IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 25 June 2023, accepted 10 July 2023, date of publication 13 July 2023, date of current version 26 July 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3295242

RESEARCH ARTICLE

Enhancing Grey Wolf Optimizer With Levy Flight for Engineering Applications

WU LEI [1], WU JIAWEI [2], AND MENG ZEZHOU [1]
[1]Information College, North China University of Technology, Beijing 100144, China
[2]Faculty of Architecture, Civil and Transportation Engineering, Beijing University of Technology, Beijing 100124, China

Corresponding author: Wu Lei (stone.wu@ncut.edu.cn)

This work was supported by the Beijing Municipal Government Fund Project 18JYB015299 and Project BJSZ2021ZC65.

ABSTRACT Since Grey Wolf Optimizer (GWO) first introduction, it continues to be used extensively today, owing to its simplicity, easy handling, and applicability to a wide range of problems. Although there are many different GWO variants in the literature, the problem that the GWO produces early convergence and inefficient results have still continued to emerge in their variants. In order to overcome the drawbacks of the GWO, the GWO integrated together with Levy Flight (LFGWO) is proposed. In order to demonstrate the overall performance of the LFGWO, experiments are conducted using the 23 standard benchmark functions and 10 composition functions of CEC 2019 compared with the other eight state-of-art algorithms. The 28 out of 33 average and 27 out of 33 standard deviation values obtained by LFGWO are all less than those obtained by the other eight optimization algorithms, which verified and demonstrated the performance, stability, and robustness of the LFGWO. The extensibility test with different scales of dimensions 50, 100, 300, and 500, is undertaken by comparing LFGWO with GWO and IGWO to assess the dimensional influence on problem consistency and optimization quality. Moreover, the performance of the LFGWO has also been tested on five real-world problems and infinite impulse response (IIR) challenging model identification, experimental results and statistical tests demonstrate that the performance of LFGWO is significantly better than the other compared algorithms, and the LFGWO is capable of solving real-world problems.

INDEX TERMS Benchmark function, global convergence, grey wolf optimizer (GWO), levy flight.

I. INTRODUCTION

A. OPTIMIZATION TECHNIQUES

Optimization is defined as the selection of the best elements or actions from a set of feasible alternatives. More precisely, optimization consists of finding the set of variables that produce the best values of objective functions in which the feasible domain of the variables is restricted by constraints [1]. During the past several decades, various domain-specific problems increase the complexity because of the high dimensionality and various constraints that may fail to solve using exact algorithms. To solve complex problems, numerous metaheuristic optimization algorithms have been designed to tackle a plurality of more complex optimization problems.

In [2], the recent well-regarded Manta Ray Foraging Optimization (MRFO) was developed. However, the MRFO suffers from the deficiencies of decreasing population diversity and low accuracy of exploitation in the late iteration. As soon as the MRFO appeared in publications, variants of MRFO appeared quickly, in [3], Tang, et al. used the ESP, ACP, and DES strategy to modify Manta ray foraging optimization (m-MRFO), and achieved more or less extensive success. But, the crucial drawback of the m-MRFO didn't pay more attention to the infeasible region. Abd Elaziz et al. in [4] proposed MRFO with the triangular mutation operator and orthogonal learning strategy (MRTMO) focus on paying more attention to the infeasible region, however, which leads to MRTMO attracting the solution to a local point and the final output's quality is degradation. In [5], Yousri et al. proposed a novel variant of the Manta ray foraging optimizer, named FCMRFO, which has better addressed the flaw that the MRFO's exploitation ability is weaker than the exploration ability.

In [6], Band et al. introduces a new colonial competitive optimizer using three modified RAO metaphor-less

The associate editor coordinating the review of this manuscript and approving it for publication was Ines Domingues.

algorithms, called CCRAO. Even though they tested CCRAO on 30 standard functions of CEC2014 with 50 dimensions compared with several well-known algorithms and on five popular engineering problems, the CCRAO all achieved better results. But, the CCRAO didn't present a reasonable explanation about how to coordinate their powerful group algorithms and harmony with these three modified RAO metaphor-less algorithms, which may need more time to study in-depth on theory and experience. In [7], Faramarzi et al. proposed a Marine Predators Algorithm (MPA) integrated Levy and Brownian movements with optimal encounter rate policy in biological interaction between predator and prey. Even they compared the MPA with several well-regarded optimization methods using CEC 2017 benchmark functions, and the outcomes showed that the MPA has superior performance and surpasses most of the other algorithms. Crucially, moreover, it has not been tested on how to coordinate and harmony Levy and Brownian movements to maximize the performance of the MPA. In [8], Hua et al. proposed a unique MCSA algorithm and validated their MCSA on more benchmark test functions, such as 23 benchmark test functions, CEC2017 and CEC2019 test suites by comparing with the other competitive optimization algorithms, and six real-world engineering problems. It will be better; if their paper listed more statistical results (for example, comparing the mean and standard deviation of MCSA with other algorithms in solving six optimization problems).

In the past decades, more researchers are dedicated to the improvement of the GWO by presenting various variants of the GWO. However, there are still limited innovations in enhancing population diversity and global search capabilities by changing population structure and searching mechanisms.

In [9], Ma et al., an improved version of the grey wolf optimizer based on Aquila exploration method (AGWO) was proposed to solve the global optimization and freely adjust its exploitation and exploration capabilities. The main idea emphasizes the exploitation ability of the grey wolf and the exploration ability of Aquila. Needless to say, even though the AGWO is more accurate with a faster convergence rate compared to GWO, however, the AGWO needs to be validated by composition benchmark functions. Saxena et al. in [10] presented E-GWO, which utilizes a sinusoidal bridging technique with novel selection, crossover and mutation operators to enhance the exploration ability and to avoid local optima stagnation. However, it lags in the intensification of unimodal functions and fails to balance the search process for hybrid functions. In addition to that, it shows a better exploration ability for landscapes with many local optima, but weak exploitation in unimodal problems and does not strike a proper balance between intensification and diversification in hybrid functions ineffective global search is still its major problem. Dhargupta et al. in [11] combine the opposition-based learning strategy and Spearman's correlation coefficient with GWO. The Selective Opposition based GWO algorithm (SOGWO) changes the distribution strategy of the population. This improvement may increase the computational complexity of the algorithm. At the same time, SOGWO can diminish unnecessary exploration and obtain a fast convergence. However, in the same context, the SOGWO may lead to premature convergence and loss of diversity. Rajakumar, et al. in [12] presented an evolution version of the GWO, namely Accelerated Grey Wolf Optimization (AGWO), which incorporates the enhanced hierarchy into the GWO technique. Although the AGWO can right balance exploitation and exploration in hybrid functions and has a better exploration in the landscapes with many local optima. But it lags to balance exploration and exploitation in solving complex problems. The I-GWO [13] proposed to improve the GWO search strategy by a new learning-based hunting search strategy to tackle imbalance exploration and exploitation and premature convergence weaknesses. The I-GWO concentrates on and obtains a good balance between exploration and exploitation. But, the I-GWO overlooks the accurate approximation of the global optimum for the composition functions. The VAGWO [14] algorithm was recently proposed to add velocity term to the position-updating mechanism of the canonical GWO. The velocity has been shown to significantly improve the GWO algorithm when attempting to explore the search space, as the velocity can continue to push the wolves to continue their global search and; prevent a significant number of good positions from being missed during the optimization process. However, bigger problem sizes for combinatorial optimization could be a challenge for the VAGWO. The VAGWO show no significant superiority when outperforming several other algorithms on most of composition functions to get the accurate approximation of the global optimum. In [15], a new variant of GWO termed Randomized Balanced Grey Wolf Optimizer (RBGWO) is introduced, which improves the overall efficiency of the search process by establishing a balance between its exploitation and exploration capability incorporating three successive enhancement strategies equipped with a social hierarchy mechanism and random walk with student's t-distributed random numbers. It is very challenging to choose different parameters of RBGWO to resolve different optimization problems and the solution requires more enhancements. In [16], a new variant of the GWO called a mutation-driven modified grey wolf optimizer and denoted by MDM-GWO is proposed. The MDM-GWO combines a new update search mechanism, modified control parameter, mutation-driven scheme, and greedy approach of selection in the search procedure of the GWO. Therefore, it is necessary to study the influence trend of parameters on the MDM-GWO, which will inevitably increase calculation and operation costs. In [17], a new variant of the GWO named GWOCMALOL is proposed, which uses covariance matrix adaptation evolution strategy, Levy flight mechanism, and orthogonal learning strategy to bring more effective exploratory inclinations. However, the inability to flexibly adjust parameters defined by many mechanisms is challenging for GWOCMALOL, and different choices of control parameters have different effects on the optimization results. In [18], a called Fast-Dynamic GWO (FDGWO) is proposed. In the FDGWO, 8 fixed coefficients must be determined before implementation. Needless to say, even though the FDGWO is more accurate with a faster convergence rate compared to GWO, however, the FDGWO requires 8 parameters to be adjusted; the determination and coordination of these 8 parameters are still not easy. To improve the performance of the GWO, the Diversity Enhanced Strategy-based Grey Wolf Optimizer (DSGWO) is proposed in [19]. The number of leading wolves changing from three to six is the mechanism of the Diversity Enhanced Strategy in the DSGWO. And the exploration-exploitation balance mechanism of the DSGWO divides the hunting process into two stages: in the first stage, the position of the omega wolf is between two leading wolves randomly selected from the six leading wolves. In the second stage, the updating process of the population is the same as classical GWO. However, as the hunting mechanism of DSGWO is the same as the GWO; needless to say, even the number of leading wolves change from three to six thus, the premature convergence and local optima trapping have still remained. In [20], an improved variant of the GWO named gaze cues learning-based grey wolf optimizer (GGWO) is proposed. By two new search strategies of neighbour gaze cues learning (NGCL) and random gaze cues learning (RGCL), the GGWO improves diversification, exploration, and exploitation. Despite achieving these results, there are still chances for improving the GGWO in terms of updating solutions and accelerating the convergence rate.

Till date, practitioners or researchers in various fields of science and engineering also have presented and experimented with various GWO variants and hybrid variants optimization algorithms to the main four application scenes.

1) FEATURE SELECTION

Feature selection problem is one of the main difficulties to find the smaller number of informative features among a huge amount of feature space which guides the maximum classification ratio. In [21], Preeti, Kusum Deep proposed a Random Walk Grey Wolf Optimizer based on dispersion factor (RWGWO) used in wrapper feature selection method. To demonstrate their methodology, they conducted a set of classification measures experiments on eighteen different chronic disease data. In [22], Wang et al. proposed an Adaptively Balanced Grey Wolf Optimization (ABGWO) algorithm to seek out the optimal feature subset for high-dimensional classification. In [23] Hu et al. proposed Binary Grey Wolf Optimizer (BGWO) to extends the application of the previous GWO algorithm and conducted a comprehensive study on utilizing five transfer functions of Binary GWO in feature selection. Their BGWO showed promising results in terms of the feature selection in the UCI datasets and acquired low classification errors with few features. Geetha and Deepa in [24] proposed a Fisher kernel based PCA dimensionality reduction algorithm and grey wolf optimizer based weight dropped BiLSTM classifier (FKPCA-GWO WDBiLSTM) for intrusion detection. In [25], a hybrid GWO with CSA, namely GWOCSA is proposed by Arora, S et al. which combines the strengths of both the algorithms effectively with the aim to generate promising candidate solutions in order to solve the feature selection problem. Their results reveal that the GWOCSA has comprehensive superiority in solving the feature selection problem.

2) TRAINING NEURAL NETWORKS

Artificial neural networks (ANNs) are information processing models inspired by the biological nervous systems. ANNs are widely applied in research and practice due to their high capability for capturing nonlinearity and dynamicity models. However, the performance of ANNs is highly affected by their structure and connection weights. Recently, various GWO variants and hybrid variants optimization algorithms applied to optimize the weight and biases of (ANNs). In [26], Meng et al. proposed an Advanced Grey Wolf Optimization algorithm (AGWO) with elastic, circling and attacking mechanisms to alleviate local stagnation and premature convergence problems. Mohakud et al. [27] successfully applied Grey Wolf Optimization algorithm for optimizing the hyper parameters of CNN, by adopting a proper encoding scheme. Ali Asghar Heidari et al. [28] proposed boosted grey wolf optimizer for global optimization and kernel extreme learning machines.

3) OPTIMIZING SUPPORT VECTOR MACHINES

Support vector machine (SVM) is recognized as one of several powerful machine learning algorithms, and it is utilized for a wide search space of real-world problems. The effectiveness of the SVM algorithm and its production chiefly rely on the kernel model and its main adjusting parameters. To maximize the performance of SVM, two hyper parameters should be tuned; the error penalty parameter C and the kernel parameters, however, which needed long running time for evaluating all possible combinations by using a simple or exhaustive grid search. Recently, various GWO variants and hybrid variants optimization algorithms were applied for tuning the hyper parameters of SVM in different publications. In [29], Kumar and Singh. attempted to extract the valuable information by selecting the relevant features using their proposed EGWO-SVM (enhanced grey wolf optimization in combination with support vector machine) approach. In [30], Badr et al. proposed hybrid GWO-SVM model and tested and compared their model on two different datasets, Wisconsin diagnosis breast cancer (WDBC) dataset and Electronic Health Records (EHR). In [31], Kamel et al. used data mining as a combination of feature selection method by Grey Wolf Optimization (GWO) and support vector machine (SVM), which increased the accuracy of diagnosis by 27.68%, tested and compared with numerous existing former works in the field.

4) CLUSTERING APPLICATIONS

Clustering is a common machine learning and deep learning where the goal is to divide data instances into a number of groups that have similar characteristics in some sense. Various GWO variants and hybrid variants optimization algorithms have been widely used and applied for clustering tasks and as an alternative to the classical k-means algorithm which is one of the most famous clustering approaches. In [32], Ahmadi et al. proposed a Modified Grey Wolf Optimizer to tackle data clustering. Ghorbanvirdi et al. in [33] proposed a centralized multiple clustering based on GWO that uses both energy and distance in cluster head selection. Zhang et al. [34] proposed a novel hybrid algorithm based on PSO and GWO (HGWOP). Experimental results on K-means clustering optimization reveal that HGWOP has obvious advantages over the comparison algorithms and can productively find minimum feature subset on the selected feature conundrum. In [35], Purushothaman et al. proposed a novel hybrid algorithm based on GWO and GOA for text feature selection and clustering.

B. GREY WOLF OPTIMIZER (GWO)

Initially, GWO was introduced by Mirjalili et al. [36]. The GWO is nature-inspired by the leadership hierarchy and hunting mechanism of grey wolves. The GWO is a meta-heuristic algorithm different from others in terms of model structure and is based on the social hierarchy of grey wolves as well as their hunting and cooperation strategies. The GWO has been successfully widely applied in engineering applications realms [13], [37], [38], [39], as the control parameters of the GWO need to be turned is less.

C. THE DEFECTIVE OF THE GWO ALGORITHM

Although the GWO has better theoretical architecture, the GWO has the disadvantage of premature convergence and low quality of the optimal solution. In recent years, Niu et al. gave an in-depth study of the GWO. As described in [40]: one of the main drawbacks of the GWO for optimizing real-world problems is ''When GWO solves the same optimization function, the farther the function's optimal solution is from 0, the worse its performance''. They showed that GWO's performance degrades as the optimal solution of the problem diverges from 0. As a matter of fact, the GWO has good performance for the optimization problem whose optimal solution is 0, however, for other problems, its advantage is not as obvious as before or even worse [41].

D. REMARK

The GWO was cited by 8741 papers (2023-01-06) and employed by a large number of researchers and designers, such that the number of citations to the GWO paper far exceeded many other meta-heuristic and swarm intelligence algorithms. On the one hand, which reveals that GWO is one of the top mainstream algorithms and is popularly compared to many state-of-the-art algorithms. On the other hand, a large number of practitioners or researchers in various fields of science and engineering cited the GWO to integrate the GWO with their developed algorithm and focus on a variety of practical application problems and have achieved more or less extensive success. Originates and benefits from the GWO, according to the literature, in recent years, there emerged rapidly a variety of GWO variants, each covering different applications and aspects. To the best of our knowledge, as soon as the GWO as an open-source optimization tool was developed by Seyedali Mirjalili in 2014, hundreds and thousands of GWO variants, such as hybridizing and compositing other algorithms with GWO or integrating a variety of evolutionary operators into the GWO, not only the simple mathematical formulation and structure, easily understandable, and high performance in terms of convergence and acceptable quality of solutions about the GWO, and the underlying comprehensive reasons are indeed the GWO scheme being open to various improvements simplicity in implementation and being very flexible and scalable to be extended.

Near decades have cited new GWO variants across various disciplines are being developed by researchers or designers and have successfully achieved a large variety of unique optimizations in both scientific and industrial problems, without any doubt, the GWO variants certainly inherit the foremost features and advantages of the GWO. From the theoretical perspective, the model structure of the GWO forms the basis of proposing many GWO variants now and then. Based on this conclusion, all GWO variants are looks similar to each other or, at least, nearly equivalently in terms of the model structure and the effective mechanisms maintain a good balance between exploration and exploitation, in most cases, all the GWO variants preserve the searching advantages of the GWO. The reasons appear to be two-fold. On one hand, the majority of the GWO variants gained immense popularity and have been extensively successfully applied in a variety of application domains including complex optimization problems largely depending on the GWO. On the other hand, the GWO variants achieved a lot of competitive performances and significantly outperforms the majority of the other variants based on the meta-heuristic and swarm intelligence algorithms in the literature, however, contributed and correspond to the effective mechanisms of the GWO.

As we know, the overall efficiency of a meta-heuristic optimization algorithm largely depends on a sound balance between exploration and exploitation (intensification and diversification). Recent years have seen a burgeoning stronger interest by researchers and practitioners from different fields in fusing meta-heuristic search methods with Levy flight for the majority of the complex non-line constrained optimization problems. The probability of obtaining the global optimization value is much higher by adopting the meta-heuristic algorithm of Levy flight mode than by adopting other methods (uniform or Gaussian). As soon as a meta-heuristic or swarm intelligence algorithm incorporates the concept of Levy Flight proving to be more efficient

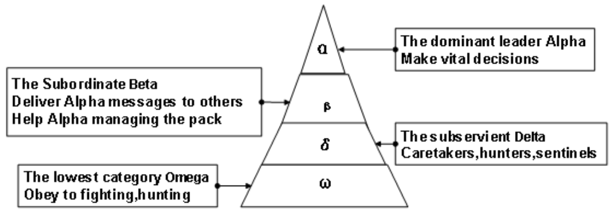


FIGURE 1. The social hierarchy of Grey wolves.

than the same optimization algorithms without Levy Flight. The comprehensive reason behind the effectiveness of the meta-heuristic or swarm intelligence algorithms embedded with Levy Flight is that the Levy Flight-based jumps can effectively redistribute the search agents to enhance their diversity and to emphasize more explorative steps in case of immature convergence to low optimization. As reported in numerous publications and many successful experiments suggest that it is the mainstream trend to incorporate Levy Flight with meta-heuristic and swarm intelligence algorithms to boost the efficacy of the meta-heuristic and swarm intelligence algorithms.

Based on the theorem of ''no free lunch'' (NFL) [42], many nature-inspired optimization algorithms now and then will be appeared, and a universal best optimizer for the specific problems in various fields of science and engineering does not exist.

The main contribution of this paper can be summarized as follows:

- The levy flight strategy is properly embedded with GWO.
- The LFGWO was validated by 23 mathematical benchmark functions and 10 composition functions of CEC 2019 in comparison with the eight well-known meta-heuristic algorithms.
- Comparing LFGWO with GWO and IGWO using 4 dimensions (D = 50, 100, 300, and 500) to assess the dimensional influence on consistency and optimization quality.
- The LFGWO was employed to resolve five real-world problems and achieved promising results.

II. THE CONCEPT OF THE GWO

A. THE SOCIAL HIERARCHY OF GREY WOLVES

GWO is a population-based meta-heuristics algorithm that simulates grey wolves as considered apex predators, which are at the top of the food chain.

- Grey wolves prefer to live in groups (packs), each group containing 5-12 individuals on average.
- All the individuals in the group have a very strict social dominance hierarchy as demonstrated in the accompanying figure 1.

1) Alpha wolf is considered the dominant wolf in the pack and his/her orders should be followed by the pack members.

