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RESEARCH ARTICLE

Roaming Navigation: Diverse Constrained Paths Using Heuristic Search

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ABSTRACT Navigation has become an indispensable technology, especially when exploring unfamiliar environments. However, the existing shortest route-based navigation systems only focus on route effectiveness, which may deprive users of the opportunity to explore new areas. Hence, there is a need for a new navigation service that enriches our walking activities and maximizes their benefits in our daily lives by stimulating our natural tendency to explore. In this paper, we revisit navigation services from two orthogonal technological perspectives: 1) navigation using non-shortest routes; and 2) navigation across multiple days. Based on these two perspectives, we explain the relationship among related studies and develop a new heuristic search-based method to create diverse multiday routes. This could help construct a new navigation service to increase the graph roaming entropy values on those generated routes.

INDEX TERMS Heuristic search, pedestrian navigation, roaming entropy.

I. INTRODUCTION

We tend to visit unfamiliar landmarks and landscapes in our lifetime. Previously, tourists traveling to unfamiliar cities would use printed maps and/or guidebooks. Nowadays, we can easily utilize navigation functions provided by mobile devices, such as smartphones and smartwatches, in the form of mobile applications, which we call pedestrian navigation services (PNSs). Although such navigation functions were initially developed for car driving as global positioning system (GPS)-based positioning technologies prevailed [1], [2], [3], we can now access them wherever and whenever using mobile devices. These functions could increase travel efficiency as they provide the most efficient routes in terms of some primary measures, such as travel distances and travel times.

The efficiency of the navigated routes displayed in navigation does *not* always coincide with the freedom of personal exploration and emotional pleasure since walkers have their subjective purposes for walking. Several researchers investigated various aspects of navigation and routes and their connection to our health and well-being. Quercia et al. proposed a method to create beautiful, quiet, and happy travel routes rather than just offering efficient routes [4]. Siriaraya et al. categorized quality-aware route navigation systems from multiple perspectives using the SWEEP[1] taxonomy [5]. Lee and Buchner discussed the importance of walking in our lives [6]. Heller et al. experimentally evaluated the relationship between traveling and happiness [7]. Sharker et al. studied the intersection of the health-based and routing-based aspects of navigation services [8]. However, no practical navigation applications have been used in the existing literature to support the exploring activities (e.g., the use of navigation in our lives is still limited [9]) despite the importance of these activities in our daily lives.

A technical backbone supporting PNS is the calculation of (efficient) routes to explore a target area, such as a sightseeing district. In this study, we call an exploring activity between a departure location and a destination location *roaming*.[2] To implement our new PNS, we revisited two aspects of routing: (1) navigation using *non-shortest* routes and (2) navigation *across multiple days*. Fig. 1 describes the concept of our roaming navigation, and Fig. 2 highlights our work and the related studies from the two perspectives. From the

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[1] Acronym for Safety, Well-being, Effort, Exploration, and Pleasure.
[2] 'Roaming' was originally used for an activity *without any specific destination*, but we use it to express more general exploring walking activities.

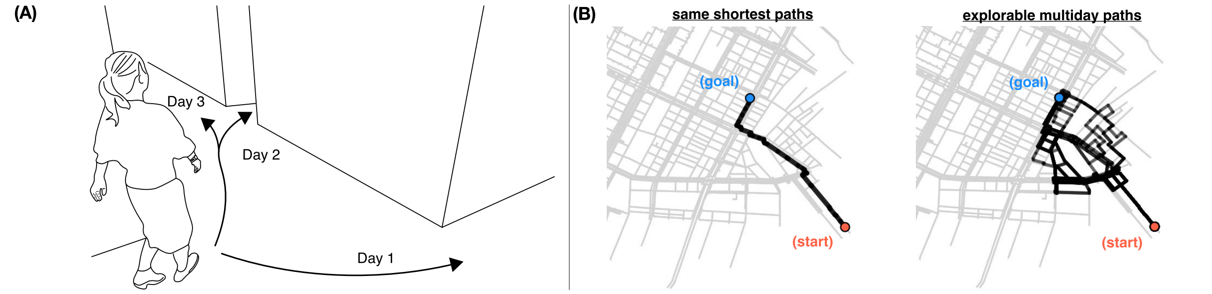


FIGURE 1. **Roaming navigation: Different routes supporting users' exploring activities. (A) Exploring within an unfamiliar environment using different routes over multiple days is a promising aspect when walking. (B) A heuristic search-based algorithm that performs within a reasonable time on real road networks.**

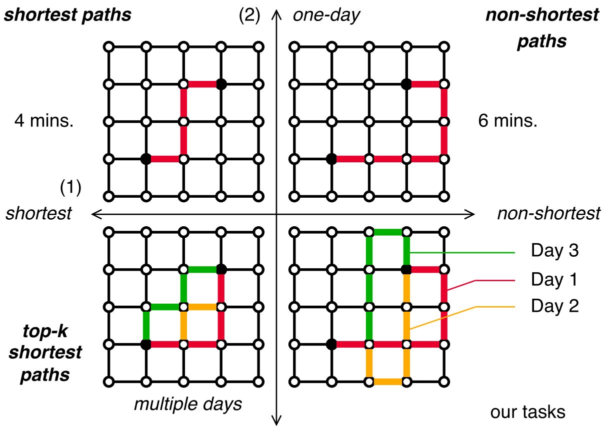


FIGURE 2. **Two technical aspects of navigation: (1) navigation using non-shortest paths (on the horizontal axis) and (2) navigation across multiple days (on the vertical axis). We focus on the right-bottom region.**

first perspective, most navigation services usually adopt the most efficient routes according to some primary measures (e.g., distance and time). However, the use of shortest paths is not suitable for exploration. Thus, we must focus on the non-shortest paths (e.g., constrained shortest paths [10], [11]). From the second perspective, we consider a use case of PNS for exploring an unfamiliar city during a multiday sightseeing trip. Users should roam between POIs on diverse routes daily to make exploring the city and visiting points of interest (POIs) compatible. Pursuing path diversity on graphs has also been investigated (e.g., top-k paths [12], [13], [14], [15], [16]). Users, like tourists, could adopt such varied routes to explore an unknown city. Although some researchers tackled the two aspects separately, combining them to build a PNS supporting "exploring activities" remains challenging, and this is the main focus of our study.

