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THEORY

Distributed Observer Approach to Cooperative Output Regulation of Multi-Agent Systems Without Exchange of Controller States

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ABSTRACT In this paper, the cooperative output regulation (COR) of heterogeneous linear multi-agent systems is studied. We propose a novel distributed observer approach that ensures synchronization of agents' outputs to a reference trajectory generated by a leader, while rejecting disturbance. A unified framework based on tools from $\mathcal{H}_\infty$ theory is established which allows treatment of both networks of non-introspective and networks of introspective agents. The proposed protocols do not require exchange of the controller states, which reduces or completely eliminates communication costs. A sufficient local stability condition is derived and a novel controller gain design method is provided to satisfy that condition. It is proven that the solvability of the COR problem can be guaranteed in advance for: i) introspective agents with arbitrary dynamics, ii) non-introspective agents with stable dynamics, under the assumption that the poles of exosystems lie on the imaginary axis. The effectiveness of the proposed approach is verified through numerical simulations.

INDEX TERMS Cooperative output regulation, leader-following consensus, observer-based approach, multi-agent systems (MASs), $\mathcal{H}_\infty$ static output feedback.

I. INTRODUCTION

During the past two decades, cooperative control of multi-agent systems (MASs) has received significant attention from researchers, see [1], [2], [3], [4], [5] and references therein. It has been demonstrated that in order to achieve collective goals, dynamic agents need to mutually interact, which leads to new theoretical challenges that require extension of the classical methods in control. In the process, the cooperative output regulation (COR) has stood out as an important problem, where the main challenge is to design a distributed control law such that agents asymptotically track the reference trajectory generated by a leader while simultaneously rejecting disturbances. A variety of cooperative control problems, such as leaderless synchronization, leader-following consensus, formation and containment control [5], [6], [7], [8], [9], [10], [11], [12] can be seen as a special case or extension of the COR problem.

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There are two well-known methods for tackling the COR problem for heterogeneous MASs. The first method is based on a distributed observer and relies on the assumption that a solution of the corresponding regulator equations exists. The pioneering work in this area was done in [13], where the dynamic compensator in form of a distributed observer was introduced. This work was extended in [14] to the output feedback case. The second method is based on the distributed internal model, which is more robust against plant parameters variation, but requires the transmission-zero condition to be satisfied [15]. Both design methods have been extensively investigated in recent years. For instance, in [16], the authors have analyzed global output regulation problem under the communication constraint of limited bandwidth. The COR problem under switching graphs and time-delays is considered in [17], [18], and [19], while adaptive protocols that solve the COR problem in the case when the model of a leader is not available to all agents can be found in [20] and [21].

All results in the area of cooperative control can be categorized in regard to information available to every agent in the network. More specifically, if agents possess some type of self-knowledge, such as measurement of their own state or output, then we refer to them as *introspective agents*. In this case, the possibilities of manipulating agents' internal dynamics are broad and various control schemes can be utilized. Design of protocols based on the full state information has been carried out in [13], [15], [18], [19], [20], and [21], while the protocols requiring only output information were addressed in [14], [16], and [22]. Unlike the introspective agents, if the agents in the network cannot measure their own state or output, then they are called *non-introspective agents* [23]. Therefore, the controller of each agent must be based only on relative measurements and information acquired through the communication channels from the neighboring agents in the network. In [24], the authors have proposed the neighborhood controller–neighborhood observer protocol for output synchronization of homogeneous networks, while the heterogeneous networks are considered in [23]. The COR problem for linear heterogeneous non-introspective agents is investigated in [25], and for nonlinear agents in [26].

In majority of the existing consensus and COR protocols [12], [13], [14], [16], [17], [18], [20], [21], the internal controller (observer) states are transmitted between agents via the communication network and used as inputs for the distributed observer. Some protocols, such as those in [7], [22], [23], and [25], even require the additional exchange of output or state measurements. Recently, significant research efforts have been devoted to developing consensus and COR protocols that rely solely on the exchange of output measurements. Such protocols can greatly reduce the communication burden since output measurements typically have a lower dimension than controller states [27], [28]. Additionally, when agents can measure their neighbors' relative output information, these protocols can be implemented without establishing the communication network [29], [30].

However, the limited exchange of information between agents gives rise to difficulties in analyzing the stability of the closed-loop system. The dynamics of the distributed observer becomes coupled with that of the heterogeneous multi-agent system, and as a consequence, the controller and observer gains cannot be independently designed. In [31] and [32], the authors developed a low-gain technique based on the small-gain condition to design a distributed observer for networks of introspective agents. Nonetheless, this method cannot guarantee the solvability of the COR problem in the presence of external disturbances, thus limiting its practical applications. More recently, the COR problem without controller state exchange was investigated in [33], and a stability condition based on the dynamics of the overall multi-agent system has been derived. However, the proposed design algorithm does not directly incorporate this condition into the design process. Instead, the controller and observer gains are independently calculated using Riccati equations, and the process is iteratively repeated until the stability condition is fulfilled. Lately, tools from $\mathcal{H}_\infty$ control theory have proven to be of great use for the design of the distributed internal-model-based protocols [34], [35], [36], while their potential in the design of distributed observer-based protocols is not yet fully exploited.

In networks of non-introspective agents, both agent and exosystem states need to be estimated in a distributed manner since local output measurements are not available. Consequently, designing distributed observers without knowledge of neighboring agents' controller states becomes an even more challenging task than it is in networks of introspective agents. In [27] and [28], the authors solve the COR problem for networks of heterogeneous agents with minimum-phase dynamics and identical relative degrees. The tracking problem in homogeneous MASs with general linear dynamics is solved in [29] and [30] by introducing the local observer that estimates synchronization error. Further, the heterogeneous MASs with general linear dynamics have been studied in [37], but no guarantees for the solvability of the COR problem by the proposed design method are established, even for the agents with minimum-phase dynamics.

Motivated by above discussion, we propose a novel distributed observer-based approach that solves the COR problem in heterogeneous linear MASs without requiring agents to exchange the internal controller states. Unlike other papers in the literature which address either the case of non-introspective or introspective agents, here we consider both cases in a unified framework based on tools from $\mathcal{H}_\infty$ control. An extensive solvability analysis is carried out from both the agents' and exosystem's perspective. Under the assumption that the poles of exosystems lie on the imaginary axis, it is proven that the solvability of the COR problem can be guaranteed in advance for: i) introspective agents with arbitrary dynamics, ii) non-introspective agents with stable dynamics.

The main contributions of the paper can be summarized as follows:

1) Contrary to the observer-type protocols [12], [13], [14], [16], [17], [18], [20], [21], [22], [23], [25], the approach proposed in this paper does not require the exchange of the controller states among the agents. Instead, only the output information needs to be shared, which considerably reduces the communication burden. Moreover, if the followers are equipped with sensors that provide them with relative output measurements of the neighboring agents, then the proposed protocols can be implemented without establishing a communication network, making a MAS more secure [30].
2) A novel controller design method that combines the advantages of parametric algebraic Riccati equations (ARE) and tools from $\mathcal{H}_\infty$ theory is devel-

oped. Compared to the low-gain method [31], [32], the proposed approach provides greater flexibility in designing the controller parameters, thus enabling better tuning of the system performance. Moreover, the solvability of the COR problem is guaranteed for introspective linear agents with arbitrary dynamics in the presence of disturbances, which extends results in [31] and [32]. Furthermore, contrary to [33], the stability condition in our paper depends on the individual dynamics of each agent, rather than the dynamics of the overall MAS.

3) In addition to introspective agents, a more challenging case of the COR problem in the networks of non-introspective agents is also considered. Compared to the protocols [29], [30] that are devised for homogeneous agents, in this paper the agents are allowed to be heterogeneous with a general linear dynamics. Furthermore, the existence of a solution to the COR problem is guaranteed for agents with poles in the closed left half-plane, which extends the results in [27] and [28], where heterogeneous agents are assumed to be minimum-phase with an identical relative degree.

The rest of the paper is organized as follows. In Section II, the preliminaries are given and the COR problem is formally stated. In Sections III and IV, COR protocols for non-introspective and introspective agents are proposed, respectively. Section V deals with extensive solvability analysis. Finally, numerical simulations and concluding remarks are given in Sections VI and VII, respectively.

Notation: I is an identity matrix with appropriate dimension. For a square matrix A, $\lambda(A)$ denotes its spectrum, and $\rho(A)$ its spectral radius. The absolute value of a matrix is defined as $|A| = [|a_{ij}|]$, where $A = [a_{ij}]$. Moreover, $A > 0$ (≥ 0) means that A is positive definite (semidefinite). The Kronecker product of two matrices A and B is denoted as $A \otimes B$. For a stable system $G(s)$, $\|G\|_\infty$ denotes its $\mathcal{H}_\infty$ norm, while $\|G(j\omega)\|$ denotes its largest singular value with respect to the frequency. The operator diag$(\cdot)$ builds a (block) diagonal matrix from its arguments.

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider a multi-agent system consisting of N linear heterogeneous agents described by the following dynamics:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i \omega \\ y_i &= C_i x_i + Q\omega, \; i = 1, \ldots, N, \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state, $u_i(t) \in \mathbb{R}^{m_i}$ is the control input, $y_i(t) \in \mathbb{R}^p$ is the output of the agent i, and $\omega(t) \in \mathbb{R}^{q_\omega}$ is the state of an exosystem Σ_ω that represents disturbance to be rejected. The disturbance is generated as follows

$$\Sigma_\omega : \dot{\omega} = P\omega, \tag{2}$$

where P is a constant matrix. Furthermore, suppose that the reference signal, denoted by $y_0(t)$, is generated by an exosystem

$$\Sigma_\nu : \begin{cases} \dot{\nu} = S\nu \\ y_0 = F\nu \end{cases}, \tag{3}$$

where $\nu(t) \in \mathbb{R}^{q_\nu}$ is the state, and $y_0(t) \in \mathbb{R}^p$ is the output of the exosystem Σ_ν. Note that the time index t has been dropped in the equations for sake of clarity.

The N agents with the dynamics (1), called the *followers*, and an agent described by the exosystem (3), termed the *leader*, can be represented by a node set $\mathcal{V} = \{0, 1, 2, \ldots, N\}$, with 0 corresponding to the leader. The interactions among the agents are modeled by an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where the ordered pair $(i, j) \in \mathcal{E}$ indicates the existence of a directed link from node i to node j. In such case, we say that the node i is a neighbor of the node j. The set of all neighbors of the node i is denoted by $\mathcal{N}_i$. An adjacency matrix associated with a directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is denoted as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$, where $a_{ij} > 0$ for $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Without loss of generality, we assume $\sum_{j=0}^N a_{ij} = 1, \; i = 0, \ldots, N$. Then, Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is defined as $\mathcal{L} = I - \mathcal{A}$. The Laplacian and adjacency matrices can be partitioned in the following way

$$\mathcal{A} = \begin{bmatrix} 1 & 0 \\ a_0 & \bar{\mathcal{A}} \end{bmatrix}, \; \mathcal{L} = \begin{bmatrix} 0 & 0 \\ -a_0 & \bar{\mathcal{L}} \end{bmatrix}, \tag{4}$$

where $a_0 = [a_{10}, a_{20}, \ldots, a_{N0}]^T$.

