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RESEARCH ARTICLE

Distributed Event-Triggered Output Consensus for General Linear Heterogeneous Multi-Agent Systems With System Uncertainties

XINGHAN LIN[1], ZHIGANG HUANG[1], KEYOU GUO[1], GANG LI[1], AND CHANGKUN DU[2], (Member, IEEE)

[1]School of Artificial Intelligence, Beijing Technology and Business University, Beijing 100048, China
[2]School of Mechatronical Engineering, Beijing Institute of Technology, Beijing 100081, China

Corresponding author: Changkun Du (duchangkun88@gmail.com)

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ABSTRACT This paper is devoted to study the distributed event-triggered output consensus (ETOC) of heterogeneous multi-agent systems (MASs) with general linear dynamics subject to system uncertainties over digraphs. To account for the practical case where accurate system model cannot be obtained in advance, an event-triggered output consensus control method is studied based on the internal model principle such that the output consensus error approaches to a small adjustable bounded set related to the mismatch level between accurate and inaccurate model in a distributed way. To improve the triggering performance, a novel resilient state-independent threshold is introduced in the state-dependent threshold, which endows the piecewise continuous mixed threshold a feature of reset to a greater value when an event is triggered. Within the proposed ETOC method, the circumvent of continuous neighbouring state exchange is ensured. Consensus stability and Zeno phenomenon are analyzed to ensure the theoretical correctness of the proposed ETOC method. Numerical simulations are carried out.

INDEX TERMS Heterogeneous multi-agent systems, event-triggered output consensus, system uncertainties.

I. INTRODUCTION

The idea on consensus of multi-agent systems (MASs) is to construct an effective control algorithm in a distributed fashion so as to achieve an agreement on a common state of interest for all agents. Early related works on this problem can be found in [1] and [2]. Then, plentiful works have been extensively investigated and contributed to this topic (e.g., [3], [4], [5], [6], [7], [8], [9]). The feature of requirement on uninterruptable information flow limits the application scope of the aforementioned results since communication energy and bandwidth are usually restrictive in practice, which impels to study consensus problem of MASs with efficient utilization of limited communication resource.

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As an effective way to save communication resource, event-triggered mechanism (ETM) energizes the development of cooperative control of MASs in past decades. Within the framework of the ETM, communication actions are performed by detecting a pre-designed triggering function. To this end, both centralized and decentralized ETMs are presented in [10]. Then, the combined measurement approach and an exponential-function-based triggering function are proposed to reach consensus in [11] and [12], respectively. To further extend the results focused on integrator-type MASs in [10], [11], and [12] to MASs with more general dynamics, the event-triggered consensus of MASs with general linear dynamics are studied in [13], [14], and [15]. To remove continuous state exchange in controller, broadcast state [13], [14] or estimated state determined based on the system model and broadcast state [15] from agent itself and its local neighbors (instead of continuous states directly obtained from neighbors) are used to design controller. Compared to controller design in ETMs, to design the triggering law with a triggering condition integrated is more challenge. Moreover, a dynamic event-triggering law is designed in [16] based on the combined measurement approach such that the consensus of homogeneous MASs with general linear dynamics can be achieved. However, the requirement on continuous communication is still needed in triggering condition design in [13], [14], [15], and [16].

Then, efforts to overcome continuous communication in triggering condition design have arisen. By using the sum of relative difference of estimated states from agent itself and the one from its neighbors to construct the triggering threshold, continuous access to neighbors' states is avoided in [17], [18], and [19] where general linear MASs are considered. Besides, by using a threshold based on the exponential function, both event/self-triggered schemes without continuous state transmission are discussed in [20]. Considering MASs with identical general linear dynamic over undirected graphs, the event-triggered control protocol with constructing an observer is studied in [21] such that the secure consensus can be guaranteed under DoS attack of cyber-physical systems. Finite-time consensus problem via ETMs for integrator-type and general linear MASs over digraphs with fully intermittent communication are studied in [22] and [23], respectively. Whereas, the aforementioned results have a particularly focus on the homogeneous MASs featured with identical agent dynamics. Noteworthily, compared with homogeneous MASs, heterogeneous MASs where agent dynamics are allowed to be different due to inherent unique physical characteristics and environmental uncertainties are more appropriate to describe the networked systems in practical applications. Therefore, it is necessary to study the event-triggered output consensus (ETOC) problem for heterogeneous MASs since it is more common and realistic in practice that agents have different models due to different unique physical characteristics and uncertainties. Along this line, [24], [25], [26] investigate ETOC problem of heterogeneous general linear MASs under digraphs. Within the framework of dynamic ETMs, consensus control of heterogeneous strict-feedback MASs subject to nonlinear-in-parameter uncertainties is studied in a distributed manner in [27]. Noteworthily, to implement the aforementioned algorithms proposed for MASs with general linear dynamics, it is necessary to obtain the prior knowledge on the accurate system model information. However, the accurate system model information might not be known beforehand. Hence, it is imperative to study ETOC without accurate system model and continuous state transmission.

Motivated by the above discussions, a distributed ETOC control method is studied for general linear heterogeneous MASs over digraphs in the absence of accurate system model information. The main contribution is emphasized as follows.

First, an innovative point of our research is that different uncertainties between the unknown accurate system information and the obtained inaccurate system information are considered for a collection of networked agents with different dynamics, which is of obvious physical significance in practice. Thus, the ETOC problem is investigated in a framework of general linear heterogeneous MAS without a priori knowledge of the accurate system information over digraphs, which in hence brings about more challenges. Most works on event-triggered consensus for general linear homogeneous/heterogeneous MAS in the existing literature require the accurate system model information no matter whether the continuous state transmission is avoided or not. Therefore, the methods proposed in the aforementioned literature cannot be directly adopted in this paper. To deal with the challenges of inaccurately known system information, a distributed ETOC without the requirement on accurate system model information is proposed and the practical output consensus can be achieved with fully avoiding continuous state exchange in not only controller updates but also triggering detections. It is worth mentioning that there exist significant challenges in the analysis of convergence in our framework.

Second, a resilient state-independent threshold is introduced into the state-dependent threshold, which endows the piecewise continuous mixed threshold a feature of resetting to a greater value when an event is triggered. The proposed novel mixed threshold combines the benefits of state-dependent and -independent thresholds and helps improve triggering performance. Moreover, in contrast to the continuous state-independent triggering threshold studied in [20], the proposed mixed triggering threshold with the resilient feature is more effective. Thus, more challenges have emerged in the analysis of the consensus stability and the Zeno behavior.

The outline of this paper is organized as follows. Problem statement and some useful lemmas are introduced in Section II. The main result of this study is given in Section III. Simulation examples are illustrated in Section IV. Finally, conclusions are drawn in Section V.

Notations: $\mathbb{R}^{N\times N}$ and $\mathbb{R}^N$ are real matrices and vectors. $P < 0$ ($P > 0$) represents that P is negative (positive)-definite. diag$\{\cdot\}$ means a diagonal matrix. $\|A_i\|_{\max}$ denote the maximum $\|A_i\|$. $\sigma(\cdot)$, $\overline{\mathbb{C}}_-$, and $j\mathbb{R}$ denote the spectrum of a square matrix, the left-half complex plane, and the imaginary axis, respectively.