This study proposes a navigation service to facilitate pedestrian exploratory activities beyond the shortest path-based PNSs. We develop a new heuristic search method for *non-shortest path-based, multiday PNSs*. The displayed routes should be adaptive according to the previously navigated ones for multiday use. We measure the diversity of computed routes based on previously computed ones and thus enable the user to explore other new routes daily. We apply our method to navigation tasks on synthetic graphs and real road networks to demonstrate our approach.

In summary, we tackle the following task to depict a new pedestrian navigation service.

- We formulate a new navigation task named *multiday explorable non-shortest navigation*. This task is based on two orthogonal technical perspectives: (1) navigation using shortest or non-shortest paths and (2) navigation for a single day or multiple days. We discuss the relationship between our task and existing studies, clarifying the computational difficulty of the task.

The contributions of this study are summarized as follows:

- We propose a new measure called *GraphRE* (graph roaming entropy), which evaluates the diversity of daily activities by users based on an existing measure [7]. We discuss our navigation task as the problem involving increasing GraphRE values.
- We show that a simple heuristic search procedure could create diverse explorable paths with time constraints.
- The experiments demonstrate our method regarding GraphRE values; the proposed approach can enhance the GraphRE values. Furthermore, we compare our heuristic-based method with baselines designed for the targeting task. We confirm that our method generates more explorable routes.

II. RELATED WORK

Using navigation systems is common in exploratory activities. The widespread use of mobile devices with GPS raises various research questions for both navigation systems and human beings: how to display navigation information (e.g., [17], [18], [19], [20]), how to navigate users (e.g., [21]), and how navigation relates to (public) health and happiness (e.g., [4], [6], [8], [22]). We herein clarify the two aspects of our study.

A. NAVIGATION AND WELL-BEING

Promoting exploration instead of just navigating the most efficient routes is promising for the human-computer interaction (HCI) community. 'I did it my way' by Robinson et al.

allows users to select their routes and arrive at their destinations in an exploratory manner by providing haptic vibrations to guide them toward the direction of their destination [23]. Using the haptic stimulus as a cue, this navigation method satisfies the user's desire to explore without disturbing the user's exploratory behavior, such as sightseeing. Devices supporting explorative activities have been an emerging topic in the HCI community for decades. Examples are vibrotactile navigation with smartphone [24], smartwatches [25], bicycles [26], and auditory navigation [27], [28].

Recommending quality-aware routes, such as beautiful routes, rather than just providing the shortest paths is challenging since it depends on subjective measurements [4]. Besides, it has been shown that exploring unfamiliar cities by walking has health benefits. As mentioned earlier, Heller et al. [7] adopted roaming entropy (RE) values to measure the diversity of participants' locations and evaluate its association with their happiness. In addition, well-being is known to be an important measure of pedestrian routes as in [6], [8], and [5]. We can conjecture that explorable routes–based PNSs efficiently enrich users' experience as pedestrians.

B. ROUTE PLANNING FOR NAVIGATION

Our navigation task, *multiday explorable non-shortest navigation* (formally defined in Sec. III-C), is related to two orthogonal research fields: (1) computing (most efficient) paths on graphs and (2) computing multiple paths for navigation across multiple days, corresponding to the two axes in Fig. 2.

Computing paths on graphs is a fundamental task. The shortest path problem is a fundamental combinatorial problem studied in graph theory and algorithms (e.g., [29]). Many researchers have made contributions concerning engineering to efficiently compute shortest paths on large-scale road networks (e.g., [30], [31], [32], [33]). Other studies involve heuristic search (e.g., [34], [35]), which focuses on building a generic search procedure on search spaces. Together with additional (resource) constraints beyond the primary ones, constrained shortest path problems ($\mathcal{CSP}$ problems) have been studied as a variant of shortest path problems (e.g., [10], [11], [36], [37], [38], [39]). An example of $\mathcal{CSP}$ problems is the problem of finding the cheapest travel path using public transportation under total travel time constraints. We can model non-shortest routes for PNSs by imposing constraints on resources (e.g., minimum travel distances, as used in Sec. III-B1),

Another research topic is pursuing the diversity of multiple paths. For shortest paths, two well-known algorithms by Yen [12] and by Eppstein [13] are recognized as fundamental approaches to computing top-k shortest paths. Evaluating k solutions as a set (e.g., the diversity of solutions) is also a critical issue (e.g., [14]). Moreover, computing diverse top-k shortest paths is an active research topic (e.g., [15], [16], [40]). User studies on alternatives have also been discussed (e.g., [41]). However, few studies have addressed the problem of computing a set of *non-shortest* paths because of its computational intractability, an essential aspect of our roaming navigation application.

III. HEURISTIC SEARCH ON GRAPHS FOR ROAMING NAVIGATION

A. GRAPHS AND SHORTEST PATHS

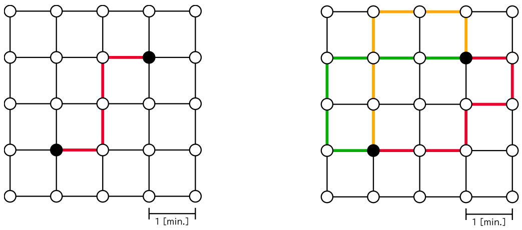
Let $G = (V, E, w)$ be an undirected weighted simple graph representing a target service area. The set V is a set of vertices corresponding to intersections and/or intermediate points on road segments. The set $E \subseteq \{\{u, v\} \mid u, v \in V\}$ is a set of (undirected) edges representing walkable road segments for pedestrians. The function $w : E \to \mathbb{R}_{\geq 0}$ defines the cost $w(e)$ of traveling $e \in E$. Herein, we assume that an undirected graph models an area, but our study can be generalized to models using directed graphs (e.g., areas with one-way streets). A path between two vertices s and t is a sequence of distinct vertices, denoted by $p = \langle p_1 = s, p_2, \ldots, p_{|p|} = t \rangle$, where $|p|$ is the length of p. Herein, we assume that all paths are simple .[3] Such a path, starting from s and ending at t for given two vertices s and t, is called an s-t path. Note that the terms *path* and *route* will be used interchangeably hereafter. For convenience, $\Pi(v_1, v_2)$ represents the set of all paths starting from $v_1 \in V$ and ending at $v_2 \in V$ if $v_1 \neq v_2$. The travel cost $cost(p)$ along with p is defined by $cost(p) := \sum_{j=1}^{|p|-1} w(\{p_j, p_{j+1}\})$. For a shortest path $\pi \in \Pi(v_1, v_2)$ between two vertices v_1 and v_2, we denote by $d(v_1, v_2) = cost(\pi)$ the shortest path distance from v_1 to v_2. Providing the shortest path to pedestrians is a fundamental approach for PNSs.

