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RESEARCH ARTICLE

Observer-Based Output Feedback Tracking Consensus for Multi-Agent Systems With Periodic Intermittent Communication and Input Saturation

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ABSTRACT This paper studies the semi-global tracking consensus problem for general multi-agent networks with input saturation and periodic intermittent communication. The considered problem is solved by proposing an efficient distributed observer-based periodic intermittent control protocol, where the control inputs of follower agents are subjected to saturation. First, a low-gain feedback method is introduced to suppress the input saturation constraint, and the output feedback control protocol for agents are defined based on the algebraic Riccati equation. Next, a state observer is designed to ensure that observer can realize state reconstruction under the condition that the state of agents cannot be measured directly in the process of information interaction. Then, according to the Lyapunov second method and related linear inequalities, the sufficient conditions which are necessary for a state observation system for the coordinated tracking consensus, are defined under the conditions of input saturation and periodic intermittent communication. Finally, a simulation is performed to prove the expectant algorithm.

INDEX TERMS Multi-agent systems, consensus, input saturation, periodic intermittent control.

I. INTRODUCTION

Recently, multi-agent systems have attracted increasing attention from researchers, due to their excellent results and wide application prospect in different fields, including distributed optimization [1], [2], spacecraft formation control [3], [4], sensor network communication [5], [6], and unmanned aerial vehicle swarm combat [7], [8]. As the basis for cooperative control, a consensus has been a central academic issue [9], [10], [11].

The consensus has been widely studied in recent decade [12], [13], [14], [15]. Rehák and Lynnyk [12] designed an algorithm to achieve linear system synchronization tracking with a time delay. Cai et al. [13] studied the fixed-time consensus problem in multi-dimensional nonlinear multi-agent networks and proposed a distributed static fixed-time control protocol. Wei et al. [14] designed a distributed controller with time-varying coupled weights to achieve the consensus in heterogeneous disturbances. Hu et al. [15] proposed a hybrid periodic protocol to solve the consensus tracking problems in multi-agent networks.

At present, most of the previous studies on leader-following consensus have focused on continuous communication between agents and their neighbors. However, in practice, it can be challenging to maintain continuous communication between agents. In addition, compared with a continuous control strategy, an intermittent control strategy has the advantage of adjusting a suitable control width dynamically, which improves system stability and increases the fault tolerance rate. Therefore, it is more realistic to study consensus considering intermittent communication. Liu et al. [16] used periodic intermittent control to study stochastic complex-valued coupled systems. Yu et al. [17] proposed an improved decentralized periodic intermittent

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control technology, which has been used to study the stability problems in interconnected fractional-order systems. Wang et al. [18] designed an intermittent strategy to investigate the output formation-containment consensus in coupled heterogeneous linear systems. Wang et al. [19] proposed an innovative aperiodic intermittent distributed control strategy to study the exponential consensus in nonlinear systems with a time-varying delay. Guo et al. [20] studied the nonlinear systems consensus tracking with aperiodic intermittent communication and a time delay.

However, in the aforementioned literature, the consensus has been explored based on the same assumption, that control input has no limits. Still, in practical engineering applications, the control input is finite, and when the control input of a system reaches the maximum, it is considered that input saturation is achieved. Therefore, neglecting the effect of input saturation can result in performance degradation and even cause system instability. Thus, considering the input saturation has been of material concern in the research on multi-agent system consensus. Chen et al. [21] achieved the second-order consensus with input saturation and external disturbances in finite time. Wang et al. [22] studied the semi-global consensus subjected to input saturation and developed a fully distributed event-triggered strategy. Inspired by Wang et al. [22], Chang et al. [23] extended the fully distributed event-triggered strategy to a dynamic fully distributed event-triggered strategy, and minimized the number of event-triggered situations by adjusting the adaptive parameters. Wang et al. [24] adopted an adaptive metamorphic low-gain feedback strategy to realize robust consensus with input saturation. Zhu et al. [25] considered a class of linear singular systems and designed a low-gain feedback method to address the bipartite consensus problem under the condition of input saturation.

Most of the existing research has studied the multi-agent systems considering only the condition of input saturation or periodic intermittent control, and there have been fewer studies on the multi-agent system consensus considering both conditions. Recently, promising results have been achieved in this aspect. Su et al. [26] studied the consensus with intermittent containment control and input saturation. Yin et al. [27] designed an improved distributed adaptive event-triggered protocol for nonlinear systems with input saturation. Fan et al. [28] investigated aperiodic intermittent saturation actuators to achieve the linear multi-agent system consensus. However, there have still been certain problems to be solved in this aspect. For instance, how to realize the system consensus under both periodical intermittent control and input saturation could be a further research direction.

Notably, most of the aforementioned works on tracking consensus have been focused on multi-agent systems whose all state information is known. However, the research on the multi-agent system consensus becomes more complex if the system state is unknown. To tackle this problem, extensive studies have introduced an observer into the analysis of consensus, and promising results have been achieved. Chang et al. [29] studied the output consensus in nonlinear multi-agent systems, and designed an output observer based on the periodic sampling strategy. Wang et al. [30] developed a disturbance observer-based controller to achieve the nonlinear bipartite consensus subjected to external disturbances. Xu et al. [31] addressed a semi-global observer-based event-triggered bipartite consensus problem with input saturation. Li et al. [32] constructed a distributed state observer to solve the fuzzy adaptive nonlinear consensus tracking problem. Furthermore, Hu et al. [33] studied the H_∞ robust consensus problem in uncertain linear systems with external disturbances under directed networks. Nevertheless, none of the abovementioned studies have analyzed the consensus considering both input saturation and periodic intermittent control strategy, and none of them have considered the effect of the presence of observers.

To address the shortcomings of the existing research, this paper studies semi-global tracking subjected to input saturation and periodic intermittent control. To that end, a distributed observer-based output feedback law is defined under the assumption that the system state cannot be measured directly.

In sum, the major superiorities of this paper are four following points.

(1) An efficient control protocol that considers periodic intermittent control with input saturation is defined to investigate the tracking consensus problem of linear multi-agent systems.

(2) A complex application environment where the system state information is unknown is considered, and an improved Lyapunov function is derived to prove system stability under the condition of input saturation and periodic intermittent control.

(3) Inspired by [16], [17], [18], [19], [20], [25], [34], [35], and [36], this paper combines the low gain output feedback algorithm with periodic intermittent control, which has rarely been studied in previous literature.

(4) Compared with the existing literature [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], the designed consensus protocol can flexibly cope with the situation when the control input reaches saturation, and the anti-interference capability is improved. The use of periodic intermittent control can actively select the appropriate control bandwidth, and given the control bandwidth is kept within the range we seek, system stability can be guaranteed, the resource occupation of the network is reduced, and it is also closer to the actual application situation.

