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RESEARCH ARTICLE

Gaussian Sampling Guided Differential Evolution Based on Elites for Global Optimization

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ABSTRACT Mutation takes a vital part in assisting differential evolution (DE) to achieve satisfactory performance. The most crucial factor for a good mutation scheme is to mutate individuals dispersedly but with fast convergence to optimal regions. With this purpose, this paper designs a novel mutation approach, termed as "DE/current-to-gselite/1", by utilizing the Gaussian distribution to sample guiding exemplars around elites in the population to evolve individuals. Accordingly, a Gaussian sampling guided differential evolution (GSGDE) is devised to hopefully tackle optimization problems effectively. With the assistance of the Gaussian distribution, GSGDE mutates distinct individuals with very different guiding exemplars. Hence, high mutation diversity is expectedly maintained, which leads to that individuals could traverse the problem space in diverse directions. Thanks to the narrow sampling range of the Gaussian distribution, the generated guiding exemplars are likely better and thus individuals in the population are anticipated to move towards optimal regions fast. This is of great profit for fast convergence to high-quality solutions. Further, a dynamic parameter adjustment strategy is proposed to dynamically regulate the number of elites. Hereafter, GSGDE gradually shifts from concentrating on exploring problem space to focusing on exploiting found optimal areas. Cooperated with an existing adaptive parameter strategy, GSGDE is anticipated to strike a good balance between exploitation and exploration to traverse the problem space and hence likely obtain satisfactory performance. Experiments have been extensively carried out on the latest CEC2014 and CEC2017 problem suites with three settings of the dimensionality. Experimental results substantiate that GSGDE has a good scalability and attains highly competitive performance with or even significantly superior performance to 11 latest and representative DE methods. Particularly, its superiority becomes more and more significant as the dimensionality increases.

INDEX TERMS Global optimization, differential evolution, Gaussian sampling guided mutation, Gaussian distribution, elite learning.

I. INTRODUCTION

As a kind of evolutionary algorithms, differential evolution (DE) originally devised by Storn and Price [1], has received plenty of attention in the community of evolutionary computation thanks to its fast convergence to optimal solutions and easy implementation [2], [3], [4]. Consequently, DE has been taken advantage of to solve various optimization problems, like unimodal problems [5], [6], [7], [8], multimodal problems [9], [10], [11], [12], and multi-objective problems [13], [14], [15]. Meanwhile, due to its strong global search ability, DE has also been popularly applied to tackle many practical optimization problems in the real world, including ship-unloading scheduling [16], feature selection [15], [17], and face recognition [18].

In particular, DE evolves a number of individuals to traverse the problem space with three major operators, namely mutation, crossover, and selection [2], [19], [20]. To promote the optimization performance of DE in handling complex problems, abundant researchers have poured attention to

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devising effective techniques involved in the three operators to aid DE to tackle optimization problems effectively and efficiently. Consequently, hundreds and thousands of remarkable DE algorithms [21], [22], [23] have emerged. From a broad perspective, the research on DE principally concentrates on two main directions, proposing effective mutation strategies [24], [25], [26], [27] and devising effective parameter adaptation strategies [22], [23], [28], [29].

Mutation is the most critical operator in DE, because it introduces new values into the population [27], [30], [31]. Therefore, in the literature on DE, the research on mutation has drawn the most attention from researchers. Consequently, many remarkable mutation strategies have been developed [21], [24], [26], [32], [33]. At first, researchers mainly focused on developing a single mutation framework, which is shared by all individuals [7], [27], [30], [34], [35]. In this mutation framework, the key is usually the selection of the parent individuals, especially the leading exemplars, involved in the mutation strategy, which generally determines the mutation diversity and the convergence of individuals to optimal regions [27], [36], [37]. Then, since distinct mutation methods usually own different properties and advantages in tackling different kinds of optimization problems, some research fellows have tried to employ multiple distinct mutation methods to mutate the population. In this manner, the advantages of these mutation schemes can be assembled to help DE better balance exploration and exploitation to traverse the problem space [9], [14], [26], [31], [32]. In this mutation framework, the key is how to effectively utilize different mutation strategies to mutate individuals [32], [38], [39].

Besides the mutation operation, the parameter settings in DE also make significant influence on its optimization performance [22], [23], [29]. In particular, the crossover rate CR associated with the crossover operator and the scaling factor F accompanied with the mutation operator heavily impact the optimization effectiveness of DE. Specifically, in the mutation operator, F controls the effect of the difference vectors on the base individual, while in the crossover operator, CR significantly influences the difference between the parents and their offspring [7], [23], [40]. As a consequence, the optimal values of the two parameters are not just distinct for the same crossover and mutation operators to solve different optimization problems, but also distinct for different crossover and mutation operators to solve the same problem [10], [22], [28]. In other words, the optimization effectiveness of DE heavily relies on the settings of the two parameters. To circumvent this predicament, researchers have paid a good deal of devotion to devising adaptive parameter regulation techniques to relieve DE from the sensitivity to the two key parameters. Therefore, in the literature on DE, a lot of outstanding parameter adaption methods [22], [28], [29], [40] have been proposed to help DE achieve good performance. As far as we are concerned, the most commonly used parameter adaptation mechanisms can be summarized into two main types: population-level adaptation strategies [23], [33], [34], [36], [40] and individual-level adaptation methods [7], [28], [38], [41], [42]. In the first type of methods, all individuals share identical parameter settings and the settings are dynamically regulated during the evolution on the basis of historical evolutionary information of the population or the individuals [23], [36], [40]. By contrast, in the second type of methods, each individual has its own parameter settings and different individuals usually have different settings [6], [41], [43]. Besides, the settings of each individual are dynamically regulated based on the evolution state of the individual. Both types of parameter adaption methods have been widely employed to help newly developed DE variants to achieve satisfactory performance [37], [44], [45], [46], [47].

The above research on DE has largely promoted its optimization ability in seeking global optima of optimization problems. However, most existing advanced mutation strategies utilize a small number of elite individuals in the population to direct the mutation of the whole population [27], [32]. On the one hand, the mutation diversity is limited on account of the small selection range of candidate guiding exemplars. On the other hand, these already seen information usually provides limited assistance in mutating individuals diversely. As a result, individuals in most existing DE variants are in great danger of trapping into local regions once the elite individuals stagnate. Consequently, most existing advanced DE variants still encounter challenges in coping with complicated problems that are increasingly ubiquitous in the world of Internet of Things [20], [48], [49], [50]. In particular, due to the increasingly complex correlations among variables, the landscape of complicated optimization problems is usually very intricate and considerably difficult for the population to traverse [51], [52], [53], [54]. In this situation, high search diversity is usually required for DE to gain satisfactory performance.

To let individuals be mutated with high diversity and then improve the optimization effectiveness of DE, we devise a novel mutation strategy, named "DE/current-to-gselite/1", by utilizing the Gaussian distribution to sample guiding exemplars to mutate individuals. Incorporating it into the DE framework along with one widely utilized parameter adaption scheme for CR and F, a novel DE, called Gaussian sampling guided differential evolution (GSGDE) is developed to cope with optimization problems effectively and efficiently.

Specifically, the main novelty of this paper and the major components of GSGDE are summarized in the following:

1) A Gaussian sampling guided mutation mechanism, named "DE/current-to-gselite/1", is designed. Unlike most existing studies [27], [32], which directly utilize the elites in the population to mutate individuals, "DE/current-to-gselite/1" randomly generates one guiding exemplar around the elites in the current population according to the Gaussian distribution for each individual. In this way, on the one hand, the leading exemplars of distinct individuals are very different, which contributes largely to the promotion of mutation

diversity; on the other hand, better exemplars are likely generated around the elites by the Gaussian distribution model with a small standard deviation thanks to the narrow sampling range of the Gaussian distribution, which is of great benefit for individuals to move toward optimal regions fast. Therefore, with this mutation strategy, GSGDE is anticipated to explore the problem space dispersedly and subtly exploit the located optimal zones.

2) A dynamic adjustment scheme for the number of elites is proposed. As the evolution continues, the number of elites becomes smaller and smaller. As a result, the sampling range of the Gaussian distribution becomes narrower and narrower. This indicates that in the early period, the sampled guiding exemplars by the Gaussian distribution for individuals are expectedly scattered dispersedly and thus individuals concentrate more on exploring the search space. By contrast, in the late period, the generated leading exemplars by the Gaussian distribution are hopefully around the increasingly better elites and therefore individuals focus more on exploiting the optimal regions where the elites locate. Hence, with this dynamic strategy, GSGDE is anticipated to first explore the problem space dispersedly and then gradually tend to exploit the located optimal regions.

With the close collaboration between the above two methods along with the widely used parameter adaption scheme for *CR* and *F* in SHADE [41], GSGDE is anticipated to evolve the population with a promising balance between search convergence and search diversity to appropriately explore and exploit the problem space. To substantiate its effectiveness and efficiency, we carry out comparative experiments on the latest CEC2014 [55] and CEC2017 [56] benchmark problem suites with three settings of the dimensionality, which have been widely used to test the performance of various evolutionary algorithms [57], [58], [59]. For comparisons, we compare GSGDE with totally 11 latest and well-performed DE methods. At last, we also carry out experiments on the CEC2017 set to observe the effect of the devised two techniques on the optimization performance of GSGDE.

The remainder of this paper is structured as follows. Brief review of the classical DE and its representative advanced variants are elucidated in Section II. Subsequently, Section III elaborates the proposed GSGDE in detail. Then, Section IV verifies the effectiveness and efficiency of GSGDE by extensive experiments. At last, Section V affords the conclusion of this paper.

II. RELATED WORK

For better understanding of the development of DE, this section first reviews the working principle of the basic DE in Section II-A. Then, the main research on DE is briefly reviewed in two major directions by introducing latest and representative methods in Section II-B.

A. CLASSICAL DIFFERENTIAL EVOLUTION

DE [1] continuously updates a number of individuals to search the problem space via the difference vectors between them. Specifically, a classical DE has the following four steps:

1) INITIALIZATION

In the literature [7], [9], [16], the most commonly used initialization strategy is to randomly generate individuals with the uniform distribution in the search domain [60], [61], [62]. To be concrete, each individual is generated randomly in the following way:

$$x_{i,j} = LB_j + rand(0, 1) * (UB_j - LB_j) \tag{1}$$

where $x_{i,j}$ is the jth ($j = 1, 2, \ldots, D$) dimension value of the ith individual ($i = 1, 2, \ldots, NP$), D denotes the dimensionality of the optimized problem, and NP is the population size. rand(0, 1) uniformly generates a real random value within [0,1]. UB_j and LB_j represent the upper limit and the lower limit of the jth variable, respectively.

After all individuals are initialized, their fitness values are computed, and then the global best individual is identified. After that, DE enters the main evolution which involves three crucial operations: mutation, crossover, and selection.

2) MUTATION

The mutation operation is to create a mutation vector $\boldsymbol{v_i} = [v_{i,1}, v_{i,2}, \ldots, v_{i,D}]$ for each parent $\boldsymbol{x_i}$ by using the difference vectors between individuals [6], [7], [24]. Lots of mutation schemes have been proposed from different perspectives. Some typical and representative mutation schemes are listed in the following:

DE/rand/1 [1]:

$$\boldsymbol{v}_i = \boldsymbol{x}_{r1} + F \times (\boldsymbol{x}_{r2} - \boldsymbol{x}_{r3}) \tag{2}$$

DE/best/1 [63]:

$$\boldsymbol{v}_i = \boldsymbol{x}_{\text{best}} + F \times (\boldsymbol{x}_{r1} - \boldsymbol{x}_{r2}) \tag{3}$$

DE/current-to-best/1 [64]:

$$\boldsymbol{v}_i = \boldsymbol{x}_i + F \times (\boldsymbol{x}_{\text{best}} - \boldsymbol{x}_i) + F \times (\boldsymbol{x}_{r1} - \boldsymbol{x}_{r2}) \tag{4}$$

DE/current-to-pbest/1 [21]:

$$\boldsymbol{v}_i = \boldsymbol{x}_i + F \times (\boldsymbol{x}_{\text{pbest}} - \boldsymbol{x}_i) + F \times (\boldsymbol{x}_{r1} - \boldsymbol{x}_{r2}) \tag{5}$$

where $\boldsymbol{x}_{r1}$, $\boldsymbol{x}_{r2}$, and $\boldsymbol{x}_{r3}$ are three distinct individuals chosen randomly from the population and they are also distinct from $\boldsymbol{x}_i$, namely $i \neq r1 \neq r2 \neq r3$. F is a real parameter in [0,1], which controls the effect of the difference vectors. $\boldsymbol{x_{best}}$ is the best solution found so far; $\boldsymbol{x_{pbest}}$ is an elite selected randomly from the best p ones in the population.

3) CROSSOVER

The crossover operation aims to create a trial vector $\boldsymbol{u_i} = [u_{i,1}, u_{i,2}, \ldots, u_{i,D}]$ for each individual $\boldsymbol{x_i}$ by recombining the dimensions of $\boldsymbol{x_i}$ and $\boldsymbol{v_i}$ [10], [23], [31]. Whether each component of $\boldsymbol{u_i}$ is inherited from $\boldsymbol{x_i}$ or $\boldsymbol{v_i}$ is regulated by the crossover rate (CR), which is another critical parameter in DE. In the literature [7], [23], [33], [36], [37], the most popular crossover is the binomial crossover, which works as follows:

$$u_{i,j} = \begin{cases} v_{i,j}, & \text{if rand}(0,1) \leqslant CR \text{ or } j = j_{rand} \\ x_{i,j}, & \text{otherwise} \end{cases} \tag{6}$$

where rand(0, 1) returns a real random value uniformly from [0,1]. CR denotes the crossover rate belonging to [0,1], and j_{rand} is the index of a random dimension uniformly sampled from $[1, D]$, and is utilized to guarantee that $\boldsymbol{u_i}$ inherits more than one dimension from $\boldsymbol{v_i}$.

4) SELECTION

The selection operator aims to make comparison between each parent individual $\boldsymbol{x_i}$ and its offspring $\boldsymbol{u_i}$ and then choose the better one to go into the next iteration [22], [28], [33]. It works as follows:

$$\boldsymbol{x_i} = \begin{cases} \boldsymbol{u_i}, & \text{if } f(\boldsymbol{u_i}) < f(\boldsymbol{x_i}) \\ \boldsymbol{x_i}, & \text{otherwise} \end{cases} \tag{7}$$

where $f(\boldsymbol{x_i})$ is the fitness value of parent $\boldsymbol{x_i}$, while $f(\boldsymbol{u_i})$ is that of offspring $\boldsymbol{u_i}$. $f(\boldsymbol{u_i}) < f(\boldsymbol{x_i})$ implies that the lower the fitness value of one individual is, the better that individual is. As a result, this selection strategy is fit for dealing with minimization optimization problems.

The above three operators are sequentially executed repeatedly to update the population until the predefined termination criterion is met, which is usually that the given number of fitness evaluations runs out [10], [33], [35], [40], [42]. After the termination, the best solution found by DE is output.

B. MAIN RESEARCH ON DE

To promote the optimization ability of DE, lots of researchers have devoted extensive effort to devising novel DE variants. As far as we know, the research on DE proceeds in two major directions, namely developing novel effective mutation schemes [24], [26], [33], [35], [36] and devising effective parameter adaption methods [7], [10], [19], [23], [29].

1) RESEARCH ON MUTATION STRATEGIES

In the literature, a large number of researchers have been committed to devising effective mutation schemes to create high-quality offspring [21], [41], [65]. As a result, a lot of remarkable mutation mechanisms have sprung up [24], [27], [30], [31], [43]. In a broad sense, the research on mutation strategies is roughly divided into two types: adopting only one single mutation strategy to mutate all individuals [7], [21], [23], [30], [38] and assembling multiple mutation strategies to evolve individuals [28], [31], [32], [43], [66].

In the early research on mutation, researchers mainly focused on developing a single mutation framework for DE to evolve all individuals [7], [21], [23], [27], [38]. In general, the key to this mutation framework is to choose benefiting parent individuals to take part in the mutation operation. In this direction, numerous mutation strategies have been designed [7], [23], [24], [27], [30]. Since it is hardly possible to review them all, this paper only introduces some latest and representative mutation schemes in the following.

To augment the mutation diversity of individuals, Yang et al. [27] designed a neighbor elite directed mutation scheme, termed as "DE/current-to-rnbest/1". In such a mutation scheme, a number of distinct individuals are randomly chosen from the population to build a neighbor area for each individual. Subsequently, the best individual in the area is used as the leading exemplar to guide the evolution of this individual. In [30], a fitness and diversity ranking-based mutation mechanism was designed. In particular, instead of only using the fitness as the measurement to select parent individuals, this strategy defines a new measurement by integrating the fitness and the diversity contribution of individuals and then chooses parent individuals according to the rankings of all individuals after they are ranked from the best to the worst regarding the new measurement. In [23], Zou et al. proposed a consecutive unsuccessful updates-based mutation strategy. Specifically, this mutation scheme first records the times that an individual remains unchanged, and then it computes the choosing probability of each individual according to this number. After that, the base individual along with the guiding exemplar involved in the mutation is randomly chosen based on the calculated probabilities. In [38], Qiao et al. designed a level-based learning scheme for individuals to mutate. Particularly, this mutation scheme first divides individuals into several levels concerning their fitness, and subsequently chooses individuals in higher levels randomly to direct the mutation of those in lower levels. In [35], Stanovov et al. proposed a novel individual selection mechanism by introducing the selection pressure. Specifically, individuals are first ranked according to their fitness, and next, the parents taking part in the mutation are chosen by using the tournament selection scheme together with the roulette wheel selection scheme with the selection probabilities calculated according to the ranks of individuals. In [37], Xia et al. designed a novelty and fitness driving mutation strategy. Specifically, they first defined a new measure called novelty to assess the contribution of each individual to diversity maintenance. Then, the devised mutation scheme randomly chooses the parental individuals on the basis of the combination of the fitness value and the novelty value of each individual. In [67], Mohamed et al. developed an order-based mutation mechanism, termed as "DE/current-to-ord_best/1". Specifically, three distinct individuals are first stochastically selected from the population and then they are compared in terms of their fitness. Subsequently, the best individual is utilized as the leading exemplar and the other two are employed to create the random difference vector. In [68], an adaptive mutation strategy was devised. In particular, it first divides all individuals into three clusters denoted as the best cluster, the better cluster, and the worst cluster respectively. Then, it randomly selects two individuals from the better and worst clusters respectively to construct the random difference vector and another one from the best cluster to serve as the leading exemplar to mutate each individual. In [69], a mutation scheme termed as "DE/current-to-ci_mbest/1" was designed. In this mutation strategy, a collective vector is first built with a linear combination of m individuals with the top ranks in the population. Then, this collective vector is employed to mutate each individual. In [52], Deng et al. designed a dynamic combination based mutation operator where the base vector of each individual is constructed by the dynamic combination of the best individual in the population and the best one among three randomly chosen individuals. In [53], Deng et al. designed a new neighborhood mutation scheme termed as "DE/neighbor-to-neighbor/1". In particular, this mutation strategy uses an individual chosen stochastically from the neighborhood as the base vector and then adopts another high-quality individual in the neighborhood as the leading exemplar to mutate each individual.

As the research on mutation progresses, it is well recognized that distinct mutation schemes usually have different advantages in tackling different kinds of optimization problems [24]. Hence, a natural way is to assemble multiple different mutation schemes to update individuals. In this direction, there are also a lot of ensemble strategies in the literature [22], [28], [31], [32], [36]. Likewise, it is also hardly possible to review them all and thus we only introduce some representative and state-of-the-art ensemble methods below.

Wang et al. [22] proposed an adaptive parameter strategy DE (APSDE). In this DE variant, six different mutation schemes are maintained and an accompanying population is kept to cooperate with the main population to traverse the problem space. In addition, the evolution is further separated into three stages, and then in different stages, the two populations adaptively select different mutation strategies to evolve individuals to appropriately explore and exploit the problem space. In [10], Meng and Yang proposed a two-stage DE (TDE). In this DE variant, they developed two mutation strategies, namely the historical-solution based mutation and the inferior-solution based mutation. Subsequently, to ensemble these two strategies, they first partitioned the evolution of the population into two stages. Then, in the first evolution period, the historical-information based mutation is employed to evolve individuals to find optimal regions. Then, in the second evolution period, the inferior-individual based mutation is utilized to update individuals to exploit the located optimal zones with a good compromise between diversity and convergence. In [28], Sun et al. designed a Gaussian mutation and dynamic parameter adjustment based DE (GPDE). In this DE method, a new Gaussian distribution based mutation and an improved classical mutation called "DE/rand-worst/1" are collaboratively used to mutate individuals based on their cumulative performance. In [32], Li et al. designed a dual mutation schemes collaboration DE (DMCDE). In this method, an elite direction scheme was first proposed and then embedded into "DE/best/2" and "DE/rand/2" to develop two new mutation strategies. Subsequently, a collaboration mechanism between the dual mutation schemes was developed to mutate individuals, such that a good compromise between local exploitation and global exploration is attained. In [40], a self-adaptive ensemble-based DE (SEDE) was devised by assembling three different mutation schemes to collaboratively mutate individuals. In particular, these three mutation schemes are adaptively chosen to mutate individuals based on their optimization performance. In [36], a fitness-based adaptive DE (FADE) was designed. In this method, all individuals are separated into three categories, namely the best individuals, the worst individuals, and the medium individuals. Next, three classical mutation strategies are adaptively chosen by these individuals to mutate based on their fitness. In [31], Deng et al. designed a tri-population DE (TPDE). To be concrete, this method first separates the population into three subpopulations based on a zonal-constraint stepped division strategy. Then, it employs three different elite-guided mutation strategies to evolve individuals in the three subpopulations. In [25], Tan et al. designed an adaptive mutation selection mechanism according to fitness landscape analysis. Specifically, a mutation scheme selector based on random forest is first trained on different optimization problems. Then, when confronted with an optimization problem, the trained mutation selector is first used to adaptively choose a mutation scheme for DE to solve this problem. On the foundation of this idea, they further developed a dynamic fitness landscape-based DE (DFLDE) in [66] according to the dynamic fitness landscape characteristics of the problem to be solved. In [65], a historical and heuristic-based DE (HHDE) was designed. Particularly, in this algorithm, three mutation strategies are maintained. In each generation, each individual adaptively chooses one mutation strategy according to the heuristic information of the individual and the historical information of the population. In [43], Cui et al. proposed to use two different mutation mechanisms to create two mutation vectors for each parent and accordingly generate two offspring to compete with the parent. In [70], Guo et al. devised an improved triangular Gaussian mutation based DE (ITGDE). Concretely, they first designed a Gaussian distribution based mutation scheme by using the positions and the fitness differences of three individuals organized in the triangular structure. Subsequently, each individual adaptively chooses the devised Gaussian mutation scheme and "DE/rand/1" to mutate with a probability. In [71], Chen and Shen designed a self-adaptive DE with Gaussian–Cauchy mutation (SDEGCM). Particularly, they alternatively employed the Cauchy distribution and the Gaussian distribution to mutate individuals, so that the strong local exploitation ability of the Gaussian distribution and the powerful global exploration ability of the Cauchy distribution can be assembled to aid DE sustain a promising compromise between exploration and exploitation. Further, they also combined "DE/rand/1" and let each individual select a mutation strategy randomly from the three mutation schemes with equal probabilities to mutate. In [72], Chen et al. divided the population into three subpopulations, namely the exploration subpopulation, the exploitation subpopulation, and the balanced subpopulation. Then, they designed a modified hunger games search operator to evolve the exploration subpopulation to enhance the global exploration, utilized "DE/current-to-pbest/1" with a Gaussian tail to update the exploitation subpopulation to promote the local exploitation, and devised a distance-based multi-population algorithm to update the balanced subpopulation to make full use of feedback information from the former two subpopulations. In [73], the authors proposed a Gaussian bare-bones DE (GBDE). Specifically, they first designed a Gaussian mutation scheme by using the linear combination of each individual and the global best individual as the mean and their Euclidean distance as the variance to randomly sample a mutation vector for the individual. Then, they alternatively took advantage of the Gaussian mutation strategy and "DE/best/1" to mutate each individual.

Though the above developed mutation strategies have been substantiated to help DE solve certain kinds of optimization problems effectively, the performance of most existing DE variants is still not as satisfactory as anticipated in solving complicated optimization problems. Due to the increasingly complex correlations among variables, the landscape of complicated optimization problems is usually very intricate and considerably difficult for the population to traverse [74], [75], [76], [77]. In this situation, high search diversity is usually required for DE to achieve satisfactory optimization performance. However, in most existing mutation strategies, it is found that individuals are guided by relatively superior ones in the current population. This already seen information usually provides limited assistance in mutating individuals diversely. To further enhance the mutation diversity of individuals, this paper develops a Gaussian sampling guided mutation scheme, termed as "DE/current-to-gselite/1" for DE to mutate individuals, such that they can traverse the problem space in various directions and at the same time explore the complex landscape with slight intensification to find optimal zones and mine the located promising zones with slight diversification to escape from local basins.

2) RESEARCH ON CONTROL PARAMETER ADAPTION

In DE, two parameters, namely the crossover probability (CR) in the crossover and the scaling parameter (F) in the mutation, also play a crucial role in generating high-quality offspring, and thus significantly affect the optimization ability of DE [29]. However, the best settings of the two parameters are generally distinct for the same crossover and mutation schemes in coping with different optimization problems and they are also distinct for different mutation and crossover schemes in solving the same problem. To circumvent this predicament, researchers have been dedicated to proposing parameter adaption schemes for F and CR, and consequently, a lot of parameter adaption strategies have sprung up [7], [22], [29], [40], [41], [78]. Comprehensively speaking, most existing parameter adaption methods could be roughly summarized into two categories, namely the population-level parameter adaption strategies [23], [33], [34], [36], [40] and the individual-level parameter adaption strategies [7], [10], [22], [41], [42].

In the population-level parameter adaption schemes, all individuals usually adopt identical settings of F and CR, but the settings are regulated dynamically during the evolution [33], [34], [36]. To afford diverse population-level parameter settings, some researchers even took advantage of the type-2 fuzzy systems to design dynamic parameter adaption methods for CR and F to aid DE to achieve good performance [79], [80].

Different from the population-level parameter adaption methods, the individual-level parameter adaption strategies generally afford different settings of CR and F for distinct individuals [10], [21], [41]. Besides, these settings of distinct individuals are dynamically adjusted according to their optimization performance and evolutionary states. Compared with the population-level parameter adaption methods, the individual-level ones provide higher parameter diversity and thus they usually offer more effective help for DE to achieve better optimization performance [10], [21], [26], [32], [33]. As a result, we mainly introduce some state-of-the-art and representative individual-level parameter adaption strategies in the following.

The most representative and popular individual-level parameter adaption method is the one in the success-history based adaptive DE (SHADE) [41]. Specifically, in this method, two archives, denoted as $\boldsymbol{M_F}$ and $\boldsymbol{M_{CR}}$, respectively, are maintained to store the mean values of the historically successful F and CR. Then, during the evolution, for each individual $\boldsymbol{x_i}$, a mean value of F denoted as M_{F,r_i} is randomly chosen from $\boldsymbol{M_F}$ and a mean value of CR denoted as M_{CR,r_i} is randomly chosen from $\boldsymbol{M_{CR}}$. Subsequently, the settings of F and CR for individual $\boldsymbol{x_i}$ are randomly generated as follows:

$$F_i = randc_i(M_{F,r_i}, 0.1) \tag{8}$$

$$CR_i = randn_i(M_{CR,r_i}, 0.1) \tag{9}$$

where $\mathrm{randn}_i(M_{CR,r_i}, 0.1)$ returns a real random value sampled by the Gaussian distribution model, whose mean value is M_{CR,r_i} and standard deviation is 0.1 and $\mathrm{randc}_i(M_{F,r_i}, 0.1)$ returns a real random value sampled by the Cauchy distribution model, whose position parameter is M_{F,r_i} and scaling factor is 0.1. r_i is a random index uniformly chosen from $[1, H]$, where H is the archive size of $\boldsymbol{M_F}$ and $\boldsymbol{M_{CR}}$.

If the generated value of CR_i by Eq. (9) is outside the range of [0,1], it is then regenerated until it is within [0,1]. If the generated $F_i \leq 0$, it is resampled by Eq. (8) until $F_i > 0$. If $F_i > 1$, it is truncated to 1. As for the two archives $\boldsymbol{M_F}$ and

$\boldsymbol{M_{CR}}$, in the beginning, all elements $M_{CR,i}$ and $M_{F,i}$ ($i = 1, 2, 3, \ldots, H$) are set as 0.5 in [41]. Then, during the evolution, those successful F_i and CR_i values that help the mutation and the crossover strategies generate better offspring to take place of the associated parents are stored into $\boldsymbol{S_F}$ and $\boldsymbol{S_{CR}}$, respectively. Subsequently, at the end of each iteration, the mean values of these successful F and CR are computed as follows:

$$M_{CR} = \sum_{k=1}^{|S_{CR}|} w_k \cdot S_{CR,k} \tag{10}$$

$$M_F = \frac{\sum_{k=1}^{|S_F|} w_k \cdot S_{F,k}^2}{\sum_{k=1}^{|S_F|} w_k \cdot S_{F,k}} \tag{11}$$

$$w_k = \frac{\Delta f_k}{\sum_{j=1}^{|S_{CR}|} \Delta f_j} \tag{12}$$

where $\Delta f_j = \left| f(\boldsymbol{u_j}) - f(\boldsymbol{x_j}) \right|$ is the fitness improvement of the better offspring u_j to the associated parent x_j.

Then, the computed mean values of CR and F are used to replace the oldest values in the two archives $\boldsymbol{M_{CR}}$ and $\boldsymbol{M_F}$, respectively. It should be noted that when all generated offspring fail to replace the associated parents, $\boldsymbol{S_{CR}} = S_F = \emptyset$. In this situation, the two archives $\boldsymbol{M_F}$ and $\boldsymbol{M_{CR}}$ are not updated. With the above mechanism, this adaptive parameter strategy provides high parameter setting diversity for individuals and thus effectively assists SHADE to attain good performance in tackling optimization problems. By means of such superiority, this adaptive scheme has been widely utilized in many advanced DE variants to cooperate with the devised mutation strategies to solve optimization problems [7], [26], [42].

Besides, inspired by such a good parameter adaption scheme, researchers have designed other individual-level adaptive schemes for the two parameters as well [10], [22], [26], [33], [34]. To name a few, in [32], the settings of F and CR for each individual are adjusted based on its evolutionary state. Specifically, if the configurations of F and CR for an individual help it generate a better offspring, they remain unchanged; if such settings cannot generate a better offspring for a given number of consecutive generations, they are randomly initialized. In [31], three groups of parameters were designed according to a triangular wave function, the Cauchy distribution model and the Gaussian distribution model, respectively. In [7], an adaptive parameter adjustment scheme was devised by integrating the population information with respect to the standard deviation of fitness values and the sum of the standard deviations of each dimension of the population. In [42], an improved adaptive parameter strategy in terms of the one in SHADE [41] was designed. Particularly, this adaptive method incorporates the information of the problem landscape into the adaptation of F and CR by constructing spatial-distance-based neighborhoods for each individual and then only considering the values of CR and F related to the successful neighborhoods to adjust the parameter settings of this individual.

These adaptive methods have aided DE to gain promising performance in solving optimization problems [10], [22], [32], [42], [49]. As a result, they are frequently utilized by many researchers to cooperate with their newly designed DE variants to tackle optimization problems [14], [27], [37], [42], [67]. Likewise, this paper also directly uses the parameter adaption scheme in SHADE [41] to collaborate with the devised GSGDE to solve optimization problems.

III. PROPOSED GAUSSIAN SAMPLING GUIDED DE

In most existing mutation strategies [24], [27], [31], [32], [34], [35], individuals are usually evolved by the ones in the population or historical positions. All these positions are actually seen or visited by the population. Such information may provide limited guidance for individuals to traverse the problem space diversely once all individuals fall into local regions. Such a phenomenon is particularly common in the complex problem space with complicated landscape containing numerous saddle areas and local basins. To alleviate this dilemma, high mutation diversity is highly required, such that individuals can traverse the problem space in diverse directions and at the same time explore the complex landscape with slight intensification to approach optimal regions and exploit the found promising regions with slight diversification to escape from local basins.

