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RESEARCH ARTICLE

Simulated Annealing Aided Artificial Hummingbird Optimizer for Infinite Impulse Response System Identification

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ABSTRACT Infinite impulse response (IIR) systems, with their ability to model intricate system behaviors, have proven to be a powerful class of digital filters. However, accurately identifying the optimal filter parameters for system emulation remains a challenge. Metaheuristic algorithms have emerged as effective tools for parameter optimization in IIR filter design, allowing for the exploration of parameter spaces and the discovery of suitable filter sets. This paper introduces a novel adaptive algorithm, named simulated annealing aided artificial hummingbird optimizer (AHA-SA), which combines the strengths of the artificial hummingbird algorithm (AHA) and simulated annealing (SA). The synergistic integration of AHA and SA in the AHA-SA optimizer enables efficient search space exploration, rapid convergence, and the attainment of precise solutions. Extensive experiments demonstrate the superiority of the AHA-SA optimizer over competitive algorithms, both in terms of solution quality and convergence speed. The proposed optimizer presents a promising solution for optimization problems in various domains, with its simplicity, intuitive workflow, and potential for widespread adoption.

INDEX TERMS Artificial hummingbird algorithm, simulated annealing algorithm, digital filter design, metaheuristics.

I. INTRODUCTION

In the realm of digital filters, infinite impulse response (IIR) systems stand out as a powerful class that incorporates feedback loops to account for past inputs and outputs. This unique characteristic enables IIR filters to effectively model intricate system behaviors like resonances and decays, surpassing the limitations of finite impulse response (FIR) filters [1]. To accurately emulate a given system, system identification via IIR filters involves determining the optimal filter parameters. This process typically entails applying an input signal, measuring the output, and utilizing optimization algorithms to minimize the disparity between the predicted and actual outputs [2].

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Optimizing the parameter selection for IIR filter design often calls for the application of metaheuristic algorithms, which excel at solving complex problems through stochastic search strategies [3]. Within the context of system identification, metaheuristic algorithms allow for the exploration of potential filter parameter spaces to discover the most suitable set that captures the system's characteristics [4], [5], [6]. One particularly popular algorithm for IIR filter design is the particle swarm optimization (PSO) algorithm [7]. PSO simulates a swarm of particles navigating the parameter space, with each particle representing a potential solution. By adjusting their position and velocity based on personal and swarm-wide best solutions, the particles converge on an optimal filter design. In another work, an enhanced version of golden jackal optimization (EJGO) was proposed [8]. The EJGO utilizes the elite opposition-based learning technique and the simplex technique to improve its search and optimization abilities for adaptive IIR system identification. The experimental results demonstrated that EGJO outperforms other competitive optimization methods in terms of optimization efficiency, recognition accuracy, convergence rate, computation precision, control parameters, and fitness value and was proven to be stable and resilient in solving the IIR system identification problem. In [9], the water cycle algorithm (WCA) was utilized to optimize the design of low pass and high pass filters with IIR. The WCA, based on evaporation and raining processes, balances diversification and intensification abilities and the stability of the IIR filter was ensured using a lattice equivalent approach. Experimental results showed that the WCA achieves a high-quality design with low objective function values, minimal passband and stopband errors, and superior stopband attenuation compared to other techniques, establishing its effectiveness in finding optimal solutions. To overcome the difficulty of minimizing complex multimodal error surfaces regarding IIR models, a novel hybrid algorithm called chimp-cuckoo search (ChCS) [10] was also proposed. The effectiveness of ChCS was demonstrated through three types of IIR models, showcasing its superiority over other competing methods. Other metaheuristic algorithms applicable to IIR filter design include adaptive simulated annealing [11], teacher learner-based optimization [12], whale optimization [13], tabu search [14] and seeker optimization [15].

In recent research, optimal modeling of IIR filtering systems has emerged as a prominent topic. Consequently, various metaheuristic structures have been proposed for system identification problems involving IIR filters. While paying special attention to the filter order is crucial for effective IIR filter design, it is equally important to utilize adaptive algorithms that reduce design complexity. This work introduces a novel and efficient adaptive algorithm for designing digital IIR filters used in system identification. Combining the artificial hummingbird algorithm (AHA) [16] and simulated annealing (SA) algorithm [17], the proposed approach aims to present a compelling metaheuristic optimization procedure.

The AHA has demonstrated remarkable performance in solving diverse optimization problems. However, like any optimization algorithm, AHA has limitations such as susceptibility to local optima and slow convergence rates. To address these challenges, this paper proposes a hybrid optimization algorithm called SA aided AHA (AHA-SA) optimizer, which combines the strengths of AHA and SA. SA is a local search metaheuristic that escapes local minima by employing hill-climbing techniques to discover global solutions [18], [19]. AHA, on the other hand, mimics the flight patterns of hummingbirds to efficiently explore the search space. The synergistic integration of AHA and SA in the AHA-SA optimizer enables effective search space exploration, rapid convergence, and the attainment of more precise solutions.

