that (11a) and (11b) are fulfilled. Consequently, one can say:

$$[B\ \ A] \begin{bmatrix} K_1 \\ I_n \end{bmatrix} = [B\ \ A] \begin{bmatrix} \mathbb{U}_{[0\ \ T-1]} \\ \mathbb{X}_{[0\ \ T-1]} \end{bmatrix} G_1, \tag{13a}$$

$$[B\ \ A] \begin{bmatrix} K_2 \\ \mathbf{0}_n \end{bmatrix} = [B\ \ A] \begin{bmatrix} \mathbb{U}_{[0\ \ T-1]} \\ \mathbb{X}_{[0\ \ T-1]} \end{bmatrix} G_2. \tag{13b}$$

Based on the dynamics in (1), it straightforwardly satisfies the following condition [12]:

$$[B\ \ A] \begin{bmatrix} \mathbb{U}_{[0\ \ T-1]} \\ \mathbb{X}_{[0\ \ T-1]} \end{bmatrix} = \mathbb{X}_{[1\ \ T]}. \tag{14}$$

According to (13) and (14), (10) is obtained and the proof is complete. ■

In the next section, the design process of the disturbance observer-based data driven model predictive tracking control of LTI systems with move blocking structure is presented.

III. MAIN RESULTS

This section presents the design steps of the proposed move blocking data driven MPC of LTI discrete-time systems. The controller consists of two components that are acquired through offline and online procedures, utilizing data-driven approaches. The tracking performance is firstly guaranteed in the presence of external disturbance by an offline controller based on the model matching condition. The data-driven-based estimation of the external disturbance is utilized to reduce the effects of the external disturbance. In addition, to constraint satisfaction and optimality, an online part is also designed. The data-driven MPC is configured based on the stabilized nominal system in a move blocking scheme. The move blocking matrix influences on the considered ET set in such a way that the recursive feasibility of the optimization problem and closed-loop stability of system are guaranteed.

The overall control signal is provided as follows:

$$\boldsymbol{u}(k) = u(k) + c(k), \tag{15a}$$

$$u(k) = K_1 x(k) + K_2 r(k) + u_1(k), \tag{15b}$$

$$u_1(k) = -B^{\dagger} B \widehat{\omega}(k), \tag{15c}$$

where $u(k)$ is obtained based on the data matrices and estimation of the external disturbance in an offline scheme. The part $c(k)$ is also calculated in an online scheme based on the data driven move blocking MPC. To do this, the design procedure of the suggested controller is presented in two steps. In the first step, an offline data driven tracking controller is designed. Then, to reach the optimality and constrained performance, the data driven MPC based on the stabilized system is provided in a move blocking structure.

Here, the disturbance observer (DO) is designed based on the data driven representation (10).

A. DATA DRIVEN-BASED DISTURBANCE OBSERVER DESIGN

Consider the estimation of the external disturbance $\omega(k)$ in (6) and the data driven representation (10), one can say:

$$\widehat{\omega}(k) = K_{\omega} x(k) - \mathfrak{s}(k), \tag{16a}$$

$$\mathfrak{s}(k+1) = \mathfrak{s}(k) + K_{\omega}\left(\mathcal{A}x(k) + \mathcal{B}r(k) + B\widehat{\omega}(k) - x(k)\right). \tag{16b}$$

The DO gain K_{ω} is chosen as stated in Lemma 2 to reach the stability of the estimation error $e_{\omega}(k)$.

Remark 1: To obtain the estimation of the $\omega(k)$ based on the available data matrices, matrix B must be known. Therefore, it is assumed that the input matrix B is known in the proposed method. If the input matrix B was not available, one can obtain the estimation of it based on the following *least-squares* optimization problem [32]:

$$\left[\widehat{B}\quad \widehat{A}\right] = \underset{B,A}{\operatorname{argmin}} \left\| \mathbb{X}_{[1\ \ T]} - [B\ \ A] \begin{bmatrix} \mathbb{U}_{[0\ \ T-1]} \\ \mathbb{X}_{[0\ \ T-1]} \end{bmatrix} \right\|. \tag{17}$$

B. DESIGNING THE DO-BASED MOVE BLOCKING DATA DRIVEN MPC

According to the data driven-based DO (16), the control signal $\boldsymbol{u}(k)$ will be designed in the next subsection. The designing process includes two offline and online steps. In the first step, the tracking performance is guaranteed due to the external disturbance. Then, to reach the optimal and constrained performance, the move blocking data driven MPC is developed based on the stabilized system.

1) FIRST STEP: DATA DRIVEN-BASED OFFLINE TRACKING CONTROLLER DESIGN

To reach the tracking performance in the presence of external disturbance, the data driven-based offline tracking controller is developed. By considering the LTI system (5) and the control signal $u(k)$ (15b), we have:

$$\begin{aligned} &x(k+1) \\ &= Ax(k) + B\left(K_1 x(k) + K_2 r(k) - B^{\dagger} B \widehat{\omega}(k)\right) + B\omega(k) \\ &= Ax(k) + BK_1 x(k) + BK_2 r(k) - B\widehat{\omega}(k) + B\omega(k) \\ &= (A + BK_1)\, x(k) + BK_2 r(k) + Be_{\omega}(k). \end{aligned} \tag{18}$$