From an application viewpoint, let $w(e)$ represent the travel distance, and $cost(p)$ indicates the total travel distance along path p. Similarly, let $w(e)$ represent the required walking time, and $cost(p)$ indicates the total walking time. Therefore, assuming some fixed walking speed (e.g., $4[km/h]$), time does not require to be distinguished from a distance. Conversely, we can independently set distance and time (e.g., assuming slopes and travel times per direction). In such a case, multi-objective functions to evaluate paths are defined according to problem formulations. This study implicitly assumes some walking speed and discusses the feasibility of using only travel distances. In another approach similar to $\mathcal{CSP}$ problems, we can use either the distance or time as the primary objective function and adopt another as a defining constraint. As explained earlier and in Sec. III-B1, this study adopts $\mathcal{CSP}$ problems to model non-shortest paths.

B. EXISTING NAVIGATION TASKS

We explain our two perspectives: (1) non-shortest paths (Sec III-B1) and (2) multiday navigation (Sec III-B2).

[3] A simple path is a path on G, which does not possess repeating vertices.



(a) Shortest path (b) Feasible paths

FIGURE 3. Comparison of shortest paths and feasible paths on 5 × 5 grid graphs: (3a) shortest path with travel time of 4 minutes. (3b) three feasible paths with travel time of 6 minutes with additional $\epsilon = 2$ minutes to explore areas.

1) FEASIBLE AND NON-SHORTEST PATHS

This paper adopts $\mathcal{CSP}$ formulations to formulate the feasibility while computing non-shortest paths. Without loss of generality, an s-t path $p \in \Pi(s, t)$ is *feasible* if and only if p satisfies user-designed constraints on a navigation application. An example of such constraints regarding travel times is as follows.

> **Time-constrained path** (start s, goal t): A user on $s \in V$ at time T_s has his/her *desired* arrival time on $t \in V$, i.e., until T_t. If $w(e)$ represents the travel time required on edge $e \in E$; we need to find a path $p \in \Pi(s, t)$ such that $T_s + cost(p) \approx T_t$. With any tolerance parameter $\epsilon > 0$, we can define the above arrival time constraint as $|T_s + cost(p) - T_t| \leq \epsilon$, which is an example of user-designed constraints in terms of travel time.

We write a set of all feasible paths by $\Pi^{\text{feasible}}(s, t)$. For some infeasible path $p \in \Pi(s, t) \setminus \Pi^{\text{feasible}}(s, t)$, traveling along with p is too fast or late for arriving t on time for users.

a: EXAMPLES

Let us assume that an edge in the grid graphs requires unit time (1 min.) to travel. The shortest travel time is 4 minutes, as shown in Fig. 3a, whereas the non-shortest paths require 6 minutes, as shown in Fig. 3b. Based on the time-constrained paths, the path of travel time of 6 mins. (Fig. 3c) is feasible if $\epsilon = 2$. Users can set ϵ according to their willingness to explore on G. They can utilize the non-shortest paths if they have an additional 2 minutes to walk.

b: CHALLENGES

Computing non-shortest paths is computationally challenging. For example, computing a time-constrained shortest-distance path $\arg\min_{\hat{p} \in \Pi^{\text{feasible}}(s,t)} cost(\hat{p})$ is known to be NP-hard. This computational intractability can be shown from the relationship between the well-known knapsack problems and $\mathcal{CSP}$ problems having upper bounds of resource consumption (e.g., Chapter 3 in [11]). We can generalize such constrained shortest path problems having both *lower bounds* and upper bounds of resource consumption. This means computing roaming routes with time constraints (e.g., 30 minutes non-shortest walking) is challenging.

2) MULTIDAY NAVIGATION TASK

Beyond single day navigation tasks, we are interested in a scenario where users access our PNS for multiple days (e.g., every day in a month). Herein, just computing a feasible path $p \in \Pi^{\text{feasible}}(s, t)$ is insufficient because following the same path p daily is unpleasant for exploring unknown cities. We then study a *multiday navigation task* with a route set evaluation function for this scenario. We use p_j to index daily routes, meaning that the route p_j was navigated on the day j, and denote by $\mathcal{R}_k = \{p_1, \ldots, p_k\}$ the route set until day k. The function $m : \mathcal{R}_k \mapsto \mathbb{R}_{\geq 0}$ evaluates the set $\mathcal{R}_k$, where a higher value is more pleasant to users.

Our multiday navigation task is described as follows: On each day $k \in \{1, \ldots, M\}$, the task involves navigating a user with a feasible route p_k maximizing $m(\mathcal{R}_k)$. Defining $m(\cdot)$ should depend on the purpose of the PNS. Liu et al. described the dissimilarity based on given similarity functions and discussed top-k shortest diversified paths [14]. Chondrogiannis et al. [15] adopted the minimum collective length based on the Jaccard index for top-k shortest paths.

From the connection between roaming activity and well-being, we focus on *roaming entropy* (RE), which was originally used in [42] to evaluate the GPS log traces and adopted in previous study [7]. In their setting, the authors measured n GPS traces of all participants and evaluated the RE value as $\text{RE} := \frac{\sum_{j=1}^{K} \mathbf{p}_{ij} \log_2 \mathbf{p}_{ij}}{\log K}$, where $\mathbf{p}_{ij}$ is the empirical probability that a participant i visits a point $j \in \{1, \ldots, K\}$, and K is the total number of points by discretizing GPS traces. Dividing the Shannon entropy by the factor $\log(K)$ scales the value from zero to one. The challenge of adopting RE values for our task is that we need to model multiday activities.

