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RESEARCH ARTICLE

Practical Validation of a New Analytical Method for the Analysis of Power Transmission Lines at Steady State

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ABSTRACT For basic studies of power transmission lines, in steady state, any line is usually modeled through a single-line pi-circuit, which intrinsically represents all the conductors of the three phases, as well as the grounding wires, and thus, allows analytical approaches that facilitate the understanding of the theoretical fundamentals of the operation of any transmission line. However, for practical applications, when any unavoidable degree of unbalance is present, this method does not give accurate results and thus alternative and more complex methods of analysis are naturally used. On the other hand, although most of the usual alternative methods are quite effective in the analysis of the behavior of the line, they require the obvious use of computer tools and calculations that make difficult any of the important analytical approaches. Thus, based on these concerns and with the aim of giving emphasis on allowing the analytical approach, this article proposes a new analytical method for the analysis of the behavior of a power transmission line, in a steady state, based on the application of the Laplace transform. To illustrate the proposed method, it is introduced and validated through an actual case in the Swedish transmission grid with a basis on real data of phasor measurement units (PMU) systems. The accuracy of the proposed method is evidenced by the comparison with actual data from PMU measurements along with the prediction of the voltage and current profile for each phase, over the line length.

I. INTRODUCTION

Due to its fundamental importance in power systems, the accurate modeling of a power transmission line plays a very important role in the analyses of power systems [1]. The most important steady-state operation of any power transmission line, both in the planning phase as well as during operation, needs to be carefully evaluated through modeling, which characterizes the study of contingencies [2]. The aim is to evaluate the severity of some limiting loading conditions, in view of checking the stability of not only the studied power transmission line as well as of the whole power system [3].

To perform these evaluations, the adoption of computer programs is quite usual and consolidated, thanks to the wide availability of software with a high degree of accuracy. On the

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other hand, for some practical circumstances, the analytical evaluation of the behavior of a power transmission line, at a steady state, is preferred. For example, this could be the case of a conceptual analysis of the differences between capacitive and inductive loads.

Modeling a power transmission line by an equivalent single-phase circuit, compressing the effects of all the phase conductors and grounding wires in the nominal π-model is the usual preference due to its significant simplicity [4]. However, the obtained results are not always sufficiently accurate due to several assumptions that are hard to realize. For example, the assumption of a balanced three-phase voltage and current phasors is almost never fully true in most conditions. In the following study, a more detailed method of analyzing three-phased transmission systems is proposed, based on the availability of phasor data of voltage to ground and current of each phase. This more accurate steady-state analysis may be used to analyze real three-phase power transmission lines, without using any drastically simplifying assumptions.

Phasor data of voltage to ground and current of each phase is becoming increasingly available to system operators as new phasor measurement units (PMU) are being adopted in the power system on a larger scale [5]. The PMU equipment provides time-referenced values of the amplitude of voltage and current for each phase [6], as well as their phase angle, within pre-defined time intervals, which perfectly serve for the purposes of the proposed method.

This data in combination with the proposed method allows accurate evaluation of the behavior of each of the phases of a transmission line. To demonstrate the advantages of the proposed method, this work shows its application to a real transmission line that is operated by the Swedish Transmission System Operator (TSO) Svenska Kraftnät, between the cities of Alvesta and Tenhult.

II. PROPOSED ANALYTICAL METHOD

The analytical method to more accurately analyze an unsymmetrical three-phase system is derived in the following section. Considering the following two basic phasor matrix equations that rule the behavior of voltage and current of each conductor of a transmission line along its length (x) at steady state [7]:

$$\frac{d}{dx}\left[\dot{V}_{r,s,t}(x)\right] = -\left[\dot{Z}_{r,s,t}\right]\left[\dot{I}_{r,s,t}(x)\right] \tag{1}$$

$$\frac{d}{dx}\left[\dot{I}_{r,s,t}(x)\right] = -\left[\dot{Y}_{r,s,t}\right]\left[\dot{V}_{r,s,t}(x)\right] \tag{2}$$

where $[\dot{V}_{a,b,c}(x)]$ is the reduced three-phase matrix of phasor voltage to the ground for each conductor of the line, $[\dot{I}_{a,b,c}(x)]$ is the respective reduced three-phase matrix of phasor current, $[\dot{Z}_{a,b,c}]$ is the reduced three-phase matrix of impedances per unit of length and $[\dot{Y}_{a,b,c}]$ is the reduced three-phase matrix of admittances per unit of length.