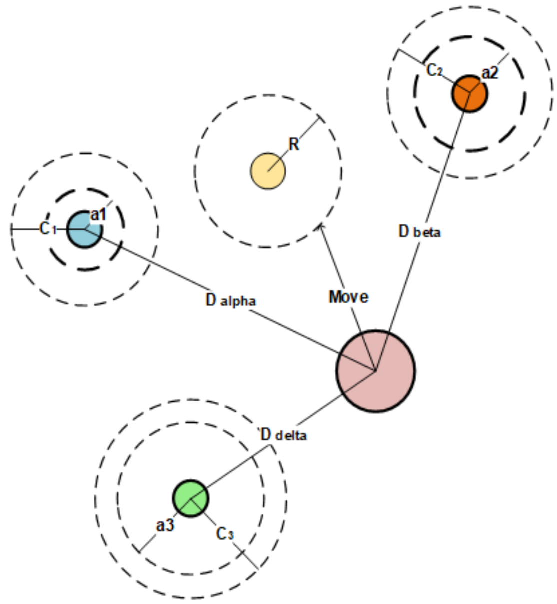


FIGURE 2. Evolution of position in GWO.

2) Beta wolves are subordinate wolves, which help the alpha wolf in decision-making and are considered the best candidate to be the alpha wolf.
3) Delta wolves have to submit to the alpha and beta, but they dominate the omega.
4) Omega wolves are considered the scapegoat in the pack, are the least important individuals in the pack, and are only allowed to eat at last.

B. GREY WOLF HUNTING AND GREY WOLF OPTIMIZER

1) ENCIRCLING THE PREY

When the prey location is captured by the grey wolves, encircling of prey is performed. In the process of encircling, grey wolf individuals should first determine the distances between themselves and the prey according to Eq. (1) and then update their positions through Eq. (2):

$$\vec{D} = |\vec{C} \cdot \vec{X_p}(t) - \vec{X}(t)| \tag{1}$$

$$\vec{X}(t+1) = \vec{X_p}(t) - \vec{A} \cdot \vec{D} \tag{2}$$

where t indicates the current iteration, $\vec{A}$ and $\vec{C}$ are defined as coefficient vectors, $\vec{X_p}$ is the best solution position vector that the prey has been observed so far, and $\vec{X}$ indicates the position vector of a grey wolf. In Figure. 2, the $\vec{D}$ is the difference vector which decides the movement of wolf either towards the neighborhood regions of prey or opposite of it.

Both $\vec{A}$ and $\vec{C}$ are changed over iterations as following:

$$\vec{C} = 2\vec{r_2} \tag{3}$$

$$\vec{A} = 2\vec{a} \cdot \vec{r_1} - \vec{a} \tag{4}$$

where $\vec{r_1}$ and $\vec{r_2}$ are randomly generated stochastic vectors from the interval [0, 1]. $\vec{C}$ and $\vec{A}$ are coefficients that are determined by Eq. (3) and (4). The components of vector $\vec{a}$

are linearly decreased from 2 to 0 over the course of iterations and can be formulated as Eq. (5):

$$a = 2 - 2 * \frac{t}{T} \tag{5}$$

where t indicates the current iteration and T denotes the maximum number of iterations.

2) HUNTING THE PREY

In GWO, for the global optimums of an optimization problem are unknown, the first three grey wolves of Alpha, Beta, and Delta are always assumed as the closest solutions to the optimal value. In the hunting strategy, the positions of each search agent (wolf) are adjusted based on the three best positions of Alpha, Beta, and Delta. The following equations are used to mimic the hunting process and to identify the better optimum in the boundary space. Therefore, the remaining wolves are supposed to update their positions following the leading wolves which can be calculated by Eq. (6)-(8).

$$\begin{aligned} \overrightarrow{D_\alpha} &= \left|\overrightarrow{C_1} \cdot \overrightarrow{X_\alpha}(\mathrm{t}) - \vec{X}(\mathrm{t})\right| \\ \overrightarrow{D_\beta} &= \left|\overrightarrow{C_1} \cdot \overrightarrow{X_\beta}(t) - \vec{X}(t)\right| \\ \overrightarrow{D_\delta} &= \left|\overrightarrow{C_1} \cdot \overrightarrow{X_\delta}(t) - \vec{X}(t)\right| \end{aligned} \tag{6}$$

$$\begin{aligned} \vec{X_1} &= \vec{X_\alpha} - \vec{A_1} \cdot (\vec{D_\alpha}) \\ \overrightarrow{X_2} &= \overrightarrow{X_\beta} - \overrightarrow{A_2} \cdot (\overrightarrow{D_\beta}) \\ \overrightarrow{X_3} &= \overrightarrow{X_\delta} - \overrightarrow{A_3} \cdot (\overrightarrow{D_\delta}). \end{aligned} \tag{7}$$

$$\vec{X}(t+1) = \frac{\vec{X_1} + \vec{X_2} + \vec{X_3}}{3} \tag{8}$$

where $\vec{X_\alpha}$, $\vec{X_\beta}$, $\vec{X_\delta}$ are the three best positions of Alpha, Beta and Delta, $\vec{D_\alpha}$, $\vec{D_\beta}$,$\vec{D_\delta}$ are distances of search agents away from the three best solutions, $\vec{A_1}, \vec{A_2}, \vec{A_3}$ show random vectors.

3) ATTACKING PREY (EXPLOITATION PHASE)

Grey wolves diverge from each other to search for prey and converge to attack prey. Grey wolves will only attack the prey when they are no longer moving. This phase is responsible for exploitation and is handled by a linear decrement in $\vec{a}$. The linear decrement in this parameter enables grey wolves to attack the prey while it stops moving.

4) SEARCHING FOR PREY (EXPLORATION PHASE)

It is obvious that when the prey stops moving, the wolf will kill the prey and, in this way, they complete their hunting process. Grey wolves mostly search according to the position of the α,β and δ. The process of GWO can be exhibited in detail as follows Pseudo code of the GWO algorithm 1.

Algorithm 1 Pseudo Code of the GWO

```
Randomly initialize the population of grey wolves X_i(i = 1, 2, ..., n)
Initialize the value of a=2, A and C (using eq. 3, 4)
Calculate the fitness of each member of the population
X_α: member with the best fitness value
X_β: second best member (in terms of fitness value)
X_δ: third best member (in terms of fitness value)
    while (t < T (Max number of iterations))) do
      for each search agent do
        Update the position of all wolves by eq. 6, 7 and 8
        Update a, A, C (using eq. 3, 4 and 5)
      end for
      Calculate the fitness of all search individual
      Update X_α, X_β, X_δ
      t = t + 1
    end while
    return The best solutionX_α.
```

III. GWO RANDOM WALKS WITH LEVY FLIGHTS

From a mathematical point of view, the distribution function of probability density on the variation of Levy's flight length normally approximated can be defined as:

$$L(s) \sim |s|^{-1-\theta}, \quad 0 < \theta \leq 2 \tag{9}$$

The random step length of Levy's flight is S, the θ is normally set to be 1.5, which harmonies the peak sharpness of the levy distribution graph and can harmony both exploitation and exploration trends over the course of iterations. However, the true Levy distribution cannot be realised directly with computer code, but a modified version of Levy distribution or the approximate form is the Mantegna algorithm which may be the best alternative for a symmetric Levy stable distribution, where "symmetric" means that the steps can be positive and negative. Mantegna's algorithm always consists of three steps and the step length S can be calculated by the quotation.

$$S = \frac{U}{|V|^{\frac{1}{\theta}}} \tag{10}$$

The random length variable is S, the σ_U and σ_V depicted as below.

$$U \sim N\left(0,\ \sigma_U^2\right),\ V \sim \left(0, \sigma_V^2\right) \tag{11}$$

For simplicity we usually set

$$\sigma_V = 1. \tag{12}$$

V denotes a random number sampled from the Gaussian distribution V (0, 1), and U is a random number sampled from the Gaussian distribution $N(0, {\sigma_U}^2)$, the σ_U can be set presented as below.

$$\sigma_U = \left\{\frac{\Gamma(1+\theta) \times \sin(0.5\pi\theta)}{\Gamma[0.5(1+\theta)] \times \theta \times 2^{0.5(\theta-1)}}\right\}^{\frac{1}{\theta}} \tag{13}$$

where Γ represents the conventional gamma function, computed using the built-in'gamma(X)' MATLAB function. The step of the Levy flight achieved by Equation (10)-(13), which simulates many small and occasionally long-distance jumps.

Then the step size is calculated by

$$step\ size = f \times S = f \times \frac{U}{|V|^{\frac{1}{\theta}}} = 0.01 \frac{U}{|V|^{\frac{1}{\theta}}}. \quad (14)$$

Here, the factor value ($f = 0.01$) dependent on the dimension of the desired problem. The L/100 should be the typical step size of walks where L is the typical length scale; otherwise, Levy flight may become too aggressive, which makes new solutions (even) jump outside of the design domain (and thus wasting evaluations). To sum up, the procedure of the Levy flight can be presented in Algorithm 2.

Algorithm 2 Pseudo Code of the Levy Flight Function

initial: d=4;(dimensions),θ= 1.5;
Calculate σ_U by Eq.(13)
u= randn(1,d)*σ_U;
v=randn(1,d);
S=u./abs(v).^ (1/θ) by Eq.(10)
step size = 0.01*S;
end

It is particularly noteworthy that the random walk provides approximately the "same size" for every step [43], [44], [45], while Levy flight offers "varied sizes", which means in most cases the "varied sizes" is a number of small steps and occasionally a big step. On the other hand, the "step size" is the step size of the search space and be added to the updating equations of the LFGWO for finding the position of the prey. In other words, the Levy flight distribution is an effective mathematical operator for producing varied solutions in the hunting space and increasing the exploration capability of the LFGWO, which presents a natural harmony between exploration and exploitation. The flow chart of the LFGWO is given in Figure 3.

As a matter of fact, even Levy flight is a very special random walk that offers and with occasional big jumps can allow the exploration to escape from a local optimum and restart in a different region of the search space. But how to embedding the Levy Flight in a proper place will directly produce totally different results, some cases will be better and some cases will be worse. Based on the above facts, through an in-depth comprehensive study and trial-and-error experiments, the search process starts by assigning the new position with the Levy flight step, which directly influences each wolf's individual fitness and the position of each wolf individual, and influences the position of the wolf alpha. The pseudo-code of the LFGWO is submitted below.

From Figure 3 and Algorithm 3, it is worth noting the formula:

$$NewPosition = currentPosition * LFGWO\_Levy(dim)'.$$

Firstly, $LFGWO\_Levy(dim)$ represents the Levy flight function, and **dim** is the dimension. The Levy flight can consecutively generate a set of random small steps and occasionally big jumps during the process of iteration, which harmony both exploitation and exploration trends over the course of iterations.

Secondly, the above formula says that the LFGWO integrates with Levy flight not only for the top three wolves

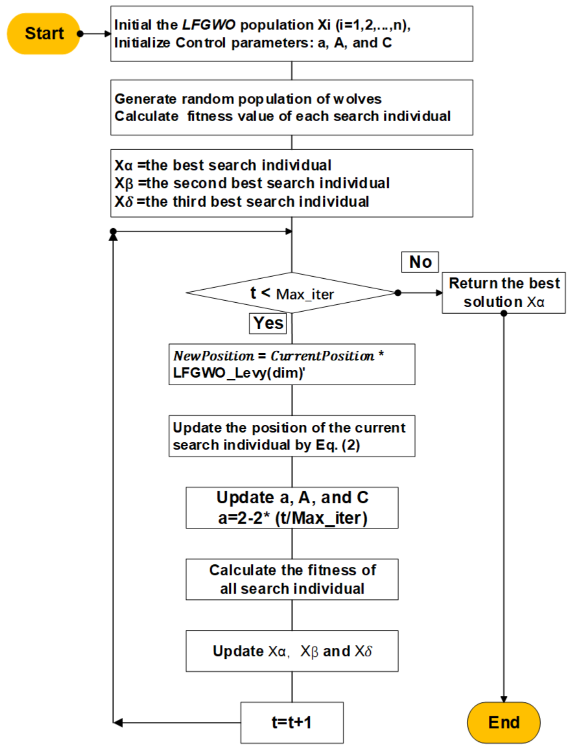


FIGURE 3. The flowchart of the LFGWO algorithm.

Algorithm 3 Pseudo Code of the LFGWO Algorithm

Randomly initialize the LFGWO population $X_i (i = 1, 2, \ldots, n)$
Initialize a, A, and C
Calculate the fitness value of each search individual
$\boldsymbol{X_\alpha}$: the best search individual
$\boldsymbol{X_\beta}$: the second best search individual
$\boldsymbol{X_\delta}$: the third best search individual
$a = 2 - 2 * (t/Max\_iter)$
while ($t < Max\ number\ of\ iterations$) **do**
  $NewPosition = CurrentPosition * LFGWO\_Levy(dim)'$
  for *each search agent* **do**
    Update the position of the current search individual by Eq. (2)
  end for
  Update a, A, and C
  $a = 2 - 2 * (t/Max\_iter)$
  Calculate the fitness of all search individual
  Update $\boldsymbol{X_\alpha}$, $\boldsymbol{X_\beta}$and$\boldsymbol{X_\delta}$
  $t = t + 1$
end while
return The best solution $\boldsymbol{X_\alpha}$.

but also for all wolves over the course of iterations. By this mechanism, the LFGWO can overcome the deficiencies of little diversity of the GWO, which is also the highlight and unique feature of the LFGWO which is totally diffrent from the other GWO variants. Consequently, all wolves' positions updating will influence the position of the wolf alpha, and which greatly increases the probability of getting the best position (solution) for the wolf alpha. In the contrary: the other GWO variants in literature always get the best position (solution) from the wolf alpha and assign new step only for the top three wolves, which mean the diversity is underdeveloped for the other GWO.

Thirdly, the LFGWO has the ability to jump out of some local optimum area in the case of stagnation by the above formula.

Additionally, initializes the position of all wolves as formulated as below:

$$X(:,i) = LFGWO\_Levy(N)'.*(high - low) + low$$

As shown in previous literature [40], [41], in some cases the GWO may be unable to find the global optimum, due to the presence of local optima that can get the search trapped. To overcome this problem, we propose modifying the GWO with the above formula that provides a large-scale deployment schema for all wolves with Levy flight values not random values at the initialization phase, which may generate diversity for LFGWO. Despite being a simple change and mixing a few parameter settings in the LFGWO, this new random distribution generates drastic changes in the optimization procedure, which increases the overall performance in terms of convergence speed and solution quality. According to obove dipicts, it is possible to say that the LFGWO can be a potential alternative in the solution of meta-heuristic optimization problems as it has high exploration and exploitation capabilities.

IV. NUMERICAL EXPERIMENTS AND DISCUSSION

In below experiments, each technique is performed using the Windows 10 OSx64 with MATLAB R2019a and the hardware platform is the Intel(R) Core (TM) i7-8700 CPU @ 3.20GHz and 8 GB main memory. The LFGWO comparing with eight well-known optimization algorithms AHA [46], AO [47], DA [48], DMOA [49], GBO [50], HGS [51], HHO [52], and MVO [53] on the 23 benchmark functions and 10 composition functions of CEC 2019 to strictly tests the performances of the LFGWO. Then non-parametric Wilcoxon, Friedman, and Nemenyi statistical tests are employed to assess the LFGWO respectively. The scalability performance of the LFGWO with the GWO and IGWO is comprehensive and thoroughly assess (in this case, for D=10, 50, 100, 300, and 500). The mathematical description of 23 benchmark functions is presented in the appendix (see Table 23), where N, T, dim, and f_{best} are respectively referred to the number of agents, the maximum iteration value, the number of dimensions, and the desired optimal value. Range denotes the interval of search space. The main details of the 10 composition functions of CEC 2019 are summarized in the appendix (see Table 24), while the complete mathematical description of each function can be found in "The 100-Digit Challenge" [54]. In the experiments, the core parameters of these nine algorithms are set up in the appendix (see Table 25).

A. NUMERICAL PERFORMANCE EVALUATION

The experimental setup includes all independent runs 30 times on each of the 23 benchmark functions and 10 composition functions of CEC 2019, the number of search agents and maximum iteration are all equal to 100 respectively under the fair condition.

Composition functions CEC04 to CEC10 of CEC 2019 are shifted and rotated, whereas composition functions CEC01 to CEC03 are not. The parameter set where defined by the CEC benchmark developer. The dimensionalities of the composition functions are different, and all of them are scalable.

The mean ('Mean' or 'Average') and standard deviations ('Std') of the best-so-far solutions are used to compare all the considered algorithms as two evaluation criteria in this experiment, which are represented follow:

$$Mean = \frac{1}{N}\sum_{i=1}^{N} g_i^* \tag{15}$$

$$Std = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(g_i^* - Mean)^2} \tag{16}$$

Here $g_i{}^*$ is the solution received in the independently run and N is the number of the independent iteration. Mean is the average value of all the solutions in the final sets obtained by an optimizer in some individual runs. The Std is used as an indicator for optimizer stability and robustness. If Std is small it depicts the optimizer converges always towards the same solution. Conversely, if Std is large it means that the results obtained are much more random and the optimizer is less reliable.

Here, the Non-parametric Friedman test is applicable to rank nine algorithms and to specify whether there exist significant differences between the results gained by the LFGWO and the other eight algorithms. Each algorithm is ranked separately the best algorithm is the one that receives the lowest rank while the worst algorithm receives the highest rank. The average and Std rankings of LFGWO in conjunction with the other eight algorithms are reported in Table 1 and Table 2, respectively.Compared with 10 composition functions of CEC 2017, the 10 composition functions of CEC 2019 case is more complicated, the complexity of the functions of CEC2019 is significantly increased.

Table 1 says the Friedman test results in which the LFGWO has the first rank compared with the other eight algorithms, and it is superior to the other eight competitors, and 28 out of 33 average values obtained by LFGWO are all less than those obtained by the other eight optimization algorithms, which further proved that the utilization of Levy Flight can effectively enhance the performance of the GWO algorithm.

TABLE 1. The values of the average and average Rank of the Friedman test (The bold font in the table indicates the optimal value).