The remainder of this paper is built as follows: Section II introduces preconditions. Section III shows the problem formulation. Section IV presents an observer-based output feedback control protocol subjected to input saturation and intermittent communication. Section V conducts a numerical simulation. Finally, Section VI summarizes the paper.

II. PRELIMINARIES

A. SYMBOL DESCRIPTION

The set of real numbers is indicated by $\mathcal{R}$. The symbol $\otimes$ means the Kronecker product. $\|\cdot\|$ and $\|\cdot\|_\infty$ are defined as the Euclidean norm and infinite form, respectively. The transpose matrix of matrix $\mathcal{A}$ is $\mathcal{A}^{\mathcal{T}}$. $diag(\cdot)$ means the diagonal matrix. $sign(\cdot)$ represents the sign function. $\mathcal{N}$ means N agents.

B. GRAPH THEORY

Let an undirected graph $\mathcal{G}^\ell = (\mathcal{V}^\ell, \mathcal{E}^\ell)$ be the network consisting of $\mathcal{N}$ agents. In this graph, $\mathcal{V}^\ell = \{1, 2, \ldots, \mathcal{N}\}$ and $\mathcal{E}^\ell \subseteq \mathcal{V}^\ell \times \mathcal{V}^\ell$ express the node set and edge set of the network topology. $A^\ell = \left(a_{ij}^\ell\right)$ is denoted as the adjacent matrix of $\mathcal{G}^\ell$, $a_{ij}^\ell = 1$ if there exists a connection between agent i and agent j, and $a_{ij}^\ell = 0$ otherwise. $\mathcal{D}^\ell = diag\left\{d_1^\ell, d_2^\ell, \ldots, d_{\mathcal{N}}^\ell\right\}$ means the diagonal matrix, $d_i^\ell = \sum_{j \in \mathcal{N}_i} a_{ij}^\ell$. The Laplacian matrix $L^\ell = \left(l_{ij}^\ell\right) \in \mathcal{R}^{\mathcal{N}\times\mathcal{N}}$ stands for $l_{ii}^\ell = \sum_{j=1, j\neq i}^{\mathcal{N}} a_{ij}^\ell$, $l_{ij}^\ell = -a_{ij}^\ell$, $i \neq j$. The eigenvalues of L^ℓ satisfy the condition of $\lambda_1\left(L^\ell\right) \leq \lambda_2\left(L^\ell\right) \leq \cdots \leq \lambda_{\mathcal{N}}\left(L^\ell\right)$.

III. PROBLEM STATEMENT

Consider a group composed of $\mathcal{N}$ followers and a leader with linear dynamics. The dynamic of the i^{th} follower in the group can be expressed by:

$$\begin{aligned} \dot{x}_i &= A^\ell x_i + B^\ell \sigma^*(u_i) \\ y_i &= C^\ell x_i, i = 1, 2, \ldots, \mathcal{N} \end{aligned} \tag{1}$$

where $x_i \in \mathcal{R}^n$ and $y_i \in \mathcal{R}^p$ are the state and corresponding output acting on agent i, $u_i \in \mathcal{R}^m$ is the control input. $\sigma^*(\,) : \mathcal{R}^m \rightarrow \mathcal{R}^m$ is a saturation function, $\sigma^*(u_i) = \{sat^*(u_{i1}), sat^*(u_{i2}), \cdots, sat^*(u_{im})\}^T$ and $sat^*(u_{ij}) = \min\left\{|u_{ij}|, \widehat{\varpi}\right\}$, for some constant $\widehat{\varpi} > 0$.

The dynamics of a leader can be described by

$$\begin{aligned} \dot{x}_{\mathcal{N}+1} &= A^\ell x_{\mathcal{N}+1} \\ y_{\mathcal{N}+1} &= C^\ell x_{\mathcal{N}+1} \end{aligned} \tag{2}$$

As explained before, in practical applications, there can exist several extrinsic elements causing the information to be conveyed discontinuously among neighboring agents. Some of these factors include the limitation of communication bandwidth capacity, energy saving considerations, and other physical conditions. Therefore, it is feasible to use a control method that considers both input saturation and period intermittent communication. The description of the proposed periodic intermittent controller is as follows:

$$u_i \begin{cases} \neq 0, & t \in [sT, sT + \vartheta) \\ = 0, & t \in [sT + \vartheta, (s+1)T) \qquad s = 0, 1, \ldots \end{cases} \tag{3}$$

where $T > 0$ indicates a whole control cycle and $\vartheta > 0$ denotes the control width.

Definition 1 ([37]): For any a priori given bounded set $\chi \subset \mathcal{R}^{\mathcal{N}+1}$ and all $x_i(0) \in \chi$, $i = 1, 2, \ldots, \mathcal{N}, \mathcal{N}+1$, there exist

$$\lim_{t\rightarrow\infty} \|x_i - x_{N+1}\| = 0, \quad i = 1, 2, \ldots, \mathcal{N}$$

Assumption 1 ([38]): The pair $\left(A^\ell, B^\ell\right)$ is asymptotically null controllable with bounded controls, and the pair $\left(A^\ell, C^\ell\right)$ is detectable.

Remark 1: Assumption 1 is a sufficient and necessary condition for a linear network to be semi-globally stable with input saturation.

Lemma 1 ([39]): If the Assumption 1 holds. For each $\widehat{\varepsilon} \in (0, 1]$, there exists a unique matrix $\Phi(\widehat{\varepsilon}) > 0$ that solves the algebraic Riccati Equation:

$$\left(A^\ell\right)^T \Phi(\widehat{\varepsilon}) + \Phi(\widehat{\varepsilon}) A^\ell - 2\Phi(\widehat{\varepsilon}) B^\ell \left(B^\ell\right)^T \Phi(\widehat{\varepsilon}) + \widehat{\varepsilon} I = 0$$

Moreover, $\lim_{\widehat{\varepsilon}\rightarrow\infty} \Phi(\widehat{\varepsilon}) = 0$

Lemma 2 ([26]): For a given real matrix $A^\ell \in \mathcal{R}^{n\times n}$, there exists a positive definite matrix $\Omega = \Omega^T \in \mathcal{R}^{n\times n}$, such that $\left(A^\ell\right)^T \Omega + \Omega A^\ell - 2\eta\Omega < 0$, where $\eta > \max_{j=1,2,\ldots,n} \sum_{i=1}^n \left|a_{ij}^\ell\right|$.

IV. DISTRIBUTED OBSERVER-BASED PERIODIC INTERMITTENT CONTROL

In this section, an output-feedback consensus protocol with input saturation and intermittent control strategy is introduced via a low-gain feedback method. The algorithm designed for systems (1)-(2) is performed in two steps.

(1) Solve the parametric algebraic Riccati equation (ARE):

$$\left(A^\ell\right)^T \Phi(\widehat{\varepsilon}) + \Phi(\widehat{\varepsilon}) A^\ell - 2\beta\Phi(\widehat{\varepsilon}) B^\ell \left(B^\ell\right)^T \Phi(\widehat{\varepsilon}) + \widehat{\varepsilon} I = 0 \tag{4}$$

where β is a positive constant satisfies $\beta \leq \left\{\lambda_1(L^\ell + W)\right\}$.