The process of matrix reduction is performed through the method of Kron [8], which allows the reduction of the order of the matrixes based on the assumption of null voltage for the grounding wires. After differentiation of (1) and application of (2), the following second-order matrix equation is obtained:

$$\frac{d^2}{dx^2}\left[\dot{V}_{r,s,t}\,(x)\right] = \left[\dot{\gamma}_{r,s,t}\right]^2\left[\dot{V}_{r,s,t}\,(x)\right] \tag{3}$$

where

$$\left[\dot{\gamma}_{r,s,t}\right] = \sqrt{\left[\dot{Z}_{r,s,t}\right]\left[\dot{Y}_{r,s,t}\right]} \tag{4}$$

is the matrix of constants of propagation. By applying the Laplace transform and its properties to (3), it becomes

$$L\left\{\frac{d^2}{dx^2}\left[\dot{V}_{r,s,t}(x)\right]\right\} = L\left\{\left[\dot{\gamma}_{r,s,t}\right]^2\left[\dot{V}_{r,s,t}(x)\right]\right\} \tag{5}$$

or

$$\begin{aligned}\left[\dot{\gamma}_{r,s,t}\right]^2\left[\dot{V}_{r,s,t}(s)\right] &= s^2\left[\dot{V}_{r,s,t}(s)\right] \\ &\quad -s\left[\dot{V}_{r,s,t}(x=0)\right] - \frac{d}{dx}\left[\dot{V}_{r,s,t}(x=0)\right]\end{aligned} \tag{6}$$

Now, by applying (1) to the last term of (5), it becomes

$$\begin{aligned}\left[\dot{\gamma}_{r,s,t}\right]^2\left[\dot{V}_{r,s,t}(s)\right] &= s^2\left[\dot{V}_{r,s,t}(s)\right] \\ &\quad -s\left[\dot{V}_{r,s,t}(x=0)\right] \\ &\quad +\left[\dot{Z}_{r,s,t}\right]\left[\dot{I}_{r,s,t}(x=0)\right]\end{aligned} \tag{7}$$

After an algebraic arrangement in (7), the expression for the voltage for each conductor, in the Laplace domain is

$$\begin{aligned}\left[\dot{V}_{r,s,t}(s)\right] &= \left\{s^2\,[i] - \left[\dot{\gamma}_{r,s,t}\right]^2\right\}^{-1} \\ &\quad \left\{s\left[\dot{V}_{r,s,t}(x=0)\right] - \left[\dot{Z}_{r,s,t}\right]\left[\dot{I}_{r,s,t}(x=0)\right]\right\}\end{aligned} \tag{8}$$

where $[i]$ is the identity matrix.

Finally, the voltage to the ground along the length for each conductor is obtained after the operation of inversion of the Laplace transform

$$\left[\dot{V}_{r,s,t}(x)\right] = L^{-1}\left\{\left[\dot{V}_{r,s,t}(s)\right]\right\} \tag{9}$$

Similarly, the expression for the current of each conductor, based on (2), is obtained by

$$\begin{aligned}\left[\dot{I}_{r,s,t}(x)\right] &= L^{-1}\left\{\left[\dot{I}_{r,s,t}(s)\right]\right\} \\ &= L^{-1}\left\{\left[\frac{\left[\dot{I}_{r,s,t}(x=0)\right] - \left[\dot{Y}_{r,s,t}\right]\left[\dot{V}_{r,s,t}(s)\right]}{s}\right]\right\}\end{aligned} \tag{10}$$

In addition, for taking into account the line transposition, an additional and basic property of the Laplace transforms is used. This is the multiplication of the function in the Laplace domain by the exponential term, $e^{-x_o s}$, for obtaining the inverse function in the space domain s. In turn, x_o is the distance, in km, from the origin ($x = 0$) to the point where the transposition takes place. Thus, to obtain the voltage and current in the space domain the following transformations must be performed:

$$\left[\dot{V}_{r,s,t}\left(x-x_o\right)\right]=L^{-1}\left\{e^{-x_o s}\left[\dot{V}_{r,s,t}(s)\right]\right\} \tag{11}$$

$$\left[\dot{I}_{r,s,t}\left(x-x_o\right)\right]=L^{-1}\left\{e^{-x_o s}\left[\dot{I}_{r,s,t}(s)\right]\right\} \tag{12}$$

Finally, the phasor functions to express voltage and current along the distance x may be stated as:

$$\dot{V}_{r,s,t}(x-x_o)=\sum_{i=1}^{3}(\dot{A}_i e^{\alpha_i x}+\dot{B}_i e^{-\alpha_i x})_{r,s,t} \tag{13}$$

$$\dot{I}_{r,s,t}(x-x_o)=\dot{C}_{r,s,t}+\sum_{i=1}^{3}(\dot{D}_i e^{\alpha_i x}+\dot{E}_i e^{-\alpha_i x})_{r,s,t} \tag{14}$$

where $\dot{A}_i$ and $\dot{B}_i$ are obtained from the initial values of phasor voltage, set at $x = 0$, whereas $\dot{C}_i$, $\dot{D}_i$ and $\dot{E}_i$ are similarly obtained with basis on the initial values of phasor current. α_i is a complex number of which the real component is the constant of attenuation, in p.u./km and the imaginary component is the constant of phase, in rad/km, along the line length that has transposition at $x = x_{oi}$.