C. OUR TARGETING TASK

We define our navigation task, named *multiday explorable non-shortest navigation*. Our motivation for the PNS is to provide pedestrians with enjoyable and explorable routes when navigating between two locations. Our task inherits features from (1) non-shortest paths in the form of resource-constrained paths (Sec. III-B1) and (2) multiday navigation (Sec. III-B2). In other words, we assume that a user uses our PNS daily with different navigation search settings. Based on the multiday navigation setting in Sec. III-B2, we write p_j to be the j-th day navigated route.

Following the previous study by Heller et al. [7], we adopt roaming entropy values with minor updates. Our formulation, named **GraphRE**, is evaluated on paths according to multiple M days. Owing to the computational intractability from (1), we aim to approximately computing routes for multiple days. To increase pedestrian exploration, our navigation task on day j is defined as follows:

> **Multiday explorable non-shortest navigation** (across M days): With any tolerance parameter $\epsilon > 0$ on

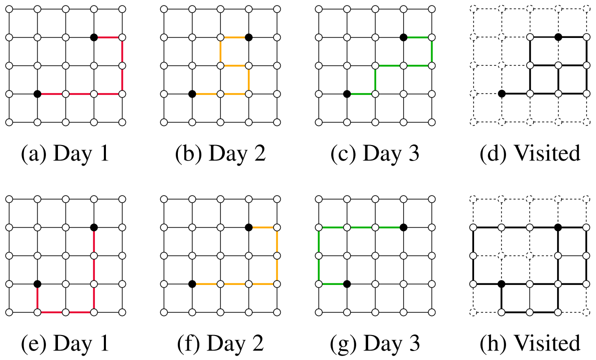


FIGURE 4. Multiday navigation using non-shortest explorable paths and visited vertices are illustrated in (4a–4c) and (4e–4g). (4d) and (4h) show visited vertices. Note that (4h) is more diverse than (4d); in (4h), all edges are distinct, but in (4d) edges are visited multiple times.

time-constrained paths, on the day $j \in \{1, \ldots, M\}$, the task is to offer the user with a feasible route p_j that maximizes GraphRE($\{p_1, \ldots, p_j\}$) computed on the navigated routes $\mathcal{R}_j := \{p_1, \ldots, p_j\}$:

$$\text{GraphRE}(\mathcal{R}_j) := -\sum_{v \in V} \mathbf{p}_v \log_2 \mathbf{p}_v, \qquad (1)$$

$$\mathbf{p}_v := |\{p_i \in \{p_1, \ldots, p_j\} \mid v \in p_i\}|/j. \qquad (2)$$

Computing such a route set $\{p_1, \ldots, p_j\}$ is challenging for the following reasons. First, computing a feasible route p_j ($1 \leq j \leq k$) remains intractable, i.e., NP-hard. Second, we could have multiple route sets for maximizing Eq. (1) as the evaluation of GraphRE only depends on (empirical) probabilities $\mathbf{p}_v$ at each $v \in V$ on G. The difficulty of such diversity issues has been proven (e.g., [43]). Therefore, we develop a heuristic search method in the following section to approximately solve the above problem. As shown in Fig. 4, it illustrates two route sets with different visited vertices, meaning they have different entropy values. Hereafter, computed routes are called *explorable routes*.

D. HEURISTIC SEARCH

1) OVERVIEW

We develop a (heuristic) search method to find approximate non-shortest routes efficiently. Our method then iteratively searches non-shortest paths for multiple M days. Search is a fundamental but effective methodology for solving *state-space search problems*. A search task involves finding a sequence of transitions from a start state to a goal state. Heuristic search algorithms employ a heuristic function $f(\cdot)$ to evaluate search states and decide which states should be expanded next using $f(\cdot)$ until goal states are found. Intuitively, values $f(\cdot)$ represent priorities of investigating states. Heuristic search algorithms expand states with higher heuristic values earlier than those with lower values using data structures, such as a priority queue.

2) PROPOSED IDEA

Our idea is to introduce customized priorities when computing feasible routes. We compute explorable routes that implicitly have large GraphRE values instead of directly optimizing them.

For our multiday explorable non-shortest navigation, we introduce the count $c(n)$ representing the number of previously navigated paths containing n for each $n \in V$. Formally, $c(n) := |\{p_i \in \{p_1, \ldots, p_{j-1}\} \mid n \in p_i\}|$ on the day $j \geq 2$. To define a state space $\mathcal{S}$, we adopt the pair of vertices and travel times required. That is, each state has the form $\mathbf{n} = (n, T(n)) \in V \times \mathbb{R}_{\geq 0}$ meaning that a user is at vertex $n \in V$ and he/she needs a travel time $T(n)$ from his/her departure from the start vertex $s \in V$. From the definition of feasibility (see Sec III-B1), for each navigation task of finding s-t path on G, we have the unique start state $\mathbf{s}_{\text{start}} := (s, 0)$. However, we have multiple goal states $\mathbf{s}_{\text{goal}}$. That is, in any state $(t, T(t))$, if a feasible path p travels from s to t, we have multiple goal states with $|cost(p) - T(t)| < \epsilon$ for given $\epsilon > 0$ in the case of using time-constraint paths. Recall that we would like to select a path p that maximizes GraphRE($\{p_1, \ldots, p_{j-1}, p\}$) on day j out of multiple feasible paths. Let $d(\mathbf{n})$ be the depth value of $\mathbf{n}$, i.e., the minimum step from $\mathbf{s}_{\text{start}}$ to $\mathbf{n}$. Using $c(n)$ and heuristic search algorithms (e.g., [35]) with $d(\mathbf{n})$, we proposed the function in Eq. (3) with a single weight parameter $\gamma > 0$ for search state $\mathbf{n} = (n, T(n))$ at vertex $n \in V$:

$$f(\mathbf{n}) = g(\mathbf{s}_{\text{start}}, \mathbf{n}) + h(\mathbf{n}, \mathbf{s}_{\text{goal}}) := d(\mathbf{n}) - \gamma \times c(n), \qquad (3)$$

where the first term in Eq. (3) inherits from DFS (Depth-First Search) in terms of steps from s to make our search process forward to the goal vertex t, and the second term in Eq. (3) is introduced to indicate that we would like to maximize GraphRE value in our multiday navigation task. To achieve larger RE, we need to generate $\mathbf{p}_v$ uniformly at random on the set V. This objective can be indirectly achieved by ensuring as uniform the counts, $c(n)$, as possible after computing M paths. To consider this idea, when computing each path on the day j, the priority of $\mathbf{n}$ must be decreased by $-\gamma \times c(n)$ to support our objective approximately but efficiently.