With the above purpose, this paper designs a Gaussian sampling guided mutation strategy, named "DE/current-to-gselite/1", which utilizes the unseen information sampled by the Gaussian distribution to guide the evolution of individuals. Cooperated with the binomial crossover scheme, a Gaussian sampling guided DE (GSGDE) is developed in this paper to solve optimization problems effectively.

A. DE/current-to-gselite/1

Specifically, the major principle of "DE/current-to-gselite/1" is to utilize an unseen guiding exemplar, which does not exist in the population, to update each individual. However, on the one side, the mutation scheme should afford high mutation diversity for the population, such that individuals could traverse the complex solution space in diverse directions. On the other side, this mutation strategy should also guide individuals to move towards promising regions with a high probability, such that fast convergence could be ensured. To achieve the above purpose, we randomly sample an unseen guiding exemplar for each individual by the Gaussian distribution model around the elites in the current population.

To be concrete, provided that NP individuals are kept, we first sort individuals from the best to the worst according to their fitness values. Next, we select the top best NEI individuals to constitute an elite group. Subsequently, for each updated individual, we first select an elite individual (denoted as $\boldsymbol{x_{randE}}$) randomly from the elite group as the mean vector of the Gaussian distribution model, and then calculate the standard deviation of the Gaussian distribution as follows:

$$\sigma_d = \frac{\varepsilon}{NEI-1}\sum_{i=1}^{NEI}\left|x_{i,d} - x_{randE,d}\right| \tag{13}$$

where *NEI* is the number of elite individuals chosen from the population and d represents one of the dimensions; $\boldsymbol{x_{randE}}$ is a randomly chosen elite individual from the elite group; $\boldsymbol{x_i}$ is the i th elite individual in the elite group, ε represents a random value used to ensure that the calculated standard deviation for the Gaussian distribution is small, so that it can randomly generate a promising leading exemplar for each individual around the selected elite. Therefore, we randomly generate a value for ε from [1.0E-04, 1.0E-03] in this paper. In addition, it should be noted that if the computed standard deviation of one dimension is 0, we reset it as 1.0E-04.

Subsequently, we randomly sample a leading exemplar for the ith individual ($\boldsymbol{x_i}$) in the population by using the Gaussian distribution model with the mean vector configured as the selected elite $\boldsymbol{x_{randE}}$ and the computed standard deviation in the following manner:

$$\boldsymbol{x_{elite}^{gs}} = Gaussian(\boldsymbol{x}_{randE}, \boldsymbol{\sigma}) \tag{14}$$

where $\boldsymbol{x_{elite}^{gs}}$ is the sampled guiding exemplar for the individual to be updated. It should be noticed that the univariate Gaussian distribution model is employed to generate a guiding exemplar thanks to its low computational complexity. Additionally, it also deserves mentioning that if the sampled value of one dimension in $\boldsymbol{x_{elite}^{gs}}$ is out of the search range of the optimization problem, it is resampled by the Gaussian distribution until it is within the search range.

With the sampled guiding exemplar $\boldsymbol{x_{elite}^{gs}}$, each individual ($\boldsymbol{x_i}$) in the current population is mutated in the following way:

$$\boldsymbol{v_i} = \boldsymbol{x_i} + F_i^*(\boldsymbol{x_{elite}^{gs}} - \boldsymbol{x_i}) + F_i^*(\boldsymbol{x}_{r1} - \hat{\boldsymbol{x}}_{r2}) \tag{15}$$

where $\boldsymbol{v_i}$ is the mutant vector, $\boldsymbol{x_{elite}^{gs}}$ is the guiding exemplar randomly sampled by the Gaussian distribution model, $\boldsymbol{x_i}$ is the ith parent in the population, $\boldsymbol{x_{r1}}$ is a random individual uniformly chosen from the population, $\hat{\boldsymbol{x}}_{\boldsymbol{r2}}$ is one individual chosen randomly from $\boldsymbol{P}\cup\boldsymbol{A}$, with $\boldsymbol{P}$ denoting the population, and $\boldsymbol{A}$ representing an external archive utilized to store the outdated parents which are replaced by their offspring. Such selection of $\hat{\boldsymbol{x}}_{\boldsymbol{r2}}$ is directly borrowed from JADE [21] to ensure high mutation diversity. It deserves notice that $r1 \neq r2 \neq i$. However, it should be mentioned that different from JADE [21], when generating the random difference vector between the two randomly chosen individuals in Eq. (15), we place the better one as $\boldsymbol{x_{r1}}$ and the worse one as $\hat{\boldsymbol{x}}_{\boldsymbol{r2}}$ to create a directional random difference vector. By this means, the convergence of the updated individual to optimal zones can likely be accelerated. In addition, it also should be paid attention to that if the value of one dimension in $\boldsymbol{v_i}$ is out of the search range of the optimization problem, it is directly set as the associated lower or upper bounds of that dimension.

Taking deep analysis of Eq. (15), we discover that the devised mutation scheme preserves the following advantages:

1) The devised mutation strategy affords high mutation diversity for the population. Consequently, the population could search the complex problem space dispersedly and preserves great chances to get out of local regions. On the one side, the leading exemplar for mutating each individual is sampled randomly by a Gaussian distribution model. This indicates that the leading exemplars of distinct individuals are likely different. Even though the mean value and the standard deviation of the Gaussian distribution are occasionally the same for some individuals, the generated guiding exemplars of these individuals are also likely different. On the other side, the mean of the Gaussian distribution model is a randomly chosen elite from the elite set. Therefore, it is highly possible that the mean of the Gaussian distribution is distinct for distinct individuals. Besides, during the calculation of the standard deviation, thanks to the random generation of ε from [1.0E-04, 1.0E-03], the standard deviation value of the Gaussian distribution model is also likely distinct for different individuals. In these two ways, high search diversity can be maintained with the help of the devised mutation strategy.
2) The designed mutation strategy also ensures fast convergence of individuals to optimal areas. On the one hand, the sampled guiding exemplars for all individuals are randomly generated around the elite ones in the current population. Since these elites are the top best ones, they likely locate at or around the optima in the solution space. Therefore, with the direction of the guiding exemplars, individuals preserve high probabilities to move towards optimal areas. On the other hand, due to the small sampling range defined by the calculated standard deviation, it is likely that the Gaussian distribution generates a more promising guiding exemplar than the associated elite randomly selected from the elite group. With the direction of such guiding exemplars, individuals are anticipated to approach optimal zones faster. In the above two manners, fast convergence of individuals to optimal solutions is expectedly ensured.
3) Together, the devised mutation is capable of maintaining a promising balance between search diversity and search convergence to traverse the complex solution space. Specifically, with the random selection of the elites along with the dynamics of the calculated standard deviation and the random sampling of the Gaussian distribution, the devised mutation strategy makes dynamic compromises between exploration of the problem space to locate promising regions and exploitation of the located optimal zones to subtly find high-accuracy solutions.

B. DYNAMIC ADJUSTMENT OF THE NUMBER OF ELITE INDIVIDUALS

In the devised mutation scheme, elites in the current population are employed to sample leading exemplars to guide the mutation of individuals. Therefore, the number of elite individuals, namely *NEI*, have great impact on the diversity of mutating individuals. Specifically, a larger *NEI* provides more elites for the Gaussian distribution to sample and consequently, higher mutation diversity is obtained. This is of great usefulness for the population to traverse the problem space in distinct directions. However, such too high diversity may do harm to the convergence of the population to optimal zones. By contrast, a smaller *NEI* leads to that fewer elites participate into the Gaussian sampling. This may result in that many guiding exemplars are sampled around the same elite. This is actually very profitable for individuals to converge to optimal zones fast and subsequently subtly exploit the zones to locate high-accuracy solutions. However, a too small *NEI* may lead to that the population moves towards optimal areas in very limited directions, and thus individuals in the population are at great danger of stepping into local regions. Such analysis indicates that a fixed *NEI* is not suitable for the designed mutation scheme to help DE achieve good performance.

Particularly, it is well recognized that at the early evolution stage, high diversity is generally preferred to traverse the problem space in various directions to locate optimal regions fast, while at the late evolution stage, fast convergence is generally favored to exploit the located optimal regions subtly to seek as high-accuracy solutions as possible. Based on this consideration, this paper designs a simple dynamic adjustment for *NEI*.

Before the dynamic adjustment, we let *NEI* be related to the population size *NP* for better adjustment as follows:

$$NEI = \lceil p^{*} NP \rceil \tag{16}$$

where p denotes the ratio of the elite individuals out of the whole population. $\lceil x \rceil$ represents the ceil function that returns the smallest integer that is larger than x.

Subsequently, instead of directly adjusting *NEI*, this paper designs the following dynamic adjustment scheme for p:

$$p = p_{max} - (p_{max} - p_{min})^{*} \frac{nfe}{FES_{max}} \tag{17}$$

where nfe denotes the number of already consumed fitness evaluations, while FES_{max} represents the given maximum number of fitness evaluations. p_{min} and p_{max} are the lower limit and the upper limit of p, respectively. In this paper, for better adjustment, we set $p_{min} = 1/2^{*} p_{max}$. Since the devised mutation scheme is a little similar with ''DE/current-to-pbest/1'' proposed in JADE [21], we set $p_{max} = 0.1$ the same as the recommended setting of p in JADE [21].

With the above dynamic adjustment of p, the number of elite individuals (*NEI*) becomes smaller and smaller as the evolution continues. As a result, not only the selection range of the mean of the Gaussian distribution model becomes smaller and smaller, but also the sampling range of the Gaussian distribution model, which is determined by the computed standard deviation, becomes narrower and narrower. Therefore, as the evolution goes on, the population gradually switches from exploration of the problem space to exploitation of the located optimal areas intensively. This indicates that the dynamic adjustment of *NEI* further helps the population sustain a good compromise between exploration and exploitation. The effectiveness of this dynamic strategy is substantiated by experiments conducted in Section IV.E.

C. THE COMPLETE GSGDE

By introducing the devised mutation strategy and the designed dynamic adjustment scheme for the number of elites into the DE procedure, the complete GSGDE is developed with the pseudocode exhibited in Algorithm 1. It should be mentioned that to relieve GSGDE from the sensitivity to the settings of *CR* and F, this paper directly adopts the adaptive parameter scheme in SHADE [41] to adjust *CR* and F during the evolution. Such an adaptive scheme is utilized because in the literature [7], [35], [42], it has been employed to successfully help many DE variants achieve good performance.

Specifically, as exhibited in Algorithm 1, *NP* random individuals are first generated and their fitness values are computed as shown in Line 1. Besides, the two parameter archives for F and *CR* in SHADE are initialized to be 0.5 and the archive to record the obsolete parents is set as an empty set (Line 2). Subsequently, GSGDE enters the main loop of the evolution (Lines 3 $\sim$ 20). During the main loop, the number of elite individuals *NEI* is first calculated (Line 5), and then all individuals are sorted from the lowest to the highest in terms of their fitness values (Line 6) to get the elites. After that, for each individual, an elite individual is randomly chosen from the elite group as the mean of the Gaussian distribution model and accordingly the standard deviation is calculated (Line 8). Subsequently, a guiding exemplar is sampled by the Gaussian distribution model randomly (Line 9), and the associated *CR* and F for the individual are randomly generated (Line 10). Then, ''DE/current-to-gselite/1'' is executed to get the mutation vector and the binomial crossover is performed to gain the trial vector (Line 11). Next, the selection operation is carried out along with the update of the archive storing the obsolete parent individuals and the adaption of the successful F and *CR* (Lines 13$\sim$15). After the update of all individuals, the two parameter archives for *CR* and F are updated accordingly (Lines 17$\sim$19). Then, the next iteration proceeds. GSGDE continues until the termination criteria is met, which is usually that the preset number of fitness evaluations runs out. In the end, the found best solution by the population is output.

From this algorithm, we can see that except for the fitness evaluation time in each iteration, it takes O(NP^{*}logNP)

TABLE 1. **The optimal population sizes of all methods on the CEC2017 suite with the three dimensionality settings.**

D	Parameter	GSGDE	GPDE (2019)	ITGDE (2020)	DMCDE (2020)	SEDE (2020)	FADE (2021)	FDDE (2021)	TPDE (2021)	CUSDE (2022)	NSHADE (2022)	PFIDE (2022)	SDEGCM (2022)
30	NP	150	40	130	170	40	NP_{init}=100	130	100	100	100	170	30
50		140	40	180	170	60	NP_{max}=NP_{init}	150	100	100	100	140	70
100		150	40	140	140	60	NP_{min}=1/3*NP_{init}	140	100	100	100	170	30

to sort the population and O($NP^*D^*NEI+NP^*D$) to generate leading exemplars for all individuals. Then, it consumes O(NP^*D) to get trial vectors of all individuals and another O(NP^*D) to update the three archives. Comprehensively speaking, the overall complexity of GSGDE is O($NP^*\log NP+NP^*D^*NEI$). In comparison with the classical DE algorithm, GSGDE does not impose severe computational burden.

Algorithm 1 The Overall Procedure of GSGDE

Input: population size NP, maximum fitness evaluations FES_{max};
Generate NP individuals randomly and calculate their fitness; $nfe = NP$;
Set all elements of $\boldsymbol{M_{CR}}$ and $\boldsymbol{M_F}$ as 0.5, and set the archive $\boldsymbol{A}=\emptyset$;
While($nfe \le FES_{max}$) **do**
  $\boldsymbol{S_F}=\emptyset, \boldsymbol{S_{CR}}=\emptyset$;
  Calculate the values of p and NEI by Eq. (17) and Eq. (16), respectively;
  Sort individuals from the lowest to the highest with respect to their fitness;
  For $i = 1{:}NP$ **do**
    Randomly choose an elite individual from the top NEI elites as $\boldsymbol{\mu}$ and calculate $\boldsymbol{\sigma}$ by Eq. (13);
    Generate $\boldsymbol{x_{elite}^{gs}}$ by the Gaussian distribution with $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$;
    Randomly select $M_{F,r}$ from $\boldsymbol{M_F}$ and $M_{CR,r}$ from $\boldsymbol{M_{CR}}$ and then generate F and CR by Eq. (8) and Eq. (9);
    Obtain the mutant vector $\boldsymbol{v_i}$ by Eq. (15) and then the trial vector $\boldsymbol{u_i}$ by Eq. (6);
    Calculate the fitness of $\boldsymbol{u_i}$ and nfe++;
    If ($f(\boldsymbol{u_i}) \le f(\boldsymbol{x_i})$) **then**
      Insert $\boldsymbol{x_i}$ into $\boldsymbol{A}$ and put the values of F and CR into $\boldsymbol{S_F}$ and $\boldsymbol{S_{CR}}$, respectively; $\boldsymbol{x_i} = u_i$;
    End If
  End For
  If ($\boldsymbol{S_F} \ne \emptyset$ and $\boldsymbol{S_{CR}} \ne \emptyset$) **then**
    Update $\boldsymbol{M_F}$ and $\boldsymbol{M_{CR}}$ by Eq. (10)-(12);
  End If
End While
Attain the best solution $\boldsymbol{x_{best}}$ and its fitness $f(\boldsymbol{x_{best}})$;

Output: $f(\boldsymbol{gbest})$ and $\boldsymbol{gbest}$

D. DIFFERENCE BETWEEN GSGDE AND EXISTING SIMILAR DE VARIANTS

The devised "DE/current-to-gselite/1" is similar to two classical mutation schemes, namely "DE/current-to-best/1" [64] and "DE/current-to-pbest/1" [21], [41]. In addition, in the literature, some Gaussian distribution based mutation schemes have also been designed [28], [71], [72], [73]. In comparison with these existing mutation schemes, GSGDE has the following major difference from them:

1) Instead of directly utilizing existing elites in the current population as the leading exemplars to update individuals as in "DE/current-to-best/1" [64] and "DE/current-to-pbest/1" [21], [41], "DE/current-to-gselite/1" utilizes an unseen guiding exemplar randomly sampled by the Gaussian distribution based on elites from the population to direct the evolution of each individual. On the one hand, since the number of elite individuals is much smaller than the population size, many individuals may share the same guiding exemplars to mutate in "DE/current-to-pbest/1" [21], [41] and "DE/current-to-best/1" [64]. As a consequence, the mutation diversity of these two classical mutation schemes is limited. However, the devised mutation scheme randomly samples a guiding exemplar for each individual with the Gaussian distribution around one randomly selected elite. Therefore, the leading exemplars are likely distinct for different individuals. Even though one elite may be selected as the mean vector of the Gaussian distribution for many individuals, the randomly generated guiding exemplars for these individuals are usually different. As a result, the devised mutation strategy could afford much higher mutation diversity for DE. On the other hand, the leading exemplars in "DE/current-to-pbest/1" [21], [41] and "DE/current-to-best/1" [64] all exist in the current population. However, the randomly sampled leading exemplars in the devised mutation strategy are all unseen positions by the population. Since they are all generated by the Gaussian distribution around the selected elites with small standard deviations, it is highly possible that the generated guiding exemplars are better than the selected elites. In this way, individuals are expectedly guided to move towards promising areas fast but with high mutation diversity. As a whole, it is anticipated that the devised mutation strategy provides much better balance between search diversity and search convergence than the two classical mutation schemes, and thus it is more effective than the two classical mutation schemes to help DE obtain better performance.
2) Unlike existing Gaussian distribution based mutation strategies [28], [70], [71], [72], [73], which directly sample a mutation vector randomly for each individual, the devised mutation scheme first randomly samples a leading exemplar for each individual and then uses such a sampled leading exemplar to mutate the individual. Specifically, in [28], the Gaussian based mutation randomly generates a mutation vector around a randomly selected individual for each target individual. Specifically, it first chooses three distinct individuals randomly from the population and then utilizes one random individual as the mean and the Euclidean distance between the other two random individuals as

the standard deviation of the Gaussian distribution to randomly sample a mutation vector randomly for each individual. In [73], for each individual to be mutated, the designed Gaussian distribution based mutation utilizes the mean position between this individual and the best individual as the mean and their difference as the standard deviation of the Gaussian distribution to randomly sample a mutation vector for the individual. In [70], for each individual to be mutated, the proposed Gaussian distribution based mutation utilizes the mean position among this individual, the best individual, and a randomly chosen individual as the mean. Then, it uses the weighted sum of their difference and fitness values to serve as the standard deviation of the Gaussian distribution to randomly sample a mutation vector for the individual. In [71] and [72], the Gaussian distribution is adopted as a local search engine to mutate each individual. Specifically, it directly utilizes each individual as the mean and a small fixed value as the standard deviation to randomly generate a mutation vector for the individual. Nevertheless, the devised mutation scheme in this paper randomly generates a guiding exemplar based on the Gaussian distribution around one randomly selected elite for each individual. Thanks to the small sampling range of the Gaussian distribution, the randomly generated guiding exemplar is likely better than the selected elite. Therefore, compared with the existing Gaussian distribution based mutation schemes, individuals mutated by the devised mutation strategy are likely guided to move towards optimal regions faster.

IV. EXPERIMENTS

This section performs experiments to substantiate the efficiency and effectiveness of GSGDE comprehensively. To be concrete, Section IV-A describes the experiment environments briefly including the used two benchmark problem suites, the evaluation measurement, and the compared algorithms. Subsequently, Section IV-B shows the comparisons between GSGDE and the compared algorithms on the CEC2017 problem set. Next, Section IV-C exhibits the comparisons between GSGDE and the compared approaches on the CEC2014 problem suite. At last, Section IV-D presents investigations on GSGDE by observing the effectiveness of the designed mutation strategy and the devised dynamic adjustment scheme for the number of elites.

A. EXPERIMENTAL SETUP

To verify the optimization effectiveness of GSGDE, we conduct experiments on the latest CEC2014 [55] and CEC2017 [56] benchmark sets, which has been widely adopted to test evolutionary computation algorithms including DE in the literature [23], [26], [27]. Specifically, the CEC2017 suite has 29 optimization problems with four types, namely the unimodal, the multimodal, the hybrid, and the composition problems, while the CEC2014 suite has 30 benchmark problems with the same four types. Tables SI and SII in the supplementary document display the summarized information of the CEC2017 suite and the CEC2014 set, respectively. For more concrete information, please refer to [55] and [56]. In the experiments, to assess the performance of GSGDE comprehensively, we adopt three distinct dimensionality settings, namely 30, 50, and 100, for both benchmark suites.

To comprehensively show the efficiency and effectiveness of GSGDE, this paper compares it with a number of latest and well-performed DE variants. Particularly, totally 11 latest and representative DE algorithms are chosen, namely GPDE [28], ITGDE [70], DMCDE [32], SEDE [40], FADE [36], FDDE [30], TPDE [31], CUSDE [23], NSHADE [42], PFIDE [7], and SDEGCM [71]. To make fair comparisons, if the compared DE algorithm has not been tested on the CEC2017 set in the associated paper, we fine-tune its population size *NP* on the CEC2017 suite with the three dimensionality settings. Otherwise, we directly set the parameters by following the recommendation in the associated paper. The concrete fine-tuning results of GSGDE and some compared DE variants for the population size are displayed in Tables SIII $\sim$ SX in the supplementary document. According to these results, Table 1 summarizes the optimal *NP* of all methods on the CEC2017 suite with the three dimensionality settings. It deserves notice that these population size settings of all methods are also utilized in the experiments on the CEC2014 benchmark suite.

Unless otherwise elucidated, the maximum number of fitness evaluations is configured as 10000^*D (where D is the dimension size) for all methods. Additionally, we execute each method independently 51 times and then employ the mean value and the standard deviation (std) value over the 51 independent executions to measure the performance of the algorithm. In addition, to show the statistical significance, we perform the Wilcoxon rank sum test at the significance level of $\alpha = 0.05$ to compare GSGDE with each algorithm on each benchmark problem. The symbols "+", "=", and "−" at the upper right corner of each p-value in the tables indicate that GSGDE obtains significantly better, equivalent and significantly worse optimization performance than the compared method on the corresponding problem, respectively. "*w*/*t*/*l*" in the tables count the numbers of "+", "=", and "−", respectively. To compare the overall optimization performance of different methods on the entire problem suite, we also conduct the Friedman test at the significance level of $\alpha = 0.05$ to attain the average ranks of all methods on the whole problem suite.

At last, it should be noted that all methods are run on the same PC with the Ubuntu 20.04 LTS 64-bit system, along with 8Gb memory and 4 Intel(R) Core(TM) i5-3470 CPUs.

B. COMPARISON WITH STATE-OF-THE-ART DE VARIANTS ON THE CEC2017 SUITE

This subsection compares GSGDE with the 11 selected latest DE algorithms on the CEC2017 problem suite with the three

TABLE 2. Optimization results of GSGDE and the 11 state-of-the-art DE variants and their comparisons on the 30 *D* CEC2017 suite.

