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RESEARCH ARTICLE

SteerVLC: A Joint Deployment and Beam-Steering Optimization for VLC-Enabled UAV Networks

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ABSTRACT Visible light communication (VLC)-enabled unmanned aerial vehicle (UAV) networks have emerged as a novel approach for simultaneous illumination and communication. Among the prominent challenges in VLC are shadowing and low reflected energy. Light-emitting diode (LED) beam-steering is a promising technique to overcome these challenges. In this paper, we consider the problem of joint UAV deployment, LED beam-steering, cell association, and power allocation to satisfy a set of illumination and data rate requirements for stochastically distributed users. Towards addressing this problem, we develop SteerVLC, a novel scheme for UAV networks equipped with steerable LEDs that jointly optimizes UAV deployment, LED beam-steering, cell association, and power allocation. While accounting for illumination interference, SteerVLC minimizes the number of UAVs needed to satisfy the illumination and data rate requirements of stochastically distributed users and minimizes the total transmission power of the steerable LEDs. Using an ϵ-controlled two-stage mixed-integer linear programming approach, SteerVLC can effectively manage the tradeoff between the required number of UAVs and the required transmission power. SteerVLC is evaluated under various system parameters, user demands, and user distributions. Our results demonstrate the superiority of SteerVLC in managing the resources of a VLC-enabled UAV network and optimizing its performance.

INDEX TERMS Beam-steering, cell association, multi-objective optimization, network deployment, power control, stochastic geometry, unmanned aerial vehicles (UAVs), visible light communication (VLC).

I. INTRODUCTION

The immense growth in demand for high data rates has redirected researchers' interest to study visible light communication (VLC). Due to the limited radio spectrum availability in radio frequency (RF), the RF spectrum suffers from limited channel capacity and transmission rate, while the data rates requested by the users continue to increase exponentially. Optical wireless and VLC systems can provide license-free, highly secure, and low-cost communications compared to RF systems [1]. However, VLC systems inherently face many challenges and limitations such as their dependence on the existence of a line-of-sight (LOS) between the transmitter and receiver and the illumination interference.

VLC is seen as a complementary technology for RF that will have a large impact on various applications in the 5G era and beyond [2], [3], [4], [5], [6]. Also, the industrial Internet of things (IIoT) field bloomed on the introduction of VLC systems playing a complementary role to the traditional wireless communication [7], which standalone may fail to serve the quality of service (QoS) requirements of the devices [8], [9]. A lot of research has been done on using VLC in IIoT [10], [11], [12], [13], [14]. VLC has been evolving in the literature for both indoor and outdoor applications. Nevertheless, the latter has progressed less due to the environmental challenges it faces [15].

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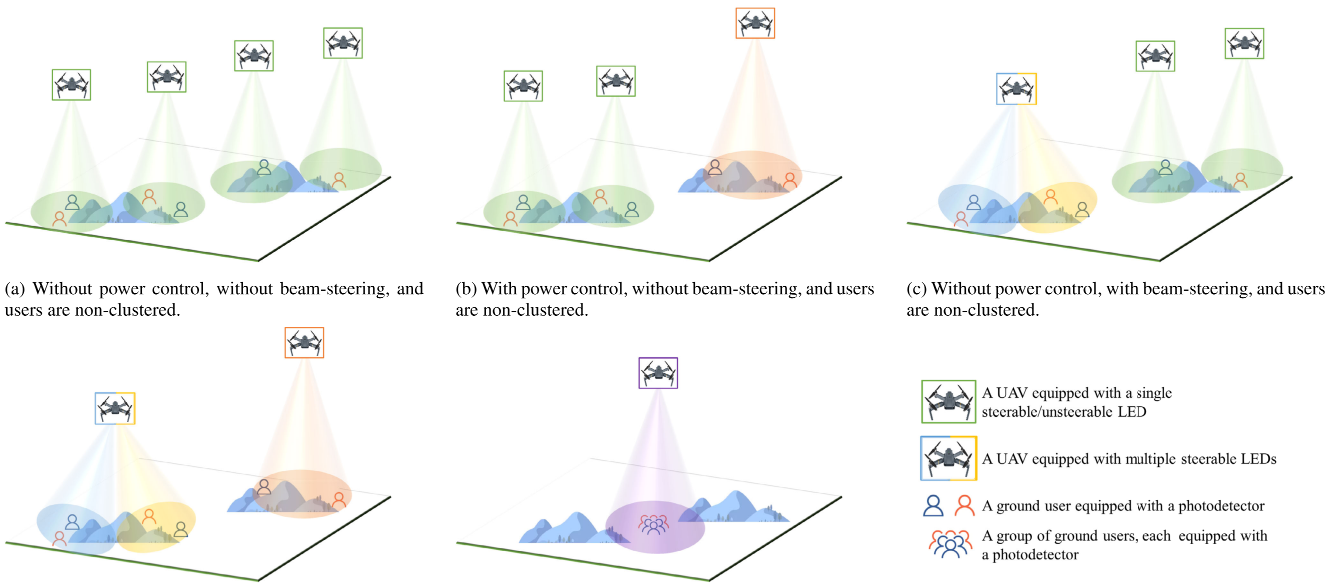

(a) Without power control, without beam-steering, and users are non-clustered.

(b) With power control, without beam-steering, and users are non-clustered.

(c) Without power control, with beam-steering, and users are non-clustered.

(d) With power control, with beam-steering, and users are non-clustered.

(e) Users are clustered.

FIGURE 1. Illustrative example of the effects of power control, beam-steering, and user clustering on the number of UAVs required to be deployed.

VLC uses visible light as a transmission medium. It utilizes light-emitting diodes (LEDs) to transmit signals and photodetectors (PDs) to receive the transmitted light signals. The VLC architecture is confined to the way the LEDs are deployed in a VLC network. LED beams are usually fixated downwards, limiting the coverage area and increasing the risk of obstacles blocking the direct LOS to the user, which degrades the network performance [16].

Combining UAV and VLC creates a reliable and flexible system that can address several outdoor-based VLC application challenges, especially in natural disasters [17]. UAV-assisted VLC (a.k.a. VLC-enabled UAV), a new paradigm that has brought together illumination and communication by mounting LEDs on UAVs [18], [19], [20], [21], [22], [23], has lately become a topic of interest for many researchers.

Few works [18], [19], [20], [23], [24], [25] have considered some of the key challenges in UAV-assisted VLC, specifically, in terms of UAV deployment and resource allocation. However, none of them considered the severe blockage of the LOS, a prominent challenge in UAV-assisted VLC that could happen for a multitude of reasons. To address this problem, several researchers proposed beam-steering techniques to overcome obstacles and improve the overall network coverage and performance for VLC systems [16], [26], [27]. Beam-steering can be used with UAV-assisted VLC systems to add another degree of freedom to further optimize the network deployment and resource allocation. This improves the overall performance of the network using fewer resources and also mitigates some of the effects of LOS blockage.

Fig. 1 illustrates the effects of beam-steering and power control on the number of UAVs required to fulfill the data rate and illumination demands of a set of ground users. In Fig. 1(a), all LED beams use the same power level (hence, all beams have the same color) and are fixated downwards (i.e., beam-steering is disabled). In this case, four UAVs are needed to be deployed. When power control is enabled in Fig. 1(b), the number of required beams/UAVs is reduced to three. In Fig. 1(c), beam-steering is enabled and power control is disabled. In this case, the number of UAVs is reduced to three compared to the case in Fig. 1(a). Both power control and beam-steering are enabled in Fig. 1(d) and the number of UAVs is now reduced to two.

Fig. 1 also demonstrates the effect of user clustering on the number of required UAVs. In all previous cases, Figs. 1(a)-1(d), users were non-clustered. User clustering has a significant impact on network deployment and resource allocation. In Fig. 1(e), users are clustered and in this case, only one UAV is required.