II. PROBLEM STATEMENT

Take into account a heterogeneous MAS composed of N connected agents. The dynamics of agents are described by

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t),\\ y_i(t) &= C_i x_i(t),\end{aligned} \tag{1}$$

where $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times p_i}$, and $C_i \in \mathbb{R}^{q\times n_i}$ are the unknown accurate system matrix, input matrix, and output matrix, respectively. $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{p_i}$, and $y_i(t) \in \mathbb{R}^q$ represent the state, the control input, and the output state, respectively. The information interaction topology is modeled by a strongly connected digraph $\mathcal{G}$ with the associated weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}_+^{N\times N}$ and Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$. (Please refer to [2] for more information on graph theory).

In real applications, due to for example measurement or identification uncertainties, it might be difficult to obtain the accurate system model in advance to design event-triggered controllers and triggering functions. With this context, the scenario where only the inaccurate model $(\tilde{A}_i, \tilde{B}_i, \tilde{C}_i)$ can be obtained is considered here. Assume that $\tilde{A}_i = A_i + \Delta_{1i}$, $\tilde{B}_i = B_i + \Delta_{2i}$, and $\tilde{C}_i = C_i + \Delta_{3i}$ where Δ_{1i}, Δ_{2i}, and Δ_{3i} denote the unknown but bounded uncertainties between the unknown accurate system information and the obtained inaccurate system information satisfying that $\|\Delta_{1i}\|_\infty \le \alpha_1$, $\|\Delta_{2i}\|_\infty \le \alpha_2$, and $\|\Delta_{3i}\|_\infty \le \alpha_3$. Moreover, assume that the matrix pairs (A_i, B_i) and $(\tilde{A}_i, \tilde{B}_i)$ are stabilizable and the matrix pairs (C_i, A_i) and $(\tilde{C}_i, \tilde{A}_i)$ are detectable. The goal is to seek for a distributed ETOC control method that drives all agents to achieve practical output consensus with only the inaccurate system model information such that the output consensus error exponentially converges to a small adjustable bounded set, mathematically,

$$\lim_{t\to\infty} \|y_i(t) - y_j(t)\| \le \Omega_i,$$

where Ω_i is a small adjustable bounded set related to the mismatch level between accurate and inaccurate model.

Lemma 1 [23]: For a strongly connected digraph $\mathcal{G}$ with the Laplacian matrix $\mathcal{L}$, the general algebraic connectivity is defined as $a(\mathcal{L}) = \min_{x^T\omega=0, x\neq 0} \frac{x^T(\Omega\mathcal{L}+\mathcal{L}^T\Omega)x}{2x^T\Omega x}$, where $\Omega = \mathrm{diag}(\omega_1, \cdots, \omega_N)$ and $\omega = [\omega_1, \cdots, \omega_N]^T > 0$, satisfying $\omega^T\mathcal{L} = 0_N$ and $\sum_{i=1}^N \omega_i = 1$.

Lemma 2 [28]: $x(t) \to 0$ as $t \to +\infty$ if x and $\dot{x}$ are both bounded satisfying $\int_0^{+\infty} x^T(\tau)x(\tau)\mathrm{d}\tau < +\infty$.

III. MAIN RESULT

In this section, the following ETOC control method is adopted to achieve practical output consensus without accurate system model information in advance.

$$\dot{\varphi}_i(t) = S\varphi_i(t) - c_1Pz_i(t), \tag{2}$$
$$u_i(t) = c_2K_io_i(t) + \Gamma_i\varphi_i(t), \quad i = 1, 2, \ldots, N \tag{3}$$

where $S \in \mathbb{R}^{m\times m}$ is a matrix to be determined with $\sigma(S) \in j\mathbb{R}$, $\varphi_i(t) \in \mathbb{R}^m$ is the compensator state, $c_1 > 0.5/a(\mathcal{L})$ with $a(\mathcal{L})$ defined in Lemma 1, and $c_2 > 0.5$. $P \in \mathbb{R}^{m\times m} > 0$ is to be calculated and $K_i = -\tilde{B}_i^T\tilde{P}_i^{-1} \in \mathbb{R}^{p_i\times n_i}$ with $\tilde{P}_i > 0$ to be determined. $z_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}(\bar{\varphi}_i(t) - \bar{\varphi}_j(t))$ with $\bar{\varphi}_i(t) = e^{S(t-t_{k_i}^i)}\varphi_i(t_{k_i}^i)$, where a_{ij} is the ijth entry of the adjacency matrix $\mathcal{A}$, $\varphi_i(t_{k_i}^i)$ is the latest broadcast state of agent i with $t_{k_i}^i$, $k_i = 1, 2, \cdots$, being the latest triggering time. The tracking error $o_i(t)$ is defined as $o_i(t) = x_i(t) - \Pi_i\varphi_i(t)$. Additionally, the matrices Π_i and Γ_i satisfy that

$$\tilde{A}_i\Pi_i + \tilde{B}_i\Gamma_i = \Pi_iS, \quad \tilde{C}_i\Pi_i = R \tag{4}$$

and (S, R) is observable. By choosing S and R according to [25], one can solve equation (4) to get Π_i and Γ_i.

Remark 1: According to [25], (4) is solvable for any given compatible matrix R if and only if there exists a matrix S such that

$$\mathrm{rank}\left(\begin{bmatrix} \tilde{A}_i - \lambda I & \tilde{B}_i \\ \tilde{C}_i & \mathbf{0} \end{bmatrix}\right) = n_i + q, \quad \forall\lambda \in \sigma(S).$$

For each agent, the triggering error is defined as

$$e_i(t) = \bar{\varphi}_i(t) - \varphi_i(t). \tag{5}$$

A novel triggering condition for each agent is designed as

$$\|e_i(t)\|^2 \le \frac{\epsilon}{\upsilon_1}\|z_i(t)\|^2 + \frac{\theta_i^{\psi_i(t)}}{\upsilon_2}e^{-\gamma(t-t_{k_i}^i)}\eta_i, \tag{6}$$

where $0 < \epsilon < \frac{\upsilon^*}{2\|\mathcal{L}\|}$, $\upsilon_1 > \upsilon^*\|\mathcal{L}\|$, $\upsilon_2 > \upsilon^*$, $0 < \theta_i < 1$, $\gamma > 0$, and $\eta_i > 0$. $\upsilon^* = -\rho\kappa\lambda_{\min}(\Omega)(1 - \frac{1}{\varepsilon_1})\|M\|^2 + \frac{c_1}{\varepsilon_2}\lambda_{\max}(\mathcal{L}^T\Omega\mathcal{L}\otimes P^2) + 2c_1\|M^T\Omega\mathcal{L}\otimes P^2\|$ where $0 < \rho < 1$, $\kappa > 0$, $0 < \varepsilon_1 < 1$, $\varepsilon_2 > 0$, Ω is defined in Lemma 1, and $M = (I_N - 1_N\omega^T)$ with ω defined in Lemma 1. Additionally, the update law of $\psi_i(t)$ is designed as

$$\psi_i(t) = \begin{cases} \psi_i(t_{k_i}^i), & t \in [t_{k_i}^i, t_{k_i+1}^i) \\ \psi_i(t_{k_i}^i) + \varpi, & t = t_{k_i+1}^i \end{cases} \tag{7}$$

with $\psi_i(0) = 1$ as the initial condition and $\varpi > 0$.