To demonstrate the efficacy of the proposed AHA-SA optimizer, extensive experiments are conducted on various unimodal and multimodal benchmark functions, as well as a fifth order IIR system identification problem. The performance of the AHA-SA optimizer is compared against several recent and competitive optimization algorithms known as reptile search algorithm (RSA) [20], Lévy flight distribution (LFD) algorithm [21] and grey wolf optimizer (GWO) [22]. The experimental results reveal that the proposed AHA-SA optimizer outperforms all other algorithms in terms of solution quality and convergence speed. Further performance comparisons are also performed using other reported best approaches of dynamic opposite learning enhanced artificial ecosystem optimization (DAEO) [23], PSO [23], inclined planes system optimization (IPO) [24], gravitational search algorithm (GSA) [24], bat algorithm (BA) [25], firefly algorithm (FFA) [26], differential evolution (DE) [26], harmony search (HS) algorithm [27], cat swarm optimization (CSO) [28] and genetic algorithm (GA) [28]. The comparisons with the best reported literature also confirm the superiority of the proposed AHA-SA optimizer for IIR system identification problem. The contributions of this work can briefly be listed as follows:

- This work introduces a novel and efficient adaptive algorithm, called AHA-SA optimizer, for designing digital IIR filters used in system identification.
- The proposed approach combines the strengths of the AHA and SA algorithms, resulting in effective search space exploration, rapid convergence, and precise solutions.
- Extensive experiments are conducted to demonstrate the efficacy of the AHA-SA optimizer, including tests on various benchmark functions and a fifth order IIR system identification problem. The results indicate that the AHA-SA optimizer outperforms several recent and competitive optimization algorithms in terms of solution quality and convergence speed.
- Comparative analyses are performed against a wide range of state-of-the-art approaches, such as RSA, LFD, GWO, DAEO, PSO, IPO, GSA, BA, FFA, DE, HS, CSO, and GA. These comparisons validate the superiority of the proposed AHA-SA optimizer for IIR system identification problems.

II. OVERVIEW OF AHA OPTIMIZER

The AHA is inspired from flight skills, foraging strategies, and memory capacity of hummingbirds [16]. Mathematically, three models are used to simulate the foraging behaviors (guided foraging, territorial foraging, migrating foraging) of the hummingbirds. This algorithm can briefly be explained as follows. A population of n hummingbirds are randomly initialized and located on n food sources using $x_i = L + r \cdot (U - L)$ where x_i is the position of the i^{th} food source, L and U are respectively the lower and upper limits of a d-dimensional problem, $i = 1, \ldots, n$, and r is a random vector within [0, 1]. The following definition is used to create a visit table:

$$VT_{i,j} = \begin{cases} 0, & i \neq j \\ null, & i = j \end{cases} \quad (1)$$

where $i = 1, \ldots, n$ and $j = 1, \ldots, n$. In the visit table defined by Eq. (1), *null* indicates a specific food source where a hummingbird takes the food whereas 0 means that the i^{th} hummingbird has just visited the j^{th} food source in the current iteration. In guided foraging, the behavior of the hummingbird visiting the food source with the highest nectar refilling rate is modeled. A hummingbird flies towards the determined food source by performing omnidirectional, diagonal, and axial flight skills [16]. The mathematical model of the guided foraging is defined as:

$$v_i(t+1) = x_{i,trg}(t) + \alpha \cdot D \cdot \left(x_i(t) - x_{i,trg}(t)\right) \quad (2)$$

where $v_i(t+1)$ represents the candidate food source position, α is the guided factor subjecting to the normal distribution with mean 0 and standard deviation 1, $x(t)$ is the i^{th} food source position at time t, $x_{i,trg}(t)$ is the targeted food source that the i^{th} hummingbird aims to visit. The position of the i^{th} food source is updated as follows where f is the fitness function value.

$$x_i(t+1) = \begin{cases} x_i(t), & f(x_i(t)) \leq f(v_i(t+1)) \\ v_i(t+1), & f(x_i(t)) > f(v_i(t+1)) \end{cases} \quad (3)$$

A hummingbird is likely to look for a new food source after visiting the target food source. This is described as territorial foraging which is explained as:

$$v_i(t+1) = x_i(t) + b \cdot D \cdot x_i(t) \quad (4)$$

where b is a territorial factor which is subjected to the normal distribution with mean 0 and standard deviation 1. After performing the territorial foraging behavior, the visit table is updated. A hummingbird may also migrate to a food source which is far away if a commonly visited region suffers from enough food supply. This behavior is described with the migration foraging behavior which is described as:

$$x_{wrst}(t+1) = L + r \cdot (U - L) \quad (5)$$

where x_{wrst} stands for the food source with the worst nectar refilling rate. The migration strategy helps the AHA to avoid local stagnation. More detailed explanation of the AHA can be found from [16].

III. SA AIDED AHA OPTIMIZER

The proposed AHA-SA optimizer is a hybridization of two highly successful optimization techniques: AHA and SA algorithms. SA algorithm is a local search metaheuristic algorithm renowned for its exceptional ability to tackle both continuous and discrete optimization problems with unparalleled efficiency [17]. The beauty of SA algorithm lies in its ability to break free from local minima through hill-climbing maneuvers, thereby enabling it to explore and ultimately discover global solutions [29], [30], [31]. To enhance the efficacy of the AHA optimizer in avoiding local minima and increasing diversity in the search space, this work proposes a hybrid approach that incorporates the SA algorithm. The result is a novel and highly effective optimizer that seamlessly integrates the lightning-fast and optimal search capability of the AHA algorithm with the hill-climbing prowess of the SA algorithm.