	Index	AHA	AO	DA	DMOA	GBO	HGS	HHO	LFGWO	MVO
F1	average	2.65E+00	1.46E+02	1.36E+03	6.97E+02	7.83E+07	6.45E+02	9.34E+02	**-2.21E-88**	7.17E+02
	Rank	2	3	8	5	9	4	7	**1**	6
F2	average	3.93E-02	5.57E-01	9.15E+00	4.21E+00	3.43E+35	9.42E+07	1.88E+08	**4.25E-89**	6.37E+00
	Rank	2	3	6	4	9	7	8	**1**	5
F3	average	2.38E+02	1.02E+02	3.76E+03	2.28E+03	9.40E+01	2.30E+03	1.51E+03	**-4.41E-74**	9.03E+02
	Rank	4	3	9	7	2	8	6	**1**	5
F4	average	1.41E-01	7.46E-01	2.77E+01	1.63E+01	4.17E+06	1.57E+00	1.61E+00	**-8.06E-87**	8.98E+00
	Rank	2	3	8	7	9	4	5	**1**	6
F5	average	7.12E+01	8.44E+04	1.61E+06	4.63E+05	2.57E+03	2.14E+06	2.02E+06	**9.74E-01**	8.94E+05
	Rank	2	3	7	5	3	9	8	**1**	6
F6	average	1.13E+02	1.41E+02	2.45E+03	8.13E+02	6.81E+05	2.11E+03	8.03E+02	**-5.10E-01**	5.66E+02
	Rank	2	3	8	6	9	7	5	**1**	4
F7	average	6.74E-02	5.60E-02	7.09E-01	1.53E-01	1.03E+01	3.38E+00	1.48E+00	**3.21E-04**	2.46E-01
	Rank	3	2	6	4	9	8	7	**1**	5
F8	average	-5.94E+03	-2.39E+03	-2.26E+03	-7.12E+12	**5.11E+01**	-1.20E+04	-1.20E+04	-3.03E+03	-2.20E+03
	Rank	6	4	3	9	1	7	8	5	2
F9	average	4.13E+00	1.60E+00	5.73E+01	5.01E+01	1.56E+01	1.11E+01	9.63E+00	**-3.75E-09**	4.75E+01
	Rank	3	2	9	8	6	5	4	**1**	7
F10	average	2.90E-01	2.77E-01	1.09E+01	6.54E+00	3.24E-01	1.15E+00	6.42E-01	**-5.90E-18**	6.25E+00
	Rank	3	2	9	8	4	6	5	**1**	7
F11	average	2.86E-01	1.37E+00	2.48E+01	7.30E+00	4.95E-03	5.31E+00	5.76E+00	**-2.36E-08**	7.25E+00
	Rank	3	4	9	8	2	5	6	**1**	7
F12	average	**8.05E-01**	3.27E+05	4.29E+06	6.53E+05	2.31E+00	4.66E+06	4.26E+06	-9.73E-01	1.63E+06
	Rank	1	4	8	5	3	9	7	2	6
F13	average	4.41E+00	3.38E+05	3.73E+06	2.33E+06	6.65E+01	1.21E+07	1.00E+07	**1.00E+00**	3.40E+06
	Rank	2	4	7	5	3	9	8	**1**	6
F14	average	1.97E+00	3.50E+00	1.22E+00	1.63E+00	**4.54E-01**	1.57E+00	2.16E+00	-3.20E+01	1.62E+00
	Rank	6	8	2	5	**1**	3	7	9	4
F15	average	2.92E-03	2.42E-03	6.09E-03	1.58E-03	**7.85E-04**	1.71E-03	4.71E-03	1.40E-01	2.91E-03
	Rank	6	4	8	2	**1**	3	7	9	5
F16	average	-1.03E+00	-1.01E+00	-1.00E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.02E+00	**-7.13E-01**	-1.01E+00
	Rank	6	3	2	7	8	9	5	**1**	4
F17	average	4.10E-01	5.22E-01	4.17E-01	4.13E-01	**4.04E-01**	4.16E-01	4.09E-01	1.23E+01	4.37E-01
	Rank	3	8	6	4	**1**	5	2	9	7
F18	average	3.11E+00	3.55E+00	4.09E+00	3.16E+00	3.59E+00	3.13E+00	3.20E+00	**-1.00E+00**	3.71E+00
	Rank	2	6	9	4	7	3	5	**1**	8
F19	average	-3.86E+00	-3.75E+00	-3.85E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.83E+00	**8.54E-01**	-3.84E+00
	Rank	6	2	5	7	8	9	3	**1**	4
F20	average	-3.26E+00	-2.67E+00	-3.06E+00	-3.25E+00	-3.29E+00	-3.26E+00	-2.98E+00	**6.56E-01**	-3.09E+00
	Rank	7	2	4	6	9	8	3	**1**	5
F21	average	-7.79E+00	-9.99E+00	-6.84E+00	-9.09E+00	-4.98E+00	-7.51E+00	-4.87E+00	**1.00E+00**	-6.15E+00
	Rank	7	9	5	8	3	6	2	**1**	4
F22	average	-7.96E+00	-1.03E+01	-7.43E+00	-9.24E+00	-9.94E+00	-9.18E+00	-4.93E+00	**9.95E-01**	-2.42E+00
	Rank	5	9	4	7	8	6	3	**1**	2
F23	average	-8.32E+00	-1.04E+01	-7.14E+00	-8.72E+00	-4.85E+00	-9.98E+00	-4.94E+00	**1.00E+00**	-6.08E+00
	Rank	6	9	5	7	2	8	3	**1**	4
cec01	Mean	1.26E+10	2.30E+10	2.29E+11	1.44E+11	1.06E+10	1.28E+11	9.95E+09	**-4.59E-03**	1.07E+11
	Rank	4	5	9	8	3	7	2	**1**	6
cec02	Mean	6.44E+01	1.41E+02	2.43E+03	3.80E+02	9.54E+01	3.13E+02	2.37E+02	**-2.46E-01**	8.54E+02
	Rank	2	4	9	7	3	6	5	**1**	8
cec03	Mean	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	1.27E+01	**4.30E-01**	1.27E+01
	Rank	2	3	4	5	6	7	8	**1**	9
cec04	Mean	2.70E+03	5.70E+03	7.34E+03	1.79E+03	7.15E+02	6.46E+02	6.47E+03	**1.02E+01**	1.33E+03
	Rank	6	7	9	5	3	2	8	**1**	4
cec05	Mean	2.24E+00	2.59E+00	3.05E+00	2.00E+00	**1.43E+00**	1.63E+00	3.79E+00	-1.13E+01	2.01E+00
	Rank	5	6	7	3	1	2	8	**9**	4
cec06	Mean	9.89E+00	1.26E+01	1.21E+01	1.14E+01	1.09E+01	7.41E+00	1.17E+01	**8.11E-03**	1.14E+01
	Rank	3	9	8	5	4	2	7	**1**	6
cec07	Mean	6.36E+02	1.16E+03	1.05E+03	9.76E+02	7.17E+02	6.08E+02	1.04E+03	**1.59E+00**	7.69E+02
	Rank	3	9	8	6	4	2	7	**1**	5
cec08	Mean	5.41E+00	5.89E+00	6.89E+00	6.77E+00	6.36E+00	4.59E+00	5.69E+00	**1.82E-01**	6.47E+00
	Rank	3	5	9	8	6	2	4	**1**	7
cec09	Mean	3.33E+02	4.73E+02	1.67E+03	2.83E+02	6.42E+01	1.91E+02	4.53E+02	**-3.87E-02**	3.55E+02
	Rank	5	8	9	4	2	3	7	**1**	6
cec10	Mean	2.03E+01	2.07E+01	2.04E+01	2.05E+01	2.04E+01	2.02E+01	2.03E+01	**1.28E+01**	2.06E+01
	Rank	3	9	5	7	6	2	4	**1**	8
Friedmans test		5.22	8.09	10.2	8.28	6	6.4	8.15	2.77	8.31

TABLE 2. The standard deviations and average Rank on nine algorithms (The bold font in the table indicates the optimal value.)

	Index	AHA	AO	DA	DMOA	GBO	HGS	HHO	LFGWO	MVO
F1	Std	1.69E+01	1.43E+03	2.43E+03	1.78E+03	4.95E+08	5.02E+03	7.37E+03	**3.42E-90**	1.89E+03
	Rank	2	3	6	4	9	7	8	**1**	5
F2	Std	2.07E-01	5.17E+00	7.58E+00	8.06E+00	3.43E+36	9.42E+08	1.88E+09	**1.85E-90**	1.46E+01
	Rank	2	3	4	5	9	7	8	**1**	6
F3	Std	2.13E+03	9.34E+02	4.18E+03	2.20E+03	4.74E+02	1.33E+04	9.35E+03	**6.86E-76**	2.27E+03
	Rank	4	3	7	5	2	9	8	**1**	6
F4	Std	8.32E-01	6.23E+00	2.22E+01	1.18E+01	3.07E+07	1.08E+01	9.66E+00	**4.56E-88**	1.10E+01
	Rank	2	3	8	7	9	5	4	**1**	6
F5	Std	4.20E+02	8.44E+05	6.66E+06	2.51E+06	1.36E+04	2.13E+07	1.77E+07	**1.39E-02**	3.96E+06
	Rank	2	4	7	5	3	9	8	**1**	6
F6	Std	1.13E+02	1.41E+03	2.99E+03	2.23E+03	6.25E+06	1.05E+04	6.55E+03	**7.62E-03**	1.44E+03
	Rank	2	3	6	5	9	8	7	**1**	4
F7	Std	5.96E-01	5.37E-01	1.79E+00	3.36E-01	2.56E+00	1.65E+01	1.35E+01	**1.50E-05**	9.85E-01
	Rank	4	3	6	2	7	9	8	**1**	5
F8	Std	1.20E+03	4.22E+02	3.82E+02	2.53E+13	4.07E+02	1.17E+03	1.96E+03	**4.28E+01**	2.95E+02
	Rank	7	5	3	9	4	6	8	**1**	2
F9	Std	2.44E+01	1.11E+01	3.44E+01	1.78E+01	1.12E+02	4.78E+01	4.94E+01	**5.28E-11**	1.58E+01
	Rank	5	2	6	4	9	7	8	**1**	3
F10	Std	1.60E+00	1.93E+00	7.42E+00	5.05E+00	1.04E+00	3.53E+00	2.94E+00	**2.67E-19**	4.19E+00
	Rank	3	4	9	8	2	6	5	**1**	7
F11	Std	2.63E+00	1.33E+01	2.56E+01	1.71E+01	4.95E-02	5.15E+01	4.76E+01	**3.38E-10**	1.82E+01
	Rank	3	4	7	5	2	9	8	**1**	6
F12	Std	3.10E+00	3.27E+06	1.24E+07	4.82E+06	6.67E+00	4.66E+07	4.23E+07	**1.34E-02**	1.08E+07
	Rank	2	4	7	5	3	9	8	**1**	6
F13	Std	1.48E+01	3.38E+06	1.44E+07	1.10E+07	3.70E+02	1.21E+08	9.32E+07	**1.28E-02**	2.26E+07
	Rank	2	4	6	5	3	9	8	**1**	7
F14	Std	1.34E+00	1.59E+00	2.25E+00	1.46E+00	2.49E+00	1.92E+00	1.05E+00	**4.53E-01**	2.91E+00
	Rank	3	5	7	4	8	6	2	**1**	9
F15	Std	7.43E-03	7.37E-03	2.15E-02	1.77E-03	1.87E-03	4.16E-03	3.19E-02	**1.59E-03**	8.15E-03
	Rank	6	5	8	2	3	4	9	**1**	7
F16	Std	**1.98E-03**	2.72E-02	1.25E-01	7.10E-03	2.56E-02	2.63E-02	1.12E-01	1.02E-02	1.27E-01
	Rank	**1**	6	8	2	4	5	7	3	9
F17	Std	**3.62E-02**	1.72E-01	9.71E-02	9.61E-02	3.65E-02	1.32E-01	7.35E-02	1.74E-01	1.44E-01
	Rank	**1**	8	5	4	2	6	3	9	7
F18	Std	4.88E-01	5.43E-01	4.64E+00	1.08E+00	5.84E+00	9.35E-01	1.49E+00	**1.28E-02**	3.90E+00
	Rank	2	3	8	5	9	4	6	**1**	7
F19	Std	6.50E-03	3.48E-02	2.18E-02	2.95E-02	3.10E-02	1.72E-02	7.80E-02	**1.03E-02**	6.82E-02
	Rank	1	7	4	5	6	3	9	**2**	8
F20	Std	1.60E-01	1.32E-01	2.23E-01	2.09E-01	1.61E-01	2.20E-01	4.77E-02	**9.82E-03**	3.67E-01
	Rank	4	3	8	6	5	7	2	**1**	9
F21	Std	1.73E+00	1.12E+00	3.62E+00	2.27E+00	3.96E-01	2.64E+00	5.42E-01	**1.33E-02**	2.71E+00
	Rank	5	4	9	6	2	7	3	**1**	8
F22	Std	2.43E+00	1.02E+00	3.31E+00	2.28E+00	1.55E+00	2.71E+00	6.57E-01	**1.20E-02**	5.30E-01
	Rank	7	4	9	6	5	8	3	**1**	2
F23	Std	1.64E+00	9.47E-01	3.28E+00	2.82E+00	8.53E-01	1.72E+00	6.13E-01	**1.45E-02**	3.39E+00
	Rank	5	4	8	7	3	6	2	**1**	9
cec01	Std	1.24E+11	1.92E+11	3.71E+11	6.29E+10	6.27E+10	6.63E+11	6.34E+10	**9.25E-03**	2.52E+11
	Rank	5	6	8	3	2	9	4	**1**	7
cec02	Std	3.77E+02	1.03E+03	4.58E+03	1.24E+03	3.25E+02	1.23E+03	1.57E+03	**4.94E-01**	1.51E+03
	Rank	3	4	9	6	2	5	8	**1**	7
cec03	Std	3.99E-04	1.18E-03	7.43E-04	6.63E-04	**2.44E-04**	5.44E-04	3.33E-04	8.65E-01	6.39E-04
	Rank	3	8	7	6	**1**	4	2	9	5
cec04	Std	4.37E+03	3.28E+03	1.05E+04	4.36E+03	2.13E+03	1.86E+03	3.64E+03	**2.04E+01**	3.69E+03
	Rank	8	4	9	7	3	2	5	**1**	6
cec05	Std	1.03E+00	**4.88E-01**	1.95E+00	6.15E-01	5.57E-01	9.86E-01	6.49E-01	2.27E+01	7.84E-01
	Rank	7	**1**	8	3	2	6	4	9	5
cec06	Std	1.52E+00	6.32E-01	5.58E-01	8.58E-01	1.09E+00	2.60E+00	9.04E-01	**1.63E-02**	5.16E-01
	Rank	8	4	3	5	7	9	6	**1**	2
cec07	Std	2.38E+02	4.19E+02	4.39E+02	2.53E+02	3.16E+02	2.46E+02	2.98E+02	**3.21E+00**	1.97E+02
	Rank	3	8	9	5	7	4	6	**1**	2
cec08	Std	6.98E-01	8.76E-01	5.11E-01	2.19E-01	**1.62E-02**	9.41E-01	4.17E-01	3.65E-01	4.83E-01
	Rank	7	8	6	2	**1**	9	4	3	5
cec09	Std	7.30E+02	7.65E+02	2.23E+03	7.23E+02	2.42E+02	6.18E+02	6.30E+02	**7.80E-02**	1.02E+03
	Rank	6	7	9	5	2	3	4	**1**	8
cec10	Std	1.12E-01	1.25E-01	2.32E-01	1.01E-01	**4.60E-02**	2.15E-01	1.88E-01	2.57E+01	1.01E-01
	Rank	4	5	8	2	**1**	7	6	9	3
Friedmans test		6.54	7.05	10.3	6.52	5.11	8.71	7.54	**4.02**	7.68

It can be observed from the last row of Table 2 that LFGWO achieves the Std ranking value of 4.02 of overall rank by the Friedman test, which is less than the other eight optimization algorithms. The 27 out of 33 standard deviation values obtained by LFGWO are all less than those obtained by the other eight optimization algorithms, which further explain that the LFGWO can obtain very competitive solutions compared to other algorithms.

Considering the two metrics of mean and standard deviation of CEC 2019 in Tables 1 and 2, among the 10 composition test functions 9 out of 10 average values and 6 out of 10 standard deviation values obtained by LFGWO are all less than those obtained by the other eight optimization algorithms. It is clear that the high performance, stability, and robustness of the LFGWO, especially when the LFGWO face composition test functions with high dimensions.

Composite benchmark functions are very challenging benchmarks for optimization algorithms. Therefore, the LFGWO's exploitation and exploration capability may be simultaneously evaluated by these composite benchmarks of the CEC 2019. Moreover, the composite benchmarks' capability of local optima avoidance can be investigated by the massive number of local optima in these composite benchmarks of the CEC 2019.

From the data of Tables 1 and 2, the LFGWO exhibits outperform on the majority of composite benchmarks. These results also uncover that the inclusive exploration propensity of the LFGWO has been significantly improved owing to the addition of the Levy flight-based components to the original GWO. The perfect results say that the balance between the exploratory and exploitative inclinations has been enhanced compared to the other eight peers. In dealing with these composite benchmark problems of the CEC 2019, the LFGWO has advantages such as various dynamic approaches for updating the positions of wolves, which permit the attribute space's exploration with diverse competencies and homogeneity between exploration and exploitation phases. Therefore, the LFGWO has reached an appropriate steadier balance amongst the intensification and diversification leanings.

Considering the two metrics of mean and standard deviation of CEC 2019 in Tables 1 and 2, among the 10 composition test functions 9 out of 10 average values obtained by LFGWO are all less than those obtained by the other eight optimization algorithms, it is clear that the solution accuracy of LFGWO is superior to other competing algorithms. The main reason why the LFGWO is superior to the other competitive algorithms on these composition functions is hidden in the unique structure of this algorithm. The LFGWO inherits some advantages from the GWO, such as having three guide leaders' wolves of Alpha, Beta, and Delta, which, in turn,

TABLE 3. The best values of nine optimization algorithms (The bold font in the table indicates the optimal value.)

F	AHA	AO	DA	DMOA	GBO	HGS	HHO	LFGWO	MVO
F1	5.27E-34	1.99E-39	4.47E-02	3.84E-02	7.49E-17	**0.00E+00**	6.22E-33	8.52E-175	1.20E-01
F2	4.34E-15	7.20E-20	5.21E-01	2.10E-02	1.20E-33	1.01E-26	2.72E-20	**2.34E-87**	8.16E-02
F3	3.32E-29	1.95E-39	1.20E+01	3.49E+02	1.61E-22	1.45E-38	4.74E-27	**8.61E-146**	2.91E-01
F4	2.48E-14	2.72E-20	3.99E+00	4.13E+00	2.63E+01	3.20E-28	2.25E-15	**1.19E-85**	1.64E-01
F5	2.87E+01	1.28E-04	1.01E+01	1.47E+02	**1.05E-22**	2.28E-02	4.26E-02	1.60E+01	5.99E+01
F6	**0.00E+00**	2.20E-06	1.52E+00	1.94E-02	2.02E-20	6.45E-06	3.88E-04	2.90E-03	1.19E-01
F7	3.63E-04	1.91E-04	7.03E-03	8.41E-03	2.18E+00	1.10E-04	7.29E-04	**7.91E-05**	3.82E-03
F8	-7.51E+03	-3.01E+03	-2.51E+03	-1.38E+14	2.25E-02	**-1.26E+04**	**-1.26E+04**	-9.07E+04	-2.57E+03
F9	**0.00E+00**	**0.00E+00**	1.64E+01	3.42E+01	3.82E-04	**0.00E+00**	**0.00E+00**	**0.00E+00**	3.79E+01
F10	4.44E-15	**8.88E-16**	2.35E+00	5.68E-01	7.90E-14	**8.88E-16**	4.44E-15	**8.88E-16**	1.86E-01
F11	**0.00E+00**	**0.00E+00**	7.60E-01	7.01E-01	2.42E-13	**0.00E+00**	**0.00E+00**	**0.00E+00**	7.12E-01
F12	6.31E-02	**4.23E-08**	3.75E-01	2.46E-01	5.73E-01	3.19E-06	8.01E-06	1.78E-04	3.79E-03
F13	2.71E+00	**7.88E-07**	1.55E-02	1.51E-01	4.81E-01	1.92E-06	3.46E-05	3.94E-03	3.54E-02
F14	**9.98E-01**	2.98E+00	**9.98E-01**	**9.98E-01**	1.01E-14	**9.98E-01**	1.99E+00	7.87E+00	9.98E-01
F15	4.17E-04	4.87E-04	1.66E-03	1.06E-03	**3.07E-04**	7.78E-04	3.25E-04	**3.08E-04**	7.55E-04
F16	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
F17	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
F18	3.00E+00	3.02E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
F19	-3.86E+00	-3.85E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
F20	**-3.32E+00**	-3.08E+00	-3.17E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.02E+00	**-3.32E+00**	-3.32E+00
F21	-9.61E+00	-1.02E+01	-1.02E+01	-1.02E+01	-5.06E+00	-1.02E+01	**-5.05E+00**	**-5.05E+00**	-1.02E+01
F22	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	**-5.09E+00**	**-5.09E+00**	-2.77E+00
F23	-1.04E+01	-1.05E+01	-1.05E+01	-1.05E+01	**-5.13E+00**	-1.05E+01	-5.11E+00	**-5.13E+00**	-1.05E+01

helps the diversity of the solutions in the search space to be considerably preserved. The other characteristic of the GWO which the LFGWO benefits from are the high exploitation capability of the GWO. These characteristics are strengthened in LFGWO by adding the Levy flight into the structure of the LFGWO to enable the LFGWO to further preserve diversity and avoid missing the good candidate solutions in the search space. In addition, the aforementioned modifications imposed on the Levy flight not only for the top three wolves but also for all wolves over the course of iterations can boost the ability of the LFGWO to both explore and exploit the promising regions in the search space. Finally, the Levy flight mechanism can intensify the convergence to the optimal point of the problems and enhance the exploitation capability of the LFGWO.

As can be seen from the 6 out of 10 standard deviation values obtained by LFGWO are all less than those obtained by the other eight optimization algorithms, but the difference is slight. This issue highlights the high complexity of the composition test functions, the solving of which is a great challenge for state-of-art algorithms. As the composition functions included in the CEC2019 are very challenging for state-of-art algorithms, the state-of-art algorithms all find these test problems hard to solve, and thus the LFGWO show no significant superiority when outperforming state-of-art algorithms on most of the composition test functions. The composition test functions can be a very good examiner of the overall eligibility of the state-of-art algorithms, as it contains the toughest problems to solve.

It can be perceived from Table 3 that LFGWO received very competitive solutions compared to other eight algorithms from 16 out of 23 benchmark functions, which reveal that the LFGWO with a better ability to harmonize both exploration and exploitation.

Empirical results expose that the success rate of the LFGWO in finding the best solutions is 70% (16/23) (Table 3). Therefore, these results verify the LFGWO's ability to maintain the balance between exploration and exploitation that causes sufficient local optima avoidance. The reason is that LFGWO using more leaders wolf of Alpha, Beta, and Delta will emphasize exploitation rather than exploration.

TABLE 4. Critical values for the two-tailed Nemenyi test.