To adequately establish a link connecting communication actions and state evolution, the event triggering condition (6) composed of the triggering error function $\|e_i(t)\|^2$ and triggering threshold $\frac{\epsilon}{\upsilon_1}\|z_i(t)\|^2 + \frac{\theta_i^{\psi_i(t)}}{\upsilon_2}e^{-\gamma(t-t_{k_i}^i)}\eta_i$ is designed. In the framework of an ETM, each agent independently decides to trigger an event by evaluating whether the triggering error exceeds the triggering threshold. The proposed ETOC control method works as follows. Each agent i monitors its own compensator state to decide its triggering by checking (6). Once (6) is violated, agent i triggers an event. Then, agent i updates $\varphi_i(t_{k_i}^i)$ in $z_i(t)$ according to its current state and broadcast it to its out-neighbors simultaneously. At the same time, the triggering error $e_i(t)$ is reset to zero. In addition, agent i also updates $\varphi_j(t_{k_j}^j)$ in $z_i(t)$ immediately when it receives new broadcast states from its in-neighbors.

Remark 2: Note that the control protocols designed in the framework of heterogeneous MAS, for example [24], [25], typically rely on the solutions of the regulator equations, where the accurate system matrices of MAS are used to solve the regulator equations. Thus, the protocols proposed in [24] and [25] might not work when the accurate system matrices cannot be obtained. Different from [24] and [25], the solutions of the regulator equations rely on the inaccurate system matrices in the proposed ETOC control method, which is more practical.

Remark 3: It is apparent that the value of $e^{-\gamma(t-t_{k_i}^i)}\eta_i$ is greater than $e^{-\gamma t}\eta_i$ between any two adjacent triggering instants due to the fact that $e^{-\gamma(t-t_{k_i}^i)}\eta_i$ is reset to η_i whenever an event is triggered. This novel design endows the triggering threshold a resilient feature, which help to improve the triggering performance and lengthen the inter-event time interval time, especially when approaching consensus.

Remark 4: The design of triggering threshold and the parameter selection therein might affect the consensus stability analysis within the framework of Lyapunov approaches and triggering performance. A monotone decreasing exponential function where no agents' states are involved is widely used to construct the triggering threshold in early works on event-triggered consensus control (e.g., [12], [20]). Whereas, the triggering performance might be influenced by the strictly time-dependent nature of this kind of state-independent triggering threshold since the network state evolution might be decoupled from the actual agents' states, which prompts the study on state-dependent triggering thresholds with using the broadcasted states of agent itself and its neighbors (e.g., [17], [18], [25]). Moreover, to further improve triggering performance, the triggering threshold mixed by the state-independent term and the state-dependent term is investigated (e.g., [14], [19]). On the other hand, the parameters selected in triggering thresholds could affect the triggering performance. Generally speaking, once ϵ/υ_1, θ_i, υ_2, and ϖ are determined, the triggering frequency would be lower by selecting larger η_i and smaller γ. Specifically, the update law (7) is designed to ensure the convergency of the resilient term in the triggering threshold in (6), which is important in the consensus stability analysis. Besides, ϖ can influence the speed of the threshold converging to zero. Concretely, the larger ϖ is, the faster the threshold would approach zero, which implies that the triggering frequency would be higher.

Remark 5: If one sets $\epsilon = 0$, $e^{-\gamma(t-t^i_{k_i})}$ as $e^{-\gamma t}$, and removes (7), the proposed threshold in (6) is degenerated into a typical state-independent threshold constructed by exponential-function proposed in [20]. In addition, the thresholds proposed in [17], [18], and citeHaikuoIJRNC can be viewed as special cases of the proposed threshold in (6) with η_i being zero.

Let $t_j^* = t^j_{k_j}$ be the latest triggering instant for agent j. Then, consider the time from t_i^* to t for all agents. Invoking (2) and (5), the closed-loop system of compensators is as follows.

$$\begin{aligned}\dot{\varphi}(t) &= (I_N \otimes S)\varphi(t) \\ &\quad - (c_1\mathcal{L} \otimes P)e^{(I_N\otimes S)[(I_N\otimes I_m)t-(T^*\otimes I_m)]}\varphi(t^*) \\ &= (I_N \otimes S - c_1\mathcal{L} \otimes P)\varphi(t) - (c_1\mathcal{L} \otimes P)e(t), \quad (8)\end{aligned}$$

where $\varphi(t) = [\varphi_1^T(t), \cdots, \varphi_N^T(t)]^T$, $\varphi(t^*) = [\varphi_1^T(t_1^*), \cdots, \varphi_N^T(t_N^*)]^T$, $T^* = \text{diag}(t_1^*, \cdots, t_N^*)$, and $e(t) = [e_1^T(t), \cdots, e_N^T(t)]^T$. The disagreement vector of compensators is defined as $\vartheta(t) = \varphi(t) - (1_N\omega^T \otimes I_m)\varphi(t)$. Note that $\vartheta(t) = (M \otimes I_m)\varphi(t)$. Revisiting (8), one has

$$\dot{\vartheta}(t) = (I_N \otimes S - c_1\mathcal{L} \otimes P)\vartheta(t) - (c_1\mathcal{L} \otimes P)e(t). \quad (9)$$

According to (1) - (4), the tracking error $o_i(t)$ satisfies

$$\begin{aligned}\dot{o}_i(t) &= A_i x_i(t) + c_2 B_i K_i o_i(t) + B_i\Gamma_i\varphi_i(t) \\ &\quad - (\tilde{A}_i\Pi_i + \tilde{B}_i\Gamma_i)\varphi_i(t) + c_1\Pi_i P z_i(t) \\ &= (\tilde{A}_i + c_2\tilde{B}_i K_i)o_i(t) - (\Delta_{1i} + c_2\Delta_{2i}K_i)o_i(t) \\ &\quad - (\Delta_{1i}\Pi_i + \Delta_{2i}\Gamma_i)\varphi_i(t) + c_1\Pi_i P z_i(t). \quad (10)\end{aligned}$$

Denote $o(t) = [o_1^T(t), \cdots, o_N^T(t)]^T$, $\tilde{\mathbb{A}} = \text{diag}\{\tilde{A}_1, \cdots, \tilde{A}_N\}$, $\tilde{\mathbb{B}} = \text{diag}\{\tilde{B}_1, \cdots, \tilde{B}_N\}$, $\mathbb{K} = \text{diag}\{K_1, \cdots, K_N\}$, $\tilde{\Delta}_1 = \text{diag}\{\Delta_{11} + c_2\Delta_{21}K_1, \cdots, \Delta_{1N} + c_2\Delta_{2N}K_N\}$, $\tilde{\Delta}_2 = \text{diag}\{\Delta_{11}\Pi_1 + \Delta_{21}\Gamma_1, \cdots, \Delta_{1N}\Pi_N + \Delta_{2N}\Gamma_N\}$, $\Pi = \text{diag}\{\Pi_1, \cdots, \Pi_N\}$. Thus, (10) can be rewritten as

$$\begin{aligned}\dot{o}(t) &= (\tilde{\mathbb{A}} + c_2\tilde{\mathbb{B}}\mathbb{K})o(t) - \tilde{\Delta}_1 o(t) - \tilde{\Delta}_2\varphi(t) \\ &\quad - c_1\Pi(I_N \otimes P)z(t). \quad (11)\end{aligned}$$

Thus, the ETOC of MAS (1) is converted to the stability problem of $\vartheta(t)$ and $o(t)$ under the proposed ETOC control method. In what follows, we present a multi-step algorithm for the proposed ETOC control method.