A visual representation of the proposed AHA-SA optimizer is presented in Fig. 1. As can be seen from the flowchart, the optimizer commences by defining the parameters of both AHA and SA algorithms, followed by initializing randomly generated initial candidate solutions. The iterations begin with the calculation of the fitness function, which is then compared with the best fitness value. If the current fitness value is better than the best one, the optimizer updates the best solution and fitness value, and proceeds with the remaining steps in the flowchart. However, if the current solution's fitness value is not better than the best solution, the proposed hybrid optimizer generates a new solution in the neighborhood of the current solution and evaluates it based on probability justification using $p = \exp(-\Delta F / T_k)$ where $\Delta F = F\left(x_i'\right) - F(x_i)$. In here, F and T denote control parameters of fitness function and temperature, respectively. The algorithm will not replace x_i by x_i' if $p < rand(0, 1)$, however, a replacement will happen on the contrary case. The SA algorithm later reduces the value of the temperature using the $T_{k+1} = \mu T_k$ where, μ denotes the cooling coefficient, which is a random constant between 0 and 1. In such cases, the SA algorithm behaves as an embedded part of the AHA algorithm and operates to justify the neighborhood solution. Therefore, the proposed AHA-SA optimizer has a similar computational complexity with respect to the original form of AHA. Overall, the AHA-SA optimizer represents a significant step forward in the field of optimization, offering a powerful and efficient solution for a wide range of optimization problems.

IV. EXPERIMENTAL RESULTS ON BENCHMARK FUNCTIONS

A. USED UNIMODAL AND MULTIMODAL BENCHMARK FUNCTIONS

The performance assessment of the constructed AHA-SA optimizer is initially performed using some of the benchmark functions. Benchmark functions are mathematical functions that are commonly used to evaluate the performance of optimization algorithms. They provide a standardized way to compare the effectiveness of different optimization techniques in finding the global minimum of a function. There are many benchmark functions available, but some of the most commonly used ones include the unimodal functions of Sphere and Rosenbrock, as well as the multimodal functions of Schwefel and Penalized.

The unimodal benchmark function of Sphere is a simple function that has a single global minimum. It is defined as

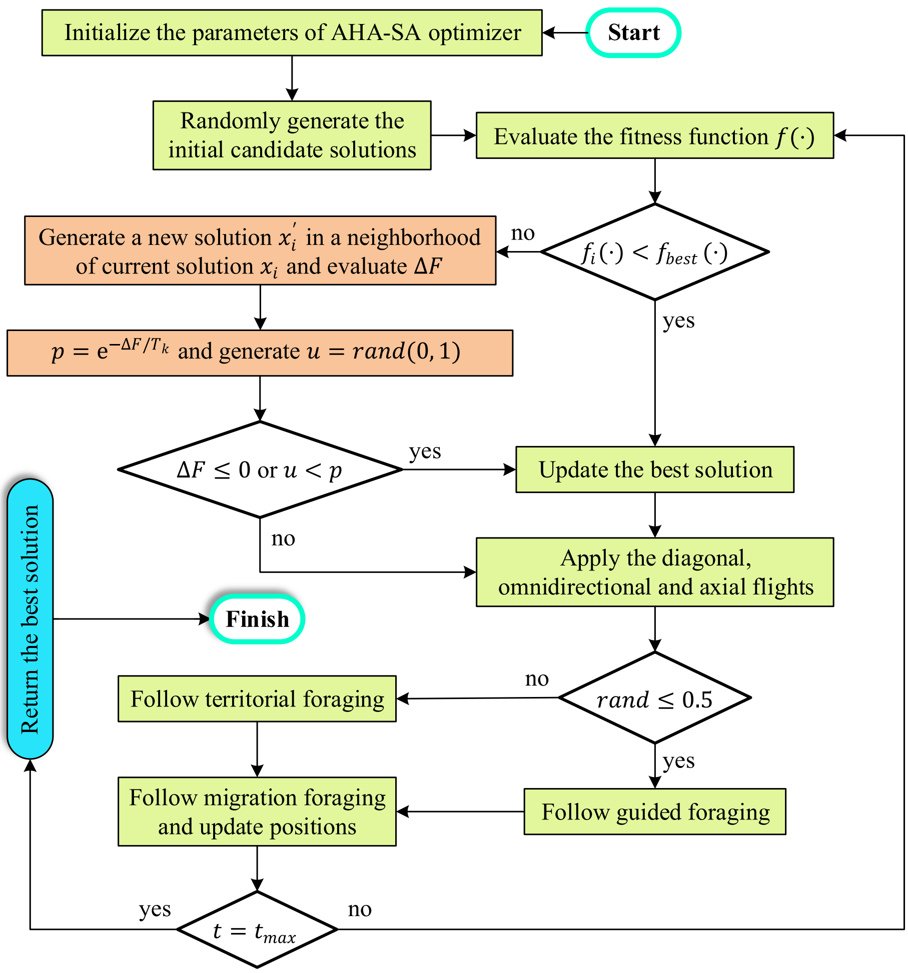

FIGURE 1. Flowchart illustrating the algorithmic structure of the recommended AHA-SA optimizer.

follows:

$$F_1(x) = \sum_{i=1}^{d} x_i^2 \tag{6}$$

where x_i is the *ith* variable in the function. The global minimum of this function is located at $F_1(x) = 0$, where all the x_i values are 0.

