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RESEARCH ARTICLE

# Distributed Finite-Time Boundedness Control for Large-Scale Networked Dynamic Systems

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**ABSTRACT** In this paper, we investigate the finite-time boundedness problems for large-scale continuous-time networked dynamical systems constituted by lots of subsystems. The interactions among these subsystems are arbitrary, and every subsystem has different dynamics. The linear time-varying and time-invariant cases are discussed respectively. Sufficient conditions for the design of finite-time boundedness state feedback controller are derived, which efficiently utilize the characteristic of system structure with the block-diagonal structure of system parameter matrices and the sparseness of the system topology. Sufficient conditions depending only on parameter matrices of the individual subsystem are also provided. Furthermore, sufficient conditions are provided for the design of the distributed output feedback controller with finite-time boundedness. Several numerical simulations have been used to show the validity of the derived conditions in the analysis of a large-scale networked system.

**INDEX TERMS** Large-scale systems, networked systems, finite-time boundedness, state feedback control, distributed output feedback control.

## I. INTRODUCTION

With the continuous development of science and technology and social productive forces, there are more and more systems with complex structures [1], [2], [3]. Meanwhile, the rapid development of computer technology and the extensive application of intelligent controllers and actuators in practical engineering make it possible to control and manage these systems. They are constituted of a large number of spatially dispersed subsystems interacting with each other, such as automated highway systems [4], underwater optical wireless communication [5], distributed satellite systems [6], gene regulatory network [7], an array of closely packed identical microcantilevers in microscope application [8] and so on. In Reference [2], the problem of state estimation for a class of nonlinear complex networks under attack is studied. A novel unified attack-defense framework of nonlinear complex networks is established, and sufficient conditions for resisting important-data-based attack and ensuring $H_\infty$ performance of augmented systems are obtained. The real-time monitoring and detection problems for load frequency control (LFC) systems of electric vehicles are studied in [3]. An $H_\infty$ sliding mode observer (SMO) is proposed to accurately estimate the internally physical state of the LFC system with electric vehicles, and a dual time-varying coding-based detection algorithm is established to ensure that the detection of covert attacks has without significant delay. There are many research results related on controller design of networked systems [9], [10], [11], [12]. In Reference [9], the distributed event-triggered output feedback control problem of discrete-time large-scale fuzzy systems in network scenario is studied. A novel distributed event-triggered control scheme is proposed and the solution of distributed event-triggering output-feedback control problem is derived. Reference [10] proposes a novel command filter-based adaptive prescribed performance control strategy to solve issues of the infinite number of time-varying actuator faults for constrained uncertain nonlinear large-scale systems. The event-based

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output-feedback control for large-scale distributed network nonlinear systems is studied in [11]. A large-scale fuzzy system with multi-rate samplers and time-driven zero order holds (ZOHs) is proposed, and nonlinear interconnection are introduced into the large-scale fuzzy systems. Reference [12] studies the adaptive torus-event-based $H_\infty$ control problem for a class of networked Takagi-Sugeno (T-S) fuzzy systems under deception attacks. The sufficient conditions for solving the asymptotically mean-square stability and disturbance suppression issues of closed-loop control systems are derived and the adaptive torus-event-based $H_\infty$ fuzzy controller gains are calculated. Traditional lumped analysis and control methods are difficult to deal with networked systems. Therefore, efficiently utilize the characteristic of system structure with the block-diagonal structure of system parameter matrices and the sparseness of the system topology, using distributed analysis and control strategy based on local information sharing can greatly reduce the amount of data transmission in the network and greatly improve the computing efficiency [13], [14], [15], [16].

The classical control concepts of Lyapunov and Poincare deal with systems that operate in infinite-time interval. In contrast, finite-time control deals with systems operating over a finite-time interval. Finite-time stability is an independent concept compared to Lyapunov asymptotic stability which is defined over an infinite-time interval. Generally, in missile systems, satellite systems and some chemical experiments where operating times are often of finite duration. In system of boost chopper, rapid current change may damage the circuit. Under normal circumstances, the overshoot is too large to be applied in many practical projects. Therefore, finite-time stability and finite-time boundedness are more practical concepts and helpful to study the transient behavior of the system within a finite interval. The concept of finite-time boundedness (FTB) was led to by the idea of finite-time stability (FTS) in [17] and [18]. A system is said to be FTS if its state does not exceed a certain threshold value during a specified time interval when given a bound on the initial condition. Correspondingly, FTB means that the state variables remain below the prescribed limit for all inputs during a specified time interval given a bound on the initial condition and a characterization of the set of admissible inputs [19].

There are some results recently dealing with FTS/FTB analysis and control problems for networked systems or multi-agent systems [20], [21], [22], [23], [24]. In [20], it deals with the synchronization of autonomous discrete-time agents and provides a distinctive controller structure. By the sliding mode control method, a new slip function is constructed for the finite-time boundedness problem of uncertain Hamiltonian systems in [21]. Reference [22] discusses the finite-time output stability for the impulse switching linear system when the norm-bounded state constraint is simultaneously considered during a scheduled finite-time period. In [23], the finite-time consensus problem is studied for heterogeneous multi-agent systems composed of first-order and second-order integrator agents. It proposes two classes of consensus protocols with and without velocity measurements by combining the homogeneous domination method with the adding a power integrator method. Reference [24] applies a nonlinear sliding mode control method for finite-time boundedness theory to the formation control problem of underdriven ships and designs a distributed controller for underdriven ships to achieve a given formation pattern in finite time.

Previous research results have mainly focused on FTS/FTB problems for multi-agent systems normally with a small scale or a specific structure, but there are difficulties in controlling finite-time boundedness for large-scale complex systems. In this paper, sufficient conditions based on finite-time boundedness are obtained for large-scale networked systems, these computationally efficient conditions utilize efficiently the block-diagonal structure of system parameter matrices and the sparseness of the subsystem connection matrix. These works avoid the inverse computation of large dimensional matrices, which makes the calculation more efficient and easier to implement in practical engineering.

The main contributions of this paper are summarized as follows.

1) The sufficient conditions for the design of finite-time boundedness state feedback controller are derived under both time-varying and time-invariant cases. Some conditions only based on the parameters of each subsystem are also derived.

2) Sufficient conditions are provided for the design of the distributed output feedback controllers with finite-time boundedness.

This paper is sketched as follows. In Section II, the problem formulation and some preliminaries are provided. In Section III, sufficient conditions for the design of finite-time boundness state feedback controllers are given. The sufficient conditions for the design of the distributed output feedback controllers with finite-time boundedness are provided in Section IV. In Section V, numerical examples are provided to illustrate the effectiveness of the proposed approaches. Finally, summarizes and prospects some characteristics of the proposed method are presented in Section VI.

We use the following symbols and expressions. $R^n$ and $R^{m\times n}$ denote the sets of $n$-dimensional real vectors and $m \times n$ real matrices, respectively. $F_u(*, \#)$ represents the upper linear fractional transformation. $\mathbf{diag}\left\{X_i|_{i=1}^{L}\right\}$ stands for a block-diagonal matrix with its $i$-th diagonal block being $X_i$, while $\mathrm{col}\left\{X_i|_{i=1}^{L}\right\}$ the vector/matrix stacked by $X_i(i = 1, 2, \cdots, L)$ with its $i$-th row block vector/matrix being $\left\{X_{ij}|_{i=1,j=1}^{i=M,j=N}\right\}$ represents a matrix with $M \times N$ blocks, where the block matrix its $i$-th row $j$-th column block matrix being $X_{ij}$. $0_m$ and $0_{m\times n}$ represent the $m$ dimensional zero column vector and the $m \times n$ dimensional zero matrix, respectively, while the subscript of dimension will be omitted without ambiguity. $I$ is the identity matrix with compatible dimension. $\mathcal{W}$ represents a constant exogenous disturbance.

$d$, $c_1$, $c_2$ and $k$ express as positive scalars. $H$, $R$ and $K$ represent symmetric matrices. $U$ and $M$ stand for diagonal matrix. $P(\cdot)$, $Q_1(\cdot)$ and $Q_2(\cdot)$ represent symmetric matrix-valued functions. $L(\cdot)$ denotes matrix-valued function. The superscript $T$ represents the transpose of a matrix/vector. When the expression of $X$ is too complex, $X^TWX$ or $XWX^T$ can be abbreviated as $(*)^TWX$ or $XW(*)^T$. In addition, for block-partitioned symmetric matrix $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}$, it can be written as $A = \begin{bmatrix} A_{11} & A_{12} \\ (*)^T & A_{22} \end{bmatrix}$ or $A = \begin{bmatrix} A_{11} & (*)^T \\ A_{12}^T & A_{22} \end{bmatrix}$. When the expression of Z is too complex, $Z+Z^T$ or $Z^T+Z$ can be abbreviated as $(Z+(\#^T))$ or $((\#^T)+Z)$. For symmetric matrices $A$ and $B$ with compatibly dimensions, $A < (\leq, >, \geq) B$ expressed as $A-B$ negative definite (negative semi-definite, positive definite, positive semi-definite).

## II. PROBLEM DESCRIPTION AND SOME PRELIMINARIES

### A. PROBLEM DESCRIPTION

Consider the networked system $\Theta$ composed of $N$ dynamic subsystems and the dynamics of the $i$-th subsystem $\Theta_i$ is described by the following state space equation,

$$\begin{bmatrix} \dot{x}(t,i) \\ z(t,i) \\ y(t,i) \end{bmatrix} = \begin{bmatrix} A_{\mathrm{TT}}(t,i) & A_{\mathrm{TS}}(t,i) & B_{\mathrm{T}}(t,i) & G_{\mathrm{T}}(t,i) \\ A_{\mathrm{ST}}(t,i) & A_{\mathrm{SS}}(t,i) & B_{\mathrm{S}}(t,i) & G_{\mathrm{S}}(t,i) \\ C_{\mathrm{T}}(t,i) & C_{\mathrm{S}}(t,i) & D_{\mathrm{T}}(t,i) & H_{\mathrm{T}}(t,i) \end{bmatrix} \times \begin{bmatrix} x(t,i) \\ v(t,i) \\ u(t,i) \\ w(t,i) \end{bmatrix}, \tag{1}$$

in which, $t$ represents the time variable and $i$ represents the subsystem number, $i = 1, 2, \cdots, N$. These subsystems are connected by

$$v(t) = \Phi(t)z(t), \tag{2}$$

where $z(t) = \mathbf{col}\left\{z(t,i)|_{i=1}^N\right\}$, $v(t) = \mathbf{col}\left\{v(t,i)|_{i=1}^N\right\}$. The called internal output $z(t,i)$ and input vectors $v(t,i)$ respectively denote the output vector to other subsystems and input vector from others, $\Phi(t)$ is called subsystem connection matrix. $y(t,i)$, $u(t,i)$ and $w(t,i)$ are external output, control input and disturbance, respectively.

The dimensions of the vectors $x(t,i)$, $v(t,i)$, $z(t,i)$, $u(t,i)$, $w(t,i)$ and $y(t,i)$ are assumed respectively $m_{xi}$, $m_{vi}$, $m_{zi}$, $m_{ui}$, $m_{wi}$ and $m_{yi}$. Without loss of generality, it is assumed in this paper that every column of the matrix $\Phi(t)$ has only one nonzero element which is equal to one and there are no rows whose elements are all zeros. In addition, the diagonal block elements of $\Phi(t)$ corresponding to the subsystem dimension are all zeros. Then, we obtain that $\Phi(t)\Phi^T(t) = \Sigma^2(t)$, in which, $\Sigma^2(t) = diag\left\{\Sigma_j^2|_{j=1}^N(t)\right\}$, $\Sigma_j^2(t) = \mathbf{diag}\{m(i)|_{i=M_{v,j-1}+1}^{M_{v,j}}\}$, $M_{v,i} = \sum_{k=1}^{i} m_{vk}$, $m(i)$ denotes the number of subsystems affecting directly the $i$-th element of the vector $z(t)$, $i = 1, 2, \cdots, \sum_{k=1}^N m_{zk}$, $j = 1, 2, \cdots, N$.