Algorithm 1: Given stabilizable $(\tilde{A}_i, \tilde{B}_i)$ and detectable $(\tilde{C}_i, \tilde{A}_i)$, the proposed ETOC control method can be constructed as follows.

1) Choose S and R by the algorithm proposed in [25]. Then, solve equation (4) to get Π_i and Γ_i.
2) Select $\kappa > 0$ and solve $PS + S^T P - P^2 \leq -\kappa I$ to get a solution P such that $-\rho\kappa\lambda_{\min}(\Omega)(1-\varepsilon_1) + c_1\varepsilon_2\lambda_{\max}(P^2) + \epsilon < 0$.
3) Select $\tilde{\kappa} > \kappa_*$ where κ_* is defined after (23). Then, solve the following linear matrix inequality $\tilde{P}_i^{-1}\tilde{A}_i + \tilde{A}_i^T\tilde{P}_i^{-1} - \tilde{P}_i^{-1}\tilde{B}_i\tilde{B}_i^T\tilde{P}_i^{-1} \leq -\tilde{\kappa}I$ to get the solution $\tilde{P}_i$. Thus, K_i can be calculated by $K_i = -\tilde{B}_i^T\tilde{P}_i^{-1}$.
4) Choose the remaining parameters.

Then, we are at the position for giving our main result.

Theorem 3: For MAS (1), under the proposed ETOC control method (2), (3), and (6), consensus of compensators and practical output consensus of agents are achieved if parameters are selected such that $-\rho\kappa\lambda_{\min}(\Omega)(1-\varepsilon_1) + c_1\varepsilon_2\lambda_{\max}(P^2) + \epsilon < 0$ and $\tilde{\kappa} > \kappa_*$ are satisfied. The output consensus error exponentially converges to a small adjustable bounded set (i.e., (25)). Furthermore, Zeno behavior is excluded.

Proof: Consider the following Lyapunov function for (9).

$$V = \vartheta^T(\Omega \otimes P)\vartheta. \quad (12)$$

By taking the time derivative of V along (9), it follows

$$\begin{aligned}\dot{V} &= 2\vartheta^T(\Omega \otimes P)\dot{\vartheta} \\ &= 2\vartheta^T(\Omega \otimes PS - c_1\Omega\mathcal{L} \otimes P^2)\vartheta - 2\vartheta^T(c_1\Omega\mathcal{L} \otimes P^2)e \\ &\leq \vartheta^T(\Omega\otimes(PS+S^TP-2c_1a(\mathcal{L})P^2))\vartheta - 2\vartheta^T(c_1\Omega\mathcal{L}\otimes P^2)e \\ &\leq -\kappa\lambda_{\min}(\Omega)\vartheta^T\vartheta - 2\vartheta^T(c_1\Omega\mathcal{L} \otimes P^2)e, \quad (13)\end{aligned}$$

where $PS + S^T P - 2c_1 a(\mathcal{L})P^2 < PS + S^T P - P^2 \leq -\kappa I$ and Lemma 1 have been used. Denote $\bar{\varphi} = [\bar{\varphi}_1^T, \cdots, \bar{\varphi}_N^T]^T$. By recalling the definition of ϑ, we denote $\bar{\vartheta} = (M \otimes I_m)\bar{\varphi}$. Then, one has that $\bar{\vartheta} = (M \otimes I_m)\bar{\varphi} = \vartheta + (M \otimes$

$I_m)e$. Based on this equation, bounding the first term in (13) yields

$$\begin{aligned}
&-\kappa\lambda_{\min}(\Omega)\vartheta^T\vartheta \\
&\quad= -\kappa\lambda_{\min}(\Omega)(\bar{\vartheta}^T\bar{\vartheta} - e^T(M^T\otimes I_m)\bar{\vartheta} - \bar{\vartheta}^T(M\otimes I_m)e \\
&\qquad + e^T(M^TM\otimes I_m)e) \\
&\quad= -\kappa\lambda_{\min}(\Omega)(\bar{\vartheta}^T\bar{\vartheta} - 2\bar{\vartheta}^T(M\otimes I_m)e + e^T(M^TM\otimes I_m)e) \\
&\quad\le -\kappa\lambda_{\min}(\Omega)[(1-\varepsilon_1)\bar{\vartheta}^T\bar{\vartheta} + (1-\frac{1}{\varepsilon_1})\|M\|^2\|e\|^2], \quad (14)
\end{aligned}$$

where $2\bar{\vartheta}^T(M\otimes I_m)e \le \varepsilon_1\bar{\vartheta}^T\bar{\vartheta} + \frac{1}{\varepsilon_1}e^T(M^TM\otimes I_m)e$ is used. Analyzing the second term in (13), we have

$$\begin{aligned}
&-2\vartheta^T(c_1\Omega\mathcal{L}\otimes P^2)e \\
&\quad= -2c_1\bar{\vartheta}^T(\Omega\mathcal{L}\otimes P^2)e + 2c_1e^T(M^T\Omega\mathcal{L}\otimes P^2)e \\
&\quad\le c_1\varepsilon_2\lambda_{\max}(P^2)\bar{\vartheta}^T\bar{\vartheta} + (\frac{c_1}{\varepsilon_2}\lambda_{\max}(\mathcal{L}^T\Omega^2\mathcal{L}\otimes P^2) \\
&\qquad + 2c_1\|M^T\Omega\mathcal{L}\otimes P^2\|)\|e\|^2. \quad (15)
\end{aligned}$$

Invoking (13), (14), and (15), it follows that $\dot{V}$ satisfies

$$\begin{aligned}
\dot{V} &\le (-\rho\kappa\lambda_{\min}(\Omega)(1-\varepsilon_1) + c_1\varepsilon_2\lambda_{\max}(P^2))\bar{\vartheta}^T\bar{\vartheta} \\
&\qquad - (1-\rho)\kappa\lambda_{\min}(\Omega)\vartheta^T\vartheta + \upsilon^*\|e\|^2. \quad (16)
\end{aligned}$$

Considering the relation $z = (\mathcal{L}\otimes I_n)\bar{\varphi} = (\mathcal{L}\otimes I_n)\bar{\vartheta}$ and $\|z\|^2 \le \|\mathcal{L}\|^2\|\bar{\vartheta}\|^2$, (16) can be rewritten as the following inequality (17) according to triggering condition (6).

