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RESEARCH ARTICLE

Inequalities and Stability of Stochastic Fuzzy Delayed Cohen–Grossberg Neural Networks

HEBING ZHANG
School of Intelligent Manufacture, Taizhou University, Jiaojiang, Taizhou, Zhejiang 318000, China
e-mail: zhanghebing729@163.com

ABSTRACT Stability is an important indicator for evaluating complex dynamic systems' performance. Many problems in practice are abstracted into the stability of networks. This study examines stochastic fuzzy Cohen-Grossberg neural networks(CGNNs) with delayed pth moment exponential stability and almost sure exponential stability. It is an improvement and supplement to existing work. Our method is based on integral inequality, differential inequality, stochastic analysis theory and Itô's formula, which discusses the system's stability, we have obtained sufficient conditions for system stability, which avoided the construction of complex Lyapunov functions. Moreover, our method does not require that the activation function be bounded, differentiable and monotone, and provides sufficient con· editions decreased conservative. At the same time, it is verified that fuzzy and stochastic terms have positive effects on system stability. Finally, the effectiveness of the results is verified by a simulation example.

INDEX TERMS Cohen–Grossberg neural network, exponential stability, fuzzy neural network, inequality.

I. INTRODUCTION

The neural network provides a new idea for solving control problems and modeling complex systems. Especially when there are uncertainties in the system, neural networks have a strong association and fault tolerance properties with adaptability the advantages of the neural network method can be better reflected. Research on neural networks is crucial to the field of artificial intelligence, and it has attracted significant attention in the fields of brain science, neuroscience, computer science, mathematics, physical science and others. Nonlinear differential equations were applied for the first time to simulate brain dynamics and characterize neural networks, Cohen and Grossberg originally suggested a model for CGNNs [1], and since then the subject has been extensively studied. CGNNs models are more realistic and universal than cellular neural networks, with more prominent advantages, greater potential, and strong self-learning ability. Therefore, a lot of research achievements have been made in many fields, such as smart home, pattern recognition, medical care and old-age care, among which the stability of neural networks is of great significance and has been widely researched by scholars [2], [3], [4], [5].

Much research shows that Fuzzy cellular neural networks (FCNN) maintain local connectivity between cells, effectively solving the contradiction between system complexity and the required accuracy. The establishment of a fuzzy cellular neural network model is a process of simplifying, abstracting and simulating the basic functions of the human brain, so as to create a machine with complete human intelligence and apply it to various fields of production and life, assisting or replacing human beings to complete some difficult and dangerous tasks. The dynamic behavior of an FCNN plays a key role in these applications, especially when stability is a concern. A lot of thoughts have given robust FCNN in recent years [6], [7], [8], [9], [10], [11], [12], [13], [14]. In references [8], the stability of networks with distributed and mixed delays is discussed. FCNN with proportional delay exhibit periodicity, which is highlighted in reference [11]. In 1996, Yang and Yang [12] introduced fuzzy operators into the CGNNs model. There is a growing number of work related to results on fuzzy neural networks (FNNs) in the literature [13], [14]. Fuzzy CGNNs have recently gained popularity in recent years due to their benefits in

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processing images and recognizing patterns [15], [16]. So far, researchers have published results on the dynamic behavior of FCGNNs, including the presence of periodic solutions, and their stability, etc. For example, Sevgen [17] created a new, readily tractable adequate condition to ensure asymptotic stability in the CGNNs model, which is represented by combining nonsingular M-matrix matrices with nonlinear Lipschitz activation functions. The type of stochastic nonlinear FCGNNs unidentified exogenous disturbance was studied by Xie and Zhu [18], and the Lyapunov function and Dynkin's formula were all used to demonstrate that the constructed event-triggered mechanism makes the system under consideration input-to-state stable. According to new findings by Zhu and Li [14], stochastic fuzzy delayed CGNNs exhibit exponential and almost exponential stability. Meng et al. [19] investigated the periodicity of FCGNNs with time delays and impulses to find a solution for the system under consideration. To do this, he used the Lyapunov method, fuzzy theory, and several standards to guarantee period and exponential stability.

Furthermore, stability analysis is an essential and critical part of the field of network synchronization control [20], [21], [22], [23]. In the actual synchronization control problem of a Complex network, due to the inconsistency of the clocks of each node, the change of the network topology and other problems, the network oscillates in the synchronization process, which is very unfavorable to the system. Therefore, determining sufficient conditions for the stability of neural networks is also extremely important in the field of network synchronization control. For example, Kong et al. [20] investigated the synchronization of delayed FCGNNs with parameter uncertainties. To complete the synchronization, they additionally established algebraic criteria, Kong et al. [21] investigated fixed-time synchronization and deduced stable time using Lyapunov-Krasovskii functional approach. Currently, the Lyapunov functional method is widely used and emphasized in network stability research. However, which makes it necessary to detect higher dimensional linear matrix inequalities and increase the computational burden. Construction of Lyapunov- Krasovskii functional approaches require more mature experience and logic. At the same time, if a differential equation under consideration has unbounded terms or unbounded attenuation properties, it is difficult to use the Lyapunov functional method. Therefore, some scholars in the study of stability moved away from Lyapunov's method and used fixed point theory and inequality for stability and reduce conservatism. In 2001, Burton et al. [24] originally tested the stability of neural networks using the fixed-point theory approach. This method was highly praised and developed rapidly. Luo [25] employed fixed-point theory and presented the requirements for the stability in the pth mean as well as in the simple route of mild solutions. Later, this approach was utilized to explore exponential stability [26]. Luo [27] applied linear matrix inequality to show the existence of some oscillatory solutions of CGNNs. Abdelaziz and Chérif [28] proposed impulsive FCGNNs with delays adequate criteria for the global exponential stability of the under-consideration model were derived utilizing an appropriate fixed point theorem. Lu et al. [29] application of generalized Halanay inequality led to the establishment of several innovative delay-dependent adequate conditions. The author used fixed point theory and did not rely on any Lyapunov function or Lyapunov functional approach. The activation function's boundedness and differentiability were not necessary for the outcomes. Based on previous work, by creating inequality structures and employing stochastic analysis methods. Ruan et al. [30] investigated the stability of Hopfield neural networks. The results did not require the construction of complex Lyapunov functions. Recently, Chen et al. [31] dealt with the stability of delayed Hopfield neural networks (SHNNs) employing the fixed-point method with discontinuous and distributed delay, and they produced some innovative findings. However, the fixed-point method had shortcomings due to the use of Hölder inequality at an inappropriate time. Additionally, Sun and Cao [32] investigated the exponential stability of the pth moment of SHNN by using variational parameters and integral inequalities. Wan and Sun [33] introduced this method. They do not require to build of Lyapunov functions. However, they needed to delay functions to be differentiable. Later, Liu et al. [34] adopted two θ methods and discussed the square stability of SHNN stochastic numerical calculations with constant delay. Liu and Zhu [35] and Rathinasamya and Narayanasamy [36] discussed the semimartingale convergence theorem-based discontinuous delay SHNNs with deterministic exponential stability.

In the view of neurophysiology, the response of neural networks to repeat receipt of the same stimulus is not the same, and its essence is random. This paper describes this Stochastic process through Brownian motion. At the same time, in the actual electronic circuit, the delay is an inevitable response. Inspired by the existing research results. This article explores the pth exponential stability and almost sure exponential stability of a class of stochastic fuzzy delayed CGNNs. The following are this paper's main contributions:

(1) The discussion results of integral inequality and delay differential inequality in reference [33] are applied to the stochastic fuzzy delayed CGNNs, which can also be seen as a further generalization of the Halanay inequality, and sufficient conditions for the p-order exponential stability and almost sure exponential stability of the system is obtained. Compared to reference [14], our conclusions are less conservative, and we can see that the method for stochastic fuzzy delayed CGNNs inequality is easier to operate than the design of the Lyapunov function and has more advantages for higher-order stability.

