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RESEARCH ARTICLE

Analytical Tuning Rules for Cascade Control Systems Based on Fractional-Order PI Controller in Frequency Domain

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ABSTRACT A design method for a cascade control system is proposed in this paper. To deal with delay times in most of process control systems, the Smith predictor is also suggested in the control structure to eliminate a delay term in closed-loop systems. The classical proportional-integral (PI) and the fractional-order PI controller are adopted for the inner loop and the outer loop respectively. The analytical tuning rules are derived based on frequency domain and also using the direct synthesis method for improving the performance for both disturbance rejection and set-point tracking problems. An illustrative example is considered to confirm the effectiveness of the proposed algorithm. In addition, the robust stability of the fractional-order system is also carried out to demonstrate that the proposed fractional-order PI controller can hold well the robustness against perturbation uncertainties in the process models.

INDEX TERMS Analytical tuning rules, cascade control systems, disturbance rejection, fractional calculus, fractional-order system, fractional-order proportional-integral controller, Smith predictor.

I. INTRODUCTION

A cascade control system is quite common in industrial processes. It is mainly used for reducing disturbances as well as enhancing the servo responses of the closed-loop system [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]. The original cascade control scheme consists of two control loops: outer and inner. It means that we have to design both controllers where the manipulated variable of the first one (the outer) plays as an input signal for the second one (the inner). As a result of that, the tuning procedures are much more sophisticated than those of other single-loop systems [4], [5], [6], [7], [8], [9], [10], [11]. In [6], the authors proposed tuning rules for both inner and outer loops using two-degree-of-freedom (2-DoF) proportional-integral/proportional-integral-derivative (PI/PID) controllers. The trade-off between system performance and robustness is considered to derive the design parameters. An enhanced cascade structure is suggested in [8] to deal with a class of integrating processes with time delay. In [10], the authors proposed a unified approach for tuning the controllers of a cascade scheme for unstable, integrating, and stable processes. The Kharitonov's theorem was adopted to justify the robustness of the controlled systems.

Recently, there has been an increasing attention paid to fractional-order calculus [12] both from the academic and control engineers for the modeling and control issues because it provides more flexibility and advancement in the computation power [13], [14], [15]. The first mention involving using a fractional structure in a feedback loop was made early by Bode [16], [17] and was extended by Barbosa and Ferreira [18]. However, this idea was not concretized and remained as a simple proposition for decades. In [19], the author proposed the fractional-order PID controller (FOPID), which is so-called $PI^{\lambda}D^{\mu}$ that involving fractional-order integrator (λ) and fractional-order differentiator (μ), and considered as the generalization of a standard PID controller. Due to its two extra parameters (λ and μ), this type of controller can be achieved the better performance than that of classical PID controllers for fractional-order processes as well as integer-order ones. It also has been justified in other works

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that fractional-order controllers provide more robustness and having been become a new trend to solve many industrial control problems [15], [20].

The research related to the tuning rules of the FOPID controller have been reported in numerous kinds of literature and is major in single-input, single-output (SISO) [20], [21], [22], [23], [24], [25], [26], [27]. For other complex control systems such as cascade structures, only a few studies use fractional-order controllers, for instance, in [28] and [29], but the authors are only concerned about the parallel cascade systems. In addition, for the primary loop with a large time delay, some available literature could not effectively solve the servomechanism problem [30], [31]. Therefore, this work suggests tuning rules for a cascade control scheme using a fractional-order PI controller to improve the system's performances.

As mentioned in the introduction section, most classical PID controllers for this kind of systems have to compromise between servomechanism and regulator problems. Therefore, in this paper, our aim is to propose an analytical tuning method of fractional-order PI controller for improving both set-point tracking and disturbance rejection of a cascade scheme with stable processes plus time delays. It is reasonable because of the flexibility in tuning rules as well as the robustness of the fractional-order controller. To eliminate a delay term in a characteristic equation of a closed-loop system, the Smith predictor [32] will be included in the control structure. Other authors also adopted this compensator to deal with time delay in parallel cascade control structures [33], [34], [35], [36], but they only considered the integer-order PID controller. Our design approach for the inner loop is mainly based on the concepts of direct synthesis method [37]. For the outer loop, by using frequency domain in combination with direct synthesis method, the proposed fractional-order PI tuning rules can be directly derived.