The unimodal benchmark function of Rosenbrock is a slightly more complex function that also has a single global minimum. It is defined as follows:

$$F_2(x) = \sum_{i=1}^{d-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2] \tag{7}$$

where x_i is the *ith* variable in the function. The global minimum of this function is located at $F_2(x) = 0$, where all the x_i values are 1.

The multimodal benchmark function of Schwefel is a function that has multiple local minima and a single global minimum. It is defined as follows:

$$F_3(x) = -\sum_{i=1}^{d} \left(x_i \sin\left(\sqrt{|x_i|}\right)\right) \tag{8}$$

where x_i is the *ith* variable in the function. The global minimum of this function is located at $F_3(x) = -418.9829d$, where all the x_i values are 420.9687.

The multimodal benchmark function of Penalized is a function that also has multiple local minima and a single global minimum. It is defined as in (9), shown at the bottom of the next page, where x_i is the *ith* variable in the function.

The global minimum of this function is located at $F_4(x) = 0$, where all the x_i values are -1. Overall, these benchmark functions are useful for testing and comparing optimization algorithms, as they provide a standardized set of challenges that can be used to evaluate the performance of different techniques.

B. STATISTICAL RESULTS AND DISCUSSION

In this study, recent and good performing optimizers known as RSA [20], LFD [21] and GWO [22] are used along with the original AHA optimizer [16] for the initial performance evaluation of the proposed AHA-SA optimizer all algorithms were run for 30 times with a maximum iteration $t_{max} = 1000$ and population size $N = 50$ for the optimization of all test functions. The statistical results for the AHA-SA algorithm, along with other algorithms such as AHA, RSA, LFD, and GWO, are provided in Table 1. The statistical results include the best, worst, mean, and standard deviation for each of the algorithms on each of the benchmark functions.

For the Sphere function, the AHA-SA optimizer and the RSA perform the best with the minimum value of 0 for all statistical measures. The LFD algorithm has a very large worst value, indicating that the algorithm is unable to converge to a good solution. For the Rosenbrock function, the AHA-SA optimizer performs the best with a minimum value of 1.2349E−29 which is followed by RSA with a minimum value of 6.8335E−29. For the Schwefel function, AHA-SA optimizer performs the best and it is followed by the AHA however, the latter one has a wider standard deviation. The standard deviation for AHA-SA optimizer is 0, indicating that the results are consistent. For the Penalized function, the AHA-SA optimizer performs the best with a minimum value of 3.1838E−14. The AHA algorithm also performs well, however, has a wider standard deviation. Overall, the AHA-SA algorithm performs the best, suggesting that it is a competitive algorithm and performs better than other state-of-the-art optimization algorithms.

C. NONPARAMETRIC TEST ANALYSIS

In order to evaluate the performance of the proposed AHA-SA optimizer against three other well-performing algorithms, RSA, LFD, and GWO, a Wilcoxon signed rank test is also conducted on adopted benchmark functions. The results are summarized in Table 2, which shows the p-values and the winner for each comparison. The p-value represents the probability that the difference between the performances of the two algorithms is due to chance [32]. A lower p-value indicates a more significant difference in performance.

The results show that AHA-SA outperformed AHA, LFD, and GWO in terms of the Sphere function with a p-value of 3.7896E-06, 1.7344E-06, and 1.7344E-06, respectively. It has a similar performance with RSA for this function. For the Rosenbrock function, AHA-SA showed a significant improvement over AHA, LFD and GWO, with a p-value of 1.7344E-06. Similar to the Sphere function, it has a similar performance with RSA on Rosenbrock function, as well. AHA-SA also outperformed all the other algorithms for the Schwefel and Penalized functions, with p-values ranging from 1.2754E-04 to 1.7344E-06.

Overall, the results suggest that AHA-SA is a promising algorithm for solving optimization problems and can outperform well-performing algorithms such as AHA, RSA, LFD, and GWO in various functions.