All of these values are different for each of the phases, r, s, and t. All these procedures of calculation are not new and details of the process of calculation are available in [9] and [10]. However, it is important to point out that the key step of such a calculation is the inversion of the Laplace transforms, which have only become available recently due to the development of powerful mathematical software.

III. RELATED WORKS

The steady state of power lines can be analyzed by the characteristics of faults in the power system, Jia et al. [11] evaluated the overvoltage caused by switch-tripping faults in ultra-high voltage lines, control measures to mitigate the overvoltage are described by them to give a baseline for restricting the overvoltage in steady-state. Polajžer et al. [12] evaluated faults considering an inverter interfaced in the steady state of transmission networks.

Yuan et al. [13] presented a unified power flow controller for steady-state modeling of power grids. Yuan et al. [14] presented a study about steady-state modeling and the optimal operation of the electrical power system with flexible alternating current transmission systems.

The study of PMU has been researched by authors focusing on the improvement of the way it is measured. Affijulla and Tripathy [15] presented a simple phasor estimator. The measured signals feed the PMU prototype with the proposed model running for the phasor calculation of signals in the electrical power system. The presented prototype has a low-cost PMU for smart grid applications.

Qin et al. [16] presented a remote field testing of the PMU using a performance analyzer. As they highlighted the PMU is necessary to ensure the precision of synchrophasor-based control. Given that several PMU testing are performed manually, in their application, it is proposed to perform PMU testing remotely. They have shown that using a field-programmable gate array was possible to measure the system under the specifications.

The evaluation of enhancing the PMU for the analysis of the power quality has been done by several authors, Pegoraro et al. [17] evaluated the compensation of systematic measurement errors in PMU for monitoring distribution power lines, Mingotti, Peretto, and Tinarelli [18] an equivalent synchronization process for the PMU, Bernard et al. [23] studied the harmonic and interharmonic phasor estimation for PMU.

Chernikova, Kosteletskii, and Zabolotsky [19] propose analytical models that accurately capture the behavior of transmission lines, considering the effects of impedance discontinuities, line length, and termination conditions. The differential and common mode voltage and current distributions along the transmission line are derived, enabling the calculation of important parameters of the grids.

The work of Pu et al. [20] provides a comprehensive overview of analytical methods for detecting and protecting against faults in direct current transmission lines, with a specific focus on single-ended protection techniques. In their work, The single-ended protection techniques utilize measurements taken at a single end of the transmission line to detect faults.

Feng et al. [21] presented how the reliability of high-voltage direct current systems can be affected by various external factors, such as temperature, wind, and pollution. To address this concern, they propose a multi-factor reliability evaluation method that incorporates these environmental parameters into the analysis. Their method utilizes probabilistic models and Monte Carlo simulations to assess the impact of each external factor on the overall reliability of the transmission power lines.

Mustafa et al. [22] presented a methodology that combines theoretical modeling with practical considerations to accurately represent the behavior of power transmission lines. Their analytical method takes into account various parameters, including line length, conductor spacing, and material properties, to determine the line's impedance, voltage drop, and power loss characteristics. The results show agreement between the modeled and measured data, validating the reliability of their method.

As presented in [24], the measure of the electric field is an alternative way to evaluate if the power system is well designed and is under the required specifications. In their application, the finite element method (FEM) is used to evaluate the optimal design of spacers of distribution power grids. In the work of Zuo et al. [25] electromagnetic field problems are considered using a combination of FEM and boundary element methods. In [26] the analysis is regarding the insulators, showing high performance when combined with optimization methods.