To express concisely, define the following vectors and matrices, $f(t) = \mathbf{col}\left\{f(t,i)|_{i=1}^N\right\}$, $f = x, y, u, w$, $A_{*\#} = \mathbf{diag}\left\{A_{*\#}|_{i=1}^N\right\}$, $B_{*\#} = \mathbf{diag}\left\{B_{*\#}|_{i=1}^N\right\}$, $C_{*\#} = \mathbf{diag}\left\{C_{*\#}|_{i=1}^N\right\}$, $D_{*\#} = \mathbf{diag}\left\{D_{*\#}|_{i=1}^N\right\}$, $G_{*\#} = \mathbf{diag}\left\{G_{*\#}|_{i=1}^N\right\}$ and $H_{*\#} = \mathbf{diag}\left\{H_{*\#}|_{i=1}^N\right\}$, where $*, \# = \mathrm{T,S}$. The dynamic system $\Theta$ can be equivalently described by the following state space form through direct algebraic operation,

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(t) & B(t) & G(t) \\ C(t) & D(t) & H(t) \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \\ w(t) \end{bmatrix}, \tag{3}$$

in which,

$$A(t) = F_u\left(\begin{bmatrix} A_{\mathrm{SS}}(t) & A_{\mathrm{ST}}(t) \\ A_{\mathrm{TS}}(t) & A_{\mathrm{TT}}(t) \end{bmatrix}, \Phi(t)\right),$$
$$B(t) = F_u\left(\begin{bmatrix} A_{\mathrm{SS}}(t) & B_{\mathrm{S}}(t) \\ A_{\mathrm{TS}}(t) & B_{\mathrm{T}}(t) \end{bmatrix}, \Phi(t)\right),$$
$$G(t) = F_u\left(\begin{bmatrix} A_{\mathrm{SS}}(t) & G_{\mathrm{S}}(t) \\ A_{\mathrm{TS}}(t) & G_{\mathrm{T}}(t) \end{bmatrix}, \Phi(t)\right),$$
$$C(t) = F_u\left(\begin{bmatrix} A_{\mathrm{SS}}(t) & A_{\mathrm{ST}}(t) \\ C_{\mathrm{S}}(t) & C_{\mathrm{T}}(t) \end{bmatrix}, \Phi(t)\right),$$
$$D(t) = F_u\left(\begin{bmatrix} A_{\mathrm{SS}}(t) & B_{\mathrm{S}}(t) \\ C_{\mathrm{S}}(t) & D_{\mathrm{T}}(t) \end{bmatrix}, \Phi(t)\right),$$
$$H(t) = F_u\left(\begin{bmatrix} A_{\mathrm{SS}}(t) & G_{\mathrm{S}}(t) \\ C_{\mathrm{S}}(t) & H_{\mathrm{T}}(t) \end{bmatrix}, \Phi(t)\right).$$

Apparently, the well-posedness of networked system $\Theta$ implies the existence of $(I - A_{\mathrm{SS}}(t)\Phi(t))^{-1}$.

### B. SOME PRELIMINARIES

The objectives of this paper are to design computationally attractive state feedback controllers and distributed output feedback controllers for the FTB analysis of System $\Theta$. The FTB concept of a networked dynamic system (1) and (2) is presented as follows, which is consistent with the concept based on (3) in [19].

*Definition 1:* The networked system (1) and (2) are said to be FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(t,i))$, with $c_1 < c_2$ and positive definite matrix functions $\Gamma(t,i) > 0, i = 1, \cdots, N$ defined over $[0, T]$ and a class of signals $\mathcal{W}$ if

$$\sum_{i=1}^N x^T(0,i)\Gamma(0,i)x(0,i) \leq c_1$$
$$\Rightarrow \sum_{i=1}^N x^T(t,i)\Gamma(t,i)x(t,i) < c_2, \forall t \in [0,T], w(\cdot) \in \mathcal{W}. \tag{4}$$

for all $w(\cdot) \in \mathcal{W}$.

In this paper, the initial condition $c_1$ and the boundary value $c_2$ are set within the specified time interval according to the actual performance requirements. To investigate the FTB problems, we first introduce the following preliminary results. The first one is the well-known Schur complement lemma and lemma 3 is Finsler lemma.

*Lemma 1 ([25]):* Given a symmetric matrix block, as shown below

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{12}^T & H_{22} \end{bmatrix}, \tag{5}$$

with compatible dimensions of submatrices, the following two conditions are equivalent.

1) $H < 0$;

2) $H_{11} < 0, H_{22} - H_{12}^T H_{11}^{-1} H_{12} < 0$.

*Lemma 2 ([26]):* Assume the diagonal matrices $M$ and $U$ with appropriate dimensions. There is a scalar $\alpha > 0$ such that,

$$MU + U^T M^T \leq \alpha M M^T + \alpha^{-1} U^T U. \tag{6}$$

*Lemma 3 ([27]):* If symmetric matrices $R$ and $K$ with proper dimensions and for every non-zero vector $v$ satisfying $v^T K v = 0$, we can get $v^T R v > 0$, then there must be a $\gamma \in R$ such that $R + \gamma K$ is positive definite.

## III. STATE FEEDBACK CONTROLLER DESIGN

The state feedback control is a characteristic of modern control theory. The state variables of a system can show the internal characteristics of the whole system without knowing the internal structure of the system. Compared with the existing output feedback control, state feedback control can make the control system more excellent and effective, and make works stably and normally.

Based on networked systems (1) and (2), a state feedback controller is designed, for each subsystem consider the following controller,

$$u(t, i) = K(t, i)x(t, i). \tag{7}$$

The whole closed-loop control system can be obtained as follows,

$$\begin{bmatrix} \dot{x}(t, i) \\ z(t, i) \\ y(t, i) \end{bmatrix} = \begin{bmatrix} A_{TT}(t, i) + B_T(t, i)K(t, i) & A_{TS}(t, i) & G_T(t, i) \\ A_{ST}(t, i) + B_S(t, i)K(t, i) & A_{SS}(t, i) & G_S(t, i) \\ C_T(t, i) + D_T(t, i)K(t, i) & C_S(t, i) & H_T(t, i) \end{bmatrix} \times \begin{bmatrix} x(t, i) \\ v(t, i) \\ w(t, i) \end{bmatrix}. \tag{8}$$

Defined matrix $K(t) = \mathbf{diag}\{K(t, i)|_{i=1}^{N}\}$. The whole closed-loop control system (8) can be equivalently described by the following state space form through direct algebraic operation,

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(t) + B(t)K(t) & G(t) \\ C(t) + D(t)K(t) & H(t) \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}. \tag{9}$$

*Definition 2:* (Finite-Time Control via State Feedback) Give Definition 1, find a state feedback controller (7) in the form (9), such that the closed loop system (9) obtained by the connection of (3) and (7) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(t, i))$.

In [28], a sufficient FTB condition for System (9) is provided as follows.

*Lemma 4 ([28]):* The following class of signals are given

$$\mathcal{W} := \left\{ w(\cdot) \mid w(\cdot) \in \mathcal{L}^2([0, T]), \int_0^T w^T(\tau)w(\tau)d\tau \leq d \right\}, \tag{10}$$

where $\mathcal{L}^2$ ([0, $T$]) is the square integrable vector-valued functions set in [0, $T$] and d is a positive scalar. Then, System (9) with $x(0) = x_0$ is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(t))$ if there exists a symmetric matrix-valued function $P(\cdot)$ and a matrix function $L(\cdot)$ such that

$$\begin{aligned} &-\dot{P}(t) + P(t)A(t)^T + A(t)P(t) + L(t)^T B(t)^T \\ &\quad + B(t)L(t) + \frac{c_1 + d}{c_2} G(t)G(t)^T < 0, \\ &P(t) \leq \Gamma^{-1}(t), \forall t \in [0, T], \\ &P(0) > \frac{c_1 + d}{c_2} \Gamma^{-1}(0). \end{aligned} \tag{11}$$

If a symmetric matrix-valued function $P(\cdot)$ and a matrix function $L(\cdot)$ satisfy the LMI condition above, then System (9) is finite-time boundedness. In this case a controller gain which FTB is $K(t) = L(t)P^{-1}(t)$.

From Lemma 4, a sufficient condition can be obtained for the FTB of System $\Theta$ based on System (9).

Take $A(t) = F_u\left(\begin{bmatrix} A_{SS}(t) & A_{ST}(t) \\ A_{TS}(t) & A_{TT}(t) \end{bmatrix}, \Phi(t)\right)$ for example, although the matrices $A_{*\#}(t), *, \# = \mathrm{T}, S$ are all block diagonal and subsystem connection matrix $\Phi(t)$ is sparse, the matrix $A(t)$ is usually dense. In this case, when the networked dynamic systems have a large number of subsystems, the calculation of FTB analysis may encounter prohibitive implementation difficulties. In addition, the inversion of high-dimensional matrix sometimes lead to numerical instability problems. A computationally attractive sufficient condition is derived for the FTB of the networked system $\Theta$.

*Theorem 1:* Given the following class of signals

$$\mathcal{W} := \left\{ \begin{array}{c} w(\cdot) = \mathrm{col}\{w(\cdot, i)|_{i=1}^{N}\} | w(\cdot, i) \in \mathcal{L}^2([0, T]), \\ \sum_{i=1}^{N} \int_0^T w^T(\tau)w(\tau)d\tau \leq d, \\ \int_0^T w^T(\tau, i)w(\tau, i)d\tau \leq d(i), \sum_{i=1}^{N} d(i) = d \end{array} \right\}, \tag{12}$$

where $\mathcal{L}^2$ ([0, $T$]) is the set of square integrable vector valued functions in [0, $T$] and $d(i)$ is a positive scalar. Then, System $\Theta$ in (2) and (8) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(t, i))$ if there exist a symmetric matrix-valued function $P(\cdot)$, a matrix function $L(\cdot)$ and a positive scalar $k$ such that

$$\begin{bmatrix} -\frac{1}{s}I & \begin{bmatrix} G_S^T(t) & G_T^T(t) \end{bmatrix} \\ (*)^T & \Psi_{22} \end{bmatrix} < 0,$$

$$\Psi_{22} = \begin{bmatrix} 0 & A_{ST}P(t) + B_S(t)L(t) \\ (*)^T & \begin{pmatrix} -\dot{P}(t) + (A_{TT}(t)P(t) + (\#)^T) \\ +(B_T(t)L(t) + (\#)^T) \end{pmatrix} \end{bmatrix} - k \begin{bmatrix} I - A_{SS}(t)\Phi(t) \\ -A_{TS}(t)\Phi(t) \end{bmatrix} (*)^T,$$

$$P(t) \leq \Gamma^{-1}(t), \forall t \in [0,T], \Gamma(t) = \mathbf{diag}\{\Gamma(t,i)|_{i=1}^{N}\},$$
$$P(0) > \frac{1}{s}\Gamma^{-1}(0), s = \frac{c_1+d}{c_2}. \tag{13}$$

**Proof**. Defining a scalar $s = \frac{c_1+d}{c_2}$, we rewrite Inequality (11) as the following expression form,

$$(*)^T \begin{bmatrix} -\dot{P}(t) & \begin{bmatrix} P(t) & L^T(t) & 0 \end{bmatrix} \\ (*)^T & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & sI \end{bmatrix} \end{bmatrix} \times \begin{bmatrix} I \\ \begin{bmatrix} A^T(t) \\ B^T(t) \\ G^T(t) \end{bmatrix} \end{bmatrix} < 0. \tag{14}$$

Replacing the expression of $A(t)$, $B(t)$ and $G(t)$, so that $A(t) = A_{\mathrm{TT}}(t) + A_{\mathrm{TS}}(t)\Phi(t)(I - A_{\mathrm{SS}}(t)\Phi(t))^{-1}A_{\mathrm{ST}}(t), B(t) = B_{\mathrm{T}}(t) + A_{\mathrm{TS}}(t)\Phi(t)(I - A_{\mathrm{SS}}(t)\Phi(t))^{-1}B_{\mathrm{S}}(t)$ and $G(t) = G_{\mathrm{T}}(t) + A_{\mathrm{TS}}(t)\Phi(t)(I - A_{\mathrm{SS}}(t)\Phi(t))^{-1}G_{\mathrm{S}}(t)$ are substituted into Inequality (14), we obtain that

$$(*)^T \begin{bmatrix} -\dot{P}(t) & \begin{bmatrix} P(t) & L^T(t) & 0 \end{bmatrix} \\ (*)^T & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & sI \end{bmatrix} \end{bmatrix} \times \begin{bmatrix} 0 & I \\ A_{\mathrm{ST}}^T(t) & A_{\mathrm{TT}}^T(t) \\ B_{\mathrm{S}}^T(t) & B_{\mathrm{T}}^T(t) \\ G_{\mathrm{S}}^T(t) & G_{\mathrm{T}}^T(t) \end{bmatrix} \times \begin{bmatrix} \Phi^T(t)\big(I - A_{\mathrm{SS}}^T(t)\Phi^T(t)\big)^{-1}A_{\mathrm{TS}}^T(t) \\ I \end{bmatrix} < 0. \tag{15}$$

The definition matrix $-F_1$ is as follows,

$$(*)^T \begin{bmatrix} -\dot{P}(t) & \begin{bmatrix} P(t) & L^T(t) & 0 \end{bmatrix} \\ (*)^T & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & sI \end{bmatrix} \end{bmatrix} \times \begin{bmatrix} 0 & I \\ A_{\mathrm{ST}}^T(t) & A_{\mathrm{TT}}^T(t) \\ B_{\mathrm{S}}^T(t) & B_{\mathrm{T}}^T(t) \\ G_{\mathrm{S}}^T(t) & G_{\mathrm{T}}^T(t) \end{bmatrix}. \tag{16}$$

Let $P_2$ and $H_1$ are expressed in the following matrix form respectively, $\begin{bmatrix} \Phi^T(t)\big(I - A_{\mathrm{SS}}^T(t)\Phi^T(t)\big)^{-1}A_{\mathrm{TS}}^T(t) \\ I \end{bmatrix}$ and $\begin{bmatrix} I & -\Phi^T(t) \end{bmatrix} \begin{bmatrix} I & 0 \\ A_{\mathrm{SS}}^T(t) & A_{\mathrm{TS}}^T(t) \end{bmatrix}$. According to $I - \Phi^T(t)A_{\mathrm{SS}}^T(t)$ is reversible, we obtain that $H_1$ is of row full rank, $P_2$ is of column full rank, and $H_1P_2 = 0$.