We assume that each follower has access to relative outputs of the neighboring agents, thus obtaining the quantity $\zeta_i = \sum_{j\in\mathcal{N}_i} a_{ij}\left(y_i - y_j\right)$ which can be written in terms of the elements of the Laplacian matrix as $\zeta_i = \sum_{j=0}^N l_{ij} y_j, \; i = 1, \ldots, N$. The regulated output for each agent is defined as follows:

$$\tilde{y}_i = y_i - y_0, \; i = 1, \ldots, N. \tag{5}$$

The main objective is to design a distributed dynamic control law that drives the regulated output to zero. Based on the information available to each agent we will distinguish the following two cases.

In the first case, we assume that the agents are non-introspective, i.e. they can only measure relative output of the neighboring agents. The goal is to design a distributed dynamic relative output feedback (ROF) control law, i.e. a control law based on a linear combination of relative output measurements $\zeta_i = \sum_{j=0}^N l_{ij} y_j, i = 1, \ldots, N$. In the second case, it is assumed that agents are introspective, i.e. each agent can measure its output signal in absolute (global) coordinates y_i. In this case, we distinguish the following two scenarios. The former requires exchange of y_i through communication channels among the neighboring agents, while the latter assumes that the introspective agents can also measure the relative outputs, thus eliminating the need for communication. The control law for this case, covering both scenarios, will be called the distributed dynamic output feedback (OF) protocol.

The cooperative output regulation (COR) problem can then be stated as follows.

Problem (COR): For the multi-agent system composed of (1), (2), and (3), design ROF (OF) controller such that the closed-loop system satisfies the following the conditions:

1) The origin of the overall closed-loop system is asymptotically stable when $\omega = 0$ and $\nu = 0$.
2) For any initial conditions $\nu(0)$, $\omega(0)$, $x_i(0)$, the regulated output satisfies $\lim_{t\to\infty} \tilde{y}_i(t) = 0$, $i = 1, \ldots, N$.

In order to solve the COR Problem, we will formalize the required assumptions:

Assumption 1: The digraph $\mathcal{G}$ contains a directed spanning tree with node 0 as its root.

Assumption 2: The matrix S has no strictly stable poles.

Assumption 3: The pairs (A_i, B_i), $i = 1, \ldots, N$, are stabilizable.

Assumption 4.1: For the ROF protocol, the pairs

$$\left(\begin{bmatrix} -C_i & -Q & F \end{bmatrix}, \begin{bmatrix} A_i & E_i & 0 \\ 0 & P & 0 \\ 0 & 0 & S \end{bmatrix}\right), \quad i = 1, \ldots, N$$

are detectable.

Assumption 4.2: For the OF protocol, the pairs

$$(F, S), \quad \left(\begin{bmatrix} C_i & Q \end{bmatrix}, \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix}\right), \quad i = 1, \ldots, N,$$

are detectable.

Assumption 5: The linear matrix equations

$$\begin{cases} \Pi_i^\omega P = A_i \Pi_i^\omega + B_i \Gamma_i^\omega + E_i \\ 0 = C_i \Pi_i^\omega + Q \end{cases}, \tag{6a}$$

$$\begin{cases} \Pi_i^\nu S = A_i \Pi_i^\nu + B_i \Gamma_i^\nu \\ 0 = C_i \Pi_i^\nu - F \end{cases}, \tag{6b}$$

have solution pairs $(\Pi_i^\omega, \Gamma_i^\omega)$ and $(\Pi_i^\nu, \Gamma_i^\nu)$ for $i = 1, \ldots, N$, respectively.

Remark 1: Assumption 2 is introduced to avoid the trivial case of strictly stable S, as eigenvalues with negative real parts exponentially decay to zero and do not affect the asymptotic behavior of the closed-loop system. Furthermore, if the COR problem is solved for a linear MAS under Assumption 4.2, then it is also solved when this assumption is violated, as stated in [38]. Assumptions 3-5 are standard in the cooperative output regulation literature [13]. The equations (6) are known as regulator equations, whose solvability is a necessary condition for solving the classical output regulation problem [38]. For the OF protocol, Assumption 4.2 can be ensured under mild conditions if the pairs (C_i, A_i) and (Q, P) are detectable. When there is no disturbance acting on the plant output (i.e., $Q = 0$), Assumption 4.2 reduces to the detectability of the pair $\left(\begin{bmatrix} C_i & 0 \end{bmatrix}, \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix}\right)$, which is always detectable if the pairs (C_i, A_i) and (E_i, A_i) are detectable [39]. For the ROF protocol, Assumption 4.1 is always ensured if the pairs $\left(\begin{bmatrix} C_i & 0 \end{bmatrix}, \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix}\right)$ and (F, S) are detectable, and matrices S and P have no common eigenvalues. If S and P have common eigenvalues, Assumption 4.1 does not hold for single-output agents, while for multi-output agents it can be ensured under mild conditions [40].

Remark 2: It is worth noting that, similar to works [17], [22], [33], we consider separate exogenous systems to model the disturbance and reference signal. The disturbance ω is assumed to be unmeasurable for all agents, while at least one agent has knowledge of the reference signal y_0. However, as noted in Remark 1, when exosystems S and P have common eigenvalues and the agents have a single output, Assumption 4.1 will not hold. A possible solution to handle this issue is simply to exclude the common eigenvalues from matrix P, as suggested in [33]. An alternative approach in the literature is to model disturbances and reference signals using a single exosystem such as in [13], [14], [15], and [16]. In this case, at least one agent must have access to the complete exosystem state, including disturbances. It is important to highlight that all the results obtained for the ROF protocol are equally applicable to this scenario. Nevertheless, for the sake of ensuring a straightforward parallel between the ROF and OF protocols, we have adopted the same approach for both introspective and non-introspective agents.

Prior to presenting the main results, we provide a lemma regarding the spectral radius of the matrix $|\mu I - \bar{\mathcal{L}}|$, where $\mu \in \mathbb{R}$. This matrix plays a crucial role in the stability analysis of MAS.

Lemma 1: Consider the Laplacian and adjacency matrix partition (4). Then, for any real scalar μ, $\rho\left(|\mu I - \bar{\mathcal{L}}|\right) = |\mu - 1| + \rho\left(\bar{\mathcal{A}}\right)$. Moreover, $\rho\left(\bar{\mathcal{A}}\right) < 1$ if and only if the digraph $\mathcal{G}$ associated with the adjacency matrix $\mathcal{A}$ contains a directed spanning tree with node 0 as a root.

Proof: In order to prove the first statement of the lemma, note that $\bar{\mathcal{L}} = I - \bar{\mathcal{A}}$. Since $\bar{\mathcal{A}}$ is a non-negative matrix with zeros on the main diagonal, it is straightforward to show that $|\mu I - \bar{\mathcal{L}}| = |\mu - 1| I + \bar{\mathcal{A}}$. Therefore, $\lambda_i\left(|\mu I - \bar{\mathcal{L}}|\right) = \lambda_i\left(|\mu - 1| I + \bar{\mathcal{A}}\right) = |\mu - 1| + \lambda_i\left(\bar{\mathcal{A}}\right)$, $i = 1, \ldots, N$. Moreover, according to Perron-Frobenius theorem, the non-negativity of $\bar{\mathcal{A}}$ implies that $\rho(\bar{\mathcal{A}})$ is its eigenvalue, from which follows $\rho(|\mu I - \bar{\mathcal{L}}|) = |\mu - 1| + \rho(\bar{\mathcal{A}})$. This completes the proof of the first statement.

For the second statement, the structure of $\mathcal{A}$ implies that it contains the eigenvalue 1 in addition to the eigenvalues of $\bar{\mathcal{A}}$. The matrix $\mathcal{A}$ is row-stochastic, thus according to (*Lemma* 3.4, [1]), it has a simple eigenvalue $\rho(\mathcal{A}) = 1$ if and only if the digraph $\mathcal{G}$ contains a directed spanning tree with node 0 as the root. Therefore, $\rho(\bar{\mathcal{A}}) < 1$. □

III. COR IN NETWORKS OF NON-INTROSPECTIVE AGENTS

In this section, the distributed ROF controller is first introduced, which is followed by a discussion on the stability analysis and controller design procedure.

Consider the following ROF controller

$$\begin{bmatrix} \dot{\hat{x}}_i \\ \dot{\hat{\omega}}_i \\ \dot{\hat{\nu}}_i \end{bmatrix} = \begin{bmatrix} A_i & E_i & 0 \\ 0 & P & 0 \\ 0 & 0 & S \end{bmatrix} \begin{bmatrix} \hat{x}_i \\ \hat{\omega}_i \\ \hat{\nu}_i \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix} u_i + \begin{bmatrix} L_i^x \\ L_i^\omega \\ L_i^\nu \end{bmatrix} \varepsilon_i,$$
$$u_i = K_i^x \hat{x}_i + K_i^\omega \hat{\omega}_i + K_i^\nu \hat{\nu}_i, \; i = 1, \ldots, N, \tag{7}$$

where $\hat{x}_i \in \mathbb{R}^{n_i}$, $\hat{\omega}_i \in \mathbb{R}^{q_\omega}$ and $\hat{\nu}_i \in \mathbb{R}^{q_\nu}$ are local estimates of x_i, ω and ν, respectively. The term $\varepsilon_i \in \mathbb{R}^p$ represents the virtual error signal defined as

$$\varepsilon_i \triangleq \sum_{j=0}^{N} l_{ij} y_j - \mu(C_i\hat{x}_i + Q\hat{\omega}_i - F\hat{\nu}_i),\ i = 1, \dots, N, \quad (8)$$

where μ is a real scalar. The first equation in (7) can be viewed as a distributed observer of the system and exosystems states. The observer gains L_i^x, L_i^ω, L_i^ν and the control law gains K_i^x, K_i^ω, K_i^ν are the parameters to be designed.

Remark 3: In addition to the relative measurements, the virtual error signal contains a term $\mu(C_i\hat{x}_i + Q\hat{\omega}_i - F\hat{\nu}_i)$ that depends on the local estimates of the system and exosystems states. This implies that the implementation of controller (7) does not require communication among agents. It should be noted that the added term is crucial for the stabilization of MAS, which will be shown later.