#classifiers	2	3	4	5	6	7	8	9	10
$q_{0.05}$	1.96	2.343	2.569	2.728	2.85	2.949	3.031	3.102	3.164
$q_{0.10}$	1.645	2.052	2.291	2.459	2.589	2.693	2.78	2.855	2.92

Therefore, the results of Table 3 show the superiority in performance of LFGWO in terms of exploiting the optimum and more proficient in regarding global optima. This is due to the proposed exploitation operators previously introduced in Eqs. 6, 7, and 8, and is that the LFGWO embedded with Levy flight mechanisms can stimulate both exploration and exploitation tendencies effectively. This mechanism is advantageous for exploiting new areas nearby the newly explored solutions. In addition to that, it was observed that the LFGWO embedded with Levy flight mechanisms can enrich the explorative behaviours by generating more lengthy jumps. In regard to the obtained results, the lower values of Std. obtained by LFGWO in Table 2, which indicates that the distribution of solutions obtained by LFGWO is more centralized and further shows that the LFGWO has better stability, which directly answers the LFGWO with high robustness and can assist wolves in generating more explorative jumps. Therefore, the results prove the appropriate ability of LFGWO in terms of maintaining the balance between exploration and exploitation.

The post-hoc Nemenyi statistical analysis consists of two parts [55]. The performance of the two algorithms is significantly different if the corresponding average ranks differ by at least the critical difference (CD). Firstly, a difference between the two algorithms is significant on the level of and corresponding critical value is pickup from Table 4, which involved calculating critical difference (CD) by quotation 17.

$$CD = q_{\alpha}\sqrt{\frac{k(k+1)}{6N}} \tag{17}$$

Here N is 33 benchmark functions and k (9) is the number of pairwise algorithms. The critical value(Table 4) is 3.102 and the corresponding CD is $3.102\sqrt{\frac{9\times 10}{6\times 33}} \approx 2.0914$.

Secondly, it is possible to gain valuable messages from the Average Rank Difference between "Average" and "Std" come from the last row of Table 1 and Table 2 about the ranking value of both Average and Std respectively for calculating the differences between LFGWO and other eight algorithms on average rank about "Average" and "Std". If the differences between LFGWO and other algorithm are less than CD, that means the LFGWO and the pairwise algorithm have similar performance in terms of average rank, which is listed in Table 5.

Next, with the aid of visuals, it's a lot easier to put the results of the data from Table 5 into intuitive visualizations in Figure 4. Any algorithm with a rank not overlapped (intersecting) with LFGWO is significantly different (on the right side of the vertical dotted line), contrary; there is similar performance in terms of average rank (on the left side of the vertical dotted line). The results of Figure 4 indicate that

TABLE 5. The differences between LFGWO and the other eight algorithms on average rank.

Algorithm	Average mean Rank	Diff with LFGWO	Std. mean Rank	Diff with LFGWO
AHA	5.22	2.45>2.0914	6.54	2.52>2.0914
AO	8.09	5.32>2.0914	7.05	3.03>2.0914
DA	10.2	7.43>2.0914	10.3	6.28>2.0914
DMOA	8.28	5.51>2.0914	6.52	2.50>2.0914
GBO	6	3.23>2.0914	5.11	1.09<2.0914
HGS	6.4	3.63>2.0914	8.71	4.69>2.0914
HHO	8.15	5.38>2.0914	7.54	5.52>2.0914
LFGWO	2.77	0	4.02	0
MVO	8.31	5.54>2.0914	7.68	3.66>2.0914

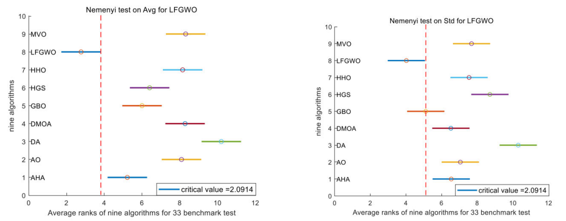


FIGURE 4. The differences between LFGWO and the other algorithms on average rank.

LFGWO versus GBO (intersecting with GBO located at the left part of the vertical dotted line, right gragh) have similar performance in terms of the average rank of Std.

B. COMPARING LFGWO WITH THE OTHER EIGHT ALGORITHMS

In order to more intuitively observe the performance and further demonstrate the superiority of the LFGWO, we performed a series of analyses of LFGWO on the 23 standard benchmark functions and 10 composition functions of CEC 2019. The multiple runs are to ensure the reliability and stability of the LFGWO. The first column of the graph in Figure 5 shows the independent convergence progress of the nine representative meta-heuristic algorithms respectively. Since there are many lines the legend attached to the figure will be particularly crowded. The convergence is a crucial indicator for understanding the exploration and exploitation performance. The second column of Figure 5 only focuses on the single convergence progress of the LFGWO. The third column of Figure 5 plots the single fitness history of the LFGWO.

In the first column of Figure 5, the LFGWO gained better results that nearly reach zero in 6 out of 7 respectively in unimodal functions, but for F5, the result was unsatisfactory for the LFGWO.

In the first column of Figure 5, except for F8, the GBO algorithm present a wrong value of positive (reference Table 3) against the value of negative that is gotten by the other eight algorithms respectively and the figure only plots them without GBO, because great difference values on two directions can't be appropriately plotted in the same figure. In the first column of Figure 5, for F14, the convergence progress of the LFGWO is not desired comparing to the other optimization algorithms. For F8-F13 and F15-F23, with the iterations increasing, the LFGWO reached satisfactory convergence progress. In the first column of Figure 5 about CEC01-CEC10, the curves show that the LFGWO has high fluctuations in the initial iterations and low variations in the last iterations. The descending trend of curves shows the wolves of the population are collaborating to improve results by updating their position to a better one as the iteration number increases. The Levy flight helps the LFGWO exhibits outperform on the majority of composite benchmarks. The first column of Figure 5 clearly intuitively observes in initial iterations that the agents have sudden changes in convergence curves until the quarter of iterations. In other means, the LFGWO covers a variety of spaces, and the LFGWO has good performance in broad search in the exploration phase. Likewise, step by step tries to achieve to optimum solution in the exploitation phase within a short time. It is the result of the LFGWO with the structure of the Levy flight. On the other hand, the leader wolf Alpha, Beta, and Delta always find the prey earlier than the other wolves in the pack. Then, in all curves of the first column of Figure 5, the LFGWO shows better convergence behavior, which can be concluded that the LFGWO strikes a balance between the exploration and exploitation in the course of iteration more than the opponent algorithms.

The curves of the second column of Figure 5 show the convergence progress of that the LFGWO has three distinct convergence behaviors for benchmark functions with different characteristics during the optimization process. First, there is a declining convergence in the initial iterations, where an approximate optimum solution is achieved, such as F1-F14, F16, F18-F19, and CEC01-CEC03. The second behavior until the half of iterations is the accelerated convergence, and the estimate of the global optimum becomes more accurate as iteration is increased, such as F15, F17, CEC05, and CEC07-CEC10. Finally, the last behavior is the gradual improvement of the solution until the final iterations, such as F20-F23, CEC04, and CEC06. By respecting the curves of the second column of Figure 5, it can be concluded that the FLGWO has the ability to strike a balance between exploration and exploitation over the course of iterations. All curves of the second column of Figure 5 show that the Levy flight has more effect on the convergence process. The main reason for this sufficient exploration and convergence of the LFGWO is introduced the Levy flight, which leads to local optima avoidance and explore the search space extensively.

The fitness history of the third column of Figure 5 demonstrated that the LFGWO can reach the approximate optimal values from the different initial directions of negative value or positive value in the non-optimal areas during the iterative process. The third column of Figure 5 shows the fitness history of all wolves during each iteration, which shows the impact of using the Levy flight in the intensification and diversification phases. The LFGWO search strategy also can find better solutions for CEC01-CEC10 of the composition function. The results verify that the LFGWO properly strikes

TABLE 6. p-Values of the Wilcoxon rank-sum tests (The bold font in the table indicates the data difference between the two groups is small.)

	AHA	AO	DA	DMOA	GBO	HGS	HHO	MVO
F1	1.85E-22	2.08E-15	2.90E-32	2.05E-31	5.37E-29	3.58E-15	8.68E-20	6.93E-32
F2	8.99E-22	2.19E-15	3.53E-32	2.42E-31	5.75E-17	2.14E-14	8.38E-21	5.76E-32
F3	8.53E-20	1.54E-13	3.08E-32	1.92E-32	5.72E-25	7.83E-18	2.02E-21	1.11E-31
F4	1.01E-22	3.06E-15	**9.03E-01**	8.85E-33	3.21E-33	4.09E-17	4.34E-20	2.86E-32
F5	1.01E-33	3.66E-32	1.78E-33	1.75E-27	**6.88E-02**	9.17E-19	8.93E-23	9.37E-33
F6	7.74E-06	3.82E-04	1.26E-32	5.30E-33	5.55E-24	8.84E-18	5.76E-17	1.02E-30
F7	7.74E-06	3.82E-04	1.26E-32	5.30E-33	5.55E-24	8.84E-18	5.76E-17	1.02E-30
F8	4.11E-29	3.45E-34	3.72E-34	4.20E-35	4.21E-35	1.11E-25	1.79E-25	2.72E-35
F9	5.59E-09	4.15E-02	1.03E-35	1.03E-36	1.07E-31	**2.92E-01**	9.29E-09	4.49E-35
F10	1.76E-22	2.41E-05	7.49E-34	4.17E-34	1.08E-26	3.68E-08	1.77E-21	1.53E-34
F11	1.00E-07	3.71E-02	5.79E-35	2.16E-34	3.40E-24	3.35E-07	3.23E-07	8.11E-35
F12	6.22E-32	7.29E-35	8.95E-33	9.28E-33	3.85E-32	2.73E-33	2.73E-33	1.05E-32
F13	4.47E-32	9.40E-34	1.53E-32	8.51E-32	3.38E-32	1.82E-21	6.85E-27	1.51E-21
F14	1.76E-24	7.93E-05	**8.95E-01**	3.44E-02	2.11E-25	8.60E-29	7.86E-27	1.92E-24
F15	**6.46E-02**	2.90E-32	2.33E-33	1.41E-31	2.48E-32	1.72E-25	2.27E-11	9.79E-28
F16	**8.83E-01**	2.22E-38	1.14E-13	6.66E-13	3.83E-11	1.76E-21	7.21E-08	1.83E-30
F17	5.99E-12	6.12E-08	9.03E-01	1.05E-24	1.52E-20	9.27E-26	2.03E-21	**7.26E-01**
F18	**6.53E-02**	5.46E-35	1.62E-12	2.21E-07	2.43E-08	2.74E-13	1.81E-04	3.67E-29
F19	1.68E-34	2.73E-34	**8.65E-01**	1.15E-23	1.09E-29	6.39E-25	3.64E-26	3.84E-09
F20	1.54E-14	2.73E-34	**8.65E-01**	1.15E-23	1.09E-29	6.39E-25	3.64E-26	3.84E-09
F21	7.93E-20	4.99E-35	1.55E-02	7.66E-14	1.70E-31	3.98E-27	1.97E-18	3.43E-33
F22	3.72E-23	6.05E-39	9.52E-11	2.98E-33	5.73E-23	3.34E-32	4.98E-23	2.52E-14
F23	3.03E-23	6.05E-39	9.52E-11	2.98E-33	5.73E-23	3.34E-32	4.98E-23	2.52E-14

TABLE 7. The average values of dimensions are equal to 50 and 100 (The bold font in the table indicates the optimal value.)

	Average (D=50)			Average (D=100)		
	GWO	IGWO	LFGWO	GWO	IGWO	LFGWO
F1	4.0683E+03	2.9234E+03	**4.8711E-44**	1.9837E+03	1.8555E+03	**-1.3883E-88**
F2	4.6503E+08	8.1422E+06	**-1.8097E-45**	7.1651E+10	1.0865E+08	**3.7349E-90**
F3	1.9831E+04	9.7617E+03	**1.2316E-34**	7.2415E+03	7.5046E+03	**-6.5191E-74**
F4	1.7528E+01	1.9763E+01	**-3.9989E-42**	9.7441E+00	8.8069E+00	**2.8926E-85**
F5	1.0930E+07	8.2610E+06	**-1.5382E-03**	7.8370E+06	3.0140E+06	**-4.9936E-03**
F6	3.8632E+03	3.5474E+03	**-4.9782E-01**	2.2514E+03	1.6589E+03	**-4.9730E-01**
F7	4.1525E+00	4.4653E+00	**-4.6074E-03**	2.5779E+00	1.5765E+00	**-3.2317E-04**
F8	-3.7479E+03	-4.2326E+03	**-9.3234E+02**	**-4.4132E+03**	-4.5423E+03	-6.5284E+03
F9	1.7578E+02	1.7748E+02	**-5.7013E-10**	9.1916E+01	1.4252E+02	**1.4059E-09**
F10	5.2844E+00	4.5342E+00	**2.6635E-16**	2.2355E+00	2.2308E+00	**-2.3796E-16**
F11	3.8827E+01	2.5368E+01	**1.7954E-09**	1.9010E+01	1.6558E+01	**-3.2533E-09**
F12	2.2087E+07	1.1143E+07	**-9.5882E-01**	9.1657E+06	5.6944E+06	**-1.0180E+00**
F13	5.1681E+07	3.1832E+07	**1.0306E+00**	2.6309E+07	9.0119E+06	**9.8256E-01**
F14	**1.9989E+00**	3.5888E+00	-3.1963E+01	**1.0974E+01**	1.4850E+00	-3.1943E+01
F15	5.0339E-03	**1.5035E-03**1	1.3714E-01	1.8923E-03	**7.8051E-04**7	1.6109E-01
F16	-1.0173E+00	-1.0104E+00	**7.1364E-01**	-1.0219E+00	-1.0304E+00	**-7.1234E-01**
F17	4.2225E-01	**4.0270E-01**	1.2359E+01	4.0995E-01	**4.0384E-01**	1.2303E+01
F18	3.1716E+00	3.1537E+00	**-9.9644E-01**	3.9724E+00	3.4212E+00	**-9.9990E-01**
F19	-3.8444E+00	-3.8516E+00	**8.5401E-01**	-3.8499E+00	-3.8557E+00	**8.4961E-01**
F20	-3.1131E+00	-3.1005E+00	**6.6227E-01**	-3.1127E+00	-3.1979E+00	**6.5793E-01**
F21	-5.1406E+00	-6.6849E+00	**1.0097E+00**	-2.2825E+00	-8.5652E+00	**1.0003E+00**
F22	-7.3712E+00	-6.9178E+00	**1.0001E+00**	-8.5219E+00	-8.4466E+00	**9.9178E-01**
F23	-7.7333E+00	-8.0745E+00	**1.0025E+00**	-6.7378E+00	-9.1730E+00	**1.0033E+00**

a proper balance between exploration and exploitation in the complex composition test functions of CEC-2019, leading to high local optima avoidance. In addition, it preserves the diversity which can handle difficulties in complex functions. This good ability of the LFGWO is because of the unique convergence behaviour of the Levy flight's jumping behaviour. The Levy flight's jumping behaviour help the LFGWO to exhibit acceptable exploitation, exploration, and local optima avoidance capability, simultaneously.

All in all, for the comprehensive result of the iteration progress in the Figure 5, the LFGWO is superior to the other eight optimization algorithms, which suggest that the LFGWO can explore the search space extensively and find promising regions of the search space.

C. LFGWO VS PAIRWISE ALGORITHMS ON THE P-VALUES OF THE WILCOXON

For further substantiation of the beneficial attributes of LFGWO, independent sets of data are checked at the confidence level of 0.05 using pairwise comparison tests. Statistical-based Wilcoxon signed-rank test is a non-parametric test, which can verify whether there is a significant difference between the two sets of data [56]. Because of the randomness of the metaheuristic algorithm, a similar statistical experiment comparison is necessary to ensure the validity of the data. When the p-values less than 0.05, which indicates that there is a significant difference between the data of the two pairwise algorithms. On the contrary, the p-values greater than 0.05, which means that there is no significant difference between the data of the two pairwise algorithms. The p values comparison results of LFGWO and pairwise algorithms are tabulated in Table 6.

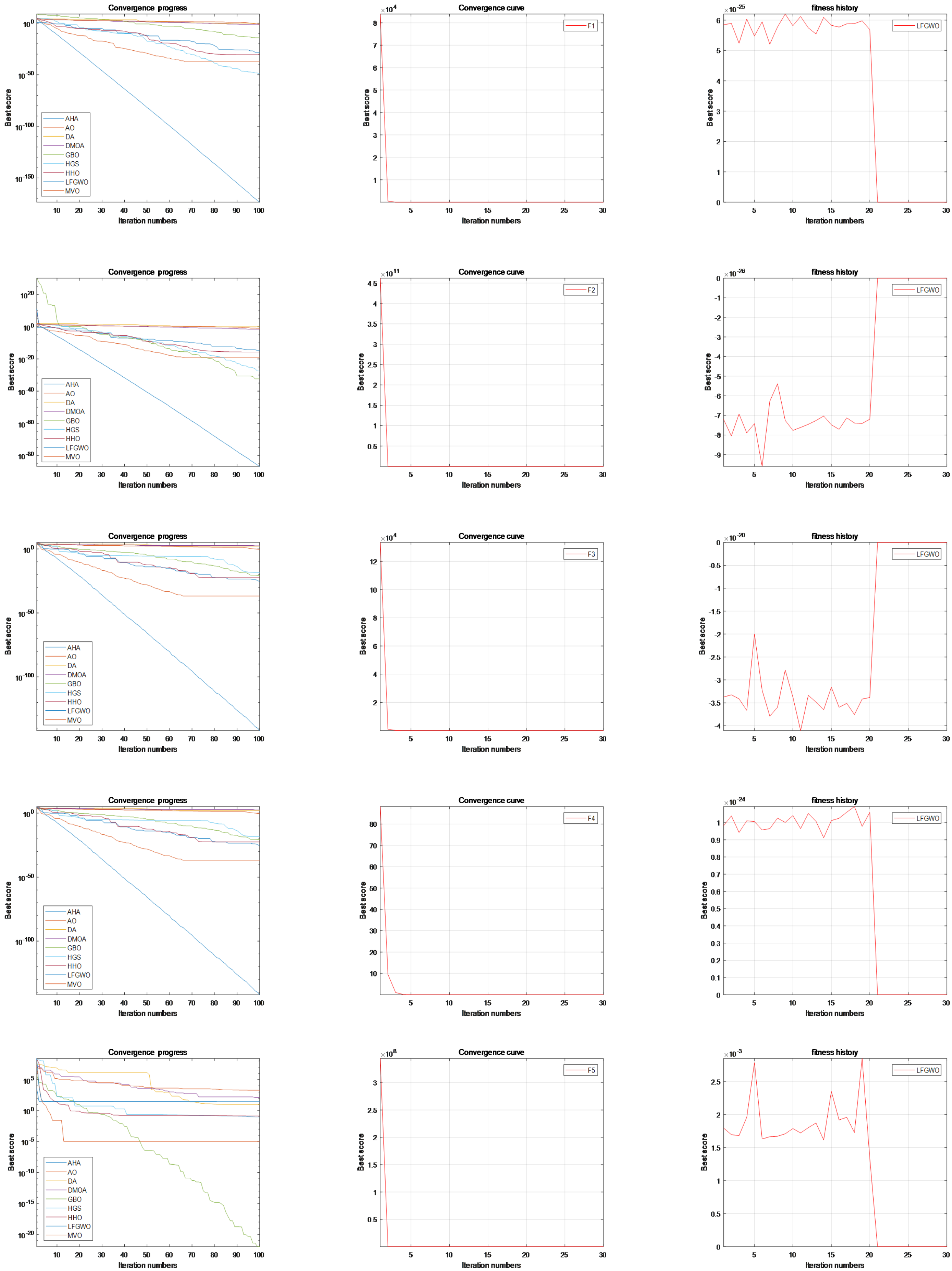
The P-values more than 0.05 are ascertained in Table 6: LFGWO/AHA in F15, F16, and F18; LFGWO/DA in F4, F14, F19, and F20; LFGWO/GBO in F5; LFGWO/HGS in F9; LFGWO/MVO in F17. In most of the comparisons in Table 6, the p-values are smaller than 0.05. From the p-values in Table 6, we can understand the superiority of the LFGWO compared to the other algorithms. The superior results do not mean that the LFGWO can tackle all the optimization problems efficiently. As per the NFL theorem, all optimization algorithms demonstrate identical performance when employed to solve all classes of optimization problems [42].