II. PRELIMINARIES

Some basic fundamentals of fractional calculus are briefly introduced in this section.

A. FRACTIONAL CALCULUS

Fractional calculus [12] is generalization of the ordinary calculus. The main idea is to develop a functional operator D, associated to an order v not restricted to integer numbers that generalizes the usual notions of derivative (for a positive v) and integral (for a negative v).

There are various definitions for fractional derivative. However, the most commonly use is the Riemann-Liouville definition [15], which is shown in the following equation:

$$ {}_aD_t^v f(t) = \frac{1}{\Gamma(n-v)}\frac{d}{dt^n}\int_a^t \frac{f(\tau)}{(t-\tau)^{v-n+1}}dt,\ n-1 < v < n \tag{1}$$

where $\Gamma(\bullet)$ represents the Euler's gamma function. It is important to note that the generalized definition of D becomes ${}_aD_t^v f(t)$. The Laplace transform of D pursues the well-known rule for zero initial condition a $L\left[{}_aD_t^v f(t)\right] = s^v F(s)$. It is implied that under initial condition, the system with a dynamic behavior described by differential equations involving fractional-order derivative transforms to a transfer function with fractional power of s.

B. INTEGER ORDER APPROXIMATION

In order to use the fractional-order controller, in both simulations and hardware, of the transfer functions involving fractional order of s, the transfer function of this controller should be perfectly approximated into the integer order transfer function with a similar behavior, which includes an infinite number of poles and zeros. Nevertheless, it can be obtained reasonable approximations with a finite number of poles and zeros. In this case, the Oustaloup continuous integer order approximation [38], which is based on a recursive distribution of poles and zeros, has been employed here:

$$ s^v \cong k\prod_{n=1}^{N}\frac{1+\left(\frac{s}{\omega_{z,n}}\right)}{1+\left(\frac{s}{\omega_{p,n}}\right)} \tag{2}$$

Equation (2) is valid in a frequency range $[\omega_l, \omega_h]$, where the gain k is adjusted for the both sides of (2) has unity gain at the gain crossover frequency of s^v(i.e., it is denoted as ω_c and commonly given as 1 rad/s). The number of poles and zeros $N = 8$ is chosen, since ω_l and ω_h have been respectively selected as $0.001\omega_c$ and $1000\omega_c$. It is important to note that the most wanted performance of the approximation strongly depends on: low values cause simpler approximations, but may cause ripples in both gain and phase behaviors. Such ripples may be functionally neglected by increasing N. However, in that case, the approximation will be become computationally heavier.

C. FRACTIONAL LINEAR MODEL

According to a SISO linear time invariant (LTI) system, the fractional-order differential equation (FODE), provided both input and output signals $u(t)$ and $y(t)$ that is relaxed at $t = 0$, can be expressed by the following differential equation:

$$ \sum_{i=0}^{n} a_i D_0^{\alpha_i} y(t) = \sum_{j=0}^{m} b_j D_0^{\beta_j} u(t) \tag{3}$$

As a result, Eq. (3) can be described in the Laplace domain by the following transfer function:

$$ G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1}s^{\beta_{m-1}} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1}s^{\alpha_{n-1}} + \ldots + a_0 s^{\alpha_0}} \tag{4}$$

where α_i and β_i are arbitrary real positive.

III. THE ANALYTICAL TUNING RULES FOR CASCADE CONTROL SYSTEMS

A. THE FOPI CONTROLLER IN FREQUENCY DOMAIN

Fractional calculus gives the fractional integro-differential equation of a fractional-order proportional integral (FOPI) controller as:

$$u(t) = K_C e(t) + K_I D_t^{-\lambda} e(t), (\lambda > 0) \tag{5}$$

where K_C and K_I respectively represent the proportional and integral terms of the FOPI controller. λ is the fractional order of the integral.