Let K_i^ω and K_i^ν be designed as follows:

$$K_i^\omega = \Gamma_i^\omega - K_i^x\Pi_i^\omega,\ K_i^\nu = \Gamma_i^\nu - K_i^x\Pi_i^\nu. \quad (9)$$

Define the tracking and estimation errors as $e_i = x_i - \Pi_i^\omega\omega - \Pi_i^\nu\nu$, $\tilde{x}_i = \hat{x}_i - x_i$, $\tilde{\omega}_i = \hat{\omega}_i - \omega$, $\tilde{\nu}_i = \hat{\nu}_i - \nu$. By taking into account the regulator equations (6), the error dynamics of each subsystem can be written as

$$\begin{bmatrix} \dot{e}_i \\ \dot{\tilde{x}}_i \\ \dot{\tilde{\omega}}_i \\ \dot{\tilde{\nu}}_i \end{bmatrix} = \begin{bmatrix} A_i+B_iK_i^x & B_iK_i^x & B_iK_i^\omega & B_iK_i^\nu \\ 0 & A_i & E_i & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & S \end{bmatrix} \begin{bmatrix} e_i \\ \tilde{x}_i \\ \tilde{\omega}_i \\ \tilde{\nu}_i \end{bmatrix} + \begin{bmatrix} 0 \\ L_i^x \\ L_i^\omega \\ L_i^\nu \end{bmatrix} \varepsilon_i.$$

Furthermore, let $\tilde{e}_i = \begin{bmatrix} \tilde{x}_i^T & \tilde{\omega}_i^T & \tilde{\nu}_i^T \end{bmatrix}^T$ and define variable $\xi_i = \sum_{j=1}^N l_{ij}C_je_j - \mu C_ie_i$. Then, ε_i can be expressed as $\varepsilon_i = \xi_i + \mu\begin{bmatrix} -C_i & -Q & F \end{bmatrix}\tilde{e}_i$. Introduce the following matrices

$$H_i = \begin{bmatrix} A_i & E_i & 0 \\ 0 & P & 0 \\ 0 & 0 & S \end{bmatrix},\ G_i = \begin{bmatrix} -C_i & -Q & F \end{bmatrix},$$

$$K_i = \begin{bmatrix} K_i^x & K_i^\omega & K_i^\nu \end{bmatrix},\ L_i = \begin{bmatrix} L_i^x \\ L_i^\omega \\ L_i^\nu \end{bmatrix},$$

which in turn gives the closed-loop dynamics

$$\begin{bmatrix} \dot{e}_i \\ \dot{\tilde{e}}_i \end{bmatrix} = \begin{bmatrix} A_i + B_iK_i^x & B_iK_i \\ 0 & H_i + \mu L_iG_i \end{bmatrix} \begin{bmatrix} e_i \\ \tilde{e}_i \end{bmatrix} + \begin{bmatrix} 0 \\ L_i \end{bmatrix} \xi_i. \quad (10)$$

Denote

$$\begin{aligned} \phi &= col\{\phi_i\},\ (\phi_i = e_i, \tilde{e}_i, \xi_i) \\ \Phi &= diag\{\Phi_i\},\ (\Phi_i = A_i, B_i, C_i, H_i, G_i, K_i^x, K_i, L_i) \\ \tilde{\mathcal{L}} &= \bar{\mathcal{L}} \otimes I_p,, \end{aligned} \quad (11)$$

then the overall system dynamics becomes $\begin{bmatrix} \dot{e} \\ \dot{\tilde{e}} \end{bmatrix} = A_{CL}\begin{bmatrix} e \\ \tilde{e} \end{bmatrix}$, where the closed-loop state matrix is

$$A_{CL} = \begin{bmatrix} A + BK^x & BK \\ L(\tilde{\mathcal{L}} - \mu I)C & H + \mu LG \end{bmatrix}. \quad (12)$$

Note that due to (6b), $\tilde{y}_i$ can be written as $\tilde{y}_i = C_ix_i + Q\omega - F\nu = C_ie_i$. It can be concluded that $\lim_{t\to\infty} e_i(t) = 0$ implies $\lim_{t\to\infty} \tilde{y}_i(t) = 0$. Therefore, ensuring that A_{CL} is Hurwitz stable is equivalent to solving the COR problem.

A. STABILITY ANALYSIS

In this subsection, we derive local stability conditions that, if satisfied, ensure the stability of the closed-loop matrix A_{CL}.

Define the following matrices

$$\hat{A}_i = \begin{bmatrix} A_i + B_iK_i^x & B_iK_i \\ 0 & H_i + \mu L_iG_i \end{bmatrix},\ \hat{B}_i = \begin{bmatrix} 0 \\ L_i \end{bmatrix},\ \hat{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}, \quad (13)$$

with the corresponding transfer function

$$T_i(s) = \hat{C}_i(sI - \hat{A}_i)^{-1}\hat{B}_i,\ i = 1, \dots, N. \quad (14)$$

Theorem 1: Consider a multi-agent system composed of (1), (2) and (3). Then, under the Assumptions 1-5, the ROF protocol (7) solves the COR problem if the following condition holds

$$\|T_i\|_\infty < \gamma^*,\ i = 1, \dots, N, \quad (15)$$

where $\gamma^* = \frac{1}{\rho(|\mu I - \bar{\mathcal{L}}|)}$.

Proof: The closed-loop system matrix A_{CL} in (12) can be rewritten as $A_{CL} = \hat{A} + \hat{B}(\tilde{\mathcal{L}} - \mu I)\hat{C}$, where

$$\hat{A} = \begin{bmatrix} A + BK^x & BK \\ 0 & H + \mu LG \end{bmatrix},\ \hat{B} = \begin{bmatrix} 0 \\ L \end{bmatrix},\ \hat{C} = \begin{bmatrix} C & 0 \end{bmatrix}. \quad (16)$$

Assume that $\hat{A}$ is Hurwitz, which can always be achieved under the Assumptions 3 and 4.1. Then, the matrix determinant lemma gives

$$\begin{aligned} &\det(sI - A_{CL}) \\ &\quad = \det(sI - \hat{A})\det(I - (sI - \hat{A})^{-1}\hat{B}(\tilde{\mathcal{L}} - \mu I)\hat{C}) \\ &\quad = \det(sI - \hat{A})\det(I + (\mu I - \tilde{\mathcal{L}})\hat{C}(sI - \hat{A})^{-1}\hat{B}). \end{aligned}$$

Note that A_{CL} is Hurwitz stable if $\det(sI - A_{CL}) \neq 0,\ \forall s \in \bar{\mathbb{C}}^+$. This means that the matrix $I + (\mu I - \tilde{\mathcal{L}})\hat{C}(sI - \hat{A})^{-1}\hat{B}$ must not have zero eigenvalues for any $s \in \bar{\mathbb{C}}^+$, otherwise its determinant will be equal to zero. Therefore, A_{CL} is stable if

$$\rho((\mu I - \tilde{\mathcal{L}})\hat{C}(sI - \hat{A})^{-1}\hat{B}) < 1,\ \forall s \in \bar{\mathbb{C}}^+. \quad (17)$$

Taking (13) and (16) into account, the transfer function $\hat{C}(sI - \hat{A})^{-1}\hat{B}$ can be expressed as

$$\begin{aligned} &\hat{C}(sI - \hat{A})^{-1}\hat{B} \\ &= C(sI - (A + BK^x))^{-1}BK(sI - (H + \mu LG))^{-1}L \\ &= diag(C_i(sI - (A_i + B_iK_i^x))^{-1}B_iK_i(sI - (H_i + \mu L_iG_i))^{-1}L_i) \\ &= \mathrm{diag}(T_i(s)). \end{aligned}$$

Furthermore, by using the block-norm matrix inequality [34] we can write

$$\rho((\mu I - \tilde{\mathcal{L}})\mathrm{diag}(T_i(s))) \le \rho(|\mu I - \bar{\mathcal{L}}|\mathrm{diag}(\|T_i\|_\infty)).$$

According to Lemma 8 in [34], the inequality $\rho((\mu I - \bar{\mathcal{L}})\,\mathrm{diag}(\|T_i\|_\infty)) \leq \rho(|\mu I - \bar{\mathcal{L}}|)\max_i \|T_i\|_\infty$ *holds, which allows the stability condition to be written as*

$$\rho(|\mu I - \bar{\mathcal{L}}|)\max_i \|T_i\|_\infty < 1.$$

The above condition is equivalent to the condition (15), which completes the proof. □

Remark 4: It should be noted that the proposed controller relies on knowledge of a global information and is not fully distributed. Actually, most of the existing protocols, for instance [28], [31], [32], [34], [35], [36], also rely on knowledge of global information. It is worth noting that in many practical situations, it is possible to know or predict the lower bound of a spectral radius of $|\mu I - \bar{\mathcal{L}}|$. *One possible approach to develop a fully distributed protocol is by employing adaptive gain methods, as demonstrated in [2] and [29].*

In the following lemma, we establish the lower bound for $\|T_i\|_\infty$.

Lemma 2: Suppose that the transfer function $T_i(s)$ *is stable. Then,* μ^{-1} *is a lower bound of* $\|T_i\|_\infty$, *i.e.*

$$\|T_i\|_\infty \geq \frac{1}{\mu},\ i = 1, \ldots, N. \tag{18}$$

Proof: Introduce the following coordinate transformation matrix

$$M_i = \begin{bmatrix} I & I & -\Pi_i^\omega & -\Pi_i^\nu \\ 0 & I & -\Pi_i^\omega & -\Pi_i^\nu \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

In the new state coordinates, the systems matrices become $\check{A}_i = M_i\hat{A}_iM_i^{-1}$, $\check{B}_i = M_i\hat{B}_i$ and $\check{C}_i = \hat{C}_iM_i^{-1}$. Taking into account (9), it can be shown that these matrices are

$$\check{A}_i = \begin{bmatrix} A_i + B_iK_i^x & -\mu\,[\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]\,L_iC_i & 0 & 0 \\ 0 & A_i - \mu\,[\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]\,L_iC_i & -B_i\Gamma_i^\omega & -B_i\Gamma_i^\nu \\ 0 & -\mu L_i^\omega C_i & P & 0 \\ 0 & -\mu L_i^\nu C_i & 0 & S \end{bmatrix},$$

$$\check{B}_i = \begin{bmatrix} [\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]L_i \\ [\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]L_i \\ L_i^\omega \\ L_i^\nu \end{bmatrix},\ \check{C}_i = \begin{bmatrix} C_i & -C_i & 0 & 0 \end{bmatrix}. \tag{19}$$

Suppose that $\|T_i\|_\infty < \mu^{-1}$ holds. Then, by the means of the small-gain theorem, the feedback controller $\check{u}_i = -\mu\check{y}_i$ stabilizes the system (19). The resulting state matrix is then equal to

$$\check{A}_i - \mu\check{B}_i\check{C}_i = \begin{bmatrix} A_i + B_iK_i^x - \mu[\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]L_iC_i & 0 & 0 & 0 \\ -\mu[\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]L_iC_i & A_i & -B_i\Gamma_i^\omega & -B_i\Gamma_i^\nu \\ -\mu L_i^\omega C_i & 0 & P & 0 \\ -\mu L_i^\nu C_i & 0 & 0 & S \end{bmatrix}.$$

However, since $\lambda(\check{A}_i - \mu\check{B}_i\check{C}_i) = \lambda(A_i) \cup \lambda(P) \cup \lambda(S) \cup \lambda(A_i + B_iK_i^x - \mu(L_i^x - \Pi_i^\omega L_i^\omega - \Pi_i^\nu L_i^\nu)C_i)$, it can be concluded that it is impossible to stabilize the system with the controller $\check{u}_i = -\mu\check{y}_i$. Thus, (18) must hold. □

So far we have not justified the necessity of the Assumption 1. This will be done in the following corollary.

Corollary 1: The stability condition (15) can be satisfied if and only if Assumption 4.1 holds and

$$\mu > \frac{1 + \rho(\bar{\mathcal{A}})}{2}. \tag{20}$$

Proof: In order for the condition (15) to be satisfiable, it is clear that γ^* must be greater than the lower bound of $\|T_i\|_\infty$, i.e. the following must hold

$$\rho(|\mu I - \bar{\mathcal{L}}|) < \mu. \tag{21}$$

If part: According to *Lemma 1*, if Assumption 4.1 holds, then $\rho\left(\left|\mu I - \bar{\mathcal{L}}\right|\right) = |\mu - 1| + \rho\left(\bar{\mathcal{A}}\right)$ and $\rho(\bar{\mathcal{A}}) < 1$. Therefore, (21) can be rewritten as $|\mu - 1| + \rho(\bar{\mathcal{A}}) < \mu$. It can be easily checked that this inequality holds for any μ that satisfies (20).