F	Category	Quality	GSGDE	GPDE	ITGDE	DMCDE	SEDE	FADE	FDDE	TPDE	CUSDE	NSHADE	PFIDE	SDEGCM
F_1	Unimodal Problems	Mean	9.47E-16	8.43E-06	5.40E-02	7.58E-15	0.00E+00	0.00E+00	9.95E-15	9.47E-16	5.02E-08	0.00E+00	2.84E-15	1.61E-14
		Std	3.61E-15	4.48E-05	5.10E-02	7.21E-15	0.00E+00	0.00E+00	7.60E-15	3.61E-15	2.52E-07	0.00E+00	5.78E-15	7.21E-15
		p-value	-	**2.36E-12**$^+$	**2.36E-12**$^+$	**9.55E-05**$^+$	1.61E-01$^=$	1.61E-01$^=$	**1.85E-06**$^+$	9.59E-01$^=$	**8.43E-05**$^+$	1.61E-01$^=$	1.35E-01$^=$	**1.30E-11**$^+$
F_3		Mean	5.68E-14	6.85E+04	1.52E+01	2.82E-08	0.00E+00	1.33E-07	6.25E+02	4.12E-02	1.34E-10	2.33E+04	4.93E-14	1.92E+03
		Std	2.11E-14	1.18E+04	7.78E+00	4.90E-08	0.00E+00	5.71E-07	3.42E+03	7.24E-02	7.29E-10	3.69E+04	1.97E-14	1.74E+03
		p-value	-	**4.10E-12**$^+$	**4.10E-12**$^+$	**4.10E-12**$^+$	1.43E-12$^-$	**1.99E-09**$^+$	2.59E-01$^=$	**4.10E-12**$^+$	**6.11E-12**$^+$	**1.16E-02**$^+$	1.64E-01$^=$	**4.10E-12**$^+$
$F_{1,3}$		*w/t/l*		2/0/0	2/0/0	2/0/0	0/1/1	1/1/0	1/1/0	1/1/0	2/0/0	1/1/0	0/2/0	2/0/0
F_4	Simple Multimodal Problems	Mean	5.38E+01	8.25E+01	3.52E+01	6.08E+01	5.81E+01	2.65E+01	5.01E+01	5.86E+01	5.87E+01	4.62E+00	4.90E+01	8.40E+01
		Std	1.84E+01	4.29E+00	3.25E+01	1.38E+01	1.12E+01	2.97E+01	2.29E+01	3.61E-14	1.01E+00	1.08E+01	2.37E+01	1.00E+01
		p-value	-	**2.01E-11**$^+$	7.91E-01$^=$	**1.75E-02**$^+$	**3.34E-03**$^+$	5.01E-04$^-$	9.65E-01$^=$	**5.56E-09**$^+$	4.79E-01$^=$	5.46E-08$^-$	8.40E-01$^=$	**2.01E-11**$^+$
F_5		Mean	2.23E+01	1.04E+02	9.79E+01	1.06E+02	3.71E+01	3.11E+01	2.71E+01	3.82E+01	2.98E+01	3.42E+01	2.90E+01	4.80E+01
		Std	3.38E+00	4.01E+01	2.06E+01	5.30E+01	1.12E+01	8.53E+00	3.82E+00	7.25E+00	1.22E+01	6.04E+00	4.18E+00	2.02E+01
		p-value	-	**1.55E-09**$^+$	**3.02E-11**$^+$	**3.32E-06**$^+$	**1.85E-08**$^+$	**4.11E-06**$^+$	**9.51E-06**$^+$	**8.10E-10**$^+$	**2.41E-02**$^+$	**1.96E-10**$^+$	**1.47E-07**$^+$	**8.09E-10**$^+$
F_6		Mean	1.14E-13	1.62E-03	2.53E-01	2.94E-07	0.00E+00	1.84E-03	1.14E-13	1.14E-13	4.56E-09	0.00E+00	1.14E-13	2.47E-07
		Std	0.00E+00	9.01E-04	3.79E-01	8.86E-07	0.00E+00	3.86E-03	0.00E+00	5.14E-29	2.50E-08	0.00E+00	0.00E+00	8.80E-07
		p-value	-	**1.21E-12**$^+$	**1.21E-12**$^+$	**1.36E-03**$^+$	1.69E-14$^-$	**1.21E-12**$^+$	NaN$^=$	**1.69E-14**$^+$	3.34E-01$^=$	1.69E-14$^-$	NaN$^=$	**5.56E-03**$^+$
F_7		Mean	5.47E+01	1.69E+02	1.28E+02	1.89E+02	6.34E+01	5.64E+01	5.71E+01	6.74E+01	6.21E+01	6.41E+01	6.11E+01	8.62E+01
		Std	3.19E+00	1.44E+01	2.12E+01	1.13E+01	1.12E+01	7.26E+00	3.64E+00	7.32E+00	1.48E+01	4.92E+00	4.19E+00	1.71E+01
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**1.11E-03**$^+$	3.33E-01$^=$	**8.68E-03**$^+$	**6.52E-09**$^+$	7.24E-02$^=$	**1.01E-08**$^+$	**3.26E-07**$^+$	**3.47E-10**$^+$
F_8		Mean	2.34E+01	1.06E+02	7.28E+01	1.12E+02	3.68E+01	2.81E+01	2.58E+01	4.16E+01	2.81E+01	3.44E+01	2.94E+01	4.21E+01
		Std	3.75E+00	4.04E+01	1.18E+01	5.35E+01	1.35E+01	8.98E+00	3.93E+00	6.49E+00	1.26E+01	6.47E+00	4.06E+00	1.74E+01
		p-value	-	**3.34E-11**$^+$	**3.01E-11**$^+$	**2.49E-06**$^+$	**3.83E-05**$^+$	8.50E-02$^=$	**3.15E-02**$^+$	**3.69E-11**$^+$	2.64E-01$^=$	**3.08E-08**$^+$	**1.61E-06**$^+$	**2.83E-08**$^+$
F_9		Mean	0.00E+00	1.81E-02	1.90E-01	9.96E-02	9.38E-02	1.09E+00	2.98E-03	0.00E+00	3.79E-15	5.80E+00	0.00E+00	5.12E-01
		Std	0.00E+00	8.40E-02	7.22E-01	1.82E-01	3.34E-01	1.24E+00	1.63E-02	0.00E+00	2.08E-14	1.34E+01	0.00E+00	1.48E+00
		p-value	-	**4.08E-12**$^+$	**5.37E-06**$^+$	**3.08E-04**$^+$	**4.19E-02**$^+$	**1.79E-09**$^+$	3.34E-01$^=$	NaN$^=$	3.34E-01$^=$	**1.93E-10**$^+$	NaN$^=$	**8.29E-07**$^+$
F_{10}		Mean	2.26E+03	3.83E+03	3.80E+03	6.28E+03	5.25E+03	2.93E+03	2.47E+03	2.46E+03	2.45E+03	1.94E+03	2.96E+03	3.12E+03
		Std	2.79E+02	1.07E+03	6.21E+02	5.22E+02	5.49E+02	1.61E+03	2.96E+02	3.24E+02	8.30E+02	1.96E+02	2.18E+02	6.61E+02
		p-value	-	**2.92E-09**$^+$	**6.12E-10**$^+$	**3.02E-11**$^+$	**3.34E-11**$^+$	2.34E-01$^=$	**6.38E-03**$^+$	**2.61E-02**$^+$	6.52E-01$^=$	2.96E-05$^-$	**2.15E-10**$^+$	**2.57E-07**$^+$
$F_{4\text{-}10}$		*w/t/l*		7/0/0	6/1/0	7/0/0	6/0/1	3/3/1	4/3/0	6/1/0	1/6/0	4/0/3	4/3/0	7/0/0
F_{11}	Hybrid Problems	Mean	3.56E+01	7.61E+01	7.65E+01	3.54E+01	2.28E+01	1.43E+01	3.77E+01	3.05E+01	2.03E+01	7.55E+01	3.53E+01	3.66E+01
		Std	2.89E+01	3.20E+01	2.41E+01	3.25E+01	2.28E+01	1.50E+01	2.96E+01	2.27E+01	2.31E+01	2.85E+01	2.92E+01	2.82E+01
		p-value	-	**9.53E-07**$^+$	**4.35E-05**$^+$	2.84E-01$^=$	8.65E-01$^=$	6.78E-02$^=$	1.91E-01$^=$	4.04E-01$^=$	2.40E-01$^=$	**4.22E-04**$^+$	2.23E-01$^=$	1.76E-01$^=$
F_{12}		Mean	1.23E+03	1.20E+05	1.87E+04	1.56E+04	1.31E+04	8.09E+03	3.96E+03	9.84E+02	2.01E+04	1.10E+04	2.63E+03	5.62E+04
		Std	3.76E+02	3.95E+05	9.44E+03	1.46E+04	1.15E+04	9.30E+03	5.17E+03	6.30E+02	1.80E+04	1.12E+04	1.99E+03	4.32E+04
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**8.89E-10**$^+$	**8.48E-09**$^+$	**1.73E-07**$^+$	**2.00E-06**$^+$	5.87E-04$^-$	**3.02E-11**$^+$	**2.03E-09**$^+$	**2.39E-04**$^+$	**3.02E-11**$^+$
F_{13}		Mean	1.95E+01	1.91E+04	1.76E+03	2.32E+03	2.72E+01	2.27E+01	5.65E+01	1.60E+03	2.88E+01	1.27E+02	8.36E+01	4.72E+03
		Std	8.39E+00	2.37E+04	1.85E+03	1.11E+04	6.49E+00	7.93E+00	3.97E+01	4.86E+03	7.73E+00	7.72E+01	9.08E+01	1.53E+04
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**6.07E-11**$^+$	**3.56E-04**$^+$	7.24E-02$^=$	**6.28E-06**$^+$	**2.61E-10**$^+$	**6.74E-06**$^+$	**4.98E-11**$^+$	**1.86E-09**$^+$	**3.02E-11**$^+$
F_{14}		Mean	2.52E+01	8.56E+01	4.77E+01	3.80E+01	2.00E+01	1.17E+01	3.37E+01	3.46E+01	3.17E+01	5.65E+01	3.05E+01	5.68E+01
		Std	1.93E+00	6.95E+00	6.38E+00	1.22E+01	1.09E+01	5.30E+00	5.88E+00	4.19E+00	8.75E+00	1.71E+01	3.30E+00	1.74E+01
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**5.00E-09**$^+$	6.52E-01$^=$	2.44E-09$^-$	**1.09E-10**$^+$	**8.99E-11**$^+$	**2.00E-06**$^+$	**3.02E-11**$^+$	**2.23E-09**$^+$	**3.02E-11**$^+$
F_{15}		Mean	5.56E+00	3.41E+03	5.56E+01	3.85E+01	1.09E+01	8.21E+00	2.26E+01	1.60E+01	7.84E+00	6.55E+01	1.40E+01	4.82E+01
		Std	5.80E+00	1.03E+04	2.91E+01	4.32E+01	6.02E+00	2.98E+00	1.81E+01	3.72E+00	3.50E+00	4.07E+01	1.27E+01	2.20E+01
		p-value	-	**3.02E-11**$^+$	**6.70E-11**$^+$	**5.97E-09**$^+$	**6.28E-06**$^+$	**1.64E-05**$^+$	**4.18E-09**$^+$	**8.89E-10**$^+$	**1.78E-04**$^+$	**2.61E-10**$^+$	**2.49E-06**$^+$	**5.49E-11**$^+$
F_{16}		Mean	2.17E+02	5.05E+02	5.63E+02	3.63E+02	5.06E+02	5.34E+02	3.48E+02	2.90E+02	4.71E+02	4.19E+02	3.29E+02	4.57E+02
		Std	7.93E+01	2.40E+02	1.62E+02	3.79E+02	2.19E+02	2.54E+02	1.18E+02	1.40E+02	3.28E+02	1.51E+02	1.56E+02	2.80E+02
		p-value	-	**1.19E-06**$^+$	**6.12E-10**$^+$	8.19E-01$^=$	**2.03E-07**$^+$	**1.39E-06**$^+$	**4.08E-05**$^+$	**7.62E-03**$^+$	**1.95E-03**$^+$	**4.80E-07**$^+$	**2.89E-03**$^+$	**1.00E-03**$^+$
F_{17}		Mean	6.64E+01	1.56E+02	1.84E+02	7.33E+01	1.64E+02	5.75E+01	8.96E+01	6.42E+01	1.20E+02	6.52E+01	7.35E+01	1.53E+02
		Std	1.01E+01	1.12E+02	1.02E+02	3.76E+01	9.89E+01	6.49E+01	2.13E+01	1.73E+01	1.36E+02	1.32E+01	1.28E+01	1.14E+02
		p-value	-	**1.86E-03**$^+$	**1.19E-06**$^+$	9.12E-01$^=$	**3.18E-04**$^+$	1.73E-06$^-$	**3.09E-06**$^+$	2.40E-01$^=$	4.29E-01$^=$	7.84E-01$^=$	**2.92E-02**$^+$	**5.83E-03**$^+$
F_{18}		Mean	5.11E+01	3.42E+05	5.19E+02	1.37E+02	3.55E+01	1.63E+01	2.44E+03	3.15E+01	2.82E+01	6.04E+01	3.33E+01	1.07E+04
		Std	3.89E+01	2.54E+05	7.04E+02	1.77E+02	1.49E+01	1.12E+01	1.30E+04	2.79E+00	3.61E+00	3.18E+01	1.34E+01	1.33E+04
		p-value	-	**3.02E-11**$^+$	**3.82E-10**$^+$	**1.09E-05**$^+$	2.64E-01$^=$	9.53E-07$^-$	2.28E-01$^=$	1.15E-01$^=$	7.29E-03$^-$	5.01E-02$^=$	4.51E-02$^-$	**3.02E-11**$^+$
F_{19}		Mean	6.02E+00	4.72E+03	2.37E+01	1.81E+01	5.41E+00	5.60E+00	1.82E+01	1.38E+01	8.39E+00	4.08E+01	1.37E+01	2.55E+01
		Std	1.91E+00	1.41E+04	3.39E+00	1.90E+01	1.80E+00	1.85E+00	1.11E+01	1.76E+00	2.04E+00	1.54E+01	3.77E+00	7.96E+00
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**5.53E-08**$^+$	1.41E-01$^=$	3.04E-01$^=$	**6.70E-11**$^+$	**3.02E-11**$^+$	**8.15E-05**$^+$	**3.02E-11**$^+$	**3.47E-10**$^+$	**3.02E-11**$^+$
F_{20}		Mean	7.65E+01	2.85E+02	1.48E+02	4.75E+01	1.72E+02	5.89E+01	9.18E+01	4.01E+01	1.14E+02	1.36E+02	7.42E+01	1.19E+02
		Std	3.39E+01	1.98E+02	4.91E+01	8.13E+01	1.16E+02	6.16E+01	4.33E+01	1.41E+01	1.44E+02	5.63E+01	2.12E+01	1.15E+02
		p-value	-	**3.57E-06**$^+$	**5.97E-05**$^+$	1.07E-07$^-$	**1.17E-03**$^+$	3.27E-02$^-$	**4.36E-02**$^+$	3.65E-08$^-$	**4.51E-02**$^+$	**8.56E-04**$^+$	3.95E-01$^=$	9.47E-01$^=$
$F_{11\text{-}20}$		*w/t/l*		10/0/0	10/0/0	6/3/1	6/4/0	3/3/4	8/2/0	5/3/2	7/2/1	8/2/0	7/2/1	8/2/0
F_{21}	Composition Problems	Mean	2.23E+02	3.23E+02	2.67E+02	2.72E+02	2.39E+02	2.28E+02	2.27E+02	2.42E+02	2.28E+02	2.33E+02	2.29E+02	2.49E+02
		Std	3.21E+00	2.92E+01	1.72E+01	5.12E+01	1.11E+01	8.16E+00	4.25E+00	6.93E+00	1.42E+01	6.03E+00	5.27E+00	1.86E+01
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**2.43E-05**$^+$	**2.19E-08**$^+$	**3.18E-03**$^+$	**6.91E-04**$^+$	**3.02E-11**$^+$	6.79E-02$^=$	**5.46E-09**$^+$	**1.86E-06**$^+$	**3.16E-10**$^+$
F_{22}		Mean	1.00E+02	5.67E+03	1.00E+02	1.78E+03	4.91E+03	1.00E+02	1.00E+02	5.14E+02	2.24E+03	1.01E+02	1.00E+02	1.65E+03
		Std	0.00E+00	1.05E+03	0.00E+00	2.83E+03	1.95E+03	0.00E+00	0.00E+00	9.47E+02	1.26E+03	2.76E+00	0.00E+00	1.70E+03
		p-value	-	**1.21E-12**$^+$	NaN$^=$	**2.79E-03**$^+$	**1.93E-10**$^+$	NaN$^=$	NaN$^=$	**2.16E-02**$^+$	**6.25E-10**$^+$	**5.58E-03**$^+$	NaN$^=$	**1.27E-05**$^+$
F_{23}		Mean	3.69E+02	4.44E+02	4.55E+02	3.80E+02	3.95E+02	3.76E+02	3.74E+02	3.91E+02	3.77E+02	3.81E+02	3.77E+02	3.93E+02
		Std	4.01E+00	4.70E+01	2.73E+01	8.12E+00	1.52E+01	9.78E+00	6.60E+00	7.43E+00	9.31E+00	7.08E+00	5.93E+00	1.38E+01
		p-value	-	**9.76E-10**$^+$	**3.02E-11**$^+$	**7.09E-08**$^+$	**8.89E-10**$^+$	**9.52E-04**$^+$	**7.30E-04**$^+$	**4.08E-11**$^+$	**4.94E-05**$^+$	**3.35E-08**$^+$	**4.42E-06**$^+$	**3.82E-10**$^+$
F_{24}		Mean	4.40E+02	5.93E+02	5.54E+02	4.51E+02	4.58E+02	4.47E+02	4.42E+02	4.63E+02	4.51E+02	4.49E+02	4.46E+02	4.82E+02
		Std	2.57E+00	1.05E+01	4.23E+01	7.57E+00	1.20E+01	1.00E+01	4.37E+00	6.74E+00	1.13E+01	9.79E+00	4.68E+00	2.16E+01
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.01E-07**$^+$	**4.20E-10**$^+$	**6.55E-04**$^+$	**2.51E-02**$^+$	**3.02E-11**$^+$	**8.29E-06**$^+$	**1.04E-04**$^+$	**9.53E-07**$^+$	**3.02E-11**$^+$
F_{25}		Mean	3.87E+02	3.87E+02	3.87E+02	3.87E+02	3.87E+02	3.87E+02	3.87E+02	3.87E+02	3.87E+02	3.89E+02	3.87E+02	3.87E+02
		Std	9.91E-02	9.71E-02	3.49E+00	4.28E-01	6.39E-01	9.31E-02	1.68E-01	9.24E-02	2.13E-02	8.28E+00	1.81E-01	9.01E-01
		p-value	-	2.48E-06$^-$	6.00E-01$^=$	**6.23E-03**$^+$	8.77E-02$^=$	8.77E-02$^=$	**1.44E-06**$^+$	9.94E-01$^=$	5.21E-11$^-$	**6.36E-05**$^+$	**6.97E-03**$^+$	9.41E-01$^=$
F_{26}		Mean	1.11E+03	1.92E+03	2.49E+03	1.30E+03	1.53E+03	1.31E+03	1.19E+03	1.47E+03	1.27E+03	1.18E+03	1.21E+03	1.48E+03
		Std	7.31E+01	4.23E+02	1.49E+03	9.42E+01	1.40E+02	1.59E+02	6.55E+01	8.88E+01	9.76E+01	5.11E+02	6.50E+01	1.51E+02
		p-value	-	**5.49E-11**$^+$	**1.95E-03**$^+$	**8.48E-09**$^+$	**3.02E-11**$^+$	**9.83E-08**$^+$	**6.35E-05**$^+$	**3.69E-11**$^+$	**9.83E-08**$^+$	**4.98E-04**$^+$	**2.49E-06**$^+$	**5.49E-11**$^+$
F_{27}		Mean	5.06E+02	5.03E+02	5.66E+02	5.07E+02	5.00E+02	4.98E+02	5.04E+02	5.01E+02	4.98E+02	5.18E+02	5.04E+02	5.05E+02
		Std	6.23E+00	6.08E+00	1.90E+01	8.86E+00	9.80E-05	9.81E+00	5.59E+00	6.14E+00	9.24E+00	6.56E+00	6.20E+00	8.13E+00
		p-value	-	5.75E-02$^=$	**3.02E-11**$^+$	4.04E-01$^=$	1.11E-06$^-$	3.85E-03$^-$	5.40E-01$^=$	8.31E-03$^-$	9.03E-04$^-$	**9.06E-08**$^+$	2.58E-01$^=$	4.20E-01$^=$
F_{28}		Mean	3.29E+02	3.91E+02	3.17E+02	3.84E+02	5.00E+02	3.35E+02	3.35E+02	3.27E+02	3.12E+02	3.28E+02	3.25E+02	3.89E+02
		Std	5.51E+01	7.91E+01	3.40E+01	6.51E+01	1.41E+00	5.60E+01	5.47E+01	5.28E+01	3.92E+01	5.28E+01	4.58E+01	5.49E+01
		p-value	-	**8.40E-05**$^+$	4.52E-03$^-$	**1.11E-03**$^+$	**7.82E-12**$^+$	6.48E-01$^=$	5.94E-01$^=$	8.11E-01$^=$	1.71E-01$^=$	8.73E-01$^=$	9.07E-01$^=$	**4.23E-05**$^+$
F_{29}		Mean	4.84E+02	5.50E+02	5.76E+02	4.99E+02	4.20E+02	4.40E+02	4.88E+02	5.53E+02	5.15E+02	5.00E+02	5.11E+02	4.72E+02
		Std	1.78E+01	1.59E+02	1.14E+02	9.26E+01	9.86E+01	1.66E+01	2.47E+01	6.87E+01	1.03E+02	4.28E+01	2.23E+01	7.45E+01
		p-value	-	1.33E-01$^=$	**3.16E-05**$^+$	3.87E-01$^=$	1.89E-04$^-$	1.17E-09$^-$	**3.95E-01**$^=$	**6.74E-06**$^+$	3.55E-01$^=$	7.98E-02$^=$	**6.74E-06**$^+$	1.63E-02$^-$
F_{30}		Mean	2.05E+03	5.35E+03	3.39E+03	2.47E+03	1.28E+03	2.22E+03	2.13E+03	1.98E+03	2.04E+03	2.46E+03	2.08E+03	4.11E+03
		Std	1.01E+02	1.85E+03	7.85E+02	1.88E+03	8.99E+02	1.36E+02	1.27E+02	4.39E+01	8.43E+01	3.46E+02	1.41E+02	1.77E+03
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.50E-03**$^+$	1.22E-02$^-$	**8.20E-07**$^+$	**1.86E-03**$^+$	1.06E-03$^-$	9.23E-01$^=$	**3.20E-09**$^+$	5.20E-01$^=$	**4.50E-11**$^+$
$F_{21\text{-}30}$		*w/t/l*		7/2/1	7/2/1	8/2/0	6/1/3	5/3/2	7/3/0	6/2/2	4/4/2	8/2/0	6/4/0	7/2/1
	w/t/l			26/2/1	25/3/1	23/5/1	18/6/5	12/10/7	20/9/0	18/7/4	14/12/3	21/5/3	17/11/1	24/4/1
	Rank		**3.24**	10.65	9.44	8.01	6.05	4.24	5.25	5.55	5.22	6.77	4.50	9.03

TABLE 3. Optimization results of GSGDE and the 11 state-of-the-art DE variants and their comparisons on the 50 *D* CEC2017 suite.

F	Category	Quality	GSGDE	GPDE	ITGDE	DMCDE	SEDE	FADE	FDDE	TPDE	CUSDE	NSHADE	PFIDE	SDEGCM
F_1	Unimodal Problems	Mean	3.08E-14	8.94E+03	8.77E+02	9.09E-02	8.36E-03	6.75E+01	3.13E-14	2.36E+01	1.75E+01	1.30E-01	1.71E-14	1.37E+03
		Std	1.30E-14	9.32E+03	1.38E+03	4.06E-01	3.69E-02	2.01E+02	1.14E-14	1.20E+02	4.74E+01	5.34E-01	5.78E-15	2.19E+03
		p-value	-	**1.25E-11**$^{+}$	**1.25E-11**$^{+}$	**1.25E-11**$^{+}$	**2.56E-10**$^{+}$	**1.25E-11**$^{+}$	6.79E-01$^{=}$	**1.25E-11**$^{+}$	**1.25E-11**$^{+}$	**1.25E-11**$^{+}$	2.05E-07^{-}	**1.25E-11**$^{+}$
F_3		Mean	2.73E-13	2.09E+05	2.43E+03	9.66E+01	0.00E+00	1.98E+00	9.33E+03	7.37E+01	2.00E+03	3.97E+04	2.16E-11	1.06E+05
		Std	7.81E-14	2.05E+04	9.28E+02	2.95E+02	0.00E+00	3.44E+00	2.17E+04	6.71E+01	6.50E+03	8.08E+04	9.53E-11	2.84E+04
		p-value	-	**2.50E-11**$^{+}$	**2.50E-11**$^{+}$	**2.50E-11**$^{+}$	9.46E-13^{-}	**2.50E-11**$^{+}$	3.48E-01$^{=}$	**2.50E-11**$^{+}$	**2.50E-11**$^{+}$	**2.50E-11**$^{+}$	2.20E-01$^{=}$	**2.50E-11**$^{+}$
$F_{1\text{-}3}$		*w/t/l*		2/0/0	2/0/0	2/0/0	1/0/1	2/0/0	0/2/0	2/0/0	2/0/0	2/0/0	0/1/1	2/0/0
F_4	Simple Multimodal Problems	Mean	5.01E+01	9.90E+01	1.26E+02	8.00E+01	6.04E+01	5.66E+01	5.40E+01	8.40E+01	7.14E+01	6.63E+01	4.80E+01	1.01E+02
		Std	4.38E+01	5.30E+01	4.32E+01	4.89E+01	4.61E+01	4.56E+01	3.61E+01	5.81E+01	4.34E+01	5.03E+01	3.89E+01	5.14E+01
		p-value	-	**2.36E-04**$^{+}$	**1.37E-07**$^{+}$	**3.58E-03**$^{+}$	**3.83E-02**$^{+}$	1.54E-01$^{=}$	4.66E-01$^{=}$	**3.23E-04**$^{+}$	7.56E-02$^{=}$	9.95E-02$^{=}$	8.10E-01$^{=}$	**1.10E-04**$^{+}$
F_5		Mean	3.97E+01	2.32E+02	1.91E+02	5.68E+01	7.72E+01	6.13E+01	5.72E+01	8.61E+01	4.36E+01	9.09E+01	6.53E+01	1.93E+02
		Std	4.42E+00	9.25E+01	2.45E+01	4.02E+01	2.21E+01	1.12E+01	7.47E+00	1.29E+01	8.65E+00	1.40E+01	8.28E+00	2.39E+01
		p-value	-	**3.02E-11**$^{+}$	**2.99E-11**$^{+}$	**5.08E-06**$^{+}$	**3.02E-11**$^{+}$	**1.77E-10**$^{+}$	**6.07E-11**$^{+}$	**3.02E-11**$^{+}$	5.94E-02$^{=}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_6		Mean	1.63E-07	3.98E-03	1.76E+00	3.02E-03	6.67E-08	5.36E-02	2.04E-07	1.89E-07	4.74E-06	0.00E+00	3.05E-06	2.88E-13
		Std	2.69E-07	1.77E-03	1.11E+00	1.05E-02	3.65E-07	5.21E-02	9.23E-07	6.81E-07	1.24E-05	0.00E+00	4.15E-06	1.95E-13
		p-value	-	**2.54E-11**$^{+}$	**2.54E-11**$^{+}$	**2.56E-08**$^{+}$	1.28E-10^{-}	**2.54E-11**$^{+}$	**1.00E-03**$^{+}$	1.97E-01$^{=}$	**1.12E-06**$^{+}$	3.67E-12^{-}	**3.43E-08**$^{+}$	1.70E-01$^{=}$
F_7		Mean	9.32E+01	3.31E+02	3.37E+02	3.19E+02	1.24E+02	1.09E+02	1.10E+02	1.34E+02	9.78E+01	1.50E+02	1.15E+02	2.45E+02
		Std	5.03E+00	5.13E+01	4.30E+01	1.04E+02	1.59E+01	1.64E+01	7.79E+00	1.50E+01	5.44E+01	1.31E+01	6.49E+00	1.98E+01
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**2.92E-09**$^{+}$	**3.02E-11**$^{+}$	**3.18E-04**$^{+}$	**1.46E-10**$^{+}$	**3.02E-11**$^{+}$	**2.05E-03**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_8		Mean	4.15E+01	2.37E+02	1.91E+02	6.58E+01	7.67E+01	5.81E+01	5.59E+01	8.71E+01	4.11E+01	9.07E+01	6.46E+01	1.95E+02
		Std	4.87E+00	8.69E+01	2.36E+01	6.45E+01	1.62E+01	1.32E+01	6.29E+00	1.39E+01	8.62E+00	1.34E+01	8.87E+00	1.94E+01
		p-value	-	**2.61E-10**$^{+}$	**3.00E-11**$^{+}$	**6.20E-04**$^{+}$	**3.47E-10**$^{+}$	**5.18E-07**$^{+}$	**2.37E-10**$^{+}$	**3.02E-11**$^{+}$	8.77E-01$^{=}$	**3.02E-11**$^{+}$	**8.99E-11**$^{+}$	**3.02E-11**$^{+}$
F_9		Mean	3.30E-02	1.24E-01	1.75E+03	1.03E+00	4.90E+00	2.22E+01	4.50E-01	9.06E-02	1.91E-01	2.28E+02	1.69E-01	1.81E-02
		Std	1.02E-01	2.24E-01	1.42E+03	1.43E+00	8.92E+00	2.53E+01	4.56E-01	1.77E-01	5.29E-01	2.40E+02	2.55E-01	8.40E-02
		p-value	-	**1.12E-06**$^{+}$	**3.23E-11**$^{+}$	**1.30E-08**$^{+}$	**6.52E-09**$^{+}$	**9.26E-12**$^{+}$	**1.49E-08**$^{+}$	**5.20E-03**$^{+}$	3.19E-01$^{=}$	**9.26E-12**$^{+}$	**1.30E-03**$^{+}$	4.94E-01$^{=}$
F_{10}		Mean	4.08E+03	1.12E+04	7.93E+03	1.29E+04	1.08E+04	4.80E+03	4.97E+03	4.81E+03	5.07E+03	3.59E+03	5.56E+03	8.22E+03
		Std	3.06E+02	1.38E+03	5.30E+02	3.56E+02	4.04E+02	7.43E+02	3.48E+02	4.32E+02	3.56E+03	3.05E+02	3.15E+02	5.59E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**4.94E-05**$^{+}$	**5.57E-10**$^{+}$	**9.06E-08**$^{+}$	7.48E-02$^{=}$	1.49E-06^{-}	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
$F_{4\text{-}10}$		*w/t/l*		7/0/0	7/0/0	7/0/0	6/0/1	6/1/0	6/1/0	6/1/0	2/5/0	4/1/2	6/1/0	5/2/0
F_{11}	Hybrid Problems	Mean	5.81E+01	1.37E+02	1.28E+02	9.29E+01	7.20E+01	4.53E+01	1.43E+02	5.03E+01	3.94E+01	1.39E+02	9.07E+01	9.69E+01
		Std	1.87E+01	3.58E+01	1.84E+01	3.47E+01	2.80E+01	7.79E+00	3.55E+01	8.74E+00	7.99E+00	2.98E+01	1.58E+01	1.89E+01
		p-value	-	**1.17E-09**$^{+}$	**2.37E-10**$^{+}$	**2.43E-05**$^{+}$	**2.07E-02**$^{+}$	1.58E-04^{-}	**1.20E-10**$^{+}$	5.75E-02$^{=}$	2.65E-07^{-}	**1.46E-10**$^{+}$	**5.46E-09**$^{+}$	**7.77E-09**$^{+}$
F_{12}		Mean	5.24E+03	1.36E+06	4.44E+05	5.47E+04	3.89E+04	4.72E+04	6.51E+03	1.61E+04	4.18E+04	1.15E+05	5.71E+03	2.05E+06
		Std	2.68E+03	8.81E+05	2.35E+05	4.70E+04	2.45E+04	2.16E+04	4.23E+03	9.97E+03	2.21E+04	6.89E+04	2.86E+03	1.62E+06
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**5.49E-11**$^{+}$	**8.89E-10**$^{+}$	**3.34E-11**$^{+}$	3.79E-01$^{=}$	**5.53E-08**$^{+}$	**3.02E-11**$^{+}$	**3.34E-11**$^{+}$	4.20E-01$^{=}$	**3.02E-11**$^{+}$
F_{13}		Mean	1.78E+02	1.50E+04	1.92E+03	8.93E+02	1.38E+03	1.10E+03	1.95E+02	6.75E+03	6.41E+03	2.75E+03	1.47E+02	1.10E+04
		Std	1.35E+02	1.29E+04	2.10E+03	1.37E+03	1.78E+03	1.70E+03	1.23E+02	7.34E+03	8.25E+03	4.98E+03	6.96E+01	6.77E+03
		p-value	-	**1.09E-10**$^{+}$	**8.48E-09**$^{+}$	**4.94E-05**$^{+}$	**9.83E-08**$^{+}$	**1.30E-03**$^{+}$	2.52E-01$^{=}$	**5.57E-10**$^{+}$	**1.49E-06**$^{+}$	**8.99E-11**$^{+}$	7.39E-01$^{=}$	**3.02E-11**$^{+}$
F_{14}		Mean	1.38E+02	4.72E+04	1.45E+02	1.93E+02	5.16E+01	4.09E+01	2.02E+02	5.70E+01	3.72E+01	1.59E+02	1.43E+02	2.25E+02
		Std	6.18E+01	1.77E+05	1.72E+01	1.54E+02	1.45E+01	9.10E+00	5.65E+01	1.17E+01	6.36E+00	5.57E+01	5.12E+01	3.67E+01
		p-value	-	**3.02E-11**$^{+}$	8.50E-02$^{=}$	1.37E-01$^{=}$	1.41E-09^{-}	1.09E-10^{-}	**2.53E-04**$^{+}$	4.57E-09^{-}	3.34E-11^{-}	1.12E-01$^{=}$	5.49E-01$^{=}$	**1.86E-06**$^{+}$
F_{15}		Mean	1.57E+02	1.57E+04	2.58E+03	2.48E+02	6.05E+01	3.93E+01	2.10E+02	3.57E+03	3.03E+01	2.93E+02	1.56E+02	9.34E+02
		Std	8.23E+01	1.43E+04	2.65E+03	1.51E+02	2.14E+01	7.88E+00	1.11E+02	8.74E+03	6.22E+00	1.00E+02	7.56E+01	4.20E+02
		p-value	-	**1.09E-10**$^{+}$	**4.98E-11**$^{+}$	**2.32E-02**$^{+}$	3.96E-08^{-}	1.55E-09^{-}	**4.68E-02**$^{+}$	1.26E-01$^{=}$	1.46E-10^{-}	**8.84E-07**$^{+}$	9.47E-01$^{=}$	**3.34E-11**$^{+}$
F_{16}		Mean	7.72E+02	1.94E+03	1.10E+03	1.09E+03	1.19E+03	8.24E+02	7.74E+02	9.95E+02	8.27E+02	8.11E+02	8.18E+02	1.48E+03
		Std	1.54E+02	6.05E+02	3.30E+02	1.03E+03	3.94E+02	3.15E+02	1.69E+02	2.15E+02	4.98E+02	1.88E+02	1.45E+02	2.98E+02
		p-value	-	**3.08E-08**$^{+}$	**3.16E-05**$^{+}$	2.64E-01$^{=}$	**2.77E-05**$^{+}$	5.69E-01$^{=}$	9.82E-01$^{=}$	**1.04E-04**$^{+}$	8.88E-01$^{=}$	3.87E-01$^{=}$	1.45E-01$^{=}$	**3.82E-10**$^{+}$
F_{17}		Mean	4.97E+02	6.96E+02	9.56E+02	7.87E+02	7.50E+02	6.66E+02	5.69E+02	6.69E+02	6.64E+02	6.67E+02	5.45E+02	7.22E+02
		Std	8.60E+01	2.08E+02	2.44E+02	6.18E+02	3.00E+02	2.48E+02	1.32E+02	1.30E+02	4.61E+02	1.31E+02	1.01E+02	1.63E+02
		p-value	-	**1.49E-04**$^{+}$	**1.96E-10**$^{+}$	7.28E-01$^{=}$	**1.04E-04**$^{+}$	**1.00E-03**$^{+}$	**2.07E-02**$^{+}$	**2.88E-06**$^{+}$	3.18E-01$^{=}$	**7.60E-07**$^{+}$	**2.71E-02**$^{+}$	**7.60E-07**$^{+}$
F_{18}		Mean	2.11E+02	3.21E+06	1.73E+04	7.96E+03	1.98E+03	8.88E+02	1.99E+02	1.57E+02	1.38E+03	4.04E+03	1.16E+02	5.82E+04
		Std	1.30E+02	1.27E+06	1.00E+04	1.20E+04	2.20E+03	6.06E+02	1.19E+02	5.93E+01	1.18E+03	4.12E+03	7.37E+01	2.99E+04
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**4.08E-11**$^{+}$	**5.00E-09**$^{+}$	**5.46E-09**$^{+}$	6.20E-01$^{=}$	1.76E-01$^{=}$	**1.70E-08**$^{+}$	**1.69E-09**$^{+}$	1.52E-03^{-}	**3.02E-11**$^{+}$
F_{19}		Mean	9.76E+01	2.06E+03	2.42E+03	8.47E+01	2.69E+01	1.67E+01	1.28E+02	9.95E+02	1.36E+01	6.84E+01	1.02E+02	1.87E+02
		Std	4.12E+01	4.42E+03	4.57E+03	5.17E+01	1.54E+01	5.04E+00	4.36E+01	4.45E+03	4.23E+00	2.86E+01	3.52E+01	8.47E+01
		p-value	-	**4.50E-11**$^{+}$	**8.10E-10**$^{+}$	1.30E-01$^{=}$	3.16E-10^{-}	3.02E-11^{-}	**1.12E-02**$^{+}$	**1.34E-05**$^{+}$	3.02E-11^{-}	5.32E-03^{-}	5.89E-01$^{=}$	**2.32E-06**$^{+}$
F_{20}		Mean	3.75E+02	7.55E+02	3.03E+02	1.19E+03	7.14E+02	5.10E+02	4.40E+02	5.90E+02	7.62E+02	4.75E+02	4.37E+02	5.29E+02
		Std	1.12E+02	2.47E+02	1.12E+02	4.39E+02	2.51E+02	2.51E+02	1.29E+02	1.35E+02	4.12E+02	1.45E+02	1.17E+02	2.15E+02
		p-value	-	**6.53E-08**$^{+}$	3.03E-02^{-}	**3.26E-07**$^{+}$	**4.31E-08**$^{+}$	**4.36E-02**$^{+}$	**4.68E-02**$^{+}$	**9.83E-08**$^{+}$	**3.37E-04**$^{+}$	**3.34E-03**$^{+}$	**4.68E-02**$^{+}$	**3.18E-03**$^{+}$
$F_{11\text{-}20}$		*w/t/l*		10/0/0	8/1/1	6/4/0	7/0/3	5/1/4	6/4/0	6/3/1	4/2/4	7/2/1	3/6/1	10/0/0
F_{21}	Composition Problems	Mean	2.41E+02	4.85E+02	3.57E+02	2.51E+02	2.76E+02	2.59E+02	2.56E+02	2.87E+02	2.41E+02	2.82E+02	2.62E+02	4.07E+02
		Std	4.69E+00	6.09E+01	2.34E+01	1.33E+01	2.14E+01	1.21E+01	7.05E+00	1.42E+01	1.17E+01	1.27E+01	6.63E+00	2.31E+01
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**1.34E-05**$^{+}$	**6.07E-11**$^{+}$	**1.31E-08**$^{+}$	**1.96E-10**$^{+}$	**3.34E-11**$^{+}$	6.95E-01$^{=}$	**3.02E-11**$^{+}$	**4.08E-11**$^{+}$	**3.02E-11**$^{+}$
F_{22}		Mean	4.15E+03	1.27E+04	3.96E+03	1.30E+04	1.10E+04	3.88E+03	4.94E+03	5.26E+03	8.06E+03	1.76E+03	5.38E+03	8.72E+03
		Std	1.34E+03	4.34E+02	4.28E+03	3.61E+02	2.11E+03	2.48E+03	1.78E+03	4.43E+02	3.91E+03	2.19E+03	1.95E+03	5.89E+02
		p-value	-	**3.02E-11**$^{+}$	5.08E-01$^{=}$	**3.02E-11**$^{+}$	**5.57E-10**$^{+}$	4.64E-01$^{=}$	**1.49E-06**$^{+}$	**8.35E-08**$^{+}$	**8.56E-04**$^{+}$	1.11E-04^{-}	**6.05E-07**$^{+}$	**3.02E-11**$^{+}$
F_{23}		Mean	4.63E+02	6.56E+02	6.54E+02	4.81E+02	5.00E+02	4.82E+02	4.83E+02	5.23E+02	4.63E+02	5.17E+02	4.90E+02	6.21E+02
		Std	1.06E+01	8.42E+01	4.34E+01	1.73E+01	2.10E+01	1.69E+01	1.11E+01	1.55E+01	1.36E+01	1.67E+01	9.89E+00	1.92E+01
		p-value	-	**3.34E-11**$^{+}$	**3.02E-11**$^{+}$	**9.21E-05**$^{+}$	**5.46E-09**$^{+}$	**1.29E-06**$^{+}$	**1.87E-07**$^{+}$	**3.69E-11**$^{+}$	9.12E-01$^{=}$	**4.08E-11**$^{+}$	**1.07E-09**$^{+}$	**3.02E-11**$^{+}$
F_{24}		Mean	5.36E+02	8.51E+02	8.10E+02	5.64E+02	5.76E+02	5.59E+02	5.44E+02	5.87E+02	5.42E+02	5.90E+02	5.52E+02	7.13E+02
		Std	4.90E+00	2.01E+01	7.39E+01	1.59E+01	2.19E+01	2.17E+01	5.45E+00	1.13E+01	1.25E+01	1.53E+01	1.02E+01	2.85E+01
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**1.78E-10**$^{+}$	**3.02E-11**$^{+}$	**1.36E-07**$^{+}$	**6.04E-07**$^{+}$	**3.02E-11**$^{+}$	**4.84E-02**$^{+}$	**3.02E-11**$^{+}$	**2.44E-09**$^{+}$	**3.02E-11**$^{+}$
F_{25}		Mean	5.16E+02	5.06E+02	5.70E+02	5.12E+02	5.22E+02	5.20E+02	5.22E+02	4.80E+02	4.96E+02	5.59E+02	5.30E+02	4.84E+02
		Std	3.54E+01	3.95E+01	1.24E+01	3.81E+01	2.81E+01	3.55E+01	4.10E+01	3.01E-02	3.10E+01	4.16E+01	2.95E+01	1.51E+01
		p-value	-	9.59E-01$^{=}$	**1.01E-08**$^{+}$	6.31E-01$^{=}$	7.62E-01$^{=}$	9.47E-01$^{=}$	5.11E-01$^{=}$	7.26E-03^{-}	1.03E-02^{-}	**3.37E-05**$^{+}$	1.60E-01$^{=}$	6.71E-04^{-}
F_{26}		Mean	1.46E+03	2.61E+03	6.70E+03	1.81E+03	2.00E+03	1.87E+03	1.63E+03	2.21E+03	1.51E+03	2.46E+03	1.66E+03	3.08E+03
		Std	7.26E+01	8.89E+02	1.94E+03	1.44E+02	1.89E+02	1.98E+02	1.19E+02	1.34E+02	1.17E+02	6.64E+02	1.08E+02	2.56E+02
		p-value	-	**3.02E-11**$^{+}$	**8.48E-09**$^{+}$	**1.09E-10**$^{+}$	**3.02E-11**$^{+}$	**3.34E-11**$^{+}$	**1.07E-07**$^{+}$	**3.02E-11**$^{+}$	6.79E-02$^{=}$	**1.07E-07**$^{+}$	**5.00E-09**$^{+}$	**3.02E-11**$^{+}$
F_{27}		Mean	5.46E+02	6.11E+02	9.86E+02	6.10E+02	5.00E+02	5.64E+02	5.53E+02	5.41E+02	5.41E+02	6.95E+02	5.58E+02	5.46E+02
		Std	2.25E+01	5.32E+01	8.20E+01	4.65E+01	1.09E-04	2.59E+01	2.36E+01	2.19E+01	3.49E+01	4.52E+01	3.23E+01	2.13E+01
		p-value	-	**4.80E-07**$^{+}$	**3.02E-11**$^{+}$	**4.69E-08**$^{+}$	3.02E-11^{-}	**1.08E-02**$^{+}$	1.91E-01$^{=}$	3.63E-01$^{=}$	2.17E-01$^{=}$	**3.02E-11**$^{+}$	1.54E-01$^{=}$	8.07E-01$^{=}$
F_{28}		Mean	4.93E+02	7.96E+02	5.12E+02	4.94E+02	5.00E+02	4.79E+02	4.91E+02	4.67E+02	4.60E+02	4.98E+02	4.96E+02	4.68E+02
		Std	2.13E+01	7.31E+02	4.92E+00	2.76E+01	8.47E-05	2.08E+01	2.06E+01	1.85E+01	8.92E+00	2.80E+01	1.88E+01	1.77E+01
		p-value	-	1.01E-01$^{=}$	**9.38E-11**$^{+}$	7.67E-01$^{=}$	3.73E-01$^{=}$	2.82E-04^{-}	5.39E-01$^{=}$	2.15E-09^{-}	7.96E-08^{-}	1.36E-01$^{=}$	5.67E-01$^{=}$	3.91E-05^{-}
F_{29}		Mean	4.37E+02	7.70E+02	1.32E+03	4.97E+02	7.34E+02	4.66E+02	4.79E+02	8.18E+02	3.62E+02	5.91E+02	5.30E+02	8.35E+02
		Std	5.81E+01	3.81E+02	3.19E+02	1.61E+02	2.40E+02	1.29E+02	6.31E+01	2.00E+02	6.28E+01	9.18E+01	8.39E+01	1.82E+02
		p-value	-	**1.03E-06**$^{+}$	**3.02E-11**$^{+}$	5.01E-01$^{=}$	**2.49E-06**$^{+}$	4.64E-01$^{=}$	**3.18E-04**$^{+}$	**6.70E-11**$^{+}$	1.01E-08^{-}	**9.26E-09**$^{+}$	**1.29E-06**$^{+}$	**6.70E-11**$^{+}$
F_{30}		Mean	6.37E+05	8.85E+05	7.23E+05	6.98E+05	2.67E+03	6.61E+05	6.65E+05	6.09E+05	5.95E+05	7.65E+05	6.45E+05	7.06E+05
		Std	6.53E+04	6.34E+05	7.98E+04	1.25E+05	2.50E+03	6.01E+04	8.62E+04	4.16E+04	2.70E+04	9.66E+04	7.45E+04	5.85E+04
		p-value	-	**3.51E-02**$^{+}$	**5.59E-05**$^{+}$	**1.10E-02**$^{+}$	2.99E-11^{-}	**3.26E-02**$^{-}$	1.35E-01$^{=}$	2.22E-01$^{=}$	5.95E-03^{-}	**1.19E-06**$^{+}$	4.86E-01$^{=}$	**6.34E-05**$^{+}$
$F_{21\text{-}30}$		*w/t/l*		8/2/0	9/1/0	7/3/0	6/2/2	6/3/1	6/4/0	6/2/2	2/4/4	8/1/1	6/4/0	7/1/2
w/t/l				27/2/0	26/2/1	22/7/0	20/2/7	19/5/5	18/11/0	20/6/3	10/11/8	21/4/4	15/12/2	24/3/2
Rank			**2.77**	10.62	9.82	7.24	6.18	4.96	5.12	6.25	4.01	7.41	4.79	8.77