3) SUMMARY

We summarize our proposed heuristic method and baseline search algorithms. We prepared the methods for computing each route p_j on the day j.

- **(Baseline 1) DFS(F)**: The depth first search ($\gamma = 0$ in Eq. (3)) without any tie-breaking strategy.
- **(Baseline 2) DFS(T)**: The depth first search like DFS(F) with a random selection-based tie-breaking of $f(\mathbf{n})$.
- **(Proposed) Heuristic Search**: The priority-based heuristic search using Eq. (3).

To determine routes for M days, we repeatedly use the selected baseline method from the three above-mentioned search methods, as our method does not compute optimal feasible paths. In the $\mathcal{CSP}$ literature, recent algorithms,

TABLE 1. Grid graphs. Degrees are averages on all vertices and $w(e) = 1$ for $e \in E$.

Graphs	$\lvert V \rvert$	$\lvert E \rvert$	Deg.	Diam.
`N10grid`	100	180	18	3.6
`N20grid`	400	760	38	3.8

TABLE 2. Real graphs. distances and degrees are averages on all edges and vertices.

Graphs	$\lvert V \rvert$	$\lvert E \rvert$	Dist.	Deg.	Diam.
`shinjuku`	2,773	4,195	39.18	3.03	3654.91
`nihonbashi`	3,278	4,982	35.50	3.04	3698.30

including the *Pulse* algorithm (e.g., [36]), are known to perform efficiently with pruning techniques from numerical experimental results. By adopting the pruning techniques used in those algorithms, we expect our algorithms to perform reasonably well for PNSs.

The worst time complexity of our solver can be estimated using the results of DFS/BFS search methods. The worst time complexity of traditional DFS/BFS search methods is $O(|V| + |E|)$. Furthermore, our heuristic search follows the same strategy of DFS by using the priority value defined in Eq. (3), and our exportable paths remain simple. Therefore, after computing M explorable paths, the worst time complexity of all computations should be $O(M(|V| + |E|))$.

IV. EXPERIMENTS

The experimental results demonstrate our heuristic search functions for multiday and explorable route navigation. Some of the results are illustrated through the figures in the appendix.

A. SETUP

For evaluating our heuristic search method to generate routes for pedestrians, we prepared synthetic grid graphs and real graphs obtained from OpenStreetMap (OSM).[4] For synthetic graphs, we let $N \in \{10, 20\}$ and generate $N \times N$ grid graphs on the 2D space, where distances and times are equivalent; one edge requires one unit of distance and time. For real graphs, we selected `shinjuku` and `nihonbashi`, two representative urban tourism areas in Tokyo, Japan. The details of the extracted graphs are summarized in Table 1 for synthetic grid graphs with sizes of $N = 10$ and $N = 20$ and Table 2 for real graphs of `shinjuku` and `nihonbashi`. In `shinjuku`, latitudes and longitudes are [35.660, 35.730] and [139.684, 139.750]. In `nihonbashi`, they are [35.654, 35.707] and [139.742, 139.835].

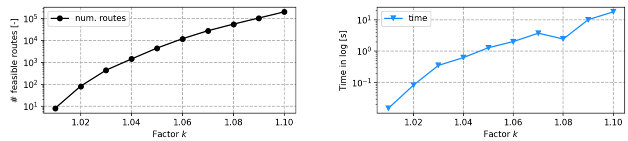
1) PRE-PROCESSING OF ROAD NETWORK DATA

After downloading an XML file from OSM, we pre-process the XML file to extract a graph $G = (V, E, w)$ as follows.

[4]https://www.openstreetmap.org/. Road networks can be downloaded from the web page or APIs, such as https://overpass-api.de/

TABLE 3. Target OSM highway tags.

Extracted `highway` tag values
`trunk`, `primary`, `secondary`, `tertiary`, `unclassified`, `residential`, `road`, `pedestrian`, `trunk_link`, `primary_link`, `secondary_link`, `tertiary_link`



(a) Numbers of feasible routes (b) Computational times

FIGURE 5. Results in `nihonbashi` using naive DFS-based enumeration of all feasible routes when varying k, i.e., the ratio regarding the shortest path distance.

First, we extract all vertices (`<node>` tags) from the file and prepare the superset $\tilde{V} \supset V$ of G, where $\tilde{V}$ should contain redundant nodes such as the borders of buildings.

Second, to only represent walkable ways for our navigation task by filtering $\tilde{V}$, we extract all edges (`<way>` tags) if (1) the way contains some references to nodes (`<nd ref>` tags) and (2) the way has an attribute `highway` tag with the values summarized in Table 3. From each way having a node reference sequence such as $v_1, v_2, \ldots, v_k$ with k nodes, we prepare a set of edges as $\{\{v_i, v_{i+1}\} \mid 1 \leq i \leq k-1\}$. Collecting all sets, we compute the set E of undirected edges.

Third, we build the set V using E as $V = \{v \in \tilde{V} \mid \exists\{u_1, u_2\} \in E \text{ s.t. } u_1 = v \text{ or } u_2 = v\}$, where each v has latitude lat_v and longitude lon_v extracted from the XML file. Moreover, we incrementally modify V and E by removing degree 2 vertices to simplify the output graph structure.

Fourth, we compute the weight $w(e)$ for each edge $e = \{u_1, u_2\} \in E$ by the haversine formula to compute the distance between two vertices u_1 and u_2.