Only if part: Suppose that Assumption 4.1 does not hold. Then, from *Lemma 1* it follows that $\rho(\bar{\mathcal{A}}) = 1$. Therefore, $\rho\left(\left|\mu I - \bar{\mathcal{L}}\right|\right) = |\mu - 1| + 1$, which is always greater or equal to μ. □

B. CONTROL LAW SYNTHESIS

Generally, it is difficult to find K_i and L_i such that $\|T_i\|_\infty < \gamma^*$, since the gain matrix L_i is embedded in both the state and input matrix, thus a general $\mathcal{H}_\infty$ design method cannot be applied. Instead, we will first determine the gain L_i such that $H_i + \mu L_iG_i$ is stable. Note that it is always possible to find such L_i since the pair (H_i, G_i) is detectable by Assumption 4.1. After L_i is determined, the problem reduces to finding the gain K_i^x such that (15) holds, since $K_i^\omega = \Gamma_i^\omega - K_i^x\Pi_i^\omega$, $K_i^\nu = \Gamma_i^\nu - K_i^x\Pi_i^\nu$.

The problem of determining the gain matrix L_i such that $H_i + \mu L_iG_i$ is Hurwitz stable is well-studied in the traditional control literature. In this paper, we introduce the following algebraic parametric Ricatti equation (ARE)

$$X_i(\epsilon_i)H_i^T + H_iX_i(\epsilon_i) - \delta_iX_i(\epsilon_i)G_i^TG_iX_i(\epsilon_i) + \epsilon_iI = 0, \tag{22}$$

where δ_i is a small positive constant and $\epsilon_i > 0$ is an adjustable parameter. After solving (22) for $X_i(\epsilon_i)$, the gain L_i is calculated as $L_i = -\mu^{-1}X_i(\epsilon_i)G_i^T$.

For the fixed L_i, the problem can be converted to the standard $\mathcal{H}_\infty$ static output feedback (SOF) problem. Namely, the state matrix $\hat{A}_i$ can be rewritten as $\hat{A}_i = \bar{A}_i + \bar{B}_iK_i^x\bar{C}_i$, where

$$\bar{A}_i = \begin{bmatrix} A_i & [0\ B_i\Gamma_i^\omega\ B_i\Gamma_i^\nu] \\ 0 & H_i + \mu L_iG_i \end{bmatrix}, \tag{23}$$

$$\bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix},\ \bar{C}_i = \begin{bmatrix} I & [I\ -\Pi_i^\omega\ -\Pi_i^\nu] \end{bmatrix}. \tag{24}$$

The task is to find the output feedback gain K_i^x such that the transfer function $T_i(s) = \bar{C}_i(sI - \bar{A}_i - \bar{B}_iK_i^x\bar{C}_i)^{-1}\hat{B}_i$ is stable and $\|T_i\|_\infty < \gamma_i$, for $i = 1, \ldots, N$. An additional variable $\gamma_i \leq \gamma^*$ has been introduced, which gives more freedom for setting the upper bound of $\|T_i\|_\infty$. Necessary condition

Algorithm 1 $\mathcal{H}_\infty$ Static Output Feedback (SOF) Algorithm

1: Set $\mathcal{P}_0 = I$ and $\mathcal{Q}_0 = I$

2: Solve the following optimization problem for $\mathcal{P}, \mathcal{Q}, \mathcal{V}_1, \mathcal{V}_2$

$$\min \quad \operatorname{trace}(\mathcal{P}\mathcal{Q}_0 - \mathcal{Q}\mathcal{P}_0), \text{ s.t. constraints}$$

$$\begin{pmatrix} \mathcal{P}\bar{A}_i + \bar{A}_i^T\mathcal{P} + \mathcal{V}_1\bar{C}_i + \bar{C}_i^T\mathcal{V}_1^T & \mathcal{P}\hat{B}_i & \hat{C}_i^T \\ \hat{B}_i^T\mathcal{P} & -\gamma_i I & 0 \\ \hat{C}_i & 0 & -\gamma_i I \end{pmatrix} < 0$$

$$\begin{pmatrix} \bar{A}_i\mathcal{Q} + \mathcal{Q}\bar{A}_i^T + \bar{B}_i\mathcal{V}_2 + \mathcal{V}_2^T\bar{B}_i^T & \hat{B}_i & \hat{C}_i^T\mathcal{Q} \\ \hat{B}_i^T & -\gamma_i I & 0 \\ \mathcal{Q}\hat{C}_i & 0 & -\gamma_i I \end{pmatrix} < 0$$

$$\begin{pmatrix} \mathcal{P} & I \\ I & \mathcal{Q} \end{pmatrix} \geq 0, \mathcal{P} > 0, \mathcal{Q} > 0$$

3: Check the following conditions:
   1) if $\operatorname{trace}(\mathcal{P}\mathcal{Q}) - n < \varepsilon_1$, a prescribed tolerance, go to Step 4
   2) if $\operatorname{trace}(\mathcal{P}\mathcal{Q}) - \operatorname{trace}(\mathcal{P}_0\mathcal{Q}_0) < \varepsilon_2$, a prescribed tolerance, initial $\mathcal{P}$ may not be found, EXIT
   3) otherwise set $\mathcal{P}_0 = \mathcal{P}$, $\mathcal{Q}_0 = \mathcal{Q}$ and go to Step 2.

4: Set $\mathcal{P}_0 = \mathcal{P}$. Solve the following optimization problem for K_i^x with given $\mathcal{P}$

$$\min \quad \alpha, \text{ s.t. constraint}$$

$$\Phi = \mathcal{P}\bar{A}_i + \bar{A}_i^T\mathcal{P} + \mathcal{P}\bar{B}_iK_i^x\bar{C}_i + \bar{C}_i^TK_i^{xT}\bar{B}_i^T\mathcal{P} - \alpha\mathcal{P}$$

$$\begin{pmatrix} \Phi & \mathcal{P}\hat{B}_i & \hat{C}_i^T \\ \hat{B}_i^T\mathcal{P} & -\gamma_i I & 0 \\ \hat{C}_i & 0 & -\gamma_i I \end{pmatrix} < 0$$

5: if $\alpha \leq 0$, the stabilizing K_i^x is found, EXIT

6: Solve the following optimization for $\mathcal{P}$ problem with given K_i^x:

$$\min \quad \alpha, \text{ s.t constraint from the Step 4.}$$

7: if $\alpha \leq 0$, the stabilizing K_i^x is found, EXIT

8: Solve the following optimization problem for $\mathcal{P}$ with given K_i^x and α:

$$\min \operatorname{trace}(\mathcal{P}) \quad \text{s.t constraint from the Step 4.}$$

9: Check the following conditions:
   1) if $\|\mathcal{P} - \mathcal{P}_0\| / \|\mathcal{P}\| < \delta$, the solution may not exists, EXIT
   2) otherwise go to Step 4

for solving the $\mathcal{H}_\infty$ SOF problem is that Assumption 3 holds.

There are many available algorithms in the literature for solving the $\mathcal{H}_\infty$ SOF problem [41], [42], [43]. In this paper, we have adopted the iterative LMI (ILMI) approach developed in [43], which utilizes a separate algorithm to optimize initial values of the variables and thus is very effective in finding the solution. The ILMI method is presented in *Algorithm* 1, where the notation is adapted to one used in this paper.

Algorithm 2 Design of ROF Controllers

1: Set μ according to Corollary 1. Initialize ϵ_i and δ_i.

2: Compute $L_i = -\mu^{-1}X_i(\epsilon_i)G_i^T$, where $X_i(\epsilon_i)$ is the solution of the ARE:

$$X_i(\epsilon_i)H_i^T + H_iX_i(\epsilon_i) - \delta_iX_i(\epsilon_i)G_i^TG_iX_i(\epsilon_i) + \epsilon_iI = 0 \tag{25}$$

3: Compute the gain K_i^x by Algorithm 1.

4: If the Algorithm 1 does not return the stabilizing solution, return to Step 1 and decrease ϵ_i. If the Algorithm 1 returns the stabilizing solution, but performance is not satisfactory, return to Step 1 and increase ϵ_i.

Further discussion regarding the solvability and important properties of ARE (22) will be provided in Section V. It should be noted that although fixing the gain L_i generally reduces the freedom for determining K_i^x, it will be shown that the proposed approach always ensures the solvability of the COR problem for certain types of plants and exosystems.

The complete procedure for determining the gains L_i and K_i^x is summarized in *Algorithm* 2.

IV. COR IN NETWORKS OF INTROSPECTIVE AGENTS

This section presents a distributed OF controller and discusses the MAS stability, as well as the controller design procedure.

Consider the following OF controller

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}}_i \\ \dot{\hat{\omega}}_i \end{bmatrix} &= \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix}\begin{bmatrix} \hat{x}_i \\ \hat{\omega}_i \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix}u_i \\ &\quad + \begin{bmatrix} L_i^x \\ L_i^\omega \end{bmatrix}\left(y_i - \begin{bmatrix} C_i & Q \end{bmatrix}\begin{bmatrix} \hat{x}_i \\ \hat{\omega}_i \end{bmatrix}\right), \\ \dot{\hat{\nu}}_i &= S\hat{\nu}_i + L_i^\nu\varepsilon_i, \\ u_i &= K_i^x\hat{x}_i + K_i^\omega\hat{\omega}_i + K_i^\nu\hat{\nu}_i, \; i = 1, \ldots, N, \end{aligned} \tag{26}$$

where $\hat{x}_i \in \mathbb{R}^{n_i}$, $\hat{\omega}_i \in \mathbb{R}^{q_\omega}$ and $\hat{\nu}_i \in \mathbb{R}^{q_\nu}$ are the local estimates of x_i, ω and ν, respectively. The observer gains L_i^x, L_i^ω, L_i^ν and the control law gains K_i^x, K_i^ω, K_i^ν are the parameters to be designed.

In the case of the introspective agents, the virtual error signal $\varepsilon_i \in \mathbb{R}^p$ is defined as

$$\varepsilon_i \triangleq \sum_{j=0}^{N} l_{ij}y_j - \mu(y_i - F\hat{\nu}_i), \; i = 1, \ldots, N, \tag{27}$$

where μ is a real scalar. Note that the additional term in equation (27) is different from that in equation (8) since the ith agent has access to its own output y_i.

Remark 5: The availability of the local output y_i allows for the design of more efficient control strategies for the introspective agents. In (26), the reference signal is estimated by a distributed observer using a virtual error signal, while the disturbance and system states are estimated based on locally available information. This approach substantially differs from the approaches [31], [32], where both the reference and disturbance signals are estimated by a distributed observer based on information diffused through the network. The benefit of constructing a local observer is that it facilitates the design of the observer gains.