(2) Design a linear observer-based output feedback protocol for followers:

$$\begin{aligned} \dot{z}_i &= A^\ell z_i + B^\ell \sigma^*(u_i) - M\left(y_i - C^\ell z_i\right) \\ u_i &= -\left(B^\ell\right)^T \Phi(\widehat{\varepsilon}) \left[\sum_{j=1}^{\mathcal{N}} a_{ij}^\ell (z_i - z_j) + w_i (z_i - z_{\mathcal{N}+1})\right] \\ & \qquad i = 1, 2, \ldots, \mathcal{N} \\ \dot{z}_{\mathcal{N}+1} &= A^\ell z_{\mathcal{N}+1} - M\left(y_{\mathcal{N}+1} - C^\ell z_{\mathcal{N}+1}\right) \end{aligned} \tag{5}$$

where z_i is the i^{th} observer state, $z_{\mathcal{N}+1}$ is the observer state of leader. w_i denotes the connection between follower i and the leader, if follower i is a neighbor of leader, then $w_i = 1$, otherwise $w_i = 0$. $M \in \mathcal{R}^{n\times p}$ is the output feedback gain matrix chosen to ensure $\left(A^\ell + MC^\ell\right)$ is asymptotically stable.

Remark 2: Based on the traditional low-gain feedback algorithm, a new controller (5) is constructed with a state observer. In this protocol, agent i needs only information on the local observation state of its neighbors without the need for information on the network topology. Therefore, the protocol is fully distributed and has higher flexibility in practical applications.

Theorem 1: Suppose Assumptions 1 and 2 hold. Consider a system with dynamics (1) and (2). The periodic intermittent control input u_i is described by (5). The control width is described by $g = \frac{\vartheta}{T}$. If the control width is given by:

$$g > \frac{\eta}{\eta + \mu} + \frac{\ln \psi}{(\eta + \mu) T}$$

then the semi-global leader-following consensus can be realized, where $\mu = \frac{1}{2} \min \left\{ \frac{\widehat{\varepsilon}}{2\lambda_{\max}\left(\Phi(\widehat{\varepsilon})\right)}, \frac{99\left(99+\frac{q^2}{\beta}\right)^{-1}}{\lambda_{\max}(P_e)} \right\}$, $\psi = \max \left\{ \frac{\max\left\{\lambda_{\max}\left(\Phi(\widehat{\varepsilon})\right), \lambda_{\max}(P_e)\right\}}{\min\{\lambda_{\min}(\Omega), \lambda_{\min}(P_e)\}}, \frac{\max\{\lambda_{\max}(\Omega), \lambda_{\max}(P_e)\}}{\min\left\{\lambda_{\min}\left(\Phi(\widehat{\varepsilon})\right), \lambda_{\min}(P_e)\right\}} \right\}$; μ is a positive parameter; q is a positive constant satisfying the condition of $q \geq \lambda_{\max}\left(L^{\ell} + W\right)$.

According to the periodic intermittent control (3), (1) can be converted into the following form:

$$\dot{x}_i = \begin{cases} A^{\ell} x_i + B^{\ell} \sigma^* (u_i), t \in [sT, sT + \vartheta) \\ A^{\ell} x_i + B^{\ell} \sigma^* (0), t \in [sT + \vartheta, (s+1) T) \end{cases} \tag{6}$$

Denote $\hbar_i = x_i - x_{\mathcal{N}+1}$, $\tilde{z}_i = z_i - z_{\mathcal{N}+1}$, $e_i = \hbar_i - \tilde{z}_i$.Then, the Lyapunov function is considered as follows:

$$\begin{aligned} &V(\hbar, e) \\ &= \begin{cases} \sum_{i=1}^{\mathcal{N}} \hbar_i^T \Phi(\widehat{\varepsilon}) \hbar_i + k \sum_{i=1}^{\mathcal{N}} e_i^T P_e e_i, t \in [sT, sT + \vartheta) \\ \sum_{i=1}^{\mathcal{N}} \hbar_i^T \Omega \hbar_i + k \sum_{i=1}^{\mathcal{N}} e_i^T P_e e_i, t \in [sT + \vartheta, (s+1) T) \end{cases} \end{aligned} \tag{7}$$

where $k = \lambda_{\max}(\Delta)(\gamma + 99) > 0$. $\gamma = \frac{q^2}{\beta}$, $\Delta = \Phi(\widehat{\varepsilon}) B^{\ell} \left(B^{\ell}\right)^T \Phi(\widehat{\varepsilon})$. P_e is a symmetric positive definite matrix satisfies $\left(A^{\ell} + MC^{\ell}\right)^T P_e + P_e \left(A^{\ell} + MC^{\ell}\right) = -I$. The matrix $\left(A^{\ell} + MC^{\ell}\right)$ is asymptotically stable. The matrix Ω satisfies the Lemma 2.

There exists a constant $\bar{c} > 0$ satisfies:

$$\begin{aligned} \bar{c} \geq \sup_{\substack{\varepsilon \in (0,1], x_i(0) \in \chi, \\ \hat{x}_i(0) \in \chi, \\ 1,2,\ldots,N}} &\left\{ \max \left\{ \sum_{i=1}^{\mathcal{N}} \hbar_i^T(0) \Phi(\widehat{\varepsilon}) \hbar_i(0), \right.\right. \\ &\left.\left. \sum_{i=1}^{\mathcal{N}} \hbar_i^T(0) \Omega \hbar_i(0) \right\} + k \sum_{i=1}^{N} e_i^T(0) P_e e_i(0) \right\} \end{aligned} \tag{8}$$

for all $i = 1, 2, \ldots \mathcal{N}, \mathcal{N}+1$, χ is bounded and $\lim_{\widehat{\varepsilon} \to \infty} \Phi(\widehat{\varepsilon}) = 0$. Let $L_V(\bar{c}) := \left\{\hbar, e \in \mathcal{R}^{\mathcal{N}n} : V(\hbar, e) \leq \bar{c}\right\}$, $\varepsilon^* \in (0, 1]$, for each $\widehat{\varepsilon} \in (0, \varepsilon^*]$, $(\hbar, e) \in L_V(\bar{c})$, $i = 1, 2, \ldots \mathcal{N}$ implies that:

$$\left\| \left(B^{\ell}\right)^T \Phi(\widehat{\varepsilon}) \left[\sum_{j=1}^{\mathcal{N}} a_{ij}^{\ell} \left(z_i - z_j\right) + w_i \left(z_i - z_{\mathcal{N}+1}\right) \right] \right\|_{\infty} \leq \widehat{\varpi} \tag{9}$$

Thus, for any $\widehat{\varepsilon} \in (0, \varepsilon^*]$, (6) remains linear within $L_V(\bar{c})$.