D. THE SCALABILITY OF LFGWO

Furthermore, the GWO, IGWO, and LFGWO conducted the scalability test with functions F1-F23 as a complementary study of comparing the property. The purpose of this study is to evaluate the impact of dimension on the capability of the solution and the efficacy of the LFGWO. In the experiments on 50, 100, 300, and 500 dimensions, the performances of each algorithm are inspected by independent execution. The whole circumstances have remained consistent, and each algorithm uses 100 search agents and runs 30 times respectively. The metrics of the mean, standard deviation and best values of the considered benchmarks' functions were commonly employed by the GWO, IGWO, and LFGWO.

From Table 7 (D=50) (D=100), it is observed that the performance of LFGWO compared to the other algorithms, there are 20 out of 23 (D=50) and 19 out of 23 (D=100) average values obtained by LFGWO, which far less than by the other algorithms.

According to the results from Table 8 (D=50) (D=100), LFGWO can be considered one of the best algorithms compared to the other two contending optimizers in terms of stability and robustness. The results of 21 out of 23 (D=50)



F1

F2

F3

F4

F5

FIGURE 5. The performance of LFGOA on F1-F23 and CEC01-CEC10.

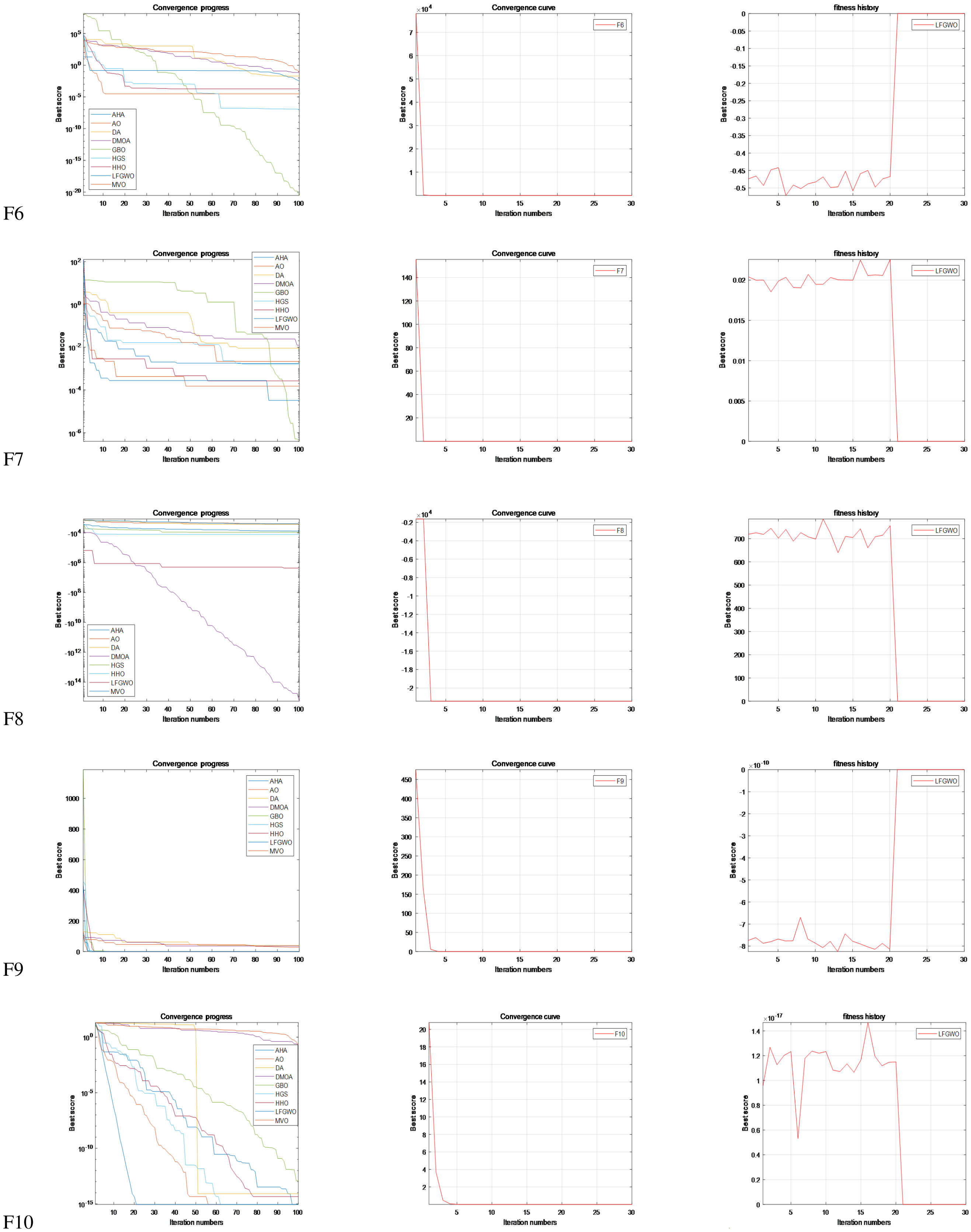


FIGURE 5. *(Continued.)* The performance of LFGOA on F1-F23 and CEC01-CEC10.

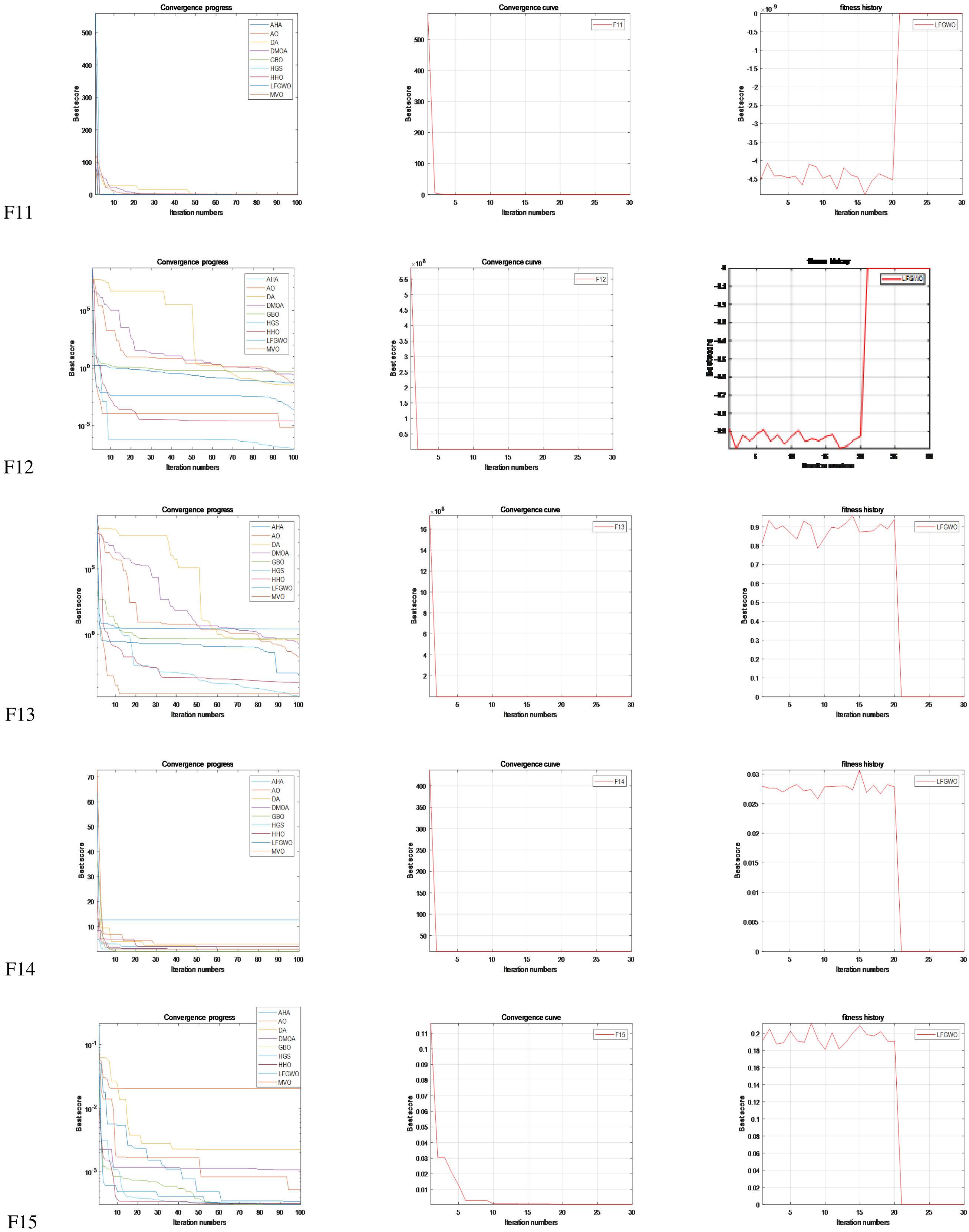
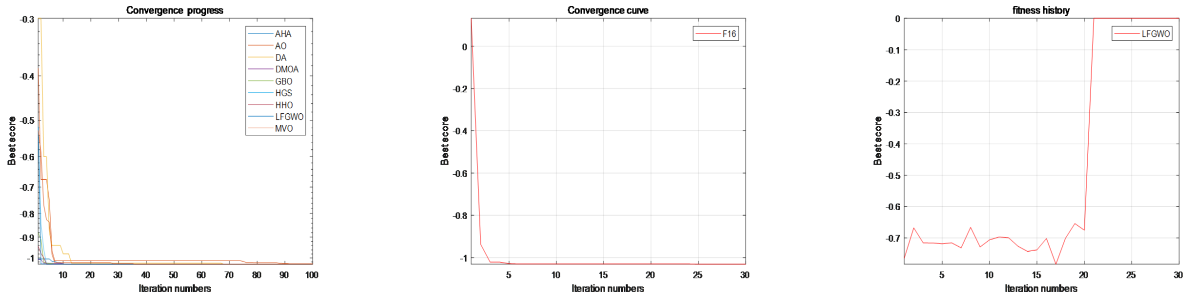
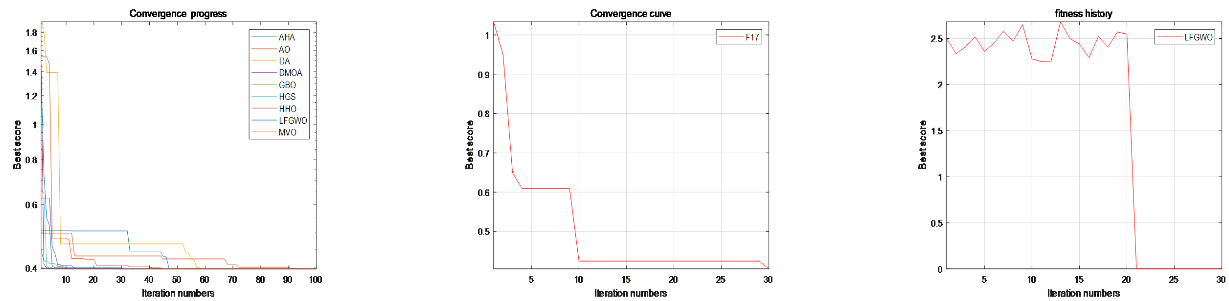


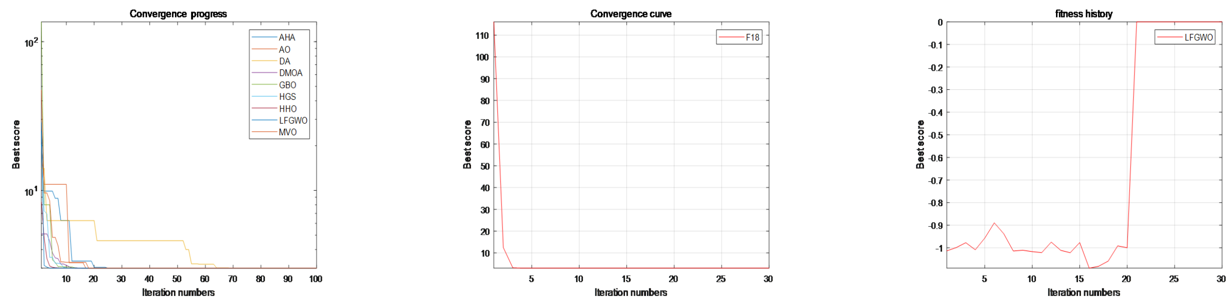
FIGURE 5. *(Continued.)* The performance of LFGOA on F1-F23 and CEC01-CEC10.



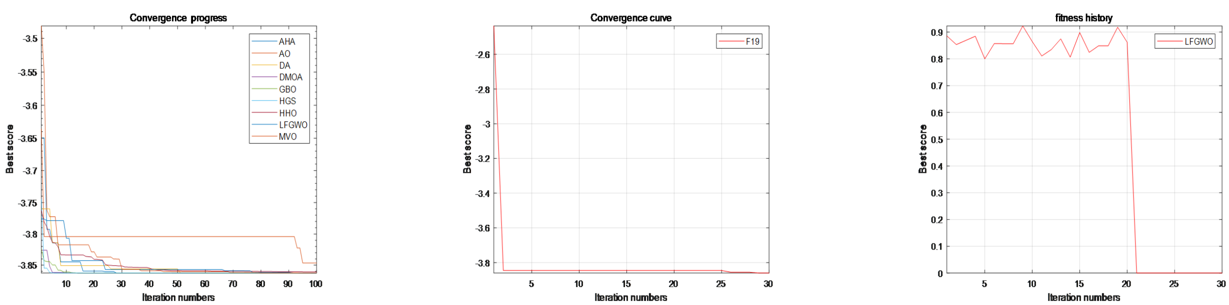
F16



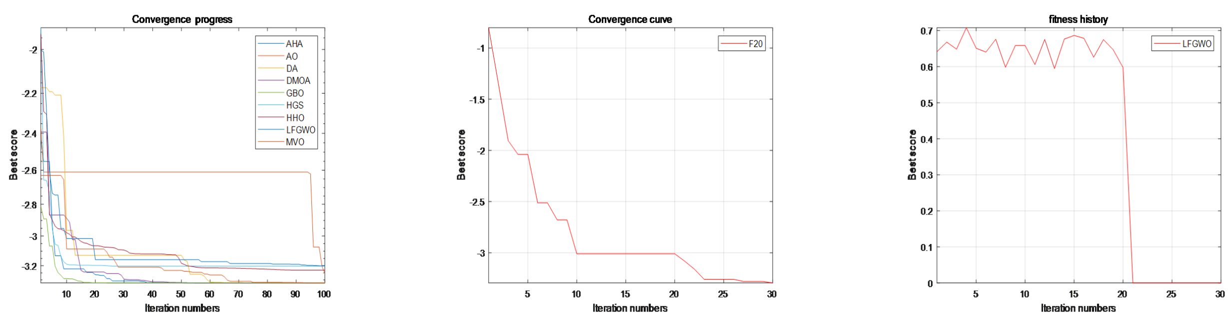
F17



F18



F19



F20

FIGURE 5. ***(Continued.)*** **The performance of LFGOA on F1-F23 and CEC01-CEC10.**

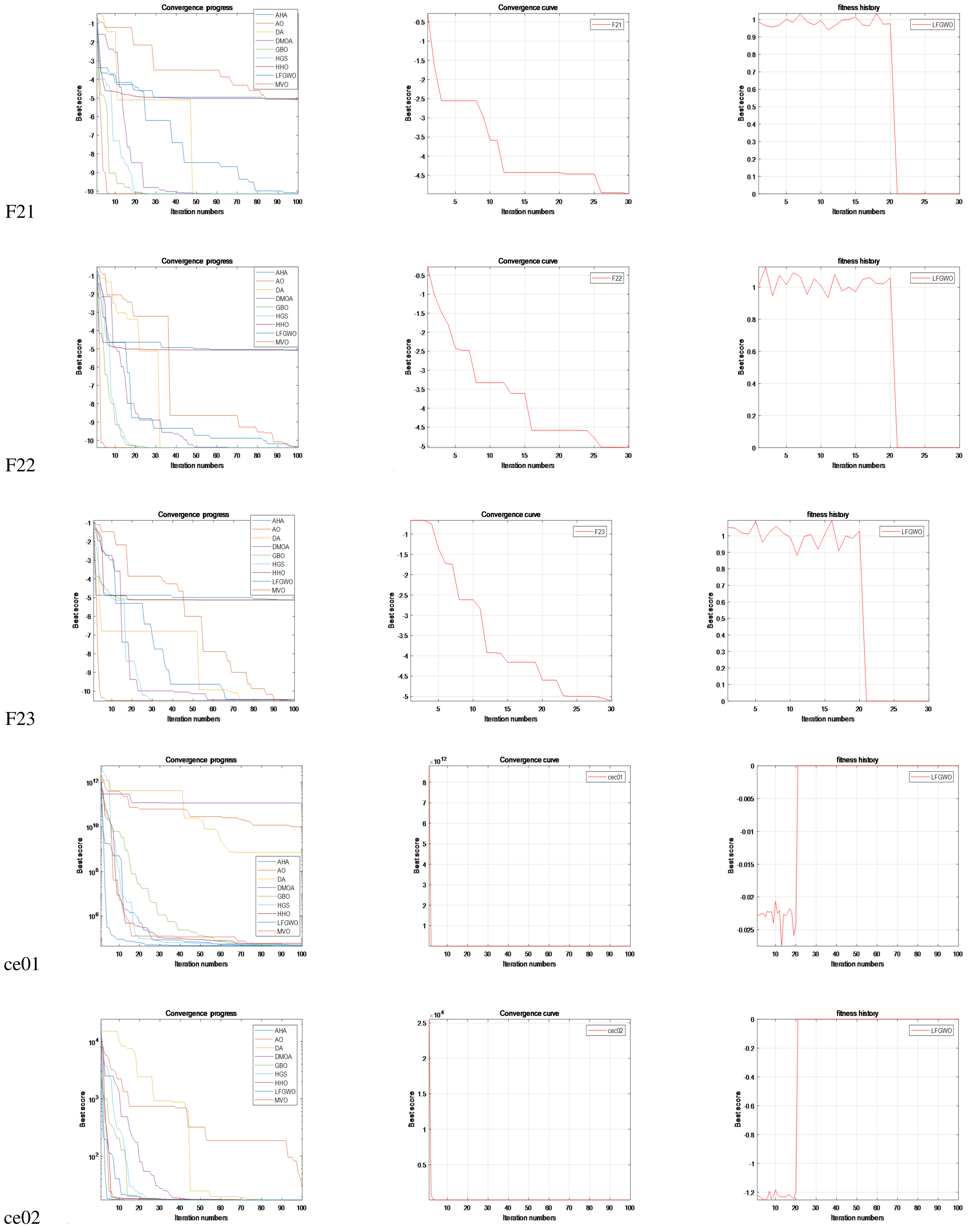


FIGURE 5. *(Continued.)* The performance of LFGOA on F1-F23 and CEC01-CEC10.