Let K_i^ω and K_i^ν be designed in the same way as in (9). Define the tracking error, local observer error and distributed observer error as

$$e_i = x_i - \Pi_i^\omega \omega - \Pi_i^\nu \nu,\ \tilde{e}_i = \begin{bmatrix} \tilde{x}_i \\ \tilde{\omega}_i \end{bmatrix} = \begin{bmatrix} \hat{x}_i - x_i \\ \hat{\omega}_i - \omega \end{bmatrix},$$
$$\tilde{\nu}_i = \hat{\nu}_i - \nu.$$

Substituting e_i, $\tilde{e}_i$, $\tilde{\nu}_i$ into (26) yields the following closed-loop dynamics

$$\begin{bmatrix} \dot{e}_i \\ \dot{\tilde{e}}_i \\ \dot{\tilde{\nu}}_i \end{bmatrix} = \begin{bmatrix} A_i + B_iK_i^x & B_iK_i^{x\omega} & B_iK_i^\nu \\ 0 & H_i + L_i^{x\omega}G_i & 0 \\ 0 & 0 & S + \mu L_i^\nu F \end{bmatrix} \begin{bmatrix} e_i \\ \tilde{e}_i \\ \tilde{\nu}_i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ L_i^\nu \end{bmatrix} \xi_i.$$

Similarly to the procedure in the previous section, define $\xi_i = \sum_{j=1}^N l_{ij}C_je_j - \mu C_ie_i$ and

$$H_i = \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix},\ G_i = \begin{bmatrix} C_i & Q \end{bmatrix},$$
$$K_i^{x\omega} = \begin{bmatrix} K_i^x & K_i^\omega \end{bmatrix},\ L_i^{x\omega} = \begin{bmatrix} L_i^x \\ L_i^\omega \end{bmatrix}.$$

Following the rule (11) for notations, the closed-loop state matrix can be written as

$$A_{CL} = \begin{bmatrix} A + BK^x & BK^{x\omega} & BK^\nu \\ 0 & H + L^{x\omega}G & 0 \\ L^\nu(\check{\mathcal{L}} - \mu I)C & 0 & \tilde{S} + \mu L^\nu \tilde{F} \end{bmatrix}, \tag{28}$$

where $\tilde{S} = I_N \otimes S$, $\tilde{F} = I_N \otimes F$. Under the same arguments as in the previous section, it can be concluded that A_{CL} being Hurwitz is equivalent to solving the COR problem.

A. STABILITY ANALYSIS

Let us first define $\bar{A}_{CL} = \begin{bmatrix} A+BK^x & BK^\nu \\ L^\nu(\check{\mathcal{L}}-\mu I)C & \tilde{S}+\mu L^\nu \tilde{F} \end{bmatrix}$. Then, the following lemma is of particular importance for the stability analysis.

Lemma 3: The closed-loop state matrix A_{CL} in (28) is stable if and only if the matrices $H+L^{x\omega}G$ and $\bar{A}_{CL}$ are stable.

Proof: The proof follows from the similarity relation

$$A_{CL} \sim \begin{bmatrix} H + L^{x\omega}G & 0 \\ \begin{smallmatrix} BK^{x\omega} \\ 0 \end{smallmatrix} & \bar{A}_{CL} \end{bmatrix},$$

where the structure of the matrix allows the separation principle to be applied. □

Under the Assumption 4.2, it is always possible to find the gain matrix $L_i^{x\omega}$ such that $H_i + L_i^{x\omega}G_i$ is Hurwitz stable for $i = 1, \ldots, N$, which further implies stability of the matrix $H + L^{x\omega}G$.

In order to analyze stability properties of $\bar{A}_{CL}$, let us first introduce the matrices

$$\hat{A}_i = \begin{bmatrix} A_i + B_iK_i^x & B_iK_i^\nu \\ 0 & S + \mu L_i^\nu F \end{bmatrix},\ \hat{B}_i = \begin{bmatrix} 0 \\ L_i^\nu \end{bmatrix},\ \hat{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}, \tag{29}$$

with the corresponding transfer function

$$T_i(s) = \hat{C}_i(sI - \hat{A}_i)^{-1}\hat{B}_i. \tag{30}$$

Then, we present the stability condition for the multi-agent system under the OF protocol.

Theorem 2: Consider a multi-agent composed of (1), (2) and (3). Then, if $H_i + L_i^{x\omega}G_i$ is Hurwitz stable and Assumptions 1-5 hold, the OF protocol (26) solves the COR problem provided that the following condition holds

$$\|T_i\|_\infty < \gamma^*,\ i = 1, \ldots, N, \tag{31}$$

where $\gamma^* = \frac{1}{\rho(|\mu I - \check{\mathcal{L}}|)}$.

Proof: Given the Hurwitzness of $H_i + L_i^{x\omega}G_i$, it remains to ensure the stability of $\bar{A}_{CL}$ according to *Lemma 3*. The stability analysis of $\bar{A}_{CL}$ is analogous to the proof of *Theorem 1*, with respect to the newly defined matrices $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$ and the corresponding transfer function $T_i(s) = \hat{C}_i(sI - \hat{A}_i)^{-1}\hat{B}_i$. □

Similarly to the previous section, we establish the lower bound for $||T_i||_\infty$.

Lemma 4: Suppose that the transfer function $T_i(s)$ is stable. Then, μ^{-1} is a lower bound of $\|T_i\|_\infty$, i.e.

$$\|T_i\|_\infty \geq \frac{1}{\mu},\ i = 1, \ldots, N, \tag{32}$$

Proof: Introduce the following coordinate transformation matrix

$$M_i = \begin{bmatrix} I & -\Pi_i^\nu \\ 0 & I \end{bmatrix},$$

Then, the new system matrices can be expressed as $\check{A}_i = M_i\hat{A}_iM_i^{-1}$, $\check{B}_i = M_i\hat{B}_i$ and $\check{C}_i = \hat{C}_iM_i^{-1}$, that is

$$\check{A}_i = \begin{bmatrix} A_i + B_iK_i^x & -\mu\Pi_i^\nu L_i^\nu F \\ 0 & S + \mu L_i^\nu F \end{bmatrix},$$
$$\check{B}_i = \begin{bmatrix} -\Pi_i^\nu L_i^\nu \\ L_i^\nu \end{bmatrix},\ \check{C}_i = \begin{bmatrix} C_i & F \end{bmatrix}. \tag{33}$$

The rest of the proof is analogous to the proof of *Lemma 2*. □

Corollary 2: The stability condition (31) can be satisfied if and only if Assumption 4.1 holds and $\mu > \frac{1+\rho(\bar{\mathcal{A}})}{2}$.

Proof: The proof is analogous to the proof of *Corollary 1*. □

Remark 6: A special type of graphs that are extensively investigated in the literature of cooperative control are *acyclic graphs [44]. Acyclic graphs are characterized by a lower triangular matrix $\bar{\mathcal{L}}$ with ones on the main diagonal. The consequence of this structure is that for $\mu = 1$ one gets $\gamma^* \to \infty$, which means that for acyclic graphs both the ROF and OF protocols solve the COR problem with a sufficient condition being that local transfer functions are stable.*

B. CONTROLLER SYNTHESIS

In this subsection, an algorithm for synthesis of the OF controller is provided. Here we encounter the same difficulty as in the case of the ROF protocol, which is the simultaneous calculation of K_i and L_i^ν. Therefore, we first find the gains $L_i^{x\omega}$ and L_i^ν such that $H_i + L_i^{x\omega} G_i$ and $S + \mu L_i^\nu F$ are Hurwitz stable, which is always possible under Assumption 4.2. Then, under Assumption 3, we will determine K_i^x such that $A_i + B_i K_i^x$ is stable and $\|T_i\|_\infty < \gamma_i$, where $\gamma_i \leq \gamma^*$.

Observe that, according to *Lemma 3*, the eigenvalues of the matrix $H_i + L_i^{x\omega} G_i$ can be assigned independently of the remaining eigenvalues of the matrix A_{CL}. Therefore, $L_i^{x\omega}$ can be obtained using standard ARE-based methods. In this paper, we use the following ARE

$$Y_i(\kappa_i)H_i^T + H_i Y_i(\kappa_i) - Y_i(\kappa_i)G_i^T G_i Y_i(\kappa_i) + \kappa_i I = 0, \quad (34)$$

which always has a solution for $\kappa_i > 0$ under Assumption 4.2 [45]. The stabilizing gain is then computed as $L_i^{x\omega} = -Y_i(\kappa_i)G_i^T$.

On the other hand, the gain L_i^ν is computed as $L_i^\nu = -\mu^{-1} X_i(\epsilon_i) F^T$, where $X_i(\epsilon_i)$ is the solution of the ARE introduced in the Section III-B:

$$X_i(\epsilon_i)S^T + SX_i(\epsilon_i) - \delta_i X_i(\epsilon_i)F^T F X_i(\epsilon_i) + \epsilon_i I = 0. \quad (35)$$

In the previous equation, δ_i represents a small positive constant, and $\epsilon_i > 0$ is an adjustable parameter.

Under the OF protocol, the state matrix $\hat{A}_i$ can be rewritten as $\hat{A}_i = \bar{A}_i + \bar{B}_i K_i^x \bar{C}_i$, where

$$\bar{A}_i = \begin{bmatrix} A_i & B_i \Gamma_i^\nu \\ 0 & S + \mu L_i^\nu F \end{bmatrix}, \quad (36)$$

$$\bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \ \bar{C}_i = \begin{bmatrix} I & -\Pi_i^\nu \end{bmatrix}. \quad (37)$$

Therefore, K_i^x can be determined by using the *Algorithm* 1. The other feedforward gains are computed as $K_i^\omega = \Gamma_i^\omega - K_i^x \Pi_i^\omega$ and $K_i^\nu = \Gamma_i^\nu - K_i^x \Pi_i^\nu$.

The complete procedure for control synthesis is summarized in *Algorithm* 3. Further discussion regarding the solvability of the COR problem by the proposed approach is provided in Section V.

V. SOLVABILITY ANALYSIS

In this section, under some additional assumptions for leader's dynamics, we provide further analysis regarding the solvability of the COR problem. It is shown that under the ROF protocol, the existence of a solution to the COR problem can be guaranteed whenever $\lambda(A_i) \in \bar{\mathbb{C}}^-$, $i = 1, \ldots, N$. Furthermore, under the OF protocol, a solution to the COR problem always exists.

Algorithm 3 Design of OF Controllers

1: Set μ according to Corollary 2. Initialize κ_i,ϵ_i and δ_i.
2: Compute $L_i^{x\omega} = -Y_i(\kappa_i)G_i^T$, where $Y_i(\kappa_i)$ is the solution of the ARE:

$$Y_i(\kappa_i)H_i^T + H_i Y_i(\kappa_i) - Y_i(\kappa_i)G_i^T G_i Y_i(\kappa_i) + \kappa_i I = 0 \quad (38)$$

3: Compute $L_i^\nu = -\mu^{-1} X_i(\epsilon_i) F^T$, where $X_i(\epsilon_i)$ is the solution of the ARE:

$$X_i(\epsilon_i)S^T + SX_i(\epsilon_i) - \delta_i X_i(\epsilon_i)F^T F X_i(\epsilon_i) + \epsilon_i I = 0, \quad (39)$$

4: Compute the gain K_i^x by Algorithm 1.
5: If the Algorithm 1 does not return the stabilizing solution, return to Step 1 and decrease ϵ_i. If the Algorithm 1 returns stabilizing solution, but performance is not satisfactory, return to Step 2 and increase ϵ_i.