TABLE 4. Optimization results of GSGDE and the 11 state-of-the-art DE variants and their comparisons on the 100 *D* CEC2017 suite.

F	Category	Quality	GSGDE	GPDE	ITGDE	DMCDE	SEDE	FADE	FDDE	TPDE	CUSDE	NSHADE	PFIDE	SDEGCM
F_1	Unimodal Problems	Mean	5.13E-12	1.40E+04	4.46E+03	1.83E+00	3.46E+03	4.38E+03	7.76E-10	3.93E+02	1.17E+03	5.24E+03	9.51E-11	4.10E+02
		Std	1.95E-11	1.80E+04	3.30E+03	9.98E+00	3.90E+03	4.41E+03	1.30E-09	8.69E+02	2.20E+03	4.39E+03	2.43E-10	1.06E+03
		p-value	-	**2.92E-11**$^{+}$	**2.92E-11**$^{+}$	**2.92E-11**$^{+}$	**2.92E-11**$^{+}$	**2.92E-11**$^{+}$	**4.58E-08**$^{+}$	**2.92E-11**$^{+}$	**2.92E-11**$^{+}$	**2.92E-11**$^{+}$	**1.69E-03**$^{+}$	**2.92E-11**$^{+}$
F_3		Mean	2.84E-09	6.86E+05	1.55E+04	7.14E+04	1.00E-01	5.44E+03	5.92E+04	7.59E+03	4.93E+05	4.17E+04	8.15E-02	2.68E+05
		Std	7.37E-09	5.80E+04	3.65E+03	3.08E+04	2.30E-01	1.99E+03	1.09E+05	5.96E+03	5.91E+04	1.26E+05	1.25E-01	6.14E+04
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
$F_{1\text{-}3}$		*w/t/l*		2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0
F_4	Simple Multimodal Problems	Mean	1.12E+02	2.19E+02	2.48E+02	2.20E+02	1.77E+02	2.02E+02	1.66E+02	2.08E+02	2.09E+02	2.03E+02	1.08E+02	2.13E+02
		Std	8.07E+01	2.08E+01	4.60E+01	3.24E+01	4.11E+01	4.43E+01	5.46E+01	8.03E+00	1.67E+01	4.72E+01	5.87E+01	3.35E+01
		p-value	-	**6.51E-08**$^{+}$	**1.41E-09**$^{+}$	**1.15E-07**$^{+}$	**2.50E-03**$^{+}$	**2.96E-05**$^{+}$	**1.76E-02**$^{+}$	**5.27E-06**$^{+}$	**1.79E-06**$^{+}$	**2.43E-05**$^{+}$	8.19E-01$^{=}$	**2.74E-06**$^{+}$
F_5		Mean	1.04E+02	6.80E+02	5.90E+02	1.41E+02	2.31E+02	1.74E+02	1.74E+02	2.24E+02	8.24E+02	2.99E+02	2.05E+02	2.83E+02
		Std	9.27E+00	1.90E+02	4.81E+01	1.97E+01	3.40E+01	2.15E+01	1.57E+01	3.78E+01	1.54E+01	2.92E+01	1.92E+01	1.13E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**5.46E-09**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_6		Mean	2.18E-03	1.70E-02	1.50E+01	2.39E-02	4.82E-03	1.13E+00	4.32E-03	3.29E-06	7.00E-04	1.37E-06	3.36E-02	5.80E-02
		Std	3.73E-03	1.10E-02	9.28E+00	5.95E-02	6.47E-03	5.07E-01	7.89E-03	2.04E-06	8.55E-04	7.49E-06	3.92E-02	1.39E-01
		p-value	-	**6.12E-10**$^{+}$	**3.02E-11**$^{+}$	**1.77E-03**$^{+}$	**2.51E-02**$^{+}$	**3.02E-11**$^{+}$	9.23E-01$^{=}$	3.02E-11^{-}	6.67E-03^{-}	1.72E-12^{-}	**5.57E-10**$^{+}$	**1.11E-06**$^{+}$
F_7		Mean	2.11E+02	8.51E+02	7.85E+02	2.45E+02	3.76E+02	3.21E+02	2.92E+02	3.19E+02	9.28E+02	5.80E+02	3.12E+02	5.12E+02
		Std	1.30E+01	1.27E+02	1.58E+02	2.74E+01	5.09E+01	3.54E+01	2.24E+01	3.61E+01	2.57E+01	6.41E+01	2.06E+01	1.12E+02
		p-value	-	**5.49E-11**$^{+}$	**3.02E-11**$^{+}$	**1.03E-06**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_8		Mean	1.03E+02	6.90E+02	6.35E+02	1.40E+02	2.21E+02	1.86E+02	1.68E+02	2.18E+02	8.24E+02	3.12E+02	1.94E+02	2.76E+02
		Std	9.12E+00	1.57E+02	5.28E+01	2.54E+01	2.57E+01	2.84E+01	1.88E+01	3.34E+01	2.07E+01	3.16E+01	1.75E+01	1.33E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**2.39E-08**$^{+}$	**3.02E-11**$^{+}$	**3.69E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**4.98E-11**$^{+}$
F_9		Mean	4.42E+00	1.72E+01	2.37E+04	6.45E+01	1.05E+03	4.50E+02	2.74E+01	7.92E-01	3.13E+00	6.39E+03	2.11E+01	2.34E+02
		Std	2.03E+00	6.69E+01	3.54E+03	6.45E+01	7.22E+02	3.19E+02	9.64E+00	7.95E-01	2.98E+00	1.59E+03	1.08E+01	3.72E+02
		p-value	-	**3.64E-02**$^{+}$	**3.00E-11**$^{+}$	**9.86E-11**$^{+}$	**3.00E-11**$^{+}$	**3.00E-11**$^{+}$	**3.32E-11**$^{+}$	1.91E-10^{-}	7.44E-03^{-}	**3.00E-11**$^{+}$	**7.34E-11**$^{+}$	**1.85E-09**$^{+}$
F_{10}		Mean	1.08E+04	3.02E+04	1.34E+04	2.95E+04	2.79E+04	1.17E+04	1.38E+04	1.22E+04	2.99E+04	9.61E+03	1.52E+04	1.84E+04
		Std	5.53E+02	4.40E+02	3.07E+03	7.53E+02	6.11E+02	1.22E+03	6.33E+02	8.38E+02	4.34E+02	3.81E+02	5.22E+02	1.76E+03
		p-value	-	**3.02E-11**$^{+}$	**1.25E-04**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**1.52E-03**$^{+}$	**3.02E-11**$^{+}$	**3.96E-08**$^{+}$	**3.02E-11**$^{+}$	6.12E-10^{-}	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
$F_{4\text{-}10}$		*w/t/l*		7/0/0	7/0/0	7/0/0	7/0/0	7/0/0	6/1/0	5/0/2	5/0/2	5/0/2	6/1/0	7/0/0
F_{11}	Hybrid Problems	Mean	9.63E+02	1.09E+03	9.55E+02	7.66E+02	3.31E+02	1.77E+02	1.12E+03	2.89E+02	6.02E+02	1.91E+03	9.48E+02	4.39E+02
		Std	1.89E+02	2.48E+02	1.52E+02	2.66E+02	1.57E+02	5.46E+01	3.70E+02	8.86E+01	6.64E+01	7.44E+03	1.65E+02	1.31E+02
		p-value	-	5.94E-02$^{=}$	8.82E-01$^{=}$	7.30E-04^{-}	6.07E-11^{-}	3.02E-11^{-}	1.15E-01$^{=}$	3.02E-11^{-}	1.78E-10^{-}	**8.48E-09**$^{+}$	1.00E+00$^{=}$	5.49E-11^{-}
F_{12}		Mean	1.45E+04	2.48E+06	1.64E+06	2.40E+05	1.84E+05	2.84E+05	4.07E+04	3.32E+05	3.08E+05	4.26E+05	3.11E+04	3.79E+06
		Std	6.89E+03	1.10E+06	5.59E+05	1.84E+05	6.52E+04	1.03E+05	3.34E+04	1.57E+05	1.18E+05	1.49E+05	3.25E+04	1.95E+06
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**4.11E-07**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**7.22E-06**$^{+}$	**3.02E-11**$^{+}$
F_{13}		Mean	3.72E+03	8.55E+03	2.78E+03	4.59E+03	3.53E+03	2.08E+03	3.80E+03	4.07E+03	9.15E+03	3.65E+03	4.22E+03	6.88E+03
		Std	2.66E+03	8.49E+03	1.89E+03	7.39E+03	5.38E+03	1.60E+03	2.99E+03	6.11E+03	9.42E+03	4.05E+03	3.44E+03	6.39E+03
		p-value	-	**4.68E-02**$^{+}$	2.01E-01$^{=}$	1.71E-01$^{=}$	2.71E-02^{-}	1.17E-02^{-}	9.82E-01$^{=}$	7.24E-02$^{=}$	**1.91E-02**$^{+}$	4.04E-01$^{=}$	9.00E-01$^{=}$	6.15E-02$^{=}$
F_{14}		Mean	6.78E+02	3.55E+06	4.48E+04	3.28E+04	4.45E+03	1.75E+03	4.84E+02	3.15E+02	1.43E+04	3.05E+04	4.61E+02	3.06E+05
		Std	1.98E+02	2.16E+06	3.67E+04	5.33E+04	4.04E+03	1.97E+03	1.69E+02	6.04E+01	1.44E+04	2.56E+04	1.69E+02	2.83E+05
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**6.01E-08**$^{+}$	**3.56E-04**$^{+}$	4.22E-04^{-}	3.47E-10^{-}	**3.34E-11**$^{+}$	**3.02E-11**$^{+}$	3.37E-05^{-}	**3.02E-11**$^{+}$
F_{15}		Mean	4.03E+02	6.50E+03	7.91E+02	5.77E+03	5.87E+03	1.51E+03	5.19E+02	3.95E+03	8.40E+03	9.21E+02	5.30E+02	5.21E+03
		Std	1.95E+02	8.09E+03	7.49E+02	7.99E+03	8.00E+03	1.94E+03	3.87E+02	4.59E+03	8.57E+03	6.95E+02	4.54E+02	6.82E+03
		p-value	-	**6.53E-07**$^{+}$	6.15E-02$^{=}$	**2.19E-08**$^{+}$	**3.09E-06**$^{+}$	2.06E-01$^{=}$	1.15E-01$^{=}$	**4.74E-06**$^{+}$	**8.15E-05**$^{+}$	**1.49E-04**$^{+}$	4.04E-01$^{=}$	**1.29E-06**$^{+}$
F_{16}		Mean	2.44E+03	7.69E+03	3.74E+03	2.30E+03	3.10E+03	2.62E+03	2.65E+03	2.79E+03	7.54E+03	2.53E+03	2.72E+03	3.11E+03
		Std	3.09E+02	3.51E+02	6.71E+02	5.97E+02	7.38E+02	6.54E+02	2.68E+02	3.97E+02	3.54E+02	2.60E+02	2.72E+02	5.66E+02
		p-value	-	**3.02E-11**$^{+}$	**3.16E-10**$^{+}$	2.46E-01$^{=}$	**6.77E-05**$^{+}$	2.06E-01$^{=}$	**8.31E-03**$^{+}$	**3.18E-04**$^{+}$	**3.02E-11**$^{+}$	1.91E-01$^{=}$	**5.41E-04**$^{+}$	**2.32E-06**$^{+}$
F_{17}		Mean	1.84E+03	4.66E+03	3.18E+03	2.75E+03	2.15E+03	1.88E+03	1.86E+03	2.24E+03	4.52E+03	2.04E+03	1.87E+03	2.26E+03
		Std	2.16E+02	2.87E+02	5.40E+02	1.40E+03	5.13E+02	6.21E+02	2.86E+02	2.97E+02	7.34E+02	2.37E+02	2.41E+02	5.24E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**4.84E-02**$^{+}$	**1.27E-02**$^{+}$	7.51E-01$^{=}$	8.30E-01$^{=}$	**1.11E-06**$^{+}$	**7.39E-11**$^{+}$	**3.50E-03**$^{+}$	5.40E-01$^{=}$	**7.70E-04**$^{+}$
F_{18}		Mean	1.25E+03	2.03E+07	2.12E+05	1.66E+05	3.64E+04	3.46E+04	2.91E+03	1.20E+04	2.05E+05	5.36E+04	4.49E+03	8.13E+05
		Std	1.11E+03	7.91E+06	1.07E+05	1.17E+05	1.94E+04	1.63E+04	2.48E+03	8.36E+03	1.08E+05	2.16E+04	4.48E+03	5.33E+05
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**2.13E-04**$^{+}$	**4.08E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**2.00E-05**$^{+}$	**3.02E-11**$^{+}$
F_{19}		Mean	2.50E+02	7.52E+03	1.26E+03	1.26E+03	7.32E+03	1.83E+03	1.27E+03	4.68E+03	1.12E+04	1.48E+03	8.41E+02	9.12E+03
		Std	5.03E+01	9.16E+03	1.50E+03	4.60E+03	7.65E+03	2.44E+03	2.59E+03	6.81E+03	1.06E+04	2.21E+03	1.79E+03	9.43E+03
		p-value	-	**6.53E-08**$^{+}$	**2.75E-03**$^{+}$	**4.84E-02**$^{+}$	**7.09E-08**$^{+}$	**2.25E-04**$^{+}$	**9.03E-04**$^{+}$	**2.77E-05**$^{+}$	**1.01E-08**$^{+}$	**2.53E-04**$^{+}$	9.12E-01$^{=}$	**9.26E-09**$^{+}$
F_{20}		Mean	1.77E+03	4.57E+03	1.90E+03	3.46E+03	2.12E+03	2.02E+03	1.91E+03	2.14E+03	4.27E+03	1.91E+03	1.82E+03	2.15E+03
		Std	2.23E+02	3.63E+02	3.74E+02	1.35E+03	5.42E+02	4.80E+02	2.30E+02	2.25E+02	8.79E+02	1.92E+02	2.38E+02	3.94E+02
		p-value	-	**3.02E-11**$^{+}$	8.77E-02$^{=}$	**5.61E-05**$^{+}$	**8.12E-04**$^{+}$	**3.27E-02**$^{+}$	**1.33E-02**$^{+}$	**6.53E-07**$^{+}$	**7.39E-11**$^{+}$	**2.24E-02**$^{+}$	5.89E-01$^{=}$	**1.64E-05**$^{+}$
$F_{11\text{-}20}$		*w/t/l*		9/1/0	6/4/0	7/2/1	8/0/2	5/3/2	5/4/1	7/1/2	9/0/1	8/2/0	3/6/1	8/1/1
F_{21}	Composition Problems	Mean	3.33E+02	8.88E+02	7.62E+02	3.69E+02	4.34E+02	4.11E+02	3.96E+02	4.62E+02	1.04E+03	4.99E+02	4.14E+02	5.04E+02
		Std	1.07E+01	1.71E+02	8.81E+01	2.17E+01	2.75E+01	2.55E+01	2.13E+01	3.03E+01	2.18E+01	3.00E+01	1.42E+01	1.15E+02
		p-value	-	**6.07E-11**$^{+}$	**3.02E-11**$^{+}$	**2.02E-08**$^{+}$	**3.02E-11**$^{+}$	**4.50E-11**$^{+}$	**4.50E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_{22}		Mean	1.19E+04	3.06E+04	1.52E+04	2.88E+04	2.84E+04	1.28E+04	1.50E+04	1.32E+04	3.02E+04	1.09E+04	1.62E+04	1.94E+04
		Std	6.03E+02	4.67E+02	2.80E+03	4.42E+03	6.93E+02	1.35E+03	5.52E+02	9.50E+02	6.10E+02	2.11E+03	6.68E+02	1.20E+03
		p-value	-	**3.02E-11**$^{+}$	**2.39E-08**$^{+}$	**4.98E-11**$^{+}$	**3.02E-11**$^{+}$	**8.12E-04**$^{+}$	**3.02E-11**$^{+}$	**8.84E-07**$^{+}$	**3.02E-11**$^{+}$	2.60E-05^{-}	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_{23}		Mean	6.15E+02	6.56E+02	1.18E+03	6.71E+02	6.85E+02	7.63E+02	7.00E+02	7.82E+02	8.11E+02	7.19E+02	7.13E+02	7.30E+02
		Std	1.31E+01	2.29E+01	1.10E+02	2.22E+01	3.29E+01	3.86E+01	1.54E+01	3.41E+01	2.86E+02	2.36E+01	1.42E+01	4.52E+01
		p-value	-	**7.12E-09**$^{+}$	**3.02E-11**$^{+}$	**5.49E-11**$^{+}$	**6.07E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	6.20E-01$^{=}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.69E-11**$^{+}$
F_{24}		Mean	9.72E+02	1.11E+03	2.41E+03	1.03E+03	1.08E+03	1.11E+03	1.04E+03	1.12E+03	1.53E+03	1.24E+03	1.06E+03	1.12E+03
		Std	1.35E+01	1.57E+02	3.20E+02	2.56E+01	3.60E+01	3.70E+01	1.87E+01	3.50E+01	2.11E+02	4.60E+01	1.75E+01	6.16E+01
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**6.07E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**1.01E-08**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_{25}		Mean	7.55E+02	7.81E+02	7.67E+02	7.87E+02	7.49E+02	7.38E+02	7.77E+02	7.68E+02	7.23E+02	7.85E+02	7.62E+02	7.65E+02
		Std	4.79E+01	4.74E+01	5.32E+01	5.39E+01	6.32E+01	6.19E+01	6.96E+01	5.04E+01	4.93E+01	5.64E+01	4.82E+01	6.99E+01
		p-value	-	5.94E-02$^{=}$	5.49E-01$^{=}$	**1.56E-02**$^{+}$	6.95E-01$^{=}$	4.92E-01$^{=}$	8.50E-02$^{=}$	3.33E-01$^{=}$	4.64E-03^{-}	**1.70E-02**$^{+}$	7.51E-01$^{=}$	6.95E-01$^{=}$
F_{26}		Mean	4.10E+03	6.11E+03	2.30E+04	5.03E+03	5.67E+03	5.69E+03	4.85E+03	5.74E+03	8.08E+03	9.17E+03	4.96E+03	5.88E+03
		Std	2.12E+02	1.59E+03	1.17E+03	2.81E+02	4.01E+02	4.00E+02	2.47E+02	3.60E+02	3.18E+03	1.40E+03	2.75E+02	7.63E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.34E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**6.07E-11**$^{+}$	**3.02E-11**$^{+}$	**5.08E-03**$^{+}$	**3.02E-11**$^{+}$	**5.49E-11**$^{+}$	**3.02E-11**$^{+}$
F_{27}		Mean	6.59E+02	6.66E+02	1.76E+03	7.03E+02	5.00E+02	7.39E+02	7.36E+02	6.45E+02	6.08E+02	9.47E+02	6.95E+02	6.87E+02
		Std	1.93E+01	1.75E+01	1.94E+02	3.52E+01	1.25E-04	2.66E+01	4.01E+01	2.19E+01	1.66E+01	6.47E+01	3.40E+01	3.39E+01
		p-value	-	2.52E-01$^{=}$	**3.02E-11**$^{+}$	**3.52E-07**$^{+}$	3.02E-11^{-}	**9.92E-11**$^{+}$	**1.78E-10**$^{+}$	1.38E-02^{-}	1.96E-10^{-}	**3.02E-11**$^{+}$	**2.32E-06**$^{+}$	**1.44E-03**$^{+}$
F_{28}		Mean	5.39E+02	1.38E+03	5.62E+02	5.70E+02	5.00E+02	5.66E+02	5.62E+02	5.30E+02	5.36E+02	5.55E+02	5.43E+02	5.47E+02
		Std	3.64E+01	3.04E+03	2.67E+01	2.97E+01	1.04E-04	2.14E+01	3.41E+01	1.71E+01	3.16E+01	2.43E+01	2.71E+01	2.26E+01
		p-value	-	4.38E-01$^{=}$	**2.05E-03**$^{+}$	**5.56E-04**$^{+}$	6.76E-05^{-}	**4.71E-04**$^{+}$	**1.03E-02**$^{+}$	2.12E-01$^{=}$	7.84E-01$^{=}$	**1.22E-02**$^{+}$	4.87E-01$^{=}$	2.17E-01$^{=}$
F_{29}		Mean	1.90E+03	3.04E+03	4.22E+03	1.86E+03	2.25E+03	1.95E+03	2.23E+03	2.73E+03	3.87E+03	2.58E+03	2.16E+03	2.27E+03
		Std	3.46E+02	1.10E+03	6.41E+02	5.23E+02	5.13E+02	3.90E+02	2.63E+02	2.44E+02	1.38E+03	2.76E+02	2.22E+02	4.40E+02
		p-value	-	**1.09E-05**$^{+}$	**3.02E-11**$^{+}$	2.97E-01$^{=}$	**8.68E-03**$^{+}$	7.17E-01$^{=}$	**2.53E-04**$^{+}$	**3.16E-10**$^{+}$	**1.61E-06**$^{+}$	**4.18E-09**$^{+}$	**1.52E-03**$^{+}$	**1.44E-03**$^{+}$
F_{30}		Mean	2.94E+03	5.59E+03	6.36E+03	3.35E+03	3.79E+03	4.65E+03	5.11E+03	3.32E+03	4.82E+03	6.77E+03	4.80E+03	5.92E+03
		Std	1.14E+03	3.57E+03	2.03E+03	3.01E+02	3.78E+03	1.88E+03	2.72E+03	8.31E+02	3.25E+03	3.03E+03	2.53E+03	4.09E+03
		p-value	-	**5.19E-07**$^{+}$	**8.89E-10**$^{+}$	**5.09E-06**$^{+}$	9.23E-01$^{=}$	**2.57E-07**$^{+}$	**1.64E-05**$^{+}$	**1.06E-03**$^{+}$	**3.77E-04**$^{+}$	**1.17E-09**$^{+}$	**3.37E-04**$^{+}$	**2.60E-08**$^{+}$
$F_{21\text{-}30}$		*w/t/l*		7/3/0	9/1/0	9/1/0	6/2/2	8/2/0	9/1/0	7/2/1	6/2/2	9/0/1	8/2/0	8/2/0
w/t/l				25/4/0	24/5/0	25/3/1	23/2/4	22/5/2	22/6/1	21/3/5	22/2/5	24/2/3	19/9/1	25/3/1
Rank			**2.17**	9.98	8.96	6.22	5.75	5.44	4.87	5.53	8.75	7.36	4.48	8.43

TABLE 5. Statistical comparison results between GSGDE and the 11 compared DE methods concerning "*w/t/l*" and the average rank on the CEC2017 suite with the three dimensionality settings.