(2) Sufficient conditions of the pth exponential stability and almost sure exponential stability are verified through simulation experiments, and the influence of time delay changes on system stability is observed, which provides a degree of insight for subsequent

research on more complex systems. At the same time, the simulation finds that the system becomes unstable after we remove the fuzzy and stochastic items, proving the advantages of fuzzy systems. Therefore, subsequent research should design improved fuzzy operators to provide increased stability in the system.

The remainder of this essay is structured as follows: we introduce the stochastic FCGNNs model in Section II and provide some essential premises and lemmas. Section III, we provide sufficient criteria for the system's stability using integral inequality and differential inequality. We simulated Section IV to demonstrate the value of our findings, and Section V presents our conclusion. Data Center Infrastructure and Power Consumption.

II. DESCRIPTION OF THE MODEL

In this study, we take into account the class of stochastic FCGNNs that includes:

$$\begin{cases} dx_i(t) = -a_i(x_i(t))\Big[b_i(x_i(t)) - \wedge_{j=1}^{n} c_{ij} f_j(t, x_j(t)) \\ \qquad - \vee_{j=1}^{n} d_{ij} f_j(t, x_j(t)) \\ \qquad - \wedge_{j=1}^{n} \alpha_{ij} g_j(t, x_j(t-\tau(t))) \\ \qquad - \vee_{j=1}^{n} \beta_{ij} g_j(t, x_j(t-\tau(t)))\Big] dt \\ \qquad + \sum_{j=1}^{n} \sigma_{ij}(t, x_j(t), x_j(t, t-\tau(t))) dw_j(t) \\ x_i(s) = \phi_i(s), \quad s \in [-\tau, 0] \end{cases} \tag{1}$$

The structural diagram is shown in Figure 1:

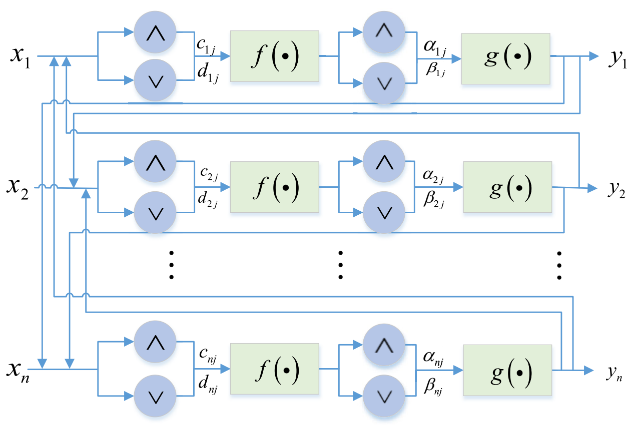

FIGURE 1. Fuzzy cohen grossberg neural network structure diagram.

For all $i, j = 1, 2, \cdots, n$, $t \geq 0$, where $x_i(t)$ represents the ith neuron's state at time t, $a_i(\cdot)$ is a representation of the ith unit's amplification function at time t. and $b_i(\cdot)$ is the behaved function. The connection weight strengths of the jth unit on the ith unit at time t are represented by the constants $c_{ij}, d_{ij}, \alpha_{ij}$ and β_{ij}. $f_j(\cdot)$ and $g_j(\cdot)$ represent the jth unit's neuron activation processes at time t. $\sigma_{ij} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ Borel-measurable function describes the noise perturbation. $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operations, respectively. $w_j(t)$ is a complete probability space $(\Omega, \mathscr{F}, P)$ as the scalar standard Brownian motions with a natural filtration $\{\mathscr{F}_t\}_{t\geq 0}$. The filtration is right continuous and $\mathscr{F}_0$ contains all $\mathbb{P}$-null sets. The time delay $\tau > 0$. Let $\mathscr{C} \triangleq \mathscr{C}([-\tau, 0]; \mathbb{R}^n)$ be the family of all continuous $\mathbb{R}^n-$valued functions and ϕ is a Banach space with P-norm defined on $[-\tau, 0]$. Define:$\|\phi\| = \left(\sup_{s\in[-\tau,0]} \sum_{i=1}^{n} |\phi_i(s)|^p\right)^{\frac{1}{p}}$, and let $\mathscr{C}_{F_0}^{B}([-\tau, 0]; \mathbb{R}^n)$ be the family of all $\{\mathscr{F}_0\}$-measurable C-value stochastic process $\xi = \{\xi(\theta) : \theta \in [-\tau, 0]\}$.

Definition 1 ([37]): The trivial solution of FCGNNs (1) is pth moment exponentially stable, if there exist $\gamma > 0$ and $\xi > 0$ such that $\mathbb{E}|x_i(t, \phi)|^p \leq \xi \max_{j\in J_n} \left\{\mathbb{E}\|\phi_j\|^p\right\} e^{-\gamma t}, t \geq 0$, holds for any $\phi_i \in \mathscr{C}_{F_0}^{B}([-\tau_i, 0]; \mathbb{R}), i \in J_n$.

Definition 2 ([37]): The trivial solution of FCGNNs (1) is almost surely exponentially stable, If there exist $\gamma > 0$ such that $\limsup_{t\to+\infty} \frac{1}{t} \ln|x_i(t, \phi)| \leq -\gamma$, holds for any $\phi_i \in \mathscr{C}_{\mathscr{F}_0}^{B}([-\tau_i, 0]; \mathbb{R}), i \in J_n$.

Assumption 1: There exist $\widehat{a}_i$, $\breve{a}_i$ and θ_i such that $0 < \widehat{a}_i \leq a_i(v_i) \leq \breve{\alpha}_i$ and $v_i b_i(v_i) \geq \theta_i v_i^2$, for any $v_i \in \mathbb{R}$, $i = 1, 2, \cdots, n$.

Assumption 2: f and g are bounded functions, there exist constants $M_i > 0$ and $N_i > 0$ satisfying the Lipschitz condition $\frac{|f(u)-f(v)|}{|u-v|} \leq M_i$, $\frac{|g(u)-g(v)|}{|u-v|} \leq N_i$, hold for any $u, v \in \mathbb{R}$, $i = 1, 2, \cdots, n$.

Assumption 3: There are constants $\mu_{ij} \geq 0$ and $\nu_{ij} \geq 0$ such that

$$\sigma_{ij}^2(t, x_j(t), x_j(t-\tau(t))) \leq \mu_{ij} x_j^2(t) + \upsilon_{ij} x_j^2(t-\tau(t)) \tag{2}$$

for each $x_j(t), x_j(t-\tau(t)) \in \mathbb{R}$, $\sigma_{ij}(t, 0, 0) = 0$, $\sigma_{ij}(0, 0, 0) \equiv 0$, $i, j \in J_n$, $t \geq 0$.

Assumption 4: $a_j(0) \equiv 0$, or $b_j(0) = f_j(0, 0) = g_j(0, 0) \equiv 0$.

Assumption 5:

$$5^{p-1}\left[\widehat{a}_i^{2-p}\left(\sum_{j=1}^{n} c_{ij} M_j\right)^P + \widehat{a}_i^{2-p}\left(\sum_{j=1}^{n} \alpha_{ij} N_j\right)^P + \widehat{a}_i^{2-p}\left(\sum_{j=1}^{n} d_{ij} M_j\right)^P + \widehat{a}_i^{2-p}\left(\sum_{j=1}^{n} \beta_{ij} N_j\right)^P + \left(2\widehat{a}_i\right)^{-(\frac{p}{2}-1)} n^{p-1} \sum_{j=1}^{n} (\mu_{ij} + \upsilon_{ij})^{\frac{p}{2}}\right] < 1 \tag{3}$$