V. APPLICATION OF AHA-SA OPTIMIZER TO HIGH-ORDER IIR SYSTEM IDENTIFICATION

A. DESIGN FORMULATION

System identification involves finding the mathematical representation of an unknown system by analyzing its input and output data. An optimization algorithm is used to minimize the error between the output of a candidate model and the actual output of the system, in order to obtain an optimal model for the unknown system. IIR models can use fewer model parameters to accurately represent physical systems for real-world applications [33]. An IIR system's output and input relationship can be given as follows where b_j is the numerator and a_i is the denominator of the IIR filter whereas M and N (N$\geq M$) are the respective orders of the numerator and denominator of the IIR model.

$$y(k) + \sum_{i=1}^{N} a_i \cdot y(k-i) = \sum_{j=0}^{M} b_j \cdot x(k-j) \tag{10}$$

In an IIR system modelling problem, the actual system is assumed to be known by considering standard IIR plants and the IIR filter tends to have a transfer function that can replicate the unknown system. With the assumption of the coefficient of $a_0 = 1$, the following transfer function can be obtained for an IIR digital filter.

$$H(z) = \frac{\sum_{j=0}^{M} b_j z^{-j}}{1 + \sum_{i=1}^{N} a_i z^{-i}} \tag{11}$$

As can be observed from the latter equation, the z transform is used to represent the digital IIR filters since they are discrete time systems [24]. In this work, the optimal IIR filter coefficients are attempted to be determined as vector $[a_i, b_j]$ to approach to the actual system response with the adaptive IIR filter. Fig. 2 demonstrates the block diagram of an adaptive infinite impulse response filter designed via the proposed AHA-SA optimizer for the system identification purpose where x(k) and $y(k)$ represent the input and the output of the filter, respectively. In here, $e(k)$ represents the error between the model and the actual plant as $e(k) = d(k) - y(k)$ which can be used for considering the infinite impulse response model identification problem as minimization problem described by the following mean squared error (MSE) function which is used as a cost function

$$F_4(x) = \frac{\pi}{d}\left\{10\sin^2(\pi y_1) + \sum_{i=1}^{d-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right] + (y_d - 1)^2\right\} + \sum_{i=1}^{d} u(x_i, 10, 100, 4) \tag{9}$$

TABLE 1. Statistical results for the used benchmark functions.

Function	Metric	AHA-SA	AHA	RSA	LFD	GWO
Sphere	Best	0	4.9407E−324	0	2.2489E−08	4.1775E−73
	Worst	0	9.4700E−287	0	8.9651E−08	3.9259E−69
	Mean	0	3.1632E−288	0	4.8339E−08	2.4824E−70
	Standard Deviation	0	0	0	1.3705E−08	7.1039E−70
Rosenbrock	Best	1.2349E−29	2.4657E+01	6.8335E−29	2.6840E+01	2.5216E+01
	Worst	9.7313E−11	2.5516E+01	2.8992E+01	2.7280E+01	2.8561E+01
	Mean	1.9961E−11	2.5099E+01	1.0604E+01	2.7075E+01	2.6527E+01
	Standard Deviation	2.7112E−11	2.0812E−01	1.4175E+01	1.4992E−01	7.3199E−01
Schwefel	Best	−1.2569E+04	−1.2569E+04	−5.7141E+03	−5.1992E+03	−8.1742E+03
	Worst	−1.2569E+04	−1.1739E+04	−5.0033E+03	−4.1108E+03	−3.9883E+03
	Mean	−1.2569E+04	−1.2413E+04	−5.5238E+03	−4.6020E+03	−6.1474E+03
	Standard Deviation	0	2.1632E+02	1.5760E+02	2.7489E+02	8.1665E+02
Penalized	Best	3.1838E−14	4.6552E−08	6.6541E−01	2.2321E−01	8.8908E−07
	Worst	1.4318E−11	4.2856E−06	1.6142E+00	5.2662E−01	7.2782E−02
	Mean	4.8457E−12	4.1187E−07	1.3315E+00	3.2943E−01	2.4153E−02
	Standard Deviation	3.9910E−12	7.8075E−07	3.0112E−01	7.2391E−02	1.7244E−02

TABLE 2. Wilcoxon signed rank test.

Function	AHA-SA vs AHA		AHA-SA vs RSA		AHA-SA vs LFD		AHA-SA vs GWO	
	p-value	Winner	p-value	Winner	p-value	Winner	p-value	Winner
Sphere	3.7896E−06	+	1	=	1.7344E−06	+	1.7344E−06	+
Rosenbrock	1.7344E−06	+	1.9152E−01	=	1.7344E−06	+	1.7344E−06	+
Schwefel	1.2754E−04	+	1.7333E−06	+	1.7344E−06	+	1.7344E−06	+
Penalized	1.7344E−06	+	1.7344E−06	+	1.7344E−06	+	1.7344E−06	+

where S represents the number of input samples.

$$MSE = \frac{1}{S}\sum_{\mathrm{k}=1}^{S}(d(k)-y(k))^2 \tag{12}$$

B. SIMULATION RESULTS AND DISCUSSION

In the simulations performed in this study, the system input $x(k)$ is taken as a uniformly distributed white sequence that takes values from $(-0.5, 0.5)$ range. The data length used to calculate the MSE cost function is $S = 200$. In the experiments, the parameters of the algorithm were set as 30 runs, 500 total iteration number (t_{max}) and 100 population size (N). The implementations were carried out on MATLAB/Simulink software package that is installed on a windows computer with 12th Gen Intel i5-12400, 2.50 GHz processor and 16.00 GB RAM. AHA, RSA, LFD and GWO optimizers are used for the comparative performance assessment of the proposed AHA-SA optimizer. In the experiments, a fifth-order plant with the same and reduced order IIR filters is examined as it is highly a highly complex benchmark example [23], [24], [25], [26], [27], [28]. The transfer function of the fifth-order plant is given by the expression in (13), shown at the bottom of the next page.