Problem Set	Problem Property	Index	GSGDE	GPDE	ITGDE	DMCDE	SEDE	FADE	FDDE	TPDE	CUSDE	NSHADE	PFIDE	SDEGCM
CEC2017-30D	Unimodal Problems	*w/t/l*	-	2/0/0	2/0/0	2/0/0	0/1/1	1/1/0	1/1/0	1/1/0	2/0/0	1/1/0	0/2/0	2/0/0
	Simple Multimodal Problems		-	7/0/0	6/1/0	7/0/0	6/0/1	3/3/1	4/3/0	6/1/0	1/6/0	4/0/3	4/3/0	7/0/0
	Hybrid Problems		-	10/0/0	10/0/0	6/3/1	6/4/0	3/3/4	8/2/0	5/3/2	7/2/1	8/2/0	7/2/1	8/2/0
	Composition Problems		-	7/2/1	7/2/1	8/2/0	6/1/3	5/3/2	7/3/0	6/2/2	4/4/2	8/2/0	6/4/0	7/2/1
	Overall		-	26/2/1	25/3/1	23/5/1	18/6/5	12/10/7	20/9/0	18/7/4	14/12/3	21/5/3	17/11/1	24/4/1
	Overall	*Rank*	**3.24**	10.65	9.44	8.01	6.05	4.24	5.25	5.55	5.22	6.77	4.50	9.03
CEC2017-50D	Unimodal Problems	*w/t/l*	-	2/0/0	2/0/0	2/0/0	1/0/1	2/0/0	0/2/0	2/0/0	2/0/0	2/0/0	0/1/1	2/0/0
	Simple Multimodal Problems		-	7/0/0	7/0/0	7/0/0	6/0/1	6/1/0	6/1/0	6/1/0	2/5/0	4/1/2	6/1/0	5/2/0
	Hybrid Problems		-	10/0/0	8/1/1	6/4/0	7/0/3	5/1/4	6/4/0	6/3/1	4/2/4	7/2/1	3/6/1	10/0/0
	Composition Problems		-	8/2/0	9/1/0	7/3/0	6/2/2	6/3/1	6/4/0	6/2/2	2/4/4	8/1/1	6/4/0	7/1/2
	Overall		-	27/2/0	26/2/1	22/7/0	20/2/7	19/5/5	18/11/0	20/6/3	10/11/8	21/4/4	15/12/2	24/3/2
	Overall	*Rank*	**2.77**	10.62	9.82	7.24	6.18	4.96	5.12	6.25	4.01	7.41	4.79	8.77
CEC2017-100D	Unimodal Problems	*w/t/l*	-	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0	2/0/0
	Simple Multimodal Problems		-	7/0/0	7/0/0	7/0/0	7/0/0	7/0/0	6/1/0	5/0/2	5/0/2	5/0/2	6/1/0	7/0/0
	Hybrid Problems		-	9/1/0	6/4/0	7/2/1	8/0/2	5/3/2	5/4/1	7/1/2	9/0/1	8/2/0	3/6/1	8/1/1
	Composition Problems		-	7/3/0	9/1/0	9/1/0	6/2/2	8/2/0	9/1/0	7/2/1	6/2/2	9/0/1	8/2/0	8/2/0
	Overall		-	25/4/0	24/5/0	25/3/1	23/2/4	22/5/2	22/6/1	21/3/5	22/2/5	24/2/3	19/9/1	25/3/1
	Overall	*Rank*	**2.17**	9.98	8.96	6.22	5.75	5.44	4.87	5.53	8.75	7.36	4.48	8.43

settings of the dimension size, namely 30, 50, and 100. With the optimal population sizes of all methods listed in Table 1, the detailed optimization results of all methods on the 30D, 50D, and 100D CEC2017 sets are presented in Tables 2∼4, respectively. For convenient observation, we summarize the statistical comparison results between GSGDE and the 11 compared DE variants on the CEC2017 suite with the three settings of the dimensionality in Table 5 concerning the average rank and "$w/t/l$". In particular, in these tables, "$w/t/l$" means that in comparison with the associated compared DE variants, GSGDE performs significantly better on w problems, equivalently on t problems, and significantly worse on l problems, respectively. "*Rank*" represents the average rank of each method attained from the Friedman test. From Tables 2 ∼ 4 and Table 5, the results in terms of the comparisons between GSGDE and the 11 compared methods are summarized in the following:

1) As shown in Table 5, concerning "*Rank*", GSGDE consistently gets the first rank among all algorithms on the 30D, 50D, and 100D CEC2017 benchmark sets. This implies that GSGDE consistently gains the best overall optimization results among all methods on the CEC2017 problems with the three dimensionality settings. Furthermore, it is also found that the rank values of GSGDE on the 30D, 50D and 100D CEC2017 benchmark sets are much lower than those of the 11 compared algorithms. This implies that GSGDE consistently is significantly superior to the 11 compared DE algorithms on the CEC2017 set with the three dimensionality settings. Further observation shows that the rank value of GSGDE becomes smaller and smaller as the dimensionality increases. This implies that GSGDE achieves better and better overall optimization performance with the dimensionality increasing. This demonstrates that GSGDE has a good scalability to solve optimization problems.
2) As displayed in Table 5, in view of "$w/t/l$", on the 30D CEC2017 benchmark suite, GSGDE gains significantly better optimization performance than the 11 compared DE algorithms on at least 12 problems, and is significantly inferior to them on no more than 7 problems. Particularly, compared with GPDE, ITGDE, DMCDE, FDDE, NSHADE and SDEGCM, GSGDE obtains significantly superior performance on more than 20 problems. In competition with SEDE and TPDE, GSGDE is much better both on 18 problems. On the 50D CEC2017 benchmark set, except for CUSDE, GSGDE significantly outperforms the rest 10 compared DE algorithms on at least 15 problems. Particularly, compared with GPDE, ITGDE, DMCDE, SEDE, TPDE, NSHADE and SDEGCM, GSGDE shows significant superiority on more than 20 problems. In comparison with FADE and FDDE, GSGDE presents significantly superior performance on at least 18 problems. On the 100D CEC2017 benchmark set, GSGDE shows much superior performance to the 11 compared DE approaches on more than 19 problems. Particularly, except for PFIDE, GSGDE is significantly better than the rest 10 compared DE algorithms on at least 21 problems. Further observation shows that with the dimension size increasing, the superiority of GSGDE to most compared DE variants becomes more and more significant. This indicates that GSGDE is more effective in dealing with high dimensional optimization problems. Therefore, GSGDE has a good scalability in problem optimization.
3) As exhibited in Table 5, with respect to the 4 types of benchmark problems, (a) on the two unimodal problems ($F_1$and F_3), GSGDE significantly beats 5 compared DE variants down on the two 30D unimodal problems. Confronted with the two 50D unimodal problems, GSGDE significantly outperforms

8 compared DE variants on both problems. On the two $100D$ unimodal problems, GSGDE is significantly superior to all 11 compared DE variants on both problems. Therefore, competed with the 11 compared DE variants, GSGDE presents increasing superiority as the dimension size grows. This implies that GSGDE is effective in addressing unimodal optimization problems, especially those with high dimensionality. (b) On the 7 multimodal problems (F_4-F_{10}), we discover that with the dimension size increasing, GSGDE gains increasingly superior performance to the 11 compared DE approaches. Specifically, on the seven $30D$ multimodal optimization problems, apart from FADE, FDDE, CUSDE, NSHADE, and PFIDE, GSGDE shows significant dominance to the rest 6 compared DE approaches on at least 6 problems. On the seven $50D$ multimodal problems, excluding CUSDE and NSHADE, GSGDE presents significant superiority to the rest 9 compared DE algorithms on at least 5 problems and presents inferiority on at most 1 problem. On the seven $100D$ problems, GSGDE significantly outperforms the 11 compared approaches on at least 5 problems and exhibits inferiority to them on at most 2 problems. These discoveries demonstrate that GSGDE is more effective to optimize multimodal problems and its superiority to the 11 compared DE methods becomes more and more significant as the dimensionality increases. (c) On the 10 hybrid functions (F_{11}-F_{20}), when the dimension size is 30, except for FADE, GSGDE significantly beats the rest 10 compared algorithms down on more than 5 problems and presents slight inferiority on at most 2 problems. When the dimension size is 50, GSGDE achieves equivalent performance with FADE, CUSDE, and PFIDE, but displays dominance to the rest 8 compared algorithms on more than 6 problems and presents inferior performance to them on at most 3 problems. When the dimension size is 100, except for PFIDE, GSGDE shows significantly superior performance to the rest 10 compared methods on at least 5 problems and presents inferiority on at most 2 problems. These observations demonstrate that GSGDE could solve the hybrid problems effectively and its dominance to most of the 11 compared algorithms becomes more and more significant as the dimensionality increases. (d) On the 10 composition functions (F_{21}-F_{30}), GSGDE shows no inferior performance to the 11 compared approaches on more than 7 of the ten $30D$ problems, on at least 6 of the ten $50D$ problems and on at least 8 of the ten $100D$ problems. These observations prove that GSGDE is effective in tackling the complicated composition problems and its superiority to most of the 11 compared algorithms becomes more and more significant with the dimensionality increasing. In conclusion, with the dimension size growing, GSGDE shows increasingly superior performance to the 11 compared DE methods in coping with the 4 types of optimization problems. This further demonstrates that GSGDE has a good scalability to tackle optimization problems.

The above experimental results have verified the great effectiveness of GSGDE in dealing with the 29 CEC2017 problems with the three settings of the dimensionality. To further exhibit its efficiency in dealing with optimization problems, we compare the convergence behaviors of GSGDE with those of the 11 compared latest and well-performed DE approaches on the CEC2017 problem suite with the three dimensionality settings. Figs. 1 $\sim$ 3 show the convergence behaviors of GSGDE and the 11 compared DE methods on the $30D$, $50D$, and $100D$ CEC2017 sets, respectively. From these three figures, we attain the following findings:

1) From Fig. 1, on the $30D$ CEC2017 problems, GSGDE achieves slower convergence than some compared methods at the early stage but at last converges to much better solutions than all 11 compared DE variants on 12 problems (F_1, F_3, F_5, F_8, F_9, F_{13}, F_{15}, F_{16}, F_{21}, F_{23}, F_{24}, and F_{26}). On the other 17 problems, GSGDE obtains very competitive or even much quicker convergence and higher-accuracy solutions than most of the compared 11 DE approaches.
2) From Fig. 2, on the $50D$ CEC2017 problems, GSGDE obtains much higher-quality solutions and much faster convergence than the 11 compared DE variants on 6 problems (F_3, F_5, F_7, F_{12}, F_{17}, and F_{26}). On the other 23 problems, GSGDE shows superior performance concerning the convergence speed and the solution quality to most of the 11 compared DE algorithms.
3) From Fig. 3, on the $100D$ CEC2017 problems, GSGDE performs better than all 11 compared DE methods in regard to the solution quality and the convergence speed on 16 problems (F_1, F_3-F_5, F_7, F_8, F_{12}, F_{15}, F_{18}-F_{21}, F_{23}, F_{24}, F_{26}, and F_{30}). On the other 13 problems, it gains very competitive performance with a few compared methods, but shows great dominance to most of the 11 compared DE methods.
4) In-depth observation shows that with the dimensionality increasing, the dominance of GSGDE in the convergence and the solution quality becomes more and more significant. This further demonstrates that GSGDE preserves a good scalability in dealing with optimization problems with higher dimensionality.

According to the above experimental results, GSGDE is capable of solving different types of optimization problems and has a good scalability to tackle optimization problems. The effectiveness and efficiency of GSGDE is mainly attributed to "DE/current-to-gselite/1". Specifically, this novel mutation scheme affords a good compromise between exploration of the problem space to locate optimal zones fast and exploitation of the located optimal areas subtly to find high-quality solutions. This is mainly attributed to that "DE/current-to-gselite/1" randomly samples a guiding exemplar for each individual according to the Gaussian

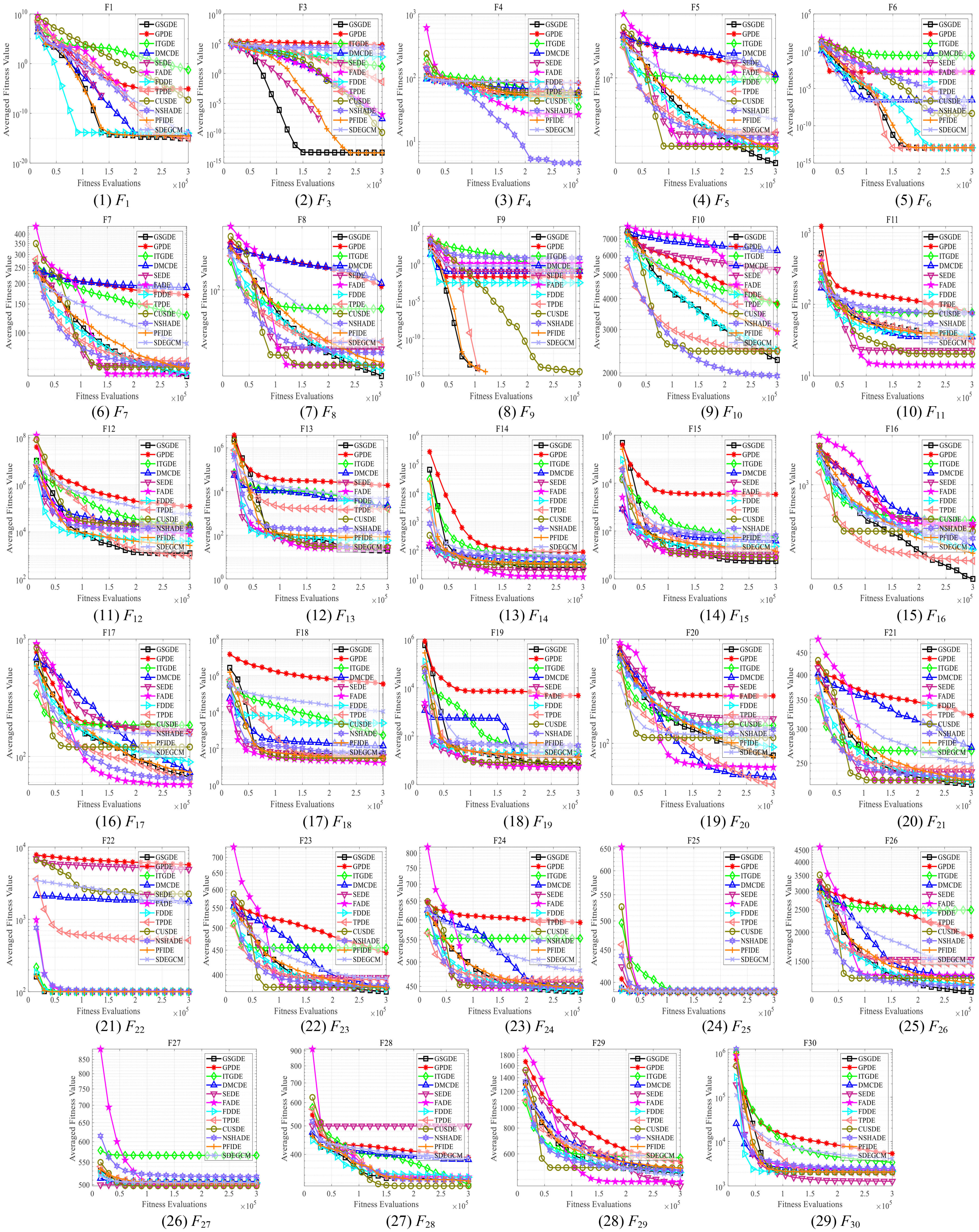

FIGURE 1. Convergence behaviors of GSGDE and the 11 compared DE variants on the 30D CEC2017 benchmark set.

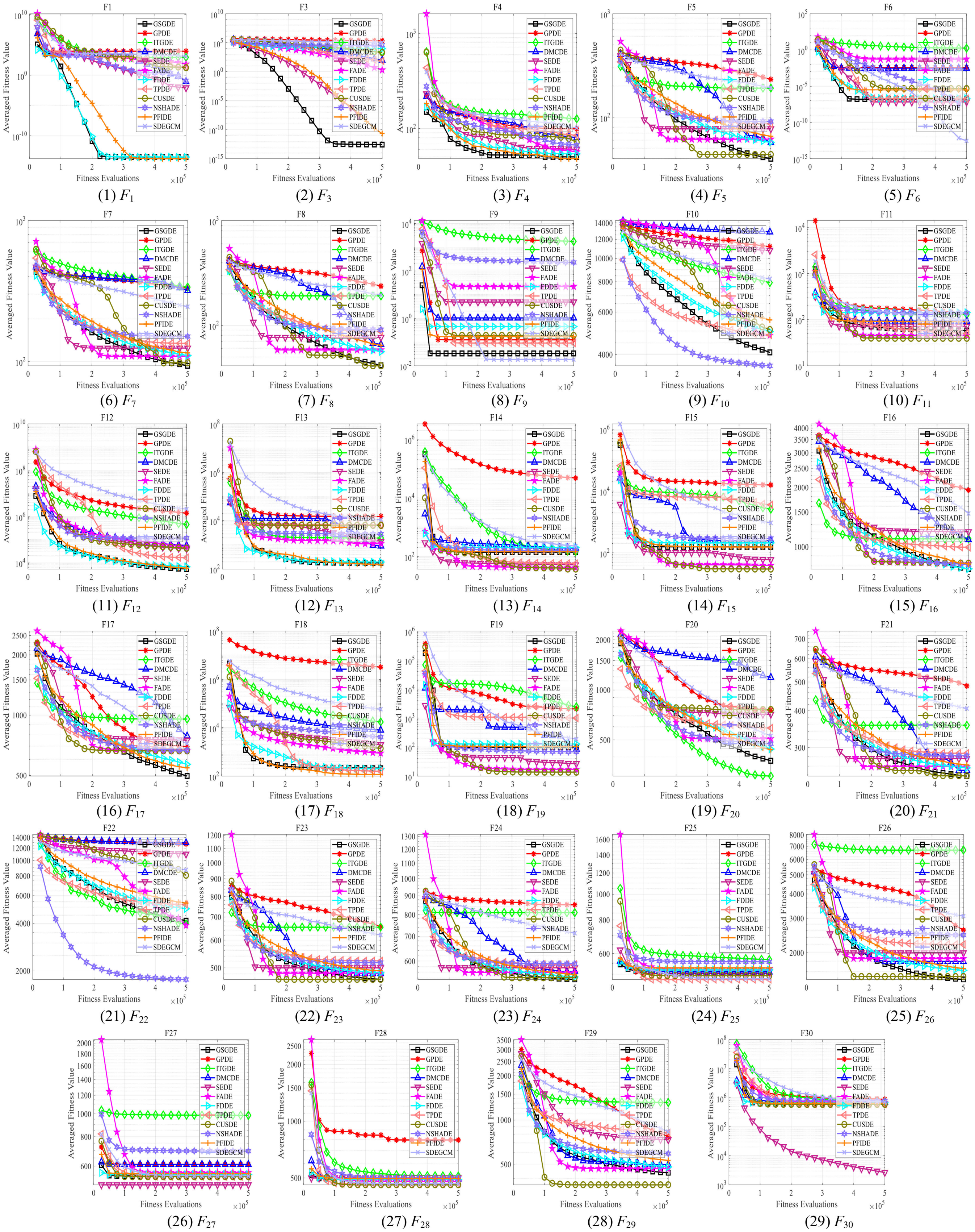

FIGURE 2. Convergence behaviors of GSGDE and the 11 compared DE variants on the 50D CEC2017 benchmark set.

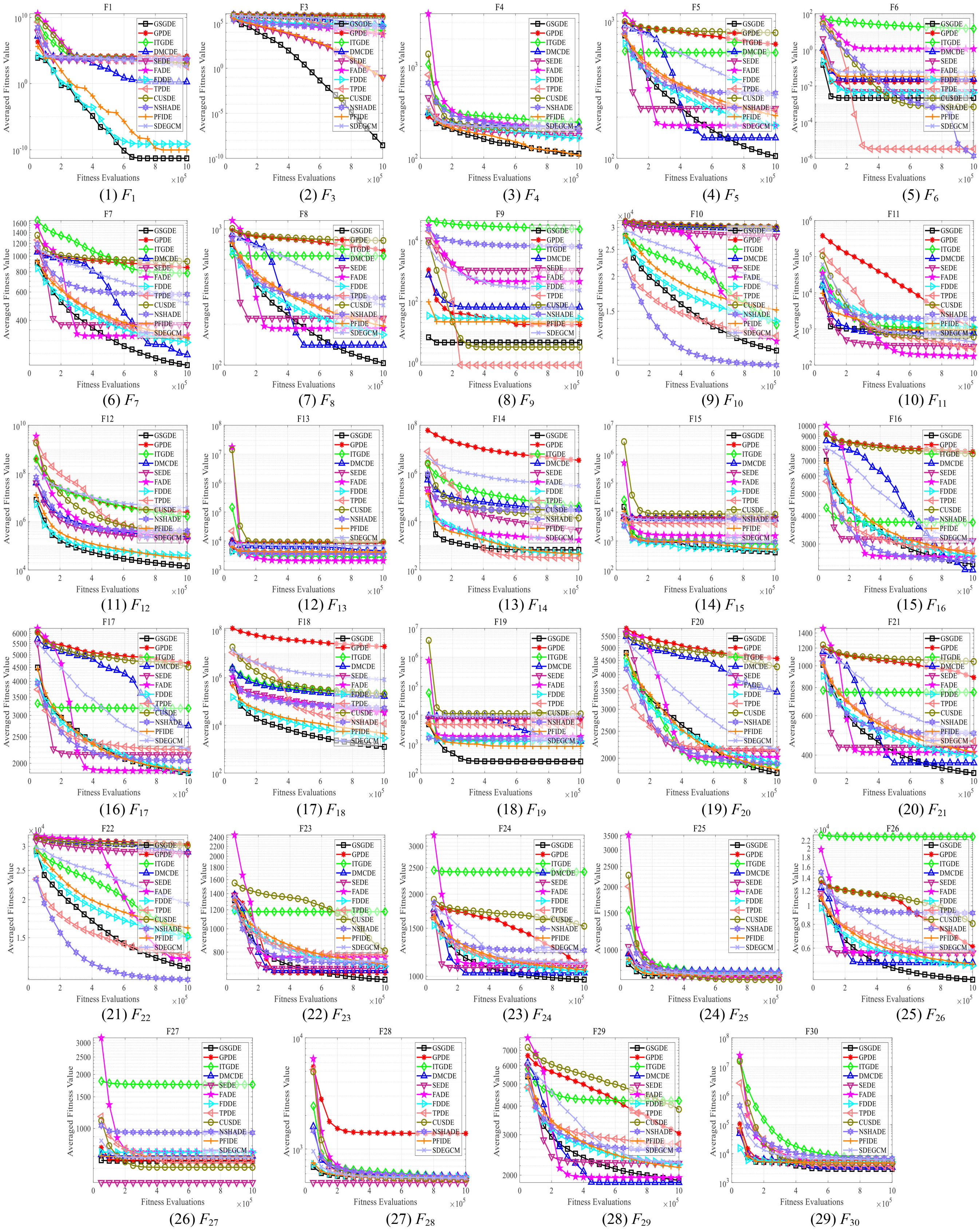

FIGURE 3. Convergence behaviors of GSGDE and the 11 compared DE variants on the 100D CEC2017 benchmark set.

distribution around one stochastically chosen elite from the elite group in the population. With this mutation mechanism, different individuals likely have different guiding exemplars, which is of great benefit for high diversity maintenance. Besides, thanks to the small sampling range of the Gaussian distribution, the randomly sampled guiding exemplars around elite individuals in the population are expectedly better than the associated elites with a high probability. This results in that individuals are mutated by following the guidance of better exemplars, which is very profitable for individuals to move towards optimal regions fast. In addition, the devised dynamic strategy for the number of elites further provides an additional compromise between exploitation and exploration for GSGDE. With the evolution proceeding, the population gradually switches from exploration of the problem space with more elites to exploitation of the located optimal zones subtly with fewer elites. The cooperation between the two major components contributes to the good performance of GSGDE, which is further demonstrated by the following experiments.

C. COMPARISON WITH STATE-OF-THE-ART DE VARIANTS ON THE CEC2014 SUITE

This subsection compares GSGDE with the 11 selected latest DE algorithms by further conducting experiments on the CEC2014 problem suite with three settings of the dimension size, namely 30, 50, and 100, to demonstrate the effectiveness of GSGDE. Since the CEC2017 set shares similarities with the CEC2014 suite, we employ the fine-tuned settings of the population size (as listed in Table 1) for all algorithms on the CEC2017 set to configure them to optimize the CEC2014 benchmark problems. The detailed optimization results of all methods on the 30D, 50D, and 100D CEC2014 sets are presented in Tables 6 $\sim$ 8, respectively. Table 9 summarizes the statistical comparison results concerning "$w/t/l$" and the average rank.

From Tables 6 $\sim$ 8 and Table 9, the comparison results between GSGDE and the 11 compared methods are summarized in the following:

1) Concerning "*Rank*", GSGDE consistently gains the first rank among all approaches on the 30D, 50D, and 100D CEC2014 benchmark sets. This implies that GSGDE consistently gains the best overall optimization results among all methods on the CEC2014 problems with the three dimensionality settings. Furthermore, the rank values of GSGDE on the 30D, 50D and 100D CEC2014 benchmark sets are much lower than those of the 11 compared algorithms. This indicates that GSGDE consistently displays significant superiority to the 11 compared methods on the CEC2014 suite with the three dimensionality settings.
2) In view of "$w/t/l$", on the 30D CEC2014 benchmark suite, GSGDE gains better optimization performance than the 11 compared DE approaches on at least 12 problems, and is significantly inferior to them on no more than 6 problems. Particularly, compared with GPDE, ITGDE, DMCDE, FDDE and SDEGCM, GSGDE is significantly superior on more than 24 problems. In competition with SEDE, FADE, TPDE, CUSDE, and NSHADE, GSGDE is much better on more than 15 problems. On the 50D CEC2014 benchmark set, GSGDE significantly beats the 11 compared algorithms down on at least 15 problems. Particularly, compared with GPDE, ITGDE, DMCDE, FADE, NSHADE and SDEGCM, GSGDE shows significant superiority on more than 20 problems. Competed with SEDE, FDDE, TPDE, CUSDE, and PFIDE, GSGDE gains superior performance on at least 15 problems. On the 100D CEC2014 benchmark set, GSGDE shows significantly superior performance to the 11 compared DE variants on at least 14 problems. Particularly, GSGDE performs significantly better than GPDE, ITGDE, DMCDE, FADE, FDDE, CUSDE, NSHADE, and SDEGCM on more than 20 problems. Compared with SEDE and TPDE, GSGDE is much better on 18 and 17 problems respectively. These findings indicate that GSGDE preserves a good scalability in problem optimization.
3) With respect to the 4 types of benchmark problems, (a) on the three unimodal problems (F_1, F_2, and F_3), GSGDE is superior to 5 compared DE approaches on all the three 30D unimodal problems and shows no inferior performance to the 11 compared DE variants on any problems. On the three 50D unimodal problems, GSGDE shows significant superiority to 8 compared DE approaches on all the three problems and shows no inferior performance to 10 compared methods on any problems. On the three 100D unimodal problems, GSGDE is superior to 9 compared DE approaches on all the three problems and displays no inferiority to 10 compared algorithms on any problems. As a whole, in comparisons with the 11 compared DE approaches, GSGDE presents increasing superiority as the dimension size grows. This implies that GSGDE is effective in addressing unimodal optimization problems, especially those with high dimensionality. (b) On the 13 multimodal problems (F_4-F_{16}), we discover that with the dimension size increasing, GSGDE gains increasingly superior performance to the 11 compared DE approaches. Specifically, on these problems with the three different dimension sizes, apart from TPDE, CUSDE, and NSHADE, GSGDE presents great dominance to the rest 8 compared DE approaches on more than 10 problems and exhibits inferior performance to them on at most 3 problems. These discoveries demonstrate that GSGDE is more effective to optimize multimodal problems. (c) On the 6 hybrid functions (F_{17}-F_{22}), when the dimension size is 30, except for SEDE, FADE, TPDE, CUSDE, and PFIDE, GSGDE significantly beats the rest 6 compared algorithms down on at least 4 problems and shows slightly

TABLE 6. Optimization performance comparisons between GSGDE and the 11 state-of-the-art DE variants on the 30*D* CEC2014 suite.