Lemma 1: Assuming that u and $v$are two state variables in the system (1), the following inequality holds:

$$\left|\mathop{\wedge}_{j=1}^{n} c_{ij} f_j(u) - \mathop{\wedge}_{j=1}^{n} c_{ij} f_j(v)\right| \leq \sum_{j=1}^{n} |c_{ij}| \left|f_j(u) - f_j(v)\right|,$$

$$\left|\mathop{\wedge}_{j=1}^{n} \alpha_{ij} g_j(u) - \mathop{\wedge}_{j=1}^{n} \alpha_{ij} g_j(v)\right| \leq \sum_{j=1}^{n} |\alpha_{ij}| \left|g_j(u) - g_j(v)\right|,$$

$$\left|\bigvee_{j=1}^{n} d_{ij}f_j(u) - \bigvee_{j=1}^{n} d_{ij}f_j(v)\right| \le \sum_{j=1}^{n} |d_{ij}| \, |f_j(u) - f_j(v)|,$$

$$\left|\bigvee_{j=1}^{n} \beta_{ij}g_j(u) - \bigvee_{j=1}^{n} \beta_{ij}g_j(v)\right| \le \sum_{j=1}^{n} |\beta_{ij}| \, |g_j(u) - g_j(v)|$$

Lemma 2: [30]. For system

$$\begin{cases} x_i(t) \le K\psi_i(0)e^{-\rho_i t} + \sum_{j=1}^{n} p_{ij}\int_0^t e^{-\rho_i(t-\tau)} \\ \qquad \times \sup_{s-\zeta_{ij}(\tau)\le v\le\tau} x_j(v)\,d\tau \\ \qquad + \sum_{j=1}^{n} q_{ij}\int_0^t x_j(\tau)e^{-\rho_i(t-\tau)}d\tau \\ x_i(t) = \psi_i(t) \in \mathscr{C}([-\zeta_i, 0]; \mathbb{R}) \end{cases} \tag{4}$$

For every $i, j \in J_n$, $x_i(t) \ge 0$, $\zeta_{ij}(\tau) \in [0, \zeta_{ij}]$ and $\tau \ge 0$, $\zeta_{ij} \le \zeta_j$, and $p_{ij} \ge 0$, $q_{ij} \ge 0$, $\rho_i > 0$. We complementarily define $\zeta \triangleq \max_{i,j\in J_n}\{\zeta_{ij}\}$. Suppose, $-\rho_i + \sum_{j=1}^{n} p_{ij} + \sum_{j=1}^{n} q_{ij} < 0$, then there exist a constant $\lambda^* > 0$ such that $\max_{i\in J_n}\{|x_i(t)|\} \le \max_{j\in J_n}\{\|\psi_j\|\}e^{-\lambda^* t}$, $t \in [-\zeta, +\infty)$.

Lemma 3: [30]. For system

$$\begin{cases} D^+x_i(t) \le \sum_{j=1}^{n} p_{ij}x_j(t) + \sum_{j=1}^{n} q_{ij}\sup_{t-\zeta_{ij}(t)\le s\le t} x_j(s) \\ \qquad - \rho_i x_i(t),\ t \ge 0 \\ x_i(t) = \psi_i(t) \in \mathscr{C}([-\zeta_i, 0]; \mathbb{R}),\ -\zeta_i \le t \le 0 \end{cases} \tag{5}$$

where D^+ is the Dini derivative. We complimentary define $x_i(t) \triangleq \psi_i(-\zeta_i)$ for $-\zeta \le t \le -\zeta_i$, where $\zeta \triangleq \max_{i,j\in J_n}\{\zeta_{ij}\}$. Suppose $-\rho_i + \sum_{j=1}^{n} p_{ij} + \sum_{j=1}^{n} q_{ij} < 0$, then there exist $\lambda^* > 0$ such that $\max_{i\in J_n}\{|x_i(t)|\} \le \max_{j\in J_n}\{\|\psi_j\|\}e^{-\lambda^* t}$, $t \in [-\zeta, +\infty)$.

Remark 1: In [14] and [38], complex Lyapunov functions were constructed and complex matrix norms were defined, respectively, and the results were not easy to verify. We quoted the inequality conclusion given in reference [37] and tried to apply this conclusion to the stability analysis of fuzzy delayed CGNNs, avoiding the construction of complex Lyapunov functions and the definition of complex matrix norms. Sufficient conditions for stability with lower conservatism are obtained.

III. INEQUALITIES AND STABILITY

Be sure that the symbols in your equation have been defined before the equation appears or immediately following. Italicize symbols (T might refer to temperature, but T is the unit tesla). Refer to "(1)," not "Eq. (1)" or "equation (1)," except at the beginning of a sentence: "Equation (1) is ...."

Theorem 1: Then FCGNNs (1) is exponentially stable at the pth ($p \ge 2$) moment, assuming assumptions 1-4 hold.

Proof: From Assumption 1, we have

$$\begin{aligned} dx_i(t) \le -\widehat{a}_i\Big[&\theta x_i(t) - \wedge_{j=1}^{n} c_{ij}f_j(t, x_j(t)) - \vee_{j=1}^{n} d_{ij}f_j \\ &\times (t, x_j(t)) - \wedge_{j=1}^{n}\alpha_{ij}g_j(t, x_j(t-\tau)) \\ &- \vee_{j=1}^{n}\beta_{ij}g_j(t, x_j(t-\tau))\Big]dt \\ &+ \sum\nolimits_{j=1}^{n}\sigma_{ij}(t, x_j(t, t-\tau(t)))\,dw_j(t) \end{aligned} \tag{6}$$

Multiply both system (6) sides by $e^{\widehat{a}_i t}$, and integrate between 0 and t, which gives

$$\begin{aligned} x_i(t) \le\ & e^{-\widehat{a}_i t}\phi_i(0) + \widehat{a}_i\int_0^t e^{-\widehat{a}_i(t-s)} \wedge_{j=1}^{n} c_{ij}f_j(s, x_j(s))\,ds \\ &+ \widehat{a}_i\int_0^t e^{-\widehat{a}_i(t-s)} \wedge_{j=1}^{n}\alpha_{ij}g_j(s, x_j(s-\tau))\,ds \\ &+ \widehat{a}_i\int_0^t e^{-\widehat{a}_i(t-s)} \vee_{j=1}^{n} d_{ij}f_j(s, x_j(s))\,ds \\ &+ \widehat{a}_i\int_0^t e^{-\widehat{a}_i(t-s)} \vee_{j=1}^{n}\beta_{ij}g_j(s, x_j(s-\tau))\,ds \\ &+ \int_0^t e^{-\widehat{a}_i(t-s)}\sum\nolimits_{j=1}^{n}\sigma_{ij}(s, x_j(s, s-\tau))dw_j(s) \\ \triangleq\ & S_{i1}(t) + S_{i2}(t) + S_{i3}(t) + S_{i4}(t) + S_{i5}(t) + S_{i6}(t) \end{aligned} \tag{7}$$

There exists $0 < k < 1$ such that

$$\begin{aligned} \frac{5^{p-1}}{(1-k)^{p-1}}\Bigg[&\widehat{a}_i^{2-p}\left(\sum_{j=1}^{n} c_{ij}M_j\right)^p + \widehat{a}_i^{2-p}\left(\sum_{j=1}^{n}\alpha_{ij}N_j\right)^p \\ &+ \widehat{a}_i^{2-p}\left(\sum_{j=1}^{n} d_{ij}M_j\right)^p + \widehat{a}_i^{2-p}\left(\sum_{j=1}^{n}\beta_{ij}N_j\right)^p \\ &+ \left(2\widehat{a}_i\right)^{-(\frac{p}{2}-1)} n^{p-1}\sum_{j=1}^{n}(\mu_{ij} + \nu_{ij})^{\frac{p}{2}}\Bigg] < 1 \end{aligned} \tag{8}$$

Apply Hölder inequality

$$\begin{aligned} &\mathbb{E}|x_i(t)|^p \\ &\le \frac{\mathbb{E}|S_{i1}(t)|^p}{k^{p-1}} + \frac{\mathbb{E}|S_{i2}(t) + S_{i3}(t) + S_{i4}(t) + S_{i5}(t) + S_{i6}(t)|^p}{(1-k)^{p-1}} \\ &\le \frac{\mathbb{E}|S_{i1}(t)|^p}{k^{p-1}} + 5^{p-1}\frac{\mathbb{E}|S_{i2}(t)|^p}{k^{p-1}} + 5^{p-1}\frac{\mathbb{E}|S_{i3}(t)|^p}{k^{p-1}} \\ &\quad + 5^{p-1}\frac{\mathbb{E}|S_{i1}(t)|^p}{k^{p-1}} + 5^{p-1}\frac{\mathbb{E}|S_{i4}(t)|^p}{k^{p-1}} + 5^{p-1}\frac{\mathbb{E}|S_{i5}(t)|^p}{k^{p-1}} \\ &\quad + 5^{p-1}\frac{\mathbb{E}|S_{i6}(t)|^p}{k^{p-1}} \end{aligned} \tag{9}$$