When $v = P_2\xi, \xi \in R^{\#}$, for any $v \neq 0$, we can get that $H_1v = 0$ implies $v^TF_1v > 0$. Therefore, according to Lemma 3, there exists a real scalar $k \in R$ such that

$$(*)^T \begin{bmatrix} -\dot{P}(t) & \begin{bmatrix} P(t) & L^T(t) & 0 \end{bmatrix} \\ (*)^T & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & sI \end{bmatrix} \end{bmatrix} \times \begin{bmatrix} 0 & I \\ A_{\mathrm{ST}}^T(t) & A_{\mathrm{TT}}^T(t) \\ B_{\mathrm{S}}^T(t) & B_{\mathrm{T}}^T(t) \\ G_{\mathrm{S}}^T(t) & G_{\mathrm{T}}^T(t) \end{bmatrix} - k \times (*)^T \begin{bmatrix} I - \Phi^T(t) \end{bmatrix} \times \begin{bmatrix} I & 0 \\ A_{\mathrm{SS}}^T(t) & A_{\mathrm{TS}}^T(t) \end{bmatrix} < 0. \tag{17}$$

Combined with Lemma 1, the half part proof is accomplished.

On the other hand, suppose Inequality (13) is established. Multiplying $v^T$ and $v$ from the left and right sides of Inequality (13) respectively, we obtain that

$$(*)^T \begin{bmatrix} -\dot{P}(t) & \begin{bmatrix} P(t) & L^T(t) & 0 \end{bmatrix} \\ (*)^T & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & sI \end{bmatrix} \end{bmatrix} \times \begin{bmatrix} 0 & I \\ A_{\mathrm{ST}}^T(t) & A_{\mathrm{TT}}^T(t) \\ B_{\mathrm{S}}^T(t) & B_{\mathrm{T}}^T(t) \\ G_{\mathrm{S}}^T(t) & G_{\mathrm{T}}^T(t) \end{bmatrix} \times \begin{bmatrix} \Phi^T(t)\big(I - A_{\mathrm{SS}}^T(t)\Phi^T(t)\big)^{-1}A_{\mathrm{TS}}^T(t) \\ I \end{bmatrix}$$
$$-k \times (*)^T \begin{bmatrix} I - \Phi^T(t) \end{bmatrix} \begin{bmatrix} I & 0 \\ A_{\mathrm{SS}}^T(t) & A_{\mathrm{TS}}^T(t) \end{bmatrix} \times \begin{bmatrix} \Phi^T(t)\big(I - A_{\mathrm{SS}}^T(t)\Phi^T(t)\big)^{-1}A_{\mathrm{TS}}^T(t) \\ I \end{bmatrix} < 0. \tag{18}$$

Noting that $H_1P_2 = 0$, we can obtain Inequality (11). The sufficiency proof is accomplished.

*Remark 1:* Compared with the available results (11), an attractive feature of Theorem 1 is that the left side of the first inequality in (13) is linearly dependent on $\dot{P}(t)$, $P(t)$, $L(t)$, $L^T(t)$ and $k$. For a small-scale problem, it is usually easy to verify the feasibility of the matrix inequality using existing DLMI solvers. On the other hand, note that all matrices $A_{*\#}(t)$ with $*, \# =$ T,S are block diagonal, and the subsystem connection matrix $\Phi(t)$ is sparse. Therefore, based on the existing results about solving sparse semi-definite programming problems [29], it can be seen that the condition in Theorem 1 is also effective for a moderate size problem.

*Remark 2:* When the matrix $\Sigma^2(t)$ of Inequality (13) is replaced by $\Phi(t)\Phi^T(t)$, results of Theorem 1 become valid for an arbitrary subsystem connection matrix $\Phi(t)$.

For a networked system with a very large scale, numerical computation prohibitions may still arise in verifying the condition of Theorem 1. Therefore, a condition that depends only on each subsystem parameter is more attractive. The following lemma is introduced and its proof is omitted.

Note that for any $\beta > 0$, from Lemma 2 we get

$$\begin{bmatrix} I & -\Phi^T \\ -\Phi & \Phi\Phi^T \end{bmatrix} \geq (1-\beta)\begin{bmatrix} I \\ 0 \end{bmatrix}(*)^T + (1 - \frac{1}{\beta})\begin{bmatrix} 0 \\ \Phi \end{bmatrix}(*)^T. \tag{19}$$

According to $\Phi(t)\Phi^T(t) = \Sigma^2(t)$ and the first inequality in (13) of Theorem 1, another sufficient condition for the FTB of System $\Theta$ is derived as follows.

*Theorem 2:* Given the following class of signals

$$\mathcal{W} := \left\{ \begin{array}{c} w(\cdot) = col\{w(\cdot,i)|_{i=1}^{N}\} | w(\cdot,i) \in \mathcal{L}^2([0,T]), \\ \sum_{i=1}^{N}\int_0^T w^T(\tau)w(\tau)d\tau \le d, \\ \int_0^T w^T(\tau,i)w(\tau,i)d\tau \le d(i), \sum_{i=1}^N d(i) = d \end{array} \right\}, \tag{20}$$

where $\mathcal{L}^2$ ($[0, T]$) is the set of square integrable vector valued functions in $[0, T]$ and $d(i)$ is a positive scalar. Then, System $\Theta$ in (2) and (8) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(t, i))$ if there exist a symmetric matrix-valued function $P(\cdot, i)$, a matrix function $L(\cdot, i)$ and two scalars $\alpha > 0, 0 < \beta < 1$ such that

$$\begin{aligned}
&\begin{bmatrix} -\frac{1}{s}I & \left[G_{\mathrm{S}}^T(t,i)\ G_{\mathrm{T}}^T(t,i)\right] \\ (*)^T & \Upsilon_{22}(t,i)\end{bmatrix} < 0,\\
&P(t,i) \ge \Gamma(t,i), \forall t\in[0,T],\\
&P(0,i) < \frac{1}{s}\Gamma(0,i), s = \frac{c_1+d}{c_2},\\
&\Upsilon_{22}(t,i)\\
&= \begin{bmatrix} 0 & A_{\mathrm{ST}}(t,i)P(t,i)+B_{\mathrm{S}}(t,i)L(t,i) \\ (*)^T & \begin{pmatrix} -\dot P(t,i) + (A_{\mathrm{TT}}(t,i)P(t,i)+(\#)^T) \\ +(B_{\mathrm{T}}^T(t,i)L(t,i)+(\#)^T)\end{pmatrix}\end{bmatrix}\\
&+\alpha\left(\begin{bmatrix} A_{\mathrm{SS}}(t,i)\Sigma_i^2(t)A_{\mathrm{SS}}^T(t,i)-\beta I & A_{\mathrm{SS}}(t,i)\Sigma_i^2(t)A_{\mathrm{TS}}^TP(t,i) \\ A_{\mathrm{TS}}(t,i)\Sigma_i^2(t)A_{\mathrm{SS}}^TP(t,i) & A_{\mathrm{TS}}(t,i)\Sigma_i^2(t)A_{\mathrm{TS}}^T(t,i)\end{bmatrix}\right),\\
&P(t,i)\ge\Gamma(t,i),\forall t\in[0,T],\\
&P(0,i) < \frac{1}{s}\Gamma(0,i), s=\frac{c_1+d}{c_2}, i=1,\cdots,N,
\end{aligned} \tag{21}$$

or there exist symmetric matrix-valued functions $P(\cdot, i)$, a matrix function $L(\cdot, i)$ and two scalars $\alpha > 0, \beta > 1$ such that

$$\begin{aligned}
&\begin{bmatrix} -\frac{1}{s}I & \left[G_{\mathrm{S}}^T(t,i)\ G_{\mathrm{T}}^T(t,i)\right] \\ (*)^T & \Pi_{22}(t,i)\end{bmatrix} < 0,\\
&P(t,i) \ge \Gamma(t,i), \forall t\in[0,T],\\
&P(0,i) < \frac{1}{s}\Gamma(0,i), s = \frac{c_1+d}{c_2},\\
&\Pi_{22}(t,i)\\
&= \begin{bmatrix} 0 & A_{\mathrm{ST}}(t,i)P(t,i)+B_{\mathrm{S}}(t,i)L(t,i) \\ (*)^T & \begin{pmatrix} -\dot P(t,i) + (A_{\mathrm{TT}}(t,i)P(t,i)+(\#)^T) \\ +(B_{\mathrm{T}}^T(t,i)L(t,i)+(\#)^T)\end{pmatrix}\end{bmatrix}\\
&-\alpha\left(\begin{bmatrix} A_{\mathrm{SS}}(t,i)\Sigma_i^2(t)A_{\mathrm{SS}}^T(t,i)-\beta I & A_{\mathrm{SS}}(t,i)\Sigma_i^2(t)A_{\mathrm{TS}}^TP(t,i) \\ A_{\mathrm{TS}}(t,i)\Sigma_i^2(t)A_{\mathrm{SS}}^TP(t,i) & A_{\mathrm{TS}}(t,i)\Sigma_i^2(t)A_{\mathrm{TS}}^T(t,i)\end{bmatrix}\right),\\
&P(t,i)\ge\Gamma(t,i),\forall t\in[0,T],\\
&P(0,i) < \frac{1}{s}\Gamma(0,i), s=\frac{c_1+d}{c_2}, i=1,\cdots,N.
\end{aligned} \tag{22}$$

*Proof:* From Inequality (19) and the proof of Theorem 1, a sufficient condition of the FTB of System $\Theta$ can be obtained that there exist matrix $P(t)$, $L(t)$ and two real numbers $k, \beta > 0$ such that

$$\begin{aligned}
&(*)^T\begin{bmatrix} -\dot P(t) & \left[P(t)\ L^T(t)\ 0\right] \\ (*)^T & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & sI \end{bmatrix}\end{bmatrix}\\
&\times\begin{bmatrix} 0 & I \\ A_{\mathrm{ST}}^T(t) & A_{\mathrm{TT}}^T(t) \\ B_{\mathrm{S}}^T(t) & B_{\mathrm{T}}^T(t) \\ G_{\mathrm{S}}^T(t) & G_{\mathrm{T}}^T(t)\end{bmatrix}\\
&-k\times(*)^T\left((1-\beta)\begin{bmatrix} I \\ 0\end{bmatrix}(*)^T + \left(1-\frac{1}{\beta}\right)\begin{bmatrix} 0 \\ \Phi(t)\end{bmatrix}(*)^T\right)\\
&\times\begin{bmatrix} I & 0 \\ A_{\mathrm{SS}}^T(t) & A_{\mathrm{TS}}^T(t)\end{bmatrix} < 0,\\
&P(t) \le \Gamma^{-1}(t), \forall t\in[0,T], \Gamma(t) = \mathbf{diag}\{\Gamma(t,i)|_{i=1}^N\},\\
&P(0) > \frac{1}{s}\Gamma^{-1}(0), s = \frac{c_1+d}{c_2}.
\end{aligned} \tag{23}$$

Since $k$ is a variable, let $\alpha = k \times \frac{1-\beta}{\beta}$. Combined Lemmas 1 and 2, with $P(t) = \mathbf{diag}\left\{P(t,i)|_{i=1}^N\right\}$, the proof can be accomplished.

*Remark 3:* Compared with Lemma 4 and Theorem 1, the inequations in Theorem 2 depend on the parameters of each individual subsystem when $\alpha$ is fixed. This means that for a large-scale networked system, the calculation cost of Theorem 2 is significantly lower than that of Theorem 1, which has been verified by numerical simulations. Inequation (19) is a sufficient condition and the resulting Theorem 2 based on parameters of each individual subsystem is also a sufficient condition, so Theorem 2 is more conservative with respect to Lemma 4 and Theorem 1, and more practical in many engineering problems.

*Remark 4:* Generally, for a large-scale networked system, the parameters of some subsystems are considered to be the same, in this case, the computational efficiency of the conditions in Theorem 2 is further highlighted.

In the following we discuss the FTB analysis for a networked dynamic system constituted by many linear time invariant (LTI) subsystems. A sufficient FTB condition for System (9) with LTI dynamic and $u(t)$ is a constant input is provided in [19] based on which we begin the derivation for the LTI case of System (2) and (8).

*Lemma 5 ([19]):* Consider the following kind of constant signals

$$\mathcal{W} := \left\{w | w^T w \le d\right\},$$

where $d$ is a nonnegative scalar. Then, System (9) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, R)$ if, letting $\tilde{Q}_1 = R^{-1/2}Q_1R^{-1/2}$, there exist a nonnegative scalar $\alpha$, two symmetric positive definite matrices $Q_1$, $Q_2$ and a matrix $L$ such that

$$\begin{bmatrix} A\tilde{Q}_1 + \tilde{Q}_1 A^T + BL + L^T B^T - \alpha\tilde{Q}_1 & GQ_2 \\ Q_2 G^T & -\alpha Q_2 \end{bmatrix} < 0, \quad (24)$$

$$\frac{c_1}{\lambda_{\min}(Q_1)} + \frac{d}{\lambda_{\min}(Q_2)} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q_1)}, \quad (25)$$

where $\lambda_{max}(Q)$, $\lambda_{min}(Q)$ are the maximum eigenvalue and minimum eigenvalue of the parameter, respectively. In this case a controller gain which FTB is $K(t) = L(t)Q^{-1}(t)$.

The condition (25) can be guaranteed by imposing the inequality

$$I < Q < \frac{c_2}{c_1} e^{-\alpha T} I, \quad (26)$$

which is converted into an LMI based condition.

Similar to Lemma 4, when the networked system have a large number of subsystems, FTB computation may encounter prohibitive implementation difficulties. We provide a computationally attractive sufficient condition for FTB analysis of the networked system $\Theta$. Its derivations are omitted for the similarity with Theorem 1.