According to Equation (7), the derivative of $V(\hbar, e)$ at $t \in [sT, sT + \vartheta)$ and $t \in [sT + \vartheta, (s+1) T)$ can be obtained.

(1) For $t \in [sT, sT + \vartheta)$,

$$\begin{aligned} \dot{\hbar}_i &= \dot{x}_i - \dot{x}_{\mathcal{N}+1} \\ &= A^{\ell} \hbar_i - B^{\ell} \left(B^{\ell}\right)^T \Phi(\widehat{\varepsilon}) \left(\sum_{j=1}^{\mathcal{N}} a_{ij}^{\ell} \left(\hbar_i - \hbar_j + e_j - e_i\right) \right) \\ &\quad - w_i B^{\ell} \left(B^{\ell}\right)^T \Phi(\widehat{\varepsilon}) (\hbar_i - e_i) \\ \dot{\tilde{z}}_i &= \dot{z}_i - \dot{z}_{\mathcal{N}+1} \\ &= A^{\ell} z_i - B^{\ell} \left(B^{\ell}\right)^T \Phi(\widehat{\varepsilon}) \left(\sum_{j=1}^{\mathcal{N}} a_{ij}^{\ell} \left(\hbar_i - \hbar_j + e_j - e_i\right) \right) \\ &\quad - w_i B^{\ell} \left(B^{\ell}\right)^T \Phi(\widehat{\varepsilon}) (\hbar_i - e_i) - MC^{\ell} e_i \\ \dot{e}_i &= \dot{\hbar}_i - \dot{\tilde{z}}_i = \left(A^{\ell} + MC^{\ell}\right) e_i \end{aligned} \tag{10}$$

From (10), the derivatives of the Lyapunov function can be obtained as follows:

$$\dot{V}(\hbar, e) = 2 \sum_{i=1}^{\mathcal{N}} \hbar_i^T \Phi(\widehat{\varepsilon}) \dot{\hbar}_i + k \sum_{i=1}^{\mathcal{N}} \left(\dot{e}_i^T P_e e_i + e_i^T P_e \dot{e}_i\right) \tag{11}$$

The first items in (11) are expressed by:

$$\begin{aligned} &2 \sum_{i=1}^{\mathcal{N}} \hbar_i^T \Phi(\widehat{\varepsilon}) \dot{\hbar}_i \\ &= \sum_{i=1}^{\mathcal{N}} \hbar_i^T \left(\left(A^{\ell}\right)^T \Phi(\widehat{\varepsilon}) + \Phi(\widehat{\varepsilon}) A^{\ell} \right) \hbar_i \\ &\quad - 2 \sum_{i=1}^{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} a_{ij}^{\ell} \hbar_i^T \Delta \left(\hbar_i - \hbar_j\right) - 2 \sum_{i=1}^{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} a_{ij}^{\ell} \hbar_i^T \Delta \left(e_j - e_i\right) \\ &\quad - 2 \sum_{i=1}^{\mathcal{N}} w_i \hbar_i^T \Delta \hbar_i + 2 \sum_{i=1}^{\mathcal{N}} w_i \hbar_i^T \Delta e_i \\ &= \sum_{i=1}^{\mathcal{N}} \hbar_i^T \left(\left(A^{\ell}\right)^T \Phi(\widehat{\varepsilon}) + \Phi(\widehat{\varepsilon}) A^{\ell} \right) \hbar_i \\ &\quad - \sum_{i=1}^{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} a_{ij}^{\ell} \left(\hbar_i - \hbar_j\right)^T \Delta \left(\hbar_i - \hbar_j\right) \\ &\quad - \sum_{i=1}^{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} a_{ij}^{\ell} \left(\hbar_i - \hbar_j\right)^T \Delta \left(e_j - e_i\right) \end{aligned}$$

$$-2\sum_{i=1}^{\mathcal{N}} w_i \hbar_i^T \Delta \hbar_i + 2\sum_{i=1}^{\mathcal{N}} w_i \hbar_i^T \Delta e_i \tag{12}$$

The remainder of (11) is given by:

$$\begin{aligned} &k\sum_{i=1}^{\mathcal{N}}\left(\dot{e}_i^T P_e e_i + e_i^T P_e \dot{e}_i\right)\\ &\quad = k\sum_{i=1}^{\mathcal{N}} e_i^T \left[\left(A^\ell + MC^\ell\right)^T P_e + P_e\left(A^\ell + MC^\ell\right)\right] e_i\\ &\quad = -k\sum_{i=1}^{\mathcal{N}} e_i^T e_i \end{aligned} \tag{13}$$

For convenience of easier notation, $\hbar = [\hbar_1\ \hbar_2\ \hbar_3 \cdots \hbar_{\mathcal{N}}]^T$ and $e = [e_1\ e_2\ e_3 \cdots e_{\mathcal{N}}]^T$ are defined.

Denote $\alpha = \left(A^\ell\right)^T \Phi(\widehat{\varepsilon}) + \Phi(\widehat{\varepsilon})A^\ell$, $W = diag\{w_1, w_2, \ldots, w_{\mathcal{N}}\}$. Based on (12) and (13), (11) can be expressed as follows:

$$\begin{aligned} &\dot{V}(\hbar, e)\\ &\quad = \hbar^T [I_N \otimes \alpha]\hbar - 2\hbar^T\left[\left(L^\ell + W\right)\otimes\Delta\right]\hbar\\ &\qquad + 2\hbar^T\left[\left(L^\ell + W\right)\otimes\Delta\right]e - \lambda_{\max}(\Delta)(\gamma + 99)e^T e\\ &\quad = \hbar^T\left[I_N\otimes\alpha - \left(L^\ell + W\right)\otimes(2\Delta)\right]\hbar\\ &\qquad + 2\hbar^T\left[\left(L^\ell + W\right)\otimes\Delta\right]e - \lambda_{\max}(\Delta)(\gamma + 99)e^T e \end{aligned} \tag{14}$$

Because matrix $L^\ell + W$ is symmetrical, it is clear that there exists an orthogonal matrix $\mathcal{Q} \in \mathcal{R}^{\mathcal{N}\times\mathcal{N}}$,

$$L^\ell + W = \mathcal{Q}^T diag\{\lambda_1, \lambda_2, \ldots, \lambda_{\mathcal{N}}\}\mathcal{Q}. \tag{15}$$