Assumption 2 and lemma 1

$$\mathbb{E}|S_{i1}(t)|^p = e^{-p\widehat{a}_i t}\mathbb{E}|\phi_i(0)|^p \le e^{-\widehat{a}_i t}\mathbb{E}|\phi_i(0)|^p,\ t \ge 0 \tag{10}$$

$$
\begin{aligned}
&\mathbb{E}\left|S_{i2}(t)\right|^{p}\\
&=\mathbb{E}\left|\widehat{a}_i\int_0^t e^{-\widehat{a}_i(t-s)}\wedge_{j=1}^{n}c_{ij}f_j\left(t,x_j(s)\right)ds\right|^{p}\\
&\leq\mathbb{E}\left[\widehat{a}_i\int_0^t e^{-\widehat{a}_i(t-s)}\wedge_{j=1}^{n}\left|c_{ij}\right|\left|f_j\left(t,x_j(s)\right)-f_j(t,0)\right|ds\right]^{p}\\
&\leq\mathbb{E}\left[\widehat{a}_i\int_0^t e^{-\widehat{a}_i(t-s)}\sum_{j=1}^{n}\left|c_{ij}\right|M_j\left|x_j(s)\right|ds\right]^{p}\\
&=\mathbb{E}\left[\widehat{a}_i\sum_{j=1}^{n}\left|c_{ij}\right|M_j\int_0^t e^{-\widehat{a}_i(t-s)}\left|x_j(s)\right|ds\right]^{p}\\
&=\widehat{a}_i\left(\sum_{j=1}^{n}\left(\left|c_{ij}\right|M_j\right)\right)^{p-1}\\
&\quad\times\left(\sum_{j=1}^{n}\left(\left|c_{ij}\right|M_j\right)\mathbb{E}\left[\int_0^t e^{-\widehat{a}_i(t-s)}\left|x_j(s)\right|ds\right]^{p}\right)\\
&=\widehat{a}_i\left(\sum_{j=1}^{n}\left(\left|c_{ij}\right|M_j\right)\right)^{p-1}\\
&\quad\times\left(\sum_{j=1}^{n}\left(\left|c_{ij}\right|M_j\right)\mathbb{E}\left[\int_0^t e^{\frac{(1-p)\widehat{a}_i(t-s)}{p}}e^{\frac{-\widehat{a}_i(t-s)}{p}}\left|x_j(s)\right|ds\right]^{p}\right)\\
&=\widehat{a}_i\left(\sum_{j=1}^{n}\left(\left|c_{ij}\right|M_j\right)\right)^{p-1}\left(\int_0^t e^{-\widehat{a}_i(t-s)}ds\right)^{p-1}\\
&\quad\times\left(\sum_{j=1}^{n}\left|c_{ij}\right|M_j\int_0^t\mathbb{E}\left|x_j(s)\right|^{p}e^{-\widehat{a}_i(t-s)}ds\right)\\
&=\widehat{a}_i^{2-p}\left(\sum_{j=1}^{n}\left(\left|c_{ij}\right|M_j\right)\right)^{p}\int_0^t\mathbb{E}\left|x_j(s)\right|^{p}e^{-\widehat{a}_i(t-s)}ds
\end{aligned}\tag{11}
$$

Similarly, we get

$$
\begin{aligned}
\mathbb{E}\left|S_{i3}(t)\right|^{p}\leq\widehat{a}_i^{2-p}&\left(\sum_{j=1}^{n}\left(\left|d_{ij}\right|N_j\right)\right)^{p}\int_0^t e^{-\widehat{a}(t-s)}\\
&\times\sup_{s-\tau_{ij}^{(1)}\leq v\leq s}\mathbb{E}\left|x_j(s)\right|^{p}ds
\end{aligned}\tag{12}
$$

$$
\begin{aligned}
\mathbb{E}\left|S_{i4}(t)\right|^{p}\leq\widehat{a}^{2-p}&\left(\sum_{j=1}^{n}\left(\left|\alpha_{ij}\right|M_j\right)\right)^{p}\int_0^t e^{-\widehat{a}(t-s)}\\
&\times\mathbb{E}\left|x_j(s)\right|^{p}ds
\end{aligned}\tag{13}
$$

$$
\begin{aligned}
\mathbb{E}\left|S_{i5}(t)\right|^{p}\leq\widehat{a}^{2-p}&\left(\sum_{j=1}^{n}\left(\left|\beta_{ij}\right|N_j\right)\right)^{p}\int_0^t e^{-\widehat{a}(t-s)}\\
&\times\sup_{s-\tau_{ij}^{(1)}\leq v\leq s}\mathbb{E}\left|x_j(s)\right|^{p}ds
\end{aligned}\tag{14}
$$

For $p\geq 2$, assumption 3 and Holder inequality lead to the following conclusion:

$$
\begin{aligned}
&\mathbb{E}\left|I_{i6}(t)\right|^{p}\\
&=\mathbb{E}\left|\sum_{j=1}^{n}\int_0^t e^{-\widehat{a}_i(t-s)}\sigma_{ij}\left(x_j(t,t-\tau(t))\right)dw_j(s)\right|^{p}\\
&\leq n^{p-1}\sum_{j=1}^{n}\mathbb{E}\left|\int_0^t e^{-\widehat{a}_i(t-s)}\sigma_{ij}\left(x_j(t,t-\tau(t))\right)dw_j(s)\right|^{p}\\
&=n^{p-1}\sum_{j=1}^{n}\mathbb{E}\left[\left|\int_0^t e^{-\widehat{a}_i(t-s)}\sigma_{ij}\left(x_j(t,t-\tau(t))\right)dw_j(s)\right|^{2}\right]^{\frac{p}{2}}\\
&\leq n^{p-1}\sum_{j=1}^{n}\mathbb{E}\left[\int_0^t e^{-2\widehat{a}_i(t-s)}\left|\sigma_{ij}\left(x_j(t,t-\tau(t))\right)\right|^{2}ds\right]^{\frac{p}{2}}\\
&\leq n^{p-1}\sum_{j=1}^{n}\mathbb{E}\left[\int_0^t e^{-2\widehat{a}_i(t-s)}\sum_{j=1}^{n}\left(\mu_{ij}x_j^2(s)+\upsilon_{ij}x_j^2(s-\tau)\right)ds\right]^{\frac{p}{2}}\\
&=n^{p-1}\sum_{j=1}^{n}\mathbb{E}\left[\int_0^t e^{-\frac{2p-4}{p}\widehat{a}_i(t-s)}e^{-\frac{4}{p}\widehat{a}_i(t-s)}\sum_{j=1}^{n}\right.\\
&\qquad\left.\times\left(\mu_{ij}x_j^2(s)+\upsilon_{ij}x_j^2(s-\tau)\right)ds\right]^{\frac{p}{2}}\\
&\leq n^{p-1}\left(2\widehat{a}_i\right)^{-\left(\frac{p}{2}-1\right)}\\
&\quad\times\left[\int_0^t e^{-2\widehat{a}_i(t-s)}\sum_{j=1}^{n}\mathbb{E}\left(\mu_{ij}x_j^2(s)+\upsilon_{ij}x_j^2(s-\tau)\right)^{\frac{p}{2}}ds\right]\\
&\leq n^{p-1}\left(2\widehat{a}_i\right)^{-\left(\frac{p}{2}-1\right)}\sum_{j=1}^{n}\left(\mu_{ij}+\upsilon_{ij}\right)^{\frac{p}{2}}\int_0^t\\
&\quad\times\sup_{s-\tau\leq v\leq s}\mathbb{E}\left|x_j(s)\right|^{p}e^{-\widehat{a}_i(t-s)}ds
\end{aligned}\tag{15}
$$