$$\begin{aligned}
\dot{V} &\le (-\rho\kappa\lambda_{\min}(\Omega)(1-\varepsilon_1) + c_1\varepsilon_2\lambda_{\max}(P^2) + \epsilon)\|\bar{\vartheta}\|^2 \\
&\quad - (1-\rho)\kappa\lambda_{\min}(\Omega)\vartheta^T\vartheta + \sum_{i=1}^N \theta_i^{\psi_i(t)}e^{-\gamma(t-t^i_{k_i})}\eta_i, \\
&\le a^*\vartheta^T\vartheta + \sum_{i=1}^N \theta_i^{\psi_i(t)}e^{-\gamma(t-t^i_{k_i})}\eta_i, \quad (17)
\end{aligned}$$

where $a^* = (\rho-1)\kappa\lambda_{\min}(\Omega) < 0$. According to (17), one has

$$V(t) - V(0) \le \sum_{i=1}^N \int_0^t \theta_i^{\psi_i(s)}e^{-\gamma(s-t^i_{k_i})}\eta_i ds. \quad (18)$$

For simplicity, denote by $g(s) = \theta_i^{\psi_i(s)}e^{-\gamma(s-t^i_{k_i})}\eta_i$. According to the triggering law (6) and update law (7), it is noted that the value of $g(s)$ might jump to another value when agent i triggers an event, which implies that $g(s)$ is piecewise continuous. Hence, without loss of generality, suppose that an event for agent i is triggered at $t = t^i_{k_i}$. Thus,

$$\begin{aligned}
&\sum_{i=1}^N \int_0^t g(s)ds \\
&= \sum_{i=1}^N [\int_0^{t^i_1} g(s)ds + \cdots + \int_{t^i_{k_i(t)-1}}^{t^i_{k_i(t)}} g(s)ds + \int_{t^i_{k_i(t)}}^{t} g(s)ds] \\
&= -\frac{1}{\gamma}\sum_{i=1}^N\sum_{k_i=0}^{k_i(t)} \theta_i^{1+k_i\varpi}\eta_i(e^{-\gamma(t^i_{k_i+1}-t^i_{k_i})} - 1) \\
&\le \frac{1}{\gamma}\sum_{i=1}^N\sum_{k_i=0}^{k_i(t)} \theta_i^{1+k_i\varpi}\eta_i, \quad (19)
\end{aligned}$$

where $k_i(t)$ represents the number of triggered events for agent i before time t. Therefore, according to (18) and (19), it follows that $V(\infty) \le V(0) + \frac{1}{\gamma}\sum_{i=1}^N\sum_{k_i=0}^{\infty}\theta_i^{1+k_i\varpi}\eta_i$. It is apparent that $\frac{1}{\gamma}\sum_{i=1}^N\sum_{k_i=0}^{\infty}\theta_i^{1+k_i\varpi}\eta_i$ is bounded by recalling the designed update law (7). Therefore, V is bounded. Invoking (12), ϑ is bounded which implies that $(\mathcal{L}\otimes I_m)\varphi$ is also bounded due to the relation that $(\mathcal{L}\otimes I_m)\varphi = (\mathcal{L}\otimes I_m)\vartheta$. We denote a_* as the upper bound of $\|(\mathcal{L}\otimes I_m)\varphi\|$. Based on (5), we have $(\mathcal{L}\otimes I_m)\varphi = z - (\mathcal{L}\otimes I_m)e$. Therefore, $\|z\| - \|\mathcal{L}\|\|e\| \le a_*$. Then, by recalling the triggering condition (6), one can get

$$\begin{aligned}
\|z\|^2 &\le 2\|\mathcal{L}\|^2\|e\|^2 + 2a_*^2 \\
&\le 2\|\mathcal{L}\|^2(\frac{\epsilon}{\upsilon_1}\|z\|^2 + \sum_{i=1}^N \frac{\theta_i^{\psi_i(t)}}{\upsilon_2}e^{-\gamma(t-t^i_{k_i})}\varphi_i) + 2a_*^2 \\
&\le \frac{2\|\mathcal{L}\|\epsilon}{\upsilon^*}\|z\|^2 + \frac{2N\theta_{\max}\varphi_{\max}\|\mathcal{L}\|^2}{\upsilon_2} + 2a_*^2, \quad (20)
\end{aligned}$$

where $\theta_{\max} = \max\{\theta_i | i = 1,\cdots,N\}$ and $\eta_{\max} = \max\{\eta_i | i = 1,\cdots,N\}$. It follows from this inequality that $\|z\|^2 \le b_*$ with $b_* = \frac{\upsilon^*(2N\theta_{\max}\eta_{\max}\|\mathcal{L}\|^2 + 2a_*^2\upsilon_2)}{\upsilon_2(\upsilon^* - 2\|\mathcal{L}\|\epsilon)}$. Therefore, z is bounded. Besides, it is obvious that e is bounded. According to (9), one can have $\dot{\vartheta}$ is bounded. By above-mentioned analysis, $\sum_{i=1}^N\int_0^\infty \theta_i^{\psi_i(s)}e^{-\gamma(s-t^i_{k_i})}\eta_i ds$ is bounded and the upper bound is that $d_* = \frac{N\eta_{\max}\theta_{\max}}{\gamma(1-\theta_{\max}^{\varpi})}$. Besides, (17) enforces that

$$V(\infty) - V(0) \le a^*\int_0^\infty \vartheta(s)^T\vartheta(s)ds + d_*. \quad (21)$$

Therefore,

$$\int_0^\infty \vartheta(s)^T\vartheta(s)ds \le \frac{V(\infty) - V(0) - d_*}{a^*} < +\infty, \quad (22)$$

where we have used the fact that $V(\infty) - V(0) \le d_*$

Quoting Lemma 2, it follows from (22) that $\vartheta \to 0_{Nm}$ as $t \to \infty$. It enforces that $\varphi_i - \varphi_j \to 0_m$ as $t \to \infty$, which means that asymptotic consensus of compensators can be achieved.

Next, we will analysis the tracking error. Consider the Lyapunov function $W = o^T\tilde{\mathbb{P}}o$ for (11), where $\tilde{\mathbb{P}} = \text{diag}\{\tilde{P}_1^{-1},\cdots,\tilde{P}_N^{-1}\}$ with $\tilde{P}_i^{-1}$ being defined in the controller design. According to (11), one has

$$\begin{aligned}
\dot{W} &= 2o^T\tilde{\mathbb{P}}\dot{o} \\
&= o^T(\tilde{\mathbb{P}}\tilde{\mathbb{A}} + \tilde{\mathbb{A}}^T\tilde{\mathbb{P}} - 2c_2\tilde{\mathbb{P}}\tilde{\mathbb{B}}\tilde{\mathbb{B}}^T\tilde{\mathbb{P}})o - 2o^T\tilde{\mathbb{P}}\tilde{\Delta}_1 o \\
&\quad - 2o^T\tilde{\mathbb{P}}\tilde{\Delta}_2\varphi - 2c_1 o^T\Pi(I_N\otimes P)z \\
&\le (-\tilde{\kappa} + 2\|\tilde{\mathbb{P}}\tilde{\Delta}_1\| + \|\tilde{\mathbb{P}}\tilde{\Delta}_2\|^2)o^To + \varphi^T\varphi \\
&\quad - 2c_1 o^T\Pi(I_N\otimes P)z \\
&\le \zeta W + \varphi^T\varphi - 2c_1 o^T\Pi(I_N\otimes P)z, \quad (23)
\end{aligned}$$