By choosing the $K_{\omega} = \left(I_p - \mathfrak{D}\right)(MB)^{\dagger} M$, the estimation error $e_{\omega}(k)$ is stabilized. In addition, based on proposition 1, the data driven representation of (18) is obtained as follows:

$$x\,(k+1)=\mathbb{X}_{[1\ T]}G_1x\,(k)+\mathbb{X}_{[1\ T]}G_2r\,(k)+Be_{\omega}\,(k)\,. \quad (19)$$

Due to matching condition (Lemma 1), the tracking performance is guaranteed if the following conditions satisfy:

$$\mathbb{X}_{[1\ T]}G_1=\bar{A}, \quad (20a)$$
$$\mathbb{X}_{[1\ T]}G_2=\bar{B}, \quad (20b)$$
$$\mathbb{X}_{[0\ T-1]}G_1=I_n, \quad (20c)$$
$$\mathbb{X}_{[0\ T-1]}G_2=\mathbf{0}_n. \quad (20d)$$

By satisfying the feasibility of the matching problem, the controller gains K_1 and K_2 are obtained as follows:

$$K_1=\mathbb{U}_{[0\ T-1]}G_1, \quad (21a)$$
$$K_2=\mathbb{U}_{[0\ T-1]}G_2. \quad (21b)$$

To reduce the conservatism of the equality constraints (20a) and (20b), we can solve the following optimization problem (with $\gamma>0$) to obtain the matrices G_1 and G_1:

$$\begin{aligned}&\min \left\|\mathbb{X}_{[1\ T]}G_1-\bar{A}\right\|+\gamma\left\|\mathbb{X}_{[1\ T]}G_2-\bar{B}\right\|\\ &\text{Subject to}: \quad \mathbb{X}_{[0\ T-1]}G_1=I_n,\\ &\qquad\qquad\qquad \mathbb{X}_{[0\ T-1]}G_2=\mathbf{0}_n. \end{aligned} \quad (22)$$

Remark 2: The feasibility of the minimization problem (22) guarantees that the stabilizing control gain K_1 exists when the matching problem satisfied. Therefore, to decrease the conservatism of the minimization problem (22), we can incorporate a stability condition to (22) even the matching problem is not completely satisfied [28]. To do this, by considering the proposition 1 and the Lyapunov inequality, we have:

$$\begin{aligned}&\min \left\|\mathbb{X}_{[1\ T]}Q_1-\bar{A}P\right\|+\gamma\left\|\mathbb{X}_{[1\ T]}Q_2-\bar{B}P\right\|\\ &\text{Subject to}: \begin{bmatrix} P & \mathbb{X}_{[1\ T]}Q_1\\ \star & P\end{bmatrix}>0,\\ &\qquad\qquad\quad \mathbb{X}_{[0\ T-1]}Q_1=P,\\ &\qquad\qquad\quad \mathbb{X}_{[0\ T-1]}Q_2=\mathbf{0}_n, \end{aligned} \quad (23)$$

where $P>0$, $Q_1=G_1P$ and $Q_2=G_2P$. The obtained results are summarized in the following theorem.

Theorem 1: Consider LTI system (5), the pre-defined reference model (2) and the controller $u\,(k)$ (15b). By satisfying the optimization problem (23), the controller gains K_1 and K_2 guarantee the closed-loop stability and tracking performance.

Proof: According to matching condition (Lemma 1), the tracking performance for LTI system (5) is obtained when the conditions (20a)-(20d) be satisfied. Moreover, to reduce the conservatism of the presented controller, the controller gains can be obtained based on the minimization problem (23) as $K_1=\mathbb{U}_{[0\ T-1]}Q_1P^{-1}$ and $K_2=\mathbb{U}_{[0\ T-1]}Q_2P^{-1}$. Therefore, the closed-loop stability of system (5) with tracking performance is guaranteed. ■

The obtained results based on Theorem 1 only guarantee the tracking performance in the presence of external disturbance. Therefore, in the next step, the online part of the suggested controller $c(k)$ is designed based on the move blocking approach to reach the constrained and optimal performance of the overall control law $\boldsymbol{u}\,(k)$.

2) SECOND STEP: DATA DRIVEN-BASED MOVE BLOCKING MPC DESIGN

Considering the limitations of the control signal and the system states are very essential in practical applications. The designed controller $u\,(k)$ only guarantees the tracking performance and also the optimality is not considered in the design process. Consequently, this step develops the overall control law which includes the offline controller $u\,(k)$ and is calculated based on a stabilized closed-loop system. Moreover, to reduce the computational complexity of the online optimization problem, the move blocking method is employed.

Here, the structure of the proposed data driven-based optimization problem in a move blocking scheme is considered as follows:

$$\mathbb{J}^*\,(\tilde{x},\mathcal{L})$$
$$=\min_{\bar{c}_0,\ldots\bar{c}_{\ell-1}} \tilde{x}_N^TP\tilde{x}_N+\sum_{t=0}^{N-1}\tilde{x}_t^Tc_1\tilde{x}_t+\mathbb{u}_t^TR_2\mathbb{u}_t, \quad (24a)$$
$$s.t:\ x_{t+1}=\mathbb{X}_{[1\ T]}G_1x_t+\mathbb{X}_{[1\ T]}G_2r_t+Bc_t, \quad (24b)$$
$$\boldsymbol{u}_t=u_t+c_t,\quad \forall t\in\{0,\ldots,N-1\}, \quad (24c)$$
$$\tilde{x}_m\in\mathbb{x}_{\varsigma}, \quad (24d)$$
$$x_t\in X, \quad (24e)$$
$$\boldsymbol{u}_t\in\mathbb{U},\quad \forall t\in\{0,\ldots,N-1\}, \quad (24f)$$
$$\begin{bmatrix}c_0\\ \vdots\\ c_{m-1}\end{bmatrix}=(\mathcal{L}\otimes I_m)\begin{bmatrix}\bar{c}_0\\ \vdots\\ \bar{c}_{\ell-1}\end{bmatrix}, \quad (24f)$$
$$c_t=0,\quad \forall t\in\{\mathfrak{m},\ldots,N-1\}, \quad (24g)$$

where $\tilde{x}_t=x\,(t|k)-x_r(t|k)$, $\mathbb{u}_t=\boldsymbol{u}\,(t|k)-\boldsymbol{u}_r(t|k)$. N, $\mathcal{L}\in\mathcal{R}^{m\times\ell}$, $x_r\,(k)=\bar{x}(k)$ and $u_r(k)$ are the prediction horizon, blocking matrix, desired value, and desired input, respectively. $R_1>0$, and $R_2\geq 0$ are the constant weighting matrices and $\otimes$ denotes Kronecker product. $\tilde{x}_N^TP\tilde{x}_N$ and $\mathbb{x}_{\varsigma}$ are the terminal cost and ET set. The Lyapunov function of data driven-based offline tracking controller is chosen as the terminal cost for minimization problem (24a) to reach the recursive feasibility and closed-loop stability of the controlled system.

Moreover, the following ET constraint (25) is considered in the solving process of the optimization problem (24a). The ET constraint leads to feasibility improvement against the terminal equality-based data driven MPC [21]. Remark 3 demonstrates how to obtain the ET set $\mathbb{x}_{\varsigma}$.

$$\mathbb{x}_{\varsigma}=\left\{\tilde{x}\,(k)\in R^n\,|\,\tilde{x}^T\,(k)\,P\tilde{x}\,(k)\leq\varsigma\right\}. \quad (25)$$

Remark 3 (Calculating Ellipsoidal Set): By calculating the maximum value of ς, the largest possible set $\mathbb{x}_{\varsigma}$ can be obtained. This means the maximum value of ς must be

Algorithm 1 The Steps of Computing ς

Define variables o, $\mathfrak{r}$ as the center and radius of Chebyshev ball
Define variable ε
Calculate P due to Theorem 1
$sP = \sqrt{P}$
$iP = P^{-1}$
Objective $= \max \mathfrak{r}$
Constraints $= [\eta == \mathfrak{r} * iP, \Omega_1 (o + \eta * \varepsilon) \leq \Omega_2]$;
Solve the maximization problem
$\varsigma = \max \mathfrak{r}$

calculated such that all constraints of the system be satisfied. So, consider the following maximization problem:

$$\varsigma = \{\max \mathfrak{r} \,|\, \mathbb{x}_r \subset \Psi\}. \tag{26}$$

where the feasible set Ψ is as follows:

$$\Psi = \{\bar{c}(k) | \text{(24a), (24d) and (24e) are satisfied}\}. \tag{27}$$

To obtain the maximum value of $\mathfrak{r}$, we can reform problem (26) to the Chebyshev ball problem. Therefore, the largest possible ball which can be placed in feasible set Ψ can be determined. In the following algorithm the procedure of computing $\mathbb{x}_\varsigma$ is presented.

where Ω_1 and Ω_2 are the matrices of the limitations on the control inputs and the system states.

Remark 4 (Calculating Blocking Matrix): [33]: To obtain the blocking matrix $\mathcal{L} = \mathcal{L}(0)$, the following conditions must be satisfied: a) the number of the matrix row m should be: $\ell \leq \mathfrak{m} < N$, b) in each row of the matrix, only one element should be equal to one and the other should be zero, c) there should be no completely zero rows or columns, and d) the structure of matrix elements should be upper staircase.

Remark 5: In the first step, the data driven-based offline controller is developed to reach the robust tracking performance. Then, the provided controller $u(k)$ is considered as a known term in optimization problem. The effects of the external disturbance are decreased by $u_1(k)$ which is designed based on the estimation of the external disturbance. Therefore, in the design process of the online part, the optimization problem is structured based on the nominal stabilized system. Moreover, the value of $u(k)$ is considered in constraint (24e) to satisfy the limitations of the total control signal. In fact, the online part of the suggested controller confirms the optimal and constrained performance of the total control signal. Also, to reach the recursive feasibility and closed-loop stability, the Lyapunov function of data driven-based offline controller and the ellipsoidal set (24d) are considered as the terminal cost and terminal constraint of the optimization problem. Furthermore, to decrease the computational complexity of the online optimization problem, the control process is structured based on the move blocking scheme which leads to reduce the number of the decision variables.

Theorem 2 presents the process of the recursive feasibility and closed-loop stability proofs of the system (24b). The structure of the blocking matrix $\mathcal{L}$ at time step $k+1$ divides the proof procedure into two separate cases.