Case 1: Modeling with same order

The transfer function in (14), shown at the bottom of the next page, represents the same order IIR filter to approximate the fifth-order plant given in (13). Table 3 presents statistical metrics for five metaheuristic algorithms (AHA-SA, AHA, RSA, LFD, GWO) used to optimize a system modeled with the same order IIR filter. Based on the statistical metrics presented in the table, it is observed that the AHA-SA algorithm performs the best among the five metaheuristic algorithms evaluated. The AHA-SA algorithm has the lowest best MSE value, indicating that it found the best solution on average across all test cases. Additionally, it has the lowest mean MSE value, indicating that it produced solutions that are generally closer to the optimal solutions than the other algorithms. Furthermore, the AHA-SA algorithm has the lowest standard deviation for MSE values, which means that it is more consistent in producing high-quality solutions across different test cases. Overall, these results suggest that the AHA-SA optimizer is a promising choice for optimizing IIR system identification models.

Table 4, on the other hand, lists the obtained coefficients for the same order case. The table shows the actual values of the parameters and the estimated values obtained by five different optimization algorithms (AHA-SA, AHA, RSA, LFD, GWO) for Case 1 of parameter estimation. The parameters are divided into two categories: a_i and b_j. For each parameter, the actual value is given, followed by the estimated values obtained by the optimization algorithms. The superiority of the algorithms can be assessed by comparing their estimated values with the actual values. From the table, it can be seen that AHA-SA outperforms the other algorithms in estimating the values of most of the parameters.

Further capability of the AHA-SA for this type of the IIR system identification is demonstrated through convergence curve provided in Fig. 3. The convergence curve provides a visual representation of the performance of each algorithm in

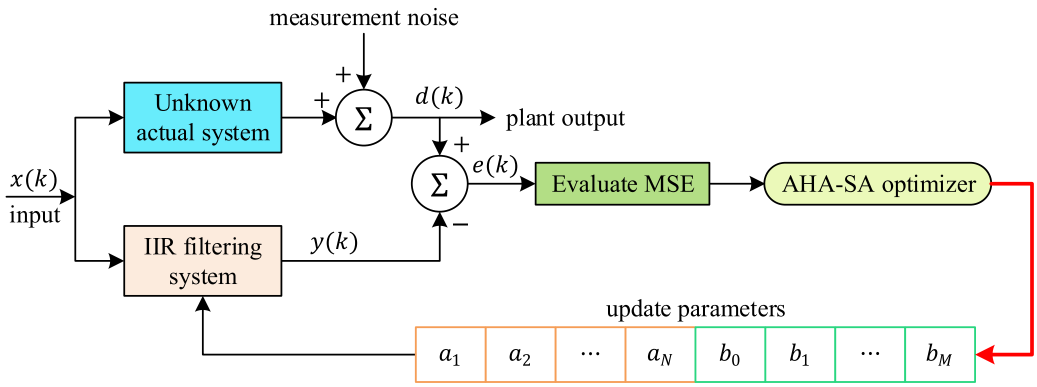

FIGURE 2. Detailed block diagram of recommended AHA-SA optimizer-based adaptive IIR system identification.

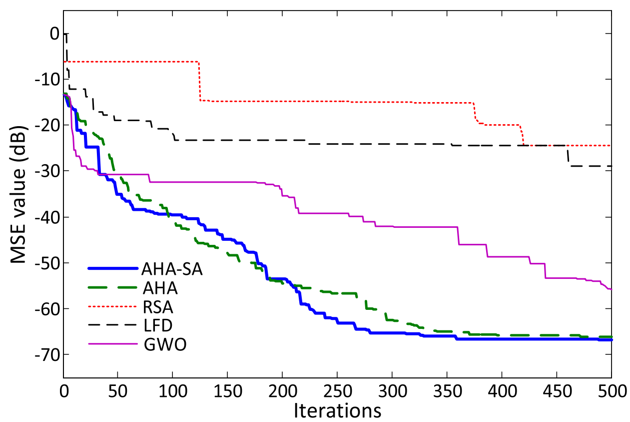

FIGURE 3. Comparative convergence curves for Case 1.

terms of optimization. As seen from this figure, the AHA-SA optimizer keeps converging to the lowest MSE values through iterations. Based on the convergence curve, it seems that AHA-SA optimizer has performed better than the other algorithms in terms of reaching the lowest MSE value. The AHA algorithm is also a good performer, but it is not able to converge to the lowest MSE value as AHA-SA. Since AHA-SA has a similar computational complexity with the original AHA, it means that AHA-SA is computationally efficient and does not require significant additional computational resources compared to AHA. This is an advantage of AHA-SA as it is able to achieve better performance in terms of MSE without sacrificing computational efficiency.