A. MAIN CONTRIBUTIONS

Considering a UAV network equipped with steerable LEDs, in this paper, we develop SteerVLC, a mixed-integer linear joint optimization framework for UAV deployment, LED beam-steering, cell association, and power allocation. SteerVLC fulfills a set of illumination and data rate requirements for a set of stochastically distributed users using the smallest set of UAVs and the least LED transmission power. To the best of our knowledge, SteerVLC is the first resource allocation scheme for VLC-enabled UAV networks that:

- Utilizes LED beam-steering.
- *Jointly* optimizes the UAV deployment, LED beam-steering, user association, and LED power allocation to deliver a set of illumination and data rate requirements to a set of users.

- Considers multiple objectives: Minimizing the number of deployed UAVs and minimizing the LED transmission power. SteerVLC manages the inherent tradeoff between these two objectives using an ϵ-controlled two-stage mixed-integer linear programming (MILP) framework.
- Is aware of the illumination interference caused by surrounding UAVs and other sources of ambient illumination.

SteerVLC is numerically evaluated under various system parameters, user demands, and user distributions. Two widely adopted stochastic geometry models are considered for user distribution, namely the Poisson point process (PPP) and Matérn Poisson cluster process (PCP) [28], [29]. PPP and PCP models are used in evaluating SteerVLC to examine the effects of user clustering on the UAV network performance and the required network resources (i.e., transmission power and UAVs), as illustrated in Section VIII. It is worth mentioning that other stochastic geometry models can also be used to generate the user locations as input data to SteerVLC.

B. PAPER ORGANIZATION

The remainder of this paper is organized as follows. The literature review is provided in Section II. The considered models and assumptions are stated in Section III, followed by our problem statement. An overview of SteerVLC, our framework for solving the problem stated in Section III, is given in Section IV. The details of SteerVLC are explained in Sections V, VI, and VII. SteerVLC is extensively evaluated in Section VIII. Finally, we conclude the paper in Section IX and provide directions for future research. The main notations used in this paper are summarized in Table 1.

TABLE 1. Notation.

Sets:

Notation	Description
$\mathcal{L}$	Set of potential locations for deploying UAVs
$\mathcal{U}$	Set of ground users
$\mathcal{K}$	Set of cells in the floor

Data:

Notation	Description
L	Number of UAV potential locations
U	Number of ground users
K	Number of cells in the floor
B	Maximum number of beams per UAV
h_{lu}	DC channel gain of an LED LOS beam from location $l \in \mathcal{L}$ to user $u \in \mathcal{U}$
$p_{\min}$	Minimum required received power at ground user u (W). $p_{\min}$ is the same for all $u \in \mathcal{U}$
C_{th}	Data rate demand of user u in bits/transition. C_{th} is the same for all $u \in \mathcal{U}$
I_u	Illumination interference received at user $u \in \mathcal{U}$ (W)
η_{th}	Illumination threshold of the receiver
$p_{\max}$	Maximum allowed transmission power by the LEDs (W)
p_{opt}	Maximum allowed received power at ground users, due to eye safety (W)
ϵ	A non-negative controllable parameter

Decision Variables:

Notation	Type	Description
x_{lku}	Binary	Equals one if user $u \in \mathcal{U}$ is to be covered by a beam generated from a UAV located at $l \in \mathcal{L}$ and steered to cover cell $k \in \mathcal{K}$, and it equals zero otherwise. The optimal value of x_{lku} obtained from solving STEERVLC (STAGE I) is denoted by x^*_{lku}
p_{lk}	Continuous	Power to be transmitted by an LED mounted on a UAV located at $l \in \mathcal{L}$ and steered to cover cell $k \in \mathcal{K}$
y_l	Binary	Equals one if a UAV is located at $l \in \mathcal{L}$ and it equals zero otherwise. The optimal value of y_l obtained from solving STEERVLC (STAGE I) is denoted by y^*_l
y_{lk}	Binary	Equals one if a UAV is located at $l \in \mathcal{L}$ and one of its beams is steered to cover cell $k \in \mathcal{K}$, and it equals zero otherwise
z_{lku}	Continuous	Equals $x_{lku}\, p_{lk}$

II. RELATED WORKS

In this section, we present the most related literature to our work, identify some of the existing research gaps, and emphasize the gaps filled in this paper.

The UAV deployment optimization problem was considered in [24] to minimize the total transmission power of the LEDs. Only UAV deployment was considered in [24]. In [20], the user association problem was considered in addition to the UAV deployment problem. Both problems were studied aiming to minimize the total transmission power of UAVs. In [23], the authors optimally addressed the problem of joint UAV deployment and user association aiming to minimize both the total transmission power and the required number of UAVs.

Ambient illumination and UAV interference-based illumination have drastic effects on the network performance in a UAV-assisted VLC system. In [18], the authors considered a UAV-assisted VLC outdoor setup and studied the UAV deployment and user association problems, aiming to minimize the transmission power of the LEDs. They solved the joint UAV deployment and user association problem sub-optimally as the two problems were solved separately. The authors extend their work in [19], where they proposed an algorithm that predicts ambient nighttime illumination. However, the LEDs mounted on UAVs are confined to a specific LOS (i.e., beam-steering was not considered), which reduces network flexibility. This makes it more susceptible to blockage.

Recently, beam-steering has been considered in UAV-assisted VLC systems. In [26] and [27], the authors investigated the effects of mobile orientation and mobility on network performance and introduced beam-steering as a technique to improve the overall network performance. The authors in [16] used beam-steering to locate the position of the obstacle, identify the shape and size of the obstacle, and then redirect the LED ray to avoid the obstacle.

The prior works did not consider the impact of nighttime illumination such as vehicle lights, streetlights, and building lights, which will cause strong interference to VLC links. In [19] the authors proposed a deep learning-based prediction approach to make accurate predictions of the future illumination distribution. They considered the effect of ambient illumination on the minimum data rate demand for each user and the illumination received by each user. However, beam-steering was not considered in [19].

To the best of our knowledge, LED beam-steering optimization for VLC-enabled UAV networks has not been studied before. Furthermore, all existing works on network deployment and resource allocation for VLC-enabled UAV networks can be included in one or more of the following categories: (i) do not provide an *optimal* solution for the *joint* network deployment and resource allocation problem, (ii) do not consider optimizing the number of required UAVs to be deployed, and (iii) do not account for illumination interference. In this paper, we develop the first multi-objective optimization framework for joint UAV deployment, LED beam-steering, user association, and power control in VLC-enabled UAV networks that accounts for illumination interference. Our framework minimizes both the number of UAVs and the transmission power of the LEDs.

III. MODELS, ASSUMPTIONS, AND PROBLEM STATEMENT

A. SYSTEM AND CHANNEL MODELS

We consider an outdoor environment with a set $\mathcal{L} = \{1, 2, \ldots, L\}$ of potential locations for locating UAVs. The deployed UAVs are equipped with steerable LEDs, to provide data communication and illumination to a set $\mathcal{U} = \{1, 2, \ldots, U\}$ of ground users. [1] The ground floor is divided into a set $\mathcal{K} = \{1, 2, \ldots, K\}$ of cells. We assume that all UAVs are at the same fixed height. Each UAV is equipped with multiple steerable LEDs and can emit up to B beams. We assume that LED beams can be steered in any given direction using a mechanical robotic arm [27].

We use $h_{lu}, l \in \mathcal{L}, u \in \mathcal{U}$, to denote the DC channel gain of a VLC LOS beam from an LED at location $l \in \mathcal{L}$ to user $u \in \mathcal{L}$. We use the same VLC channel model explained in [23] and [30].

B. COMMUNICATION AND ILLUMINATION REQUIREMENTS

The received power at user $u \in \mathcal{U}$ needs to be greater than a predetermined value, denoted by $p_{\min}$, in order to meet the data rate demand of user u, denoted by C_{th} bits/transition, and overcome the illumination interference at user u, denoted by I_u. The illumination interference is caused by surrounding UAVs and other sources of ambient illumination. Let σ_w be the standard deviation of the additive white Gaussian noise and ξ be the dimming target. Then, $p_{\min}$ can be computed from C_{th} as follows [18], [19]:

$$p_{\min} = \frac{(\sigma_w + I_u)\sqrt{\frac{2\pi}{e}\left(2^{2C_{\text{th}}} - 1\right)}}{\xi}. \tag{1}$$

Furthermore, the power received at user $u \in \mathcal{U}$ must be greater than η_{th}, the illumination threshold of the receiver. On the other hand, the LED transmission power cannot exceed $p_{\max}$, which characterizes the LED dynamic range constraint [31]. Finally, the power received at user $u \in \mathcal{U}$ must be smaller than p_{opt}, which is the maximum optical limit allowed for eye safety [31].