Theorem 2: Consider the optimization problem (24a), the ET set (25) and an admissible blocking matrix $\mathcal{L}(k)$. Let the optimization problem (24a) is feasible with solution $\bar{c}^*(x(k), \mathcal{L}(k))$ at time k. Then, the feasible set $\Psi \neq \emptyset, \forall k \in (k+1, k+2, \ldots)$ and the closed-loop stability of the system (24b) is guaranteed.

Proof: The terminal constraint (24d) is affected by the number of rows of matrix $\mathcal{L}(k+1)$. Therefore, we have:

If (Number of Rows of $\mathcal{L}(k+1) = N$): Consider the following feasible solution at time step k:

$$\begin{aligned}&\bar{c}^*(x(k), \mathcal{L}(k))\\&\quad = \left\{\bar{c}_0^*(x(k), \mathcal{L}(k)), \ldots, \bar{c}_{\ell-1}^*(x(k), \mathcal{L}(k))\right\}.\end{aligned} \tag{28}$$

Based on the receding horizon control (RHC) approach and the ET constraint (24d), the control input sequence (29) satisfies the optimization problem (24) with all constraints at time step $(k+1)$.

$$\begin{aligned}&\bar{c}(x(k+1), \mathcal{L}(k+1))\\&= \left\{\bar{c}_1^*(x(k), \mathcal{L}(k)), \ldots, \bar{c}_{\ell-1}^*(x(k+1), \mathcal{L}(k+1)), \bar{c}_r\right\}.\end{aligned} \tag{29}$$

where $\bar{c}_r$ is the desired value.

If ($1 \leq$ number of rows of $\mathcal{L}(k+1) \leq N-1$): In this case, (number of rows of $\mathcal{L}(k+1)$) = (number of rows of $\mathcal{L}(k)$) $- 1$. This means, by eliminating the first row of $\mathcal{L}(k)$, $\mathcal{L}(k+1)$ will be obtained. As a result, compared with time step k, the ET set is shifted forward at the time step $k+1$. Consequently, $\bar{c}(x(k+1), \mathcal{L}(k+1))$ will be obtained as a non-shifted sequence of $\bar{c}^*(x(k), \mathcal{L}(k))$ as follows:

$$\begin{aligned}&\bar{c}(x(k+1), \mathcal{L}(k+1))\\&\quad = \left\{\bar{c}_0^*(x(k), \mathcal{L}(k)), \ldots, \bar{c}_{\ell-1}^*(x(k), \mathcal{L}(k))\right\}.\end{aligned} \tag{30}$$

Accordingly, the recursive feasibility of the optimization problem (24) is proved. Now, to prove the closed-loop stability of the system, it will be illustrated that the cost function of the optimization problem (24) is decreased from time step k to $k+1$ based on the standard method [34]. Consider the following shifted sequence of $c^*(k)$ based on RHC policy:

$$\tilde{c}(x(k+1), \mathcal{L}(k+1)) = \left\{c_1^*(x(k), \mathcal{L}(k)), \ldots, c_r\right\}. \tag{31}$$

According to the sub-optimal solution $\tilde{c}(x(k+1), \mathcal{L}(k+1))$, we have the following sub-optimal cost function:

$$\begin{aligned}&\mathbb{J}(\tilde{x}(k+1), \mathcal{L}(k+1))\\&\quad = \tilde{x}_{N+1}^T P \tilde{x}_{N+1} + \sum_{t=1}^{N} \tilde{x}_t^T R_1 \tilde{x}_t + \mathbb{u}_t^T R_2 \mathbb{u}_t\\&\quad = \tilde{x}_{N+1}^T P \tilde{x}_{N+1} + \mathbb{J}^*(\tilde{x}(k), \mathcal{L}(k)) - \tilde{x}_0^T R_1 \tilde{x}_0 - \mathbb{u}_0^T R_2 \mathbb{u}_0\\&\qquad + \tilde{x}_N^T R_1 \tilde{x}_N + \mathbb{u}_N^T R_2 \mathbb{u}_N - \tilde{x}_N^T P \tilde{x}_N\end{aligned} \tag{32}$$

According to the lack of re-optimization, the optimal value of cost function is less than its suboptimal at time step $k+1$, therefore, we have:

$$\begin{aligned}
&\mathbb{J}^*\left(\tilde{x}(k+1), \mathcal{L}(k+1)\right) \\
&\quad \leq \tilde{\mathbb{J}}\left(\tilde{x}\left(k+1\right), \mathcal{L}\left(k+1\right)\right) \\
&\quad = \tilde{x}_{N+1}^T P \tilde{x}_{N+1} + \sum_{t=1}^{N} \tilde{x}_t^T R_1 \tilde{x}_t + \mathbb{u}_t^T R_2 \mathbb{u}_t \\
&\quad = \tilde{x}_{N+1}^T P \tilde{x}_{N+1} + \mathbb{J}^*\left(\tilde{x}(k), \mathcal{L}(k)\right) - \tilde{x}_0^T R_1 \tilde{x}_0 - \mathbb{u}_0^T R_2 \mathbb{u}_0 \\
&\qquad + \tilde{x}_N^T R_1 \tilde{x}_N + \mathbb{u}_N^T R_2 \mathbb{u}_N - \tilde{x}_N^T P \tilde{x}_N
\end{aligned} \tag{33}$$

The cost function is decaying sequence if the following condition is satisfied:

$$\tilde{x}_{N+1}^T P \tilde{x}_{N+1} - \tilde{x}_N^T P \tilde{x}_N + \tilde{x}_N^T R_1 \tilde{x}_N + \mathbb{u}_N^T R_2 \mathbb{u}_N \leq 0 \tag{34}$$

Based on the obtained results from Theorem 1, the term $\tilde{x}_N^T P \tilde{x}_N$ is as a Lyapunov function and the control law $u\left(k\right) = K_1 x\left(k\right) + K_2 r\left(k\right) + u_1(k)$ is feasible for system (24b) in $\mathbb{x}_\varsigma$. Therefore, the condition (34) is satisfied and the cost function is a decaying sequence from time step $k+1$ to k which concludes the proof.