Denote $\hat{\hbar} = (\mathcal{Q}\otimes I_{\mathcal{N}})\hbar$, $\hat{e} = (\mathcal{Q}\otimes I_{\mathcal{N}})e$. Then,

$$\begin{aligned} &\dot{V}(\hbar, e)\\ &= \hbar^T\left(\mathcal{Q}^T\otimes I_{\mathcal{N}}\right)[I_{\mathcal{N}}\otimes\alpha](\mathcal{Q}\otimes I_{\mathcal{N}})\hbar\\ &\quad - \hbar^T\left[\left(L^\ell + W\right)\otimes(2\Delta)\right]\hbar - \lambda_{\max}(\Delta)(\gamma + 99)e^T e\\ &\quad + \hbar^T\left[\left(L^\ell + W\right)\otimes(2\Delta)\right]e\\ &= \sum_{i=1}^{\mathcal{N}}\hat{\hbar}_i^T(\alpha - 2\lambda_i\Delta)\hat{\hbar}_i + 2\sum_{i=1}^{\mathcal{N}}\hat{\hbar}_i^T\lambda_i\Delta\hat{e}_i\\ &\quad - \lambda_{\max}(\Delta)(\gamma + 99)\sum_{i=1}^{\mathcal{N}} e_i^T e_i\\ &\le \sum_{i=1}^{\mathcal{N}}\hat{\hbar}_i^T\left(\left(A^\ell\right)^T\Phi(\widehat{\varepsilon}) + \Phi(\widehat{\varepsilon})A^\ell - 2\beta\Phi(\widehat{\varepsilon})B^\ell\left(B^\ell\right)^T\Phi(\widehat{\varepsilon})\right)\hat{\hbar}_i\\ &\quad - 99\lambda_{\max}(\Delta)\sum_{i=1}^{\mathcal{N}} e_i^T e_i\\ &\quad + \sum_{i=1}^{\mathcal{N}}\hat{\hbar}_i^T\left(2\lambda_i\Phi(\widehat{\varepsilon})B^\ell\left(B^\ell\right)^T\Phi(\widehat{\varepsilon})\right)\hat{e}_i\\ &\quad - \lambda_{\max}(\Delta)\frac{q^2}{\beta}\sum_{i=1}^{\mathcal{N}} e_i^T e_i\\ &< -\widehat{\varepsilon}\sum_{i=1}^{\mathcal{N}}\hbar_i^T\hbar_i - 99\lambda_{\max}(\Delta)\sum_{i=1}^{\mathcal{N}} e_i^T e_i\\ &\quad - \sum_{i=1}^{\mathcal{N}}\left(\sqrt{\beta}\left(B^\ell\right)^T\Phi(\varepsilon)\hbar_i - \frac{\lambda_i}{\sqrt{\beta}}\left(B^\ell\right)^T\Phi(\varepsilon)e_i\right)^T\\ &\quad \times\left(\sqrt{\beta}\left(B^\ell\right)^T\Phi(\varepsilon)\hbar_i - \frac{\lambda_i}{\sqrt{\beta}}\left(B^\ell\right)^T\Phi(\varepsilon)e_i\right)\\ &< -\widehat{\varepsilon}\sum_{i=1}^{\mathcal{N}}\hbar_i^T\hbar_i - 99\lambda_{\max}(\Delta)\sum_{i=1}^{\mathcal{N}} e_i^T e_i \end{aligned} \tag{16}$$

According to $\sum_{i=1}^{\mathcal{N}}\hbar_i^T\hbar_i \ge \frac{1}{\lambda_{\max}\left(\Phi(\widehat{\varepsilon})\right)}\sum_{i=1}^{\mathcal{N}}\hbar_i^T\Phi(\widehat{\varepsilon})\hbar_i$, $\sum_{i=1}^{\mathcal{N}} e_i^T e_i \ge \frac{1}{\lambda_{\max}(P_e)}\sum_{i=1}^{\mathcal{N}} e_i^T P_e e_i$, (16) can be rewritten as:

$$\dot{V}(\hbar, e) < -2\mu V(\hbar, e) \tag{17}$$

where $\mu = \frac{1}{2}\min\left\{\frac{\widehat{\varepsilon}}{2\lambda_{\max}\left(\Phi(\widehat{\varepsilon})\right)}, \frac{99\left(99 + \frac{q^2}{\beta}\right)^{-1}}{\lambda_{\max}(P_e)}\right\}$.

(2)For $t \in [sT + \vartheta, (s+1)T)$, $u_i = 0$.

$$\begin{aligned} \dot{\hbar}_i &= \dot{x}_i - \dot{x}_{\mathcal{N}+1} = A^\ell\hbar_i\\ \dot{\tilde{z}}_i &= \dot{z}_i - \dot{z}_{\mathcal{N}+1} = A^\ell\tilde{z}_i - MC^\ell e_i\\ \dot{e}_i &= \left(A^\ell + MC^\ell\right)e_i \end{aligned} \tag{18}$$

Then,

$$\begin{aligned} &\dot{V}(\hbar, e)\\ &= \sum_{i=1}^{\mathcal{N}}\dot{\hbar}_i^T\Omega\hbar_i + \sum_{i=1}^{\mathcal{N}}\hbar_i^T\Omega\dot{\hbar}_i + k\sum_{i=1}^{\mathcal{N}}\left(\dot{e}_i^T P_e e_i + e_i^T P_e\dot{e}_i\right)\\ &= \sum_{i=1}^{\mathcal{N}}\hbar_i^T\left(\left(A^\ell\right)^T\Omega + \Omega A^\ell\right)\hbar_i + k\sum_{i=1}^{\mathcal{N}} e_i^T\left(A^\ell + MC^\ell\right)^T P_e e_i\\ &\quad + k\sum_{i=1}^{\mathcal{N}} e_i^T P_e\left(A^\ell + MC^\ell\right)e_i\\ &\le 2\eta_1\sum_{i=1}^{\mathcal{N}}\hbar_i^T\Omega\hbar_i - k\sum_{i=1}^{\mathcal{N}} e_i^T e_i\\ &< 2\eta V(\hbar, e) \end{aligned} \tag{19}$$

where $\eta = \max\left\{\eta_1, -\frac{1}{2\lambda_{\max}(P_e)}\right\}$. Thus,

$$\dot{V}(\hbar, e) < \begin{cases} -2\mu V(\hbar, e), t \in [sT, sT + \vartheta)\\ 2\eta V(\hbar, e), t \in [sT + \vartheta, (s+1)T) \end{cases} \quad s = 1, 2, \ldots \tag{20}$$

From the inequality equation (20), the following results can be obtained:

(1) $0 \leq t < \vartheta$, $V(\hbar(t), e(t)) < V(\hbar(0), e(0)) e^{-2\mu t}$. Furthermore, because
$\lim_{t\to\vartheta^-}\left[\sum_{i=1}^{\mathcal{N}} \hbar_i^T(t)\,\Phi(\widehat{\varepsilon})\hbar_i(t) + k\sum_{i=1}^{\mathcal{N}} e_i^T(t)\,P_e e_i(t)\right]$
$\leq V(\hbar(0), e(0)) e^{-2\mu\vartheta}$,