In [27] the evaluation of different methods used for calculating the electric field in transmission lines is presented. The load simulation method, the FEM, and the measured electric field are compared considering a 525 kV power transmission

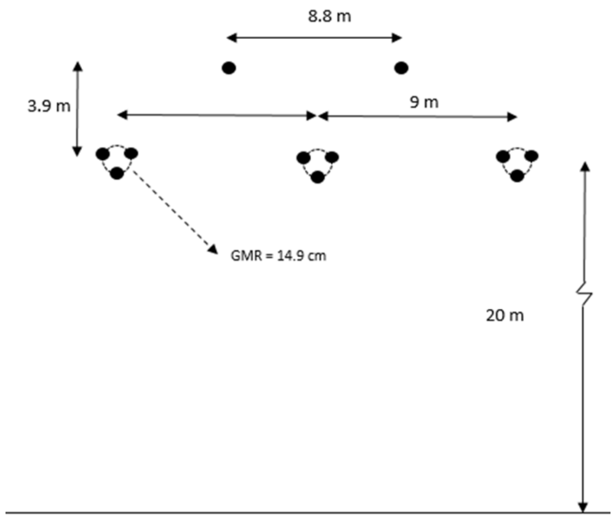

FIGURE 1. Cross-section view of the conductors with distances for the 400 kV – SvK – Alvesta/Tenhult power transmission line.

TABLE 1. Characteristics of the line conductors.

Conductors	Characteristics
Phase	3 x ACSR 54/7 Curlew 1033.5 kcmil (per phase) - GMR = 14.9 cm
Grounding wires	Two AAAC 12/7 Dotterel 176.9 kcmil

line. The results showed that the FEM is closer to the results of the measured electric field when the measure is under the cables (close to the center of the power line), after 10 m from the center of the transmission line, the load simulation method was more promising compared to the FEM.

Besides the analytical and optimization methods of analyzing power systems, there is an increasing interest in the application of artificial intelligence models especially based on deep learning for fault identification in transmission lines [28]. According to Souza et al. [29], the use of hybrid methods may outperform well-defined models.

IV. STUDY OF A PRACTICAL CASE

The application of the proposed method is illustrated through a case based on measurements performed in a 400 kV power transmission line, 97.3 km long, non-perfectly transposed, located in Southern Sweden. This line is operated by the *Swedish TSO Svenska Kraftnät* (SvK) and connects the power substations of the cities of Tenhult and Alvesta. The cross-section view of this power transmission line is shown in Figure 1, where all the important distances are indicated, whereas in Table 1 the most important characteristics of the conductors are also given.

The values of the electric resistance of all the conductors were obtained from the manufacturer's catalogs. In addition, since the average value of the electric resistivity of the local soil has been assumed as equal to 100 Ωm and the power frequency is 50 Hz, the correction factors of Carson, for the return of current through the ground, were not taken into account [30].

Based on all these data, the obtained elements of the 5 × 5 matrixes of impedances and admittances, per unit of length, are given below, in Ω/km and μS/km, respectively.

$$\begin{bmatrix} 0.02+j0.36 & j0.1048 & j0.0644 & j0.1149 & j0.0813 \\ j0.1048 & 0.02+j0.36 & j0.1048 & j0.1157 & j0.1157 \\ j0.0644 & j0.1048 & 0.02+j0.36 & j0.0813 & j0.1149 \\ j0.1149 & j0.1157 & j0.0813 & 0.32+j0.58 & j0.1230 \\ j0.0813 & j0.1157 & j0.1149 & j0.1230 & 0.32+j0.58 \end{bmatrix} \tag{15}$$

$$j\begin{bmatrix} 3.4871 & -0.7265 & -0.2417 & -0.4741 & -0.1972 \\ -0.7265 & 3.7236 & -0.7265 & -0.4125 & -0.4125 \\ -0.2417 & -0.7265 & 3.4871 & -0.1972 & -0.4741 \\ -0.4741 & -0.4125 & -0.1972 & 2.1716 & -0.2750 \\ -0.1972 & -0.4125 & -0.4741 & -0.2750 & 2.1716 \end{bmatrix} \tag{16}$$

Thus, in accordance with the method of Kron for matrix reduction, both the matrixes were reduced to 3 × 3 matrixes through the virtual elimination of the grounding wires [8] and they respectively become in

$$\begin{bmatrix} 0.0341+j0.3377 & 0.0124+j0.0780 & 0.0099+j0.0425 \\ 0.0124+j0.0780 & 0.0376+j0.3297 & 0.0124+j0.0780 \\ 0.0099+j0.0425 & 0.0124+j0.0780 & 0.0341+j0.3377 \end{bmatrix} \tag{17}$$

as the matrix of impedances, in Ω/km, and

$$j\begin{bmatrix} j3.487089 & -j0.726499 & -j0.241746 \\ -j0.726499 & j3.723602 & -j0.726500 \\ -j0.241746 & -j0.726499 & j3.487089 \end{bmatrix} \tag{18}$$

Now, by setting the reference of length, $x = 0$, as being at the Alvesta power substation, the data of the PMU System of this same substation, voltage to ground, and current of each phase, was taken as the initial condition for the analytical solution. The behavior of this transmission line at two different instants was then analyzed, based on PMU measurements data obtained on the 20th of March of 2013.