where $\tilde{\mathbb{P}}\tilde{\mathbb{A}} + \tilde{\mathbb{A}}^T\tilde{\mathbb{P}} - 2c_2\tilde{\mathbb{P}}\tilde{\mathbb{B}}\tilde{\mathbb{B}}^T\tilde{\mathbb{P}} < \tilde{\mathbb{P}}\tilde{\mathbb{A}} + \tilde{\mathbb{A}}^T\tilde{\mathbb{P}} - \tilde{\mathbb{P}}\tilde{\mathbb{B}}\tilde{\mathbb{B}}^T\tilde{\mathbb{P}} \le -\tilde{\kappa}I$ has been used. Additionally, $\zeta = \frac{\kappa_* - \tilde{\kappa}}{\lambda_{\max}(\tilde{\mathbb{P}})}$ with

$\kappa_* = \tilde{n}(\alpha_1 + c_2\alpha_2\|K_i\|_{\max}) + (\tilde{n}(\alpha_1\|\Pi_i\|_{\max} + \alpha_2\|\Gamma_i\|_{\max}))^2$. Additionally, $\tilde{n} = \lambda_{\max}(\tilde{\mathbb{P}})\max\{\sqrt{n_i}|i = 1, \cdots, N\}$.

As mentioned above, when consensus of compensators is achieved, the evolution of φ is oscillatory according to (2) and the fact that $\sigma(S) \in j\mathbb{R}$, which implies that $\|\varphi\|^2$ is bounded with the upper bound being denoted as ϱ. Obviously, it follows from (23) that $\dot{W} \leq \zeta W + \varrho$ when consensus of compensators is achieved. Therefore, the tracking error o exponentially converges to the following bounded set:

$$\mathcal{S}_1 \triangleq \{o| \ o^T o \leq -\frac{\varrho}{\zeta\lambda_{\min}(\tilde{\mathbb{P}})}\}, \tag{24}$$

By recalling the definition of tracking error o_i, (4), and (24), it can be derived that the output consensus error $\|y_i - y_j\|$ approaches to the following bounded set:

$$\mathcal{S}_2 \triangleq \{y_i - y_j| \|y_i - y_j\| \leq \beta\}, \tag{25}$$

where $\beta = 2[(\|\tilde{C}_i\|_{\max} + \tilde{n}\alpha_3)\sqrt{\frac{-\varrho}{\zeta\lambda_{\min}(\tilde{\mathbb{P}})}} + \tilde{n}\alpha_3\|\Pi_i\|_{\max}\sqrt{\varrho}]$. Although the adjustable bounded set of the output consensus error seems complex, it is reasonable since it is affected by the mismatch level between accurate and inaccurate model.

Zeno behavior means that infinite number of events are triggered in a finite time interval [29]. Once the Zeno phenomenon occurs, the purpose of saving communication energy by avoiding continuous state transmission cannot be addressed even the stability of close-loop system cannot be guaranteed. Therefore, it is necessary to exclude the Zeno behavior. Then, the following analysis will rule out the Zeno behavior.

$e_i(t)$ is reset to zero once the event is triggered, which leads to a sudden change on the value of $z_i(t)$. Thus, it implies that the triggering of agent i would be affected by the events triggered at its neighbours. Then, to rule out the Zeno behavior, the following two scenarios are considered.

Scenario 1*:* Define $t^i_{k_i}$ and $t^i_{k_i+1}$ as the time instant of two consecutive triggered events. For $t \in [t^i_{k_i}, t^i_{k_i+1})$, assume that there is no event triggered at agent i's neighbors. Revisiting (2) and (5), it is apparent that $\dot{e}_i(t) = Se_i(t) + c_1Pz_i(t)$. Thus,

$$\begin{aligned}\frac{\mathrm{d}\|e_i(t)\|}{\mathrm{d}t} &= \frac{e_i^T(t)\dot{e}_i(t)}{\|e_i(t)\|} \\ &\leq \|\dot{e}_i(t)\| \leq \|S\|\|e_i(t)\| + c_1\|P\|\|z_i(t)\|. \end{aligned} \tag{26}$$

It should be pointed out that $z_i(t)$ is bounded based on the analysis above. Thus, we denote the upper bound of $c_1\|P\|\|z_i(t)\|$ as χ. In addition, $\chi = c_1\|P\|b_*$.

Consider the following nonnegative function

$$\dot{\nu}(t) = \|S\|\nu(t) + \chi, \quad \nu(0) = 0. \tag{27}$$

Invoking (26) and (27), it follows that $\|e_i(t)\| \leq \nu(t - t^i_{k_i})$ with $\nu(t)$ being the solution of (27). It is noted that $\nu(t) = \frac{b_*}{\|S\|}(e^{\|S\|t} - 1)$. According to the triggering condition (6), it is apparent that the inter-event time interval can be lower-bounded by the solution of $\frac{b_*^2}{\|S\|^2}(e^{\|S\|\tau^i_{k_i}} - 1)^2 \geq \frac{\epsilon}{\upsilon_1}\|z_i(t^i_{k_i} + \tau^i_{k_i})\|^2 + \frac{\theta_i^{\psi_i(t^i_{k_i}+\tau^i_{k_i})}}{\upsilon_2}e^{-\gamma(\tau^i_{k_i})}\eta_i$. Then, one has

$$\tau^i_{k_i} \geq \frac{1}{\|S\|}\ln\left(\frac{\|S\|}{b_*}\left(\frac{\epsilon}{\upsilon_1}\|z_i(t^i_{k_i} + \tau^i_{k_i})\|^2 + \frac{\theta_i^{\psi_i(t^i_{k_i}+\tau^i_{k_i})}}{\upsilon_2}e^{-\gamma\tau^i_{k_i}}\eta_i\right)^{\frac{1}{2}} + 1\right) \tag{28}$$

Let $t^i_{k_i+1} - t^i_{k_i}$ be the time-interval between agent i's two consecutive triggered events. Then, we have $t^i_{k_i+1} - t^i_{k_i} \geq \tau^i_{k_i} > 0$. Note that $\tau^i_{k_i}$ always exists and is strictly positive whenever consensus of compensators is not yet achieved. Therefore, the Zeno behavior is excluded in this scenario.

Scenario 2*:* For $t \in [t^i_{k_i}, t^i_{k_i+1})$, assume that there is at least one event being triggered at agent i's certain neighbor. In this scenario, there are three cases of the sudden change of the value of $\|z_i(t)\|$ to be discussed. The value of $\|z_i(t)\|$ increases, unchange, and decreases. The change in the former two cases will not induce the triggering at agent i. For the last case, event at agent i is triggered due to the triggering at its neighbours. To analyze the Zeno behavior, the extreme situation that all agents are triggered at the same time t_* due to its neighbors, which implies that $e_i(t_*)$ becomes zero for all agents. Therefore, a nonzero time interval is necessary for the triggering error of agent i to exceed its triggering threshold again, which means that the inter-event time interval between agent i' two consecutive events is strictly greater than zero based on (28).