F	Category	Quality	GSGDE	GPDE	ITGDE	DMCDE	SEDE	FADE	FDDE	TPDE	CUSDE	NSHADE	PFIDE	SDEGCM
F_1	Unimodal Problems	Mean	9.56E+01	9.73E+06	5.39E+05	2.10E+05	2.40E+04	3.84E+04	4.41E+03	8.95E+03	1.13E+04	2.40E+04	2.84E+03	1.92E+06
		Std	3.10E+02	6.07E+06	2.74E+05	4.00E+05	1.51E+04	3.23E+04	4.16E+03	7.49E+03	1.14E+04	1.99E+04	2.88E+03	1.01E+06
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.47E-10**$^+$	**4.50E-11**$^+$	**3.34E-11**$^+$	**3.02E-11**$^+$	**3.82E-10**$^+$	**3.02E-11**$^+$
F_2		Mean	0.00E+00	6.59E-06	3.29E-07	9.47E-16	0.00E+00	0.00E+00	6.63E-15	0.00E+00	1.26E-09	0.00E+00	0.00E+00	1.80E-14
		Std	0.00E+00	3.60E-05	1.87E-07	5.19E-15	0.00E+00	0.00E+00	1.22E-14	0.00E+00	4.81E-09	0.00E+00	0.00E+00	1.39E-14
		p-value	-	**1.21E-12**$^+$	**1.21E-12**$^+$	3.34E-01$^=$	NaN$^=$	NaN$^=$	**5.47E-03**$^+$	NaN$^=$	**1.37E-03**$^+$	NaN$^=$	NaN$^=$	**1.79E-07**$^+$
F_3		Mean	0.00E+00	1.87E-07	1.59E-07	3.03E-14	0.00E+00	0.00E+00	2.23E-04	0.00E+00	2.65E-14	0.00E+00	1.89E-15	5.31E-14
		Std	0.00E+00	6.73E-07	9.46E-08	2.88E-14	0.00E+00	0.00E+00	1.22E-03	0.00E+00	8.41E-14	0.00E+00	1.04E-14	2.08E-14
		p-value	-	**1.21E-12**$^+$	**1.21E-12**$^+$	**3.80E-06**$^+$	NaN$^=$	NaN$^=$	**9.21E-09**$^+$	NaN$^=$	**5.51E-03**$^+$	NaN$^=$	3.34E-01$^=$	**5.42E-12**$^+$
$F_{1\text{-}3}$		*w/t/l*		3/0/0	3/0/0	2/1/0	1/2/0	1/2/0	3/0/0	1/2/0	3/0/0	1/2/0	1/2/0	3/0/0
F_4	Simple Multimodal Problems	Mean	4.55E-14	2.13E+01	5.53E+01	1.28E+01	3.33E-08	2.86E-02	5.42E-04	9.07E-02	2.11E+00	1.63E-01	4.20E-12	4.22E+01
		Std	2.31E-14	3.34E+01	3.81E+01	2.46E+01	1.83E-07	4.96E-02	2.47E-03	1.74E-01	1.16E+01	2.33E-01	2.09E-11	4.25E+01
		p-value	-	**6.27E-12**$^+$	**6.32E-12**$^+$	**6.30E-12**$^+$	**1.04E-08**$^+$	**6.32E-12**$^+$	**1.83E-11**$^+$	**6.32E-12**$^+$	**6.32E-12**$^+$	**1.04E-10**$^+$	5.70E-02$^=$	**6.32E-12**$^+$
F_5		Mean	2.03E+01	2.05E+01	2.06E+01	2.09E+01	2.06E+01	2.09E+01	2.04E+01	2.02E+01	2.06E+01	2.01E+01	2.04E+01	2.03E+01
		Std	3.01E-02	1.25E-01	5.08E-02	5.74E-02	4.09E-02	3.61E-02	3.48E-02	3.81E-02	1.11E-01	1.39E-02	4.10E-02	8.06E-02
		p-value	-	**2.32E-06**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**4.37E-09**$^+$	3.02E-11$^-$	**1.96E-10**$^+$	3.02E-11$^-$	**8.89E-10**$^+$	3.48E-01$^=$
F_6		Mean	7.35E-01	1.08E+01	1.26E+01	3.18E+00	1.58E+01	1.45E+00	4.60E+00	1.26E+01	4.14E+00	1.07E+01	4.88E+00	6.13E+00
		Std	1.64E+00	3.07E+00	2.88E+00	2.14E+00	9.19E+00	9.63E-01	3.53E+00	4.37E+00	2.38E+00	1.13E+00	4.52E+00	3.47E+00
		p-value	-	**2.41E-11**$^+$	**1.60E-11**$^+$	**1.05E-07**$^+$	**4.44E-11**$^+$	**2.34E-05**$^+$	**8.98E-07**$^+$	**3.28E-11**$^+$	**3.80E-08**$^+$	**1.44E-11**$^+$	**1.19E-03**$^+$	**9.20E-10**$^+$
F_7		Mean	0.00E+00	4.93E-04	2.47E-04	2.05E-03	4.11E-04	1.72E-03	6.57E-04	0.00E+00	1.14E-14	3.03E-03	0.00E+00	2.47E-04
		Std	0.00E+00	1.88E-03	1.35E-03	3.99E-03	2.25E-03	6.56E-03	2.58E-03	0.00E+00	3.47E-14	6.87E-03	0.00E+00	1.35E-03
		p-value	-	**1.20E-12**$^+$	**2.68E-07**$^+$	**5.01E-06**$^+$	3.34E-01$^=$	8.15E-02$^=$	**2.56E-05**$^+$	NaN$^=$	8.14E-02$^=$	**2.77E-03**$^+$	NaN$^=$	**1.15E-10**$^+$
F_8		Mean	1.02E-13	7.72E+00	1.77E+01	3.83E+01	3.32E-02	2.64E+01	8.38E-05	2.68E+01	2.18E+01	0.00E+00	5.80E-05	5.31E+00
		Std	3.47E-14	2.67E+00	1.59E+01	1.73E+01	1.82E-01	7.48E+00	1.41E-04	6.73E+00	7.18E+00	0.00E+00	9.21E-05	2.99E+00
		p-value	-	**3.08E-12**$^+$	**3.13E-12**$^+$	**3.15E-12**$^+$	**1.69E-10**$^+$	**3.12E-12**$^+$	**3.15E-12**$^+$	**3.15E-12**$^+$	**3.13E-12**$^+$	3.94E-12$^-$	**3.15E-12**$^+$	**6.38E-11**$^+$
F_9		Mean	2.30E+01	7.70E+01	7.97E+01	7.48E+01	3.74E+01	2.67E+01	2.37E+01	4.14E+01	3.28E+01	3.31E+01	2.64E+01	4.67E+01
		Std	2.77E+00	3.99E+01	1.51E+01	5.98E+01	1.23E+01	8.45E+00	3.04E+00	6.58E+00	1.58E+01	5.73E+00	3.86E+00	1.73E+01
		p-value	-	**3.69E-11**$^+$	**3.01E-11**$^+$	**2.81E-02**$^+$	**5.09E-08**$^+$	3.11E-01$^=$	4.04E-01$^=$	**4.98E-11**$^+$	**7.28E-04**$^+$	**4.20E-10**$^+$	**3.99E-04**$^+$	**4.19E-10**$^+$
F_{10}		Mean	3.32E-01	3.69E+02	7.64E+01	2.56E+03	4.14E+02	7.97E+02	4.92E+01	7.17E+02	8.17E+02	4.48E-01	1.18E+02	1.43E+02
		Std	7.08E-01	2.12E+02	3.13E+01	5.74E+02	1.30E+02	3.07E+02	8.22E+00	2.02E+02	3.37E+02	3.54E-01	2.82E+01	1.45E+02
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.08E-08**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$
F_{11}		Mean	1.99E+03	4.60E+03	3.50E+03	6.47E+03	5.17E+03	3.37E+03	2.16E+03	2.27E+03	2.27E+03	1.49E+03	2.36E+03	2.88E+03
		Std	1.78E+02	1.09E+03	4.56E+02	3.23E+02	6.03E+02	1.69E+03	2.35E+02	3.05E+02	6.40E+02	2.79E+02	2.46E+02	8.35E+02
		p-value	-	**1.21E-10**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.34E-11**$^+$	**5.86E-06**$^+$	**1.33E-02**$^+$	**2.25E-04**$^+$	**4.36E-02**$^+$	1.69E-09$^-$	**1.16E-07**$^+$	**1.53E-05**$^+$
F_{12}		Mean	3.47E-01	4.71E-01	9.14E-01	2.39E+00	9.42E-01	2.37E+00	4.25E-01	2.09E-01	3.78E-01	1.51E-01	4.70E-01	6.45E-01
		Std	4.66E-02	3.40E-01	1.24E-01	2.56E-01	1.25E-01	4.42E-01	5.24E-02	4.13E-02	5.12E-01	1.89E-02	5.28E-02	1.69E-01
		p-value	-	6.10E-01$^=$	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.02E-11**$^+$	**2.37E-10**$^+$	**5.60E-07**$^+$	3.16E-10$^-$	**5.57E-03**$^+$	3.02E-11$^-$	**4.20E-10**$^+$	**3.20E-09**$^+$
F_{13}		Mean	1.81E-01	3.28E-01	3.40E-01	2.46E-01	2.28E-01	2.48E-01	1.94E-01	1.95E-01	1.82E-01	2.65E-01	2.06E-01	3.10E-01
		Std	2.18E-02	4.48E-02	4.69E-02	3.58E-02	3.53E-02	5.63E-02	2.42E-02	3.81E-02	4.29E-02	4.77E-02	3.16E-02	4.45E-02
		p-value	-	**3.02E-11**$^+$	**3.34E-11**$^+$	**1.01E-08**$^+$	**5.60E-07**$^+$	**1.49E-06**$^+$	**1.76E-02**$^+$	**3.78E-02**$^+$	9.71E-01$^=$	**4.18E-09**$^+$	**5.41E-04**$^+$	**5.49E-11**$^+$
F_{14}		Mean	2.56E-01	4.92E-01	2.10E-01	3.89E-01	2.51E-01	2.72E-01	2.55E-01	2.53E-01	2.88E-01	2.18E-01	2.45E-01	3.02E-01
		Std	4.93E-02	1.86E-01	3.61E-02	1.36E-01	8.13E-02	3.89E-02	3.21E-02	8.79E-02	7.75E-02	3.60E-02	3.03E-02	9.19E-02
		p-value	-	**6.53E-08**$^+$	3.83E-05$^-$	**3.57E-06**$^+$	3.55E-01$^=$	**4.84E-02**$^+$	6.63E-01$^=$	7.73E-02$^=$	**1.08E-02**$^+$	1.24E-03$^-$	5.69E-01$^=$	**4.22E-04**$^+$
F_{15}		Mean	3.56E+00	1.38E+01	1.27E+01	1.28E+01	3.39E+00	3.35E+00	3.52E+00	3.64E+00	3.21E+00	4.36E+00	4.08E+00	7.64E+00
		Std	3.71E-01	1.21E+00	1.96E+00	2.29E+00	9.02E-01	1.05E+00	4.31E-01	5.64E-01	1.06E+00	9.59E-01	4.89E-01	1.23E+00
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**3.69E-11**$^+$	4.29E-01$^=$	3.26E-01$^=$	6.95E-01$^=$	5.20E-01$^=$	8.31E-03$^-$	**1.32E-04**$^+$	**4.64E-05**$^+$	**3.02E-11**$^+$
F_{16}		Mean	9.74E+00	1.22E+01	1.06E+01	1.20E+01	1.20E+01	1.24E+01	1.01E+01	1.08E+01	9.89E+00	9.43E+00	1.03E+01	1.07E+01
		Std	2.90E-01	3.81E-01	4.13E-01	3.50E-01	6.20E-01	2.63E-01	2.99E-01	3.30E-01	1.02E+00	4.52E-01	2.90E-01	6.05E-01
		p-value	-	**3.02E-11**$^+$	**1.86E-09**$^+$	**3.02E-11**$^+$	**3.16E-10**$^+$	**3.02E-11**$^+$	**3.15E-05**$^+$	**6.70E-11**$^+$	8.19E-01$^=$	3.50E-03$^-$	**1.25E-07**$^+$	**3.35E-08**$^+$
$F_{4\text{-}16}$		*w/t/l*		12/1/0	12/0/1	13/0/0	10/3/0	10/3/0	10/3/0	8/3/2	9/3/1	7/0/6	10/3/0	12/1/0
F_{17}	Hybrid Problems	Mean	8.54E+02	6.44E+05	3.11E+03	1.86E+03	5.06E+02	2.30E+02	1.10E+03	8.06E+02	3.65E+02	1.22E+03	7.44E+02	5.44E+04
		Std	3.50E+02	1.03E+06	1.60E+03	2.64E+03	4.20E+02	1.99E+02	3.71E+02	2.62E+02	2.69E+02	9.85E+02	3.59E+02	4.54E+04
		p-value	-	**3.02E-11**$^+$	**3.69E-11**$^+$	**7.30E-04**$^+$	1.04E-04$^-$	2.67E-09$^-$	**2.32E-02**$^+$	6.63E-01$^=$	3.52E-07$^-$	1.02E-01$^=$	2.40E-01$^=$	**3.02E-11**$^+$
18		Mean	1.86E+01	7.81E+03	2.09E+02	6.39E+01	1.45E+01	1.32E+01	8.62E+01	1.22E+01	9.51E+00	8.07E+01	2.50E+01	1.08E+02
		Std	1.04E+01	1.05E+04	4.40E+02	5.11E+01	7.29E+00	4.89E+00	5.21E+01	3.28E+00	4.34E+00	2.84E+01	1.57E+01	3.37E+01
		p-value	-	**3.02E-11**$^+$	**3.69E-11**$^+$	**1.25E-07**$^+$	1.67E-01$^=$	6.15E-02$^=$	**2.92E-09**$^+$	2.92E-02$^-$	2.53E-04$^-$	**6.70E-11**$^+$	7.24E-02$^=$	**3.34E-11**$^+$
F_{19}		Mean	4.59E+00	5.45E+00	1.20E+01	6.43E+00	3.71E+00	2.37E+00	5.25E+00	5.51E+00	3.24E+00	7.02E+00	4.72E+00	5.84E+00
		Std	8.04E-01	1.68E+00	1.55E+00	1.18E+00	1.77E+00	4.94E-01	9.40E-01	5.69E-01	9.84E-01	1.06E+01	6.01E-01	1.22E+00
		p-value	-	**4.36E-02**$^+$	**3.02E-11**$^+$	**3.08E-08**$^+$	5.87E-04$^-$	3.02E-11$^-$	**1.86E-03**$^+$	**1.73E-06**$^+$	1.19E-06$^-$	**2.38E-03**$^+$	2.06E-01$^=$	**5.46E-06**$^+$
F_{20}		Mean	4.62E+00	6.01E+01	2.43E+01	1.55E+01	1.02E+01	7.48E+00	6.70E+02	1.58E+01	6.67E+00	4.76E+01	9.09E+00	3.60E+01
		Std	2.05E+00	2.53E+01	5.25E+00	1.45E+01	4.90E+00	3.28E+00	1.08E+03	4.68E+00	2.97E+00	2.71E+01	3.63E+00	1.17E+01
		p-value	-	**3.02E-11**$^+$	**3.34E-11**$^+$	**2.78E-07**$^+$	**6.05E-07**$^+$	**1.78E-04**$^+$	**2.15E-10**$^+$	**6.07E-11**$^+$	**6.91E-04**$^+$	**3.02E-11**$^+$	**7.09E-08**$^+$	**3.02E-11**$^+$
F_{21}		Mean	1.89E+02	4.92E+04	8.71E+02	3.72E+02	1.63E+02	1.05E+02	3.33E+02	3.45E+02	1.91E+02	3.48E+02	2.39E+02	4.59E+03
		Std	1.08E+02	1.95E+05	2.27E+02	2.47E+02	1.29E+02	1.21E+02	1.34E+02	1.51E+02	2.14E+02	1.73E+02	1.41E+02	7.73E+03
		p-value	-	**3.02E-11**$^+$	**3.02E-11**$^+$	**4.23E-03**$^+$	1.91E-01$^=$	1.60E-03$^-$	**1.49E-04**$^+$	**1.58E-04**$^+$	3.87E-01$^=$	**2.68E-04**$^+$	2.52E-01$^=$	**3.02E-11**$^+$
F_{22}		Mean	9.42E+01	2.65E+02	2.70E+02	7.15E+01	1.69E+02	1.15E+02	1.26E+02	4.05E+01	1.27E+02	1.88E+02	1.24E+02	2.04E+02
		Std	5.92E+01	1.89E+02	1.06E+02	6.52E+01	9.05E+01	6.20E+01	6.71E+01	3.25E+01	1.53E+02	6.14E+01	5.85E+01	1.37E+02
		p-value	-	**3.16E-05**$^+$	**1.07E-07**$^+$	7.62E-03$^-$	**3.03E-03**$^+$	6.84E-01$^=$	**4.68E-02**$^+$	3.52E-07$^-$	3.87E-01$^=$	**1.25E-04**$^+$	8.50E-02$^=$	**3.03E-03**$^+$
$F_{17\text{-}22}$		*w/t/l*		6/0/0	6/0/0	5/0/1	2/2/2	1/2/3	6/0/0	3/1/2	1/2/3	5/1/0	1/5/0	6/0/0
F_{23}	Composition Problems	Mean	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02
		Std	5.78E-14	5.78E-14	5.78E-14	5.78E-14	2.21E-02	5.78E-14	5.78E-14	5.78E-14	5.78E-14	5.78E-14	5.78E-14	5.78E-14
		p-value	-	NaN$^=$	NaN$^=$	NaN$^=$	1.00E+00$^=$	NaN$^=$	NaN$^=$	NaN$^=$	NaN$^=$	NaN$^=$	NaN$^=$	NaN$^=$
F_{24}		Mean	2.24E+02	2.28E+02	2.20E+02	2.29E+02	2.25E+02	2.23E+02	2.26E+02	2.24E+02	2.15E+02	2.28E+02	2.24E+02	2.26E+02
		Std	1.40E+00	4.37E+00	8.12E+00	5.56E+00	2.61E+00	4.57E+00	3.76E+00	2.20E+00	1.16E+01	4.20E+00	6.08E-01	4.22E+00
		p-value	-	**5.09E-06**$^+$	2.81E-02$^-$	**2.15E-06**$^+$	4.83E-01$^=$	9.71E-01$^=$	**6.38E-03**$^+$	**2.61E-02**$^+$	3.83E-05$^-$	**8.84E-07**$^+$	4.92E-01$^=$	1.96E-01$^=$
F_{25}		Mean	2.03E+02	2.05E+02	2.07E+02	2.04E+02	2.03E+02	2.03E+02	2.04E+02	2.03E+02	2.03E+02	2.10E+02	2.03E+02	2.04E+02
		Std	5.02E-01	1.97E+00	2.85E+00	7.84E-01	8.09E-01	4.34E-01	1.17E+00	4.52E-02	3.12E-01	2.97E+00	5.74E-01	7.57E-01
		p-value	-	**4.74E-03**$^+$	**1.25E-07**$^+$	2.90E-01$^=$	8.65E-01$^=$	6.31E-01$^=$	**1.17E-05**$^+$	3.32E-11$^-$	3.09E-06$^-$	**1.69E-09**$^+$	1.71E-01$^=$	1.37E-01$^=$
F_{26}		Mean	1.00E+02	1.00E+02	1.20E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.07E+02	1.00E+02	1.24E+02
		Std	3.11E-02	4.67E-02	4.06E+01	4.78E-02	4.88E-02	5.27E-02	2.49E-02	3.01E-02	4.04E-02	2.53E+01	2.32E-02	5.81E+01
		p-value	-	**3.33E-11**$^+$	**1.39E-10**$^+$	**1.43E-06**$^+$	**6.76E-05**$^+$	**4.97E-11**$^+$	1.15E-01$^=$	2.46E-02$^-$	**2.25E-04**$^+$	**1.95E-10**$^+$	1.12E-05$^-$	**3.68E-11**$^+$
F_{27}		Mean	3.05E+02	5.16E+02	5.39E+02	4.14E+02	7.69E+02	3.88E+02	3.30E+02	3.65E+02	3.56E+02	3.96E+02	3.25E+02	4.40E+02
		Std	1.54E+01	8.35E+01	1.21E+02	5.70E+01	2.22E+02	3.37E+01	3.55E+01	3.90E+01	5.07E+01	3.66E+01	4.44E+01	6.67E+01
		p-value	-	**4.39E-12**$^+$	**3.16E-12**$^+$	**1.05E-10**$^+$	**3.16E-12**$^+$	**1.17E-11**$^+$	**2.93E-04**$^+$	**2.99E-09**$^+$	**1.57E-06**$^+$	**1.61E-11**$^+$	6.57E-02$^=$	**7.99E-12**$^+$
F_{28}		Mean	8.20E+02	8.62E+02	1.03E+03	8.57E+02	5.91E+02	8.78E+02	8.15E+02	8.31E+02	8.31E+02	8.80E+02	8.00E+02	9.05E+02
		Std	2.68E+01	3.25E+01	8.16E+01	2.95E+01	6.47E+01	3.45E+01	3.37E+01	1.85E+01	2.33E+01	4.22E+01	3.14E+01	1.27E+02
		p-value	-	**7.22E-06**$^+$	**3.02E-11**$^+$	**1.87E-05**$^+$	3.02E-11$^-$	**2.39E-08**$^+$	5.79E-01$^=$	1.45E-01$^=$	1.19E-01$^=$	**4.44E-07**$^+$	1.38E-02$^-$	**1.73E-06**$^+$
F_{29}		Mean	7.20E+02	3.30E+06	9.85E+02	2.98E+05	2.17E+02	5.57E+02	2.98E+05	2.81E+05	5.91E+05	7.02E+02	7.25E+02	6.22E+05
		Std	8.01E+00	3.85E+06	1.92E+02	1.63E+06	1.06E+00	1.94E+02	1.63E+06	1.53E+06	2.16E+06	1.52E+02	1.26E+01	2.37E+06
		p-value	-	**2.80E-11**$^+$	**3.02E-11**$^+$	**1.10E-08**$^+$	3.02E-11$^-$	1.08E-02$^-$	**9.26E-09**$^+$	**3.27E-02**$^+$	**1.56E-02**$^+$	3.33E-01$^=$	2.46E-01$^=$	**3.34E-11**$^+$
F_{30}		Mean	1.72E+03	1.27E+04	1.87E+03	3.26E+03	6.85E+02	5.24E+02	2.14E+03	2.30E+03	1.70E+03	1.43E+03	1.65E+03	3.08E+03
		Std	8.95E+02	1.53E+04	3.24E+02	1.21E+03	1.21E+02	1.42E+02	8.33E+02	9.37E+02	8.96E+02	4.60E+02	6.90E+02	1.16E+03
		p-value	-	**8.10E-10**$^+$	**1.50E-02**$^+$	**3.83E-06**$^+$	3.02E-11$^-$	3.02E-11$^-$	**2.51E-02**$^+$	**4.21E-02**$^+$	7.17E-01$^=$	4.73E-01$^=$	8.77E-01$^=$	**1.53E-05**$^+$
$F_{23\text{-}30}$		*w/t/l*		7/1/0	6/1/1	6/2/0	2/3/3	3/3/2	5/3/0	4/2/2	3/3/2	5/3/0	0/7/1	5/3/0
w/t/l				28/2/0	27/1/2	26/3/1	15/10/5	15/10/5	24/6/0	16/8/6	16/8/6	18/6/6	12/17/1	26/4/0
Rank			**3.20**	9.86	8.95	8.80	5.68	5.50	5.90	5.50	5.13	6.26	4.40	8.80

TABLE 7. Optimization performance comparisons between GSGDE and the 11 state-of-the-art DE variants on the 50*D* CEC2014 suite.

F	Category	Quality	GSGDE	GPDE	ITGDE	DMCDE	SEDE	FADE	FDDE	TPDE	CUSDE	NSHADE	PFIDE	SDEGCM
F_1		Mean	2.23E+04	2.01E+07	1.98E+06	8.46E+05	4.09E+05	2.26E+05	5.08E+04	3.35E+05	1.50E+05	2.09E+05	4.21E+04	1.39E+07
		Std	1.35E+04	1.69E+07	4.67E+05	5.54E+05	1.56E+05	8.68E+04	3.59E+04	1.51E+05	1.04E+05	1.13E+05	5.36E+04	4.19E+06
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.56E-04**$^{+}$	**3.02E-11**$^{+}$	**8.15E-11**$^{+}$	**3.02E-11**$^{+}$	**1.91E-02**$^{+}$	**3.02E-11**$^{+}$
F_2	Unimodal Problems	Mean	6.44E-14	1.01E+04	4.79E+03	1.17E-05	7.90E-06	4.79E+01	5.78E-14	5.65E-03	1.35E+00	1.70E-01	3.79E-14	1.86E+03
		Std	1.97E-14	9.67E+03	3.41E+03	6.20E-05	1.43E-05	1.42E+02	2.74E-14	8.10E-03	3.61E+00	5.10E-01	1.36E-14	2.38E+03
		p-value	-	**1.24E-11**$^{+}$	**1.24E-11**$^{+}$	**1.24E-11**$^{+}$	6.54E-01$^{=}$	**1.24E-11**$^{+}$	9.68E-02$^{=}$	**1.24E-11**$^{+}$	**1.24E-11**$^{+}$	**1.24E-11**$^{+}$	4.70E-07^{-}	**1.24E-11**$^{+}$
F_3		Mean	5.84E-13	8.22E+02	8.39E+00	8.23E-06	3.77E-02	8.61E+01	1.31E+03	1.91E-03	1.25E+00	6.47E+00	2.79E-13	1.20E+02
		Std	5.47E-13	6.73E+02	5.11E+00	2.28E-05	1.67E-01	9.15E+01	1.55E+03	9.26E-04	4.11E+00	8.70E+00	2.06E-13	1.10E+02
		p-value	-	**2.89E-11**$^{+}$	**2.89E-11**$^{+}$	**2.89E-11**$^{+}$	**2.89E-11**$^{+}$	**2.89E-11**$^{+}$	**4.82E-07**$^{+}$	**2.89E-11**$^{+}$	**2.89E-11**$^{+}$	**2.89E-11**$^{+}$	4.05E-03^{-}	**2.89E-11**$^{+}$
$F_{1\text{-}3}$		*w/t/l*		3/0/0	3/0/0	3/0/0	2/1/0	3/0/0	2/1/0	3/0/0	3/0/0	3/0/0	1/0/2	3/0/0
F_4		Mean	2.69E+01	9.67E+01	9.26E+01	8.43E+01	7.15E+01	5.38E+01	6.29E+01	9.72E+01	5.56E+01	5.18E+01	3.71E+01	9.75E+01
		Std	4.37E+01	2.57E+00	3.99E+01	1.97E+01	4.09E+01	3.57E+01	3.40E+01	3.51E+00	4.63E+01	4.23E+01	4.19E+01	1.64E+00
		p-value	-	**4.46E-06**$^{+}$	**1.83E-05**$^{+}$	**5.14E-05**$^{+}$	**4.94E-07**$^{+}$	**2.86E-03**$^{+}$	**3.13E-04**$^{+}$	**1.68E-11**$^{+}$	**6.87E-05**$^{+}$	**7.89E-03**$^{+}$	**7.62E-03**$^{+}$	**3.75E-07**$^{+}$
F_5		Mean	2.04E+01	2.09E+01	2.07E+01	2.11E+01	2.09E+01	2.11E+01	2.06E+01	2.02E+01	2.11E+01	2.01E+01	2.06E+01	2.06E+01
		Std	2.01E-02	1.16E-01	3.32E-02	4.12E-02	3.51E-02	3.26E-02	2.59E-02	4.33E-02	1.33E-01	1.78E-02	2.88E-02	8.06E-02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	3.02E-11^{-}	**3.02E-11**$^{+}$	3.02E-11^{-}	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_6		Mean	1.45E+00	2.71E+01	2.73E+01	9.32E+00	5.04E+01	6.24E+00	8.13E+00	2.23E+01	9.08E+00	2.25E+01	3.14E+00	1.14E+01
		Std	1.54E+00	6.16E+00	2.69E+00	3.46E+00	1.54E+01	2.19E+00	8.40E+00	3.96E+00	4.09E+00	2.01E+00	3.03E+00	3.89E+00
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**5.49E-11**$^{+}$	**3.02E-11**$^{+}$	**6.12E-10**$^{+}$	**1.60E-03**$^{+}$	**3.02E-11**$^{+}$	**4.62E-10**$^{+}$	**3.02E-11**$^{+}$	**1.44E-03**$^{+}$	**4.08E-11**$^{+}$
F_7		Mean	1.14E-13	5.75E-04	2.47E-04	1.31E-03	4.93E-04	5.09E-03	1.23E-03	8.72E-14	3.29E-04	7.55E-03	2.30E-03	5.13E-12
		Std	0.00E+00	2.21E-03	1.35E-03	3.47E-03	1.88E-03	7.83E-03	2.80E-03	4.89E-14	1.80E-03	9.19E-03	4.71E-03	2.72E-11
		p-value	-	**1.21E-12**$^{+}$	**1.21E-12**$^{+}$	**4.19E-02**$^{+}$	**5.86E-11**$^{+}$	5.44E-02$^{=}$	**4.90E-06**$^{+}$	1.97E-13^{-}	5.70E-01$^{=}$	3.44E-01$^{=}$	**5.56E-03**$^{+}$	**1.35E-04**$^{+}$
F_8		Mean	3.03E-14	2.48E+01	9.06E+01	2.01E+01	6.63E-02	5.88E+01	5.71E+00	5.41E+01	3.18E+01	0.00E+00	3.57E-02	4.49E+01
		Std	5.11E-14	7.21E+00	5.00E+01	4.37E+00	2.52E-01	1.11E+01	1.59E+00	1.69E+01	9.16E+00	0.00E+00	6.22E-02	9.24E+00
		p-value	-	**8.87E-12**$^{+}$	**8.86E-12**$^{+}$	**8.66E-12**$^{+}$	6.27E-02$^{=}$	**8.86E-12**$^{+}$	**8.87E-12**$^{+}$	**8.87E-12**$^{+}$	**8.73E-12**$^{+}$	2.70E-03^{-}	**8.87E-12**$^{+}$	**8.87E-12**$^{+}$
F_9		Mean	4.08E+01	1.38E+02	1.81E+02	4.91E+01	6.18E+01	6.01E+01	5.88E+01	9.18E+01	3.91E+01	8.69E+01	5.61E+01	1.90E+02
		Std	5.30E+00	9.42E+01	2.62E+01	1.20E+01	9.32E+00	1.05E+01	7.44E+00	1.38E+01	8.96E+00	1.21E+01	5.27E+00	2.07E+01
		p-value	-	**4.20E-10**	**3.02E-11**$^{+}$	**8.31E-03**$^{+}$	**2.61E-10**$^{+}$	**1.85E-09**$^{+}$	**1.46E-10**$^{+}$	**3.02E-11**$^{+}$	3.04E-01$^{=}$	**3.02E-11**$^{+}$	**6.70E-11**$^{+}$	**3.02E-11**$^{+}$
F_{10}	Simple Multimodal Problems	Mean	3.86E-01	9.56E+02	7.91E+02	3.84E+03	4.33E+03	2.21E+03	4.78E+02	1.80E+03	1.43E+03	1.55E+00	4.83E+02	1.05E+03
		Std	6.81E-01	3.16E+02	2.09E+02	3.38E+03	2.86E+02	6.32E+02	9.91E+01	4.39E+02	3.87E+02	6.23E-01	7.01E+01	5.00E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**7.12E-09**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_{11}		Mean	4.18E+03	1.16E+04	8.38E+03	1.28E+04	1.13E+04	5.35E+03	5.28E+03	4.68E+03	4.60E+03	3.61E+03	5.37E+03	8.49E+03
		Std	3.20E+02	6.03E+02	7.49E+02	4.73E+02	3.67E+02	1.95E+03	3.52E+02	5.27E+02	2.77E+03	2.91E+02	3.07E+02	4.93E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**6.74E-06**$^{+}$	**8.15E-11**$^{+}$	**4.64E-05**$^{+}$	4.12E-01$^{=}$	4.31E-08^{-}	**4.50E-11**$^{+}$	**3.02E-11**$^{+}$
F_{12}		Mean	2.95E-01	1.46E+00	1.21E+00	3.22E+00	1.59E+00	3.35E+00	5.56E-01	2.23E-01	1.90E+00	1.58E-01	4.67E-01	9.41E-01
		Std	2.26E-02	6.71E-01	1.27E-01	2.92E-01	1.51E-01	3.00E-01	7.05E-02	3.19E-02	1.21E+00	1.23E-02	7.26E-02	1.97E-01
		p-value	-	**1.78E-10**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.69E-11**$^{+}$	6.72E-10^{-}	**8.20E-07**$^{+}$	3.02E-11^{-}	**1.96E-10**$^{+}$	**3.02E-11**$^{+}$
F_{13}		Mean	2.81E-01	4.08E-01	5.07E-01	3.32E-01	3.92E-01	3.92E-01	2.90E-01	2.46E-01	3.09E-01	4.14E-01	3.12E-01	4.39E-01
		Std	3.49E-02	5.19E-02	5.07E-02	4.19E-02	7.53E-02	8.34E-02	4.32E-02	3.87E-02	3.64E-02	6.94E-02	3.84E-02	4.37E-02
		p-value	-	**2.37E-10**$^{+}$	**3.02E-11**$^{+}$	**7.22E-06**$^{+}$	**7.69E-08**$^{+}$	**2.60E-08**$^{+}$	3.63E-01$^{=}$	1.24E-03^{-}	**2.05E-03**$^{+}$	**1.41E-09**$^{+}$	**7.70E-04**$^{+}$	**4.50E-11**$^{+}$
F_{14}		Mean	2.88E-01	5.72E-01	2.82E-01	4.62E-01	2.88E-01	3.40E-01	3.06E-01	3.55E-01	3.90E-01	3.02E-01	3.00E-01	3.18E-01
		Std	2.94E-02	2.21E-01	2.44E-02	1.84E-01	3.41E-02	8.90E-02	6.87E-02	1.66E-01	1.91E-01	4.65E-02	3.39E-02	1.14E-01
		p-value	-	**5.97E-09**$^{+}$	3.18E-01$^{=}$	**2.32E-06**$^{+}$	9.35E-01$^{=}$	**1.32E-04**$^{+}$	3.04E-01$^{=}$	4.46E-01$^{=}$	**2.24E-02**$^{+}$	2.01E-01$^{=}$	1.96E-01$^{=}$	5.11E-01$^{=}$
F_{15}		Mean	7.24E+00	3.02E+01	3.31E+01	1.48E+01	6.35E+00	5.93E+00	8.90E+00	7.52E+00	1.72E+01	1.88E+01	9.11E+00	1.97E+01
		Std	5.91E-01	2.22E+00	2.33E+00	1.05E+01	1.21E+00	1.16E+00	7.51E-01	1.37E+00	1.07E+01	3.64E+00	7.01E-01	2.41E+00
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	7.39E-01$^{=}$	2.62E-03^{-}	1.17E-05^{-}	**5.57E-10**$^{+}$	1.67E-01$^{=}$	1.58E-01$^{=}$	**3.02E-11**$^{+}$	**2.37E-10**$^{+}$	**3.02E-11**$^{+}$
F_{16}		Mean	1.81E+01	2.25E+01	1.99E+01	2.17E+01	2.20E+01	2.20E+01	1.91E+01	2.00E+01	2.13E+01	1.77E+01	1.92E+01	2.02E+01
		Std	3.79E-01	2.83E-01	3.72E-01	3.45E-01	2.43E-01	4.60E-01	2.94E-01	4.28E-01	3.01E-01	4.21E-01	3.69E-01	6.05E-01
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**8.99E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	1.24E-03^{-}	**5.49E-11**$^{+}$	**3.02E-11**$^{+}$
$F_{4\text{-}16}$		*w/t/l*		13/0/0	12/1/0	12/1/0	10/2/1	11/1/1	11/2/0	7/2/4	9/4/0	6/2/5	12/1/0	12/1/0
F_{17}		Mean	2.62E+03	3.46E+06	3.93E+04	2.27E+04	9.85E+03	1.01E+04	2.70E+03	2.06E+03	1.20E+04	3.84E+04	2.53E+03	3.40E+05
		Std	6.60E+02	2.05E+06	1.39E+04	1.56E+04	7.87E+03	7.85E+03	7.59E+02	3.99E+02	1.10E+04	2.20E+04	7.50E+02	2.82E+05
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**1.61E-10**$^{+}$	**1.29E-06**$^{+}$	**3.96E-08**$^{+}$	5.59E-01$^{=}$	5.56E-04^{-}	**9.76E-10**$^{+}$	**3.02E-11**$^{+}$	5.69E-01$^{=}$	**3.02E-11**$^{+}$
18		Mean	1.90E+02	3.06E+03	1.36E+03	2.31E+03	4.83E+01	4.70E+01	1.72E+02	1.65E+03	2.20E+01	4.98E+02	1.57E+02	7.94E+02
		Std	6.68E+01	2.36E+03	8.05E+02	2.26E+03	3.15E+01	3.00E+01	5.04E+01	1.94E+03	1.11E+01	6.30E+02	5.21E+01	2.99E+02
		p-value	-	**1.56E-08**$^{+}$	**3.02E-11**$^{+}$	**1.11E-03**$^{+}$	2.15E-10^{-}	1.61E-10^{-}	3.11E-01$^{=}$	**1.68E-04**$^{+}$	3.02E-11^{-}	7.01E-02$^{=}$	1.44E-02^{-}	**3.34E-11**$^{+}$
F_{19}		Mean	1.10E+01	1.61E+01	3.73E+01	1.71E+01	1.09E+01	6.95E+00	1.70E+01	1.34E+01	6.24E+00	3.75E+01	1.57E+01	1.45E+01
	Hybrid Problems	Std	2.30E+00	3.65E+00	2.84E+01	8.43E+00	3.17E+00	1.87E+00	1.05E+01	3.98E+00	1.98E+00	2.55E+01	9.02E+00	1.39E+00
		p-value	-	**5.53E-08**$^{+}$	**3.02E-11**$^{+}$	**1.17E-03**$^{+}$	8.07E-01$^{=}$	1.87E-07^{-}	**2.61E-02**$^{+}$	**1.17E-04**$^{+}$	2.02E-08^{-}	**7.69E-08**$^{+}$	**1.56E-02**$^{+}$	**7.69E-08**$^{+}$
F_{20}		Mean	1.13E+02	1.53E+03	1.22E+02	1.24E+02	3.80E+01	3.41E+01	1.68E+03	5.15E+01	2.54E+01	7.77E+02	9.01E+01	1.99E+02
		Std	5.02E+01	2.42E+03	1.75E+01	5.66E+01	1.82E+01	2.03E+01	3.67E+03	2.09E+01	2.40E+01	3.07E+03	3.31E+01	4.13E+01
		p-value	-	**3.02E-11**$^{+}$	3.04E-01$^{=}$	4.83E-01$^{=}$	2.92E-09^{-}	1.41E-09^{-}	7.17E-01$^{=}$	3.26E-07^{-}	6.12E-10^{-}	**2.39E-08**$^{+}$	6.57E-02$^{=}$	**3.08E-08**$^{+}$
F_{21}		Mean	1.28E+03	1.01E+06	9.69E+03	4.82E+03	1.04E+03	8.00E+02	1.31E+03	1.20E+03	1.62E+03	1.48E+04	1.19E+03	3.52E+04
		Std	3.24E+02	8.12E+05	5.86E+03	4.96E+03	9.24E+02	4.36E+02	4.46E+02	2.86E+02	1.23E+03	1.41E+04	3.77E+02	2.01E+04
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**2.19E-08**$^{+}$	6.55E-04^{-}	4.42E-06^{-}	9.12E-01$^{=}$	3.55E-01$^{=}$	9.12E-01$^{=}$	**1.78E-10**$^{+}$	2.40E-01$^{=}$	**3.02E-11**$^{+}$
F_{22}		Mean	3.89E+02	7.59E+02	7.90E+02	3.86E+02	8.42E+02	5.66E+02	4.93E+02	4.11E+02	5.84E+02	5.45E+02	4.24E+02	4.88E+02
		Std	1.44E+02	2.48E+02	3.33E+02	3.81E+02	2.10E+02	2.64E+02	1.37E+02	1.65E+02	3.95E+02	1.52E+02	1.17E+02	1.91E+02
		p-value	-	**1.73E-07**$^{+}$	**2.32E-06**$^{+}$	1.15E-01$^{=}$	**1.55E-09**$^{+}$	**3.03E-03**$^{+}$	**3.64E-02**$^{+}$	7.51E-01$^{=}$	1.33E-01$^{=}$	**3.99E-04**$^{+}$	5.20E-01$^{=}$	**2.81E-02**$^{+}$
$F_{17\text{-}22}$		*w/t/l*		6/0/0	5/1/0	4/2/0	2/1/3	2/0/4	2/4/0	2/2/2	1/2/3	5/1/0	1/4/1	6/0/0
F_{23}		Mean	3.44E+02	3.44E+02	3.44E+02	3.44E+02	3.44E+02	3.44E+02	3.44E+02	3.44E+02	3.44E+02	3.44E+02	3.44E+02	3.44E+02
		Std	1.16E-13	1.16E-13	1.16E-13	1.16E-13	1.31E-01	0.00E+00	1.16E-13	1.73E-13	1.16E-13	0.00E+00	1.16E-13	1.16E-13
		p-value	-	NaN$^{=}$	NaN$^{=}$	NaN$^{=}$	**5.36E-13**$^{+}$	1.69E-14^{-}	NaN$^{=}$	**1.69E-14**$^{+}$	NaN$^{=}$	1.69E-14^{-}	NaN$^{=}$	NaN$^{=}$
F_{24}		Mean	2.73E+02	2.73E+02	2.62E+02	2.74E+02	2.69E+02	2.74E+02	2.74E+02	2.66E+02	2.70E+02	2.78E+02	2.74E+02	2.66E+02
		Std	1.82E+00	2.55E+00	7.94E+00	2.82E+00	2.35E+00	3.04E+00	2.10E+00	2.71E+00	2.34E+00	3.08E+00	2.46E+00	3.21E+00
		p-value	-	6.63E-01$^{=}$	1.29E-06^{-}	4.51E-01$^{=}$	6.04E-07^{-}	**2.51E-02**$^{+}$	**9.26E-03**$^{+}$	6.35E-11^{-}	3.32E-06^{-}	**8.48E-09**$^{+}$	5.19E-02$^{=}$	1.76E-10^{-}
F_{25}		Mean	2.09E+02	2.09E+02	2.06E+02	2.07E+02	2.06E+02	2.07E+02	2.14E+02	2.05E+02	2.06E+02	2.28E+02	2.17E+02	2.12E+02
		Std	2.61E+00	2.22E+00	9.02E+00	1.90E+00	1.14E+00	1.27E+00	4.39E+00	4.50E-01	9.31E-01	3.23E+00	4.62E+00	2.34E+00
		p-value	-	3.87E-01$^{=}$	6.68E-03^{-}	1.12E-01$^{=}$	1.78E-04^{-}	1.05E-01$^{=}$	**1.09E-05**$^{+}$	6.72E-10^{-}	2.77E-05^{-}	**3.02E-11**$^{+}$	**2.39E-08**$^{+}$	**7.66E-05**$^{+}$
F_{26}		Mean	1.00E+02	1.00E+02	1.67E+02	1.08E+02	1.00E+02	1.00E+02	1.00E+02	1.07E+02	1.35E+02	1.27E+02	1.00E+02	1.21E+02
		Std	1.37E-01	6.75E-02	4.78E+01	4.35E+01	6.84E-02	5.61E-02	1.62E-01	3.81E+01	7.95E+01	4.47E+01	1.00E-01	5.79E+01
	Composition Problems	p-value	-	2.00E-06^{-}	**2.37E-07**$^{+}$	**5.40E-04**$^{+}$	1.63E-05^{-}	9.79E-05^{-}	3.91E-01$^{=}$	**9.88E-03**$^{+}$	**2.56E-02**$^{+}$	**3.08E-08**$^{+}$	5.10E-02$^{=}$	**5.38E-07**$^{+}$
F_{27}		Mean	3.84E+02	7.11E+02	9.37E+02	5.98E+02	1.50E+03	4.90E+02	4.12E+02	6.11E+02	4.66E+02	7.40E+02	4.24E+02	5.19E+02
		Std	4.29E+01	8.14E+01	1.01E+02	1.04E+02	3.85E+02	5.77E+01	5.22E+01	1.13E+02	8.71E+01	9.79E+01	5.36E+01	1.27E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**1.33E-10**$^{+}$	**3.02E-11**$^{+}$	**2.92E-09**$^{+}$	**3.67E-03**$^{+}$	**9.92E-11**$^{+}$	**9.21E-05**$^{+}$	**3.02E-11**$^{+}$	**3.99E-04**$^{+}$	**2.39E-08**$^{+}$
F_{28}		Mean	1.13E+03	1.17E+03	1.70E+03	1.23E+03	8.42E+02	1.43E+03	1.13E+03	1.12E+03	1.13E+03	1.61E+03	1.12E+03	1.12E+03
		Std	5.16E+01	4.18E+01	2.92E+02	8.66E+01	6.49E+01	1.12E+02	6.10E+01	4.43E+01	4.13E+01	1.86E+02	5.92E+01	3.28E+01
		p-value	-	**1.11E-03**$^{+}$	**3.02E-11**$^{+}$	**8.84E-07**$^{+}$	3.02E-11^{-}	**4.08E-11**$^{+}$	9.71E-01$^{=}$	5.69E-01$^{=}$	9.23E-01$^{=}$	**3.02E-11**$^{+}$	5.11E-01$^{=}$	6.84E-01$^{=}$
F_{29}		Mean	8.82E+02	3.53E+07	1.48E+03	8.58E+06	2.46E+02	9.00E+02	9.06E+02	1.06E+07	2.68E+07	1.09E+03	8.97E+02	1.29E+07
		Std	6.72E+01	1.87E+05	3.05E+02	1.58E+07	7.44E+00	2.29E+02	8.93E+01	1.65E+07	1.65E+07	2.27E+02	7.38E+01	1.73E+07
		p-value	-	**9.33E-12**$^{+}$	**3.02E-11**$^{+}$	**2.38E-03**$^{+}$	3.02E-11^{-}	2.23E-01$^{=}$	1.12E-01$^{=}$	6.52E-01$^{=}$	**2.43E-09**$^{+}$	**8.15E-05**$^{+}$	3.33E-01$^{=}$	**2.80E-11**$^{+}$
F_{30}		Mean	8.73E+03	1.16E+04	1.13E+04	9.10E+03	1.76E+03	9.99E+03	9.60E+03	8.22E+03	8.40E+03	1.09E+04	9.16E+03	8.73E+03
		Std	4.67E+02	6.17E+03	8.57E+02	8.16E+02	1.72E+02	4.53E+02	9.91E+02	3.26E+02	4.07E+02	1.21E+03	4.98E+02	3.83E+02
		p-value	-	**3.15E-02**$^{+}$	**3.34E-11**$^{+}$	1.12E-01$^{=}$	3.02E-11^{-}	**2.15E-10**$^{+}$	**7.20E-05**$^{+}$	3.37E-05^{-}	6.97E-03^{-}	**4.98E-11**$^{+}$	**2.50E-03**$^{+}$	9.12E-01$^{=}$
$F_{23\text{-}30}$		*w/t/l*		4/3/1	5/1/2	4/4/0	2/0/6	4/2/2	4/4/0	3/2/3	3/2/3	7/0/1	3/5/0	4/3/1
w/t/l				26/3/1	25/3/2	23/7/0	16/4/10	20/3/7	19/11/0	15/6/9	16/8/6	21/3/6	17/10/3	25/4/1
Rank			**3.20**	9.55	8.45	7.75	5.83	6.63	5.75	5.30	5.98	7.15	4.51	7.88