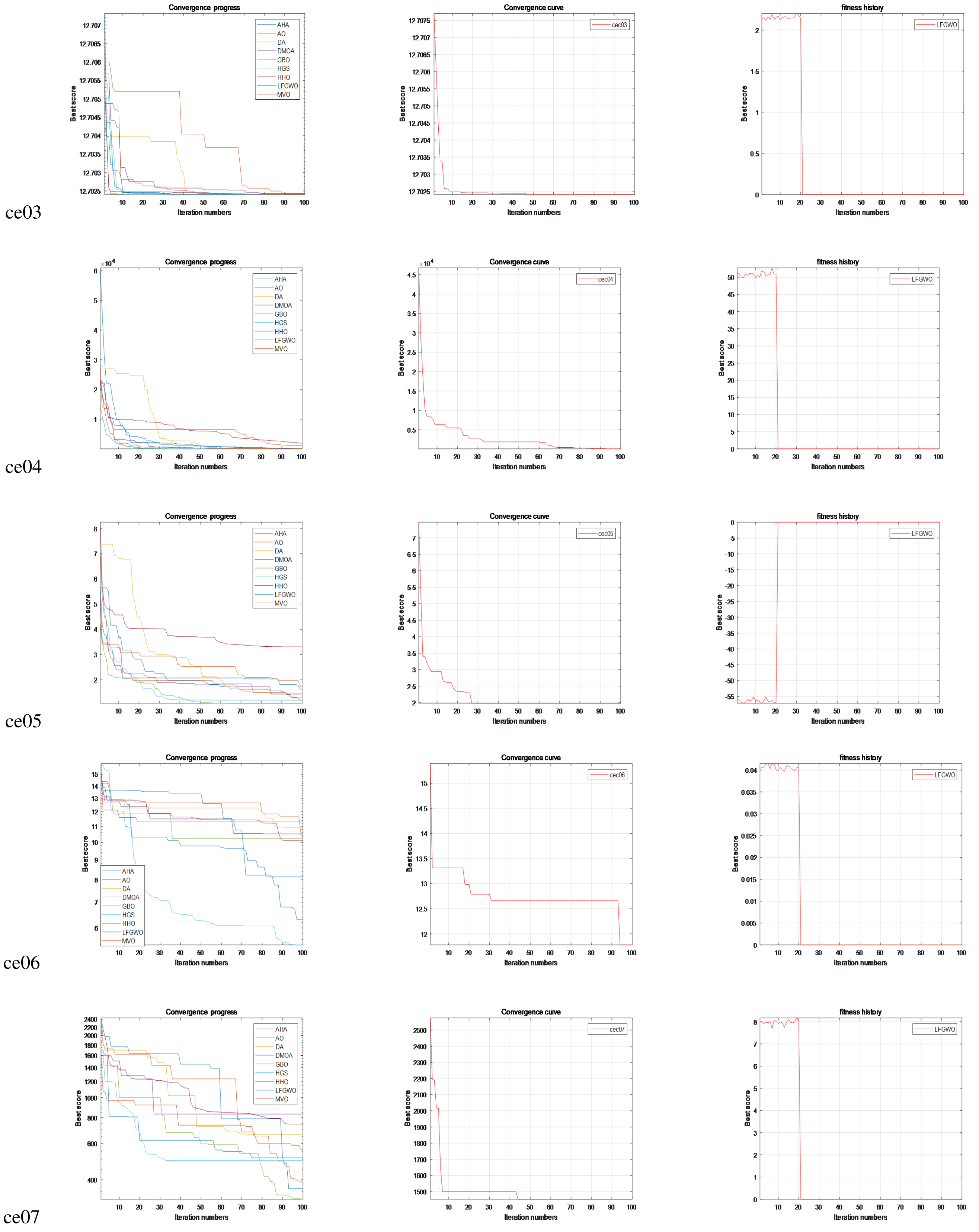


FIGURE 5. *(Continued.)* **The performance of LFGOA on F1-F23 and CEC01-CEC10.**

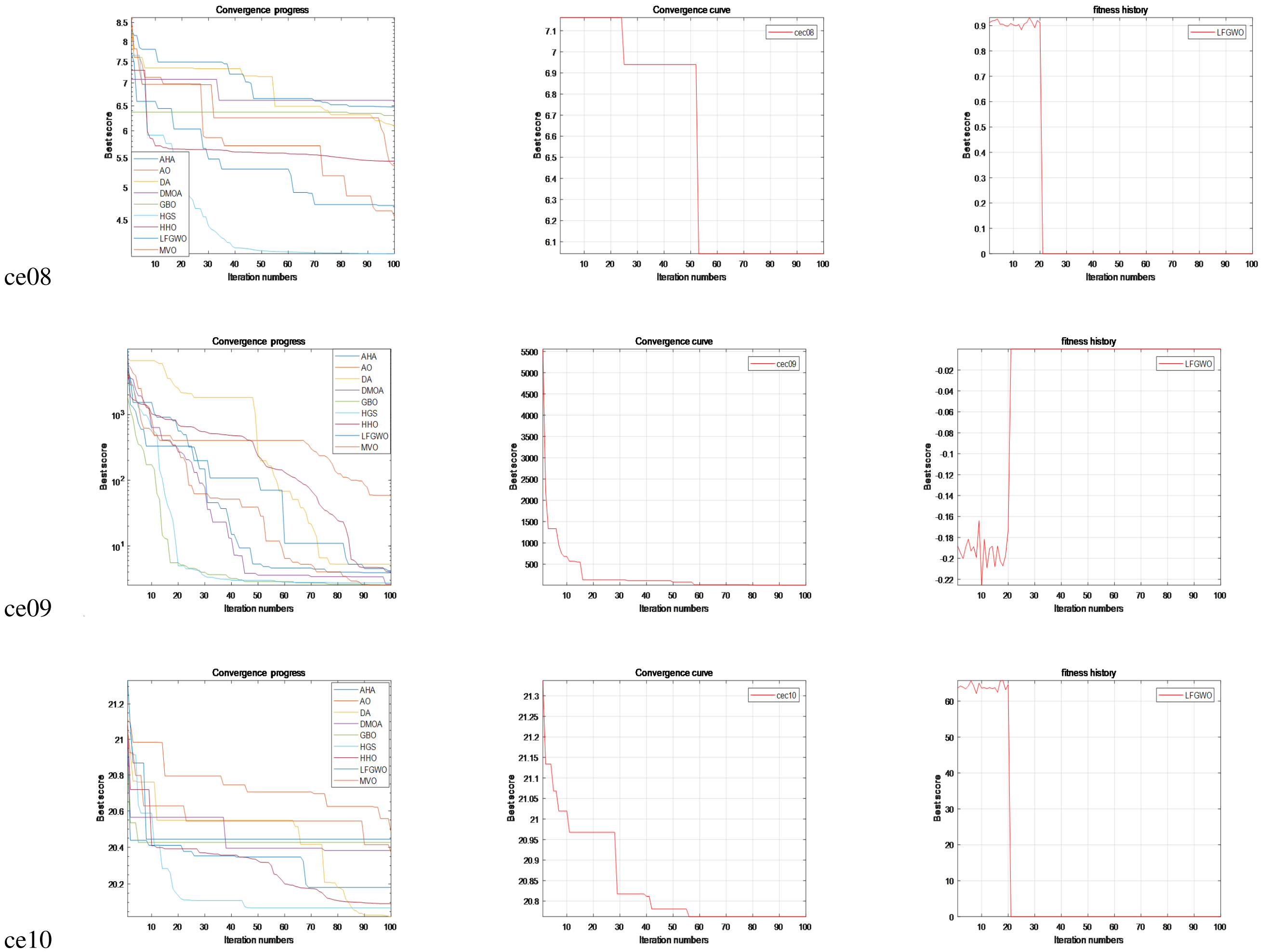


FIGURE 5. *(Continued.)* The performance of LFGOA on F1-F23 and CEC01-CEC10.

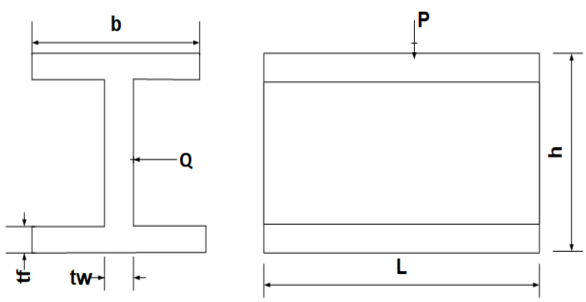


FIGURE 6. Sketch map of I-beam design.

and 20 out of 23 (D=100) Std values obtained by LFGWO are all less than other algorithms.

Based on the overall statistical results of Table 9 (D=50 and 100), there are 22 out of 23 best values obtained by LFGWO, which can be significantly better than the other two optimization algorithms.

The overall statistical results evaluated by the GWO, IGWO, and LFGWO for F1-F23 are reported in Table 10 (D=300) and (D=500). There are 21 out of 23 (D=300) and 19 out of 23 (D=500) average values received by LFGWO

TABLE 8. The Std values of dimensions are equal to 50 and 100 (The bold font in the table indicates the optimal value.)

	Std(D=50)			Std(D=100)		
	GWO	IGWO	LFGWO	GWO	IGWO	LFGWO
F1	1.1327E+04	8.9635E+03	**1.6009E-45**	8.0593E+03	7.8673E+03	**1.8947E-90**
F2	3.2874E+09	5.7119E+07	**8.7357E-46**	7.1651E+11	1.0865E+09	**1.0154E-90**
F3	2.6503E+04	2.1774E+04	**4.0771E-36**	1.6766E+04	1.6110E+04	**2.3415E-75**
F4	2.1398E+01	1.9147E+01	**1.1546E-43**	2.0292E+01	1.7002E+01	**5.3226E-87**
F5	4.6061E+07	3.6292E+07	**8.6724E-04**	3.4783E+07	1.9372E+07	**2.5634E-04**
F6	1.0502E+04	1.1545E+04	**1.2634E-02**	8.9914E+03	7.0871E+03	**7.3112E-03**
F7	1.4087E+01	1.8275E+01	**1.2285E-04**	1.2479E+01	9.3215E+00	**2.0902E-05**
F8	1.3536E+03	1.1019E+03	**2.7409E+01**	1.1775E+03	6.5360E+02	**7.7033E+01**
F9	9.3469E+01	7.6123E+01	**1.9661E-11**	9.7652E+01	8.9559E+01	**1.7922E-11**
F10	6.1837E+00	5.8112E+00	**6.1470E-18**	4.9331E+00	4.9445E+00	**3.3423E-18**
F11	1.1006E+02	7.9982E+01	**5.3934E-11**	6.9812E+01	6.9549E+01	**4.4257E-11**
F12	8.0580E+07	5.0631E+07	**2.7001E-02**	5.6258E+07	4.1160E+07	**1.4411E-02**
F13	1.9293E+08	1.4358E+08	**2.9443E-02**	1.2828E+08	6.4132E+07	**1.3647E-02**
F14	1.8715E-02	8.4491E+00	**8.0404E-01**	1.1620E+00	1.8824E+00	**4.0903E-01**
F15	1.9622E-02	**3.1359E-03**	3.3594E-03	6.9157E-03	**1.4061E-03**	1.9928E-03
F16	7.3024E-02	1.3368E-01	**1.9355E-02**	9.4517E-02	**4.8737E-03**	8.2914E-03
F17	1.5021E-01	**1.2449E-02**	3.0519E-01	6.3916E-02	**2.7781E-02**	1.5608E-01
F18	6.9452E-01	7.7623E-01	**2.6975E-02**	9.2747E+00	3.2142E+00	**1.3578E-02**
F19	5.4619E-02	4.3275E-02	**2.1965E-02**	6.1705E-02	4.2591E-02	**1.0546E-02**
F20	1.7240E-01	2.6723E-01	**1.7995E-02**	2.0322E-01	1.5280E-01	**8.4764E-03**
F21	3.2440E+00	3.1314E+00	**2.6562E-02**	3.7033E-01	2.4305E+00	**1.2497E-02**
F22	2.7883E+00	2.8991E+00	**2.6773E-02**	1.9388E+00	2.5142E+00	**1.6778E-02**
F23	3.6686E+00	2.4698E+00	**2.5149E-02**	2.5615E+00	2.4849E+00	**1.2857E-02**

TABLE 9. The best values of dimensions are equal to 50 and 100 (The bold font in the table indicates the optimal value.)

	f_{best}(D=50)			f_{best}(D=100		
	GWO	IGWO	LFGWO	GWO	IGWO	LFGWO
F1	8.1131E-01	3.2076E-01	**7.1973E-86**	2.4050E-05	4.8042E-06	**1.2196E-174**
F2	2.6109E-01	1.1183E-01	**1.4979E-42**	5.2681E-04	5.5844E-04	**3.2695E-87**
F3	1.6577E+03	1.2117E+02	**1.8278E-66**	1.1634E+01	2.2415E+01	**3.1286E-144**
F4	3.1510E+00	4.8836E+00	**7.0764E-42**	1.6332E-01	2.6008E-01	**7.8420E-85**
F5	4.2696E+01	6.5390E+01	**2.6975E+01**	2.6868E+01	2.5872E+01	**2.6969E+01**
F6	4.1088E+00	4.0568E+00	**1.0997E-02**	1.2676E+00	5.1266E-01	**3.1461E-03**
F7	8.4869E-03	1.0977E-02	**5.9585E-05**	8.8154E-03	4.3594E-03	**1.8663E-05**
F8	-6.5348E+03	-7.1838E+03	**-2.1361E+04**	-6.6950E+03	-6.2170E+03	**-1.4629E+05**
F9	4.4605E+01	5.2085E+01	**0.0000E+00**	1.7432E+01	2.8620E+01	**0.0000E+00**
F10	5.1375E-01	2.3446E-01	**8.8818E-16**	7.3879E-04	6.2300E-04	8 .8818E-16
F11	7.2681E-01	2.4479E-01	**0.0000E+00**	4.4054E-05	4.6489E-05	**0.0000E+00**
F12	5.3590E-01	2.5960E-01	**7.8971E-04**	4.3304E-02	2.7705E-02	**3.4033E-04**
F13	1.8453E+00	1.9575E+00	**1.4653E-02**	2.0176E-01	5.1661E-01	**1.4160E-02**
F14	1.9920E+00	**9.9800E-01**	2.9821E+00	1.0763E+01	**9.9800E-01**	2.9821E+00
F15	7.1836E-04	4.7088E-04	**3.1051E-04**	4.1848E-04	**3.0801E-04**	**3.1281E-04**
F16	-1.0316E+00	-1.0316E+00	**-1.0316E+00**	-1.0313E+00	-1.0316E+00	-1.0316E+00
F17	3.9792E-01	3.9789E-01	**3.9931E-01**	3.9790E-01	3.9789E-01	3.9809E-01
F18	3.0015E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00
F19	-3.8626E+00	-3.8628E+00	**-3.8625E+00**	-3.8626E+00	-3.8628E+00	-3.8622E+00
F20	-3.2016E+00	-3.2026E+00	-3.3192E+00	-3.2023E+00	-3.3219E+00	-3.3216E+00
F21	-1.0013E+01	-1.0104E+01	-5.0493E+00	-2.6273E+00	-1.0144E+01	**-5.0541E+00**
F22	-1.0286E+01	-1.0347E+01	-5.0823E+00	-1.0374E+01	-1.0398E+01	**-5.0859E+00**
F23	-1.0488E+01	-1.0452E+01	-5.1252E+00	-1.0507E+01	-1.0529E+01	**-5.1279E+00**

TABLE 10. The average values of dimensions are equal to 300 and 500 (The bold font in the table indicates the optimal value.)

	Average (D=300)			Average (D=500)		
	GWO	IGWO	LFGWO	GWO	IGWO	LFGWO
F1	6.1898E+02	5.8820E+02	**3.0775E-163**	3.8375E+02	3.5060E+02	**1.9277E-163**
F2	2.7588E+09	2.9584E+07	**4.7518E-255**	2.4010E+06	9.5034E+03	**0.0000E+00**
F3	2.7882E+03	2.7255E+03	**4.0251E-164**	1.9486E+03	2.2425E+03	**2.1840E-164**
F4	3.4073E+00	3.4246E+00	**1.9338E-245**	2.2175E+00	1.9545E+00	**0.0000E+00**
F5	1.8432E+06	1.4456E+06	**6.0946E-04**	1.1275E+06	7.3027E+05	**2.1135E-01**
F6	8.7899E+02	5.9062E+02	**-1.6388E-01**	4.4978E+02	4.4084E+02	**-9.8835E-02**
F7	9.6962E-01	5.8523E-01	**-8.4910E-05**	4.4394E-01	3.2271E-01	**2.7227E-05**
F8	-4.3229E+03	-5.5585E+03	**-2.1461E+03**	**-4.2866E+03**	-6.2742E+03	2.6760E+04
F9	1.0599E+02	1.0599E+02	**-1.1909E-09**	1.8250E+01	1.2118E+02	**1.1806E-09**
F10	9.0580E-01	8.7877E-01	**3.9958E-17**	5.0217E-01	5.0829E-01	**6.2901E-17**
F11	6.6357E+00	5.4062E+00	**3.2238E-09**	3.8562E+00	3.7406E+00	**4.8426E-10**
F12	3.4842E+06	2.2012E+06	**-3.3020E-01**	2.4667E+06	9.3121E+05	-1.9806E-01
F13	4.6954E+06	6.0376E+06	**3.3044E-01**	4.4614E+06	3.5462E+06	**2.0020E-01**
F14	1.1458E+00	1.1701E+00	**.3885E-03**9	3.0852E+00	**1.0877E+00**	-6.3959E+00
F15	1.0629E-03	**1.0627E-03**	4.5800E-02	**5.3628E-04**	8.3894E-04	2.7163E-02
F16	-1.0280E+00	-1.0303E+00	**-2.3744E-01**	-1.0300E+00	-1.0299E+00	**1.4255E-01**
F17	4.0413E-01	**4.0022E-01**	4.0918E+00	4.0129E-01	**3.9860E-01**	4.5504E-01
F18	3.1610E+00	3.1186E+00	**-3.3348E-01**	3.0388E+00	3.0032E+00	**-1.9982E-01**
F19	-3.8591E+00	-3.8624E+00	**2.8416E-01**	-3.8563E+00	-3.8616E+00	**1.7058E-01**
F20	-3.1686E+00	-3.3041E+00	**1.2467E-02**	-3.2876E+00	-3.1946E+00	**7.7073E-03**
F21	-7.6569E+00	-9.3736E+00	**3.3365E-01**	-7.7869E+00	-9.9424E+00	**2.0024E-01**
F22	-8.2073E+00	-9.6270E+00	**3.3235E-01**	-8.4590E+00	-1.0009E+01	**1.9987E-01**
F23	-7.6151E+00	-9.8311E+00	**3.3295E-01**	-7.7049E+00	-1.0014E+01	**2.0017E-01**

that are all less than the other algorithms, which confirms the facts that LFGWO still provides the best-obtained results in F1-F23 when the dimension is big.

From Table 11 (D=300) (D=500), it can be perceived that LFGWO can be more fruitful than GWO and IGWO in dealing with the F1-F23 benchmark functions, 18 out of 23 (D=300) and 16 out of 23 (D=500) Std values obtained by LFGWO, which are all less than those obtained by GWO and IGWO, which implicates that the LFGWO have better stability and robustness.

In comparison with GWO and IGWO, the LFGWO still performs meaningfully better on F1-F23 in terms of the results of 18 out of 23 (D=300) and 20 out of 23 (D=500)

TABLE 11. The Std values of dimensions are equal to 300 and 500 (The bold font in the table indicates the optimal value.)