A. INSTANT 1 – LOCAL TIME: 13:37:16

These were the values for phase voltage and current at $x = 0$, at the Alvesta power substation, with power flowing from Tenhult to Alvesta:

$$\dot{V}_{r(x=0)} = 238900.9\angle 123.95V$$
$$V = -133429.5 + j198167.1\ V \tag{19}$$
$$\dot{V}_{s(x=0)} = 239356.7\angle 4.00V$$
$$V = 238773.1 + j16704.8\ V \tag{20}$$
$$\dot{V}_{t(x=0)} = 239413.8\angle -116.11V$$
$$V = -105349.7 - j214989.3\ V \tag{21}$$
$$\dot{I}_{r(x=0)} = 146.6\angle -114.8A$$
$$A = -61.4 - j133.1\ A \tag{22}$$

TABLE 2. **Comparison between data of experimental measurement (PMU) and the proposed method (analytical) for phase voltage and current at the Tenhult power substation, at 13:37:16 - local time.**

	Tenhult PMU Data (Measured)	Tenhult Proposed Method (Analytical)
$\dot{V}_r(V)$	241118.8 ∠+124.42	241087.1 ∠+124.35
$\dot{V}_s(V)$	240383.0 ∠+004.39	241275.3 ∠+004.42
$\dot{V}_t(V)$	241682.5 ∠-115.62	241606.3 ∠-115.63
$\dot{I}_r(A)$	80.3 ∠-075.3	80.6 ∠-076.8
$\dot{I}_s(A)$	74.4 ∠+169.3	75.1 ∠+167.7
$\dot{I}_t(A)$	82.7 ∠+048.2	83.9 ∠+046.6

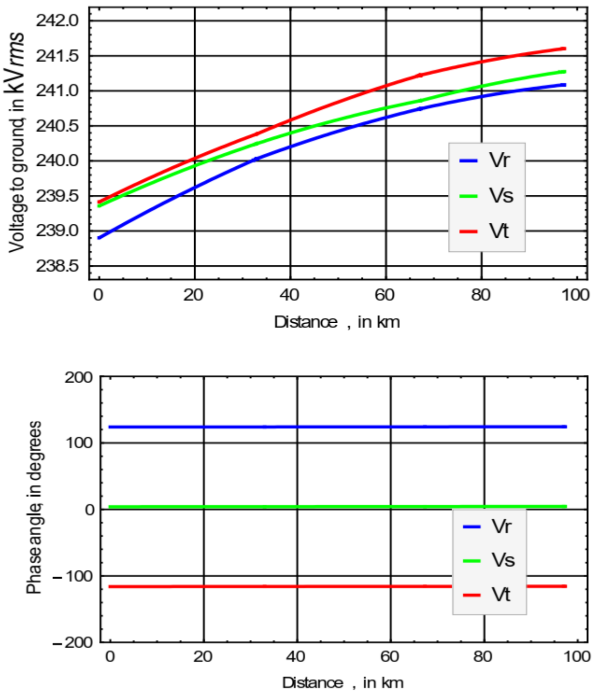

FIGURE 2. **Profile of phase voltage to the ground along the line length at the Instant 1: (a) Amplitude; (b) Phase angle.**

$$\dot{I}_{s(x=0)} = 137.8\angle 125.5A$$
$$A = -80.1 + j112.2\,A \tag{23}$$
$$\dot{I}_{t(x=0)} = 145.7\angle 7.5A$$
$$A = 144.4 + j18.9\,A \tag{24}$$

After the application of these data to the proposed method, the values predicted for voltage and current at the Tenhult power substation were evaluated and compared to the real PMU data, as shown in Table 2.

The analytical results had a very high good likelihood with the actual measured values, at the Tenhult power substation, which confirms the accuracy of the proposed method. Therefore, results like active and reactive power at both ends agree, too. The calculated phase voltage and current behavior along this line, at this same instant, may also be obtained by the proposed method and is shown in Figs. 2 and 3.