It can be obtained that:

$$\begin{aligned}
&V(\hbar(\vartheta), e(\vartheta))\\
&\leq \max\{\lambda_{\max}(\Omega), \lambda_{\max}(P_e)\}\\
&\quad\times\left[\sum_{i=1}^{\mathcal{N}} \hbar_i^T(\vartheta)\hbar_i(\vartheta) + k\sum_{i=1}^{\mathcal{N}} e_i^T(\vartheta)e_i(\vartheta)\right]\\
&\leq \frac{\max\{\lambda_{\max}(\Omega), \lambda_{\max}(P_e)\}}{\min\left\{\lambda_{\min}\left(\Phi(\widehat{\varepsilon})\right), \lambda_{\min}(P_e)\right\}} V(\hbar(0), e(0)) e^{-2\mu\vartheta}\\
&\leq \psi V(\hbar(0), e(0)) e^{-2\mu\vartheta}
\end{aligned}$$

(2) For $\vartheta \leq t < T$, $V(\hbar(t), e(t)) < V(\hbar(\vartheta), e(\vartheta)) e^{2\eta(t-\vartheta)}$.

$$\begin{aligned}
V(\hbar(T), e(T)) &\leq \frac{\max\left\{\lambda_{\max}\left(\Phi(\widehat{\varepsilon})\right), \lambda_{\max}(P_e)\right\}}{\min\{\lambda_{\min}(\Omega), \lambda_{\min}(P_e)\}}\\
&\quad\times V(\hbar(\vartheta), e(\vartheta)) e^{2\eta(T-\vartheta)}\\
&\leq V(\hbar(0), e(0)) e^{-2[(\mu+\eta)\vartheta-\eta T-\ln\psi]}
\end{aligned}$$

Denote $\Gamma = (\mu+\eta)\vartheta - \eta T - \ln\psi$, since $g = \frac{\vartheta}{T} > \frac{\eta}{\eta+\mu} + \frac{\ln\psi}{(\eta+\mu)T}$, it is clear that $\Gamma > 0$.

Then, $V(\hbar(T), e(T)) \leq V(\hbar(0), e(0)) e^{-2\Gamma}$.

Similarly,

$V(\hbar(T+\vartheta), e(T+\vartheta)) \leq \psi V(\hbar(0), e(0)) e^{-2\Gamma}e^{-2\mu\vartheta}$, $T \leq t < T+\vartheta$

$$V(\hbar(2T), e(2T)) \leq V(\hbar(0), e(0)) e^{-4\Gamma}, T+\vartheta \leq t < 2T$$

Furthermore, it can be assumed that the following results can be obtained using mathematical induction,

$$\begin{aligned}
&V(\hbar(sT), e(sT)) \leq V(\hbar(0), e(0)) e^{-2s\Gamma}\\
&V(\hbar((s+1)T), e((s+1)T)) \leq V(\hbar(0), e(0)) e^{-2(s+1)\Gamma}\\
&\qquad s = 0, 1, \ldots
\end{aligned}$$

In the following, a short proof of the abovementioned results is provided.

(1) $t \in [sT, sT+\vartheta)$,

$$\begin{aligned}
&V(\hbar(t), e(t)) \leq V(\hbar(sT), e(sT)) \leq V(\hbar(sT), e(sT)) e^{2\eta T}\\
&V(\hbar(sT+\vartheta), e(sT+\vartheta)) \leq V(\hbar(sT), e(sT)) e^{-2\mu\vartheta}
\end{aligned}$$

(2) $t \in [sT+\vartheta, (s+1)T)$,

$$\begin{aligned}
V(\hbar(t), e(t)) &\leq V(\hbar(sT+\vartheta), e(sT+\vartheta)) e^{2\eta(t-sT-\vartheta)}\\
&< V(\hbar(sT), e(sT)) e^{-2(\mu+\eta)\vartheta+2\eta T+\ln\psi}
\end{aligned}$$

Since $2(\mu+\eta)\vartheta - \ln\psi > 0$, then $e^{-2(\mu+\eta)\vartheta+\ln\psi} < 1$.

$V(\hbar(t), e(t)) \leq V(\hbar(sT), e(sT)) e^{2\eta T}$

Therefore, for all $t \in [sT, (s+1)T)$,

$$\begin{aligned}
V(\hbar(t), e(t)) &\leq V(\hbar(0), e(0)) e^{-2s\Gamma+2\eta T}\\
&\leq V(\hbar(0), e(0)) e^{-2\left(\frac{t}{T}-1\right)\Gamma+2\eta T}\\
&= e^{2\Gamma+2\eta T} V(\hbar(0), e(0)) e^{-2\frac{\Gamma}{T}t}
\end{aligned}$$

Denote $a = e^{2\Gamma+2\eta T} V(\hbar(0), e(0)) > 0$, $b = 2\frac{\Gamma}{T}$, then

$$V(\hbar(t), e(t)) \leq ae^{-bt}, t \in (0, \infty)$$

This implies that the derivative of is bounded and converges to the origin asymptotically when $t \to \infty$.This completes the proof.

Remark 3: In Theorem 1, the low gain feedback method is used to create small inputs to ensure that the system is bounded and unsaturated. It is an ingenious solution to deal with the input saturation problem in a multi-agent system. Then, the abovementioned system can be transformed into an observation system without saturation constraints.

Remark 4: In [39], the group consensus of one-sided Lipschitz nonlinear multi-agent systems with input saturation was considered by using a novel dynamic distributed event-triggered communication scheme, in which the network contains fixed topology and Markovian switching topologies respectively. In [41] and [42], the authors further considered the unmeasurable states and output-feedback consensus tracking problem. However, in [39], [41], and [42], the situation of intermittent communication was not considered, which could be beneficial to saving bandwidth capacity and controlling energy output. In view of that, this situation is studied in this work.

In the proof of Theorem 1, for system (1), the following conclusions can be reached easily.

Special case 1: When the status information of the agents can be directly obtained, the dynamics of followers and the leader are as follows:

$$\begin{aligned}
&\dot{x}_i = A^\ell x_i + B^\ell \sigma^*(u_i) \qquad i = 1, 2, \ldots, \mathcal{N}\\
&\dot{x}_{\mathcal{N}+1} = A^\ell x_{\mathcal{N}+1}
\end{aligned} \tag{21}$$

The algorithm is performed in two steps.