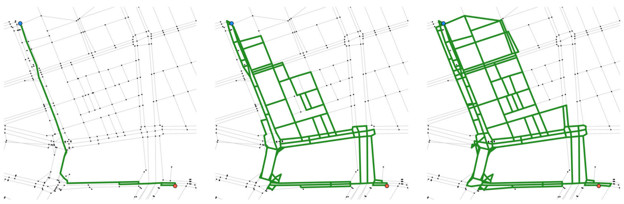
Last, we build a graph $\tilde{G} = (V, E, w)$, compute a *largest connected component* $C \subseteq V$ of G, and obtain the induced subgraph $G = \tilde{G}[C]$ as a targeting graph for navigation.

2) TWO SCENARIOS

We applied our method to the multiday navigation task using two scenarios: (Scenario 1) the same start-goal setting and (Scenario 2) different start-goal settings. In scenario (Scenario 1), we randomly sample a start-goal pair and compute routes for 30 days. Here, we expect the user explores some POIs and roads in a familiar city with available travel time. In scenario (Scenario 2), we randomly sample 30 start-goal pairs and generate a route for each pair. Here, we assume the user enjoys roaming an unfamiliar city for multiple days.

B. PRELIMINARY EXPERIMENTS

Before diving into roaming navigation, we observe the number of feasible paths on some navigation instances on `nihonbashi`. For a randomly sampled start vertex s and



(a) $k = 1.01$ (b) $k = 1.05$ (c) $k = 1.09$

FIGURE 6. Possible edges included in feasible routes with three factors ($k = 1.01, 1.05$, and 1.09) in `nihonbashi`**.**

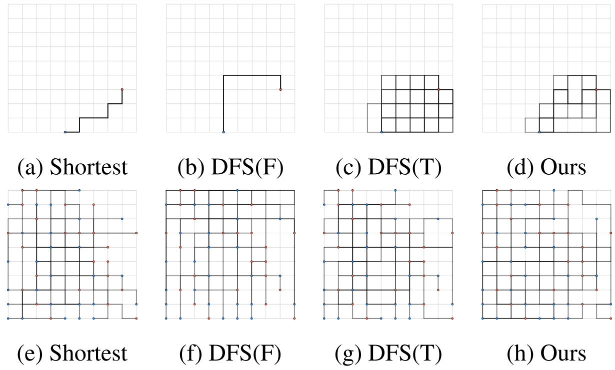
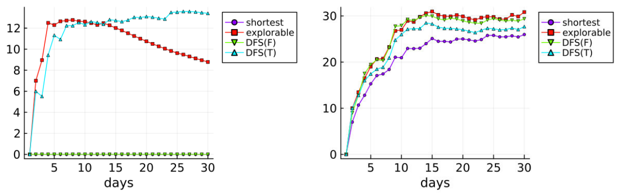
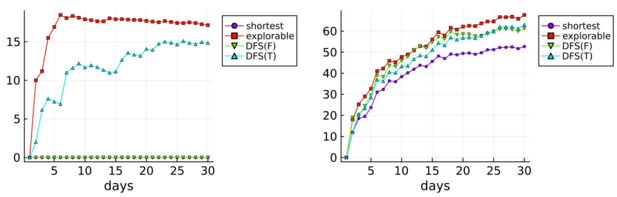


FIGURE 7. Visualized routes on `N10grid` **by competitors for (Scenario 1, above) and (Scenario 2, below).**



(a) Scenario 1 (b) Scenario 2

FIGURE 8. GraphRE values in `N10grid`**.**



(a) Scenario 1 (b) Scenario 2

FIGURE 9. GraphRE values in `N20grid`**. For more results, please see our appendix and Fig. 14.**

goal vertex t, we prepare the factor $k \in [1.01, 1.09]$ in steps of 0.01. Here, k indicates that a route $p \in \Pi(s, t)$ is feasible if and only if its distance is almost up to $k \times cost(\pi)$. By counting feasible routes up to k, we can estimate the hardness of our targeting routing problems according to the upper bounds of the size of feasible routes.

Figure 5a represents the number of feasible solutions in the logarithmic scale in the y axis, and Fig. 5b represents computational times required to enumerate all feasible routes. These results suggest that (1) computing a feasible solution is practically possible, although the problem itself is computationally intractable. Additionally, (2) the number of feasible solutions increases when the feasibility condition is expanded by k. Moreover, Fig. 6 illustrates edges possibly included in feasible solutions. Complex combinations across roads and intersections should enhance the number of feasible solutions. These phenomena could increase exponentially with the additional travel time or distance based on k. These results indicate the difficulty of naive optimal solutions by enumerating all feasible solutions and selecting M day routes to maximize GraphRE values.

We can conclude that a DFS-based search for our task works efficiently. As shown in Fig. 5b, explorable paths (i.e., non-shortest paths) can be efficiently computed. The parameter γ did not influence the results drastically during our preliminary experiments when using the current method.

C. RESULTS

Here we show experimental results. In visualized routes, red and blue circles (best viewed in color) represent the start and goal vertices on the graphs, although our graph G is undirected, and we do not need to distinguish them explicitly.

1) SYNTHETIC GRID GRAPHS

For grid graphs with sizes of $N = 10$ and $N = 20$, we compared the baseline methods and our heuristic method following the two scenarios. The results of $N = 10$ are illustrated in Fig. 7 and Fig. 8. In Fig. 7, the above row corresponds to scenario (1), and the lower row corresponds to scenario (2). Graph RE values are demonstrated in Fig. 8. Fig. 7a and 7e correspond to the shortest path-based navigation without considering multiday navigation. Other routes represent the non-shortest paths. Similarly, the results of GraphRE values on $N = 20$ graphs are illustrated in Fig. 9. For readability, individual routes are depicted in Fig. 14 in the appendix.

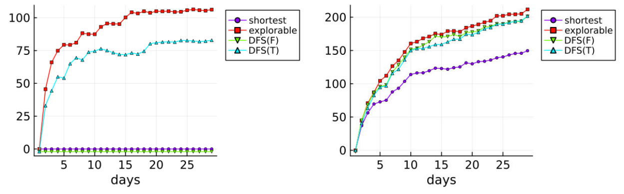
2) REAL OSM GRAPHS

We conducted the same experiments on extracted real road graphs. Fig. 10 and Fig. 11 show the experimental results of GraphRE values in `shinjuku` and `nihonbashi`, respectively. Again, individual routes in `shinjuku` and `nihonbashi` are provided in the appendix section.