Therefore, by the above analysis, the following conclusion can be obtained. Since $\tau^i_{k_i}$ approaches to zero only when $t \to \infty$, therefore, Zeno behavior can be excluded. Thus, this completes the whole proof. ∎

Corollary 4: Let $\sigma(S) \in \overline{\mathbb{C}}_-$. Under the conditions in Theorem 3, the output consensus error $\|y_i - y_j\|$ asymptotically converges to zero. In addition, if the accurate system information can be obtained which implies that there are no uncertainties, the output consensus error $\|y_i - y_j\|$ also asymptotically converges to zero.

Remark 6: Although the global information, like the Laplacian matrix $\mathcal{L}$ associated with digraphs, is involved into the calculation of the exact upper/lower bounds of the parameters to be selected, it should be emphasized that the global information is only used to determine the bounds of the parameters but not the parameters themselves. For practical applications, before running the proposed ETOC control method, the parameters can be selected more conservatively as long as the bounds are satisfied. The proposed ETOC control method itself is still distributed when it runs online.

Remark 7: The proposed ETOC control method depends on only each individual's local information including its local state and the broadcast states of itself and its neighbors. The potential requirement on continuous access to states of neighboring agents for agent's own triggering detection is no longer needed. As thus, the proposed ETOC control method

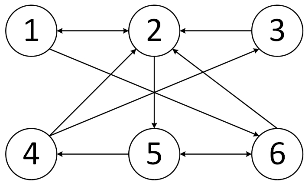

FIGURE 1. Communication topology of MASs.

can indeed reduce energy supply in practical applications by completely avoiding continuous state transmission in MASs.

Remark 8: Although the adjustable bounded set of the output consensus error seems complex, it is reasonable since it is affected by the mismatch level between accurate and inaccurate model.

IV. NUMERICAL SIMULATIONS

The numerical simulations are presented to verify the proposed ETOC control method. Consider a network composed of different mass-spring systems studied in [25]. The communication flow is modeled as the graph shown in Fig. 1, where the Laplacian matrix of the communication topology among the agents, namely $\mathcal{L}$, is given as follows.

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 2 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}.$$

The accurate system model (A_i, B_i, C_i) is unknown in advance and only the inaccurate system model $(\tilde{A}_i, \tilde{B}_i, \tilde{C}_i)$ can be obtained. Note that

$$A_1 = \begin{bmatrix} 0 & 1 \\ -4/3 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -4/3 & 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0 & 1 \\ -1.5 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 1 \\ -8/5 & 0 \end{bmatrix},$$
$$A_5 = \begin{bmatrix} 0 & 1 \\ -3 & 0 \end{bmatrix}, \quad A_6 = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \quad B_{2-6} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C_{1-6} = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Without loss of generality, assume that $\tilde{A}_1 = 1.5 * A_1, \tilde{A}_2 = 3 * A_2; \tilde{A}_3 = 1.2 * A_3, \tilde{A}_4 = A_4, \tilde{A}_5 = 0.9 * A_5, \tilde{A}_6 = 8 * A_6, \tilde{B}_{1-2,4-6} = [0; 1], \tilde{B}_3 = 3 * [0; 1]$, and $\tilde{C}_{1-6} = [0\ 1]$. According to Algorithm 1, one can calculate that

$$S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} 5.499 & 0 \\ 0 & 5.499 \end{bmatrix},$$
$$\tilde{P}_1 = \begin{bmatrix} 58.980 & 8.036 \\ 8.036 & 47.957 \end{bmatrix}, \quad \tilde{P}_2 = \begin{bmatrix} 58.980 & 8.036 \\ 8.036 & 47.957 \end{bmatrix},$$
$$\tilde{P}_3 = \begin{bmatrix} 60.751 & 7.734 \\ 7.734 & 45.090 \end{bmatrix}, \quad \tilde{P}_4 = \begin{bmatrix} 61.618 & 7.562 \\ 7.562 & 43.409 \end{bmatrix},$$
$$\tilde{P}_5 = \begin{bmatrix} 65.440 & 6.121 \\ 6.121 & 25.940 \end{bmatrix}, \quad \tilde{P}_6 = \begin{bmatrix} 64.090 & 7.045 \\ 7.045 & 37.329 \end{bmatrix}.$$

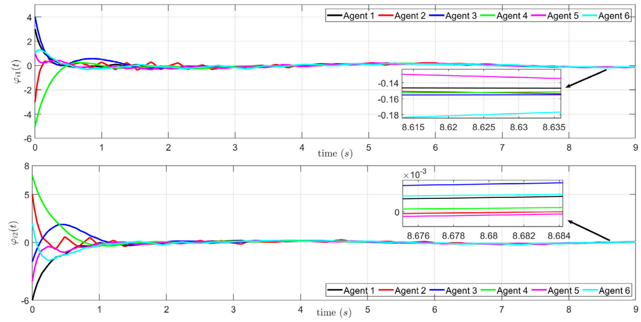

FIGURE 2. Profiles of compensators' states.

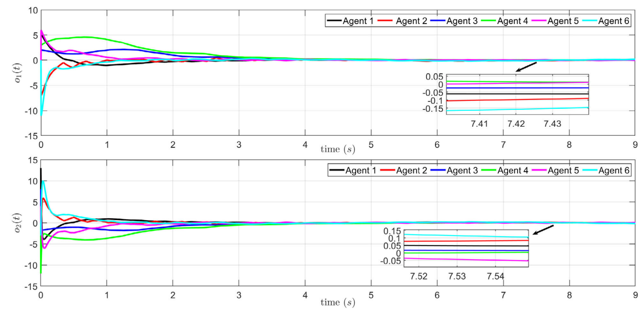

FIGURE 3. Profiles of tracking errors.

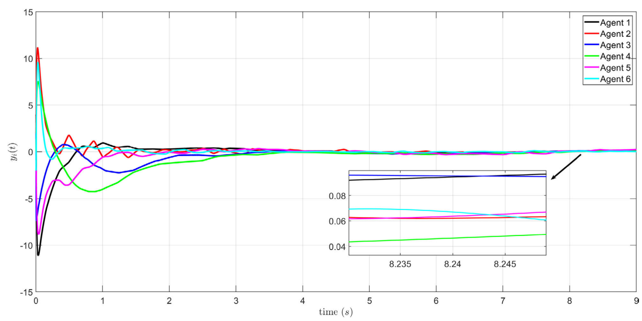

FIGURE 4. Profiles of outputs.