C. PROBLEM STATEMENT

Consider a set $\mathcal{U}$ of users stochastically distributed in a geographical area (according to a stochastic geometry model), each with a data rate demand of C_{th} and requires a minimum illumination of η_{th}. Using a UAV network equipped with steerable LEDs and given all data defined in Table 1, we need to *jointly* determine (i) the smallest set of UAVs, (ii) their optimal locations, (iii) the optimal beam-steering of the LEDs mounted on each UAV, (iv) the optimal LED beam-user assignment, and (v) the least transmission power needed from each UAV to satisfy the data rate and illumination demands of the users in $\mathcal{U}$.

IV. OVERVIEW OF SteerVLC

In this section, we provide an overview of SteerVLC, our optimization framework that jointly optimizes the placement of UAVs, their beam directions, their transmission powers, and their assignment to the ground users.

A. DECISION VARIABLES OF SteerVLC

In this subsection, we define the decision variables used in SteerVLC. These decision variables are summarized in Table 1.

Let x_{lku}, $l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$, be a binary decision variable; x_{lku} determines whether a UAV is located at location l, beam-steered to cover cell k, and assigned to user u. Each user can be associated with one beam only. The optimal values of $x_{lku}, l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$, denoted by x^*_{lku}, provide the answers for the first four questions raised in subsection III-C. Furthermore, let $p_{lk} \in [0, p_{\max}], l \in \mathcal{L}, k \in \mathcal{K}$, be a continuous decision variable representing the transmission power of a steerable LED located at l and steered to cover cell k. The optimal values of $p_{lk}, l \in \mathcal{L}, k \in \mathcal{K}$, denoted by p^*_{lk}, answers the fifth question raised in subsection III-C.

In SteerVLC, we adopt an ϵ-controlled two-stage optimization framework that jointly answers the questions raised in subsection III-C. The two stages of SteerVLC will be explained in the following sections.

B. FLOWCHART OF SteerVLC

Fig. 2 depicts a flowchart of SteerVLC. After obtaining the values of all single, one-dimensional, and two-dimensional

[1] $\mathcal{L}$ is formed based on the coverage areas of the steerable LEDs mounted on the UAVs and considering a minimum distance between neighboring UAVs.

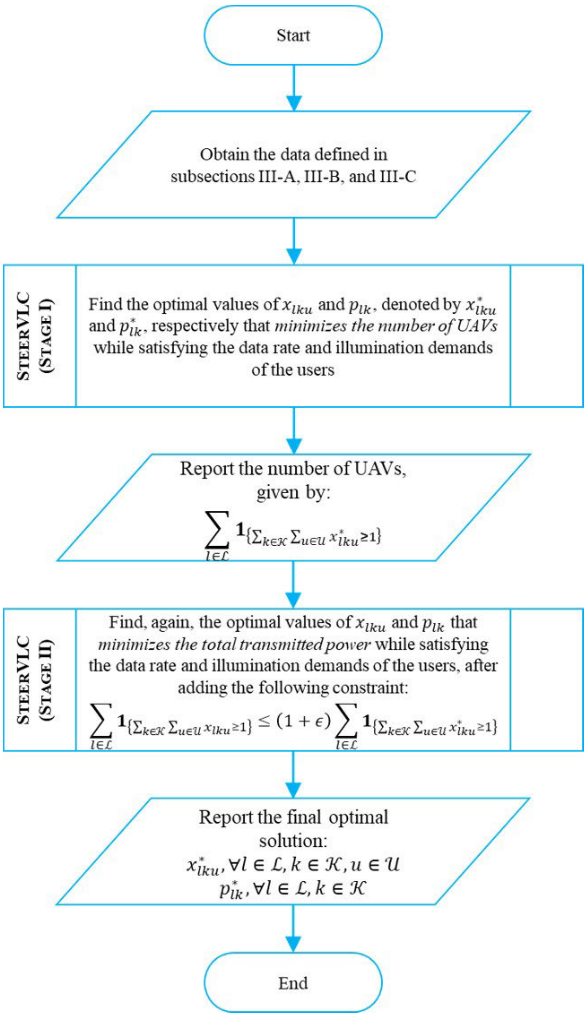

FIGURE 2. Flowchart of SteerVLC.

data defined in subsections III-A and III-B, the first stage of SteerVLC is executed to determine x^*_{lku}, $l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$ and p^*_{lk}, $l \in \mathcal{L}, k \in \mathcal{K}$, aiming to attain the smallest set of UAVs that satisfies the data rate and illumination demands of the ground users. The minimum number of required UAVs is then computed from x^*_{lku}, $l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$ as follows:

$$\text{Number of UAVs} = \sum_{l\in\mathcal{L}} \mathbb{1}_{\{\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x^*_{lku}\geq 1\}}, \tag{2}$$

where $\mathbb{1}_{\{\cdot\}}$ is an indicator function; $\mathbb{1}_{\{\cdot\}}$ equals one if the condition inside $\{\cdot\}$ is satisfied, and it equals zero otherwise.

After computing the minimum number of required UAVs, as obtained from SteerVLC (Stage I), the second stage of SteerVLC is executed to recompute x^*_{lku}, $l \in \mathcal{L}, k \in \mathcal{K}$, $u \in \mathcal{U}$ and p^*_{lk}, $l \in \mathcal{L}, k \in \mathcal{K}$, aiming now at minimizing the transmission power of the LEDs. x^*_{lku}, $l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$ and p^*_{lk}, $l \in \mathcal{L}, k \in \mathcal{K}$ are computed in SteerVLC (Stage II) while satisfying the data rate and illumination demands of the ground users, and *ensuring that the number of used UAVs is no more than* $(1+\epsilon)\times 100\%$ *of the minimum number of UAVs obtained from* SteerVLC (Stage I), where ϵ is a non-negative controllable parameter. ϵ is a critical parameter of SteerVLC used to control the tradeoff between the number of UAVs and the transmission power of the LEDs.

V. FIRST STAGE OF SteerVLC

In this section, we develop the mathematical formulation of SteerVLC (Stage I), which answers the questions stated in subsection III-C with the objective of minimizing the number of needed UAVs.

A. OBJECTIVE FUNCTION

The objective function of SteerVLC (Stage I) is the total number of deployed UAVs, which should be minimized. Given the decision variables defined in subsection IV-A, the objective function can be expressed mathematically as follows:

$$\underset{\substack{\{x_{lku}, p_{lk},\\ l\in\mathcal{L}, k\in\mathcal{K}, u\in\mathcal{U}\}}}{\text{minimize}} \sum_{l\in\mathcal{L}} 1_{\{\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku}\geq 1\}}. \tag{3}$$

B. CONSTRAINTS

The selection of the values of x_{lku}, $l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$ and p_{lk}, $l \in \mathcal{L}, k \in \mathcal{K}$ is constrained by several sets of constraints, which will be explained and mathematically formulated in the following:

1) DATA RATE DEMAND CONSTRAINTS

Each user $u \in \mathcal{U}$ should receive a minimum data rate of C_{th} bits/transition. This necessitates that each user $u \in \mathcal{U}$ receives a minimum power of $p_{\min}$ Watts, as explained in subsection III-A. This set of constraints (one for each user) can be expressed mathematically as follows:

$$\sum_{l\in\mathcal{L}}\sum_{k\in\mathcal{K}} h_{lu}\, x_{lku}\, p_{lk} \geq p_{\min}, \ \forall u \in \mathcal{U}. \tag{4}$$