	Std(D=300)			Std(D=500)		
	GWO	IGWO	LFGWO	GWO	IGWO	LFGWO
F1	4.4292E+03	4.0591E+03	**0.0000E+00**	3.5498E+03	3.3029E+03	**0.0000E+00**
F2	4.7781E+10	5.1239E+08	**0.0000E+00**	5.1435E+07	2.0534E+05	**0.0000E+00**
F3	1.1503E+04	1.1695E+04	**0.0000E+00**	1.1126E+04	8.5465E+03	**0.0000E+00**
F4	1.2426E+01	1.1837E+01	**0.0000E+00**	1.0119E+01	8.6511E+00	**0.0000E+00**
F5	1.5075E+07	1.5050E+07	**8.6939E-04**	1.2134E+07	9.4120E+06	**4.2313E-01**
F6	5.6127E+03	4.3409E+03	**2.3216E-01**	3.7059E+03	3.8566E+03	**1.9787E-01**
F7	9.3306E+00	6.0370E+00	**1.2028E-04**	5.1754E+00	3.8877E+00	**5.4510E-05**
F8	1.1949E+03	**9.3462E+02**	3.0402E+03	1.0787E+03	1.1149E+03	5.3573E+04
F9	7.1008E+01	7.1008E+01	**1.6871E-09**	5.9745E+01	9.0402E+01	**2.3635E-09**
F10	3.3806E+00	3.2188E+00	**5.6604E-17**	2.4403E+00	2.4159E+00	**1.2593E-16**
F11	4.3758E+01	3.8863E+01	**4.5668E-09**	3.6515E+01	3.3450E+01	**9.6949E-10**
F12	3.3034E+07	3.0417E+07	**4.6776E-01**	3.1844E+07	1.4615E+07	**3.9651E-01**
F13	5.5951E+07	6.6778E+07	**4.6810E-01**	5.4190E+07	5.1304E+07	**4.0081E-01**
F14	5.0589E-01	1.8929E+00	**1.3300E-02**	9.4097E-01	**8.7238E-01**	1.2805E+01
F15	**4.0703E-03**	8.0801E-03	6.4880E-02	**1.0355E-03**	6.3719E-03	5.4381E-02
F16	5.8293E-02	**1.4715E-02**	3.3635E-01	**2.9317E-02**	3.0661E-02	2.8539E-01
F17	5.9193E-02	**1.2582E-02**	5.7964E+00	**1.4938E-02**	**6.4575E-03**	9.1100E-01
F18	2.2993E+00	1.5867E+00	**4.7240E-01**	7.9795E-01	**5.4988E-02**	4.0003E-01
F19	1.5570E-02	**2.2155E-03**	4.0254E-01	**6.1671E-03**	1.4594E-02	3.4151E-01
F20	7.0428E-02	6.7756E-02	**1.7661E-02**	7.9353E-02	8.5121E-02	**1.5430E-02**
F21	2.4990E+00	2.0350E+00	**4.7264E-01**	2.5390E+00	9.7958E-01	**4.0088E-01**
F22	2.3241E+00	1.8335E+00	**4.7080E-01**	1.9401E+00	1.2707E+00	**4.0014E-01**
F23	2.3327E+00	1.6492E+00	**4.7165E-01**	2.5090E+00	1.5696E+00	**4.0075E-01**

TABLE 12. The best values of dimensions are equal to 300 and 500 (The bold font in the table indicates the optimal value.)

	f_{best}(D=300)			f_{best}(D=500)		
	GWO	IGWO	LFGWO	GWO	IGWO	LFGWO
F1	1.8389E-23	4.7566E-23	**0.0000E+00**	1.0691E-42	1.3404E-41	**0.0000E+00**
F2	6.8608E-14	3.4298E-14	**2.7068E-253**	6.9968E-24	1.5091E-24	**0.0000E+00**
F3	8.5492E-06	5.3998E-05	**0.0000E+00**	7.2896E-12	6.7519E-10	**0.0000E+00**
F4	6.2932E-06	4.7091E-05	**1.0188E-244**	2.9455E-10	4.6946E-10	**0.0000E+00**
F5	2.5458E+01	**2.4404E+01**	2.6102E+01	2.7135E+01	2.3305E+01	**1.4351E+01**
F6	7.5788E-01	4.7462E-01	**4.6304E-04**	2.7218E-05	2.3773E-05	**1.7800E-04**
F7	2.8721E-04	5.5223E-04	**9.2920E-06**	5.5584E-04	8.1498E-04	**1.3809E-05**
F8	-6.2635E+03	-8.0777E+03	**-1.9112E+05**	-6.1280E+03	-9.0703E+03	**-3.8773E+06**
F9	2.8272E+01	2.8272E+01	**0.0000E+00**	0.0000E+00	2.3170E+01	**0.0000E+00**
F10	1.4397E-12	6.3327E-13	**8.8818E-16**	2.2204E-14	2.2204E-14	**8.8818E-16**
F11	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
F12	3.1586E-02	**7.6610E-06**	2.3495E-05	1.9895E-02	**2.1701E-06**	1.2773E-05
F13	2.9932E-01	**8.2508E-05**	3.6969E-04	1.0019E-01	**2.4071E-05**	1.0269E-04
F14	**9.9800E-01**	**9.9800E-01**	1.2671E+01	2.9821E+00	**9.9800E-01**	**9.9800E-01**
F15	3.0787E-04	3.0749E-04	3.0759E-04	3.0751E-04	3.0749E-04	3.0749E-04
F16	-1.0316E+00	-1.0316E+00	-1.0316E+00	-1.0316E+00	-1.0316E+00	-1.0316E+00
F17	3.9789E-01	3.9789E-01	3.9790E-01	3.9789E-01	3.9789E-01	3.9789E-01
F18	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00
F19	-3.8628E+00	-3.8628E+00	-3.8627E+00	-3.8602E+00	-3.8628E+00	-3.8628E+00
F20	-3.2030E+00	**-3.3220E+00**	-3.2029E+00	**-3.3220E+00**	-3.2032E+00	-3.2008E+00
F21	-1.0150E+01	-1.0153E+01	**-5.0551E+00**	-1.0153E+01	-1.0153E+01	**-5.0551E+00**
F22	-1.0401E+01	-1.0403E+01	**-5.0874E+00**	-1.0403E+01	-1.0403E+01	**-5.0876E+00**
F23	-1.0534E+01	-1.0536E+01	**-5.1284E+00**	-1.0536E+01	-1.0536E+01	**-5.1284E+00**

in Table 12 best values obtained by LFGWO, and such that the number is far exceeded GWO and IGWO.

As shown in the above tables, it can be observed that LFGWO has obtained obvious advantages on most functions relative to the GWO and IGWO respectively in all dimensions. While these three algorithms are meta-heuristic algorithms, a tendency can be found to enhance the minimum mean value as the dimension increases. In short, loosely speaking, a conclusion can be drawn through the scalability test that LFGWO has better scalability to control optimization tasks. Based on comparative experiments in scalability in Section IV-D, according to the strict numerical statistic, some consequences can be inferred. Firstly, the tests and comparison with other competitors on F1–F23 show that the proposed LFGWO based on the Levy flight methodology can solve these complex optimization problems on average, standard deviation, and best values, respectively. The LFGWO can, therefore, be used to solve certain real-life and complex optimisation problems.

E. THE TIME COMPLEXITY OF THE LFGWO

In general, metaheuristic schemes are complex systems that include random processes. Under such conditions, performing a complexity study from a deterministic perspective is impractical. For this reason, this paper uses the Big-O notation to express the complexity of the algorithm. The evaluation of an algorithm is mainly inspected by computational complexity. The execution time of the GWO is faster than the LFGWO. However, the LFGWO's success in catching the hunt is much more successful than the GWO and eliminates some of the weaknesses of the GWO method (as mentioned earlier, not being able to quickly circle around the prey, and allow the prey to escape in some cases).

The computational complexity mainly includes the following parts in terms of the LFGWO. The initialization of the agent is O (d) (d=dimension size). The fitness value evaluation of the initial population is O (s×d) (s=population size). The position of updating agent is O (s×d + s). Producing a new solution by levy flight strege is O (s). The fitness value evaluated for all agents after updating their position is O (s). Therefore, the total computational complexity of LFGWO is simplified as O (d + s×d + s×d + s + s + s) = O (d+3×s + 2×s×d).

Besides, Ref [57] mathematically proved that using Levy flight does not increase the time complexity of optimization algorithms. Therefore, both the GWO and the LFGWO have the same time complexity so their differences can be ignored computationally.

F. DISCUSSION

Even the frameworks of the Levy Flight and the GWO are excellent, but how to infuse Levy Flight with the GWO will directly produce a different result, sometimes even presenting a not disired result. In [37], Levy flight and greedy selection strategies are integrated into GWO to boost its performance of LGWO. In [37], new positions are determined by:

$$\vec{X}_{new}(t) = \begin{cases} 0.5 \times (\vec{X}_a - \vec{A}_1\vec{D}_\alpha + \vec{X}_\beta - \vec{A}_2\vec{D}_\beta) \\ \quad + \alpha \oplus Levy(\beta) & |A| > 0.5 \\ 0.5 \times (\vec{X}_a - \vec{A}_1\vec{D}_\alpha + \vec{X}_\beta - \vec{A}_2\vec{D}_\beta) & |A| < 0.5 \end{cases} \quad (18)$$

Based on the value of |A| in every iteration, the random value of A inside the [0, 1], if |A|>0.5, then the new position is to be set by Levy Flight step length, other with to be set with random values. Next, in [37] the author "referring to the greedy selection (GS) strategy from DE algorithm," new operator is formulated as::

$$\vec{X}(t+1) = \begin{cases} \vec{X}(t) & f(\vec{X}_{new}(t)) > f(\vec{X}(t)\ and\ r_{new} < P) \\ \vec{X}_{new}(t) & otherwise \end{cases} \quad (19)$$

The new position whether to be held or updated depends on the combination conditions: the next new position to be held with the last position under the condition of the fitness value of $f(X_{new}(t))$ great than $f(X(t))$ and the value of P also great than r_{new}(r_{new} and P are random values inside [0, 1]), other with to be set with the new position.

However, different real-world problems often have different constraints, so a suitable approach is demanded to deal with such problems. We designed the LFGWO with the aim of being as simple as possible with few parameters to be tuned and employed the LFGWO to solve as many as possible in both non-constrained and contained real-world optimization problems in engineering and other fields. Through an in-depth comprehensive study and more extensive comparison trial-and-error experiments, we embedded Levy flight into the GWO to execution position-updated by the following simple but effetely mechanisms. Initializes the position of agents in the search space by Levy flight as the below formula:

$X(:, i) = LFGWO - Levy(N)' * (high - Low) + Low$

At the initialization stage, all wolves (not only for the top three wolves) are assigned Levy flight values not random numbers between [0, 1] from the Gaussian distribution, which provides a large-scale deployment schema for the LFGWO and directly increases the wide diversity of the LFGWO. Secondly, randomization is more efficient as the step length is heavy-tailed, and any large step is possible, which effectively increases the probability of LFGWO's global search ability and precision.

$NewPosition = curretPosition * LFGWO - Levy(dim)'$.

By the above formula, without any doubt, on the one hand, the scheme of LFGWO has considerable differences LGWO of [37]. On the other hand, with such mechanisms, the LFGWO presents a harmony and proper balance between exploration and exploitation, an increment in the accuracy and the ability to avoid the convergence in local minima.

The LFGWO gets a highlight performance compared with LGWO of [37], LF-IGWO of [39] in terms of key values of Mean on 23 well-known benchmark functions, the following Tables 13 messages come from [37] and [39], and Table 1. The Table 14 messages come from LGWO of [37] and Tables 2. The comparing results of Tables 13 and 14 answers whether there is a significant difference between the LFGWO with LGWO in [37] and LF-IGWO in [39].

G. CONCLUSION

Summarizing the experimental results of three different numerical tests in this section: the LFGWO significantly outperforms other competing algorithms. There are two reasons why the LFGWO performs so well. Firstly, the Levy

TABLE 13. Mean cached by LFGWO, LGWO, and LF-IGWO on 23 benchmark functions.

Mean	F1	F2	F3	F4	F5	F6	F7	F8
LFGWO	-2.21E-88	4.25E-89	-4.41E-74	-8.06E-87	9.74E-01	-5.10E-01	3.21E-04	-3.03E+03
LGWO	3.17E-30	5.39E-19	8.12E-08	1.17E-08	8.35E+00	2.69E-04	3.02E-03	-3.37E+03
LF-IGWO	7.37E-61	7.72E-38	9.07E-15	5.56E-13	22.65	3.65E-6	6.61E-4	-1.02E+4
	F9	F10	F11	F12	F13	F14	F15	F16
LFGWO	-3.75E-09	-5.90E-18	-2.36E-08	-9.73E-01	1.00E+00	-3.20E+01	1.40E-01	-7.13E-01
LGWO	9.46E-02	2.12E-15	2.42E-05	7.12E-04	3.94E-07	1.15E+00	2.53E-06	-1.03E+00
LF-IGWO	7.96E-0	7.99E-15	0	1.96E-7	8.98E-6	9.98E-1	3.07E-4	-1.03E+0
	F17	F18	F19	F20	F21	F22	F23	
LFGWO	1.23E+01	-1.00E+00	8.54E-01	6.56E-01	1.00E+00	9.95E-01	1.00E+00	
LGWO	3.98E-01	3.00E+00	-3.86E+00	-3.32E+00	-1.02E+01	1.04E+01	-1.05E+01	
LF-IGWO	3.10E-1	3.00E-0	-3.86E+0	-3.32E-0	-1.02E+1	-1.04E+1	-1.05E+1	

TABLE 14. Std cached by LFGWO and LGWO on 23 well-known benchmark functions.

Std	F1	F2	F3	F4	F5	F6	F7	F8
LFGWO	3.42E-90	1.85E-90	6.86E-76	4.56E-88	1.39E-02	7.62E-03	1.50E-05	4.28E+01
LGWO	4.07E-20	1.07E-02	2.05E+00	1.32E+00	5.34E+00	2.30E-05	1.10E-03	2.96E+02
	F9	F10	F11	F12	F13	F14	F15	F16
LFGWO	5.28E-11	2.67E-19	3.38E-10	1.34E-02	1.28E-02	4.53E-01	1.59E-03	1.02E-02
LGWO	2.16E+01	4.30E-02	8.39E-05	3.00E-03	2.10E-04	2.91E+00	3.95E-04	3.20E-12
	F17	F18	F19	F20	F21	F22	F23	
LFGWO	1.74E-01	1.28E-02	1.03E-02	9.82E-03	1.33E-02	1.20E-02	1.45E-02	
LGWO	0.00E+00	2.40E-02	1.47E-05	3.86E-02	5.14E+00	2.20E+00	2.01E+00	

flight is utilized to generate the initial candidate solution to increase the diversity of the population and lay the foundation for the LFGWO's global search. Meanwhile, the initial position of the candidate solutions is random of Levy flight, ergodic, avoid stagnant local optima and find the global optimal, thereby avoiding premature convergence effectively to a certain extent. Secondly, the LFGWO integrates with Levy flight not only for the top three wolves but also for all wolves over the course of iterations, so as to improve the diversity of the population and seek the optimal solution.

V. APPLICABILITY OF LFGWO FOR SOLVING NON-LINE-CONSTRAINED OPTIMIZATION PROBLEMS

Most real-life constrained optimization problems are complex and non-line, sometimes even having many local optimal due to the constraints of the problem. Solving these problems has always been a challenge for researchers and practitioners. To see how the effectiveness and the performance of LFGWO, we investigated the infinite impulse response (IIR) challenging model identification and five challenging non-line constrained optimization problems, and selected well-known standards and modified optimizers proposed by other researchers in prior work for comparison.

A. I-BEAM DESIGN

The I-shape beam is a famous problem that many researchers and practitioners use to test the capabilities of metaheuristic algorithms. The purpose of this problem is to minimize the vertical deflection of the I-beam by optimizing the dimensions of four unknown variables including the flange's width b ($=x_1$), the height of section h ($=x_2$), the thicknesses of the beam web $t_w(x_3)$, and the flange's thickness $t_f(x_4)$ of the beam. The I-beam design is subject to the load's cross-sectional area and stress constraints under the applied loads. As shown in Figure. 6, minimize the vertical deflection $f(x)(= \frac{PL^3}{48EI})$when the length of the beam (L) and modulus of elasticity (E) are respectively 5,200cm and 523,104 kN/cm^2. This problem with the variable vector $x=(b, h, t_w, t_f)=(x_1, x_2, x_3, x_4)$, then the problem can be written as following.

Minimize:

$$f(x) = \frac{500}{\left(\frac{x_3(x_2-2x_4)^3}{12} + \frac{x_1x_4^3}{6} + 2x_1x_4(x_2-x_4)^2\right)}$$

Subject to cross-section area less than 300 cm^2

$$g_1(x) = 2x_1x_3 + x_3(x_2 - 2x_4) \leq 300$$

If the allowable bending stress of the beam is 56 kN/cm^2, the stress constraint is as follows:

$$\begin{aligned} g_2(x) = &(18x_2 \times 10^4)/(x_3(x_2-2x_4)^3 \\ &+ 2x_1x_3(4x_4^2 + 3x_2(x_2-2x_4))) \\ &+ (15x_1 \times 10^3)/((x_2-2x_4)x_3^2 + 2x_3x_1^3) \leq 6 \end{aligned}$$

where the initial design spaces are:

$10 \leq x_1 \leq 50$,
$10 \leq x_2 \leq 80$,
$0.9 \leq x_3 \leq 5$,
$0.9 \leq x_4 \leq 5$.

This nonlinearly constrained problem has been solved with other optimization methods such as Special Relativity Search (SRS) [58], Ranked-based mechanism-assisted Biogeography optimization (RBBO) [59], Atomic Orbital Search Algorithm (AOS) [60], Dwarf Mongoose Optimization Algorithm (DMO) [49], efficient hybridized CS-PSO algorithm (ECS-AGQPSO) [61], Cuckoo search algorithm (CS) [62], and Cooperation search algorithm (CSA) [63].

TABLE 15. Statistical and Comparative results for I-beam design.

Algorithms	x1	x2	x3	x4	Best	Mean	Std
LFGWO	50.0000	80.0000	1.7647	5.0000	0.0066	0.0066	0.00E+00
SRS	50.0000	79.9800	0.9000	2.3211	0.0131	0.0131	3.37E-05
RBBO	50.0000	80.0000	0.9000	2.3218	0.0131	0.0131	8.10E-08
AOS	50.0000	80.0000	0.9000	2.3218	0.0131	0.0138	1.56E-04
DMO	50.0000	80.0000	0.9000	2.3218	0.0131	0.0131	5.95E-13
ECS-AGQPSO	50.0000	80.0000	1.7647	5.0000	0.0066	0.0066	1.76E-18
CS	50.0000	80.0000	0.9000	2.3217	0.0131	0.0132	1.35E-04
CSA	50.0000	80.0000	0.9000	2.3218	0.0131	0.0131	9.37E-05

The corresponding statistical results of these methods and LFGWO are listed in Table 15. From the data in Table 15, it can be concluded that LFGWO can touch the optimal function value (with 6.6260E-03). And LFGWO has the lower standard deviations for this problem. This symbolizes that LFGWO has highly competitive performance in terms of stability and solution accuracy on the engineering problem. It can be found that the LFGWO is superior to the other methods, demonstrating its engineering practicability.

Figure.7 shows the convergence performance of LFGWO to an ideal spot, which demonstrates that the convergence rate of LFGWO is better. The results of the experiments reveal that LFGWO has a more effective searching ability and can locate and develop the target solution space in a small number of iterations.

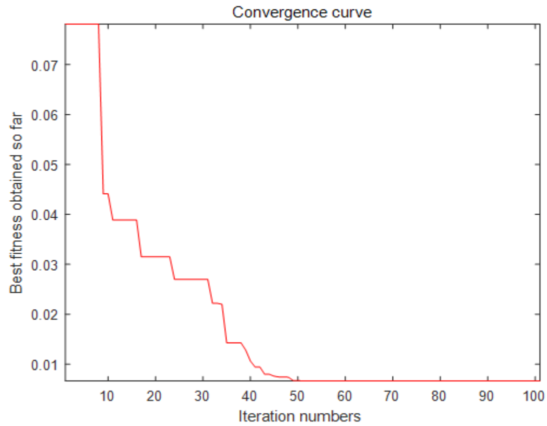


FIGURE 7. The convergence graph of the LFGWO.

B. HEAT EXCHANGER DESIGN

The primary aim of this design is to obtain a high heat transfer rate without exceeding the allowable pressure drop. Heat Exchanger Design is a benchmark minimization problem that is regarded as a difficult problem since all the constraints are binding.

The heat exchanger design deals with the optimal configuration by minimizing a linear objective function subjected to eight design variables and six inequality constraints (three linear and three non-linear). All six constraints are active at this solution. Mathematical model is defining the problem objectively can be formalized by the following expression.

Minimize:

$$f(x) = x_1 + x_2 + x_3$$

Subject to:

$$g_1(x) = 0.0025(x_4 + x_6) - 1 \leq 0,$$
$$g_2(x) = 0.0025(x_5 + x_7 - x_4) - 1 \leq 0,$$
$$g_3(x) = 0.01(x_8 - x_5) - 1 \leq 0,$$
$$g_4(x) = 100x_1 + 833.33252x_4 - x_1x_6 - 83333.333 \leq 0,$$
$$g_5(x) = 1250(x_5 - x_4) + x_2x_4 - x_2x_7 \leq 0,$$
$$g_6(x) = 1250000 + x_3x_5 - 2500x_5 - x_3x_8 \leq 0,$$

where:

$$100 \leq x_1 \leq 10000,$$
$$1000 \leq x_2 \leq 10000,$$
$$1000 \leq x_3 \leq 10000,$$
$$10 \leq x_4 \leq 1000,$$
$$10 \leq x_5 \leq 1000,$$
$$10 \leq x_6 \leq 1000,$$
$$10 \leq x_7 \leq 1000,$$
$$10 \leq x_8 \leq 1000.$$

Many algorithms have been designed to solve the heat exchanger design, including non-equidistant grey prediction evolution algorithm (NeGPE-s) [64], Local search enhanced Aquila optimization algorithm ameliorated (DAQUILA) [65], and Bat algorithm (BA) [66] reported in the literatures. Statistical results of LFGWO and other three algorithms for heat exchanger design are portrayed in Table 16, and the solutions found by the four algorithms were somewhat close to each other on the optimal solution. As shown in the Figure 8, the convergence curve of the LFGWO is quick and the solutions were obtained instantly under satisfy all constraints.

TABLE 16. Statistical results of the heat exchanger example by different models.