The figures indicate that the results of the real graphs are similar to those of the synthetic grid graphs. Note that the required computational time for each route is a few seconds, so our heuristic non-optimal search is effective in building mobile navigation applications.

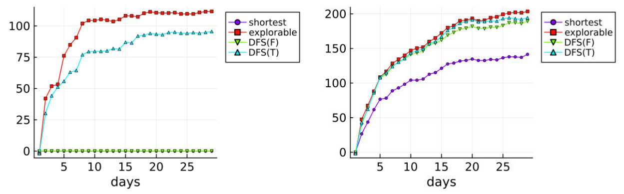
D. QUANTITATIVE ANALYSIS

Here, we used our proposed method along with DFS(T) for (Scenario 1) because we are interested in the case where users access our PNS in their daily lives. We applied



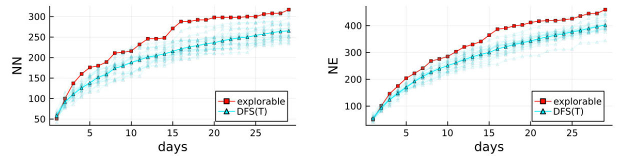
(a) Scenario 1 (b) Scenario 2

FIGURE 10. **GraphRE values in** `nihonbashi`**. For more results, please see our appendix and Fig. 15.**

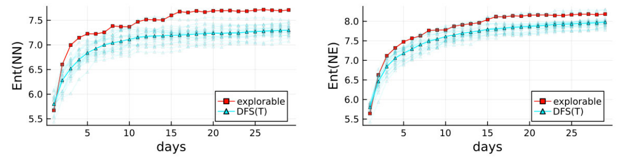


(a) Scenario 1 (b) Scenario 2

FIGURE 11. **GraphRE values in** `shinjuku`**. For more results, please see our appendix and Fig. 16.**

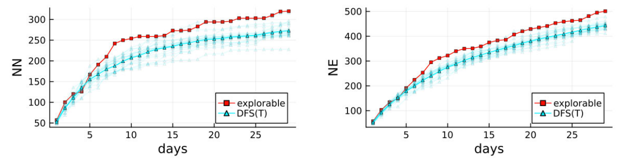


(a) # visited nodes (b) # visited edges

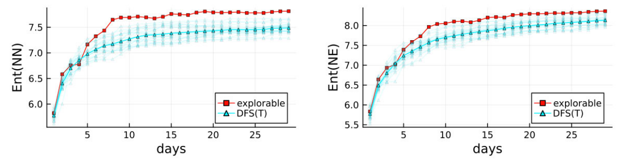


(c) Entropy of # visited nodes (d) Entropy of # visited edges

FIGURE 12. **Additional evaluations in** `nihonbashi`**.**



(a) # visited nodes (b) # visited edges



(c) Entropy of # visited nodes (d) Entropy of # visited edges

FIGURE 13. **Additional evaluations in** `shinjuku`**.**

these methods to OSM graphs and evaluate the computed routes from multiple aspects. Furthermore, experiments using DFS(T) were repeated as the results displayed randomness. Moreover, we measured the number of visited vertices and edges to observe generated routes. We also computed entropy values according to the number of visited vertices and edges.

Results in `nihonbashi` and `shijuku` are illustrated in Fig. 12 and Fig. 13, respectively. Notably, our heuristic search method does not directly optimize GraphRE values. However, our routes can provide diversity in users' explorations. Regarding the number of visited nodes and edges, our heuristic method is better than the mean values of randomized DFS. Similar results can be observed for the entropy values. These results confirm that our method can create more explorable routes for our target scenario.

E. DISCUSSION AND FUTURE DIRECTIONS

1) EXPERIMENTAL RESULTS

Compared with shortest path-based routes, multiday explorable routes described in Fig. 7d and Fig. 7h cover wider areas on grid graphs. Similar results can be observed in `N20grid` (Fig. 14d and Fig. 14h), and real OSM graphs of `nihonbashi` (Fig. 15d and Fig. 15h) and `shinjuku` (Fig. 16d and Fig. 16h). In most cases, our heuristics achieved better GraphRE values than those by DFSs. Only in a few cases (as in Fig. 7c, Fig. 7d, and Fig. 8a), our method achieved lower values than those by baseline methods; we conjecture that these are due to tie-breaking issues as many nodes in grid graphs have the same priority values. Further, due to their symmetry, many routes in grid graphs have the same distances.

2) SEARCH METHODS AND FUTURE DIRECTIONS

The experiments demonstrate the effectiveness of our approach regarding the extent of the explored area via navigation. Particularly, heuristic search-based methods are effective in terms of computational time. Moreover, our explorable navigation can find exploration routes in a few seconds, even in realistic conditions, and therefore can be integrated with mobile applications and personal mobility vehicles. We have diverse routes even if the same constraints are given when computing feasible paths by using baseline methods.

Specifically, the results of GraphRE (e.g., in Fig. 8) confirm that our method can provide users with more explorable routes, similar to the other metrics discussed in Sec. IV-D. The increments of GraphRE can be visually confirmed as well. These results are also supported by additional evaluations illustrated in Fig. 12 and Fig. 13. We conclude that people can explore larger areas using our roaming-oriented navigation method. Our GraphRE measure, as defined in Eq. (1), is now the proxy of this expansion; higher values mean users could walk wider areas.

As mentioned in Sec. III-D3, our method is a non-optimal heuristic search-based one because it does not aim to find

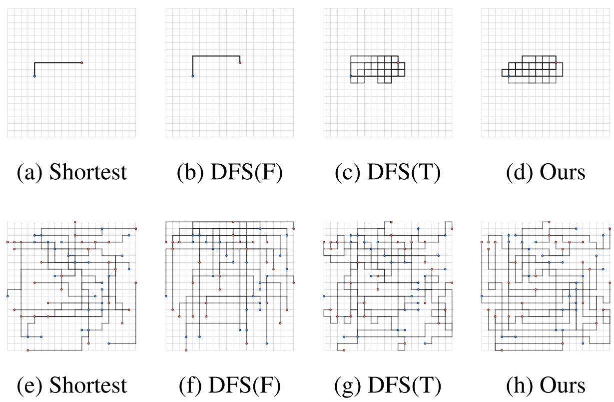


FIGURE 14. Visualized routes from `N20grid` for (Scenario 1, above) and (Scenario 2, below).