2) ILLUMINATION CONSTRAINTS

Given the illumination threshold of the receiver, η_{th}, and the illumination interference at each user, I_u, $u \in \mathcal{U}$, each user u should receive a minimum power of $\eta_{\text{th}} - I_u$ Watts. This set of constraints can be expressed mathematically as follows:

$$\sum_{l\in\mathcal{L}}\sum_{k\in\mathcal{K}} h_{lu}\, x_{lku}\, p_{lk} \geq \eta_{\text{th}} - I_u, \ \forall u \in \mathcal{U}. \tag{5}$$

3) EYE SAFETY CONSTRAINTS

Given the maximum optical limit allowed for eye safety, p_{opt}, each user $u \in \mathcal{U}$ should not receive a power higher than p_{opt} Watts. This set of constraints can be expressed mathematically as follows:

$$\sum_{l\in\mathcal{L}}\sum_{k\in\mathcal{K}} h_{lu}\, x_{lku}\, p_{lk} \leq p_{\text{opt}}, \ \forall u \in \mathcal{U}. \tag{6}$$

4) SINGLE UAV ASSOCIATION CONSTRAINTS

Each user $u \in \mathcal{U}$ can be associated with one LED beam only. This set of constraints can be expressed mathematically as follows:

$$\sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} x_{lku} \leq 1, \ \forall u \in \mathcal{U}. \tag{7}$$

5) NUMBER OF BEAMS PER UAV CONSTRAINTS

Each UAV, regardless of its deployed location, cannot generate more than B beams. This set of constraints can be expressed mathematically as follows:

$$\sum_{k \in \mathcal{K}} \mathbb{1}_{\{\sum_{u \in \mathcal{U}} x_{lku} \geq 1\}} \leq B, \ \forall l \in \mathcal{L}. \tag{8}$$

6) MAXIMUM POWER CONSTRAINTS

The transmission power of any LED cannot exceed $p_{\max}$, regardless of its UAV, deployed location, and beam index. This set of constraints can be expressed mathematically as follows:

$$0 \leq p_{lk} \leq p_{\max}, \ \forall l \in \mathcal{L}, \forall k \in \mathcal{K}. \tag{9}$$

C. FORMULATION

The complete mathematical formulation of SteerVLC (Stage I) can be stated as follows:

SteerVLC (Stage I):

$$\underset{\substack{x_{lku}, p_{lk}, \\ l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}}}{\text{minimize}} \ \sum_{l \in \mathcal{L}} \mathbb{1}_{\{\sum_{k \in \mathcal{K}} \sum_{u \in \mathcal{U}} x_{lku} \geq 1\}} \tag{10}$$

subject to:

$$\sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} h_{lu} \, x_{lku} \, p_{lk} \geq p_{\min}, \ \forall u \in \mathcal{U} \tag{11}$$

$$\sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} h_{lu} \, x_{lku} \, p_{lk} \geq \eta_{\text{th}} - I_u, \ \forall u \in \mathcal{U} \tag{12}$$

$$\sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} h_{lu} \, x_{lku} \, p_{lk} \leq p_{\text{opt}}, \ \forall u \in \mathcal{U} \tag{13}$$

$$\sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} x_{lku} \leq 1, \ \forall u \in \mathcal{U} \tag{14}$$

$$\sum_{k \in \mathcal{K}} \mathbb{1}_{\{\sum_{u \in \mathcal{U}} x_{lku} \geq 1\}} \leq B, \ \forall l \in \mathcal{L} \tag{15}$$

$$0 \leq p_{lk} \leq p_{\max}, \ \forall l \in \mathcal{L}, \forall k \in \mathcal{K} \tag{16}$$

$$x_{lku} \in \{0, 1\}, \ \forall l \in \mathcal{L}, \forall k \in \mathcal{K}, \forall u \in \mathcal{U}. \tag{17}$$

VI. SECOND STAGE OF SteerVLC

Solving SteerVLC (Stage I), we obtain (i) $x^*_{lku}, l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$, the optimal UAV deployment, LED beam steering, and user-beam association, and (ii) the minimum number of required UAVs, given by (2).

Given the minimum number of required UAVs, the goal of the second stage of SteerVLC is to compute the optimal values of $x_{lku}, l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$ and $p_{lk}, l \in \mathcal{L}, k \in \mathcal{K}$ that minimize the total transmitted power by the LEDs, satisfy the sets of constraints explained in subsection V-B, and ensure the following additional constraint:

$$\sum_{l \in \mathcal{L}} \mathbb{1}_{\{\sum_{k \in \mathcal{K}} \sum_{u \in \mathcal{U}} x_{lku} \geq 1\}} \leq (1 + \epsilon) \sum_{l \in \mathcal{L}} \mathbb{1}_{\{\sum_{k \in \mathcal{K}} \sum_{u \in \mathcal{U}} x^*_{lku} \geq 1\}}, \tag{18}$$

where ϵ is a non-negative controllable parameter. Constraint (18) ensures that the optimal UAV deployment, LED beam steering, and user-beam association that aims at minimizing the total transmitted power by the LEDs should not use more than $(1 + \epsilon) \times 100\%$ of the minimum number of UAVs obtained from SteerVLC (Stage I). As mentioned in Section IV, ϵ is a critical parameter used in SteerVLC to control the tradeoff between the number of UAVs and the transmission power of the LEDs.

A. OBJECTIVE FUNCTION

The objective function of SteerVLC (Stage II) is the total transmitted power by the LEDs mounted on the UAVs, which should be minimized. Given the decision variables defined in subsection IV-A, the objective function can be expressed mathematically as follows:

$$\underset{\substack{x_{lku}, p_{lk}, \\ l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}}}{\text{minimize}} \ \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} p_{lk}. \tag{19}$$

B. CONSTRAINTS

The selection of the values of $x_{lku}, l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$ and $p_{lk}, l \in \mathcal{L}, k \in \mathcal{K}$ in SteerVLC (Stage II) is constrained by the six sets of constraints explained in subsection V-B, in addition to constraint (18) explained above.

C. FORMULATION

The complete mathematical formulation of SteerVLC (Stage II) can be stated as follows:

SteerVLC (Stage II)

$$\underset{\substack{x_{lku}, p_{lk}, \\ l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}}}{\text{minimize}} \ \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} p_{lk} \tag{20}$$

subject to:

$$\sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} h_{lu} \, x_{lku} \, p_{lk} \geq p_{\min}, \ \forall u \in \mathcal{U} \tag{21}$$

$$\sum_{l\in\mathcal{L}}\sum_{k\in\mathcal{K}} h_{lu}\, x_{lku}\, p_{lk} \geq \eta_{\text{th}} - I_u,\ \forall u \in \mathcal{U} \tag{22}$$

$$\sum_{l\in\mathcal{L}}\sum_{k\in\mathcal{K}} h_{lu}\, x_{lku}\, p_{lk} \leq p_{\text{opt}},\ \forall u \in \mathcal{U} \tag{23}$$

$$\sum_{l\in\mathcal{L}}\sum_{k\in\mathcal{K}} x_{lku} \leq 1,\ \forall u \in \mathcal{U} \tag{24}$$

$$\sum_{l\in\mathcal{L}} \mathbb{1}_{\{\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku}\geq 1\}} \leq (1+\epsilon)\sum_{l\in\mathcal{L}} \mathbb{1}_{\{\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x^*_{lku}\geq 1\}} \tag{25}$$

$$\sum_{k\in\mathcal{K}} \mathbb{1}_{\{\sum_{u\in\mathcal{U}} x_{lku}\geq 1\}} \leq B,\ \forall l \in \mathcal{L} \tag{26}$$

$$0 \leq p_{lk} \leq p_{\max},\ \forall l \in \mathcal{L}, \forall k \in \mathcal{K} \tag{27}$$

$$x_{lku} \in \{0,1\},\ \forall l \in \mathcal{L}, \forall k \in \mathcal{K}, \forall u \in \mathcal{U}. \tag{28}$$

VII. EQUIVALENT LINEAR REFORMULATION OF SteerVLC

The mathematical formulation of SteerVLC developed in the previous sections is non-linear. In this section, we develop an *equivalent* linear reformulation of SteerVLC. SteerVLC has two types of non-linearity: The indicator function and the product of decision variables.