Thus, from Assumption 4, we have

$$
\begin{aligned}
&\mathbb{E}\left|x_i(t)\right|^{p}\\
&\leq\frac{\mathbb{E}\left|S_{i1}(t)\right|^{p}}{k^{p-1}}\\
&\quad+\frac{5^{p-1}\mathbb{E}\left|S_{i2}(t)+S_{i3}(t)+S_{i4}(t)+S_{i5}(t)+S_{i6}(t)\right|^{p}}{(1-k)^{p-1}}\\
&\leq\frac{e^{-\widehat{a}_it}\mathbb{E}\left|\phi_i(0)\right|^{p}}{k^{p-1}}+\frac{5^{p-1}}{(1-k)^{p-1}}\widehat{a}_i^{2-p}\\
&\quad\times\left[\left(\sum_{j=1}^{n}\left(\left|c_{ij}\right|M_j\right)\right)^{p}\int_0^t\mathbb{E}\left|x_j(s)\right|^{p}e^{-\widehat{a}_i(t-s)}ds\right.\\
&\qquad+\left(\sum_{j=1}^{n}\left(\left|d_{ij}\right|N_j\right)\right)^{p}\int_0^t\sup_{s-\tau_{ij}^{(1)}\leq v\leq s}\mathbb{E}\left|x_j(s)\right|^{p}
\end{aligned}
$$

$$\times e^{-\widehat{a}_i(t-s)}ds + \left(\sum_{j=1}^{n}\left(\left|\alpha_{ij}\right| M_j\right)\right)^{p-1} \int_0^t \mathbb{E}\left|x_j(s)\right|^p$$
$$\times e^{-\widehat{a}_i(t-s)}ds + \left(\sum_{j=1}^{n}\left(\left|\beta_{ij}\right| N_j\right)\right)^{p}$$
$$\left.\times \int_0^t \sup_{s-\tau_{ij}^{(1)}\le v\le s} \mathbb{E}\left|x_j(s)\right|^p e^{-\widehat{a}_i(t-s)}ds\right]$$
$$+ \frac{5^{p-1}}{(1-k)^{p-1}} n^{p-1}\left(2\widehat{a}_i\right)^{-\left(\frac{p}{2}-1\right)} \sum_{j=1}^{n}\left(\mu_{ij}+\upsilon_{ij}\right)^{\frac{p}{2}}$$
$$\times \int_0^t e^{-\widehat{a}_i(t-s)} \sup_{s-\tau\le v\le s} \mathbb{E}\left|x_j(s)\right|^p ds \tag{16}$$

We are aware that Lemma 2's prerequisites have all been met. Next, there are $\xi > 0$ and $\gamma > 0$ such that

$$\max_{i\in J_n}\left\{\mathbb{E}\left|x_i(t)\right|^p\right\} \le \xi e^{-\gamma t} \max_{j\in J_n}\left\{\mathbb{E}\left\|\phi_j\right\|^p\right\}, \quad t \ge -\tau \tag{17}$$

Remark 2: By constructing Lyapunov Krasovskii functions, references [14] and [38] proved that fuzzy delayed CGNNs are exponentially stable in different cases, that is, they are only applicable to the case of $p = 2$. Higher-order exponential stability requires higher requirements for the design of Lyapunov Krasovskii functions. We also apply the inequality method to $p > 2$, so we can see the ease of use of the method in this paper.

Remark 3: Zhu and Li [14] considering the special case of equation (1), degenerate $f_j\left(t, x_j(t)\right)$ to $f_j\left(x_j(t)\right)$, degenerate $g_j\left(t, x_j(t-\tau(t))\right)$ to $g_j\left(x_j(t-\tau(t))\right)$, sufficient conditions for the stability of the mean square index are given:

$$2m_i\widehat{a}_i\theta_i - \lambda m_i - n_i - m_i\breve{a}_i\sum_{j=1}^{n}\left|c_{ij}\right|M_j - \sum_{j=1}^{n} m_j\breve{a}_j\left|c_{ji}\right|M_i$$
$$- m_i\breve{a}_i\sum_{j=1}^{n}\left|d_{ij}\right|N_j - m_i\breve{a}_i\sum_{j=1}^{n}\left|d_{ij}\right|M_j - \sum_{j=1}^{n} m_j\breve{a}_j\left|d_{ji}\right|M_i$$
$$- m_i\breve{a}_i\sum_{j=1}^{n}\left|\beta_{ij}\right|N_j - \sum_{j=1}^{n} m_j\mu_{ji} > 0$$

and

$$e^{-\rho\tau}n_i - \sum_{j=1}^{n} m_i\left|d_{ji}\right|N_i - \sum_{j=1}^{n} m_j\left|\beta_{ji}\right|N_i - \sum_{j=1}^{n} m_i\upsilon_{ji} > 0,$$

It can be seen that our results are easier to achieve. Mao [37] considered the special case where $a_i\left(x_i(t)\right) = 1$, Our results are equivalent to generalizing literature [30].

Theorem 2: Under assumptions 1–3 and

$$-\widehat{a}_i + \sum_{j=1}^{n}\left(c_{ij}M_j + \alpha_{ij}N_j + d_{ij}M_j + \beta_{ij}N_j + \frac{p-1}{2}\left(\mu_{ij}+v_{ij}\right)\right) < 0, i \in J_n,$$

then $\max_{i\in J_n}\left\{\mathbb{E}\left|x_i(t)\right|^p\right\} \le \max_{j\in J_n}\left\{\mathbb{E}\left\|\phi_j\right\|^p\right\} e^{-\lambda^* t}$, where λ^* is the following equations' minimal solution

$$\lambda - p\widehat{a}_i + \sum_{j=1}^{n}\Big((p-1)\left(\alpha_{ij}+\beta_{ij}\right)N_j + p\left(c_{ij}+d_{ij}\right)M_j$$
$$+\frac{\mu_{ij}}{2}p(p-1) + \frac{\upsilon_{ij}}{2}(p-1)(p-2)\Big)$$
$$+ \sum_{j=1}^{n}\left(\alpha_{ij}N_j + \beta_{ij}N_j + (p-1)\upsilon_{ij}\right)e^{\lambda\tau_{ij}} = 0, i \in J_n \tag{18}$$

Proof: Using Itô's formula, we obtain

$$E\left|x_i(t)\right|^p = \int_0^t E\mathscr{L}\left|x_i(s)\right|^p ds + E\left|\phi_i(0)\right|^p \tag{19}$$