(1) Solve the parametric *ARE* by (4);

(2) Design state feedback laws via periodically intermittent control:

$$u_i = \begin{cases} -\left(B^\ell\right)^T \Phi(\widehat{\varepsilon})\left[\sum_{j=1}^{\mathcal{N}} a_{ij}^\ell\left(x_i - x_j\right) + w_i\left(x_i - x_{\mathcal{N}+1}\right)\right], \\ \quad t \in [sT, sT+\vartheta) \\ 0, t \in [sT+\vartheta, (s+1)T) \end{cases} \tag{22}$$

Corollary 1: Suppose Assumptions 1 holds. Consider a system with dynamics (21). The periodic intermittent control input u_i is described by (22). If the control width satisfies:

$$g > \frac{\ln\psi}{\eta},$$

where $\eta = \frac{\widehat{\varepsilon}}{2\lambda_{\max}\left(\Phi(\widehat{\varepsilon})\right)} > 0$, $\psi = \max\left\{\frac{\lambda_{\max}\left(\Phi(\widehat{\varepsilon})\right)}{\lambda_{\min}(\Omega)}, \frac{\lambda_{\max}(\Omega)}{\lambda_{\min}\left(\Phi(\widehat{\varepsilon})\right)}\right\}$. Then the semi-global leader-following consensus can be realized.

Special case 2:

Corollary 2: Let Assumptions 1 and 2 hold. Consider a system with dynamics (1) and (2). The control input (5) could achieve the semi-global leader-following consensus.

Special case 3: There is no leader in the system (1) when $x_{\mathcal{N}+1} = 0$. The dynamics of agents are described by (1). The algorithm is performed in two steps.

(1) Solve the parametric *ARE* by (4);

(2) Design a linear observer-based output feedback law:

$$\dot{z}_i = A^\ell z_i + B^\ell \sigma^*(u_i) - M\left(y_i - C^\ell z_i\right)$$
$$u_i = \begin{cases} -\left(B^\ell\right)^T \Phi(\widehat{\varepsilon}) \sum_{j=1}^{N} a_{ij}^\ell \left(z_i - z_j\right), t \in [sT, sT + \vartheta) \\ 0, t \in [sT + \vartheta, (s+1)T) \end{cases} \tag{23}$$

Corollary 3: Suppose Assumptions 1 and 2 hold. Consider a system with dynamics (1). The periodic intermittent control input is described by (23). If the control width is given by:

$$g > \frac{\eta}{\eta + \mu} + \frac{2\ln\psi}{(\eta+\mu)T},$$

where

$$\psi = \max\left\{\frac{\max\left\{\lambda_{\max}\left(\Phi(\widehat{\varepsilon})\right), \lambda_{\max}(P_e)\right\}}{\min\{\lambda_{\min}(\Omega), \lambda_{\min}(P_e)\}}, \frac{\max\{\lambda_{\max}(\Omega), \lambda_{\max}(P_e)\}}{\min\left\{\lambda_{\min}\left(\Phi(\widehat{\varepsilon})\right), \lambda_{\min}(P_e)\right\}}\right\},$$

and $\mu = \frac{1}{2}\min\left\{\frac{\widehat{\varepsilon}}{2\lambda_{\max}\left(\Phi(\widehat{\varepsilon})\right)}, \frac{100\left(100+\frac{q^2}{\beta}\right)^{-1}\lambda_{\min}(P_e^{-1})}{2}\right\}$.

Then the semi-global consensus can be realized.

Remark 5: This paper studies the semi-global consensus tracking problem with input saturation under the condition of periodic intermittent control. However, how to implement semi-global tracking consensus under the conditions of aperiodic intermittent control and input saturation has been an unsolved problem. Meanwhile, how to realize the semi-global consensus without leaders is also worth studying.

V. NUMERICAL SIMULATION

In this section, a simulation is conducted to prove the validity of the expected analysis. Consider a multi-agent system consisting of 3 followers and a leader. The network topology is shown in Fig.1.

The node 0 is designed as a leader and nodes 1×3 denote followers. Next, the appropriate parameters can be chosen.

$$A^\ell = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}, B^\ell = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C^\ell = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

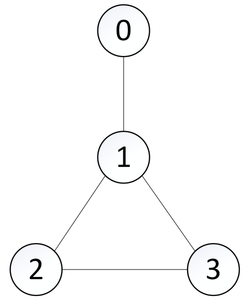

FIGURE 1. The network topology.

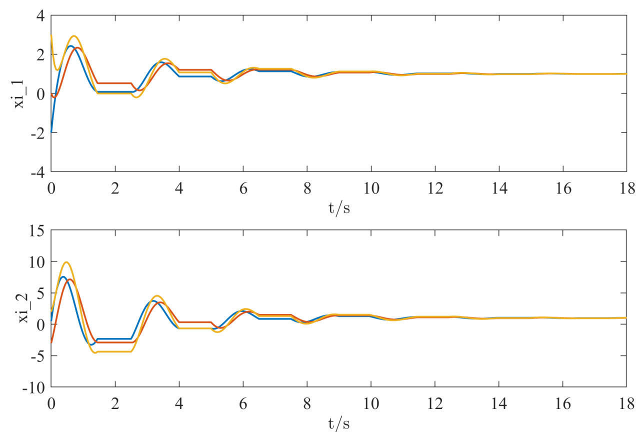

FIGURE 2. The state trajectories with $\varepsilon = 0.01$ and $T = 2.5$.

Moreover, $M = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ is chosen to ensure $\left(A^\ell + MC^\ell\right)$ is asymptotically stable. $x_{i1}(0)$, $x_{i2}(0)$ are stochastically chosen from region $[-5, 5] \times [-5, 5]$, respectively. $\hat{x}_i(0)$ are selected as $[0.5, 0.5]$. Similarly, $x_{N+1}(0)$ is selected as $[1, 1]$, while $\hat{x}_{N+1}(0)$ is chosen as $[0.75, 0.75]$.

The Laplacian matrix of the system can be obtained from the network topology

$$L^\ell = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

Choosing $W = diag\{1, 0, 1\}$, we can obtain $\lambda_1\left(L^\ell + W\right) = 0.5$. Since $\beta \leq \lambda_1\left(L^\ell + W\right)$, we choose $\beta = 0.4$. Because $\eta > \max_{j=1,2\ldots n} \sum_{i=1}^{n} \left|a_{ij}^\ell\right|$, we select $\eta = 0.05$. The symmetric matrix Ω and P_e can be calculated:

$$\Omega = \begin{pmatrix} 0.1667 & -0.0849 \\ -0.0849 & 0.3739 \end{pmatrix}, P_e = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.5 \end{pmatrix},$$

and $\lambda_{\min}(\Omega) = 0.1364$, $\lambda_{\max}(\Omega) = 0.4042$.

Let $\widehat{\varepsilon} = 0.1$, then $\Phi(\varepsilon)$ is:

$$\Phi\left(\widehat{\varepsilon} = 0.1\right) = \begin{pmatrix} 0.0976 & 0.1549 \\ 0.1549 & 0.3536 \end{pmatrix},$$

and $\lambda_{\min}\left(\Phi\left(\widehat{\varepsilon} = 0.1\right)\right) = 0.0247$, $\lambda_{\max}\left(\Phi\left(\widehat{\varepsilon} = 0.1\right)\right) = 0.4265$.