Prior to the further discussion, we introduce the two lemmas that are fundamental for establishing subsequent results.

Lemma 5: Consider a system with the state-space realization

$$\dot{x} = (A + \mu LC)x + Lu$$
$$y = Cx, \quad (40)$$

and corresponding transfer function

$$G(s) = C(sI - A - \mu LC)^{-1}L, \quad (41)$$

where A is an anti-Hurwitz stable matrix (i.e. it has at least one eigenvalue in $\bar{\mathbb{C}}^+$), and the pair (C, A) is detectable. Then, for every $\chi > \mu^{-1}$ there exists a gain matrix L such that $\|G\|_\infty < \chi$. Moreover, the gain matrix can be obtained as $L = -\mu^{-1}PC^T$, where $P > 0$ is a solution of the following ARE

$$PA^T + AP + \epsilon I + \left(\frac{1}{\mu^2\chi^2} - 1\right)PC^T CP = 0, \quad (42)$$

with ϵ being a positive constant.

Proof: According to the bounded real lemma (BRL) [46], $A + \mu LC$ is Hurwitz stable and $\|G\|_\infty < \chi$, if there exists $P > 0$ satisfying

$$P(A + \mu LC)^T + (A + \mu LC)P + \frac{1}{\chi^2}LL^T + PC^T CP < 0. \quad (43)$$

Let $L = -\mu^{-1}PC^T$. Then, the inequality (43) becomes

$$PA^T + AP + \left(\frac{1}{\mu^2\chi^2} - 1\right)PC^T CP < 0. \quad (44)$$

Since A has eigenvalues with nonnegative real parts, there does not exist $P > 0$ that satisfies the inequality (44) whenever $\mu^{-2}\chi^{-2} - 1 \geq 0$, thus $\chi > \mu^{-1}$ must hold.

The inequality (44) is satisfied for every $P > 0$ that is a solution of the ARE (42). The existence of such P is ensured by detectability of (C, A) [45]. □

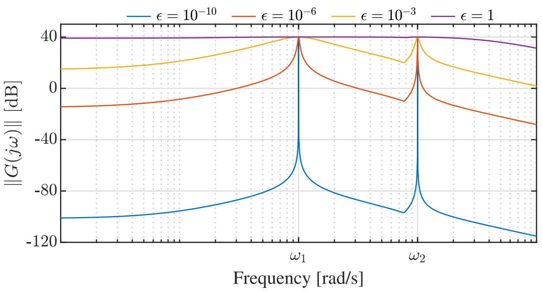


FIGURE 1. Maximal singular value with respect to the frequency ($||G(j\omega)||$) of two-input, two-output system for $\mu^{-1} = 100$, $\chi = \mu^{-1} + 0.01$ and different values of ϵ. The system is of the seventh order, with two pairs of poles on the imaginary axis, at $\pm j\omega_1$ and $\pm j\omega_2$.

It should be noted that $-\mu^{-1}PC^T$ is not a unique substitution for the gain L in terms of P. However, this choice of L can lead to the minimum possible norm of $G(s)$. In the following, we establish the value of that minimum norm.

Lemma 6: Consider the system given by (40), (41), where A is an anti-Hurwitz stable matrix. Then, $||G||_\infty \geq \mu^{-1}$.

Proof: Under the the feedback controller $u = -\mu y$, the closed loop matrix of (41) is equal to A, which is anti-Hurwitz stable by assumption. Therefore, according to the small-gain theorem, $||G||_\infty \geq \mu^{-1}$ must hold. □

Remark 7: In the case when A is an anti-Hurwitz stable matrix, Lemma 5 and Lemma 6 imply that there always exists a gain L such that $\|G\|_\infty \in [\mu^{-1}, \chi)$, where χ can be chosen to be arbitrarily close to μ^{-1}. It should be noted, that when A is Hurwitz, there is no restriction on the lower bound of $\|G\|_\infty$ since the inequality can be satisfied for any $\chi > 0$.

Remark 8: Suppose that χ is chosen as $\chi = \mu^{-1} + \Delta$, where $\Delta > 0$ is a small constant. Then, for any $\epsilon > 0$, we have $\|G\|_\infty \approx \mu^{-1}$. Furthermore, it is well-known that the solution $P(\epsilon)$ of the ARE (42) is monotonically non-decreasing with respect to ϵ. In the special case, when state matrix has eigenvalues in the closed left half-plane, $\epsilon \to 0$ corresponds to $P(\epsilon) \to 0$ [47], [48], thus leading to $L \to 0$.

In order to illustrate the scenario in Remark 8, Fig. 1 shows the largest singular value with respect to frequency, i.e. $||G(j\omega)||$, for a marginally stable MIMO system. The system is of the seventh order, with two pairs of poles at $\pm j\omega_1$ and $\pm j\omega_2$. As it can be seen, $\|G\|_\infty \to \mu^{-1}$ for $\epsilon = 10^{-10}$, where the largest singular values occur at the frequencies ω_1 and ω_2. On the rest of the frequency range, $||G(j\omega)||$ decreases as frequencies get further from ω_1 and ω_2, due to $L \to 0$. An increase of the value of ϵ causes a larger gain matrix L, which further increases $||G(j\omega)||$ on the rest of the frequency range. However, $||G||_\infty$ remains the same.

A. SOLVABILITY ANALYSIS UNDER ROF PROTOCOL

In this subsection we provide some additional results regarding the solvability of the COR problem under the ROF protocol.

First, introduce the following matrices

$$\check{H}_i = \begin{bmatrix} A_i & -B_i\Gamma_i^\omega & -B_i\Gamma_i^\nu \\ 0 & P & 0 \\ 0 & 0 & S \end{bmatrix}, \quad \check{L}_i = \begin{bmatrix} I \;-\Pi_i^\omega \;-\Pi_i^\nu \, L_i \\ L_i^\omega \\ L_i^\nu \end{bmatrix},$$
$$\check{G}_i = \begin{bmatrix} -C_i & 0 & 0 \end{bmatrix}.$$

Then, by taking into account (19), the transfer function (14) can be rewritten in terms of these matrices as

$$\begin{aligned} T_i(s) &= C_i(sI - A_i - B_iK_i^x)^{-1}\left[I \;-\Pi_i^\omega \;-\Pi_i^\nu\right]L_i \\ &\quad + \left(I + \mu C_i(sI - A_i - B_iK_i^x)^{-1}\left[I \;-\Pi_i^\omega \;-\Pi_i^\nu\right]L_i\right) \\ &\quad \times \check{G}_i(sI - \check{H}_i - \mu\check{L}_i\check{G}_i)^{-1}\check{L}_i. \end{aligned} \tag{45}$$

Define the new transfer function $\bar{T}_i(s) = \check{G}_i(sI - \check{H}_i - \mu\check{L}_i\check{G}_i)^{-1}\check{L}_i$, which can be shown to be identical to $\bar{T}_i(s) = G_i(sI - H_i - \mu L_iG_i)^{-1}L_i$. This leads to a more concise representation of (45) in terms of matrices in original coordinates

$$\begin{aligned} T_i(s) &= C_i(sI - A_i - B_iK_i^x)^{-1}\left[I \;-\Pi_i^\omega \;-\Pi_i^\nu\right]L_i \\ &\quad + \left(I + \mu C_i(sI - A_i - B_iK_i^x)^{-1}\left[I \;-\Pi_i^\omega \;-\Pi_i^\nu\right]L_i\right) \\ &\quad \times \bar{T}_i(s). \end{aligned} \tag{46}$$

Assumption 6: The eigenvalues of A_i, $i = 1, \ldots, N$, belong to the closed left half-plane of the complex plane, while the eigenvalues of the matrices P and S lie on the imaginary axis.

Theorem 3: Suppose that μ is chosen such that the condition (20) holds. Then, under Assumptions 1-6, the COR problem is always solvable by ROF protocol (7). Moreover, for a sufficiently small ϵ_i, the values of K_i and L_i, $i = 1, \ldots, N$, can be obtained by Algorithm 2.

Proof: The ARE (25) in *Algorithm 2* corresponds to the ARE (42) when $\delta_i = 1 - \mu^{-2}\chi^{-2}$. The Assumption 6 implies that H_i has eigenvalues in the closed left half-plane, thus by Remark 8, $L_i^\nu \to 0$ as $\epsilon_i \to 0$. Therefore, for any gain K_i^x that stabilizes $A_i + B_iK_i^x$ we can write $T_i(s) \to \bar{T}_i(s)$. Choosing a sufficiently small δ_i in *Algorithm 2* leads to $\|\bar{T}_i\|_\infty \to \mu^{-1}$, thus implying $||T_i||_\infty \to \mu^{-1}$. The rest of the proof follows from *Corollary 1*. □

Remark 9: The Theorem 3 guarantees the existence of a solution for agents with poles in the closed left half-plane for $\epsilon_i \to 0$. On the other side, the system response becomes faster as ϵ_i increases because $X_i(\epsilon_i)$, and hence the control input, also increase with ϵ_i. Even though for a larger ϵ_i we still have $||\bar{T}_i||_\infty \to \mu^{-1}$, the other terms in $T_i(s)$ become non-negligible, leading to a larger $||T_i||_\infty$. This makes it less likely to find K_i^x that solves the $\mathcal{H}_\infty$ SOF problem. Furthermore, it should be noted that if either an agent or an exosystem has poles in the open right half-plane, the existence of a solution cannot be guaranteed in advance, as it is not possible to achieve $L_i \to 0$ for any ϵ_i. Namely, as the unstable poles of the exosystems or agents move further right in the complex plane, the required stabilizing gain L_i increases. Consequently, minimizing $\|T_i\|_\infty$, which depends on both L_i and the follower state model, becomes more difficult. It is *important to note that, in general, the $\mathcal{H}_\infty$ norm of linear systems cannot be arbitrarily reduced by using state or output feedback [49].*

Remark 10: Although seemingly restrictive, the Assumption 6 covers a wide variety of important and common agents' dynamics in multi-agent systems. An example are agents with first and second order integrator dynamics [10], [11]. The assumption that the the poles of exosystems lie on imaginary axis is a common assumption in many existing results such as [7], [16], [18], [21], [31], [32], [39], and [37]. Under the assumption, the exosystems associated with matrices S and P can generate a diverse range of reference and disturbance signals that are interesting in practice. This includes step signals, polynomial signals, sinusoidal signals of various frequencies, and their linear combinations [31]. Possible practical applications include cooperative tracking and formation control of mobile vehicles [2], [39], control of unmanned aerial vehicles [33], control of robotic manipulators [20], and so on.

Remark 11: Solvability of the COR problem for certain classes of agents' dynamics has also been discussed in related works. For example, in [27] it is assumed that the agents are minimum-phase and with an identical relative degrees. In [37], an $\mathcal{H}_\infty$ based design method is proposed, but it does not guarantee that the stabilizing controller gains can be found, even when the agents' dynamics is stable. Similarly, [28] considers the output synchronization of the non-introspective agents that are minimum-phase and SISO.

B. SOLVABILITY ANALYSIS UNDER OF PROTOCOL

In this subsection, we analyze solvability of the COR problem under the OF protocol.