*Theorem 3:* Given the following class of constant signals

$$\mathcal{W} := \left\{ \begin{array}{c} w(\cdot) = col\{w(\cdot, i)|_{i=1}^{N}\} | \sum_{i=1}^{N} w^T w \le d, \\ w^T(i)\, w(i) \le d(i),\ \sum_{i=1}^{N} d(i) = d \end{array} \right\}. \quad (27)$$

Then, System $\Theta$ in (2) and (8) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(t, i))$ if, letting $\tilde{Q}_1 = \Gamma^{-1/2} Q_1 \Gamma^{-1/2}$, $\Gamma = \mathbf{diag}\{\Gamma(i)|_{i=1}^{N}\}$, there exist a nonnegative scalar $\alpha$, two symmetric positive definite matrices $Q_1$, $Q_2$ and a matrix $L$ such that

$$\begin{bmatrix} \frac{1}{\alpha} G_S Q_2 G_S^T + (A_{SS}\Phi + (\#)^T) - I - A_{SS}\Sigma^2 A_{SS}^T & (*)^T \\ \tilde{Q}_1 A_{ST}^T + L^T B_S^T + \frac{1}{\alpha} G_T Q_2 G_S^T + A_{TS}\Phi - A_{TS}\Sigma^2 A_{SS}^T & \begin{array}{c} (-\alpha\tilde{Q}_1 + (A_{TT}\tilde{Q}_1 + (\#)^T) + (B_T L \\ +(\#)^T) + \frac{1}{\alpha} G_T Q_2 G_T^T - A_{TS}\Sigma^2 A_{TS}^T) \end{array} \end{bmatrix} < 0, \quad (28)$$

$$\frac{c_1}{\lambda_{\min}(Q_1)} + \frac{d}{\lambda_{\min}(Q_2)} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q_1)}. \quad (29)$$

The following Theorem 4 provides a sufficient FTB condition for the system $\Theta$ that depends on the parameters of each individual subsystem, which is computationally valid. Based on the inequality in (19) and Theorem 3, the proof can be obtained straightly and omitted here.

*Theorem 4:* Given the following class of constant signals

$$\mathcal{W} := \left\{ \begin{array}{c} w(\cdot) = col\{w(\cdot, i)|_{i=1}^{N}\} | \sum_{i=1}^{N} w^T w \le d, \\ w^T(i)\, w(i) \le d(i),\ \sum_{i=1}^{N} d(i) = d \end{array} \right\}. \quad (30)$$

Then, System $\Theta$ in (2) and (8) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(i))$ if, letting $\tilde{Q}_1 = \Gamma(i)^{-1/2} Q_1 \Gamma(i)^{-1/2}$, there exist two scalars $\alpha > 0, \beta > 1$, two positive definite matrices $Q_1(i)$, $Q_2(i)$ and a matrix $L(i)$ for each subsystem, such that

$$\begin{bmatrix} \begin{array}{l} \frac{1}{\alpha} G_S(i) Q_2(i) G_S^T(i) + \beta I - A_{SS}(i)\Sigma_i^2 A_{SS}^T(i) \end{array} & (*)^T \\ \begin{array}{l} (\tilde{Q}_1(i) A_{ST}^T(i) + L^T(i) B_S^T(i) + \frac{1}{\alpha} G_T(i) Q_2(i) \\ \times G_S^T(i) - A_{TS}(i)\Sigma_i^2 A_{SS}^T(i)) \end{array} & \begin{array}{l} (-\alpha\tilde{Q}_1(i) + A_{TT}(i)\tilde{Q}_1(i) + B_T(i)L(i) \\ +\tilde{Q}_1(i) A_{TT}^T(i) + L^T(i) B_T^T(i) + \frac{1}{\alpha} G_T(i) \\ \times Q_2(i) G_T^T(i) - A_{TS}(i)\Sigma_i^2(i) A_{TS}^T(i)) \end{array} \end{bmatrix} < 0, \quad (31)$$

$$\frac{c_1}{\lambda_{\min}(Q_1(i))} + \frac{d}{\lambda_{\min}(Q_2(i))} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q_1(i))}, \quad (32)$$

or there exist two scalars $\alpha > 0, 0 < \beta < 1$, two positive definite matrices $Q_1(i)$, $Q_2(i)$ and a matrix $L(i)$ for each subsystem, such that

$$\begin{bmatrix} \begin{array}{l} \frac{1}{\alpha} G_S(i) Q_2(i) G_S^T(i) - \beta I + A_{SS}(i)\Sigma_i^2 A_{SS}^T(i) \end{array} & (*)^T \\ \begin{array}{l} (\tilde{Q}_1(i) A_{ST}^T(i) + L^T(i) B_S^T(i) + \frac{1}{\alpha} G_T(i) \\ \times Q_2(i) G_S^T(i) + A_{TS}(i)\Sigma_i^2 A_{SS}^T(i)) \end{array} & \begin{array}{l} (-\alpha\tilde{Q}_1(i) + A_{TT}(i)\tilde{Q}_1(i) + B_T(i)L(i) \\ +\tilde{Q}_1(i) A_{TT}^T(i) + L^T(i) B_T^T(i) + \frac{1}{\alpha} G_T(i) \\ \times Q_2(i) G_T^T(i) + A_{TS}(i)\Sigma_i^2 A_{TS}^T(i)) \end{array} \end{bmatrix} < 0, \quad (33)$$

$$\frac{c_1}{\lambda_{\min}(Q_1(i))} + \frac{d}{\lambda_{\min}(Q_2(i))} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q_1(i))}. \quad (34)$$

## IV. DISTRIBUTED OUTPUT FEEDBACK CONTROLLER DESIGN

In the distributed networked control system, each subsystem has a coupling relationship with the whole system, and one sub-controller will synthesize its own information and that of other coupling subsystems to generate the control quantity. Distributed output feedback control can obtain better signal accuracy than traditional output feedback control, so it will produce better control effect than traditional output feedback control. The distributed output feedback controller designed in this paper can make the system that cannot be judged to be finite-time boundedness regain finite-time boundedness.

We expand the state space model of subsystem $\Theta_i$, a control input signal $d(t, i)$ and a measurement output signal $g(t, i)$ are introduced, and the following state space description is obtained, (To simplify the formula writing, $(t, i)$ in some time-varying systems is abbreviated as $(i)$)

$$\begin{bmatrix} \dot{x}(t, i) \\ z(t, i) \\ y(t, i) \\ g(t, i) \end{bmatrix} = \begin{bmatrix} A_{TT}(i) & A_{TS}(i) & B_{TD}(i) & B_{TU}(i) \\ A_{ST}(i) & A_{SS}(i) & B_{SD}(i) & B_{SU}(i) \\ C_{TZ}(i) & C_{SC}(i) & D_{ZD}(i) & D_{ZU}(i) \\ C_{TY}(i) & C_{SY}(i) & D_{YD}(i) & D_{YU}(i) \end{bmatrix}$$

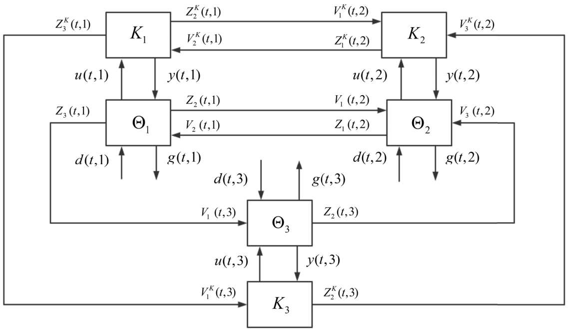


**FIGURE 1.** **The distributed output feedback control structure diagram of networked system $\Theta$ ($N \mathcal{D} 3$).**

$$\times \begin{bmatrix} x(t,i) \\ v(t,i) \\ u(t,i) \\ d(t,i) \end{bmatrix}, \tag{35}$$

these subsystems are also connected by

$$v(t) = \Phi(t)z(t). \tag{36}$$

We design a distributed output feedback controller $K$ which is a networked interconnection system with $N$ subsystems. The state space model of the distributed output feedback controller subsystem $K_i$ connected to System (35) can be expressed as follows,

$$\begin{bmatrix} \dot{x}^K(t,i) \\ z^K(t,i) \\ d(t,i) \end{bmatrix} = \begin{bmatrix} A_{\mathrm{TT}}^K(i) & A_{\mathrm{TS}}^K(i) & B_{\mathrm{T}}^K(i) \\ A_{\mathrm{ST}}^K(i) & A_{\mathrm{SS}}^K(i) & B_{\mathrm{S}}^K(i) \\ C_{\mathrm{T}}^K(i) & C_{\mathrm{S}}^K(i) & D_{\mathrm{T}}^K(i) \end{bmatrix} \times \begin{bmatrix} x^K(t,i) \\ v^K(t,i) \\ g(t,i) \end{bmatrix}, \tag{37}$$

in which, $x^K(t,i)$ is the state vector of the controller subsystem, $z^K(t,i)$ and $v^K(t,i)$ are internal output and internal input vectors of the controller subsystem, respectively. Information can be exchanged between subsystems through internal input and output signals.

When the connection structure between subsystems is the same as that of the controlled networked system, the controller subsystem $K_i$ and subsystem $\Theta_i$ have the same set of incoming and outgoing neighbors. The structure of distributed output feedback control system is shown in Figure 1.

Through connecting the dynamic subsystem (35) with the controller subsystem (37), we get the extended closed-loop subsystem state space model, which is described as follows,

$$\begin{bmatrix} \dot{x}^C(t,i) \\ z^C(t,i) \\ y^C(t,i) \end{bmatrix} = \begin{bmatrix} \bar{A}_{\mathrm{TT}}(i) + \bar{B}_{\mathrm{TU}}(i)K(i)\bar{C}_{\mathrm{TY}}(i) & \bar{A}_{\mathrm{TS}}(i) + \bar{B}_{\mathrm{TU}}(i) \times K(i)\bar{C}_{\mathrm{SY}}(i) & \bar{B}_{\mathrm{TD}}(i) + \bar{B}_{\mathrm{TU}}(i)K(i)\bar{D}_{\mathrm{YD}}(i) \\ \bar{A}_{\mathrm{ST}}(i) + \bar{B}_{\mathrm{SU}}(i)K(i)\bar{C}_{\mathrm{TY}}(i) & \bar{A}_{\mathrm{SS}}(i) + \bar{B}_{\mathrm{SU}}(i) \times K(i)\bar{C}_{\mathrm{SY}}(i) & \bar{B}_{\mathrm{SD}}(i) + \bar{B}_{\mathrm{SU}}(i)K(i)\bar{D}_{\mathrm{YD}}(i) \\ \bar{C}_{\mathrm{TZ}}(i) + \bar{D}_{\mathrm{ZU}}(i)K(i)\bar{C}_{\mathrm{TY}}(i) & \bar{C}_{\mathrm{SZ}}(i) + \bar{D}_{\mathrm{ZU}}(i) \times K(i)\bar{C}_{\mathrm{SY}}(i) & D_{\mathrm{ZD}}(i) + \bar{D}_{\mathrm{ZU}}(i)K(i)\bar{D}_{\mathrm{YD}}(i) \end{bmatrix} \times \begin{bmatrix} x^C(t,i) \\ v^C(t,i) \\ u^C(t,i) \end{bmatrix}, \tag{38}$$

where

$$K(i) = \begin{bmatrix} A_{\mathrm{TT}}^K(i) & A_{\mathrm{TS}}^K(i) & B_{\mathrm{T}}^K(i) \\ A_{\mathrm{ST}}^K(i) & A_{\mathrm{SS}}^K(i) & B_{\mathrm{S}}^K(i) \\ C_{\mathrm{T}}^K(i) & C_{\mathrm{S}}^K(i) & D_{\mathrm{T}}^K(i) \end{bmatrix}, \tag{39}$$

$$\begin{aligned} \bar{A}_{\mathrm{TT}}(i) &= \begin{bmatrix} A_{\mathrm{TT}}(i) & 0 \\ 0 & 0 \end{bmatrix}, \bar{A}_{\mathrm{TS}}(i) = \begin{bmatrix} A_{\mathrm{TS}}(i) & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{A}_{\mathrm{ST}}(i) &= \begin{bmatrix} A_{\mathrm{ST}}(i) & 0 \\ 0 & 0 \end{bmatrix}, \bar{A}_{\mathrm{SS}}(i) = \begin{bmatrix} A_{\mathrm{SS}}(i) & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{B}_{\mathrm{TU}}(i) &= \begin{bmatrix} 0 & 0 & B_{\mathrm{TU}}(i) \\ I & 0 & 0 \end{bmatrix}, \bar{B}_{\mathrm{TD}}(i) = \begin{bmatrix} B_{\mathrm{TD}}(i) \\ 0 \end{bmatrix}, \\ \bar{C}_{\mathrm{TY}}(i) &= \begin{bmatrix} 0 & I \\ 0 & 0 \\ C_{\mathrm{TY}}(i) & 0 \end{bmatrix}, \bar{C}_{\mathrm{SY}}(i) = \begin{bmatrix} 0 & 0 \\ 0 & I \\ C_{\mathrm{SY}}(i) & 0 \end{bmatrix}, \\ \bar{B}_{\mathrm{SU}}(i) &= \begin{bmatrix} 0 & 0 & B_{\mathrm{SU}}(i) \\ 0 & I & 0 \end{bmatrix}, \bar{B}_{\mathrm{SD}}(i) = \begin{bmatrix} B_{\mathrm{SD}}(i) \\ 0 \end{bmatrix}, \\ \bar{C}_{\mathrm{TZ}}(i) &= \begin{bmatrix} C_{\mathrm{TZ}}(i) & 0 \end{bmatrix}, \bar{C}_{\mathrm{SZ}}(i) = \begin{bmatrix} C_{\mathrm{SZ}}(i) & 0 \end{bmatrix}, \\ \bar{D}_{\mathrm{ZU}}(i) &= \begin{bmatrix} 0 & 0 & D_{\mathrm{ZU}}(i) \end{bmatrix}, \bar{D}_{\mathrm{YD}}(i) = \begin{bmatrix} 0 \\ 0 \\ D_{\mathrm{YD}}(i) \end{bmatrix}. \end{aligned} \tag{40}$$

*Definition 3.* (Finite-Time Control via Output Feedback) Consider the linear system (35), find a dynamic output feedback controller (37) in the form (38), where $x^C(t)$ has the same dimension of $x(t)$, such that the closed loop system obtained by the connection of (35) and (38) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(t,i), \Gamma^K(t,i))$.