For sufficiently small Δt and any $t \ge 0$, we get

$$D^+E\left|x_i(t)\right|^p = E\mathscr{L}\left|x_i(t)\right|^p \tag{20}$$

Itô's formula, Young's inequality and Assumptions 1–3 yield

$$D^+\mathbb{E}\left|x_i(t)\right|^p$$
$$\le \mathbb{E}\Big[-p\widehat{a}_i\left|x_i(t)\right|^p$$
$$+ \wedge_{j=1}^{n} c_{ij}f_j\left(t, x_j(t)\right)\operatorname{sgn}\left\{x_i(t)\right\}p\left(x_i(t)\right)^{p-1}$$
$$+ \wedge_{j=1}^{n}\alpha_{ij}g_j\left(t, x_j(t-\tau)\right)p\left(x_i(t)\right)^{p-1}\operatorname{sgn}\left\{x_i(t)\right\}$$
$$+ \vee_{j=1}^{n} d_{ij}f_j\left(t, x_j(t)\right)\operatorname{sgn}\left\{x_i(t)\right\}p\left(x_i(t)\right)^{p-1}$$
$$+ \vee_{j=1}^{n}\beta_{ij}g_j\left(t, x_j(t-\tau)\right)p\left(x_i(t)\right)^{p-1}\operatorname{sgn}\left\{x_i(t)\right\}$$
$$+ \sum_{j=1}^{n}\frac{p(p-1)}{2}\left|\sigma_{ij}(s, x(s, s-\tau))\right|^2\left|x_i(t)\right|^{p-2}\Big]$$
$$\le -p\widehat{a}_i\mathbb{E}\left|x_i(t)\right|^p + \sum_{j=1}^{n} c_{ij}M_j\mathbb{E}\left[\left|x_j(s)\right|p\left|x_i(t)\right|^{p-1}\right]$$
$$+ \sum_{j=1}^{n}\alpha_{ij}N_j\mathbb{E}\left[\left|x_j(t-\tau)\right|p\left|x_i(t)\right|^{p-1}\right]$$
$$+ \sum_{j=1}^{n} d_{ij}M_j\mathbb{E}\left[\left|x_j(s)\right|p\left|x_i(t)\right|^{p-1}\right]$$
$$+ \sum_{j=1}^{n}\beta_{ij}N_j\mathbb{E}\left[\left|x_j(t-\tau)\right|p\left|x_i(t)\right|^{p-1}\right]$$
$$+ \sum_{j=1}^{n}\mathbb{E}\Big[\Big(\left|x_j(t)\right|^2\mu_{ij} + \frac{p(p-1)}{2}\left|x_i(t)\right|^{p-2}$$
$$\times\left|x_j(t-\tau)\right|^2\upsilon_{ij}\Big)\Big]$$
$$\le \Big\{-p\widehat{a}_i + \sum_{j=1}^{n}(p-1)\Big[c_{ij}M_j + \alpha_{ij}N_j + d_{ij}M_j + \beta_{ij}N_j$$
$$+\frac{(p-2)\left(\mu_{ij}+\upsilon_{ij}\right)}{2}\Big]\Big\}\mathbb{E}\left|x_i(t)\right|^p$$

$$+\sum_{j=1}^{n}\left(c_{ij}M_j+d_{ij}M_j+\mathbb{E}\left|x_j(t)\right|^p\mu_{ij}(p-1)\right)$$
$$+\sum_{j=1}^{n}\left((p-1)\upsilon_{ij}+\left(\alpha_{ij}+\beta_{ij}\right)N_j\right)\sup_{t-\tau\le s\le t}\mathbb{E}\left|x_j(s)\right|^p \tag{21}$$

As a result, Lemma 2's presumptions are all met. Thus, we have

$$\max_{i\in J_n}\left\{E\left|x_i(t)\right|^p\right\}\le\max_{j\in J_n}\left\{E\left\|\phi_J\right\|^p\right\}e^{-\lambda^* t},t\in[-\tau,+\infty) \tag{22}$$

This suggests that FCGNNs (1) is pth instant exponentially stable. The evidence is finished.

Remark 4: The condition of Theorem 2 is weaker than that of Theorem 1, and the result is sharper than that of Theorem 1.

Remark 5: Theorems 1 and 2 reveal that stochastic fuzzy delayed CGNNs are exponentially stable at the p-order level. The criteria given in [39], [40], and [41] are invalid in our results because the part of fuzzy logic is not considered, and [28], [42], and [43] ignores the Brownian mot ion existing in the system. The criteria obtained in [18] are not valid in Theorem 1 and Theorem 2 because exponential stability has not been studied.

In the proof of Theorem 2, we used Lemma 1, however, we replace the inequalities with if

$$\left|\mathop{\wedge}_{j=1}^{n}\eta f_j(u)-\mathop{\wedge}_{j=1}^{n}\eta f_j(v)\right|\le\min_{1\le j\le n}|\eta|\left|f_j(u)-f_j(v)\right| \tag{23}$$
$$\left|\mathop{\vee}_{j=1}^{n}\eta f_j(u)-\mathop{\vee}_{j=1}^{n}\eta f_j(v)\right|\le\max_{1\le j\le n}|\eta|\left|f_j(u)-f_j(v)\right| \tag{24}$$

is used in place of the aforementioned disparities.

Corollary 1: Suppose Assumptions 1–3 and

$$-\widehat{a}_i+\max_{1\le j\le n}\left(c_{ij}M_j+\alpha_{ij}N_j\right)+\min_{1\le j\le n}\left(d_{ij}M_j+\beta_{ij}N_j\right)+\frac{p-1}{2}\sum_{j=1}^{n}\left(\mu_{ij}+\upsilon_{ij}\right)<0,i\in J_n,$$

then $\max_{i\in J_n}\{E\left|x_i(t)\right|^p\}\le\max_{j\in J_n}\left\{E\left\|\phi_j\right\|^p\right\}e^{-\lambda^* t}$, where λ^* is the following equations' minimal solution.

$$\lambda-p\widehat{a}_i+\min_{1\le j\le n}\left(pc_{ij}M_j+(p-1)\alpha_{ij}N_j\right)+\min_{1\le j\le n}\alpha_{ij}N_j$$
$$+\max_{1\le j\le n}\beta_{ij}N_j\sum_{j=1}^{n}\left((p-1)\upsilon_{ij}\right)e^{\lambda\tau}$$
$$+\max_{1\le j\le n}\left((p-1)\beta_{ij}N_j+pd_{ij}M_j\right)$$
$$\times\left(\frac{p(p-1)}{2}\sum_{j=1}^{n}\mu_{ij}+\frac{(p-1)(p-2)}{2}\sum_{j=1}^{n}\upsilon_{ij}\right)=0 \tag{25}$$

Remark 6: Replacing Lemma 1 with Equations (23) – (24), the result is obvious, and the proof process is the same as Theorem 2.

Theorem 3: Assume that assumptions 1-3 and

$$-\widehat{a}_i+\sum_{j=1}^{n}\left(c_{ij}M_j+\alpha_{ij}N_j+b_{ij}M_j+\beta_{ij}N_j+\frac{1}{2}\left(\mu_{ij}+\upsilon_{ij}\right)\right)<0,i\in J_n$$

are true., then it is almost surely exponentially stable for system(2. 1).

Proof: Where $t\in[N,N+1]$, assuming N is a big enough number, then

$$x_i(t)\le e^{-\widehat{a}_i t}x_i(N)+\widehat{a}_i\int_N^t e^{-\widehat{a}_i(t-s)}\wedge_{j=1}^{n}c_{ij}f_j\left(s,x_j(s)\right)ds$$
$$+\widehat{a}_i\int_N^t e^{-\widehat{a}_i(t-s)}\wedge_{j=1}^{n}\alpha_{ij}g_j\left(s,x_j(s-\tau)\right)ds$$
$$+\widehat{a}_i\int_N^t e^{-\widehat{a}_i(t-s)}\vee_{j=1}^{n}d_{ij}f_j\left(s,x_j(s)\right)ds$$
$$+\widehat{a}_i\int_N^t e^{-\widehat{a}_i(t-s)}\vee_{j=1}^{n}\beta_{ij}g_j\left(s,x_j(s-\tau)\right)ds$$
$$+\int_N^t e^{-\widehat{a}_i(t-s)}\sum_{j=1}^{n}\sigma_{ij}\left(s,x_j(s,s-\tau)\right)dw_j(s)$$
$$\triangleq K_{i1}(t)+K_{i2}(t)+K_{i3}(t)+K_{i4}(t)+K_{i5}(t)+K_{i6}(t)\ i\in J_n \tag{26}$$