TABLE 8. Optimization performance comparisons between GSGDE and the 11 state-of-the-art DE variants on the 100*D* CEC2014 suite.

F	Category	Quality	GSGDE	GPDE	ITGDE	DMCDE	SEDE	FADE	FDDE	TPDE	CUSDE	NSHADE	PFIDE	SDEGCM
F_1	Unimodal Problems	Mean	1.25E+05	2.00E+07	1.09E+07	3.09E+06	1.01E+06	1.03E+06	3.39E+05	2.70E+06	7.58E+06	1.03E+06	3.84E+05	2.21E+07
		Std	6.13E+04	1.29E+07	1.86E+06	2.84E+06	3.23E+05	2.75E+05	1.11E+05	7.08E+05	2.28E+06	2.74E+05	1.32E+05	6.57E+06
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.50E-09**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**6.12E-10**$^{+}$	**3.02E-11**$^{+}$
F_2		Mean	1.78E-10	2.94E+04	1.37E+04	8.18E-04	1.33E-01	1.30E+04	5.14E-10	6.37E+03	1.45E+03	5.64E+03	2.10E-11	3.15E+01
		Std	4.94E-10	3.38E+04	6.05E+03	3.93E-03	2.81E-01	1.46E+04	1.25E-09	8.97E+03	2.39E+03	9.86E+03	5.95E-11	6.62E+01
		p-value	-	**2.99E-11**$^{+}$	**2.99E-11**$^{+}$	**4.46E-11**$^{+}$	**2.99E-11**$^{+}$	**2.99E-11**$^{+}$	1.17E-01$^{=}$	**2.99E-11**$^{+}$	**2.99E-11**$^{+}$	**2.99E-11**$^{+}$	2.07E-04^{-}	**2.99E-11**$^{+}$
F_3		Mean	1.10E-03	1.47E+04	4.51E+01	4.19E-03	3.07E+02	1.11E+03	1.85E+03	1.57E-01	2.22E+03	4.23E+03	7.17E-09	3.32E+02
		Std	4.06E-03	6.22E+03	2.13E+01	1.11E-02	3.35E+02	8.25E+02	2.29E+03	8.99E-02	2.11E+03	5.36E+03	3.79E-08	4.69E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**4.36E-02**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	9.35E-01$^{=}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	3.02E-11^{-}	**3.02E-11**$^{+}$
$F_{1\text{-}3}$		*w/t/l*		3/0/0	3/0/0	3/0/0	3/0/0	3/0/0	1/2/0	3/0/0	3/0/0	3/0/0	1/0/2	3/0/0
F_4	Simple Multimodal Problems	Mean	1.15E+02	2.08E+02	2.77E+02	1.79E+02	1.57E+02	2.07E+02	1.59E+02	1.83E+02	1.80E+02	1.97E+02	1.08E+02	1.84E+02
		Std	5.08E+01	3.42E+01	5.25E+01	2.91E+01	3.28E+01	3.24E+01	5.33E+01	3.12E+01	3.24E+01	5.25E+01	5.73E+01	2.96E+01
		p-value	-	**1.41E-09**$^{+}$	**2.37E-10**$^{+}$	**4.79E-07**$^{+}$	**5.26E-04**$^{+}$	**1.69E-09**$^{+}$	**4.22E-03**$^{+}$	**2.51E-07**$^{+}$	**6.03E-07**$^{+}$	**6.04E-07**$^{+}$	7.79E-01$^{=}$	**2.19E-07**$^{+}$
F_5		Mean	2.05E+01	2.13E+01	2.09E+01	2.13E+01	2.12E+01	2.13E+01	2.08E+01	2.04E+01	2.13E+01	2.02E+01	2.08E+01	2.08E+01
		Std	3.26E-02	2.23E-02	2.94E-02	2.12E-02	2.28E-02	2.20E-02	3.29E-02	5.66E-02	2.74E-02	2.34E-02	1.93E-02	7.67E-02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	5.46E-09^{-}	**3.02E-11**$^{+}$	3.02E-11^{-}	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_6		Mean	2.07E+01	8.32E+01	8.60E+01	4.31E+01	5.35E+01	4.35E+01	2.84E+01	6.51E+01	2.53E+01	6.63E+01	2.98E+01	4.28E+01
		Std	4.00E+00	2.33E+01	4.43E+00	9.22E+00	1.93E+01	5.34E+00	8.01E+00	1.03E+01	8.27E+00	4.89E+00	3.35E+00	1.06E+01
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**6.07E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.32E-06**$^{+}$	**3.02E-11**$^{+}$	**6.67E-03**$^{+}$	**3.02E-11**$^{+}$	**5.07E-10**$^{+}$	**4.50E-11**$^{+}$
F_7		Mean	1.23E-03	7.51E-04	7.11E-08	1.01E-02	6.57E-04	4.35E-03	3.78E-03	1.44E-13	4.93E-04	5.24E-03	5.75E-04	2.62E-03
		Std	3.28E-03	2.83E-03	6.56E-08	2.71E-02	2.50E-03	8.87E-03	6.83E-03	5.11E-14	1.88E-03	1.06E-02	2.21E-03	9.19E-03
		p-value	-	5.34E-07^{-}	8.47E-07^{-}	**4.88E-08**$^{+}$	5.24E-10^{-}	**1.78E-04**$^{+}$	**2.22E-06**$^{+}$	5.14E-12^{-}	3.14E-05^{-}	**9.80E-03**$^{+}$	6.78E-01$^{=}$	**1.13E-06**$^{+}$
F_8		Mean	1.14E-13	9.92E+01	4.27E+02	7.00E+01	9.35E+00	1.65E+02	6.23E+01	1.54E+02	1.19E+02	0.00E+00	5.96E+01	6.68E+01
		Std	0.00E+00	1.50E+01	5.75E+01	1.04E+01	2.66E+00	1.69E+01	3.32E+00	3.84E+01	8.61E+01	0.00E+00	3.12E+00	1.76E+01
		p-value	-	**1.21E-12**$^{+}$	**1.21E-12**$^{+}$	**1.21E-12**$^{+}$	**1.16E-12**$^{+}$	**1.21E-12**$^{+}$	**1.21E-12**$^{+}$	**1.21E-12**$^{+}$	**1.21E-12**$^{+}$	1.69E-14^{-}	**1.21E-12**$^{+}$	**1.21E-12**$^{+}$
F_9		Mean	1.02E+02	4.34E+02	5.22E+02	1.36E+02	1.82E+02	1.75E+02	1.67E+02	2.26E+02	8.17E+02	2.93E+02	1.76E+02	2.70E+02
		Std	1.23E+01	2.72E+02	4.55E+01	1.79E+01	3.07E+01	2.72E+01	1.56E+01	2.94E+01	1.84E+01	2.54E+01	1.35E+01	1.11E+02
		p-value	-	**3.02E-11**$^{+}$	**3.01E-11**$^{+}$	**4.18E-09**$^{+}$	**3.02E-11**$^{+}$	**6.07E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_{10}		Mean	9.81E-01	3.35E+03	1.23E+03	5.84E+03	1.56E+04	6.86E+03	3.05E+03	5.57E+03	1.74E+04	3.61E+00	3.88E+03	1.94E+03
		Std	3.45E-01	7.59E+02	2.73E+02	1.09E+03	4.47E+02	9.51E+02	2.65E+02	1.68E+03	3.60E+03	6.63E-01	2.80E+02	4.84E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.34E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_{11}		Mean	1.11E+04	3.01E+04	1.22E+04	2.83E+04	2.84E+04	1.22E+04	1.43E+04	1.23E+04	3.00E+04	1.01E+04	1.48E+04	1.89E+04
		Std	6.81E+02	6.58E+02	2.89E+03	4.88E+03	5.93E+02	3.26E+03	6.32E+02	8.61E+02	6.17E+02	5.30E+02	5.59E+02	1.24E+03
		p-value	-	**3.02E-11**$^{+}$	4.29E-01$^{=}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**8.68E-03**$^{+}$	**3.02E-11**$^{+}$	**1.19E-06**$^{+}$	**3.02E-11**$^{+}$	3.32E-06^{-}	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_{12}		Mean	3.81E-01	3.32E+00	1.45E+00	4.08E+00	2.67E+00	3.97E+00	7.88E-01	3.98E-01	4.05E+00	2.27E-01	8.20E-01	1.55E+00
		Std	3.49E-02	3.71E-01	1.23E-01	2.42E-01	1.87E-01	2.28E-01	5.37E-02	4.01E-02	2.95E-01	1.89E-02	5.59E-02	2.66E-01
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	1.09E-01$^{=}$	**3.02E-11**$^{+}$	3.02E-11^{-}	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_{13}		Mean	3.70E-01	5.36E-01	4.80E-01	4.50E-01	4.71E-01	5.42E-01	3.99E-01	3.48E-01	6.14E-01	4.58E-01	3.91E-01	5.39E-01
		Std	3.32E-02	4.12E-02	2.93E-02	5.95E-02	7.32E-02	6.22E-02	4.33E-02	4.35E-02	5.32E-02	5.78E-02	4.38E-02	5.67E-02
		p-value	-	**3.02E-11**$^{+}$	**7.39E-11**$^{+}$	**2.38E-07**$^{+}$	**7.09E-08**$^{+}$	**4.08E-11**$^{+}$	**6.67E-03**$^{+}$	4.51E-02^{-}	**3.02E-11**$^{+}$	**7.09E-08**$^{+}$	7.01E-02$^{=}$	**3.69E-11**$^{+}$
F_{14}		Mean	3.18E-01	5.60E-01	2.93E-01	5.04E-01	2.93E-01	3.50E-01	3.25E-01	4.31E-01	4.31E-01	3.04E-01	3.24E-01	3.80E-01
		Std	2.71E-02	2.38E-01	1.23E-02	2.06E-01	2.01E-02	9.00E-02	2.56E-02	1.68E-01	1.95E-01	3.09E-02	2.27E-02	1.47E-01
		p-value	-	**7.12E-09**$^{+}$	4.64E-05^{-}	**9.26E-09**$^{+}$	1.89E-04^{-}	**9.07E-03**$^{+}$	3.63E-01$^{=}$	**4.86E-03**$^{+}$	**2.60E-05**$^{+}$	3.15E-02^{-}	2.84E-01$^{=}$	**1.03E-02**$^{+}$
F_{15}		Mean	2.12E+01	7.30E+01	7.72E+01	1.84E+01	1.60E+01	1.87E+01	3.30E+01	2.17E+01	7.31E+01	7.20E+01	3.15E+01	5.58E+01
		Std	1.97E+00	3.91E+00	5.28E+00	6.17E+00	3.27E+00	3.67E+00	4.11E+00	4.17E+00	2.24E+00	1.78E+01	3.33E+00	9.61E+00
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	4.71E-04^{-}	9.83E-08^{-}	1.24E-03^{-}	**1.21E-10**$^{+}$	2.58E-01$^{=}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$
F_{16}		Mean	4.03E+01	4.72E+01	4.27E+01	4.60E+01	4.66E+01	4.64E+01	4.22E+01	4.33E+01	4.67E+01	4.01E+01	4.26E+01	4.36E+01
		Std	4.90E-01	2.21E-01	6.17E-01	6.42E-01	2.07E-01	2.49E-01	5.13E-01	4.81E-01	2.56E-01	5.54E-01	4.35E-01	8.34E-01
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.34E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	5.37E-02$^{=}$	**3.02E-11**$^{+}$	**4.98E-11**$^{+}$
$F_{4\text{-}16}$		*w/t/l*		12/0/1	10/1/2	12/0/1	10/0/3	12/0/1	12/1/0	8/2/3	12/0/1	7/1/5	9/4/0	13/0/0
F_{17}	Hybrid Problems	Mean	9.36E+03	1.71E+07	7.65E+05	3.55E+05	1.68E+05	1.30E+05	4.14E+04	8.71E+04	6.64E+05	3.38E+05	1.73E+04	4.02E+06
		Std	4.17E+03	1.03E+07	2.32E+05	3.54E+05	8.82E+04	5.01E+04	3.83E+04	4.49E+04	3.73E+05	2.01E+05	1.22E+04	2.33E+06
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.65E-08**$^{+}$	**3.34E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**4.98E-04**$^{+}$	**3.02E-11**$^{+}$
18		Mean	1.68E+03	4.39E+03	7.74E+02	2.72E+03	2.42E+03	8.09E+02	1.48E+03	1.85E+03	4.52E+03	1.28E+03	1.28E+03	2.64E+03
		Std	1.70E+03	4.19E+03	8.10E+02	2.94E+03	2.98E+03	8.07E+02	1.18E+03	1.78E+03	4.50E+03	9.91E+02	1.23E+03	3.51E+03
		p-value	-	**2.05E-03**$^{+}$	4.23E-03^{-}	1.81E-01$^{=}$	7.28E-01$^{=}$	3.03E-03^{-}	7.84E-01$^{=}$	6.84E-01$^{=}$	**1.56E-02**$^{+}$	7.51E-01$^{=}$	2.58E-01$^{=}$	3.87E-01$^{=}$
F_{19}		Mean	9.53E+01	1.01E+02	1.08E+02	9.75E+01	9.32E+01	8.03E+01	9.73E+01	9.44E+01	9.47E+01	1.08E+02	9.62E+01	1.01E+02
		Std	2.54E+00	3.54E+00	1.77E+01	3.01E+00	1.97E+00	2.01E+01	8.72E+00	1.72E+00	2.01E+00	3.29E+01	7.69E+00	6.59E+00
		p-value	-	**1.07E-07**$^{+}$	**1.11E-06**$^{+}$	**4.23E-03**$^{+}$	1.95E-03^{-}	4.74E-06^{-}	**3.18E-03**$^{+}$	1.62E-01$^{=}$	4.38E-01$^{=}$	2.12E-01$^{=}$	9.33E-02$^{=}$	**3.32E-06**$^{+}$
F_{20}		Mean	5.95E+02	2.14E+04	4.23E+02	7.24E+02	6.29E+02	2.57E+03	2.26E+03	3.47E+02	1.14E+03	4.71E+03	4.69E+02	1.91E+03
		Std	1.54E+02	7.37E+03	9.18E+01	1.59E+02	2.79E+02	1.14E+03	5.44E+03	9.36E+01	3.79E+02	5.63E+03	1.00E+02	1.58E+03
		p-value	-	**3.02E-11**$^{+}$	5.86E-06^{-}	**2.38E-03**$^{+}$	8.65E-01$^{=}$	**7.38E-10**$^{+}$	1.12E-01$^{=}$	8.48E-09^{-}	**5.97E-09**$^{+}$	**8.15E-11**$^{+}$	1.11E-03^{-}	**8.99E-11**$^{+}$
F_{21}		Mean	3.14E+03	6.89E+06	3.28E+05	1.43E+05	6.34E+04	7.69E+04	8.09E+03	3.16E+04	1.36E+05	2.13E+05	3.70E+03	1.25E+06
		Std	7.34E+02	3.84E+06	1.74E+05	1.16E+05	2.48E+04	3.01E+04	1.27E+04	1.73E+04	6.99E+04	8.76E+04	1.32E+03	6.21E+05
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.52E-07**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	1.37E-01$^{=}$	**3.02E-11**$^{+}$
F_{22}		Mean	1.47E+03	3.65E+03	2.19E+03	2.75E+03	1.86E+03	1.73E+03	1.44E+03	1.62E+03	3.98E+03	1.49E+03	1.35E+03	1.81E+03
		Std	2.13E+02	3.82E+02	5.13E+02	1.13E+03	3.12E+02	5.25E+02	3.03E+02	2.89E+02	4.14E+02	2.27E+02	2.61E+02	4.43E+02
		p-value	-	**3.02E-11**$^{+}$	**1.07E-07**$^{+}$	**3.77E-04**$^{+}$	**2.78E-07**$^{+}$	5.75E-02$^{=}$	8.53E-01$^{=}$	**9.47E-03**$^{+}$	**3.02E-11**$^{+}$	7.39E-01$^{=}$	1.12E-01$^{=}$	**1.95E-03**$^{+}$
$F_{17\text{-}22}$		*w/t/l*		6/0/0	4/0/2	5/1/0	3/2/1	3/1/2	3/3/0	3/2/1	5/1/0	3/3/0	1/4/1	5/1/0
F_{23}	Composition Problems	Mean	3.48E+02	3.48E+02	3.43E+02	3.48E+02	3.48E+02	3.48E+02	3.48E+02	3.48E+02	3.48E+02	3.48E+02	3.48E+02	3.48E+02
		Std	0.00E+00	1.49E-01	2.71E+01	0.00E+00	5.95E-04	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
		p-value	-	3.34E-01$^{=}$	3.34E-01$^{=}$	NaN$^{=}$	**2.71E-14**$^{+}$	NaN$^{=}$	NaN$^{=}$	NaN$^{=}$	NaN$^{=}$	NaN$^{=}$	NaN$^{=}$	NaN$^{=}$
F_{24}		Mean	3.91E+02	3.93E+02	3.60E+02	3.99E+02	3.77E+02	3.93E+02	4.00E+02	3.69E+02	3.87E+02	4.12E+02	3.94E+02	3.82E+02
		Std	3.33E+00	6.99E+00	2.38E+01	7.34E+00	4.14E+00	5.18E+00	5.98E+00	3.05E+00	4.02E+00	8.48E+00	3.63E+00	5.49E+00
		p-value	-	6.20E-01$^{=}$	2.37E-10^{-}	**2.00E-05**$^{+}$	4.98E-11^{-}	2.40E-01$^{=}$	**6.01E-08**$^{+}$	3.02E-11^{-}	1.53E-05^{-}	**1.61E-10**$^{+}$	**1.38E-02**$^{+}$	1.31E-08^{-}
F_{25}		Mean	2.63E+02	2.38E+02	2.08E+02	2.32E+02	2.28E+02	2.53E+02	2.71E+02	2.24E+02	2.22E+02	2.73E+02	2.54E+02	2.37E+02
		Std	5.29E+00	9.60E+00	1.93E+01	8.11E+00	2.63E+01	1.29E+01	7.98E+00	3.29E+00	2.43E+00	7.65E+00	7.30E+00	1.01E+01
		p-value	-	1.61E-10^{-}	2.57E-11^{-}	3.34E-11^{-}	2.22E-09^{-}	3.37E-05^{-}	**1.43E-05**$^{+}$	3.02E-11^{-}	3.02E-11^{-}	**1.39E-06**$^{+}$	1.53E-05^{-}	9.76E-10^{-}
F_{26}		Mean	2.00E+02	2.43E+02	2.00E+02	2.11E+02	1.00E+02	2.00E+02	2.00E+02	2.09E+02	2.00E+02	2.00E+02	2.00E+02	2.01E+02
		Std	3.94E-03	1.47E+02	3.43E-02	1.14E+02	7.01E-02	1.66E-02	7.15E-03	1.38E+02	1.01E+02	2.16E-02	7.93E-03	5.51E-01
		p-value	-	6.62E-01$^{=}$	**2.88E-11**$^{+}$	**2.69E-02**$^{+}$	2.89E-11^{-}	**2.89E-11**$^{+}$	**2.02E-09**$^{+}$	1.00E+00$^{=}$	3.79E-01$^{=}$	**2.89E-11**$^{+}$	**4.32E-05**$^{+}$	**2.89E-11**$^{+}$
F_{27}		Mean	6.64E+02	1.27E+03	2.07E+03	1.30E+03	3.51E+03	1.18E+03	9.96E+02	1.43E+03	6.28E+02	1.62E+03	8.49E+02	1.20E+03
		Std	8.18E+01	1.77E+02	1.81E+02	1.46E+02	8.23E+02	1.19E+02	8.46E+01	4.38E+02	1.42E+02	1.16E+02	9.17E+01	1.75E+02
		p-value	-	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**3.02E-11**$^{+}$	**4.50E-11**$^{+}$	**3.69E-11**$^{+}$	1.05E-01$^{=}$	**3.02E-11**$^{+}$	**4.57E-09**$^{+}$	**3.02E-11**$^{+}$
F_{28}		Mean	2.19E+03	2.27E+03	5.02E+03	2.69E+03	2.05E+03	3.30E+03	2.36E+03	2.23E+03	2.19E+03	4.48E+03	2.24E+03	2.94E+03
		Std	8.78E+01	1.35E+02	1.01E+03	4.72E+02	1.26E+02	3.85E+02	2.61E+02	4.28E+01	2.31E+02	6.67E+02	1.56E+02	5.90E+02
		p-value	-	**1.77E-03**$^{+}$	**3.02E-11**$^{+}$	**5.07E-10**$^{+}$	4.42E-06^{-}	**3.02E-11**$^{+}$	**1.37E-03**$^{+}$	**3.50E-03**$^{+}$	4.55E-01$^{=}$	**3.02E-11**$^{+}$	1.54E-01$^{=}$	**2.44E-09**$^{+}$
F_{29}		Mean	1.16E+03	8.34E+07	1.80E+03	1.46E+07	3.28E+02	1.62E+03	1.34E+03	2.77E+06	3.53E+07	1.53E+03	1.28E+03	2.38E+07
		Std	3.10E+02	3.59E+05	3.31E+02	3.32E+07	5.59E+01	2.27E+02	2.29E+02	1.52E+07	4.41E+07	2.37E+02	2.68E+02	4.05E+07
		p-value	-	**1.10E-11**$^{+}$	**2.19E-08**$^{+}$	**9.06E-08**$^{+}$	3.02E-11^{-}	**9.53E-07**$^{+}$	**2.61E-02**$^{+}$	**5.07E-10**$^{+}$	**3.02E-11**$^{+}$	**2.77E-05**$^{+}$	1.86E-01$^{=}$	**3.02E-11**$^{+}$
F_{30}		Mean	8.69E+03	1.63E+04	6.99E+03	1.10E+04	4.90E+03	6.44E+03	9.23E+03	8.65E+03	7.34E+03	1.18E+04	8.94E+03	9.38E+03
		Std	9.75E+02	1.60E+04	1.34E+03	3.38E+03	3.19E+02	1.53E+03	1.14E+03	6.97E+02	1.05E+03	2.44E+03	8.61E+02	8.92E+02
		p-value	-	**1.73E-07**$^{+}$	3.32E-06^{-}	**2.57E-07**$^{+}$	3.02E-11^{-}	1.36E-07^{-}	**3.39E-02**$^{+}$	8.88E-01$^{=}$	6.28E-06^{-}	**9.83E-08**$^{+}$	2.64E-01$^{=}$	**9.47E-03**$^{+}$
$F_{23\text{-}30}$		*w/t/l*		4/3/1	4/1/3	6/1/1	2/0/6	4/2/2	7/1/0	3/3/2	1/4/3	7/1/0	3/4/1	5/1/2
w/t/l				25/3/2	21/2/7	26/2/2	18/2/10	22/3/5	23/7/0	17/7/6	21/5/4	20/5/5	14/12/4	26/2/2
Rank			**3.28**	9.98	6.85	7.71	5.35	7.00	5.46	5.51	7.81	6.91	4.35	7.75

inferior performance on at most 1 problem. When the dimension size is 50, GSGDE achieves equivalent performance with SEDE, FADE, FDDE, TPDE, CUSDE, and PFIDE, but presents great dominance to the rest 5 compared approaches on at least 4 problems. When the dimension size is 100, excluding PFIDE, GSGDE shows better performance than the rest 10 compared approaches on more than 3 problems and exhibits inferior performance to them on at most 2 problems. These discoveries demonstrate that GSGDE could solve the hybrid problems effectively and its dominance to most of the 11 compared algorithms becomes more and more significant as the dimensionality increases. (d) On the 8 composition problems (F_{23}-F_{30}), GSGDE shows much better performance than GPDE, ITGDE, DMCDE, FDDE, NSHADE, and SDEGCM on more than 5 problems when the dimensionality is 30. When it comes to the 50D composition problems, GSGDE performs significantly better than ITGDE and NSHADE on at least 5 problems. Confronted with the 100D composition problems, GSGDE significantly outperforms DMCDE, FDDE, NSHADE, and SDEGCM on at least 5 problems. These discoveries prove that GSGDE is effective in coping with the complicated composition problems.

In summary, the above experimental results have demonstrated that with the dimension size growing, GSGDE shows increasingly superior performance to most of the 11 compared DE approaches in coping with the CEC2014 problems. This further demonstrates that GSGDE has a good scalability in coping with optimization problems.

D. DEEP INVESTIGATION ON GSGDE

This subsection carries out experiments on the 50D CEC2017 problem set to observe the effect of the two major techniques on the optimization performance of GSGDE.

1) EFFECTIVENESS OF THE DEVISED "DE/CURRENT-TO-GSELITE/1"

This subsection performs experiments extensively to testify the effectiveness of the devised mutation strategy "DE/current-to-gselite/1". For this purpose, we develop different variants of GSGDE to make comparisons.

First, since "DE/current-to-gselite/1" is a little similar with "DE/current-to-pbest/1" [21] and "DE/current-to-best/1" [64], we directly replace the devised mutation scheme with the two classical ones and thus two variants of GSGDE are developed, which are denoted as "GSGDE-pbest" and "GSGDE-best", respectively.