Based on the extended closed-loop subsystem state space expression (38), it is applied to FTB, the following sufficient conditions for FTB analysis of closed-loop system are obtained. Its derivations are omitted for the similarity with Theorem 1.

*Theorem 5:* Given the following class of signals

$$\mathcal{W} := \left\{ \begin{array}{c} w(\cdot) = \mathrm{col}\{w(\cdot,i)|_{i=1}^N\} | w(\cdot,i) \in \mathcal{L}^2([0,T]), \\ \sum_{i=1}^N \int_0^T w^T(\tau)w(\tau)d\tau \le d, \\ \int_0^T w^T(\tau,i)w(\tau,i)d\tau \le d(i), \sum_{i=1}^N d(i) = d \end{array} \right\}, \tag{41}$$

where $\mathcal{L}^2([0,T])$ is the set of square integrable vector valued functions in $[0,T]$ and $d(i)$ is a positive scalar. Then, System $\Theta$ in (2) and (8) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(t,i))$ if there exist a symmetric matrix-valued function $P(\cdot)$ such that

$$\begin{bmatrix} \Psi_{11} & \begin{bmatrix} P(t)(\bar{B}_{\mathrm{SD}}(t) + \bar{B}_{\mathrm{SU}}(t)K(t)\bar{D}_{\mathrm{YD}}(t)) \\ P(t)(\bar{B}_{\mathrm{TD}}(t) + \bar{B}_{\mathrm{TU}}(t)K(t)\bar{D}_{\mathrm{YD}}(t)) \end{bmatrix} \\ (*)^T & -\frac{1}{s}I \end{bmatrix} < 0,$$

$$
\Psi_{11} = \begin{bmatrix} \begin{pmatrix} ((\bar{A}_{SS}(t)+\bar{B}_{SU}(t)K(t)\bar{C}_{SY}(t)) \\ \times\Phi(t)+(\#)^T)-I-(\bar{A}_{SS}(t) \\ +\bar{B}_{SU}(t)K(t)\bar{C}_{SY}(t))\Sigma^2(t)(*)^T \end{pmatrix} & (*)^T \\ \begin{pmatrix} (\bar{A}_{ST}(t)+\bar{B}_{SU}(t)K(t)\bar{C}_{TY}(t))^T P(t) \\ +(\bar{A}_{TS}(t)+\bar{B}_{TU}(t)K(t)\bar{C}_{SY}(t))\Phi(t) \\ -(\bar{A}_{TS}(t)+\bar{B}_{TU}(t)K(t)\bar{C}_{SY}(t)) \\ \times\Sigma^2(t)(\bar{A}_{SS}(t)+\bar{B}_{SU}(t)K(t)\bar{C}_{SY}(t))^T \end{pmatrix} & \begin{pmatrix} \dot{P}(t)+(P(t)(\bar{A}_{TT}(t)+\bar{B}_{TU}(t) \\ \times K(t)\bar{C}_{TY}(t))+(\#)^T)-(\bar{A}_{TS}(t) \\ +\bar{B}_{TU}(t)K(t)\bar{C}_{SY}(t))\Sigma^2(t)(*)^T \end{pmatrix} \end{bmatrix}
$$

$$
\begin{aligned} & P(t)\geq\Gamma(t), \forall t\in[0,T], \Gamma(t)=\operatorname{diag}\{\Gamma(t,i)|_{i=1}^N\}, \\ & P(0)<\frac{1}{s}\Gamma(0), s=\frac{c_1+d}{c_2}. \end{aligned} \tag{42}
$$

*Remark 5:* It is noted that the matrix parameter in Theorem 5 contains the unknown parameter $k(i)$ of the controller subsystem. If the controller parameter is undetermined, the above closed-loop conditions are nonconvex optimization problems. Next, by eliminating the controller parameters $\{K(i), i=1,\cdots,N\}$ from the inequality conditions, the sufficient conditions of the closed-loop system FTB based on a single subsystem are obtained. The following lemma plays an important role in our method.

*Lemma 6 ([30]):* Given a symmetric matrix $\Omega\in R^n$ and two matrices $V\in R^{r\times n}$, $G\in R^{s\times n}$, consider the problem of finding some matrix $\Xi\in R^{r\times s}$ such that

$$
\Omega+V^T\Xi^T G+G^T\Xi V<0. \tag{43}
$$

Denote by $W_V$, $W_G$ any matrices whose columns form bases of the null bases of $V$ and $G$, respectively. Then (43) is solvable for $\Xi$ if and only if

$$
\begin{cases} W_V^T\Omega W_V<0, \\ W_G^T\Omega W_G<0. \end{cases} \tag{44}
$$

The Theorem 5 combining Lemma 6, the sufficient condition that the design of a distributed output feedback controller for each individual subsystem can be obtained as follows.

*Theorem 6:* Given the following class of signals

$$
\mathcal{W}:=\left\{ \begin{array}{c} w(\cdot)=col\{w(\cdot,i)|_{i=1}^N\}|w(\cdot,i)\in\mathcal{L}^2([0,T]), \\ \sum_{i=1}^N\int_0^T w^T(\tau)w(\tau)d\tau\leq d, \\ \int_0^T w^T(\tau,i)w(\tau,i)d\tau\leq d(i), \sum_{i=1}^N d(i)=d \end{array} \right\}, \tag{45}
$$

where $\mathcal{L}^2([0,T])$ is the set of square integrable vector valued functions in $[0,T]$ and $d(i)$ is a positive scalar. Then, System $\Theta$ in (38) is FTB with respect to $(c_1,c_2,\mathcal{W},T,\Gamma(t,i),\Gamma^K(t,i))$ if there exist a symmetric matrix-valued functions $P(\cdot)$, two block matrices $W_V(t,i)$, $W_G(t,i)$ and two scalars $0<y_1<y_2$ such that

$$
W_V^T(i)\begin{bmatrix} R_1(i)\dot{P}_1(i)R_1(i)+(A_{TT}(i)R_1(i)+(\#)^T) & R_1(i)A_{ST}^T(i) & A_{TS}(i) & B_{TD}(i) \\ A_{ST}(i)R_1(i) & -y_1 I & A_{SS}(i) & B_{SD}(i) \\ A_{TS}^T(i) & A_{SS}^T(i) & (y_2\Sigma_i^2)^{-1} & 0 \\ B_{TD}^T(i) & B_{SD}^T(i) & 0 & -\frac{1}{s}I \end{bmatrix} W_V(i)<0, \tag{46}
$$

$$
W_G^T(i)\begin{bmatrix} \dot{P}_1(i)+(P_1(i)A_{TT}(i)+(\#)^T) & A_{ST}^T(i) & P_1(i)A_{TS}(i) & P_1(i)B_{TD}(i) \\ A_{ST}(i) & -y_1 I & A_{SS}(i) & B_{SD}(i) \\ A_{TS}^T(i)P_1(i) & A_{SS}^T(i) & (y_2\Sigma_i^2)^{-1} & 0 \\ B_{TD}^T(i)P_1(i) & B_{SD}^T(i) & 0 & -\frac{1}{s}I \end{bmatrix} W_G(i)<0, \tag{47}
$$

$$
\begin{aligned} & P(t,i)\geq\Gamma(t,i), \forall t\in[0,T], \\ & P(0,i)<\frac{1}{s}\Gamma(0,i), s=\frac{c_1+d}{c_2}, i=1,\cdots,N, \end{aligned} \tag{48}
$$

$$
\begin{bmatrix} P_1(i) & I \\ I & R_1(i) \end{bmatrix}\geq 0, \tag{49}
$$

or there exist a symmetric matrix-valued functions $P(\cdot)$, two block matrices $W_V(t,i)$, $W_G(t,i)$ and two scalars $0<y_2<y_1$ such that

$$
W_V^T(i)\begin{bmatrix} R_1(i)\dot{P}_1(i)R_1(i)+(A_{TT}(i)R_1(i)+(\#)^T) & R_1(i)A_{ST}^T(i) & A_{TS}(i) & B_{TD}(i) \\ A_{ST}(i)R_1(i) & y_1 I & A_{SS}(i) & B_{SD}(i) \\ A_{TS}^T(i) & A_{SS}^T(i) & (-y_2\Sigma_i^2)^{-1} & 0 \\ B_{TD}^T(i) & B_{SD}^T(i) & 0 & -\frac{1}{s}I \end{bmatrix} W_V(i)<0, \tag{50}
$$

$$
W_G^T(i)\begin{bmatrix} \dot{P}_1(i)+(P_1(i)A_{TT}(i)+(\#)^T) & A_{ST}^T(i) & P_1(i)A_{TS}(i) & P_1(i)B_{TD}(i) \\ A_{ST}(i) & y_1 I & A_{SS}(i) & B_{SD}(i) \\ A_{TS}^T(i)P_1(i) & A_{SS}^T(i) & (-y_2\Sigma_i^2)^{-1} & 0 \\ B_{TD}^T(i)P_1(i) & B_{SD}^T(i) & 0 & -\frac{1}{s}I \end{bmatrix} W_G(i)<0, \tag{51}
$$

$$
\begin{aligned} & P(t,i)\geq\Gamma(t,i), \forall t\in[0,T], \\ & P(0,i)<\frac{1}{s}\Gamma(0,i), s=\frac{c_1+d}{c_2}, i=1,\cdots,N, \end{aligned} \tag{52}
$$

$$
\begin{bmatrix} P_1(i) & I \\ I & R_1(i) \end{bmatrix}\geq 0, \tag{53}
$$

where $P(t,i)=\begin{bmatrix} P_1(i) & P_2(i) \\ P_2^T(i) & P_3(i) \end{bmatrix}$, $P^{-1}(t,i)=\begin{bmatrix} R_1(i) & R_2(i) \\ R_2^T(i) & R_3(i) \end{bmatrix}$. The derived DLMI-based finite-time boundedness conditions only dependent on the parameters of each individual subsystem. This sufficient condition makes use of the sparseness of subsystem connection matrix and the diagonal block characteristics of system parameters. Its proof can be obtained as follows.

*Proof:* Derivation of similar Theorem 2, when $0 < y_1 < y_2$, We rewrite Theorem 5 as the following individual subsystem form,

$$\begin{bmatrix} \mathrm{E}_{11} & \mathrm{E}_{12} \\ (*)^T & \begin{bmatrix} \left(y_2\Sigma_i^2\right)^{-1} & 0 \\ 0 & -\frac{1}{s}I \end{bmatrix} \end{bmatrix} < 0,$$

$$\mathrm{E}_{11} = \begin{bmatrix} \begin{pmatrix} P^{-1}(i)\dot{P}(i)P^{-1}(i) \\ +((\bar{A}_{\mathrm{TT}}(i)+\bar{B}_{\mathrm{TU}}(i)K(i) \\ \times\bar{C}_{\mathrm{TY}}(i))P^{-1}(i)+(\#)^T) \end{pmatrix} & (*)^T \\ \left(\bar{A}_{\mathrm{ST}}(i)+\bar{B}_{\mathrm{SU}}(i)K(i)\bar{C}_{\mathrm{TY}}(i)\right)P^{-1}(i) & -y_1 I \end{bmatrix},$$

$$\mathrm{E}_{12} = \begin{bmatrix} \bar{A}_{\mathrm{TS}}(i)+\bar{B}_{\mathrm{TU}}(i)K(i)\bar{C}_{\mathrm{SY}}(i) & \bar{B}_{\mathrm{TD}}(i)+\bar{B}_{\mathrm{TU}}(i)K(i)\bar{D}_{\mathrm{YD}}(i) \\ \bar{A}_{\mathrm{SS}}(i)+\bar{B}_{\mathrm{SU}}(i)K(i)\bar{C}_{\mathrm{SY}}(i) & \bar{B}_{\mathrm{SD}}(i)+\bar{B}_{\mathrm{SU}}(i)K(i)\bar{D}_{\mathrm{YD}}(i) \end{bmatrix}. \quad (54)$$