In terms of the matrices introduced in (33), the transfer function (30) can be rewritten as

$$T_i(s) = -C_i\big(sI - (A_i + B_iK_i^x)\big)^{-1}\Pi_i^\nu L_i^\nu + \Big(I - \mu C_i\big(sI - (A_i + B_iK_i^x)\big)^{-1}\Pi_i^\nu L_i^\nu\Big)\,\bar{T}_i(s), \tag{47}$$

where $\bar{T}_i(s) = F\big(sI - (S + \mu L_i^\nu F)\big)^{-1}L_i^\nu$, which corresponds to the form in *Lemma 5*.

Assumption 7: The eigenvalues of the matrix S lie on the imaginary axis.

Theorem 4: Suppose that μ is chosen such that the condition (20) holds. Then, under Assumptions 1-5 and 7, the COR problem is always solvable by OF protocol (26). Moreover, for a sufficiently small ϵ_i, the values of K_i and L_i, $i = 1, \ldots, N$, can be obtained by Algorithm 3.

Proof: The ARE (39) in *Algorithm 3* corresponds to the ARE (42) when $\delta_i = 1 - \mu^{-2}\chi^{-2}$. Under the Assumption 7, S is marginally stable, thus by Remark 8, $L_i \to 0$ as $\epsilon_i \to 0$. Therefore, for any gain K_i^x that stabilizes $A_i + B_iK_i^x$ it follows that $T_i(s) \to \bar{T}_i(s)$. The choice of a sufficiently small δ_i in *Algorithm 3* leads to $\|\bar{T}_i\|_\infty \to \mu^{-1}$, implying $||T_i||_\infty \to \mu^{-1}$. The rest of the proof follows from *Corollary 2*. □

Remark 12: Along with Assumption 7, some existing results require additional assumptions in order to guarantee the solvability of the COR problem. For example, the observer-based low-gain method [31] guarantees the existence of a solution only when $E_i = 0$, while the approach [36] requires the agents to be right-invertible. Furthermore, contrary to some existing works, the proposed method does not impose any restrictions on the spectrum of the matrix P.

Finally, for the sake of clarity and to avoid repetition, we note that conclusions can be drawn in this section analogously to those presented in Remark 9.

VI. SIMULATION RESULTS

A. EXAMPLE 1

Consider a network consisting of a leader and six followers with the following dynamics

$$\begin{cases} \dot{\nu} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\nu, \; y_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\nu, \\ \dot{\omega} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\omega, \end{cases}$$

$$\begin{cases} \dot{x}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}x_i + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}u_i + \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}\omega, \\ y_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x_i + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\omega, \; i = 1, 4; \end{cases}$$

$$\begin{cases} \dot{x}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}x_i + \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}u_i + \begin{bmatrix} -2 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix}\omega, \\ y_i = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}x_i + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\omega, \; i = 2, 5; \end{cases}$$

$$\begin{cases} \dot{x}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -36 & 0 \end{bmatrix}x_i + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}u_i + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\omega, \\ y_i = \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & -1 & 1 \end{bmatrix}x_i + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\omega, \; i = 3, 6. \end{cases}$$

The solutions of the regulator equations (6) for agents $i = 1, 4$, $i = 2, 5$, and $i = 3, 6$ are respectively:

$$\Gamma_i^\omega = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, \; \Gamma_i^\nu = \begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix}, \; \Pi_i^\omega = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix},$$
$$\Pi_i^\nu = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\Gamma_i^\omega = \begin{bmatrix} 0 & 3.5 \\ -5 & 3 \end{bmatrix}, \; \Gamma_i^\nu = \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}, \; \Pi_i^\omega = \begin{bmatrix} 1 & 1.5 \\ -1 & -2.5 \\ 1 & 2.5 \end{bmatrix},$$
$$\Pi_i^\nu = \begin{bmatrix} -1 & 1 \\ 2 & -1 \\ -2 & 2 \end{bmatrix},$$
$$\Gamma_i^\omega = \begin{bmatrix} 0.67 & 0.33 \\ 0 & -13.33 \end{bmatrix}, \; \Gamma_i^\nu = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \; \Pi_i^\omega = \begin{bmatrix} 0 & -0.83 \\ 0 & -0.33 \\ 0 & -0.33 \end{bmatrix},$$
$$\Pi_i^\nu = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The network topology among agents is described by

$$\bar{\mathcal{L}} = \begin{bmatrix} [r]1 & -0.10 & 0 & -0.1 & 0 & -0.8 \\ -0.12 & 1 & -0.12 & 0 & -0.12 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -0.67 & -0.33 & 0 & 1 & 0 & 0 \\ 0 & -0.8 & 0 & 0 & 1 & -0.2 \\ 0 & -0.33 & 0 & 0 & -0.33 & 1 \end{bmatrix}.$$

By setting $\mu = 1$, for the network topology defined by $\bar{\mathcal{L}}$, we obtain $\gamma^* = \rho(\mu I - \bar{\mathcal{L}})^{-1} = 1.51$. Based on Theorems 1 and 2, in order to guarantee the stability of the MAS, the $\mathcal{H}_\infty$ norms $\|T_i(s)\|_\infty$, for $i = 1, \ldots, N$, must be smaller than γ^*.

1) ROF PROTOCOL

The observer gains are calculated by solving ARE (25) in Algorithm 2 for $\epsilon_i = 10$ and $\delta_i = 0.1$. The resulting observer gains are:

$$L_i = \begin{bmatrix} 19.17 & 9.99 & -9.25 & 1.97 & -6.32 & 6.18 \\ -2.59 & 9.09 & -9.30 & 4.91 & -1.49 & -10.94 \end{bmatrix}^T, \ i = 1, 4,$$
$$L_i = \begin{bmatrix} 28.11 & 15.48 & -9.98 & -2.01 & 10.58 & -0.25 & 8.85 \\ -16.60 & 11.01 & 0.32 & 3.48 & -8.48 & 1.25 & -10.95 \end{bmatrix}^T, \ i = 2, 5,$$
$$L_i = \begin{bmatrix} 5.07 & 1.46 & 0.56 & -8.24 & 10.98 & -6.69 & 2.04 \\ -5.81 & -6.55 & 45.76 & -2.40 & 2.41 & -1.31 & -12.22 \end{bmatrix}^T, \ i = 3, 6.$$

Then, the gain K_i^x for each agent is determined using Algorithm 1, which is implemented in YALMIP, a MATLAB optimization toolbox. For $\gamma_i = 1.2$, the following gains are obtained:

$$K_i^x = \begin{bmatrix} -4.95 & -0.60 \\ -0.60 & 2.26 \end{bmatrix}, \ i = 1, 4,$$
$$K_i^x = \begin{bmatrix} -2.39 & -3.38 & -0.59 \\ 2.55 & 3.50 & -0.15 \end{bmatrix}, \ i = 2, 5,$$
$$K_i^x = \begin{bmatrix} -2.49 & -3.10 & 0.00 \\ 2.39 & 39.35 & -4.71 \end{bmatrix}, \ i = 3, 6.$$

In the final step, the gains K_i^ω and K_i^v are calculated as $K_i^\omega = \Gamma_i^\omega - K_i^x \Pi_i^\omega$ and $K_i^v = \Gamma_i^v - K_i^x \Pi_i^v$, respectively. The resulting $\mathcal{H}_\infty$ norms are: $\|T_i\|_\infty = 1.14$ for $i = 1, 3$, $\|T_i\|_\infty = 1.12$ for $i = 2, 5$ and $\|T_i\|_\infty = 1.15$ for $i = 3, 6$. Since these norms are smaller than γ^*, we can conclude that the COR problem is solved.

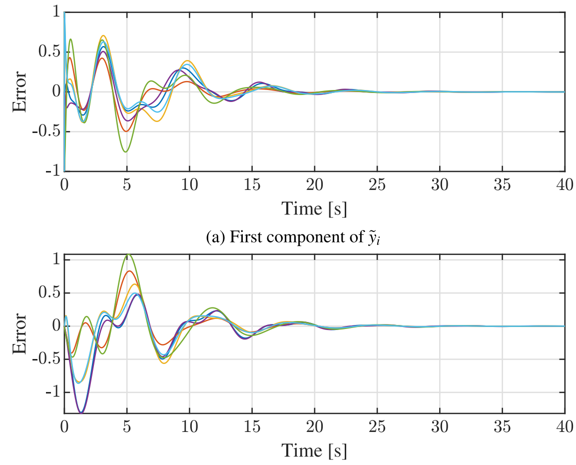
In Fig. 2, the output regulation errors of the agents under the designed controller are shown. It can be seen that the ROF protocol ensures tracking of the reference signal, even in the presence of disturbance. Furthermore, in Fig. 3 the output trajectories are depicted, demonstrating that all outputs synchronize with the reference trajectory.

2) OF PROTOCOL

In the case of the OF protocol, by solving the ARE (39) in Algorithm 3 for $\epsilon_i = 10$ and $\delta_i = 0.1$, we obtain the following distributed observer gains:

$$L_i^v = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}, i = 1, \ldots, N.$$

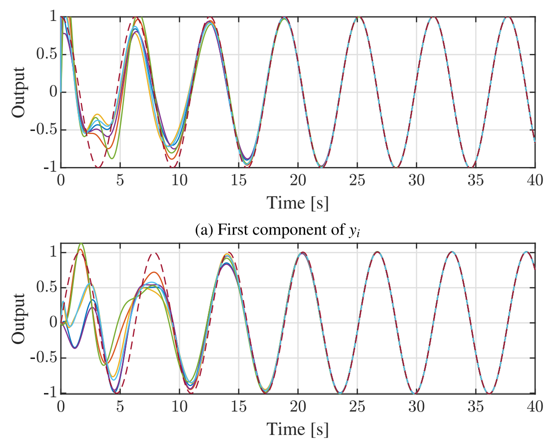
Similarly, the gains of the local observer are obtained by solving the ARE (38) for $\kappa_i = 1$, but their specific values are not presented for the sake of brevity. For $\gamma_i = 1.2$, the



(a) First component of $\tilde{y}_i$

(b) Second component of $\tilde{y}_i$.

FIGURE 2. Output regulation errors of the followers under the ROF protocol.



(a) First component of y_i

(b) Second component of y_i.

FIGURE 3. Output trajectories of the followers (solid lines) and reference trajectory (dashed line) under the ROF protocol.

Algorithm 1 gives the following controller gains:

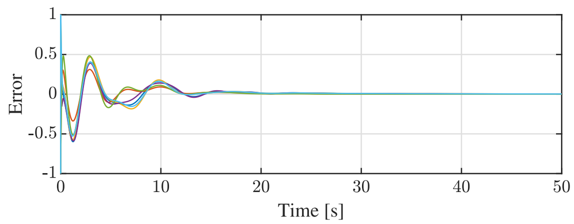
$$K_i^x = \begin{bmatrix} -4.27 & -0.67 \\ -0.67 & 1.78 \end{bmatrix}, \ i = 1, 4,$$
$$K_i^x = \begin{bmatrix} 0.35 & 0.47 & 0.72 \\ 1.88 & 3.00 & 0.57 \end{bmatrix}, \ i = 2, 5,$$
$$K_i^x = \begin{bmatrix} -0.55 & -1.13 & -1.00 \\ 1.55 & 37.96 & 0.44 \end{bmatrix}, \ i = 3, 6.$$

The remaining gains are determined as $K_i^\omega = \Gamma_i^\omega - K_i^x \Pi_i^\omega$ and $K_i^v = \Gamma_i^v - K_i^x \Pi_i^v$. As a result, we have: $\|T_i\|_\infty = 1.19$ for $i = 1, 3$, $\|T_i\|_\infty = 1.18$, for $i = 2, 5$ and $\|T_i\|_\infty = 1.003$ for $i = 3, 6$. Since the resulting norms are smaller than γ^*, we conclude that the COR problem is solved.