Second, in the devised mutation strategy, for each individual, we select an elite randomly from the elite group as the mean vector of the Gaussian distribution model. To demonstrate the effectiveness of this scheme, we utilize the mean position of all elites in the elite group as the mean of the Gaussian distribution to sample guiding exemplars. In particular, the mean of all elites is computed as follows:

$$\boldsymbol{\mu}_1 = \frac{1}{NEI}\sum_{i=1}^{NEI}\boldsymbol{x_i} \tag{18}$$

where $\boldsymbol{\mu}_1$ denotes the mean of all elites, $\boldsymbol{x_i}$ is the ith elite in the elite group, and NEI is the number of elite individuals. Replacing the randomly selected elite in Eq. (14) with the computed $\boldsymbol{\mu}_1$, we develop another variant of GSGDE, which is denoted as "GSGDE-mean".

Third, in the devised mutation strategy, we utilize the Manhattan distance to calculate the standard deviation of the Gaussian distribution as shown in Eq. (13). To demonstrate the effectiveness of this scheme, we utilize another two popular distance measures to compute the standard deviation of the Gaussian distribution, namely the Euclidean distance and the Chebyshev distance. Specifically, the Euclidean distance based standard deviation is computed as follows:

$$\sigma_{1,d} = \varepsilon^* \sqrt{\frac{1}{NEI-1}\sum_{i=1}^{NEI}(x_{i,d} - x_{randE,d})^2} \tag{19}$$

where $\boldsymbol{x_{randE}}$ is the associated elite which is chosen from the elite group randomly and is the same as the one in Eq. (14). $\boldsymbol{x_i}$ is the ith elite in the elite group, d represents one of the dimensions, NEI is the number of elite individuals, and ε is the same as the one in Eq. (13) and also randomly sampled from [1.0E-04, 1.0E-03].

The Chebyshev distance based standard deviation is calculated as follows:

$$\sigma_{2,d} = \varepsilon^* \max_{i,j\in[1,NEI]} \left|x_{i,d} - x_{j,d}\right| \tag{20}$$

where $\boldsymbol{x_i}$ and $\boldsymbol{x_j}$ are the ith and the jth elites in the elite group, respectively. d represents one of the dimensions. ε is the same as the one in Eq. (13) and also randomly sampled from [1.0E-04, 1.0E-03].

By replacing the standard deviation of the Gaussian distribution in GSGDE with these two new ones, another two variants of GSGDE are developed, which we denote as "GSGDE-Eul" and "GSGDE-Che", respectively.

Fourth, in the devised mutation strategy, for the scaling parameter ε as shown in Eq. (13), we randomly sample a value for it from [1.0E-04, 1.0E-03] for diversity maintenance. To demonstrate the effectiveness of this scheme, we develop different variants of GSGDE by using different fixed values of ε from 1.0E-05 to 1.0E-01.

Fifth, in the devised mutation strategy as shown in Eq. (15), to generate the random difference vector, we randomly choose $\hat{x}_{\boldsymbol{r2}}$ from $\boldsymbol{P}\cup\boldsymbol{A}$ and randomly select $\boldsymbol{x_{r1}}$ from the population $\boldsymbol{P}$. To demonstrate the effectiveness of this scheme, we develop another two variants of GSGDE. In the first variant, both $\boldsymbol{x_{r1}}$ and $\hat{x}_{\boldsymbol{r2}}$ are stochastically chosen from the population $\boldsymbol{P}$. That is, the archive $\boldsymbol{A}$ is removed in this version. We denote it as "GSGDE-WA". In the second variant, both $\boldsymbol{x_{r1}}$ and $\hat{x}_{\boldsymbol{r2}}$ are randomly selected from $\boldsymbol{P}\cup\boldsymbol{A}$. This version is represented as "GSGDE-2PA"

TABLE 9. Statistical comparison results between GSGDE and the 11 compared DE algorithms concerning "$w/t/l$" and the average rank on the CEC2014 set with the three different dimensionality settings.

Problem Set	Problem Property	Index	GSGDE	GPDE	ITGDE	DMCDE	SEDE	FADE	FDDE	TPDE	CUSDE	NSHADE	PFIDE	SDEGCM
CEC2014-30*D*	Unimodal Problems	*w/t/l*	-	3/0/0	3/0/0	2/1/0	1/2/0	1/2/0	3/0/0	1/2/0	3/0/0	1/2/0	1/2/0	3/0/0
	Simple Multimodal Problems		-	12/1/0	12/0/1	13/0/0	10/3/0	10/3/0	10/3/0	8/3/2	9/3/1	7/0/6	10/3/0	12/1/0
	Hybrid Problems		-	6/0/0	6/0/0	5/0/1	2/2/2	1/2/3	6/0/0	3/1/2	1/2/3	5/1/0	1/5/0	6/0/0
	Composition Problems		-	7/1/0	6/1/1	6/2/0	2/3/3	3/3/2	5/3/0	4/2/2	3/3/2	5/3/0	0/7/1	5/3/0
	Overall		-	28/2/0	27/1/2	26/3/1	15/10/5	15/10/5	24/6/0	16/8/6	16/8/6	18/6/6	12/17/1	26/4/0
	Overall	*Rank*	**3.20**	9.86	8.95	8.80	5.68	5.50	5.90	5.50	5.13	6.26	4.40	8.80
CEC2014-50*D*	Unimodal Problems	*w/t/l*	-	3/0/0	3/0/0	3/0/0	2/1/0	3/0/0	2/1/0	3/0/0	3/0/0	3/0/0	1/0/2	3/0/0
	Simple Multimodal Problems		-	13/0/0	12/1/0	12/1/0	10/2/1	11/1/1	11/2/0	7/2/4	9/4/0	6/2/5	12/1/0	12/1/0
	Hybrid Problems		-	6/0/0	5/1/0	4/2/0	2/1/3	2/0/4	2/4/0	2/2/2	1/2/3	5/1/0	1/4/1	6/0/0
	Composition Problems		-	4/3/1	5/1/2	4/4/0	2/0/6	4/2/2	4/4/0	3/2/3	3/2/3	7/0/1	3/5/0	4/3/1
	Overall		-	26/3/1	25/3/2	23/7/0	16/4/10	20/3/7	19/11/0	15/6/9	16/8/6	21/3/6	17/10/3	25/4/1
	Overall	*Rank*	**3.20**	9.55	8.45	7.75	5.83	6.63	5.75	5.30	5.98	7.15	4.51	7.88
CEC2014-100*D*	Unimodal Problems	*w/t/l*	-	3/0/0	3/0/0	3/0/0	3/0/0	3/0/0	1/2/0	3/0/0	3/0/0	3/0/0	1/0/2	3/0/0
	Simple Multimodal Problems		-	12/0/1	10/1/2	12/0/1	10/0/3	12/0/1	12/1/0	8/2/3	12/0/1	7/1/5	9/4/0	13/0/0
	Hybrid Problems		-	6/0/0	4/0/2	5/1/0	3/2/1	3/1/2	3/3/0	3/2/1	5/1/0	3/3/0	1/4/1	5/1/0
	Composition Problems		-	4/3/1	4/1/3	6/1/1	2/0/6	4/2/2	7/1/0	3/3/2	1/4/3	7/1/0	3/4/1	5/1/2
	Overall		-	25/3/2	21/2/7	26/2/2	18/2/10	22/3/5	23/7/0	17/7/6	21/5/4	20/5/5	14/12/4	26/2/2
	Overall	*Rank*	**3.28**	9.98	6.85	7.71	5.35	7.00	5.46	5.51	7.81	6.91	4.35	7.75

TABLE 10. Comparison results between the devised GSGDE and its variants on the 50*D* CEC2017 set.

F	GSGDE	GSGDE-pbest	GSGDE-best	GSGDE-mean	GSGDE	GSGDE-Eul	GSGDE-Che	GSGDE	ε=1E-05	ε=1E-04	ε=1E-03	ε=1E-02	ε=1E-01	GSGDE	GSGDE-WA	GSGDE-2PA
F_1	**3.08E-14**	3.13E-14	4.22E-14	3.13E-14	**3.08E-14**	3.27E-14	3.27E-14	3.08E-14	3.27E-14	**2.79E-14**	2.94E-14	5.49E-14	1.30E+03	**3.08E-14**	5.31E-14	3.17E-14
F_3	2.73E-13	2.39E-13	3.11E-13	**2.37E-13**	2.73E-13	2.33E-13	**2.29E-13**	2.73E-13	2.56E-13	2.44E-13	2.48E-13	**2.35E-13**	2.01E-09	2.73E-13	3.58E-13	**2.35E-13**
F_4	5.01E+01	4.30E+01	4.78E+01	**2.42E+01**	5.01E+01	**3.79E+01**	4.09E+01	5.01E+01	4.70E+01	**4.44E+01**	4.64E+01	6.63E+01	5.67E+01	5.01E+01	**3.83E+01**	4.40E+01
F_5	**3.97E+01**	4.45E+01	6.62E+01	4.33E+01	**3.97E+01**	4.52E+01	4.51E+01	**3.97E+01**	4.69E+01	4.28E+01	4.56E+01	4.67E+01	4.52E+01	**3.97E+01**	4.64E+01	4.51E+01
F_6	1.63E-07	**6.45E-08**	7.69E-05	3.73E-03	1.63E-07	**1.38E-07**	3.77E-08	1.63E-07	1.03E-07	8.22E-08	9.03E-08	**8.13E-08**	9.74E-08	1.63E-07	1.26E-07	**8.71E-08**
F_7	9.32E+01	9.87E+01	1.19E+02	**8.88E+01**	**9.32E+01**	9.79E+01	9.60E+01	**9.32E+01**	9.96E+01	9.61E+01	9.70E+01	9.80E+01	9.76E+01	**9.32E+01**	9.49E+01	9.44E+01
F_8	**4.15E+01**	4.57E+01	6.78E+01	4.44E+01	**4.15E+01**	4.69E+01	4.54E+01	**4.15E+01**	4.65E+01	4.68E+01	4.67E+01	4.60E+01	4.71E+01	**4.15E+01**	4.73E+01	4.55E+01
F_9	**3.30E-02**	6.93E-02	2.43E+00	5.74E-01	**3.30E-02**	4.18E-02	8.45E-02	**3.30E-02**	6.30E-02	5.12E-02	4.20E-02	5.42E-02	6.04E-02	**3.30E-02**	2.23E-01	9.94E-02
F_{10}	**4.08E+03**	4.16E+03	4.26E+03	4.12E+03	**4.08E+03**	4.15E+03	4.25E+03	**4.08E+03**	4.23E+03	4.24E+03	4.23E+03	4.30E+03	4.19E+03	4.08E+03	4.02E+03	**3.92E+03**
F_{11}	**5.81E+01**	6.09E+01	1.36E+02	1.18E+02	**5.81E+01**	6.23E+01	6.11E+01	**5.81E+01**	6.74E+01	6.28E+01	5.85E+01	6.04E+01	6.38E+01	**5.81E+01**	6.73E+01	7.51E+01
F_{12}	5.24E+03	6.37E+03	1.74E+04	**5.03E+03**	5.24E+03	**4.96E+03**	5.30E+03	5.24E+03	6.02E+03	4.88E+03	5.69E+03	5.01E+03	**3.94E+03**	**5.24E+03**	6.84E+03	5.74E+03
F_{13}	**1.78E+02**	2.04E+02	1.26E+03	1.31E+03	1.78E+02	**1.66E+02**	1.94E+02	1.78E+02	**1.43E+02**	1.62E+02	2.15E+02	5.19E+02	3.30E+03	1.78E+02	2.01E+02	**1.65E+02**
F_{14}	1.38E+02	**1.16E+02**	3.12E+02	2.16E+02	1.38E+02	**1.05E+02**	**1.05E+02**	1.38E+02	1.14E+02	1.21E+02	**1.01E+02**	1.41E+02	2.04E+02	1.38E+02	**1.13E+02**	1.38E+02
F_{15}	**1.57E+02**	1.64E+02	4.63E+02	3.43E+02	1.57E+02	**1.29E+02**	1.73E+02	1.57E+02	**1.32E+02**	1.87E+02	1.44E+02	3.42E+02	1.69E+03	**1.57E+02**	1.60E+02	2.33E+02
F_{16}	7.72E+02	7.55E+02	8.12E+02	**6.41E+02**	**7.72E+02**	7.97E+02	8.30E+02	7.72E+02	**7.55E+02**	7.89E+02	7.66E+02	7.83E+02	7.93E+02	**7.72E+02**	7.75E+02	8.02E+02
F_{17}	**4.97E+02**	5.30E+02	6.56E+02	5.46E+02	**4.97E+02**	5.40E+02	5.07E+02	4.97E+02	5.16E+02	5.46E+02	5.36E+02	**4.91E+02**	5.45E+02	**4.97E+02**	5.49E+02	5.17E+02
F_{18}	**2.11E+02**	2.38E+02	3.60E+02	2.24E+02	**2.11E+02**	2.51E+02	2.64E+02	2.11E+02	**1.97E+02**	2.42E+02	2.47E+02	2.85E+02	9.73E+02	2.11E+02	**1.83E+02**	2.12E+02
F_{19}	9.76E+01	**8.95E+01**	1.85E+02	1.04E+02	9.76E+01	**8.87E+01**	1.02E+02	9.76E+01	7.95E+01	**7.57E+01**	9.55E+01	1.46E+02	1.29E+03	9.76E+01	**7.53E+01**	9.88E+01
F_{20}	3.75E+02	4.11E+02	4.45E+02	**3.24E+02**	**3.75E+02**	3.88E+02	4.02E+02	**3.75E+02**	4.02E+02	4.39E+02	3.79E+02	4.23E+02	3.92E+02	3.75E+02	3.86E+02	**3.53E+02**
F_{21}	**2.41E+02**	2.44E+02	2.73E+02	**2.41E+02**	**2.41E+02**	2.47E+02	2.46E+02	**2.41E+02**	2.46E+02	2.45E+02	2.43E+02	2.46E+02	2.43E+02	**2.41E+02**	2.45E+02	2.43E+02
F_{22}	4.15E+03	4.05E+03	4.37E+03	**3.64E+03**	4.15E+03	**3.84E+03**	4.35E+03	4.15E+03	4.25E+03	**3.64E+03**	4.16E+03	3.98E+03	3.92E+03	4.15E+03	3.97E+03	**3.40E+03**
F_{23}	**4.63E+02**	4.70E+02	5.03E+02	4.67E+02	**4.63E+02**	4.64E+02	4.69E+02	**4.63E+02**	4.64E+02	4.65E+02	4.68E+02	4.69E+02	4.66E+02	**4.63E+02**	4.64E+02	4.66E+02
F_{24}	**5.36E+02**	5.39E+02	5.70E+02	**5.36E+02**	**5.36E+02**	5.38E+02	5.38E+02	**5.36E+02**	5.37E+02	5.37E+02	5.37E+02	5.37E+02	5.38E+02	5.36E+02	**5.34E+02**	5.36E+02
F_{25}	**5.16E+02**	5.25E+02	5.23E+02	5.28E+02	**5.16E+02**	5.24E+02	5.27E+02	**5.16E+02**	5.28E+02	5.38E+02	5.31E+02	5.29E+02	5.30E+02	**5.16E+02**	5.54E+02	5.37E+02
F_{26}	**1.46E+03**	1.49E+03	1.85E+03	1.57E+03	**1.46E+03**	1.49E+03	1.51E+03	**1.46E+03**	1.47E+03	1.49E+03	1.48E+03	1.48E+03	1.48E+03	1.46E+03	1.51E+03	**1.45E+03**
F_{27}	5.46E+02	**5.44E+02**	5.87E+02	5.87E+02	5.46E+02	**5.42E+02**	5.48E+02	5.46E+02	5.47E+02	5.46E+02	5.46E+02	**5.43E+02**	5.45E+02	5.46E+02	5.49E+02	**5.41E+02**
F_{28}	4.93E+02	4.95E+02	**4.85E+02**	4.95E+02	**4.93E+02**	5.01E+02	4.99E+02	**4.93E+02**	4.95E+02	5.02E+02	4.98E+02	5.02E+02	5.00E+02	**4.93E+02**	4.97E+02	4.97E+02
F_{29}	**4.37E+02**	4.52E+02	6.06E+02	5.18E+02	**4.37E+02**	4.55E+02	4.55E+02	**4.37E+02**	4.43E+02	4.60E+02	4.73E+02	4.70E+02	**4.37E+02**	**4.37E+02**	4.43E+02	4.41E+02
F_{30}	6.37E+05	6.27E+05	7.62E+05	**6.13E+05**	6.37E+05	6.26E+05	**6.13E+05**	6.37E+05	6.18E+05	6.41E+05	6.36E+05	**6.15E+05**	6.35E+05	6.37E+05	**6.11E+05**	6.36E+05
Rank	**1.68**	2.31	3.70	2.29	**1.55**	2.03	2.41	**2.43**	3.50	3.51	3.36	3.91	4.27	**1.68**	2.32	1.98

After the above preparation, we carry out experiments on the 50*D* CEC2017 set to compare the above developed variants with GSGDE. Their comparison results are displayed in Table 10. Observing this table, we get the following findings:

1) As displayed in the first part of Table 10, it is found that GSGDE ranks the first place among the four GSGDE variants. This implies that GSGDE gains the best overall performance among the four algorithms. In particular, GSGDE attains the best performance on 16 problems, while the three variants gain the best results on no more than 10 problems. This further proves the superiority of GSGDE to its three variants. These two findings substantiate the great effectiveness of "DE/current-to-gselite/1". Specifically, compared with "DE/current-to-pbest/1" and "DE/current-to-best/1", "DE/current-to-gselite/1" shows significant superiority. This demonstrates that the superiority of using the Gaussian distribution model based on elites in the population to sample guiding exemplars for individuals to directly using the elites as the guiding exemplars. Compared with "GSGDE-mean", GSGDE is significantly superior. This demonstrates the superiority of using one random elite as the mean vector of the Gaussian distribution to using the mean position of all elites as the mean of the Gaussian distribution. Comprehensively speaking, the superiority of

TABLE 11. Comparison results between GSGDE with the dynamic adjustment mechanism for the number of elites and the ones with distinct fixed values of p on the 50D CEC2017 benchmark problems.

F	Adaptive	p=0.05	p=0.06	p=0.07	p=0.08	p=0.09	p=0.10	p=0.20	p=0.30	p=0.40	p=0.50
F_1	3.08E-14	2.79E-14	3.27E-14	3.65E-14	3.03E-14	2.94E-14	2.94E-14	2.95E-14	3.27E-14	3.13E-14	**2.61E-14**
F_3	2.73E-13	2.43E-13	2.52E-13	**2.24E-13**	2.56E-13	2.48E-13	2.57E-13	2.59E-13	2.27E-13	2.54E-13	3.52E-13
F_4	5.01E+01	5.30E+01	5.29E+01	5.11E+01	**3.88E+01**	5.36E+01	5.28E+01	5.71E+01	4.39E+01	5.53E+01	4.06E+01
F_5	**3.97E+01**	4.68E+01	4.57E+01	4.51E+01	4.68E+01	4.42E+01	4.34E+01	4.57E+01	4.77E+01	4.86E+01	4.99E+01
F_6	1.63E-07	5.12E-07	1.86E-07	1.43E-07	1.12E-07	1.52E-07	5.82E-08	**1.02E-08**	1.44E-08	1.29E-08	2.56E-08
F_7	**9.32E+01**	9.98E+01	9.90E+01	9.77E+01	9.79E+01	9.76E+01	9.79E+01	9.87E+01	9.69E+01	9.84E+01	9.91E+01
F_8	**4.15E+01**	4.68E+01	4.56E+01	4.80E+01	4.73E+01	4.59E+01	4.65E+01	4.70E+01	4.91E+01	4.91E+01	5.12E+01
F_9	3.30E-02	9.04E-02	6.61E-02	1.06E-01	4.52E-02	3.60E-02	4.82E-02	**2.69E-02**	3.90E-02	5.39E-02	8.10E-02
F_{10}	**4.08E+03**	4.23E+03	4.27E+03	4.24E+03	4.22E+03	4.15E+03	4.20E+03	4.26E+03	4.30E+03	4.29E+03	4.37E+03
F_{11}	**5.81E+01**	6.43E+01	6.56E+01	6.75E+01	6.89E+01	6.20E+01	6.80E+01	6.99E+01	6.69E+01	6.74E+01	7.16E+01
F_{12}	5.24E+03	6.06E+03	5.13E+03	5.55E+03	5.06E+03	6.06E+03	5.54E+03	3.84E+03	**3.71E+03**	5.08E+03	4.32E+03
F_{13}	1.78E+02	2.09E+02	2.22E+02	1.82E+02	1.68E+02	1.60E+02	1.43E+02	1.38E+02	**1.17E+02**	1.34E+02	1.22E+02
F_{14}	1.38E+02	1.36E+02	1.19E+02	1.35E+02	1.19E+02	1.14E+02	1.02E+02	1.06E+02	1.00E+02	9.68E+01	**8.40E+01**
F_{15}	1.57E+02	1.85E+02	1.71E+02	1.60E+02	1.58E+02	1.55E+02	1.65E+02	1.23E+02	1.39E+02	**1.20E+02**	1.39E+02
F_{16}	7.72E+02	7.80E+02	7.84E+02	7.67E+02	7.90E+02	8.00E+02	**7.42E+02**	8.00E+02	8.11E+02	8.14E+02	8.56E+02
F_{17}	**4.97E+02**	5.30E+02	5.25E+02	5.11E+02	5.16E+02	5.71E+02	5.38E+02	5.47E+02	5.52E+02	5.70E+02	5.61E+02
F_{18}	2.11E+02	2.47E+02	2.17E+02	2.72E+02	2.50E+02	2.12E+02	2.44E+02	2.05E+02	1.97E+02	1.75E+02	**1.71E+02**
F_{19}	9.76E+01	1.08E+02	9.92E+01	8.02E+01	8.65E+01	8.56E+01	8.68E+01	6.69E+01	7.16E+01	**6.32E+01**	6.70E+01
F_{20}	3.75E+02	3.88E+02	3.78E+02	**3.67E+02**	3.97E+02	3.84E+02	3.76E+02	3.75E+02	3.98E+02	3.94E+02	4.50E+02
F_{21}	**2.41E+02**	2.47E+02	2.45E+02	2.45E+02	2.46E+02	2.46E+02	2.43E+02	2.45E+02	2.47E+02	2.47E+02	2.48E+02
F_{22}	4.15E+03	3.70E+03	4.31E+03	4.30E+03	3.96E+03	4.48E+03	3.95E+03	3.86E+03	3.86E+03	3.85E+03	**3.62E+03**
F_{23}	**4.63E+02**	4.66E+02	4.67E+02	4.66E+02	4.67E+02	4.65E+02	4.67E+02	4.70E+02	4.69E+02	4.71E+02	4.72E+02
F_{24}	5.36E+02	5.39E+02	5.38E+02	5.37E+02	5.38E+02	5.36E+02	**5.35E+02**	5.36E+02	5.36E+02	5.38E+02	5.39E+02
F_{25}	**5.16E+02**	5.32E+02	5.26E+02	5.29E+02	5.41E+02	5.44E+02	5.18E+02	5.31E+02	5.39E+02	5.41E+02	5.38E+02
F_{26}	**1.46E+03**	1.51E+03	1.52E+03	1.47E+03	1.47E+03	1.47E+03	1.47E+03	1.51E+03	1.50E+03	1.50E+03	1.52E+03
F_{27}	5.46E+02	5.47E+02	**5.39E+02**	5.50E+02	5.43E+02	5.42E+02	5.44E+02	5.41E+02	5.42E+02	**5.39E+02**	5.42E+02
F_{28}	4.93E+02	4.94E+02	**4.92E+02**	4.98E+02	**4.92E+02**	4.96E+02	4.95E+02	5.00E+02	4.98E+02	5.00E+02	4.96E+02
F_{29}	**4.37E+02**	4.68E+02	4.68E+02	4.77E+02	4.60E+02	4.41E+02	4.54E+02	**4.37E+02**	4.49E+02	4.53E+02	4.77E+02
F_{30}	6.37E+05	6.22E+05	6.29E+05	6.27E+05	6.24E+05	6.36E+05	6.08E+05	6.20E+05	6.14E+05	6.07E+05	**6.04E+05**
Rank	**4.46**	7.27	6.74	6.48	6.25	5.74	5.05	5.31	5.53	6.25	6.87

"DE/current-to-gselite/1" to the three compared mutation schemes mainly benefits from that it affords much higher mutation diversity for the population, such that individuals could search the problem space in quite diverse directions to find promising regions.

2) As shown in the second part of Table 10, on the one hand, GSGDE achieves a much lower rank value than GSGDE-Eul and GSGDE-Che; on the other hand, GSGDE attains the best performance on 18 problems, whereas the compared two variants perform the best on no more than 9 problems. These two observations verify the great superiority of the Manhattan distance based standard deviation calculation for the Gaussian distribution to the compared two ways, namely the Euclidean distance based standard deviation calculation and the Chebyshev distance based standard deviation calculation. Such great superiority of the Manhattan distance based standard deviation mainly profits from that the Manhattan distance offers more accurate assessment of the difference between the selected elite and the other ones with respect to each dimension.
3) As shown in the third part of Table 10, we discover that GSGDE with the dynamic strategy for ε obtains a much smaller rank value than those with the 5 fixed values of ε. Besides, GSGDE with the dynamic strategy attains the best performance on 14 problems, whereas GSGDE with the fixed values attain the best performance on no more than 5 problems. These observations substantiate the great effectiveness of the dynamic scheme for ε. In particular, such superiority of the dynamic scheme is mainly ascribed to that it potentially affords a dynamic compromise between exploitation and exploration for the population to search the problem space appropriately. Concretely, the random generation of ε from [1.0E-04, 1.0E-03] could let the standard deviation of the Gaussian distribution for distinct individuals be different. On the one hand, the diversity with respect to the Gaussian distribution is promoted, and thus high search diversity is accordingly sustained during the iteration; on the other hand, the dynamic change of ε results in the dynamic change of the sampling range of the Gaussian distribution. When ε is small, the Gaussian distribution samples guiding exemplars around the associated elites with a narrow range, which increases the probability of generating a better exemplar than the associated elite. In this case, fast convergence to optimal regions could be achieved. By contrast, when ε is large, the Gaussian distribution samples guiding exemplars with a wide range. In this situation, the probability of generating an exemplar far from the associated elite is promoted. This is profitable for individuals to get out of local regions once the associated elites step into local zones. Hence, with the dynamic generation of ε, GSGDE dynamically maintains a promising balance between search convergence and search diversity to traverse the problem space to seek satisfactory solutions.

4) As shown in the fourth part of Table 10, we discover that GSGDE obtains a much smaller rank value than GSGDE-WA and GSGDE-2PA. Besides, GSGDE gains the best performance on 15 problems, whereas the other two variants attain the best performance on at most 8 problems. These observations show the superiority of randomly selecting $\boldsymbol{x}_{r1}$ from $\boldsymbol{P}$ and randomly choosing $\hat{x}_{r2}$ from $\boldsymbol{P}\cup\boldsymbol{A}$ to randomly choosing both $\boldsymbol{x}_{r1}$ and $\hat{x}_{r2}$ from the population $\boldsymbol{P}$ and randomly selecting them both from $\boldsymbol{P}\cup\boldsymbol{A}$. This is mainly because the former selection scheme for the selection of the two random individuals helps DE maintain a better balance between exploration and exploitation than the latter two selection schemes.

Based on the above experimental results, the effectiveness of "DE/current-to-gselite/1" including the techniques within this strategy has been verified. In particular, it makes significant contribution to the promising performance of GSGDE.

2) EFFECTIVENESS OF THE ADAPTIVE ADJUSTMENT OF THE NUMBER OF ELITES

This subsection aims to testify the effectiveness of the designed dynamic scheme for the number of elites. For this purpose, distinct fixed values (from 0.05 to 0.50) are set for p in Eq. (16) instead of the dynamic adjustment of p. Then, we make comparisons between GSGDE with the dynamic scheme and the ones with these distinct fixed p on the $50D$ CEC2017 benchmark set. The comparison results are presented in Table 11. Observing this table, we gain the following discoveries:

1) Concerning "*Rank*", GSGDE with the adaptive scheme attains the smallest rank value among all GSGDE variants. This proves that GSGDE gains the best overall performance on the entire $50D$ CEC2017 set. This proves the effectiveness of the designed adaptive mechanism for controlling the number of elites.
2) Further observation shows that GSGDE with the adaptive scheme achieves the best results on 11 problems, whereas the ones with the fixed values of p gain the best performance on no more than 5 problems. This further shows the superiority of the designed adaptive scheme for the number of elites to the fixed number of elites. Such superiority is mainly attributed to that the adaptive strategy affords a special way for GSGDE to balance exploration and exploitation. Specifically, as the evolution continues, the number of elites becomes smaller and smaller, leading to that the selection range of the mean vector of the Gaussian distribution becomes narrower and narrower. Therefore, with the evolution proceeding, GSGDE gradually switches from concentration on exploring the problem space to focus on exploiting the located optimal regions.
3) In-depth observation reveals that on different optimization problems, the best number of elites is distinct for GSGDE to achieve the best performance. This means that a lot of effort is needed to fine-tune this parameter for GSGDE to attain good optimization performance. However, with the help of the devised adaptive strategy, GSGDE could alleviate its sensitivity to this parameter, and thus effort for fine-tuning the number of elites can be saved.

On the whole, the above experimental results have proven that the devised two techniques are of great use for GSGDE to acquire good optimization performance in tackling optimization problems. With the cohesive cooperation between these two techniques, GSGDE is anticipated to keep a good and dynamic balance between search convergence and search diversity to explore and exploit the problem space to seek high-accuracy solutions.

V. CONCLUSION

This paper has designed a new mutation scheme termed as "DE/current-to-gselite/1" for DE to deal with global optimization problems. Instead of directly using existing elite individuals to mutate the population, the devised mutation scheme randomly samples a leading exemplar for each individual to direct its mutation according to the Gaussian distribution around one randomly chosen elite in the population. By this means, the guiding exemplars for distinct individuals are likely different and hence high mutation diversity can be sustained. This is of great help for DE to traverse the problem space in diverse directions. In addition, thanks to the small sampling range of the Gaussian distribution, the generated guiding exemplar is expectedly better than the associated elite and thus individuals are anticipated to be guided to promising areas fast. This is of great use for DE to converge fast to optimal solutions in the space. Besides, to relieve GSGDE from the sensitivity to the number of elites, we further developed a dynamic adjustment strategy to dynamically regulate the number of elites during the optimization. Hence, the population gradually switches from exploring the solution space for seeking optimal regions to exploiting the found optimal areas intensively for locating high-accuracy solutions. Integrating the two techniques along with an existing adaptive scheme for CR and F, we developed a new DE algorithm, called Gaussian Sampling Guided Differential Evolution (GSGDE). With the cooperation between the above two techniques, GSGDE is anticipated to sustain a promising compromise between exploitation and exploration to traverse the problem space and thus is anticipated to achieve good optimization performance.

Experiments have been comprehensively carried out on the CEC2014 and the CEC2017 problem suites with three distinct dimensionality settings to substantiate the effectiveness of GSGDE. In comparison with totally 11 latest and representative DE methods on the CEC2017 and CEC2014 sets, GSGDE performs competitively with or even significantly outperforms them on different types of benchmark problems. Particularly, it is proven that GSGDE has a good scalability to tackle optimization problems with higher dimensionality and its superiority to the 11 compared DE algorithms becomes more and more significant as the dimensionality increases. Furthermore, verification experiments on the influence of the two devised techniques on GSGDE have also been executed. Experimental results have substantiated that the devised two techniques are of great benefit to help GSGDE achieve good optimization performance.

The future works of this paper lie in two ways. The first is to replace the Gaussian distribution with the Cauchy distribution to randomly sample guiding exemplars for individuals because the Cauchy distribution is very similar to the Gaussian distribution but it preserves a wider sampling range. Besides, we can also try to hybridize these two distribution models to generate guiding exemplars for individuals, such that the advantages of both distribution models can be assembled to help GSGDE maintain a better balance between exploitation and exploration. The other direction is to employ GSGDE to tackle real-world optimization problems, like the optimization of the neural network architecture [81], the optimization of control parameters in wireless power transfer [82], expensive optimization [83], the optimization of information spreading in social network [84], etc.

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