From the graphics of Fig. 2(a), it is noticeable not only how unbalanced the voltage amplitudes are as well as the

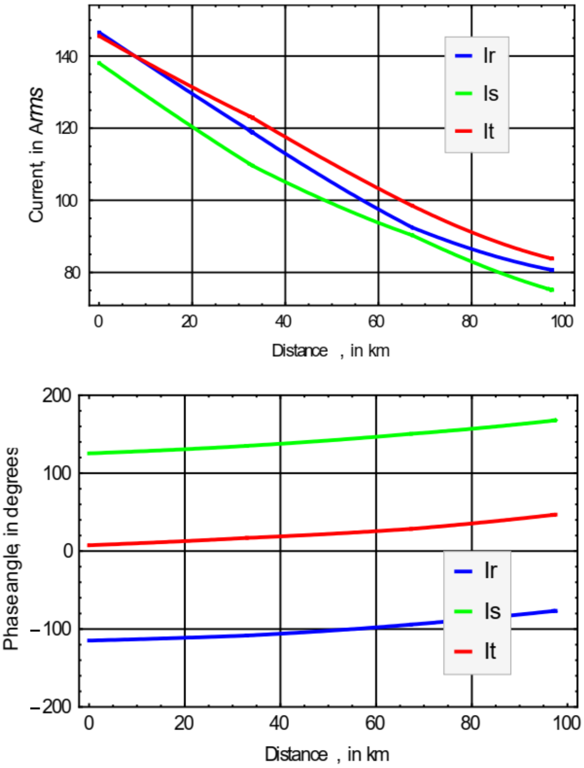

FIGURE 3. **Profile of phase current along the line length at the Instant 1: 3(a) Amplitude; 3(b) Phase angle.**

points where each of the transposition happens, along the line length. Moreover, from Fig. 2(b) it is noticeable how stable the behavior of the phase angle of each phase voltage is.

Regarding the behavior of the profiles of phase current, the graphics of Fig. 3(a) show that the current amplitude is similarly unbalanced as the voltage, whereas graphics of Fig. 3(b) show that the changes in the phase angle of current along the line length are more significant than for the phase angle of voltage.

B. INSTANT 2 – LOCAL TIME: 13:40:47

After less than three minutes, new values were gathered for the phase voltage and current at $x = 0$ at the Alvesta power substation, with power still flowing from Tenhult to Alvesta.

$$\dot{V}_{r(x=0)} = 238925.2\angle 154.42V$$
$$V = -215513.2 + j103147.1\,V \tag{25}$$
$$\dot{V}_{s(x=0)} = 239324.4\angle 34.47V$$
$$V = 197289.7 + j135473.0\,V \tag{26}$$
$$\dot{V}_{t(x=0)} = 239444.8\angle -85.62V$$
$$V = 18301.0 - j238744.4\,V \tag{27}$$
$$\dot{I}_r = 152.4\angle 281.76A$$
$$A = 31.054 - j149.198\,A \tag{28}$$
$$\dot{I}_{s(x=0)} = 144.0\angle 162.2A$$
$$A = -137.154 + j43.906\,A \tag{29}$$

TABLE 3. Comparison between data of experimental measurement (PMU) and the proposed method (analytical) for phase voltage and current at the Tenhult power substation, at 13:40:47 - local time.

	Tenhult PMU Data (Measured)	Tenhult Proposed Method (Analytical)
$\dot{V}_r(V)$	241,038.2 ∠+154.99	241,046.0 ∠+154.93
$\dot{V}_s(V)$	240,280.0 ∠+034.96	241,177.5 ∠+035.00
$\dot{V}_t(V)$	241,614.0 ∠-085.03	241,582.1 ∠-085.04
$\dot{I}_r(A)$	95.1 ∠-039.04	94.9 ∠-040.30
$\dot{I}_s(A)$	89.8 ∠-155.06	89.8 ∠-156.43
$\dot{I}_t(A)$	97.7 ∠+083.76	98.2 ∠+082.10

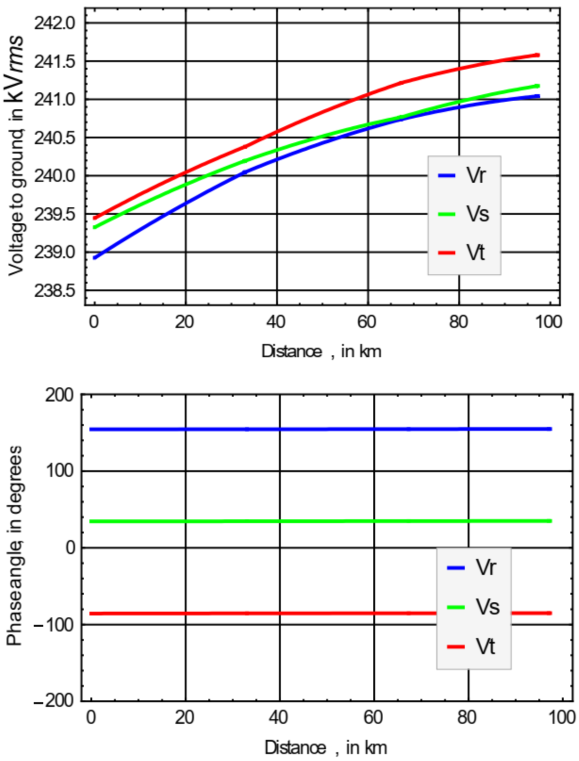

FIGURE 4. Profile of phase voltage to ground at the Instant 2: 4(a) Amplitude; 4(b) Phase angle.