Case 2: Modeling with reduced order

The following transfer function represents the reduced order IIR filter to approximate the fifth-order plant given in (13).

$$H_r(z) = \frac{b'_0 + b'_1 z^{-1} + b'_2 z^{-2} + b'_3 z^{-3} + b'_4 z^{-4}}{1 - a'_1 z^{-1} - a'_2 z^{-2} - a'_3 z^{-3} - a'_4 z^{-4}} \tag{15}$$

Table 5 presents statistical metrics for five metaheuristic algorithms (AHA-SA, AHA, RSA, LFD, GWO) used to optimize a system modeled with the reduced order IIR filter. Based on the statistical metrics presented in the table, it is observed that the AHA-SA algorithm performs the best among the five metaheuristic algorithms evaluated.

The AHA-SA algorithm has the lowest best MSE value, indicating that it found the best solution on average across all test cases. Additionally, it has the lowest mean MSE value, indicating that it produced solutions that are generally closer to the optimal solutions than the other algorithms. Overall, these results suggest that the AHA-SA optimizer is a promising choice for optimizing IIR system identification models.

Table 6 lists the obtained coefficients for the reduced order case. The better performance of the AHA-SA optimizer in terms of reaching more accurate results can be observed from this table. Further capability of the AHA-SA optimizer for this type of the IIR system identification is demonstrated through convergence curve provided in Fig. 4 where the AHA-SA optimizer is shown to reach the lowest MSE values through iterations, indicating its more efficient capability.

C. COMPARISON WITH REPORTED BEST APPROACHES

In this subsection, to further demonstrate the superior performance of the proposed AHA-SA optimizer, the performance of comparison is presented with other reported best approaches in the literature (using other reported best approaches of DAEO [23], PSO [23], IPO [24], GSA [24],

$$H_p(z) = \frac{0.1084 + 0.5419z^{-1} + 1.0837z^{-2} + 1.0837z^{-3} + 0.5419z^{-4} + 0.1084z^{-5}}{1 + 0.9853z^{-1} + 0.9738z^{-2} + 0.3864z^{-3} + 0.1112z^{-4} + 0.0113z^{-5}} \tag{13}$$

$$H_s(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5}}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - a_4 z^{-4} - a_5 z^{-5}} \tag{14}$$

TABLE 3. Statistical results of MSE values for Case 1.

Metric	AHA-SA	AHA	RSA	LFD	GWO
Best	2.1159E−07	2.4319E−07	3.6268E−03	1.2714E−03	2.5943E−06
Worst	4.8772E−06	3.1433E−05	3.2724E−02	3.2425E−02	1.2621E−03
Mean	1.6901E−06	5.3538E−06	2.0748E−02	5.4763E−03	1.9879E−04
Standard Deviation	1.4581E−06	6.8538E−06	1.1904E−02	6.2312E−03	2.3487E−04

TABLE 4. Parameter estimation for Case 1.

Parameter	Actual values	AHA-SA	AHA	RSA	LFD	GWO
a_1	−0.9853	−0.0949	−0.1063	0.0527	0.0431	0.0328
a_2	−0.9738	−0.4298	−0.4227	0.0264	−0.2058	−0.3487
a_3	−0.3864	0.2304	0.2268	0.0021	0.1865	0.3007
a_4	−0.1112	−0.0065	−0.0009	0.0373	0.1871	0.0131
a_5	−0.0113	0.0255	0.0241	0.0282	−0.1862	0.0183
b_0	0.1084	0.1082	0.1087	0.0605	0.1163	0.1077
b_1	0.5419	0.4455	0.4466	0.3424	0.3901	0.4317
b_2	1.0837	0.6373	0.6418	0.6191	0.5425	0.5722
b_3	1.0837	0.2910	0.2943	−0.0046	0.1040	0.1803
b_4	0.5419	−0.1051	−0.1070	−0.2290	−0.1848	−0.1929
b_5	0.1084	−0.1006	−0.1043	0.1129	−0.1126	−0.1199

TABLE 5. Statistical results of MSE values for Case 2.

Metric	AHA-SA	AHA	RSA	LFD	GWO
Best	4.4932E−07	1.8532E−05	1.7191E−03	1.6005E−03	1.9591E−05
Worst	4.1999E−05	5.1943E−05	3.1450E−02	1.2775E−02	6.9108E−04
Mean	2.8028E−05	3.5653E−05	1.2125E−02	3.6097E−03	1.6596E−04
Standard Deviation	1.0833E−05	9.9555E−06	1.1216E−02	2.4398E−03	1.6074E−04

TABLE 6. Parameter estimation for Case 2.