The steps of the design process of the provided data-driven controller are illustrated in Algorithm 2.

Remark 6: The main advantages of the suggested robust data-driven MPC can be summarized as follows:

1) Development of a novel data-driven MPC comprising both offline and online components to ensure optimal, constrained, and robust tracking performance.
2) The online optimization problem is formulated for a pre-stabilized system with an ellipsoidal terminal constraint, leading to a significant improvement in MPC feasibility.
3) The proposed MPC scheme incorporates a move blocking method, which reduces the degree of freedom of decision variables and ensures robustness in an offline process. Consequently, the computational complexity of the data-driven MPC is substantially reduced.

Remark 7: According to the fact that the proposed approach consists of two parts, offline and online, it has advantages in both aspects. These advantages include guaranteeing the stability of the closed-loop system, achieving robust and optimal performance, and the ability to consider physical constraints, along with low computational complexity.

IV. SIMULATION RESULTS

The performance of the proposed data-driven controller is evaluated in this section. Two scenarios are considered. In the first scenario, the feasibility of the optimization problem and the computational complexity are compared with the existing data-driven MPC [21]. Moreover, the optimal and constrained performance of the presented controller is compared with the data-driven state-feedback controller [13] in the presence of external disturbance.

Algorithm 2 The Design Process of the Robust Data-Driven MPC

Offline steps

Step 0: Initializing the following parameters:
$\mathcal{L}\left(0\right), N, R_1 > 0, R_2 \geq 0, T \geq \left(m+1\right)n + m$.
Step 1: Calculate the matrices $P > 0$, $Q_1 = G_1 P$ and $Q_2 = G_2 P$ based on the minimization problem (23) (Theorem 1). Obtain the offline controller gains $K_1 = \mathbb{U}_{[0\ \ T-1]} G_1$ and $K_2 = \mathbb{U}_{[0\ \ T-1]} G_2$.
Step 2: Calculate the disturbance observer gain $K_\omega = \left(I_p - \mathfrak{D}\right) \left(MB\right)^\dagger M$.
Obtain the maximum value of ς by Algorithm 1 to obtain the ET set $\mathbb{x}_\varsigma$.

For all k:

Online steps

Step 3: Calculating the estimation of external disturbance, $\widehat{\omega}\left(k\right)$ according to (16).
Step 4: Solve the optimization problem (24) to obtain $c(k)$.
Step 5: Calculating the overall constrained control signal $\boldsymbol{u}\left(k\right) = K_1 x\left(k\right) + K_2 r\left(k\right) + c(k) - B^\dagger B \widehat{\omega}\left(k\right)$ based on (15).
Step 6: Applying the control signal $\boldsymbol{u}\left(k\right)$ to the considered system (5) and go to step 3.

End for

A. FIRST SCENARIO (FEASIBILITY AND COMPUTATIONAL COMPLEXITY COMPARISON)

In this scenario, the feasibility and the computational complexity of the online optimization problem are considered to compare the performance of the proposed approach and the data-driven MPC [21] with $\omega\left(k\right) = 0$.

Suppose the following discrete-time LTI system [28]:

$$x\left(k+1\right) = Ax\left(k\right) + B\left(u\left(k\right) + \omega\left(k\right)\right)$$

where

$$A = \begin{bmatrix} a & 0.215 & -0.108 \\ 0.458 & 0.079 & 0.085 \\ -0.564 & -0.326 & 0.894 \end{bmatrix}, \quad B = \begin{bmatrix} 0.929 & 0.914 & -0.716 \\ -0.684 & -0.029 & -0.156 \\ 0.941 & 0.600 & b \end{bmatrix}.$$

To compare the feasibility problem, the parameters $-2 \leq a \leq 3$, $-0.8 \leq b \leq 1$ with initial condition $x\left(0\right) = [0.1\ \ 0.05\ \ 0.2]$ are considered. Figure 1 illustrates the feasibility reign of the online optimization problem (24) under the proposed data-driven controller and data-driven MPC [21]. The feasibility issue is greatly enhanced with the suggested data-driven controller due to the following factors: 1) configuration of the online optimization problem using a stabilized system, 2) consideration of an ET constraint during the solving the online optimization problem, and 3) a reduced number of decision variables compared to the data-driven MPC [21].