The continuous transfer function of the FOPI controller can be obtained by Laplace transformation:

$$G_C(s) = K_C + \frac{K_I}{s^\lambda} \tag{6}$$

The controller has three parameters (K_C, K_I, and λ) to tune, since the fractional order λ is not necessarily integer. An integer PI controller is a special case of this FOPI controller where $\lambda = 1$. This expansion provides more flexibility in achieving control objectives. However, it is often complicated by requiring a non-linear objective function and user-defined constraints to obtain controller parameters that satisfy some specified performance criterion.

By substituting $s = j\omega$ into (6), the FOPI controller is represented in the frequency domain as:

$$G_C(j\omega) = K_C + \frac{K_I}{(j\omega)^\lambda} \tag{7}$$

The fractional power of $j\omega$ can be written as

$$(j\omega)^\lambda = \omega^\lambda j^\lambda = \omega^\lambda \left[e^{j\left[\frac{\pi}{2}+2n\pi\right]}\right]^\lambda = \omega^\lambda \left[e^{j\left[\frac{\pi}{2}\lambda+2n\lambda\pi\right]}\right] \tag{8}$$

where $n = 0, \pm\frac{1}{\lambda}, \pm\frac{2}{\lambda}, \ldots, \pm\frac{m}{\lambda}$. Therefore, the following convenient form is obtained:

$$(j\omega)^\lambda = \omega^\lambda (\cos\gamma_I + j\sin\gamma_I), \gamma_I = \frac{\pi\lambda}{2} \tag{9}$$

Substituting (9) into (7) and rearranging gives a complex equation for the FOPI controller in the frequency domain:

$$G_C(j\omega) = \left(K_C + \frac{K_I\cos\gamma_I}{\omega^\lambda}\right) - j\frac{(K_I\sin\gamma_I)}{\omega^\lambda} \tag{10}$$

B. THE ANALYTICAL DESIGN FOR CASCADE CONTROL SYSTEMS

The proposed cascade control system is shown in Figure 1, where $G_{p_1}(s)$ and $G_{p_2}(s)$ are the transfer functions of the outer and inner processes respectively. $G_{c_1}(s)$ and $G_{c_2}(s)$ are the primary and secondary controllers; $\tilde{G}_p(s)$ is the free-delay term of the equivalent transfer function of the inner control loop (from the input r_2 to the output y_1).

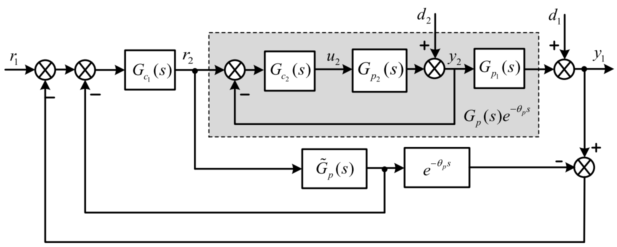

FIGURE 1. The proposed cascade control structure.

The closed-loop transfer function from the input r_1 to the primary output y_1 is as follows:

$$\frac{y_1}{r_1} = \frac{G_{c_1}(s)G_p(s)e^{-\theta_p s}}{1 + G_{c_1}(s)\tilde{G}_p(s) - G_{c_1}(s)\tilde{G}_p(s)e^{-\theta_p s} + G_{c_1}(s)G_p(s)e^{-\theta_p s}} \tag{11}$$

Assuming that the perfect model is given, $G_p(s) = \tilde{G}_p(s)$, the above equation is reduced as Eq. (12):

$$\frac{y_1}{r_1} = \frac{G_{c_1}(s)G_p(s)e^{-\theta_p s}}{1 + G_{c_1}(s)\tilde{G}_p(s)} \tag{12}$$

It is obvious that due to the Smith predictor structure, the delay term is eliminated out of the characteristic equation of the closed-loop system.