A. LINEARIZING THE INDICATOR FUNCTION

The objective function of SteerVLC (Stage I) (10), its set of constraints (15), and the sets of constraints (25) and (26) in SteerVLC (Stage II) have indicator functions, which are non-linear. In the following, we explain the linearization methodology for each of these indicator functions [32].

1) LINEARIZING SteerVLC (Stage I) OBJECTIVE FUNCTION

To linearize the indicator function in (10), we introduce new *auxiliary* binary decision variables, $y_l, \forall l \in \mathcal{L}$, which are defined in terms of the *true* decision variables x_{lku}, $l \in \mathcal{L}$, $k \in \mathcal{K}$, $u \in \mathcal{U}$, as follows:

$$y_l \overset{\text{def}}{=} 1_{\{\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku}\geq 1\}}, \forall l \in \mathcal{L}. \tag{29}$$

The objective function (10) is then rewritten as:

$$\underset{\left\{\substack{x_{lku}, p_{lk}, y_l \\ l\in\mathcal{L}, k\in\mathcal{K}, u\in\mathcal{U}}\right\}}{\text{minimize}} \sum_{l\in\mathcal{L}} y_l \tag{30}$$

subject to:

$$y_l \overset{\text{def}}{=} 1_{\{\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku}\geq 1\}}, \forall l \in \mathcal{L}. \tag{31}$$

The relationship between y_l and x_{lku}, given by (31), needs then to be linearly reformulated so that the indicator function is omitted. Equation (31) says that $y_l = 1$ if and only if $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \geq 1$, which means the following:

- If $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \geq 1$ then $y_l = 1$.
- If $y_l = 1$ then $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \geq 1$.

Each of the above if-then statements needs to be linearly expressed.

Let M be an upper bound of $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} - 1$. Then, the first if-then statement above can be reformulated as follows:

$$\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} - (M+\zeta)\, y_l \leq 1 - \zeta, \tag{32}$$

where $\zeta > 0$ is a small tolerance beyond which the condition is considered violated. Note that U (i.e., the number of ground users) is an upper bound of $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku}$. This upper bound is reached if each user $u \in \mathcal{U}$ is assigned a separate LED beam. Hence, M can be selected to be $U - 1$. Select ζ to be 1, then (32) reduces to:

$$\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \leq U y_l. \tag{33}$$

Equation (33) expresses the if-then statement "if $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \geq 1$ then $y_l = 1$" linearly. If $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \geq 1$, then, according to (33), y_l cannot be assigned to 0 and it has to be 1.

Next, we reformulate the second if-then statement above. Let m be a lower bound of $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} - 1$. Then, the second if-then statement can be reformulated as follows:

$$\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} + m\, y_l \geq m + 1. \tag{34}$$

Note that 0 is a lower bound of $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku}$. Hence, m can be selected to be -1. In this case, (34) reduces to:

$$\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \geq y_l. \tag{35}$$

Equation (35) expresses the if-then statement "if $y_l = 1$ then $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \geq 1$" linearly. If $y_l = 1$, then, according to (35), $\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \geq 1$.

Considering (33) and (35), it follows that (31) can be linearly reformulated as follows:

$$y_l \leq \sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{U}} x_{lku} \leq U y_l, \forall l \in \mathcal{L}. \tag{36}$$

2) LINEARIZING THE NUMBER OF BEAMS CONSTRAINT

Following a similar procedure to that introduced in subsubsection VII-A1, constraint (15) can be linearized by introducing an auxiliary binary decision variable for each $l \in \mathcal{L}$ and $k \in \mathcal{K}$. Let us denote this auxiliary decision variable by y_{lk}. Then, (15) can be rewritten as:

$$\sum_{k\in\mathcal{K}} y_{lk} \leq B,\ \forall l \in \mathcal{L} \tag{37}$$

$$y_{lk} \overset{\text{def}}{=} 1_{\{\sum_{u\in\mathcal{U}} x_{lku}\geq 1\}},\ \forall l \in \mathcal{L}, \forall k \in \mathcal{K}. \tag{38}$$

The relationship between y_{lk} and x_{lku} needs to be linearly reformulated. Following a similar procedure to that in subsubsection VII-A1, (38) can be linearly reformulated as follows:

$$y_{lk} \leq \sum_{u\in\mathcal{U}} x_{lku} \leq U y_{lk}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}. \tag{39}$$

3) LINEARIZING THE ϵ-CONTROLLED TRADEOFF CONSTRAINT

Constraint (25) in SteerVLC (Stage II) can be linearly reformulated as follows:

$$\sum_{l\in\mathcal{L}} y_l \leq (1+\epsilon) \sum_{l\in\mathcal{L}} y_l^* \tag{40}$$

$$y_l \leq \sum_{k\in\mathcal{K}} \sum_{u\in\mathcal{U}} x_{lku} \leq U y_l, \forall l \in \mathcal{L}, \tag{41}$$

where y_l^* is the optimal value of y_l obtained from solving SteerVLC (Stage I).

B. LINEARIZING A PRODUCT OF DECISION VARIABLES

Constraints (11)–(13) in SteerVLC (Stage I) and constraints (21)–(23) in SteerVLC (Stage II) include a product of two decision variables, $x_{lku}p_{lk}$, which is non-linear. In this subsection, we explain the linearization methodology for such products of decision variables [32].

To linearize $x_{lku}p_{lk}$, we first introduce, for each $l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}$, a new auxiliary non-negative continuous decision variable. Let us denote this new decision variable by z_{lku}. $x_{lku}p_{lk}$ is replaced by z_{lku}, after adding the following set of constraints:

$$z_{lku} = x_{lku}\, p_{lk}, \ \forall l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}. \tag{42}$$

The relationship between z_{lku}, x_{lku}, and p_{lk}, given by (42), needs then to be linearly reformulated. It can be easily shown that (42) can be equivalently expressed by the following sets of linear constraints:

$$z_{lku} \geq 0, \forall l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U} \tag{43}$$

$$z_{lku} \leq p_{\max}\, x_{lku}, \forall l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U} \tag{44}$$

$$z_{lku} \geq p_{lk} - p_{\max}\,(1 - x_{lku}), \forall l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U} \tag{45}$$

$$z_{lku} \leq p_{lk}, \forall l \in \mathcal{L}, k \in \mathcal{K}, u \in \mathcal{U}. \tag{46}$$

If $x_{lku} = 0$, then, according to (42), z_{lku} should be 0, regardless of the value of p_{lk}. Examining (43)–(46), when $x_{lku} = 0$, $z_{lku} = 0$. On the other hand, if $x_{lku} = 1$, then, according to (42), z_{lku} should be equal to p_{lk}, regardless of the value of p_{lk}. Examining (43)–(46), when $x_{lku} = 1$, $z_{lku} = p_{lk}$.

C. MILP REFORMULATION OF SteerVLC

Based on subsections VII-A and VII-B, the MILP reformulation of SteerVLC can be stated as follows:

Stage I:

$$\underset{\left\{\substack{y_l, y_{lk}, x_{lku}, p_{lk}, z_{lku},\\ l\in\mathcal{L}, k\in\mathcal{K}, u\in\mathcal{U}}\right\}}{\text{minimize}} \sum_{l\in\mathcal{L}} y_l \tag{47}$$

subject to:

$$\sum_{l\in\mathcal{L}} \sum_{k\in\mathcal{K}} h_{lu}\, z_{lku} \geq p_{\min}, \ \forall u \in \mathcal{U} \tag{48}$$

$$\sum_{l\in\mathcal{L}} \sum_{k\in\mathcal{K}} h_{lu}\, z_{lku} \geq \eta_{\text{th}} - I_u, \ \forall u \in \mathcal{U} \tag{49}$$

$$\sum_{l\in\mathcal{L}} \sum_{k\in\mathcal{K}} h_{lu}\, z_{lku} \leq p_{\text{opt}}, \forall u \in \mathcal{U} \tag{50}$$

$$y_l \leq \sum_{k\in\mathcal{K}} \sum_{u\in\mathcal{U}} x_{lku} \leq U y_l, \forall l \in \mathcal{L} \tag{51}$$

$$y_{lk} \leq \sum_{u\in\mathcal{U}} x_{lku} \leq U y_{lk}, \ \forall l \in \mathcal{L}, \forall k \in \mathcal{K} \tag{52}$$

$$0 \leq z_{lku} \leq p_{lk}, \ \forall l \in \mathcal{L}, \forall k \in \mathcal{K}, \forall u \in \mathcal{U} \tag{53}$$

$$p_{lk} - p_{\max}\,(1 - x_{lku}) \leq z_{lku} \leq p_{\max}\, x_{lku}, \ \forall l \in \mathcal{L}, \forall k \in \mathcal{K}, \forall u \in \mathcal{U} \tag{54}$$

$$\sum_{k\in\mathcal{K}} y_{lk} \leq B, \ \forall l \in \mathcal{L} \tag{55}$$

$$\sum_{l\in\mathcal{L}} \sum_{k\in\mathcal{K}} x_{lku} \leq 1, \ \forall u \in \mathcal{U} \tag{56}$$

$$0 \leq p_{lk} \leq p_{\max}, \ \forall l \in \mathcal{L}, \forall k \in \mathcal{K} \tag{57}$$

$$x_{lku} \in \{0, 1\}, \ \forall l \in \mathcal{L}, \forall k \in \mathcal{K}, \forall u \in \mathcal{U} \tag{58}$$

$$y_l \in \{0, 1\}, \ \forall l \in \mathcal{L} \tag{59}$$

$$y_{lk} \in \{0, 1\}, \ \forall l \in \mathcal{L}, \forall k \in \mathcal{K}. \tag{60}$$

Stage II:

$$\underset{\left\{\substack{y_l, y_{lk}, x_{lku}, p_{lk}, z_{lku},\\ l\in\mathcal{L}, k\in\mathcal{K}, u\in\mathcal{U}}\right\}}{\text{minimize}} \sum_{l\in\mathcal{L}} \sum_{k\in\mathcal{K}} p_{lk} \tag{61}$$

subject to:

$$(48) - (60)$$

$$\sum_{l\in\mathcal{L}} y_l \leq (1+\epsilon) \sum_{l\in\mathcal{L}} y_l^*. \tag{62}$$

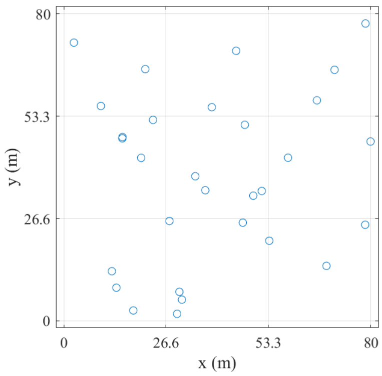

FIGURE 3. A realization of PPP-distributed users.

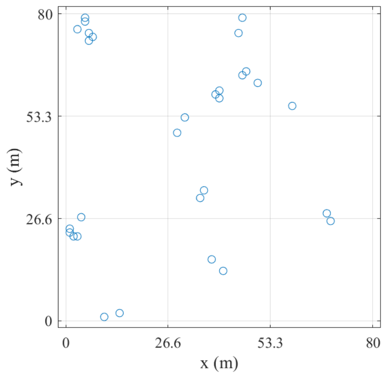

FIGURE 4. A realization of Matérn PCP-distributed users.

VIII. PERFORMANCE EVALUATION

The performance of SteerVLC is evaluated in this section based on two performance metrics: The number of required UAVs to be deployed and the total transmission power by the steerable LEDs. The effects of different system parameters on the performance of SteerVLC are examined. Finally, three variants of SteerVLC are defined and compared. These variants are:

- SteerUFPS: This variant of SteerVLC is obtained by setting ϵ in (62) to zero. SteerUFPS aims at attaining the smallest set of UAVs that satisfies the user demands First and minimizes the transmission Power of the steerable LEDs Second (i.e., power minimization is examined only when there are multiple optimal solutions that yield the same smallest set of UAVs). SteerUFPS is an enhanced steerable version of UFPS, proposed in [23], with optimized beam-steering. It is worth mentioning that UFPS was evaluated and compared with several representative benchmarks in the literature in [23].
- SteerVLC.4: This variant of SteerVLC is obtained by setting ϵ in (62) to 0.4. In this scheme, the number of deployed UAVs is allowed to exceed the minimum number needed by no more than 40%. The goal of this flexibility is to further minimize the total transmitted power.
- SteerVLCd: This variant of SteerVLC is obtained by setting ϵ in (62) to 1. In this scheme, the number of deployed UAVs is allowed to exceed the minimum number needed by no more than 100%. Similar to SteerVLC.4, the goal of this flexibility is to further minimize the total transmitted power.

A. EVALUATION SETUP

We consider an 80×80 m^2 outdoor environment, divided into $K = 9$ cells arranged as a 3×3 grid. Two stochastic geometry models are considered to represent the distribution of the ground users in our evaluation, namely, the PPP and Matérn PCP models. Figs. 3 and 4 depict, respectively, the PPP and PCP realizations of 30 users, considered in our evaluation. The achieved data rate is selected within the allowed range by the IEEE 802.15.7 standard, between 11.67 Kbps and 100 Kbps [33], [34]. Specifically, the default value of C_{th} is set to 1 bit/transition, assuming an On-Off Keying (OOK) modulation, as recommended by the IEEE 802.15.7 PHY 1 for outdoor VLC [35], [36], [37]. The transition duration (a.k.a. the clock rate) is set so that the achieved data rate is 12 Kbps. The LEDs are assumed to have the same illumination specifications as in [38]. Illumination interference is modeled as a random variable uniformly distributed between 0 and $I_{\max}$ (i.e., $I_u \sim \mathcal{U}(0, I_{\max}), \forall u \in \mathcal{U}$). Fig. 5 shows a realization of the illumination interference used in our evaluation. The number of UAV potential locations, L, is set to 49, arranged into a 7×7 grid. Typical system parameter values are used in our evaluation. Our parameter values follow closely the values used in [18], [19], [23], and [24]. Unless stated otherwise, the system and VLC channel parameters are set to the values listed in Tables 2 and 3, respectively. SteerVLC is solved using CPLEX.

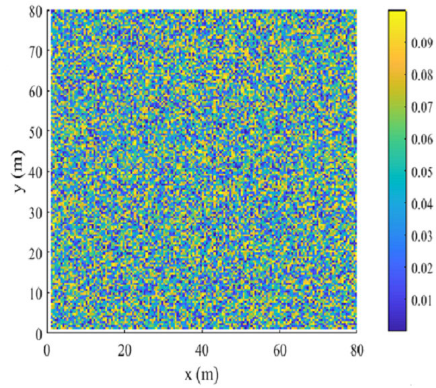

FIGURE 5. Illumination interference map.

B. SteerVLC EVALUATION

1) EFFECTS OF ϵ

Considering a single beam (i.e., $B = 1$) and the PPP-distributed users in Fig. 3, Fig. 6 shows the effects of ϵ on the number of needed UAVs and the total transmission power under different values of C_{th}. As ϵ increases, the willingness

TABLE 2. System parameters.

Parameter	Value	Parameter	Value
B	1	p_{opt}	80 W
C_{th}	1 bit/transition	$I_{\max}$	5×10^{-10} W
η_{th}	1×10^{-9} W	σ_w	1×10^{-10}
$p_{\max}$	60 W	ξ	0.8 W/Amp
UAV height	12 m	data rate	12 Kbps

TABLE 3. VLC channel parameters.