Moreover, the tracking performances of the proposed data-driven MPC and the designed data-driven MPC in [21]

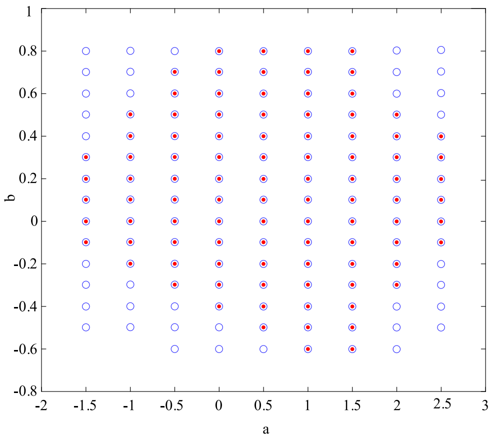

FIGURE 1. The feasibility regions of the proposed data-driven method (∘) and the data-driven MPC (*) [21].

are also evaluated with $a = 0.134$, $b = 0.793$. To do this, the matrices of reference model are considered as $\bar{A} = 0.2I_3$ and $\bar{B} = 0.8I_3$. The trajectories of $x_1(t)$, $x_2(t)$, and $x_3(t)$ under the suggested method and the data-driven MPC in [21] are illustrated in Figure 2. Reducing the conservatism of the online optimization problem of the designed controller against the data-driven MPC in [21] leads to an improvement in the system response.

Figure 3 displays the controller signals obtained from the proposed method and the data-driven MPC [21]. The feasibility enhancement of the data-driven controller leads to a decreased conservatism in the online optimization problem. As a result, the calculated control signals using the proposed approach are notably more favorable compared to [21], particularly in terms of transient response. The zoomed sections in Figure 3 highlight this improvement. Indeed, based on Theorem 1, the offline controller gains K_1 and K_2 are obtained as follows:

$$K_1 = \begin{bmatrix} 0.592 & -0.281 & 0.294 \\ -0.374 & 0.385 & -0.689 \\ 0.236 & 0.435 & -0.658 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 0.068 & -1.289 & -0.178 \\ 0.456 & 1.589 & 0.695 \\ -0.418 & 0.329 & 0.671 \end{bmatrix}.$$

The tuning parameters of the proposed controller are selected as follows:

$$\mathcal{L}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
$$N = 7,\ R_1 = 8I_3,\ R_2 = 0.1I_3,\ T = 30.$$

Moreover, the value of $\varsigma = 0.8634$ is obtained by Algorithm 1. To show the computational complexity of two data-driven controllers, the average and total CPU time are calculated and compared in Table 2. The performance of

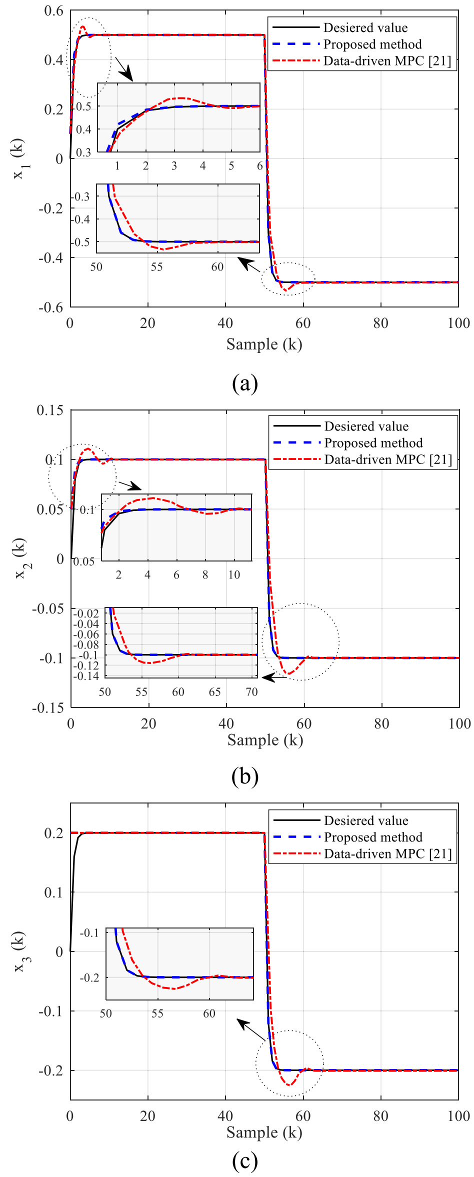

FIGURE 2. The trajectories of (a) : $x_1(t)$, (b): $x_2(t)$, and (c): $x_3(t)$.

TABLE 2. The obtained average and total CPU time.

Controller	Average CPU time	Total CPU time
The data-driven MPC [21]	0.0814 s	8.14 s
The suggested data-driven MPC	**0.0112** s	**1.12** s

the suggested controller is enhanced through the reduction of decision variables, implementation of the move blocking scheme, and utilization of a stabilized system in the online optimization problem. These actions effectively reduce the computational burden, leading to improved controller performance.

B. SECOND SCENARIO (CONSTRAINED PERFORMANCE AND OPTIMALITY COMPARISON)

In this scenario, the performance of the suggested data-driven MPC is evaluated in the presence of external disturbance