We rewrite Inequality (54) as the following expression form,

$$\begin{bmatrix} P^{-1}(i)\dot{P}(i)P^{-1}(i)+(\bar{A}_{\mathrm{TT}}(i)P^{-1}(i)+(\#)^T) & P^{-1}(i)\bar{A}_{\mathrm{ST}}^T(i) & \bar{A}_{\mathrm{TS}}(i) & \bar{B}_{\mathrm{TD}}(i) \\ \bar{A}_{\mathrm{ST}}(i)P^{-1}(i) & -y_1 I & \bar{A}_{\mathrm{SS}}(i) & \bar{B}_{\mathrm{SD}}(i) \\ \bar{A}_{\mathrm{TS}}^T(i) & \bar{A}_{\mathrm{SS}}^T(i) & \left(y_2\Sigma_i^2\right)^{-1} & 0 \\ \bar{B}_{\mathrm{TD}}^T(i) & \bar{B}_{\mathrm{SD}}^T(i) & 0 & -\frac{1}{s}I \end{bmatrix}$$
$$+\begin{bmatrix} \bar{B}_{\mathrm{TU}}(i) \\ \bar{B}_{\mathrm{SU}}(i) \\ 0 \\ 0 \end{bmatrix} K(i)\left[\bar{C}_{\mathrm{TY}}(i)P^{-1}(i)\ 0\ \bar{C}_{\mathrm{SY}}(i)\ \bar{D}_{\mathrm{YD}}(i)\right] + \left[\bar{C}_{\mathrm{TY}}(i)P^{-1}(i)\ 0\ \bar{C}_{\mathrm{SY}}(i)\ \bar{D}_{\mathrm{YD}}(i)\right]^T$$
$$\times K^T(i)\begin{bmatrix} \bar{B}_{\mathrm{TU}}(i) \\ \bar{B}_{\mathrm{SU}}(i) \\ 0 \\ 0 \end{bmatrix}^T < 0. \quad (55)$$

According to Lemma 6 and Inequality (55), matrix $\Omega$ is defined as follows,

$$\Omega(i) = \begin{bmatrix} \begin{pmatrix} P^{-1}(i)\dot{P}(i)P^{-1}(i)+\bar{A}_{\mathrm{TT}}(i)P^{-1}(i) \\ +P^{-1}(i)\bar{A}_{\mathrm{TT}}^T(i) \end{pmatrix} & P^{-1}(i)\bar{A}_{\mathrm{ST}}^T(i) & \bar{A}_{\mathrm{TS}}(i) & \bar{B}_{\mathrm{TD}}(i) \\ \bar{A}_{\mathrm{ST}}(i)P^{-1}(i) & -y_1 I & \bar{A}_{\mathrm{SS}}(i) & \bar{B}_{\mathrm{SD}}(i) \\ \bar{A}_{\mathrm{TS}}^T(i) & \bar{A}_{\mathrm{SS}}^T(i) & \left(y_2\Sigma_i^2\right)^{-1} & 0 \\ \bar{B}_{\mathrm{TD}}^T(i) & \bar{B}_{\mathrm{SD}}^T(i) & 0 & -\frac{1}{s}I \end{bmatrix}. \quad (56)$$

Let $V$ and $GP^{-1}$ denote, respectively, matrices $\left[\bar{B}_{\mathrm{TU}}^T(i)\,\bar{B}_{\mathrm{SU}}^T(i)\ 0\ 0\right]$ and $\left[\bar{C}_{\mathrm{TY}}(i)P^{-1}(i)\ 0\ \bar{C}_{\mathrm{SY}}(i)\,\bar{D}_{\mathrm{YD}}(i)\right]$. The matrix formed by defining $W_{\mathrm{V}}$ and $W_{GP^{-1}}$ as bases of null space $V$ and $GP^{-1}$, respectively. Then $W_{GP^{-1}}^T\Omega W_{GP^{-1}} < 0$ is equivalent to $W_{\mathrm{G}}^T\Omega W_{\mathrm{G}} < 0$, in which, $G(i) = \left[\bar{C}_{\mathrm{TY}}(i)\ 0\ \bar{C}_{\mathrm{SY}}(i)\ \bar{D}_{\mathrm{YD}}(i)\right]$.

When

$$V(i) = \begin{bmatrix} \bar{B}_{\mathrm{TU}}(i) \\ \bar{B}_{\mathrm{SU}}(i) \\ 0 \\ 0 \end{bmatrix}^T = \begin{bmatrix} 0 & I \\ 0 & 0 \\ B_{\mathrm{TU}}^T(i) & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & I \\ B_{\mathrm{SU}}^T(i) & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (57)$$

we can obtain the following formula,

$$W_{\mathrm{V}}(i) = \begin{bmatrix} \begin{bmatrix} V_1(i) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} V_2(i) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \end{bmatrix}. \quad (58)$$

Notice all zero rows in $W_{\mathrm{V}}$, use block matrix operation and let $P^{-1}(i) = \begin{bmatrix} R_1(i) & R_2(i) \\ R_2^T(i) & R_3(i) \end{bmatrix}$, then $W_{\mathrm{V}}^T\Omega W_{\mathrm{V}} < 0$ is equivalent to

$$W_{\mathrm{V}}^T(i)\begin{bmatrix} R_1(i)\dot{P}_1(i)R_1(i)+(A_{\mathrm{TT}}(i)R_1(i)+(\#)^T) & R_1(i)A_{\mathrm{ST}}^T(i) & A_{\mathrm{TS}}(i) & B_{\mathrm{TD}}(i) \\ A_{\mathrm{ST}}(i)R_1(i) & -y_1 I & A_{\mathrm{SS}}(i) & B_{\mathrm{SD}}(i) \\ A_{\mathrm{TS}}^T(i) & A_{\mathrm{SS}}^T(i) & \left(y_2\Sigma_i^2\right)^{-1} & 0 \\ B_{\mathrm{TD}}^T(i) & B_{\mathrm{SD}}^T(i) & 0 & -\frac{1}{s}I \end{bmatrix} W_{\mathrm{V}}(i) < 0, \quad (59)$$

in which,

$$W_{\mathrm{V}}(i) = \begin{bmatrix} \begin{bmatrix} V_1(i) & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} V_2(i) & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \end{bmatrix}.$$

On the other hand, matrix $\bar{\Omega}$ is defined as follows,

$$\bar{\Omega}(i) = \begin{bmatrix} \dot{P}(i)+(P(i)\bar{A}_{\mathrm{TT}}(i)+(\#)^T) \\ \bar{A}_{\mathrm{ST}}(i) \\ \bar{A}_{\mathrm{TS}}^T(i)P(i) \\ \bar{B}_{\mathrm{TD}}^T(i)P(i) \end{bmatrix}$$

$$\left.\begin{matrix} \bar{A}_{\mathrm{ST}}^T(i) & P(i)\bar{A}_{\mathrm{TS}}(i) & P(i)\bar{B}_{\mathrm{TD}}(i) \\ -y_1 I & \bar{A}_{\mathrm{SS}}(i) & \bar{B}_{\mathrm{SD}}(i) \\ \bar{A}_{\mathrm{SS}}^T(i) & (y_2\Sigma_i^2)^{-1} & 0 \\ \bar{B}_{\mathrm{SD}}^T(i) & 0 & -\frac{1}{s}I \end{matrix}\right]. \tag{60}$$

When

$$\begin{aligned} G(i) &= \left[\, \bar{C}_{\mathrm{TY}}(i)\ 0\ \bar{C}_{\mathrm{SY}}(i)\ \bar{D}_{\mathrm{YD}}(i) \,\right] \\ &= \begin{bmatrix} 0 & I \\ 0 & 0 \\ C_{\mathrm{TY}}(i) & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & I \\ C_{\mathrm{SY}}(i) & 0 \end{bmatrix} \\ &\quad \times \begin{bmatrix} 0 \\ 0 \\ D_{\mathrm{YD}}(i) \end{bmatrix}, \end{aligned} \tag{61}$$

we can obtain the following formula,

$$W_{\mathrm{G}}(t,i) = \begin{bmatrix} \begin{bmatrix} V_3(i) & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & I \end{bmatrix} \\ \begin{bmatrix} V_4(i) & 0 \\ 0 & 0 \\ V_5(i) & 0 \end{bmatrix} \end{bmatrix}. \tag{62}$$

Notice all zero rows in $W_{\mathrm{G}}$, use block matrix operation and let $P(i) = \begin{bmatrix} P_1(i) & P_2(i) \\ P_2^T(i) & P_3(i) \end{bmatrix}$, then $W_{\mathrm{G}}^T \bar{\Omega} W_{\mathrm{G}} < 0$ is equivalent to

$$W_{\mathrm{G}}^T(i) \begin{bmatrix} \dot{P}_1(i) + (P_1(i)A_{\mathrm{TT}}(i) + (\#)^T) & A_{\mathrm{ST}}^T(i) & P_1(i)A_{\mathrm{TS}}(i) & P_1(i)B_{\mathrm{TD}}(i) \\ A_{\mathrm{ST}}(i) & -y_1 I & A_{\mathrm{SS}}(i) & B_{\mathrm{SD}}(i) \\ A_{\mathrm{TS}}^T(i)P_1(i) & A_{\mathrm{SS}}^T(i) & (y_2\Sigma_i^2)^{-1} & 0 \\ B_{\mathrm{TD}}^T(i)P_1(i) & B_{\mathrm{SD}}^T(i) & 0 & -\frac{1}{s}I \end{bmatrix} W_{\mathrm{G}}(i) < 0, \tag{63}$$

in which,

$$W_{\mathrm{G}}(t,i) = \begin{bmatrix} \begin{bmatrix} V_3(i) & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & I \end{bmatrix} \\ \begin{bmatrix} V_4(i) & 0 \\ V_5(i) & 0 \end{bmatrix} \end{bmatrix}. \tag{64}$$

When $0 < y_2 < y_1$, the proof method is the same, so it is omitted. The sufficiency proof is accomplished.

Next, we discuss the FTB analysis and control for a networked dynamic system constituted by many LTI subsystems with LTI interaction. The derivations of Theorem 7 are omitted for the similarity with Theorem 1.

*Theorem 7:* Given the following class of constant signals

$$\mathcal{W} := \left\{ \begin{matrix} w(\cdot) = col\{w(\cdot,i)|_{i=1}^N\} | \sum_{i=1}^N w^T w \le d, \\ w^T(i)\, w(i) \le d(i),\ \sum_{i=1}^N d(i) = d \end{matrix} \right\}. \tag{65}$$

Then, System $\Theta$ in (2) and (8) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(i))$ if, letting $\tilde{Q}_1 = \Gamma^{-1/2} Q_1 \Gamma^{-1/2}$, $\Gamma = \mathbf{diag}\{\Gamma(i)|_{i=1}^N\}$, there exist a nonnegative scalar $\alpha$ and two symmetric positive definite matrices $Q_1, Q_2$, such that

$$\begin{bmatrix} \begin{pmatrix} \frac{1}{\alpha}(\bar{B}_{\mathrm{SD}} + \bar{B}_{\mathrm{SU}}K\bar{D}_{\mathrm{YD}})Q_2(*)^T \\ +((\bar{A}_{\mathrm{SS}} + \bar{B}_{\mathrm{SU}}K\bar{C}_{\mathrm{SY}})\Phi + (\#)^T) \\ -I - (\bar{A}_{\mathrm{SS}} + \bar{B}_{\mathrm{SU}}K\bar{C}_{\mathrm{SY}})\Sigma^2(*)^T \end{pmatrix} & (*)^T \\ \begin{pmatrix} \tilde{Q}_1(\bar{A}_{\mathrm{ST}} + \bar{B}_{\mathrm{SU}}K\bar{C}_{\mathrm{TY}})^T + \frac{1}{\alpha}(\bar{B}_{\mathrm{TD}} \\ +\bar{B}_{\mathrm{TU}}K\bar{D}_{\mathrm{YD}})Q_2(\bar{B}_{\mathrm{SD}} + \bar{B}_{\mathrm{SU}}K\bar{D}_{\mathrm{YD}})^T \\ +(\bar{A}_{\mathrm{TS}} + \bar{B}_{\mathrm{TU}}K\bar{C}_{\mathrm{SY}})\Phi - (\bar{A}_{\mathrm{TS}} + \bar{B}_{\mathrm{TU}} \\ K\bar{C}_{\mathrm{SY}})\Sigma^2(*)^T \end{pmatrix} & \begin{pmatrix} ((\bar{A}_{\mathrm{TT}} + \bar{B}_{\mathrm{TU}}K\bar{C}_{\mathrm{TY}})\tilde{Q}_1 + (\#)^T) \\ \alpha\tilde{Q}_1 + \frac{1}{\alpha}(\bar{B}_{\mathrm{TD}} + \bar{B}_{\mathrm{TU}}K\bar{D}_{\mathrm{YD}})Q_2(*)^T \\ -(\bar{A}_{\mathrm{TS}} + \bar{B}_{\mathrm{TU}}K\bar{C}_{\mathrm{SY}})\Sigma^2(*)^T \end{pmatrix} \end{bmatrix} < 0, \tag{66}$$

$$\frac{c_1}{\lambda_{\min}(Q_1)} + \frac{d}{\lambda_{\min}(Q_2)} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q_1)}. \tag{67}$$

Next providing a sufficient condition for FTB analysis of each individual subsystem. As it is similar to the derivation of Theorem 6, its derivation is omitted.