Parameter	Value
Detection area of the photodetector (PD)	1 cm^2
Average responsivity of the PD	1 Amp/W
Gain of the optical receiver filter	1
Reflection index of the concentrator	1.5
PD field of view semi-angle	60°

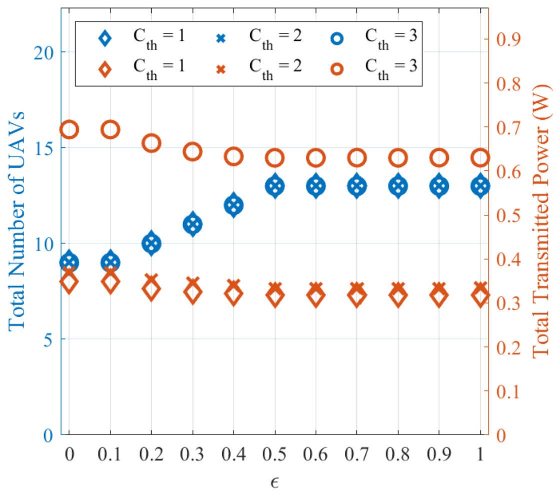

FIGURE 6. The number of deployed UAVs and the LEDs transmission power vs. ϵ for different values of C_{th}. The PPP-distributed users, shown in Fig. 3, are considered.

of SteerVLC to increase the number of UAVs in order to reduce the total transmitted power increases. In SteerVLC, the total transmitted power is minimized while permitting $\epsilon \times 100\%$ increase in the number of UAVs compared to the minimum number needed. Increasing ϵ improves the likelihood of finding better communication links between the steerable LEDs and the ground users, and thus decreases the transmission power needed to cover all users.

2) EFFECTS OF THE USERS DATA RATE DEMAND (C_{th})

Considering a single beam and the PPP-distributed users in Fig. 3, Fig. 7 shows the effects of C_{th} on the total power transmitted for SteerUFPS and SteerVLC.4. As shown in Fig. 7, the total transmitted power increases exponentially with C_{th}. Fig. 7 also shows that the growth rate of the transmitted power increases with ϵ. The total transmitted power of SteerUFPS increases faster with C_{th} than SteerVLC.4. On the other hand, our experiments show that the required number of UAVs by SteerUFPS is nine, whereas SteerVLC.4 needs twelve UAVs.

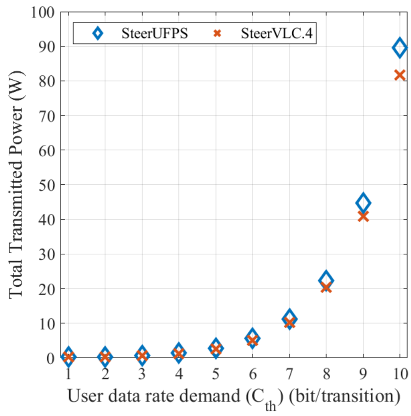

FIGURE 7. The LEDs transmission power vs. C_{th} for SteerUFPS and SteerVLC.4. The PPP-distributed users, shown in Fig. 3, are considered.

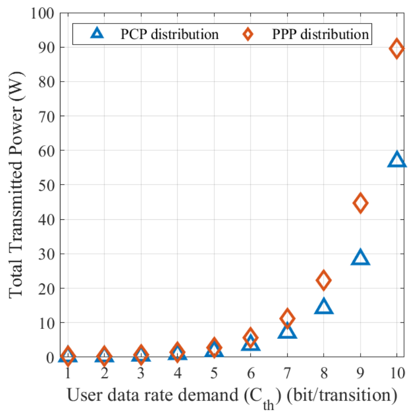

FIGURE 8. The LEDs transmission power vs. C_{th} for SteerUFPS, considering the PPP- and PCP-distributed users, shown in Figs. 3 and 4, respectively.

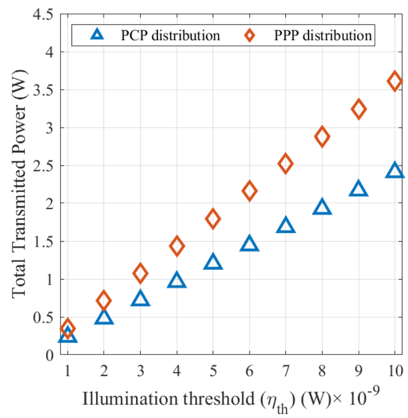

FIGURE 9. The LEDs transmission power vs. η_{th} for SteerUFPS, considering the PPP- and PCP-distributed users, shown in Figs. 3 and 4, respectively.

The number of UAVs remains the same for all considered values of C_{th} in Fig. 7.

Fig. 8 shows the effects of C_{th} on the total transmitted power for SteerUFPS, considering the PPP- and PCP-distributed users, shown in Figs. 3 and 4, respectively. Our

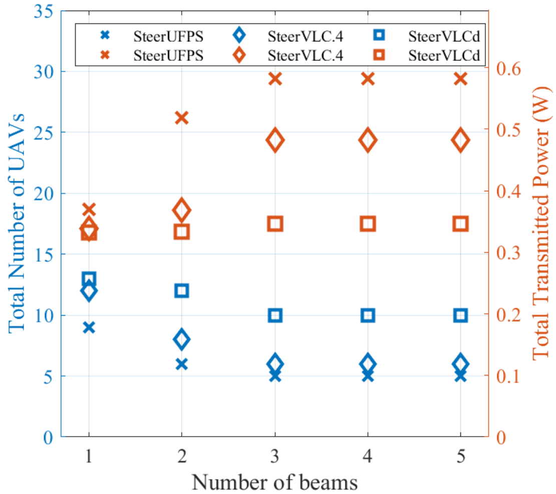

FIGURE 10. The number of deployed UAVs and the LEDs transmission power vs. the number of beams for SteerUFPS, SteerVLC.4, and SteerVLCd. The PPP-distributed users, shown in Fig. 3, are considered.

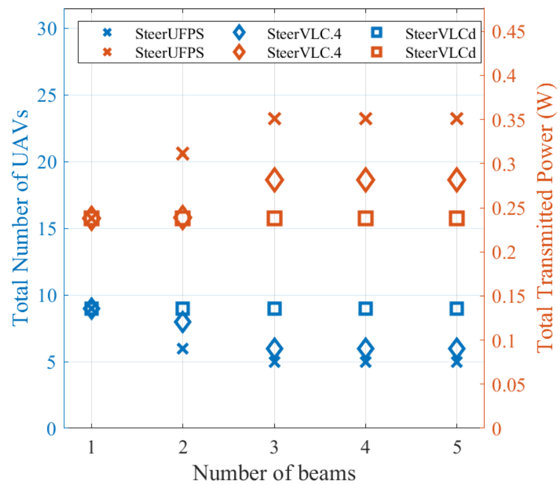

FIGURE 11. The number of deployed UAVs and the LEDs transmission power vs. the number of beams for SteerUFPS, SteerVLC.4, and SteerVLCd. The PCP-distributed users, shown in Fig. 4, are considered.

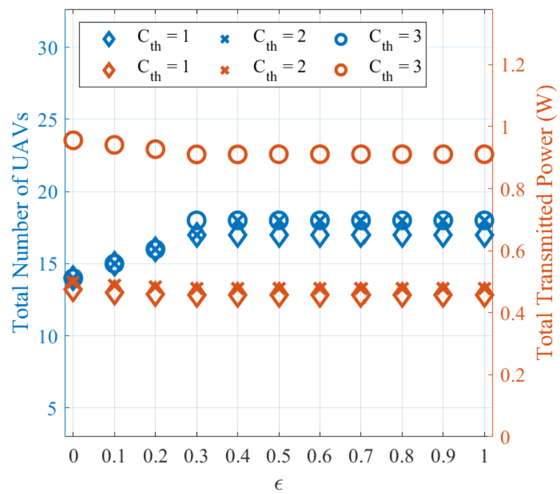

FIGURE 12. The number of deployed UAVs and the LEDs transmission power vs. ϵ for different values of C_{th}. The PPP-distributed users, shown in Fig. 3, are considered (UAV height is 8 m).

experiments show that the number of required UAVs for both user distributions is nine for all considered values of C_{th} in Fig. 8. However, Fig. 8 shows the effect of user clustering on reducing the total transmitted power. This effect becomes more apparent as C_{th} increases.

3) EFFECTS OF THE ILLUMINATION THRESHOLD (η_{th})

Fig. 9 shows the effects of η_{th} on the total transmitted power for SteerUFPS, considering the PPP- and PCP-distributed users, shown in Figs. 3 and 4, respectively. Fig. 9 shows that the total transmitted power increases linearly with η_{th}. The effect of user clustering is also evident in Fig. 9, which shows that the increasing rate of the transmitted power with η_{th} is smaller when users are clustered. The number of required UAVs for both user distributions is nine for all considered values of η_{th} in Fig. 9.