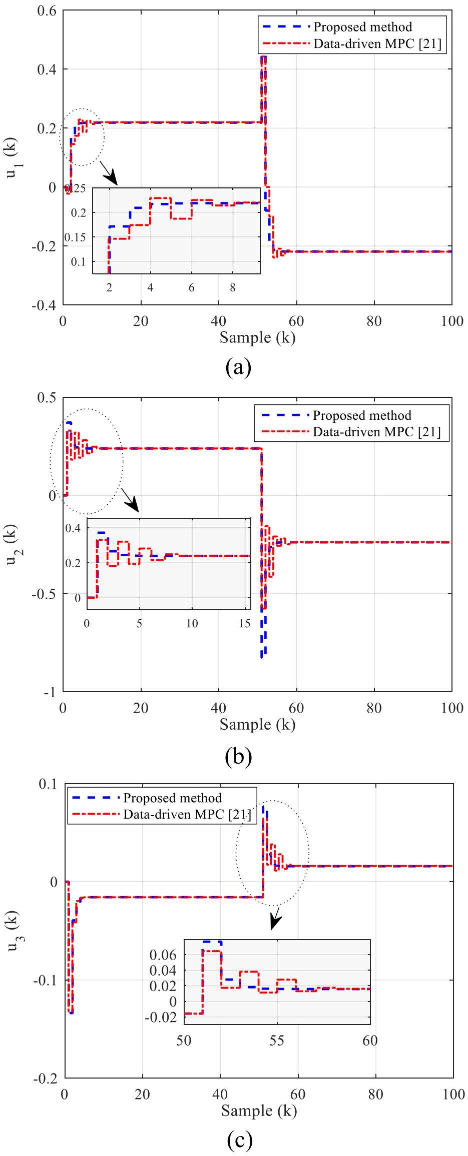

FIGURE 3. The applied data-driven MPC signals.

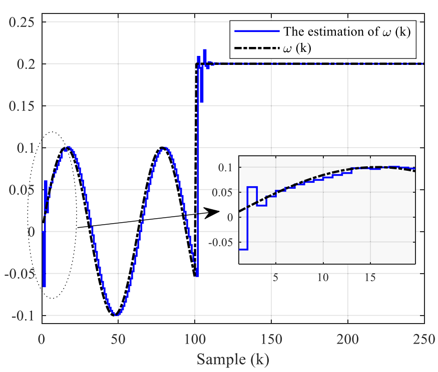

FIGURE 4. The estimation of the external disturbance.

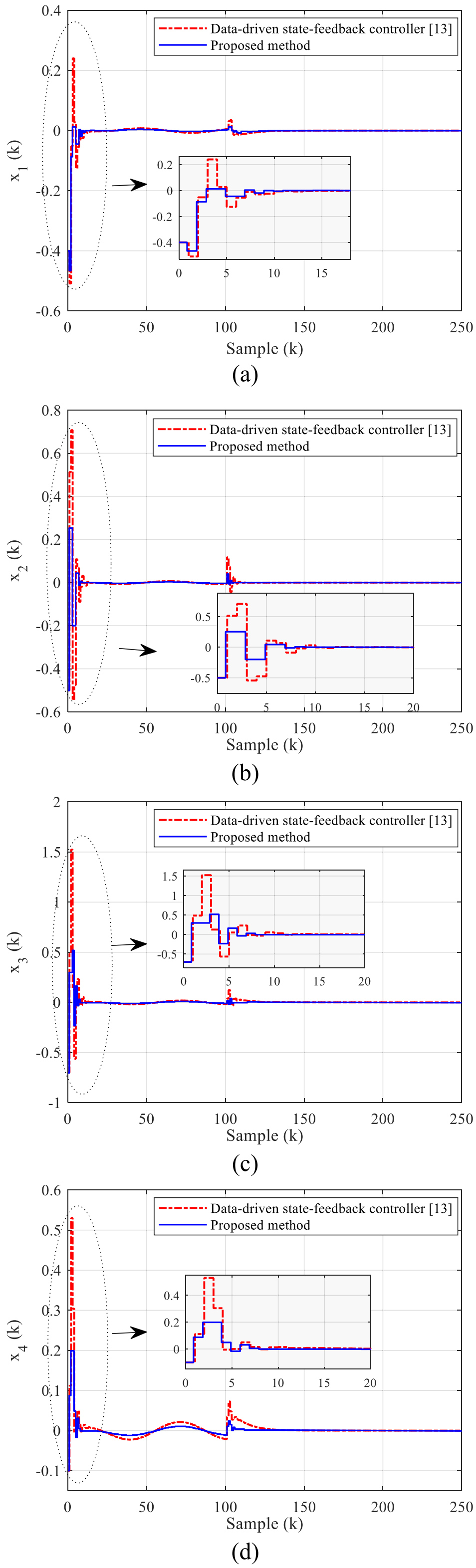

FIGURE 5. The trajectories of (a): $x_1(t)$, (b): $x_2(t)$, (c): $x_3(t)$, and (d): $x_4(t)$.

and is compared with the data-driven state-feedback controller [13]. The following discrete-time LTI system is considered:

$$x(k+1) = Ax(k) + B(u(k) + \omega(k)),$$

where

$$A = \begin{bmatrix} 1.178 & 0.001 & 0.511 & -0.403 \\ -0.051 & 0.661 & -0.011 & 0.061 \\ 0.076 & 0.0335 & 0.0560 & 0.0382 \\ 0 & 0.0335 & 0.089 & 0.0849 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.004 & -0.087 \\ 0.467 & 0.001 \\ 0.213 & -0.235 \\ 0.213 & -0.016 \end{bmatrix},$$

$$\omega_1(k) = \omega_2(k) = \begin{cases} 0.1 \sin\left(\frac{k}{10}\right) & k < 100 \\ 0.2 & k \geq 100 \end{cases}$$