The inner loop controller design

The direct synthesis (DS) method will be adopted to design the inner loop controller. The closed-loop transfer function of the inner loop (from the input r_2 to the output y_2) can be calculated:

$$\frac{y_2}{r_2} = \frac{G_{c_2}(s)G_{p_2}(s)}{1 + G_{c_2}(s)G_{p_2}(s)} \tag{13}$$

The controller G_{c_2} is derived from Eq. (13) as follows:

$$G_{c_2}(s) = \frac{1}{G_{p_2}(s)}\frac{y_2/r_2}{1 - y_2/r_2} \tag{14}$$

Assuming that the process model is prior known, $\tilde{G}_{p_2}$, and the desired closed-loop response $(y_2/r_2)_d$ is considered as the closed-loop transfer function in terms of set-point changes. Therefore, the ideal controller is obtained by rewritten:

$$G_{c_2}(s) = \frac{1}{\tilde{G}_{p_2}(s)}\frac{(y_2/r_2)_d}{1 - (y_2/r_2)_d} \tag{15}$$

Normally, $(y_2/r_2)_d$ is chosen as follows:

$$\left(\frac{y_2}{r_2}\right)_d = \frac{e^{-\theta_2 s}}{\tau_{c_2}s + 1} \tag{16}$$

where τ_{c_2} is the desired time constant of the closed-loop response and θ_2 is a time delay of the process. Replacing

Eq. (16) into Eq. (15) and using Taylor approximation for the delay term, the feedback controller is obtained:

$$G_{c_2}(s) = \frac{1}{\tilde{G}_{p_2}(s)} \frac{e^{-\theta_2 s}}{(\tau_{c_2} + \theta_2)s} \tag{17}$$

Consider the FOPDT for the inner process model:

$$\tilde{G}_{p2}(s) = \frac{K_2}{\tau_2 s + 1} e^{-\theta_2 s} \tag{18}$$

Substituting Eq. (18) into (17), the PI controller for the inner loop is obtained:

$$G_{c_2}(s) = \frac{\tau_2}{K_2(\tau_{c_2} + \theta_2)} \left[1 + \frac{1}{\tau_2 s}\right] \tag{19}$$

The outer loop controller design

The equivalent transfer function of the primary process model is calculated as follows:

$$G_p(s)\, e^{-\theta_p s} = \left(\frac{y_2}{r_2}\right)_d G_{p_1}(s) = \frac{e^{-\theta_2 s}}{\tau_{c_2} s + 1} \frac{K_1 e^{-\theta_1 s}}{\tau_1 s + 1} \tag{20}$$

where $\theta_p = \theta_1 + \theta_2$ and $G_p(s) = \frac{K_1}{(\tau_{c_2} s+1)(\tau_1 s+1)}$

The fractional-order PI controller is addressed for this control loop. According to the guideline provided by Chen et al. [15], the fractional order λ is chosen based on the relative dead time parameter of the first order plus delay time system (FOPDT).

$$\Delta = \frac{\theta_{p_1}}{\tau_p + \theta_{p_1}} \tag{21}$$

where θ_{p_1} and τ_p are calculated by using Skogestad's half rule [37] (assuming that $\tau_1 > \tau_{c_2}$)

$$\theta_{p_1} = \theta_p + \frac{\tau_c}{2} and \tau_p = \tau_1 + \frac{\tau_{c_2}}{2} \tag{22}$$

And according to [20], the value of λ can be determined as follows:

$$\lambda = \begin{cases} 1.1, \text{if} \Delta \geq 0.6 \\ 0.9, \text{if } 0.1 \leq \Delta < 0.6 \\ 0.7, \text{if} \Delta < 0.1 \end{cases} \tag{23}$$

Similar to the inner loop, the DS method is still addressed here. However, due to the Smith predictor scheme, only the free-delay process model is used in deriving the analytical tuning rules for the outer loop. The ideal feedback controller is obtained as follows:

$$G_{c_1}(s) = \frac{1}{\tilde{G}_p(s)} \frac{(y_1/r_1)_d}{1 - (y_1/r_1)_d} \tag{24}$$

In this control loop, the desired closed loop response is chosen in fractional-order form

$$\left(\frac{y_1}{r_1}\right)_d = \frac{e^{-\theta_p s}}{\tau_{c_1} s^{\gamma} + 1} \tag{25}$$

$$G_{c_1}(s) = \frac{1}{K_1} \frac{(\tau_{c_2} s + 1)(\tau_2 s + 1)}{1 + \tau_{c_1} s^{\gamma} - e^{-\theta_p s}} \tag{26}$$