Therefore, for a fixed $\varepsilon_N>0$, we get

$$P\{\sup|x_i(t)|>\varepsilon_N\}$$
$$\le P\left\{\sup_{N\le t\le N+1}\left|e^{-\widehat{a}_i t}x_i(N)\right|>\frac{\varepsilon_N}{6}\right\}$$
$$+P\left\{\sup\left|\widehat{a}_i\int_N^t e^{-\widehat{a}_i(t-s)}\wedge_{j=1}^{n}c_{ij}f_j\left(s,x_j(s)\right)ds\right|>\frac{\varepsilon_N}{6}\right\}$$
$$+P\left\{\sup\left|\widehat{a}_i\int_N^t e^{-\widehat{a}_i(t-s)}\wedge_{j=1}^{n}\alpha_{ij}g_j\left(s,x_j(s-\tau)\right)ds\right|>\frac{\varepsilon_N}{6}\right\}$$
$$+P\left\{\sup\left|\widehat{a}_i\int_N^t e^{-\widehat{a}_i(t-s)}\vee_{j=1}^{n}d_{ij}f_j\left(s,x_j(s)\right)ds\right|>\frac{\varepsilon_N}{6}\right\}$$
$$+P\left\{\sup\left|\widehat{a}_i\int_N^t e^{-\widehat{a}_i(t-s)}\vee_{j=1}^{n}\beta_{ij}g_j\left(s,x_j(s-\tau)\right)ds\right|>\frac{\varepsilon_N}{6}\right\}$$
$$+P\left\{\sup\left|\int_N^t e^{-\widehat{a}_i(t-s)}\sum_{j=1}^{n}\sigma_{ij}\left(s,x_j(s,s-\tau)\right)dw_j(s)\right|>\frac{\varepsilon_N}{6}\right\} \tag{27}$$

From Theorem 2, there exist constants $\rho>0$ and $\lambda>0$ such that

$$\max_{j\in J_n}\left\{\mathbb{E}\left|x_j(t)\right|^2\right\}\le\rho e^{-\lambda t},t\in[0,+\infty) \tag{28}$$

So we have

$$K_{i1} \leq \left(\frac{6}{\varepsilon_N}\right)^2 \mathbb{E}\left[\sup e^{-2\widehat{a}_i t}\left|x_i(N)\right|^2\right] \leq \left(\frac{6}{\varepsilon_N}\right)^2 \mathbb{E}\left|x(N)\right|^2 \leq \left(\frac{6}{\varepsilon_N}\right)^2 \rho e^{-\lambda N} \tag{29}$$

$$\begin{aligned} K_{i2} &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \mathbb{E}\left[\sup\left|\widehat{a}_i \int_N^t e^{-\widehat{a}_i t} \wedge_{j=1}^n c_{ij} f_j\left(s, x_j(s)\right) ds\right|^2\right] \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \widehat{a}_i^2 \int_N^{N+1} e^{-\widehat{a}_i t} ds \int_N^{N+1} e^{-\widehat{a}_i t} \mathbb{E} \\ &\quad \times \left|\sum_{j=1}^n c_{ij} f_j\left(s, x_j(s)\right)\right|^2 ds \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \widehat{a}_i^2 \int_N^{N+1} e^{-\widehat{a}_i t} \mathbb{E}\left|\sum_{j=1}^n c_{ij} M_j \left|x_j(s)\right|\right|^2 ds \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \widehat{a}_i^2 \left(\sum_{j=1}^n c_{ij} M_j\right)\left(\sum_{j=1}^n c_{ij} M_j \int_N^{N+1} \mathbb{E}\left|x_j(s)\right|^2 ds\right) \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i \sum_{j=1}^n c_{ij} M_j\right)^2 \rho e^{-\lambda N} \end{aligned} \tag{30}$$

Similarly, we have

$$\begin{aligned} K_{i3} &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i\right)^2 \left(\sum_{j=1}^n \alpha_{ij} N_j\right)\left(\sum_{j=1}^n \alpha_{ij} N_j \int_N^{N+1}\right. \\ &\quad \left. \times \sup_{s-\tau \leq v \leq s} \mathbb{E}\left|x_j(v)\right|^2 ds\right) \leq \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i \sum_{j=1}^n \alpha_{ij} N_j\right)^2 \\ &\quad \times e^{\lambda\tau} \rho e^{-\lambda N} \end{aligned} \tag{31}$$

$$\begin{aligned} K_{i4} &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i\right)^2 \left(\sum_{j=1}^n d_{ij} M_j\right) \\ &\quad \times \left(\sum_{j=1}^n d_{ij} M_j \int_N^{N+1} \mathbb{E}\left|x_j(s)\right|^2 ds\right) \end{aligned} \tag{32}$$

$$\leq \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i \sum_{j=1}^n d_{ij} M_j\right)^2 \rho e^{-\lambda N}$$

$$\begin{aligned} K_{i5} &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i\right)^2 \left(\sum_{j=1}^n \beta_{ij} N_j\right) \\ &\quad \times \left(\sum_{j=1}^n \beta_{ij} N_j \int_N^{N+1} \sup_{s-\tau \leq v \leq s} \mathbb{E}\left|x_j(v)\right|^2 ds\right) \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i \sum_{j=1}^n \beta_{ij} N_j\right)^2 e^{\lambda\tau} \rho e^{-\lambda N} \end{aligned} \tag{33}$$

From the independence of $\left\{w_j(t)\right\}_{j \in J_n}$ and Itô's formula, we get

$$\begin{aligned} &K_{i6} \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \mathbb{E}\left[\sup\left|\int_N^t e^{-\widehat{a}_i t} \sum\nolimits_{j=1}^n \sigma_{ij}\left(x_j(s, s-\tau)\right) dw_j(s)\right|\right] \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \mathbb{E}\left[\int_N^{N+1} e^{-2\widehat{a}_i t} \sum_{j=1}^n \left|\sigma_{ij}\left(x_j(s, s-\tau)\right)\right| ds\right] \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \sum_{j=1}^n \mathbb{E}\left[\int_N^{N+1} \mu_{ij} \mathbb{E}\left|x_j(s)\right|^2 + \upsilon_{ij} \mathbb{E}\left|x_j(s-\tau)\right|^2 ds\right] \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \left[\sum_{j=1}^n \left(\mu_{ij} + \nu_{ij}\right) \int_N^{N+1} \sup_{s-\tau \leq v \leq s} \mathbb{E}\left|x_j(v)\right|^2 ds\right] \\ &\leq \left(\frac{6}{\varepsilon_N}\right)^2 \sum_{j=1}^n \left(\mu_{ij} + \nu_{ij}\right) e^{\lambda\tau} \rho e^{-\lambda N} \end{aligned} \tag{34}$$

Thus, from Equations (29)–(34), we have

$$\begin{aligned} &P\left\{\sup\left|x_i(t)\right| > \varepsilon_N\right\} \\ &\quad \leq \left(\frac{6}{\varepsilon_N}\right)^2 \rho e^{-\lambda N} + \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i \sum_{j=1}^n c_{ij} M_j\right)^2 \rho e^{-\lambda N} \\ &\qquad + \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i \sum_{j=1}^n \alpha_{ij} N_j\right)^2 e^{\lambda\tau} \rho e^{-\lambda N} \\ &\qquad + \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i \sum_{j=1}^n d_{ij} M_j\right)^2 \rho e^{-\lambda N} \\ &\qquad + \left(\frac{6}{\varepsilon_N}\right)^2 \left(\widehat{a}_i \sum_{j=1}^n \beta_{ij} N_j\right)^2 e^{\lambda\tau} \rho e^{-\lambda N} \\ &\qquad + \left(\frac{6}{\varepsilon_N}\right)^2 \sum_{j=1}^n \left(\mu_{ij} + \nu_{ij}\right) e^{\lambda\tau} \rho e^{-\lambda N} \\ &\quad \leq \frac{D_i}{\varepsilon_N^2} e^{-\lambda N} \leq \frac{\max_{j \in J_n}\left\{D_j\right\}}{\varepsilon_N^2} e^{-\lambda N} \end{aligned} \tag{35}$$

Set $D := \max_{j \in J_n}\left\{D_j\right\}$ and $\varepsilon_N = D^{\frac{1}{2}} e^{-\frac{\lambda N}{4}}$, then

$$P\left\{\sup_{N \leq t \leq N+1}\left|x_i(t)\right| > D^{\frac{1}{2}} e^{-\frac{\lambda N}{4}}\right\} \leq e^{-\frac{\lambda N}{2}}, i \in J_n \tag{36}$$

Consequently, it follows that FCGNNs (1) is probably certainly exponentially stable.