$$\dot{I}_{t(x=0)} = 152.1\angle 43.8A$$
$$A = 109.714 + j105.275\,A \tag{30}$$

Once more, by applying the measured data to the proposed analytical method, the values for voltage to ground and current at the Tenhult power substation could be then predicted. Table 3 shows the comparison between the so predicted and the actual values.

Once again, the obtained results indicate good coherence with the measured data. Thus, in a similar way as done for Instant 1, the behavior of voltage to ground and current along the line is shown in the graphics of Figs. 4 and 5, respectively. In comparison to the respective graphics of Figs. 2 and 3 it is noticeable that, on average, the voltage has remained the same whereas the current has experienced a slight increase in its amplitude.

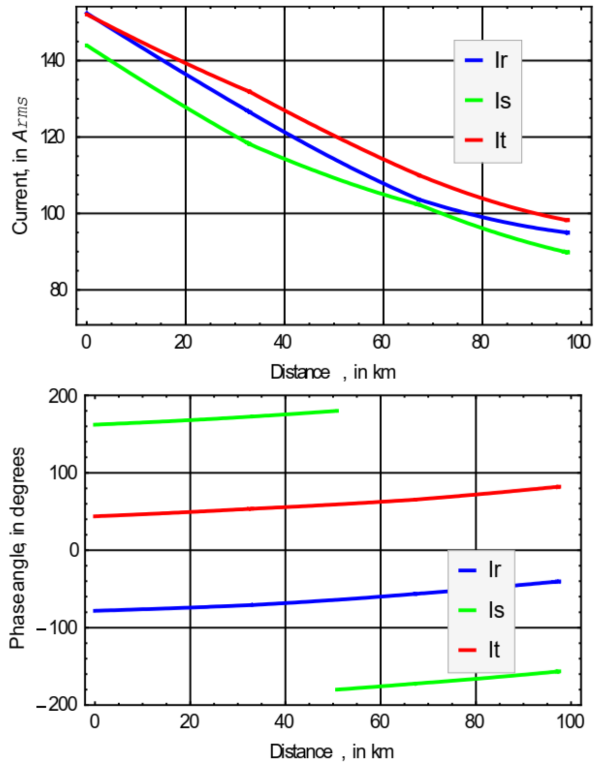

FIGURE 5. Profile of phase current at the Instant 2: 5(a) Amplitude; 5(b) Phase angle.

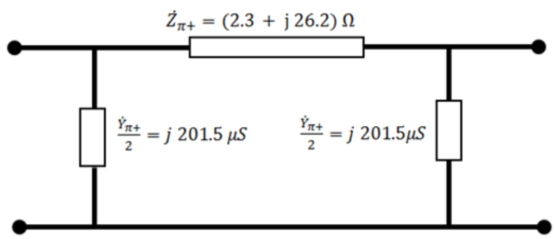

FIGURE 6. Circuit of unique and equivalent π-model for the positive sequence for the 400 kV SvK TL Alvesta-Tenhult.

Thus, since the essence of this work is to propose the use of the presented analytical model as a more complete tool of analysis instead of the usual nominal π-model, an additional comparison to this classical method is performed. In order to do this, all the same procedures to obtain the equivalent π-model to represent the studied line were performed as in [3] and in [31]. Thus, the matrixes of impedances and admittances, given by (17) and (18), respectively, were taken for obtaining the respective matrixes with average values of parameters, weighted by the length of each transposition, which made both matrices become fully symmetrical.

After that, both symmetrical matrixes were converted into the matrixes of sequences, positive, negative, and zero. Figure 6 illustrates the nominal π-model, in positive sequence, that represents the studied 400 kV transmission line at steady state operation.

TABLE 4. Comparison of results of experimental measurement (PMU), the proposed model (analytical), and a unique π circuit for the phase r at the Instant 2.

	Alvesta PMM Data (Measured/Reference)	Tenhult PMU Data (Measured)	Tenhult Proposed Method	Tenhult Nominal π-Model
$\dot{V}_r$ (V)	238,925.2 $\angle$ 154.42	241,038.2 $\angle$ 154.99	241,046.0 $\angle$ 154.93	242,300.0 $\angle$154.9
$\dot{I}_r$ (A)	152.4 $\angle$ 281.8	95.1 $\angle$ -39.0	94.9 $\angle$ -40.3	95.6 $\angle$ -41.3

For the comparison of the proposed method and based on the PMU data as the reference, phase r is considered to represent the whole transmission line. Thus, Table 4 shows the comparison of values based on data of Instant 2, from which the input voltage and current are the same for the proposed method as well as for the nominal π-model.