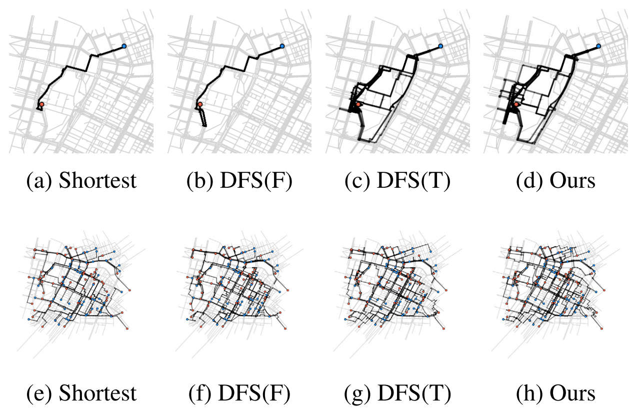


FIGURE 15. Visualized routes from `nihonbashi` for (Scenario 1, above) and (Scenario 2, below).

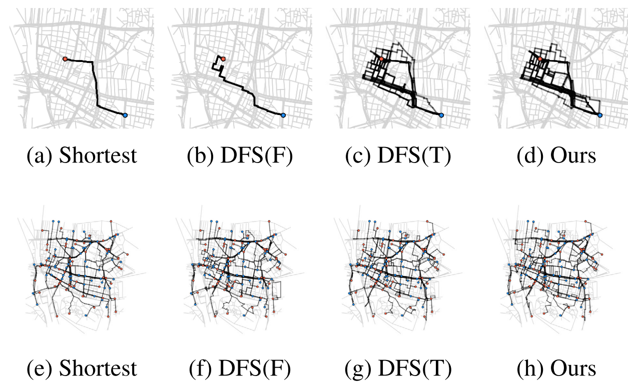


FIGURE 16. Visualized routes from `shinjuku` for (Scenario 1, above) and (Scenario 2, below).

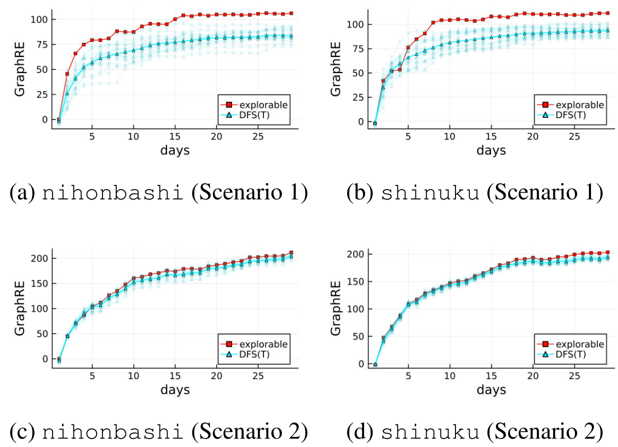


FIGURE 17. GraphRE values by our method and multiple trials of DFS(T) in `nihonbashi` and `shinjuku` on (Scenario 1, above) and (Scenario 2, below).

most efficient routes under constraints. Therefore, we could construct a navigation system for explorable routes across multiple days by developing more sophisticated search-based methods (e.g., [39] for constrained shortest path problems and [40] for computing alternative paths). Another possible future direction of our study from the computational perspective is determining the optimal solution of our provided routes regarding graph roaming entropy, although the upper bound is not estimated in the current status.

3) ROUTE QUALITIES AND FUTURE DIRECTIONS

Our approach focuses on the GraphRE value and non-optimal feasible routes. We believe that explorable navigation is promising for various applications, including pedestrian navigation and services in smart cities. Therefore, other metrics (e.g., beautiful routes like [4]) must be considered when selecting multiple routes. Researchers must also conduct user studies to assess the characteristics of human behaviors when exploring unfamiliar cities, which is also an important topic. Only a few works (e.g., [41]) have explored this direction in a limited region using limited methods. Our future work will include such developments associated with engineering involving users.

V. CONCLUSION

Navigation is an essential technology, particularly for users exploring unfamiliar environments. We revisited navigation services from two orthogonal perspectives: (1) navigation using non-shortest routes and (2) navigation across multiple days. Using graph roaming entropy, we also formulated a navigation task called *multiday explorable route navigation*. We then developed a heuristic search-based approach to generate diverse multiday routes for implementing a new navigation service. Our experimental results clearly show the functioning of the proposed method and how our multiple routes support walkers in exploring cities. The proposed simple heuristic method is a starting point for the following studies on multiple aspects.

In our future work, we could develop more sophisticated methods considering multiple objective aspects (e.g., beautiful routes). From a computational perspective, a possible future direction of our study is to estimate the optimal solution for our routes in terms of GraphRE. Additionally, analyzing the upper bound of the GraphRE value could be valuable in designing efficient heuristic search methods. Furthermore, we plan to evaluate the physical and psychological effects of exploring (unfamiliar) cities through navigation services. In conclusion, we believe that our roaming navigation approach, albeit technically simple, may open up new avenues for examining the causal role of urban exploration in real-life settings.

APPENDIX. ADDITIONAL RESULTS

A. VISUALIZED ROUTES

We provide visualized routes for other settings. Figure 14 shows computed routes on `N20grid`. Figure 15 shows computed routes on `nihonbashi`. Figure 16 shows computed routes on `shinjuku`.

B. MULTIPLE TRIALS ON GraphRE VALUES

Section IV-D presents additional evaluation metrics using DFS(T) and our method in `nihonbashi`. In this appendix, we provide results on GraphRE values through multiple trials, as shown in Fig. 17. Our approach is more effective in (Scenario 1). We conjecture these results because when the start and goal are randomly sampled from V, many unvisited nodes remain. If we use our roaming navigation continuously over a longer period, our method could become more effective in city exploration.

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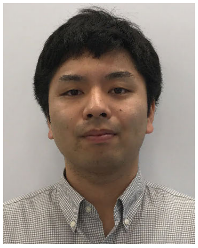

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