The parameters are selected as $c_1 = 0.5$, $c_2 = 10$, $\epsilon = 0.1$, $\upsilon_1 = 1200$, $\frac{\eta_{1,2,5}}{\upsilon_2} = 2$, $\frac{\eta_{3,4,6}}{\upsilon_2} = 3$, $\theta_{1-6} = 0.7$, $\varpi = 0.5$, and $\gamma = 2$. The initial states of compensators are given by

$$\varphi_1(0) = \begin{bmatrix} 3 \\ -6 \end{bmatrix}, \quad \varphi_2(0) = \begin{bmatrix} -3 \\ 5 \end{bmatrix}, \ \varphi_3(0) = \begin{bmatrix} 4 \\ -2 \end{bmatrix},$$
$$\varphi_4(0) = \begin{bmatrix} -5 \\ 7 \end{bmatrix}, \quad \varphi_5(0) = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \ \varphi_6(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

and the initial states of agents are given by

$$x_1(0) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \quad x_2(0) = \begin{bmatrix} -5 \\ 2 \end{bmatrix}, \ x_3(0) = \begin{bmatrix} 4 \\ 2 \end{bmatrix},$$
$$x_4(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \quad x_5(0) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \ x_6(0) = \begin{bmatrix} -8 \\ -2 \end{bmatrix}.$$

The profile of compensators' states is shown in Fig. 2, which implies that the consensus of compensators can be achieved. The tracking errors $o_i(t)$ is presented in Fig. 3. In Fig. 4 and

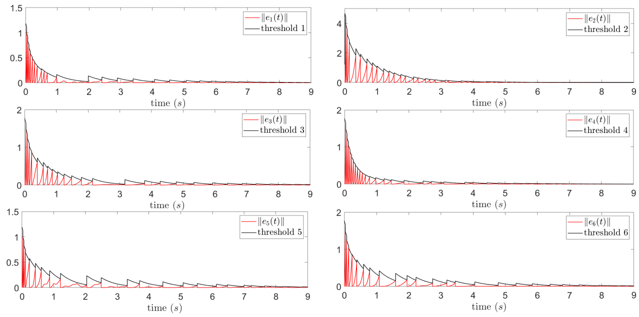

FIGURE 5. The triggering thresholds and triggering errors.

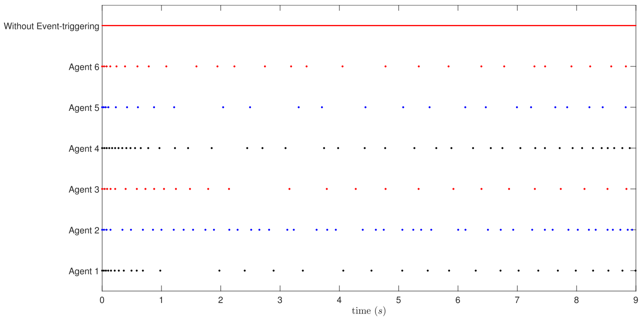

FIGURE 6. The triggering instants.

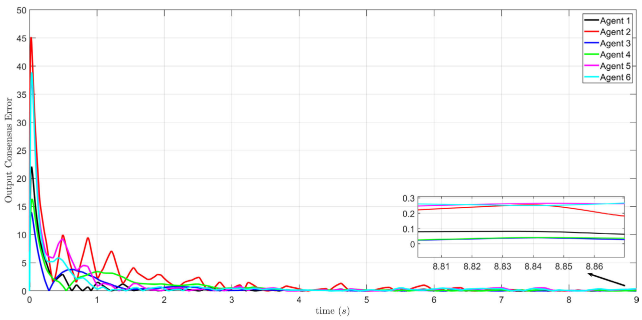

FIGURE 7. Output consensus errors of agents $\|\sum_{j\in\mathcal{N}_i} a_{ij}(y_i - y_j)\|$.

Fig. 7, the profiles of outputs and output consensus errors of agents are presented respectively, which shows the practical consensus of agents can be achieved. The triggering thresholds and triggering errors are shown in Fig. 5. It is observed that the piecewise-continuous thresholds approach zero with an overall decreasing tendency. The corresponding triggering instants are shown in Fig. 6. Evidently, the feature of communication resource saving can be observed from Fig. 6, where the number of performing communication actions is significantly reduced comparing with the case without ETM. Moreover, it can been seen from Fig. 6 that the Zeno behavior is excluded.

To further verify the effectiveness of the proposed resilient function in reducing triggering numbers, a comparison is carried out. Compared with the case where the state-independent term in (6) is replaced by the exponential-type threshold proposed in [20], the triggering numbers in 15s is reduced by 73.6% under the proposed ETOC control method. Therefore, the proposed ETOC control method can fulfill the output consensus task while possing an economic communication cost. Actually, the communication devices equipped on agents are powered by battery meanwhile possing small form factors, which implies that communication energy efficiency should be in consideration in applications, especially for MASs with large scales. As thus, the proposed ETOC control method might reduce the risk of wireless congestion in practical applications since the communication actions are no longer taken for granted and instead viewed as a scarce, globally shared resource.

V. CONCLUSION

Consider general linear heterogeneous MASs under digraphs, a distributed ETOC control method is proposed without continuous communication and accurate system model such that the output consensus error approaches to a small adjustable bounded set. To improve the triggering performance, a mixed threshold composed of a resilient state-independent term and a state-dependent one is constructed. It is shown that the proposed ETOC control method can reduce communication overheads greatly, which is beneficial for implementation of distributed consensus control protocols in practical applications since the communication resources of MASs including bandwidth and transmission rates are usually limited in practice. Moreover, to offer more feasibilities in practical applications, our future work will be focused on the fully distributed ETOC protocol design by extending the method studied in [19] into the case of directed graphs such that the global information can be completely avoided. Additionally, it is also interesting to explore the potential extension of the proposed ETOC control method to heterogeneous nonlinear MASs under more general fixed and switching topologies.

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XINGHAN LIN is currently pursuing the B.S. degree in intelligent manufacturing with the Beijing Technology and Business University, Beijing, China. His research interests include cooperation of multi-agents, artificial intelligence, control of intelligent devices, and 3D printing control. He was a recipient of the Second-Prize Winner of the National Physics Experiment Competition of China.

ZHIGANG HUANG received the M.S. degree in vibration impact noise from the Beijing Institute of Technology, Beijing, China, and the Ph.D. degree from China Agricultural University, Beijing, in 2004. He is currently a Professor with the School of Artificial Intelligence, Beijing Technology and Business University, Beijing. His research interest includes artificial intelligence.

KEYOU GUO received the Ph.D. degree from Jilin University, Changchun, China, in 2003. He is currently an Associate Professor with the School of Artificial Intelligence, Beijing Technology and Business University, Beijing, China. His research interests include deep learning and control of embedded systems.

GANG LI received the B.S. and Ph.D. degrees from China Agricultural University, Beijing, China, in 2009 and 2014, respectively. He is currently an Associate Professor with the School of Artificial Intelligence, Beijing Technology and Business University, Beijing. His research interests include artificial intelligence and energy engineering.

CHANGKUN DU (Member, IEEE) received the M.S. and Ph.D. degrees in control science and engineering from the Beijing Institute of Technology, China, in 2014 and 2019, respectively. From 2017 to 2018, he was a Visiting Scholar with the Department of Electrical and Computer Engineering, University of California at Riverside, Riverside, CA, USA. He is currently a Postdoctoral Researcher with the Beijing Institute of Technology. His research interests include multi-agent systems, event-triggered control, finite-time control, and control of connected vehicles.