The tuning parameters of the suggested data-driven MPC are chosen as follows:

$$\mathcal{L}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
$$N = 7,\ R_1 = 5I_4,\ R_2 = 1I_2,\ T = 15.$$

Due to Algorithm 1, the value of $\varsigma = 0.1676$ is obtained. Moreover, the offline controller gain K_1, and matrix P are calculated based on Theorem 1, as follows:

$$K_1 = \begin{bmatrix} -1.5514 & 1.1163 & -1.6862 & 0.6403 \\ 4.0694 & -0.5412 & 3.6088 & -1.6283 \end{bmatrix},$$
$$P = \begin{bmatrix} 0.1982 & 0.1052 & -0.1489 & -0.0021 \\ 0.1052 & 0.5246 & 0.1705 & -0.0326 \\ -0.1489 & 0.1705 & 0.5954 & 0.4648 \\ -0.0021 & -0.0326 & 0.4648 & 0.7823 \end{bmatrix}.$$

According to (16), the estimation of the external disturbance is obtained and shown in Figure 4. Therefore, the disturbance estimation-based controller part $u_1(k)$ in (15c) can be obtained. Then, according to the offline part (15b), the overall controller $\boldsymbol{u}(k)$ in (15a) is calculated and applied to the considered system. With the proposed controller and the data-driven state-feedback controller [13], the trajectories of the states are obtained and illustrated in Figure 5. Compared with the data-driven state-feedback controller [13], the performance of the proposed data-driven MPC is optimal. Therefore, the system states convergence to zero in less time which are zoomed in Figure 5. As a results, the system response is improved under the proposed data-driven controller.

By configuring the online optimization problem of the proposed approach based on a stabilized system, the conservatism of the controller is significantly reduced. In other words, the online component is designed to achieve optimality and ensure satisfaction of constraints.

Furthermore, the physical limitations of the system states and the control input are satisfied during the design procedure of the proposed approach. In this simulation, the limitations are considered as follows:

$$[-0.5\ -0.6\ -0.8\ -0.1]^T \leq x(k) \leq [0.3\ \ 0.7\ \ 1.5\ \ 0.6]^T,$$
$$[-0.5\ \ -3.2]^T \leq u(k) \leq [1.8\ \ 1.5]^T.$$

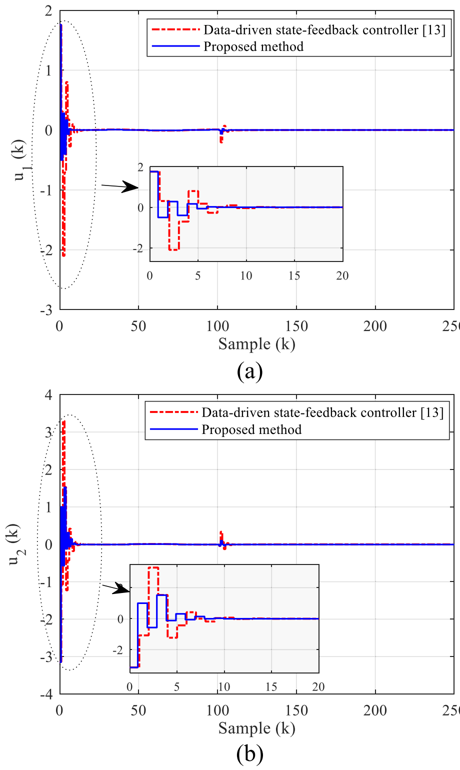

FIGURE 6. The applied data-driven control signals.

As shown in Figure 6, the control input constraints are satisfied with the proposed data-driven MPC. The zoomed parts of the control inputs demonstrate that the data-driven state-feedback controller [13] fails to satisfy the specified controller limitations. As a result, the proposed data-driven MPC, which ensures optimality and constraints satisfaction, leads to improved performance and reduced convergence time.

V. CONCLUSION AND FUTURE WORKS

In this article, a new data-driven MPC strategy was presented to robustly control the discrete-time LTI systems. The design process of the proposed data-driven controller was based on two offline and online schemes. To decrease the conservatism of the existing robust MPC methods, the robustness against the external disturbance was guaranteed based on the offline data-driven controller with tracking performance. More precisely, the offline part of the overall controller was designed based on the model matching problem and the data-driven-based estimation of the external disturbance to reach the robustly tracking of the predefined reference. Then, to satisfy the limitations of the system states and the control inputs, a data-driven MPC was provided according to the stabilized system. To do this, the optimization problem of the MPC was configured based on the stabilized system with the move blocking strategy. By considering an ET set which was affected by the proposed move blocking strategy, the feasibility enhancement resulted. In summary, the major contributions of the provided robust data-driven MPC compared with the existing data-driven MPC methods [21], [22], [24] and the data-driven state-feedback controller [13] were 1) decreasing the online computational complexity, 2) improvement of feasibility problem of the MPC, and 3) achieving to robust, constrained and optimal tracking performance. However, to design the presented controller, the input matrix must be known or estimated and the measurement noise of the used data are not considered. Therefore, for future works we consider the following topics:

1) Developing a robust data-driven MPC based on the move blocking approach and disturbance observer with noisy data.
2) Designing a move blocking robust data-driven MPC without the employing the input matrix.
3) Developing an event-triggered robust data-driven MPC based on the move blocking approach to further decrease the online computational burden and the control updates.
4) Considering the robustness against the parameter uncertainties during the design process of the data-driven MPC.

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