*Theorem 8:* Given the following class of signals

$$\mathcal{W} := \left\{ \begin{matrix} w(\cdot) = col\{w(\cdot,i)|_{i=1}^N\} | \sum_{i=1}^N w^T w \le d, \\ w^T(i)\, w(i) \le d(i),\ \sum_{i=1}^N d(i) = d \end{matrix} \right\}. \tag{68}$$

Then, System $\Theta$ in (38) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, \Gamma(t,i), \Gamma^K(t,i))$ if, letting $\tilde{Q}_1 = \Gamma(i)^{-1/2} Q_1 \Gamma(i)^{-1/2}$, there exist two positive definite matrices $Q_1(i)$, $Q_2(i)$, two block matrices $W_{\mathrm{V}}(i)$, $W_{\mathrm{G}}(i)$ and two scalars $0 < y_1 < y_2$ such that

$$W_{\mathrm{V}}^T(i) \begin{bmatrix} -\alpha\tilde{Q}_{11}(i) + (\bar{A}_{\mathrm{TT}}(i)\tilde{Q}_{11}(i) + (\#)^T) & \bar{A}_{\mathrm{ST}}^T(i)\tilde{Q}_{11}(i) & A_{\mathrm{TS}}(i) & B_{\mathrm{TD}}(i) \\ \tilde{Q}_{11}(i)\bar{A}_{\mathrm{ST}}(i) & -y_1 I & A_{\mathrm{SS}}(i) & B_{\mathrm{SD}}(i) \\ A_{\mathrm{TS}}^T(i) & A_{\mathrm{SS}}^T(i) & (y_2\Sigma_i^2)^{-1} & 0 \\ B_{\mathrm{TD}}^T(i) & B_{\mathrm{SD}}^T(i) & 0 & (-\alpha)Q_{21}^{-1}(i) \end{bmatrix} W_{\mathrm{V}}(i) < 0, \tag{69}$$

$$W_{\mathrm{G}}^T(i) \begin{bmatrix} -\alpha Z_1(i)\tilde{Q}_{11}(i)Z_1(i) + (\bar{A}_{\mathrm{TT}}(i)Z_1(i) + (\#)^T) & \bar{A}_{\mathrm{ST}}^T(i) & Z_1(i)\bar{A}_{\mathrm{TS}}(i) & Z_1(i)\bar{B}_{\mathrm{TD}}(i) \\ \bar{A}_{\mathrm{ST}}(i) & -y_1 I & \bar{A}_{\mathrm{SS}}(i) & \bar{B}_{\mathrm{SD}}(i) \\ \bar{A}_{\mathrm{TS}}^T(i)Z_1(i) & A_{\mathrm{SS}}^T(i) & (y_2\Sigma_i^2)^{-1} & 0 \\ \bar{B}_{\mathrm{TD}}^T(i)Z_1(i) & B_{\mathrm{SD}}^T(i) & 0 & (-\alpha)^{-1}Q_{21}(i) \end{bmatrix} W_{\mathrm{G}}(i) < 0, \tag{70}$$

$$\frac{c_1}{\lambda_{\min}(Q_1(i))} + \frac{d}{\lambda_{\min}(Q_2(i))} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q_1(i))}, \tag{71}$$

$$\begin{bmatrix} \tilde{Q}_{11}(i) & I \\ I & Z_1(i) \end{bmatrix} \ge 0, \tag{72}$$

or exist two positive definite matrices $Q_1(i)$, $Q_2(i)$, two block matrices $W_V(i)$, $W_G(i)$ and two scalars $0 < y_2 < y_1$, such that

$$W_V^T(i)\begin{bmatrix} -\alpha\tilde{Q}_{11}(i) + (\bar{A}_{TT}(i)\tilde{Q}_{11}(i) + (\#)^T) & \bar{A}_{ST}^T(i)\tilde{Q}_{11}(i) & A_{TS}(i) & B_{TD}(i) \\ \tilde{Q}_{11}(i)\bar{A}_{ST}^T(i) & y_1 I & A_{SS}(i) & B_{SD}(i) \\ A_{TS}^T(i) & A_{SS}^T(i) & (-y_2\Sigma_i^2)^{-1} & 0 \\ B_{TD}^T(i) & B_{SD}^T(i) & 0 & (-\alpha)^{-1}Q_2(i) \end{bmatrix} W_V(i) < 0, \tag{73}$$

$$W_G^T(i)\begin{bmatrix} -\alpha Z_1(i)\tilde{Q}_{11}(i)Z_1(i) + (\bar{A}_{TT}(i)Z_1(i) + (\#)^T) & \bar{A}_{ST}^T(i) & Z_1(i)\bar{A}_{TS}(i) & Z_1(i)\bar{B}_{TD}(i) \\ \bar{A}_{ST}(i) & y_1 I & \bar{A}_{SS}(i) & \bar{B}_{SD}(i) \\ \bar{A}_{TS}^T(i)Z_1(i) & A_{SS}^T(i) & (-y_2\Sigma_i^2)^{-1} & 0 \\ \bar{B}_{TD}^T(i)Z_1(i) & B_{SD}^T(i) & 0 & (-\alpha)^{-1}Q_2(i) \end{bmatrix} W_G(i) < 0, \tag{74}$$

$$\frac{c_1}{\lambda_{\min}(Q_1(i))} + \frac{d}{\lambda_{\min}(Q_2(i))} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q_1(i))}, \tag{75}$$

$$\begin{bmatrix} \tilde{Q}_{11}(i) & I \\ I & Z_1(i) \end{bmatrix} \geq 0, \tag{76}$$

in which, $\tilde{Q}_1(i) = \begin{bmatrix} \tilde{Q}_{11}(i) & \tilde{Q}_{12}(i) \\ \tilde{Q}_{12}^T(i) & \tilde{Q}_{13}(i) \end{bmatrix}$, $\tilde{Q}_1^{-1}(i) = \begin{bmatrix} Z_1(i) & Z_2(i) \\ Z_2^T(i) & Z_3(i) \end{bmatrix}$, $\tilde{Q}_2(i) = \begin{bmatrix} \tilde{Q}_{21}(i) & \tilde{Q}_{22}(i) \\ \tilde{Q}_{22}^T(i) & \tilde{Q}_{23}(i) \end{bmatrix}$.

## V. NUMERICAL SIMULATIONS

Several numerical simulations have been carried out to evaluate the computational efficiency of the derived conditions in this paper. The computations are performed with a personal computer with an Intel(R) Core(TM) i7-6700 CPU 3.40 GHz and 16G RAM.

*Example 1:* In these simulations, it is assumed $m_{ui} = m_{xi} = m_{vi} = m_{zi} = m_{yi} = 2$ and the system is time-invariant. The subsystem connection matrix $\Phi$ is randomly generated. Every subsystem parameter is randomly and independently generated according to a continuous uniform distribution over the interval [-1.0, 1.0], while the interval $T$ is generated randomly over [1.0, 2.0]. The rate between $c_2$ and $c_1$ is fixed to 10 and $\Gamma(i) = I$. In order to analyze the FTB of the generated system, we verify the feasibility of these LMIs using the algorithm developed in [29] based on Lemma 5, Theorems 3 and 4. Both the average and standard deviation of computation time are calculated. Tables 1 and 2 show some representative results when the number of subsystems N increases from 2 to 40. With the expansion of the system scale, when the number of subsystems is 2, 10, 20 and 30, on the premise that Lemma 5 is equivalent to Theorem 3, we can obtain the ratio of average computation time for FTB is 0.9670, 0.8787, 0.7437 and 0.5942, respectively. When the number of subsystems is great than 30, the computer can't calculate the time of Lemma 5 and Theorem 3 due to lack of memory, but Theorem 4 can still be calculated. The trend Figures 2 and 3 showing the change of average calculation time and standard deviation of calculation time with the number N of subsystems.

From the tables, the computational efficiency of Lemma 5 is comparable to that of Theorems 3 and 4 when the number of subsystems is small. However, the ratio of average computation time is smaller and smaller with the increment of the system scale which means that Theorem 3 becomes more and more computationally efficient compared to the inequality

**TABLE 1. Average of computation time for FTB.**

| Number of subsystems | Avg. CPU time(s) | | |
|---|---|---|---|
| N | Lemma 5 | Theorem 3 | Theorem 4 |
| 2 | 0.0638 | 0.0617 | 0.0538 |
| 10 | 2.4821 | 2.1810 | 0.4649 |
| 20 | 43.4134 | 32.2874 | 0.8362 |
| 22 | 68.6737 | 46.8572 | 1.0527 |
| 24 | 89.1261 | 65.8447 | 1.1327 |
| 26 | 144.2410 | 93.1770 | 1.2442 |
| 28 | 216.3734 | 135.2409 | 1.3032 |
| 30 | 338.8856 | 201.3803 | 1.5365 |
| 40 | – | – | 1.9268 |

**TABLE 2. Standard deviation of computation time for FTB.**

| Number of subsystems | Std. deviation(s) | | |
|---|---|---|---|
| N | Lemma 5 | Theorem 3 | Theorem 4 |
| 2 | 0.0024 | 0.0015 | 0.0029 |
| 10 | 0.0538 | 0.0495 | 0.0137 |
| 20 | 0.1395 | 0.1015 | 0.0143 |
| 22 | 0.4285 | 0.2753 | 0.0158 |
| 24 | 1.6492 | 0.1842 | 0.0160 |
| 26 | 3.9472 | 0.4672 | 0.0183 |
| 28 | 14.3769 | 0.7539 | 0.0215 |
| 30 | 18.2542 | 1.1639 | 0.0253 |
| 40 | - | - | 0.0312 |

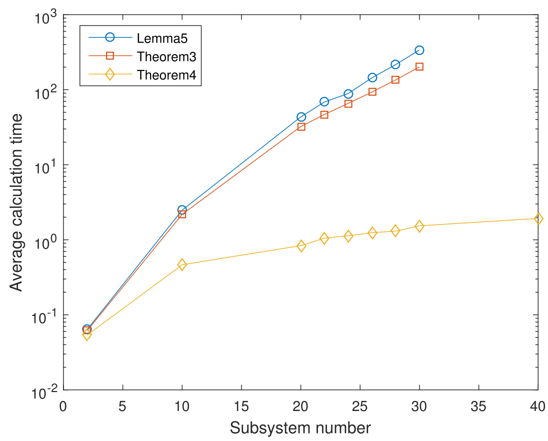


**FIGURE 2. Average calculation time.**

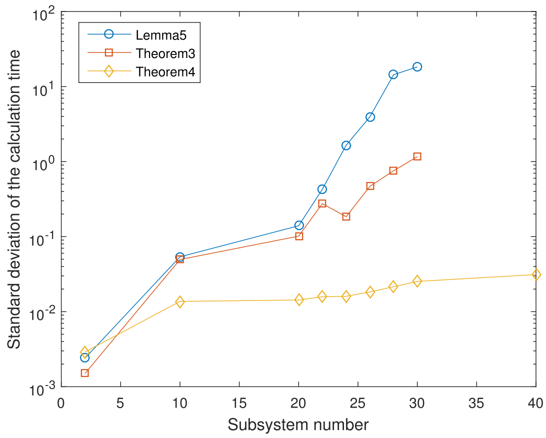


**FIGURE 3. Standard deviation time.**

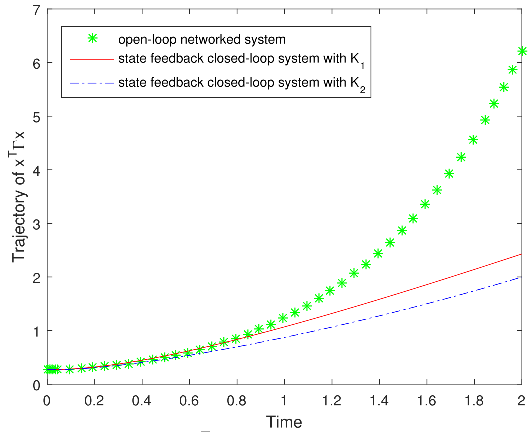


**FIGURE 4. The trajectory of $x^T \Gamma x$.**

in Lemma 5. Though Theorem 4 is more conservative, it is the most computationally efficient since it is only dependent on parameters of individual subsystem. As can be seen from Figures 2 and 3, if the number of subsystems is expanded to 100 or even more, the advantages of Theorems 3 and 4 will be further highlighted.

*Example 2:* In order to verify the applicability of the FTB obtained in this paper, the network system composed of three subsystems (N = 3) is considered, and the simulations are carried out under linear time-varying and linear time-invariant cases respectively.

1) The linear time-varying case

In the given time interval [0,2], the output of the dynamic system is required to be always in the neighborhood 3. Based on the above conditions, the parameter matrices of the networked system are generated by $a$, $a \in [-0.5, 0.5]$,

$$\Phi = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

According to the above system, the simulation parameters are selected as follows $c_1 = 1$, $c_2 = 3$, $\Gamma = I_3$, $T = 2$, $w = 1$ where $I_3$ represents a three-dimensional unit array. The FTB $(c_1, c_2, W, T, \Gamma(t, i))$ has no solution under the LMIs condition of the open-loop system, but after designing a state feedback controller, there is a feasible solution under the LMIs condition of Lemma 4 and Theorem 1. Moreover, it is FTB with respect to $(1, 3, 1, 2, I_3)$ with $K_1(t, i) = \begin{bmatrix} -0.5192 & 2.1641 & 0.1461 \end{bmatrix}$ computed by Lemma 4 and $K_2(t, i) = \begin{bmatrix} 0.1267 & 1.5527 & -0.8954 \end{bmatrix}$ by Theorem 1, respectively. The square weighted of the system state $(x^T \Gamma x)$ is shown in Figure 4, as can be seen from Figure 4, in the time interval $[0, 2]$, $x^T \Gamma x < c_2$. Therefore, the networked system is FTB with respect to $(1, 3, 1, 2, I_3)$. From Figure 4, we can also obtain that in the open-loop system, $c_2 > 3$, while in closed-loop system, $c_2 < 3$ in Lemma 4 and Theorem 1, the dynamic system is FTB, and the regulating ability of Theorem 1 is stronger than Lemma 4.