Parameter	AHA-SA	AHA	RSA	LFD	GWO
a'_1	−0.5695	−0.0942	0.0004	0.0957	−0.4073
a'_2	−0.6891	−0.5781	0.0038	0.0131	−0.6102
a'_3	−0.0967	0.1337	0.0000	0.0342	−0.0253
a'_4	−0.0468	−0.0705	0.0957	0.1369	−0.0257
b'_0	0.1085	0.1078	0.0589	0.1400	0.1070
b'_1	0.4966	0.4444	0.4643	0.3510	0.4816
b'_2	0.8718	0.6548	0.5129	0.5100	0.7947
b'_3	0.6993	0.3678	0.0001	0.0221	0.5780
b'_4	0.2202	0.0223	−0.2408	−0.2694	0.1342

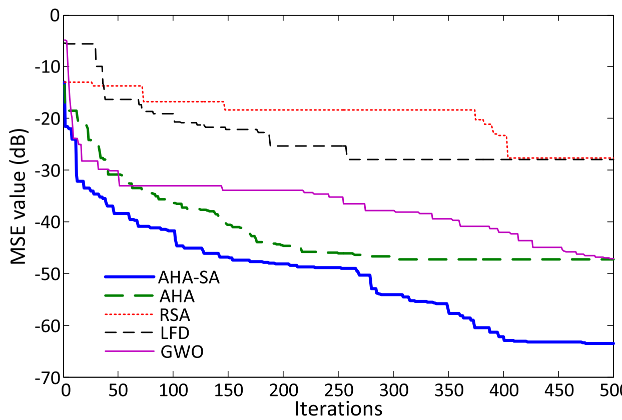

FIGURE 4. Comparative convergence curves for Case 2.

TABLE 7. Performance comparison of different reported MSE values.

Reference	Optimizer	Mean squared error (MSE) value	
		Same order	Reduced order
Present study	AHA-SA	**2.1159E−07**	**4.4932E−07**
Reference [23]	DAEO	8.32E−07	1.69E−05
	PSO	1.35E−01	3.99E−02
Reference [24]	IPO	4.8542E−05	5.8187E−05
	GSA	4.8656E−05	4.7195E−05
Reference [25]	BA	5.8182E−06	4.3986E−05
Reference [26]	FFA	1.8737E−06	5.5835E−06
	DE	6.8820E−04	0.0027
Reference [27]	HS	7.1407E−06	6.1214E−06
Reference [28]	CSO	6.35514E−05	6.9475E−05
	GA	0.013335606	0.084596041

BA [25], FFA [26], DE [26], HS [27], CSO [28] and GA [28]) for the same order and reduced-order cases. Table 7 shows the MSE values obtained from AHA-SA optimizer and the other approaches.

For the same-order case, the proposed AHA-SA approach outperforms all other approaches with an MSE value of 2.1159E-07, which is significantly lower than the MSE values obtained by the other approaches. The second-best approach is DAEO with an MSE value of 8.32E-07. Similarly, for the reduced-order case, the proposed AHA-SA approach obtains the lowest MSE value of 4.4932E-07 compared to all other approaches, indicating the superior performance of our approach. FFA has the second-best performance with an MSE value of 5.5835E-06. Overall, the results demonstrate the effectiveness of our proposed approach in achieving superior performance compared to other reported best approaches for both same-order and reduced-order cases.

VI. CONCLUSION

In conclusion, the AHA-SA optimizer represents a significant advancement in the field of optimization for IIR system identification. By combining the strengths of the AHA and SA, the AHA-SA optimizer outperforms several recent and competitive optimization algorithms. The unique contributions of this paper lie in the successful integration of the AHA and SA techniques, harnessing the exploratory capabilities of the AHA and the local search abilities of SA. This combination leads to improved solution quality and convergence speed, as demonstrated through extensive experimental comparisons against benchmark functions and a fifth order IIR system identification problem.

Theoretical implications of the AHA-SA optimizer stem from its ability to effectively navigate complex search spaces, providing insights into optimization algorithms' design and efficiency. Additionally, the successful fusion of different metaheuristic algorithms paves the way for further exploration and hybridization of optimization techniques in solving diverse optimization problems.

From a managerial perspective, the AHA-SA optimizer offers a simple and intuitive workflow, making it easily implementable and user-friendly. Its efficient search space exploration capabilities and ability to escape local optima contribute to faster convergence rates and more accurate solutions. These features make the AHA-SA optimizer a valuable tool for decision-makers and practitioners in various industries, including engineering and scientific domains.

While the AHA-SA optimizer demonstrates significant advantages, it is important to acknowledge the limitations of the research. One limitation is that the experiments were primarily focused on IIR system identification, and further investigations are needed to explore its performance in other optimization tasks. Additionally, the algorithm's parameter tuning process may require careful consideration to achieve optimal results for different problem domains. Further research and experimentation can address these limitations and expand the understanding of the AHA-SA optimizer's applicability and effectiveness in various scenarios.

In terms of future research directions, several avenues emerge from this study. Firstly, exploring the adaptability of the AHA-SA optimizer to different problem domains, such as function optimization, feature selection, or parameter estimation, would provide insights into its generalizability. Additionally, investigating the potential of combining the AHA-SA optimizer with other metaheuristic algorithms or optimization frameworks could lead to the development of even more robust and efficient optimization techniques. Finally, conducting comparative studies with other state-of-the-art optimization algorithms can help establish the AHA-SA optimizer's position among existing methods and further validate its performance.

In summary, the AHA-SA optimizer showcases remarkable performance in IIR system identification, offering theoretical advancements, managerial benefits, and great potential for future research. Its integration of the AHA and SA techniques provides a foundation for further exploration and innovation in the field of optimization, driving advancements in diverse domains and applications.

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