Remark 7: Theorem 3 gives a sufficient condition for the almost sure exponential stability of random FCGNNs (1). As Remark 4 states, the theorem is not supported by the analysis and techniques employed in [39], [40], and [41]. In addition, the criteria give in [40], [44], [45], and [46] do not apply to our results because they do not consider the problem of almost sure exponential stability. It should be emphasized that although [40] does not design the Lyapunov Krasovskii function or auxiliary function. It is necessary to test the nonsingularity of the matrix.

Remark 8: It's important to remember that references [45] and [46] also looked at the nearly certain exponential stability of random variables, but their studies were based on fuzzy cellular neural networks, which is a special case of this study, namely $a_i(x_i(t)) \equiv 1$ and $b_i(x_i(t)) = bx_i(t)$. Other scholars in recent literature [47], [48] have also researched almost sure exponential stability. The authors of [44] studied the almost inevitable exponential stability of Markov jump systems, and the study found in [46] examined stochastic Hopfield neural networks. Our research intends to complete the Cohen-Grossberg neural network's stochastic fuzzy delayed exponential stability findings.

Corollary 2: Assume that assumptions 1–3 and

$$-\widehat{a}_i + \min_{1\le j\le n}\left(c_{ij}M_j + \alpha_{ij}N_j\right) + \max_{1\le j\le n}\left(d_{ij}M_j + \beta_{ij}N_j\right) + \sum_{j=1}^{n}\frac{1}{2}\left(\mu_{ij} + \upsilon_{ij}\right) < 0, i \in J_n$$

holds, then system (1) is almost surely exponentially stable.

The same proof idea as corollary 1.

IV. EXAMPLE

Consider the second order CGNNs (1) with

$$f_j\left(t, x_j(t)\right) = \frac{0.2t\cos\left(x_j(t)\right)}{1+t}, g\left(x_j(t)\right) = 0.2x_j(t)\sin t,$$

$$a_i(x_i(t)) = 1.3 + 0.8\sin(x_i(t)), b_i(x_i(t)) = 0.3\cos(x_i(t))$$

$$\left(\sigma_{ij}t, \left(x_j(t, t-\tau)\right)\right)_{2\times2} = \begin{pmatrix} \sin t + 0.4x_1(t) & \cos t + 0.3x_1(t-\tau(t)) \\ 0.2tx_2(t-\tau(t)) & 0.2\sin t + x_2(t) \end{pmatrix}$$

Obviously, $\widehat{a}_1 = \widehat{a}_2 = \widehat{b}_1 = b_2 = 1, \breve{a}_1 = \breve{a}_1 = \breve{b}_1 = \breve{b}_2 = 2$, $M_j = N_j = 0.3$. The following are additional network (1) parameters:

$$(a_{ij})_{2\times2} = \begin{bmatrix} -3.1 & 0.4 \\ 0.6 & -0.3 \end{bmatrix}, \quad (b_{ij})_{2\times2} = \begin{bmatrix} -3.1 & 0.5 \\ -1.9 & -1.1 \end{bmatrix},$$

$$(c_{ij})_{2\times2} = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.4 \end{bmatrix}, \quad (d_{ij})_{2\times2} = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}$$

Take $\tau = 0.8\sin t$, $k = 0.5$, Using the inequality $(m+n+p+q)^2 \le 4m^2 + 4n^2 + 4p^2 + 4q^2$, we have $\mu_{11} = \mu_{12} = \upsilon_{11} = \upsilon_{12} = 1.7$ and $\mu_{21} = \mu_{22} = \upsilon_{21} = \upsilon_{22} = 0.61$. The validity of assumptions 1 through 5 can thus be easily verified. Four sets of initial conditions are given for each node.

Therefore, the conditions of Theorems 2 and 3 are satisfied. From the simulation results, we can see that the network is exponentially stable (see Figure. 1). Furthermore, we remove the fuzzy term, demonstrating the instability of the network without the fuzzy component (see Figure 2.).

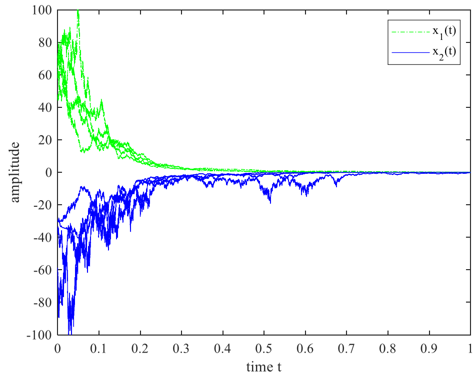

FIGURE 2. Transient responses of the state variables in the example's responses $x_1(t)$ and $x_2(t)$ to the six groups.

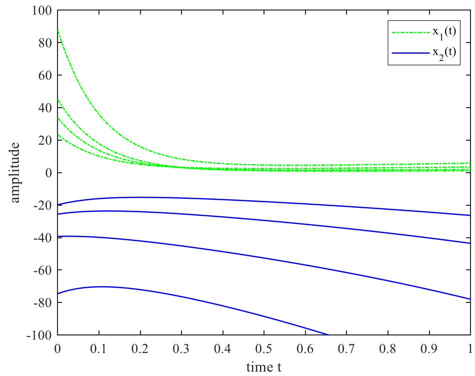

FIGURE 3. After removing fuzzy and stochastic terms, transient responses of the state variables in the example's responses $x_1(t)$ and $x_2(t)$ of example with six groups.

Furthermore, we change $(a_{ij})_{2\times2}, (b_{ij})_{2\times2}, (c_{ij})_{2\times2}, (d_{ij})_{2\times2}$ under the condition that the theorem is satisfied, proved the validity of our conclusion.

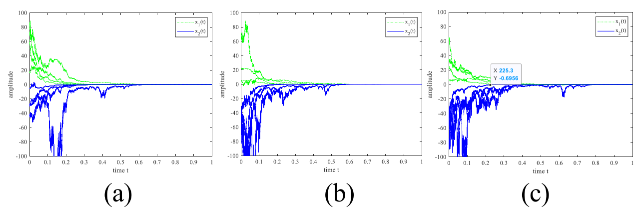

FIGURE 4. Transient responses of the state variables.

Remark 9: The above simulation shows the theorem results in this paper. Through Fig. 2, we prove that FCGNNs (1) is almost surely exponential stable and exponential stable. In contrast to the findings in the literature [24], it goes a step further and is generalized from $p = 2$ to the sufficient conditions for almost necessarily exponential stable when $p > 2$. The comparison in Fig. 2 shows that the fuzzy network is stable, so the fuzzy system has more advantages in practical applications.

V. CONCLUSION

The stability problem is a prerequisite in the actual system, and is also one of the im-portant performance indicators in the control system. It is an essential part of the dynamic analysis, and has an important position. In this paper, CGNNs with random fuzzy delay are examined in this research. To ensure the system's exponential stability and virtually certainly exponential stability, we gather enough criteria by taking into account the fixed point theory, using integral and differential inequalities, and using stochastic analysis theory. Compared with the widely used method of designing Lyapunov Krasovskii func-tionals, Our method is easier to implement, especially in practical engineering applications, reducing the conservatism of stability sufficient conditions. In addition, during the simulation process, we removed the fuzzy terms from the model and found that fuzzy systems provided more benefits than non fuzzy systems. Finally, our results can be further studied and extended to more complex systems. For example, in the current challenging field of dynamic control for soft robots, due to the need for more sensing devices, they form a more complex network and have high-dimensional nonlinear dynamic characteristics. Using the method proposed in this paper to seek sufficient conditions for their stability not only reduces the conservatism of the conditions, but also makes it easier to implement in engineering.

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HEBING ZHANG received the bachelor's degree in mathematics and applied mathematics from Harbin University, in 2007, and the master's degree in applied mathematics and the D.Phil. degree in navigation, guidance and control from Harbin Engineering University, in 2009 and 2012, respectively. He is currently an Assistant Professor with the Department of Computer Engineering, Faculty of Engineering, Taizhou University. His current research interests include stability analysis of neural networks and complex networks. He has been serving as a reviewer for many highly-respected journals.