Replacing $s = j\omega$ into the above equation and using two formulas:

$$(j\omega)^{\gamma} = \omega^{\gamma} (\cos\varphi + j\sin\varphi), \varphi = \frac{\pi\gamma}{2} \tag{27}$$

$$e^{-j\omega\theta_p} = \cos\phi - j\sin\phi, \phi = \omega\theta_p \tag{28}$$

Finally, rewritten G_{c_1} in the complex function:

$$G_{c_1}(j\omega) = \frac{a_0 + a_1}{b} - j\frac{a_2 + a_3}{b} \tag{29}$$

where $a_0 = (1 - \tau_{c2}\tau_2\omega^2)(1 - \cos\phi + \tau_{c1}\omega^{\gamma}\cos\varphi)$

$$a_1 = \omega(\tau_{c2} + \tau_2)(\tau_{c1}\omega^{\gamma}\sin\varphi + \sin\phi)$$
$$a_2 = (1 - \tau_{c2}\tau_2\omega^2)(\tau_{c1}\omega^{\gamma}\sin\varphi + \sin\phi)$$
$$a_3 = \omega(\tau_{c2} + \tau_2)(\cos\phi - \tau_{c1}\omega^{\gamma}\cos\varphi - 1)$$
$$b = (1 - \cos\phi + \tau_{c1}\omega^{\gamma}\cos\varphi)^2 + (\tau_{c1}\omega^{\gamma}\sin\varphi + \sin\phi)^2$$

By comparing Eq. (29) with standard FOPI controller in frequency domain Eq. (10), the analytical tuning rules can be found as:

$$K_C + \frac{K_I\cos(\gamma_I)}{\omega^{\lambda}} = \frac{a_0 + a_1}{b} \tag{30}$$

$$\frac{K_I\sin(\gamma_I)}{\omega^{\lambda}} = \frac{a_2 + a_3}{b} \tag{31}$$

The analytical tuning rules can be obtained by solving (30) and (31)

$$K_I = \frac{\omega^{\lambda}}{\sin\gamma_I} \frac{a_2 + a_3}{b} \tag{32}$$

$$K_C = \frac{a_0 + a_1}{b} - \frac{K_I\cos\gamma_I}{\omega^{\lambda}} \tag{33}$$

Note that τ_{c_1} and τ_{c_2} are chosen to balance the system responses and the robustness. In this case, the maximum peak of sensitivity function (M_s) is adopted to evaluate the robustness of the controlled system. The value of M_s is assigned to 1.3 for a fractional-order control system as mentioned in the previous work [39].

IV. SIMULATION STUDY

In this section, an example will be simulated and some performance indices including integral absolute error (IAE) and total variation (TV) are adopted to justify the effectiveness of the proposed method. These indices are calculated by following equations:

$$\text{IAE} = \int_0^T |e(t)|\, dt \tag{34}$$

$$\text{TV} = \sum_{k=1}^{T} |u(k+1) - u(k)| \tag{35}$$

where $e(t)$ is the error of the setpoint and the process output. Therefore, IAE is used to evaluate the system performances in terms of set-point tracking as well as disturbance rejection. TV is obtained to measure the smoothness of the control effort. Normally, these values are as small as possible.

TABLE 1. The performance indices.

	IAE	TV	M_s
Proposed	2.1743	1.234	1.3
FIMC (proposed 4)	4.905	16.75	1.7
2-DoF	7.7539	1.059	1.475

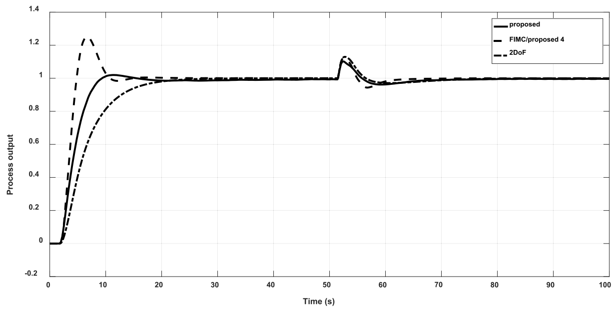

FIGURE 2. The closed-loop responses in terms of setpoint change and disturbance rejection.