2)The linear time-invariant case

In the given time interval [0,1], the output of the dynamic system is required to be always in the neighborhood 5, so we set the initial condition $c_1 = 1$ and boundary value $c_2 = 5$. Based on the above conditions, the system parameters are listed as follows,

$$A_{\mathrm{TT}}(i) = \begin{bmatrix} -0.3205 & -0.6536 & -0.4362 \\ 0.5167 & -0.0347 & -0.0215 \\ 0.4216 & -0.8172 & -0.0125 \end{bmatrix},$$

$$A_{\mathrm{TS}}(i) = \begin{bmatrix} -0.971 & -0.9474 \\ 0.6133 & -0.0624 \\ 0.5132 & -0.4316 \end{bmatrix},$$

$$A_{\mathrm{ST}}(i) = \begin{bmatrix} 0.3238 & 0.0982 & -0.4216 \\ 0.5264 & 0.6290 & -0.4216 \end{bmatrix},$$

$$A_{\mathrm{SS}}(i) = \begin{bmatrix} -0.6412 & 0.7861 \\ -0.5457 & -0.3255 \end{bmatrix},$$

$$G_{\mathrm{T}}(i) = \begin{bmatrix} 0.4216 \\ 0.8172 \\ -0.3252 \end{bmatrix},$$

$$G_{\mathrm{S}}(i) = \begin{bmatrix} 0.3205 \\ -0.5167 \end{bmatrix},$$

$$B_{\mathrm{T}}(i) = \begin{bmatrix} 2.0000 & 0.5000 & 1.0000 \end{bmatrix},$$

$$B_{\mathrm{S}}(i) = \begin{bmatrix} 0.8000 \\ -0.5000 \end{bmatrix}.$$

$c_1 = 1$, $c_2 = 5$, $\Gamma = I_3$, $T = 1$, $w = 1$. Moreover, it is FTB with respect to $(1, 5, 1, 1, I_3)$ with $K_3(i) = \begin{bmatrix} -1.6984 & 0.2651 & 0.5787 \end{bmatrix}$ computed by Lemma 5 and $K_4(i) = \begin{bmatrix} -0.3724 & 0.0140 & -0.0904 \end{bmatrix}$ by Theorem 3, respectively. The square weighted of the system state $(x^T \Gamma x)$ is shown in Figure 5, as can be seen from Figure 5, in the time interval $[0, 1]$, $x^T \Gamma x < c_2$. Therefore, the networked system is FTB with respect to $(1, 5, 1, 1, I_3)$. From Figure 5, we can also obtain that in the open-loop system, $c_2 > 5$, while in closed-loop system, $c_2 < 5$ in Lemma 5 and Theorem 3, the

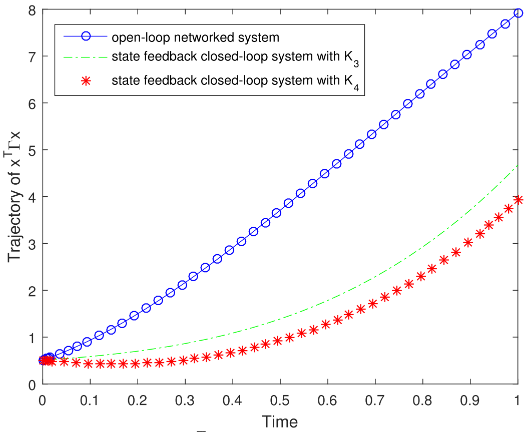


**FIGURE 5.** The trajectory of $x^T\Gamma x$.

dynamic system is FTB, and the regulating ability of Theorem 3 is stronger than Lemma 5.

*Example 3:* In this example, the proposed theory is applied to the distributed output feedback control of power network system, and the distributed output feedback controller is designed to ensure that the closed-loop system is FTB. Our model is adapted from [31] and [32].

Distributed power network system consists of a group of load-driven generators connected by transmission lines. The generator is a kind of dynamic device, the linearization model $G_i$ of the $i-th$ load-driving generator can be expressed as

$$\begin{aligned}\dot{x}(t,i) &= R(i)x(t,i) + L(i)I(t,i) + B_u(i)u(t,i),\\ V(t,i) &= K(i)x(t,i),\end{aligned} \tag{77}$$

in which, at each time $t$, $I(t,i) \in R^2$ and $V(t,i) \in R^2$ represent current deviation and voltage deviation from the selected operating point, respectively; $u(t,i) \in R$ indicates the control torque used to adjust the generator; $x(t,i) = \begin{bmatrix} x_1(t,i) \\ x_2(t,i) \end{bmatrix} \in R^2$ is the state vector of the generator, and its two components $x_1(t,i)$ and $x_2(t,i)$ respectively correspond to the deviation of the rotor angular velocity and angle from the reference value and each generator is connected with a load with admittance matrix $Y_{ii} \in R^2$. Meanwhile, each generator is also connected with other generators through transmission lines, and the admittance matrix of the corresponding transmission lines is $Y_{ij} \in R^2, j = 1, 2, \cdots, N, j \neq i$. The power network system consisting of three generators is shown in Figure 6. Using Kirchhoff's law, we derive that

$$I(t,i) = Y_{ii}V(t,i) + \sum_{i\neq j} Y_{ji}\left[V(t,i) - V(t,j)\right], \tag{78}$$

and, in turn, that

$$\dot{x}(t,i) = \left( \begin{array}{c} \left( R(i) + L(i)Y_{ii}K(i) + L(i)\sum_{i\neq j} Y_{ji}K(i) \right) \\ \times x(t,i) - L(i)\sum_{i\neq j} Y_{ji}V(t,j) + B_u(i)u(t,i) \end{array} \right). \tag{79}$$

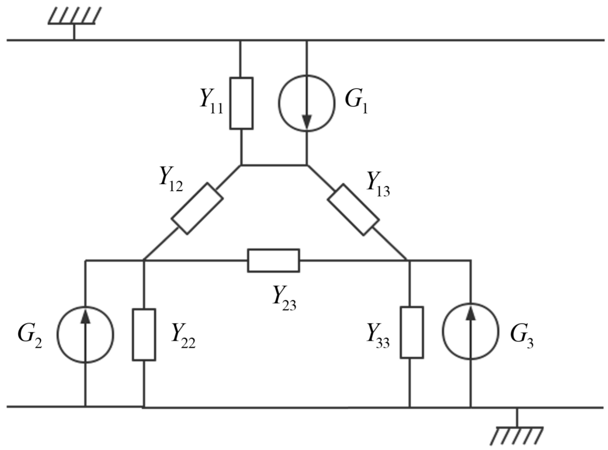


**FIGURE 6.** A power system with $N = 3$ generators.

For the generator model $G_i$ mentioned above, the disturbance signal $d(t,i) \in R$ and the measurement signal $y(t,i) \in R$ to the control torque are introduced so that $y(t,i) = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1(t,i) \\ x_2(t,i) \end{bmatrix}$, and the performance output signal $e(t,i) \in R^2$ makes $e(t,i) = x(t,i)$. Let $v(t,i) = col\{v_j(t,i)|_{j=1}^N\}$, $z(t,i) = col\{z_j(t,i)|_{j=1}^N\}$, internal output component $z_j(t,i) = V(t,i)$ and internal input component $v_j(t,i) = V(t,j)$. The following formula is expressed in the form of subsystem model. The related matrix parameters of are expressed as,

$$\begin{aligned} A_{TT}(i) &:= R(i) + L(i)Y_{ii}K(i) + L(i)\sum_{i\neq j} Y_{ji}K(i),\\ A_{TS}(i) &:= \left[ -L(i)Y_{ji} \ \cdots \ -L(i)Y_{ji} \right],\\ A_{ST}(i) &:= \begin{bmatrix} K(i) \\ \vdots \\ K(i) \end{bmatrix},\\ B_{TD}(i) &= B_{TU}(i) := Bu(i), \end{aligned}$$

the connections between subsystems are represented as follows,

$$v_j(t,i) = \Phi z_i(t,j).$$

With the above transformation, a distributed controller is designed to realize the finite-time boundedness of the closed-loop system. Figure 6 is simplified as shown in Figure 7. The characteristic parameters of each generator are given below.

$$\begin{aligned} R(i) &= \begin{bmatrix} 0.5 & 0 \\ -0.1 & 0 \end{bmatrix}, L(i) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\\ K(i) &= \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, B_u(i) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \end{aligned}$$

The admittance of the generator is set as $y_{11} = 1 + 0.1i$, $y_{22} = 1 + 0.2i$, $y_{33} = 1 + 0.3i$, and the admittance of the transmission line is $y_{ij} = 0.3 - i$, where $i$ is the imaginary

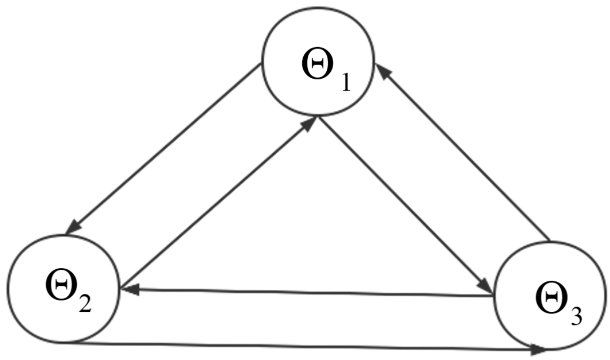


**FIGURE 7.** A power system simplification with $N \mathcal{D} 3$ generators.

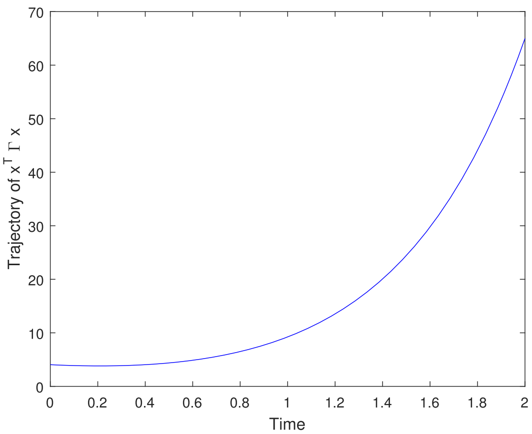


**FIGURE 8.** The $x^T \Gamma x$ trajectory of open-loop system.

unit.

$$Y_{11} = \begin{bmatrix} 1 & -0.1 \\ 0.1 & 1 \end{bmatrix}, Y_{22} = \begin{bmatrix} 1 & -0.2 \\ 0.2 & 1 \end{bmatrix},$$
$$Y_{33} = \begin{bmatrix} 1 & -0.3 \\ 0.3 & 1 \end{bmatrix}, Y_{ij} = Y_{ji} = \begin{bmatrix} 0.3 & 1 \\ -1 & 0.3 \end{bmatrix}.$$

Substitute the above conditions into Lemma 6 in [19], in the given time interval [0,2], the output of the dynamic system is required to be always in the neighborhood 40, so we set the initial condition $c_1 = 1$ and boundary value $c_2 = 40$. Let $c_1 = 1$, $c_2 = 40$, $\Gamma = I_3$, $T = 2$, $w = 1$, where $I_3$ represents a three-dimensional unit array the LMIs conditions of open-loop system has a feasible solution, but let $c_1 = 1$, $c_2 = 40$, $\Gamma = I_3$, $T = 2$, $w = 1$, the LMIs conditions of the open-loop system does not hold (This theorem is a sufficient condition for the FTB of the system, the failure of the condition does not mean that the system is not FTB). $x^T \Gamma x$ trajectory of open-loop system is shown in Figure 8.

Bring the above conditions into the distributed output feedback controller. The system is FTB with respect to $(1, 40, 1, 2, I_3)$ in the time interval $[0, 2]$, $x^T \Gamma x$ trajectory of close-loop system is shown in Figure 9. It is shown that the distributed output feedback controller design can make the system which cannot be judged as FTB regain FTB.

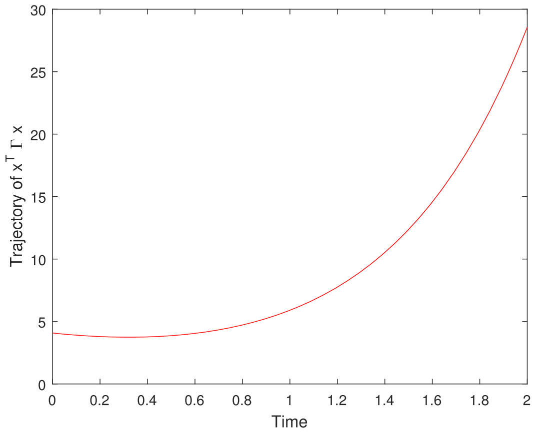


**FIGURE 9.** The $x^T \Gamma x$ trajectory of close-loop system.

## VI. CONCLUSION

In this paper, the finite-time boundedness controllers problem is investigated for networked systems composed of a large number of subsystems with arbitrary interactions. Sufficient conditions for the existence of state feedback controller and distributed output feedback controller based on some DLMI and LMI are derived, which are computationally valid for the reason that they utilize efficiently the block-diagonal structure of system parameter matrices and the sparseness of the subsystem connection matrix. Numerical simulations show that the calculation efficiency of the proposed criterions are greatly improved compared with the existing lumped criterions. Through the design of the controller, it can make the system which cannot be judged as FTB regain FTB, and the expected control purpose is achieved. The next step is to apply this algorithm to more complex systems. The next step is to apply this algorithm to more complex systems and explore more ways to reduce the conservatism of conditions.

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