4) EFFECTS OF THE NUMBER OF BEAMS

Considering the PPP-distributed users in Fig. 3, Fig. 10 shows the effects of the number of beams, B, on the number of UAVs and the total transmitted power for SteerUFPS, SteerVLC.4, and SteerVLCd. As Fig. 10 shows, as B increases the number of required UAVs decreases. Furthermore, the number of UAVs used in SteerUFPS is smaller than that used in SteerVLC.4, which is smaller than the number of UAVs used in SteerVLCd. The effect of ϵ on the number of UAVs, discussed in subsubsection VIII-B1, is also evident in Fig. 10. On the other hand, the total transmitted power is non-decreasing with B. The total transmitted power in SteerUFPS is higher than that in SteerVLC.4, which is higher than the transmitted power in SteerVLCd. The effect of ϵ on the total transmitted power, discussed in subsubsection VIII-B1, is also evident in Fig. 10.

Considering the PCP-distributed users in Fig. 4, Fig. 11 shows the effects of B on the number of UAVs and the total transmitted power for SteerUFPS, SteerVLC.4, and SteerVLCd. Fig. 4 confirms the conclusions drawn from Fig. 10. The effect of user clustering on reducing the number of UAVs needed and the total transmitted power can be clearly observed by comparing Figs. 10 and 11.

5) EFFECTS OF THE UAV HEIGHT

The experiments conducted to obtain the results in Figs. 6-11 above are repeated in this section considering a UAV height of 8 m instead of 12 m. The generated results, shown in Figs. 12-17, demonstrate the same trends explained in subsections VIII-B1-VIII-B4 and confirm all conclusions drawn from Figs. 6-11.

It is to be noted that decreasing the UAV height does not necessarily lead to decreasing the distance between a certain ground user and its associated UAV. This is because the UAV

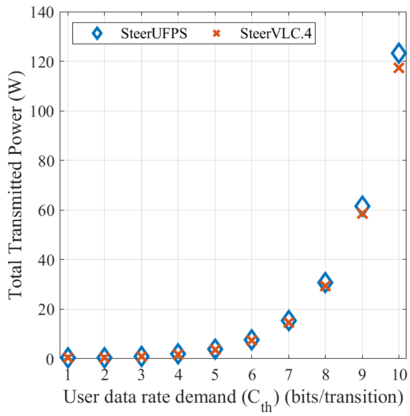

FIGURE 13. The LEDs transmission power vs. C_{th} for SteerUFPS and SteerVLC.4. The PPP-distributed users, shown in Fig. 3, are considered.

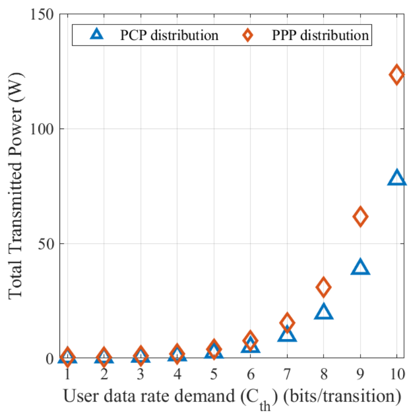

FIGURE 14. The LEDs transmission power vs. C_{th} for SteerUFPS, considering the PPP- and PCP-distributed users, shown in Figs. 3 and 4, respectively.

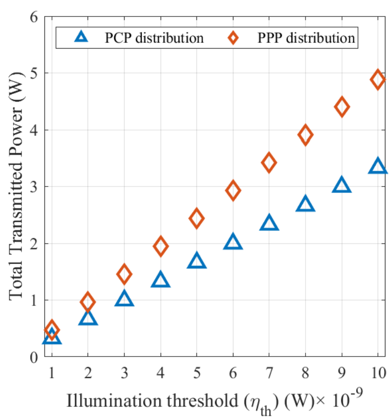

FIGURE 15. The LEDs transmission power vs. η_{th} for SteerUFPS, considering the PPP- and PCP-distributed users, shown in Figs. 3 and 4, respectively (UAV height is 8 m).

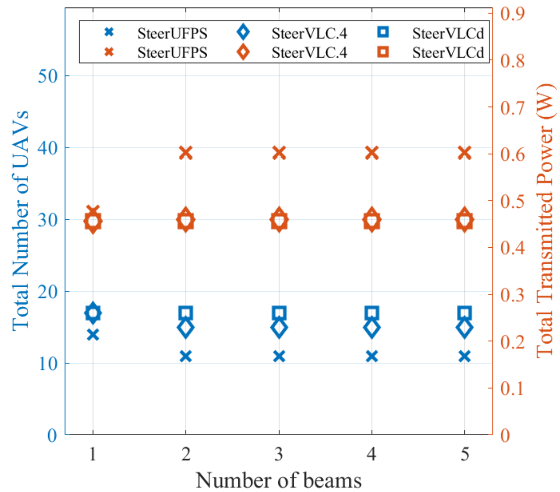

FIGURE 16. The number of deployed UAVs and the LEDs transmission power vs. the number of beams for SteerUFPS, SteerVLC.4, and SteerVLCd. The PPP-distributed users, shown in Fig. 3, are considered (UAV height is 8 m).

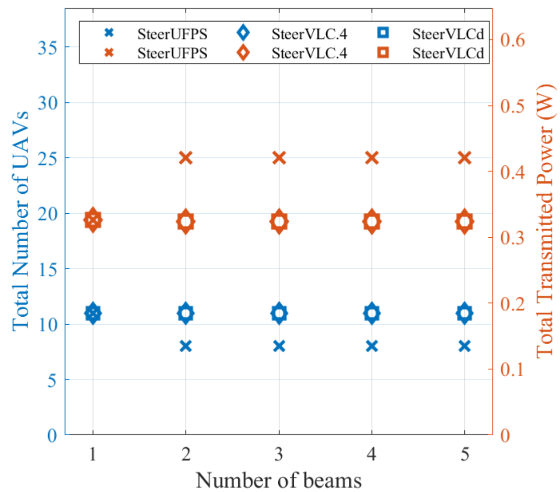

FIGURE 17. The number of deployed UAVs and the LEDs transmission power vs. the number of beams for SteerUFPS, SteerVLC.4, and SteerVLCd. The PCP-distributed users, shown in Fig. 4, are considered (UAV height is 8 m).

deployment and the UAV-user assignment, in general, change if the UAV height changes. Moreover, decreasing the distance between a user and its associated UAV does not necessarily lead to improving the VLC channel gain between the LED mounted on this UAV and the user. The reason for this is that the VLC channel gain depends not only on the distance between the LED and its associated user but also on the angles of incidence and irradiance. Finally, decreasing the UAV height does not result in increasing the UAV coverage in our experiments. The LED footprint is controlled to remain confined to one cell on the floor, to avoid adjacent-cell interference.

IX. CONCLUSION AND FUTURE RESEARCH

To overcome shadowing and low reflected energy, two major obstacles in optical wireless communications, in this paper, we developed SteerVLC, a joint optimization framework for UAV deployment, LED beam-steering, cell association, and power allocation. SteerVLC fulfills a set of illumination and data rate requirements for a set of stochastically distributed users with the minimum number of UAVs and minimum transmission power, while accounting for illumination interference. We extensively evaluated SteerVLC under different system parameters, considering PPP- and PCP-distributed users. The effects of different system parameters on network performance have been studied thoroughly. The effects of user clustering have also been numerically examined. Our results illustrated the ability of SteerVLC on managing the inherent tradeoff between minimizing the number of deployed UAVs and minimizing the transmission power of the LEDs.

Several important and challenging resource allocation problems in VLC-enabled UAV networks are still open. One interesting problem is designing an optimal resource allocation framework for intelligent reflecting surfaces (IRSs)-assisted VLC-enabled UAV networks. IRSs are needed to address the issue of having no line-of-sight transmissions.

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