The following process models are considered in this work [38]

$$G_{p_1}(s) = \frac{e^{-1.5s}}{5s+1} \tag{36}$$

$$G_{p_2}(s) = \frac{e^{-0.3s}}{s+1} \tag{37}$$

The controller parameters are obtained according to these equations (19) for G_{c2} and (32), (33) for G_{c1} as follows:

$$G_{c_1}(s) = 2.0175 + \frac{0.5137}{s^{0.9}} \tag{38}$$

$$G_{c_2}(s) = 1.6667\left(1 + \frac{1}{s}\right) \tag{39}$$

The closed-loop responses in terms of set-point tracking and disturbance rejection are shown in Figure 2. The disturbance signal is applied at t = 50s with the magnitude of 1. The proposed method is also compared with those of two-degree-of-freedom (2-DoF), which adopted the integer-order PI and PID controllers for the inner and the outer loop respectively, suggested by Alfaro et al. [6] and fractional IMC filter (FIMC) by Ranganayakulu et al. [40]. For a fair comparison, the maximum peak of sensitivity function (M_s) is adopted to keep the robustness the controlled systems being the same level.

In Figure 2, it can be seen that the proposed method has a good balance between set-point tracking response and disturbance rejection performance. The process output is fast enough to avoid overshoot and also maintain the variation under the disturbance change as low as possible. The values of performance indices of all methods are also listed in Table 1.

Figure 3 illustrates the control efforts of all methods in which the proposed method has smoother control signal compared to other ones. At time t = 50 s, when the disturbance changes, the proposed method takes least effort to maintain the output response in good set-point tracking.

To justify the robustness of the controlled system to uncertainties in process dynamics, the gains and time constants

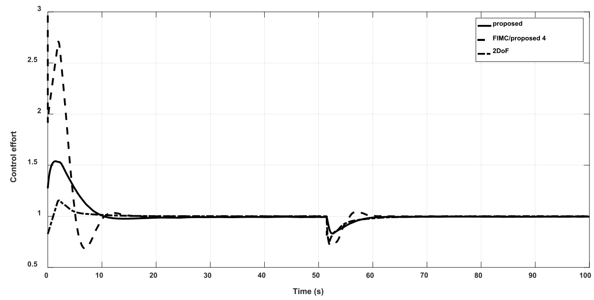

FIGURE 3. The control signals in nominal case.

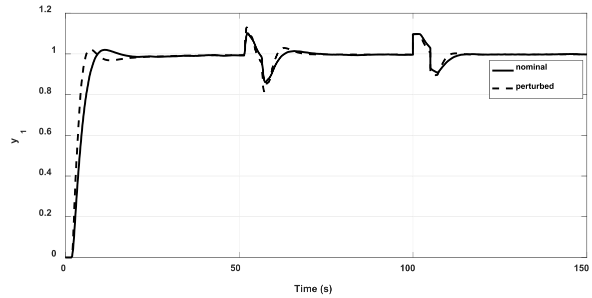

FIGURE 4. The system response in the perturbed case.

of the process models (G_{p1}, G_{p2}) are perturbed by ±20%. Figure 4 demonstrates the step responses in the presence of uncertainties and proves that the proposed controller still keeps robust stability and performs a good balance of setpoint tracking and disturbance rejection.

V. CONCLUSION

An analytical design method of FOPI controller for the cascade control systems based on fractional calculus and direct synthesis method is introduced to provide improved performance for both disturbance rejection and set-point tracking. The Smith predictor is also embedded in the control scheme to eliminate the delay time in the primary control loop. The simulation study demonstrates that the proposed approach can give the superior performance with fast and well-balanced closed-loop time responses in terms of servo tracking and regulator problem. The robustness stability is also testified by using uncertainties of process models. However, in this work, only stable processes in the form of FOPDT is considered, for other cases of industrial processes, the proposed method will be extended in next study.

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