V. DISCUSSION

From the comparison to the PMU data, for the three-phase current and voltage to ground, the obtained and shown results seem to agree very well with the actual data, which makes it as being more realistic in giving a better comprehension of the behavior of each of the three phases. Similar results could be also obtained through computer computation, widely available and in which several π-circuits, multiplied, connected, and coupled, as in [31], could be taken for modeling the transmission line also performed with several matrix calculations. Nonetheless, the essence of this work is to propose the replacement of that classical unique and elementary π-model as the basis for analytical studies of power transmission lines at steady state. Therefore, the results of Table 4 are more conclusive, since they show a frame of comparison by taking the phase r as being the representative one for the whole transmission line. Even though the results of a nominal π-model could be considered satisfactory, especially due to its simplicity, it could also be argued that using such a model would not result in enough accuracy.

If only a single phase is used to represent the whole transmission, line the phase r in this case, it would not be really representative of the whole line under most of the conditions. The same would have been the case if any of the other two phases s or t would have been used, or even the average value of all the phases. On the other hand, by taking the actual value of each of the phases and considering their effect simultaneously, as the proposed method does, it results in significantly more realistic results. This fact, by itself, is strongly motivating for the adoption of the proposed method. Furthermore, it is undeniable how elegant the analytical approach of studying a real three-phase power transmission line is, by using the proposed method, in comparison to the rather limited results given by using a unique π-circuit.

Regarding the proposed method itself, it is important to mention that the key step is the inversion of the Laplace transforms, represented by (13) and (14). This inversion has become available in a relatively recent time, thanks to the development of advanced mathematical software. Some years ago, this inversion would be restricted to scientific computers and thus the use of a unique π-circuit was the only way for the analytical approach. Fortunately, today, mathematical software able in doing this inversion is easily available for personal computers.

In addition, for a more detailed description of the steps of calculation by the proposed method, in [10] is presented the application of this same method for a real transmission line. In that work, even though the PMU data are not available, the application of this proposed method is performed with the basis of the often-desired condition of balanced three-phase voltage and current and the presented results show that this idealized condition is immediately lost when dealing with a real transmission line, which further show the advantages of the proposition in taking each of the phases in an analytical study.

In [9], an additional advantage is illustrated for analyzing the behavior of the voltage and current of each phase along the line length, which is to predict the occurrence of overvoltage or overcurrent in points of the line located between both of its ends. This characteristic also suggests that the proposed method can be applied in the online monitoring of power transmission lines. In fact, there is a vast myriad of applications for the proposed method.

VI. CONCLUSION

The results presented in this work show that the usual analytical modeling of a power transmission line through the classical and unique π-circuit has now a powerful concurrent method. The proposed method is based on the application of the Laplace transforms to the ruling phasor differential equations of the transmission line and allows more accurate results as well as inherently allows a more complete view of the behavior of the phase voltage and current along the line length. The accuracy of the proposed method is highly dependent on the availability of time-referenced phasor measurement of phase current and voltage, which may be available from PMU systems although it is still rarely found in transmission systems of today.

In the case of the unavailability of these kinds of data, the method loses some importance, however, it remains useful in some applications. For example, the method allows evaluating conditions of occurrence of overvoltage along the line length as well as showing how a real transmission line naturally adds unbalance to the power system. In any case, the proposed method may be used in order to achieve a more realistic simulation of a transmission line in conditions like under the increase in the amount of transmittable power, simulation of faults, and analyses of power harmonic and subharmonic propagation, besides various others. On the other hand, thanks to the possibility of determining the behavior of voltage and current of each phase of the line, the proposed method also allows a more accurate and realistic evaluation of the level of magnetic and electric fields along each point of the same line.

At last, besides the crucial proposition of replacing the traditional and classical unique π-model in the analytical studies of power transmission lines at steady state, this proposed method is equally concurrent to the computer method for further and more complex analyses of transmission lines.

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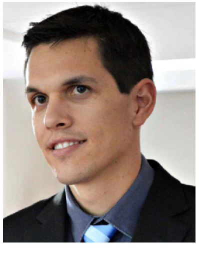

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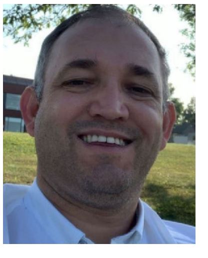

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