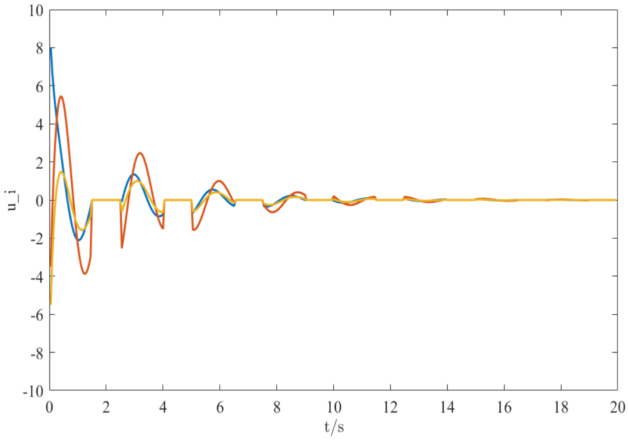

FIGURE 3. Control input with $\varepsilon = 0.01$ and $T = 2.5$.

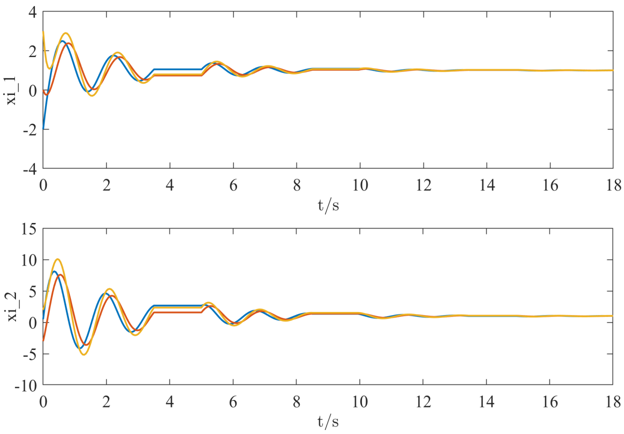

FIGURE 4. Control input with $\varepsilon = 0.01$ and $T = 5$.

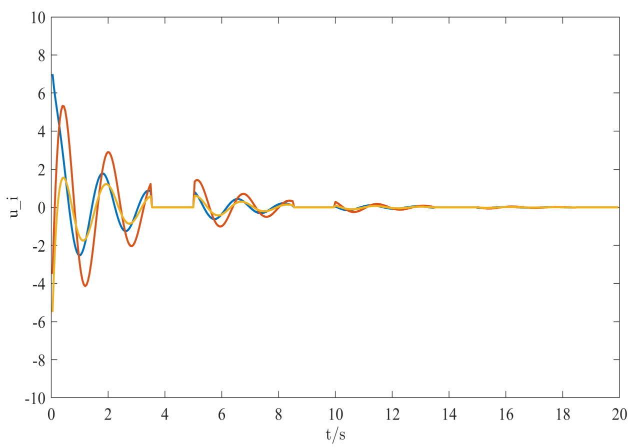

FIGURE 5. Control input with $\varepsilon = 0.01$ and $T = 5$.

When $\widehat{\varepsilon} = 0.01$,

$$\Phi\left(\widehat{\varepsilon} = 0.01\right) = \begin{pmatrix} 0.0287 & 0.0535 \\ 0.0535 & 0.1118 \end{pmatrix},$$

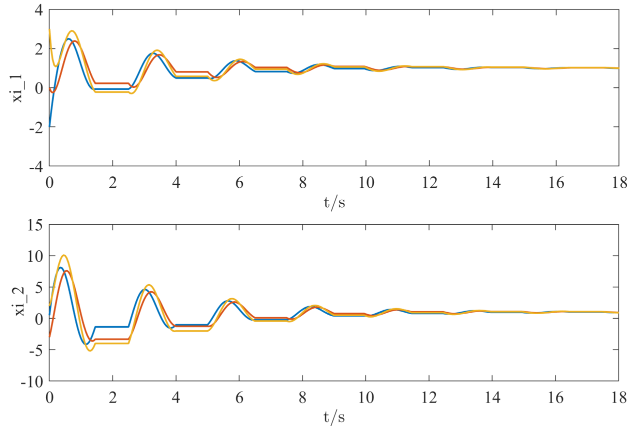

FIGURE 6. Control input with $\varepsilon = 0.1$ and $T = 5$.

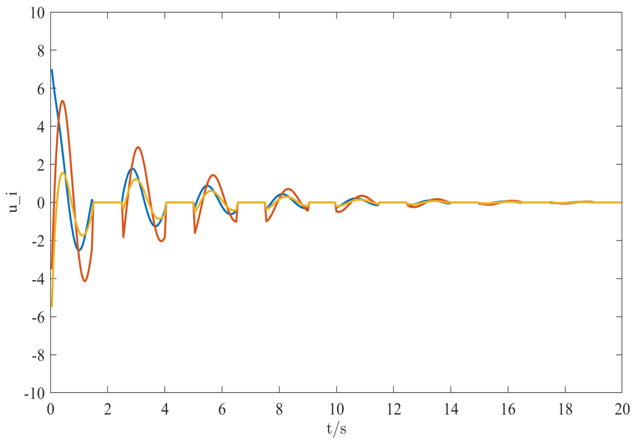

FIGURE 7. Control input with $\varepsilon = 0.1$ and $T = 5$.

$\lambda_{\min}\left(\Phi\left(\widehat{\varepsilon} = 0.01\right)\right) = 0.0025$, $\lambda_{\max}\left(\Phi\left(\widehat{\varepsilon} = 0.01\right)\right) = 0.1380$.

As shown in Figs.2-7, the control protocol (5) could achieve the semi-global tracking consensus. The results in Figs. 2-5 indicate that for the same initial states and $\widehat{\varepsilon}$, as the period increases, the difference in the control input decreases, so the consensus is reached faster. In addition, Figs. 2-3 and Figs.6-7 show that for the same initial states and T, the value of the control input u_i increases with the increase in $\widehat{\varepsilon}$, and followers quickly synchronize with the leader.

VI. CONCLUSION

This study analyzes the semi-global observer-based leader-following consensus problem in multi-agent networks with periodic intermittent communication and input saturation. To solve the considered problem, this paper proposes a new distributed output feedback consensus control protocol based on the algebraic Riccati equation, which does not need any global information of networks. At the same time, the upper bound of the control bandwidth that needs to be satisfied under the conditions of input saturation and periodic intermittent communication is defined. Furthermore, a simulation is performed to explain the validity of the suggested protocol. Based on previous research, the control protocol proposed in this paper adds input saturation, which can stably resolve the problem of control input reaching saturation in practical applications and enhance the anti-interference capability of the system. In addition, periodic intermittent control is used to actively select the appropriate control bandwidth to reduce the resource occupation of network bandwidth to guarantee system stability and grant it higher flexibility.

Considering that not all agents in the systems cooperate with each other and that competitive interaction may exist, the bipartite consensus under the condition of input saturation could be studied in the future.

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