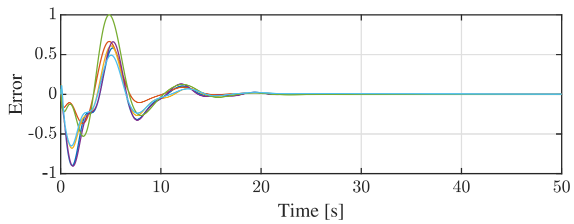
Figs. 4 and 5 show the output regulation errors and output trajectories under the designed OF protocol. It can be seen that the errors asymptotically converge to zero, while agent outputs synchronize with the reference trajectory, indicating successful output regulation.

B. EXAMPLE 2

In this example, we consider the MAS from Example 1 and investigate the influence of various design parameters on closed-loop performance for both the ROF and OF protocols.

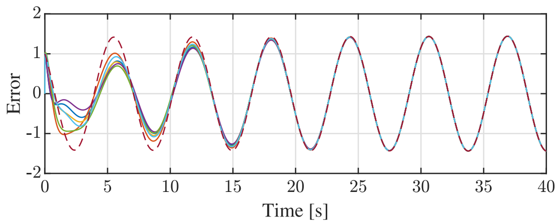


(a) First component of $\tilde{y}_i$

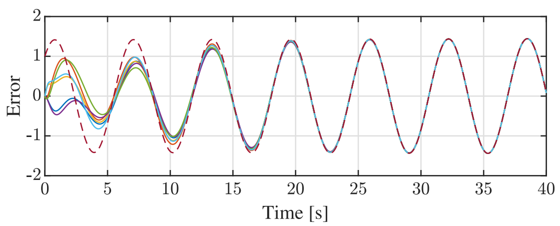


(b) Second component of $\tilde{y}_i$.

FIGURE 4. Output regulation errors of the followers under the OF protocol.

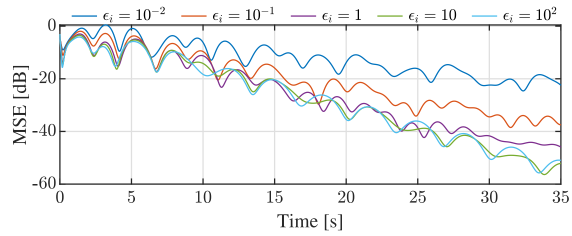


(a) First component of y_i

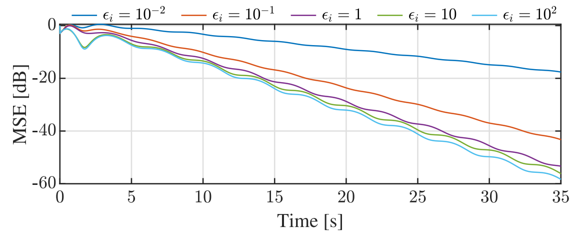


(b) Second component of y_i.

FIGURE 5. Output trajectories of the followers (solid lines) and reference trajectory (dashed line) under the OF protocol.

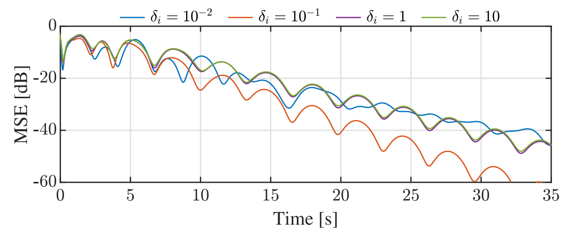


(a) ROF protocol

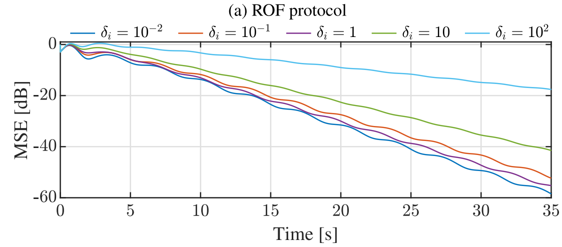


(b) OF protocol

FIGURE 6. Comparison of MSE for different values of parameter ϵ_i.



(a) ROF protocol



(b) OF protocol

FIGURE 7. Comparison of MSE for different values of parameter δ_i.

In the first scenario, the parameter ϵ_i is varied, while other parameters are the same as in the previous example. Fig. 6 shows the mean squared output regulation error for all agents, which is calculated as $\text{MSE} = \frac{1}{N}\sum_{i=1}^{N} \tilde{y}_i^T \tilde{y}_i$. The figure indicates that increasing ϵ_i enhances the performance of the MAS. However, for $\epsilon_i = 10$ and $\epsilon_i = 100$, the difference in performance is not too significant, meaning that the excessive values for ϵ_i will not improve the performance substantially. It is important to note that by increasing ϵ_i, the gains L_i (ROF) and L_i^v (OF) also increase, and for some high values (e.g., 10^4 in this example), the SOF algorithm will not be able to provide a solution.

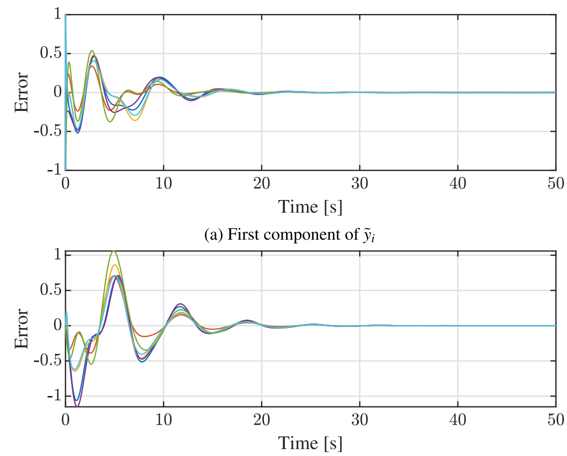
In the second scenario, the parameter δ_i is varied, while the other parameters are kept the same as in the previous example. In the case of the ROF protocol, it was not possible to find K_i^x using the SOF algorithm for $\delta_i = 100$. The resulting MSE curves are depicted in Fig. 7. Notably, for the ROF protocol, the output regulation errors decay at the fastest rate when $\delta_i = 0.1$, while in the case of the OF protocols, the best performances are obtained for $\delta_i = 0.01$. Clearly, δ_i can be used to adjust performance, but excessive values can lead to performance deterioration.

C. EXAMPLE 3

In this example, we consider exosystems that have poles with strictly positive real parts to demonstrate that even when the existence of a solution cannot be guaranteed in advance, the proposed method still may solve the COR problem, as discussed in Remark 9. The exosystem matrices are given by: $S = \begin{bmatrix} 0.5 & 1 \\ 0 & -1 \end{bmatrix}$ and $P = \begin{bmatrix} 0.5 & 2 \\ 0 & -2 \end{bmatrix}$, while the followers' model and parameters of Algorithms 1-3 remain unchanged.

In the case of the ROF protocol, the following gains and $\mathcal{H}_\infty$ norms are obtained:

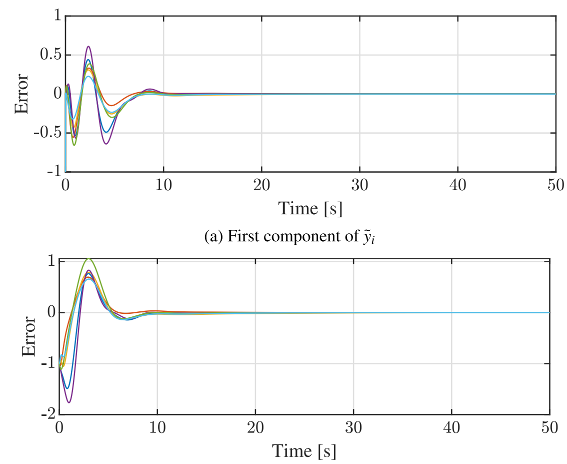
$$K_i^x = \begin{bmatrix} -9.35 & -0.38 \\ -0.38 & 5.76 \end{bmatrix}, \quad \|T_i\|_\infty = 1.19, \; i = 1, 4,$$
$$K_i^x = \begin{bmatrix} 1.25 & -1.18 & -1.66 \\ 22.14 & 15.22 & -5.70 \end{bmatrix}, \quad \|T_i\|_\infty = 1.18, \; i = 2, 5,$$
$$K_i^x = \begin{bmatrix} 1.25 & -1.18 & -1.66 \\ 22.14 & 15.22 & -5.70 \end{bmatrix}, \quad \|T_i\|_\infty = 1.20, \; i = 3, 6,$$



(a) First component of $\tilde{y}_i$

(b) Second component of $\tilde{y}_i$.

FIGURE 8. Output regulation error under the ROF protocol in the case when the exosystem matrices are unstable.



(a) First component of $\tilde{y}_i$

(b) Second component of $\tilde{y}_i$.

FIGURE 9. Output regulation error under the OF protocol in the case when the exosystem matrices are unstable.

while in the case of the OF protocol we have:

$$K_i^x = \begin{bmatrix} -6.51 & -1.17 \\ -1.17 & 6.44 \end{bmatrix}, \ \|T_i\|_\infty = 1.12, \ i = 1, 4,$$
$$K_i^x = \begin{bmatrix} 1.55 & -0.47 & -1.54 \\ 7.49 & 4.15 & -4.00 \end{bmatrix}, \ \|T_i\|_\infty = 1.17, \ i = 2, 5,$$
$$K_i^x = \begin{bmatrix} -11.04 & -7.98 & 0.54 \\ 9.84 & 47.20 & -5.98 \end{bmatrix}, \ \|T_i\|_\infty = 1.10, \ i = 3, 6.$$

Figures 8 and 9 show the output regulation errors of the followers. It can be seen that, despite the instability of the exosystem, the errors asymptotically converge to zero. The output trajectories are not shown as they exhibit exponential growth.

VII. CONCLUSION

In this paper, we have proposed a novel observer-based approach for solving the COR problem for heterogeneous MASs. Two COR protocols have been presented, for networks of non-introspective and networks of introspective agents, respectively. The proposed protocols do not require the exchange of the controller states and thus reduce the communication burden. Furthermore, algorithms based on $\mathcal{H}_\infty$ SOF theory and ARE methods are provided for determining the controller gains. It has been proven that for a large class of reference signals the solvability of the COR problem can be guaranteed in advance for: i) introspective agents with arbitrary dynamics, ii) non-introspective agents with stable dynamics.

The future work will be focused in two directions. The first direction is extension of the proposed method to dynamically switching networks as well achieving robust performance in a presence of communication delays in the case of introspective agents. The second direction is investigation of the possibility of applying the proposed method to the leaderless output synchronization problem and output synchronization problem over signed graphs.

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