Algorithms	LFGWO	NeGPE-s	DAQUILA	BA
x1	177.8034	654.7474	586.0593	579.30675
x2	1760.6847	1209.9738	1486.1896	1,359.97
x3	5117.7249	5193.7585	4988.3410	5109.9705
x4	127.7174	187.9981	182.4204	182.0177
x5	288.9599	292.2741	300.5180	295.60118
x6	226.0100	211.9998	217.2134	217.9823
x7	238.7575	295.7236	281.8297	286.41653
x8	388.9599	392.2623	400.4953	395.60118
Best	7056.2130	7049.2370	7060.5899	7049.2480
Mean	7068.4668	7134.5081	7198.1442	7049.2484
Std	8.9200	166.4778	98.3457	0.0052

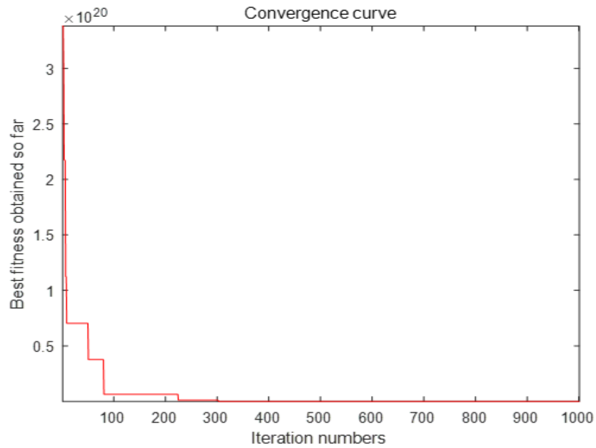


FIGURE 8. Convergence history for heat exchanger design.

C. ROLLING ELEMENT BEARINGS DESIGN

Rolling element bearings is a crucial component of any rotating machinery and has a wide variety of applications. Rolling bearings functions appear to be simple, however, their internal geometry is quite complex. Rolling element bearings design issue is considered one of the most sophisticated multidisciplinary engineering optimizations and is widely employed to prove the capability of the meta-heuristics algorithm. The primary goal of the rolling element bearings design is to increase and optimize the dynamic nature of the load-carrying capacity of rolling element bearings. The schematic diagram of structural optimization design is shown in Figure. 9.

This problem is formulated by five variables and five parameters. Nine non-linear constraints are imposed based on kinematic and manufacturing considerations. These variables are four continuous variables: ball diameter (D_b), pitch diameter (D_m), inner and outer raceway curvature coefficients (f_o and f_i), and one discrete variable: the total number of balls (Z). The parameters are $\varepsilon, e, \zeta, K_{D_{max}}, and\ K_{D_{min}}$ appeared

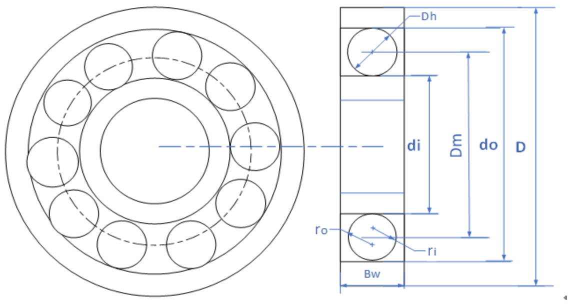


FIGURE 9. Schematic view of rolling element bearings design.

only in the constraints and indirectly affect the internal geometry and the optimum range. The mathematical model of rolling element bearings design problem is as following.

Suppose:

$$x = [x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10}]$$
$$= [\varepsilon\ e\ \zeta\ D_b\ D_m\ f_o\ f_i\ K_{D_{max}}\ K_{D_{min}}\ Z].$$

Maximize:

$$f(x) = \begin{cases} f_c x_{10}^{\frac{2}{3}} x_4^{1.8} & \text{if } D \le 25.4 \\ 3.647 f_c x_{10}^{\frac{2}{3}} x_4^{1.4} & \text{if } D > 25.4 \end{cases}$$

Subject to:

$$\begin{cases} g_1(x) = x_{10} - \dfrac{\emptyset_0}{2sin^{-1}(\frac{x_4}{x_5})} - 1 \le 0 \\ g_2(x) = x_9(D-d) - 2x_4 \le 0 \\ g_3(x) = 2x_4 - x_8(D-d) \le 0 \\ g_4(x) = x_3 B_w - x_4 \le 0 \\ g_5(x) = 0.5(D+d) - x_5 \le 0 \\ g_6(x) = (0.5+x_2)(D+d) - x_5 < 0 \\ g_7(x) = x_1 x_4 - 0.5(D - x_4 - x_5) \le 0 \\ g_8(x) = 0.515 - x_7 \le 0 \\ g_9(x) = 0.515 - x_6 \le 0 \end{cases}$$

Where:

$$f_c = 37.91\{1 + \{1.04(\tfrac{1-\gamma}{1+\gamma})^{1.72}(\tfrac{x_7(2x_6-1)}{x_6(2x_7-1)})^{0.41}\}^{10/3}\}^{-0.3}$$
$$\times[\tfrac{\gamma^{0.3}(1-\gamma)^{1.39}}{(1+\gamma)^{1/3}}][\tfrac{2f_i}{2f_i-1}]^{0.41}$$

$$\emptyset = 2\pi - 2 \times cos^{-1}(\frac{[\left\{\frac{D-d}{2}-3(\frac{T}{4})\right\}^2+\left\{\frac{D}{2}-\frac{T}{4}-x_4\right\}^2-\left\{\frac{d}{2}+\frac{T}{4}\right\}^2]}{2\left\{\frac{D-d}{2}-3(\frac{T}{4})\right\}\left\{\frac{D}{2}-\frac{T}{4}-x_4\right\}})$$

$\gamma = \frac{x_4}{x_5}, f_i = \frac{r_i}{D_b}, f_o = \frac{r_o}{D_b}, T = D - d - 2D_b$
$D = 160, d = 90, B_w = 30, r_i = r_o = 11.033$
$0.5(D+d) \le D_m \le 0.6(D+d),$
$0.15(D-d) \le D_b \le 0.45(D-d),$
$4 \le Z \le 50,$
$0.515 \le f_i,$
$f_o \le 0.6,$
$0.4 \le K_{D_{min}} \le 0.5,$
$0.6 \le K_{D_{max}} \le 0.7,$
$0.3 \le \varepsilon \le 0.4,$
$0.02 \le e \le 0.1,$
$0.6 \le \zeta \le 0.85.$

TABLE 17. Comparison results for the Rolling element bearings design.

ALGO	LFGWO	WCA	IDSE	TLBO	MRFO	AOS	DTCSMO	SSA
x1	3.97E-01	3.00E-01	3.00E-01	3.00E-01	3.00E-01	3.00E-01	3.11E-01	3.00E-01
x2	8.91E-02	4.00E-02	1.00E-01	6.89E-02	5.37E-02	2.00E-02	5.54E-02	2.00E-02
x3	6.00E-01	6.00E-01	6.00E-01	7.99E-01	6.93E-01	6.18E-01	6.00E-01	6.00E-01
x4	1.80E+01	2.14E+01	1.83E+01	2.14E+01	2.14E+01	2.19E+01	1.80E+01	2.14E+01
x5	1.25E+02	1.26E+02	1.25E+02	1.26E+02	1.26E+02	1.25E+02	1.31E+02	1.25E+02
x6	6.00E-01	5.15E-01	6.00E-01	5.15E-01	5.15E-01	5.15E-01	6.00E-01	5.15E-01
x7	5.15E-01	5.15E-01	6.00E-01	5.15E-01	5.15E-01	5.15E-01	5.95E-01	5.15E-01
x8	6.96E-01	6.59E-01	7.00E-01	6.34E-01	6.91E-01	6.58E-01	6.46E-01	7.00E-01
x9	4.03E-01	4.02E-01	4.00E-01	4.24E-01	4.05E-01	4.76E-01	4.48E-01	4.00E-01
x10	4.48E+01	1.10E+01	4.51E+00	1.10E+01	1.10E+01	1.08E+01	5.03E+00	1.09E+01
Best	3.79E+04	8.55E+04	1.84E+04	8.19E+04	8.55E+04	8.39E+04	1.70E+04	8.51E+04
Mean	2.76E+04	8.38E+04	2.38E+04	8.14E+04	2.99E+03	8.22E+04	1.70E+04	8.50E+04
Std	5.59E+03	4.88E+02	7.20E+03		1.46E-02	2.34E+01	2.24E+02	1.88E+03

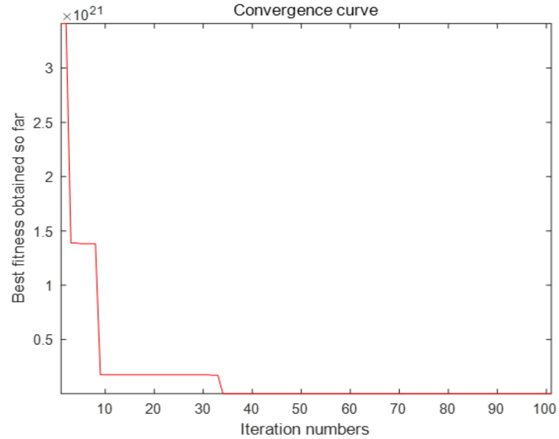


FIGURE 10. Convergence plot on the Rolling Element Bearings Design.

with bounds:
$x_1 \in [0.3, 0.4],$
$x_2 \in [0.02, 0.1],$
$x_3 \in [0.6, 0.85],$
$x_4 \in [0.15(D-d), 0.45(D-d)],$
$x_5 \in [0.5(D+d), 0.6(D+d)],$
$x_6 \in [0.515, 0.6],$
$x_7 \in [0.515, 0.6],$
$x_8 \in [0.6, 0.7],$
$x_9 \in [0.4, 0.5]$, and
$x_{10} \in [4, 50]$

Many algorithms have been designed to solve the rolling element bearings design problem, including Water cycle algorithm (WCA) [67], Information-decision searching algorithm (IDSE) [68], Teaching-learning-based optimization (TLBO) [69], Manta ray foraging optimization (MRFO) [2], Atomic Orbital Search Algorithm (AOS) [60], An efficient hybrid starling murmuration optimizer (DTCSMO) [70], and Spring Search Algorithm (SSA) [71] appeared in the literatures. In terms of statistical results in Table 17, the LFGWO offered better results (the maximum dynamic load carrying capacity of rolling element bearings) compared with the other algorithms, which suggests the significant advantages of LFGWO in optimizing the bearing capacity. It could be clearly seen from Figure 10 that the fast convergence rate of the LFGWO with increasing iterations to gain the optional solution.

In summary, these results show that the LFGWO is an effective optimizer for solving large-scale constrained

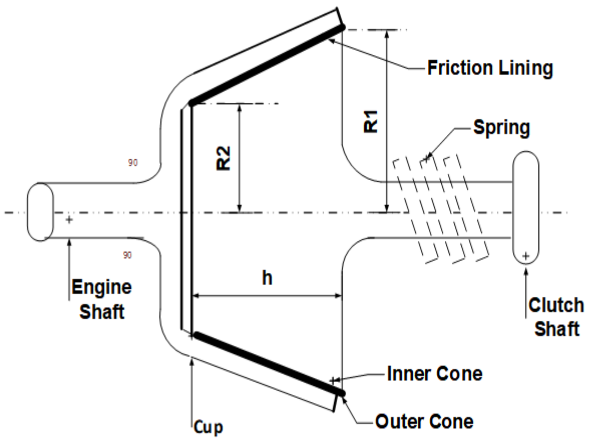


FIGURE 11. The schematic illustration of the cone clutch coupling.

engineering design problems with low computational cost and fast convergence speed.

D. CONE CLUTCH DESIGN

Cone clutch is the type of friction clutch in which the two conical shape components are used for engagement and disengagement for revolving at different speeds. It consists of Inner Cone, Outer Cone, Spring. The outer cone has friction lining on its inner conical surface and inner cone has friction lining on its outer conical surface. The outer cone is connected to the engine shaft and inner cone is connected to clutch shaft. The Inner cone is connected to clutch pedal through a linkage. Cone clutch works on the principle of friction. Friction between these two cones is cause for the power transmission from the flywheel to the gearbox. In the normal position of clutch, when a vehicle is running, the inner cone is pressed inside the outer cone. Therefore, due to the friction occurs between them, power is transmitted from the engine shaft to the clutch shaft, hence the clutch is in engaging position. The schematic illustration of the cone clutch coupling is shown in Figure 11.

In order to enable the transfer of momentum, the cone clutch must be designed to minimize the volume coupling and be subjected to two constraints. Problem variables are: inner radius of the coupling $R_1(= x_1)$, and outer radius of the coupling $R_2(= x_2)$. Goal function to be minimized is defined as:

$$f(x) = x_1^3 - x_2^3$$

Whilst the conditions to be met are:

$$g_1(x) = 2 - \frac{x_1}{x_2} \leq 0$$

$$g_2(x) = 5 - \frac{x_1^2 + x_1 x_2 + x_2^2}{x_1 + x_2} \leq 0$$

While problem variables' lower and upper bounds are:

$$1 \leq x_1, x_2 \leq 10.$$

A set of the comparison results listed in the literature [72] presented in Table 18. Analysing the results, a conclusion has

TABLE 18. Comparison of results for the design of cone clutch.

Algorithms	x1	x2	Best	Mean	Std
LFGWO	4.2857	2.1429	68.8776	68.8776	0.0000
PSO	4.2786	2.1327	68.5698	68.5698	0.0000
EHBMO	4.3250	2.1343	70.9812	71.1808	0.1025
HAS	4.3035	2.1469	69.7240	69.8060	0.1156
CSA	4.2871	2.1344	69.0700	69.0700	0.0000
CSSA	4.2962	2.1439	68.8872	69.4432	0.1615
BBCO	4.2984	2.1468	69.5242	69.5242	0.0000
GBA	4.3064	2.1437	69.9519	70.0093	0.1215
KHA	4.2961	2.1435	69.0609	69.4392	0.1424
PSA	4.2998	2.1489	69.5728	69.5728	0.0000
SSA	4.2976	2.1434	69.0154	69.5289	0.1800

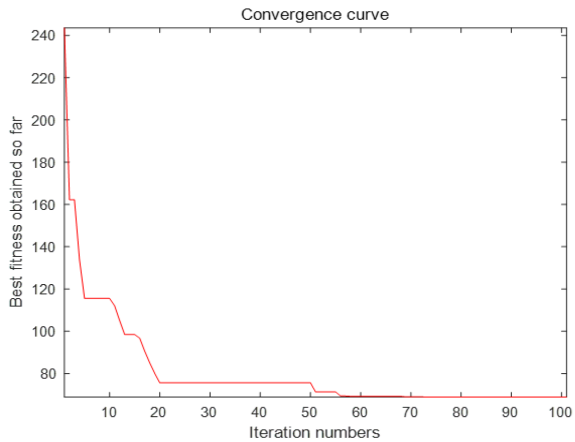


FIGURE 12. The convergence graph of cone clutch.

been drawn that the LFGWO gives better results in comparison to the other algorithms, while in comparison to LFGWO, PSO, and CSSA, the results are nearly the same. Table 18 says that the LFGWO has obtained optimal or near-optimal solutions in the case of given engineering problems. The optimal solution given by LFGWO is very close to the actual optimal solution, particularly for the variables x1 and x2. This indicates that the LFGWO has the potential for solving this class of optimization problems.

Moreover, the LFGWO fast converges towards the near-optimal solution from initial iterations can be visualized from the convergence graph of Figure.12.

E. CORRUGATED BULK DESIGN

The corrugated bulk design is used in tankers, bulkhead carriers, and product oil carriers. This optimization problem aims to minimize the weight of the tanker's corrugated bulkhead and the schematic illustration is shown in Figure 13. The design variables of this problem are the width $b(= x_1)$, depth $h(= x_2)$, length $l(= x_3)$and plate thickness $t(= x_4)$.

The mathematical model of this optimization problem is given as follows:

Minimize:

$$(x) = \frac{5.885x_4(x_1 + x_3)}{x_1 + \sqrt{(x_3^2 - x_2^2)}}$$

Subject to:

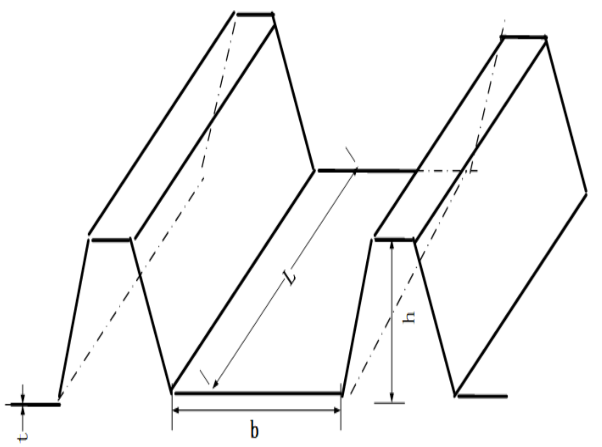


FIGURE 13. The schematic illustration of the corrugated bulk.

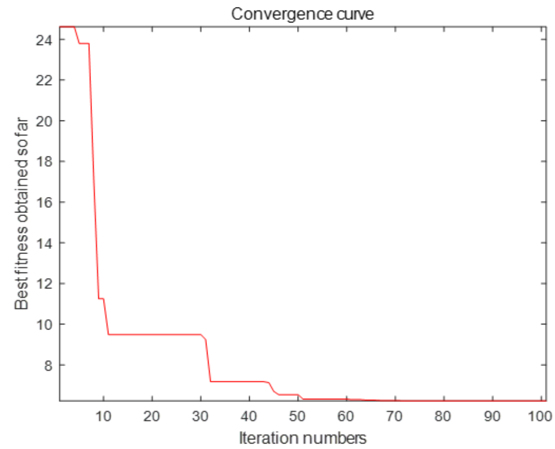


FIGURE 14. Convergence history for corrugated bulk design.

TABLE 19. Comparative results for corrugated bulk design.

Algorithms	x1	x2	x3	x4	Best	Mean	Std
LFGWO	40.1454	33.5305	39.7210	0.7763	5.9384	6.1397	0.2013
DMO	48.3119	54.7827	61.9298	0.4249	4.6972	4.6693	4.3569
CS	37.1179	33.0350	37.1939	0.7306	5.8943	5.9883	0.0644
AEFA-C	57.6928	34.1330	57.5529	1.0501	6.8458	6.8468	0.0000

$$g_1(x) = 8.94(x_1 + \sqrt{(x_3^2 - x_2^2)}) - x_2x_4(0.4x_1 + \frac{x_3}{6}) \leq 0$$
$$g_2(x) = 2.2(8.94(x_1+\sqrt{(x_3^2-x_2^2)}))^{\frac{4}{3}} - x_2^2x_4(0.2x_1+\frac{x_3}{12}) \leq 0$$
$$g_3(x) = 0.15 + 0.0156x_1 - x_4 \leq 0$$
$$g_4(x) = 0.15 + 0.0156x_3 - x_4 \leq 0$$
$$g_5(x) = 0.15 - x_4 \leq 0$$
$$g_6(x) = x_2 - x_3 \leq 0$$

Variable range:

$$0 \leq x_1, x_2, x_3 \leq 100,$$
$$0 \leq x_4 \leq 5.$$

This case was solved by a number of works in the literature such as Dwarf Mongoose Optimization Algorithm (DMO) [49], Cuckoo search algorithm (CS) [62], and Artificial electric field algorithm (AEFA-C) [73] selected from literature. This engineering case is more difficult because of so many variables and constraints, as well as a low rate of the feasible solution space to the entire search space. The statistical results of Table 19 reveal that the LFGWO and the other algorithms have nearly similar performance for this optimization problem and the LFGWO achieve the near-optimal solution. The best value of the LFGWO is only slightly inferior to the best optimal value of 4.6972 for this problem. From the figure 14, it is clearly evident that the fast convergence ability of the LFGWO towards the global minima in the later iterations.

F. IIR MODEL IDENTIFICATION BY THE LFGWO

Adaptive IIR filtering is an active area of research for many years and has been used for applications in signal processing, control systems, image processing, and communication. Since physical systems usually have infinite-impulse response (IIR) dynamics, IIR models are widely used for system identification. The infinite impulse response (IIR) models which are preferred over their equivalent finite impulse response (FIR) models since they represent more accurate real-world applications. Nevertheless, system identification is a difficult optimization problem, especially, the IIR models tend to generate multimodal error surfaces which are significantly difficult to optimize [74]. On the other hand, the adaptive LFGWO with the ability to explore complex search spaces for suitable solutions. In this section the IIR system identification task is formulated as an optimization problem and the adaptive LFGWO is introduced for IIR system identification.

TABLE 20. Best estimated values.

ALGO	a	b	Avg	Std
LFGWO	0.9105	-0.2372	9.5000E-03	5.8407E-04
PSO	0.9125	-0.3012	2.8400E-02	1.0500E-02
ABC	0.1420	-0.3525	1.9700E-02	1.5000E-03
EM	0.9034	0.3030	1.6500E-02	1.2000E-03
CS	0.9173	-0.2382	1.0100E-02	3.1180E-04
FPA	0.9364	-0.2001	1.0500E-02	5.1030E-04

TABLE 21. Best estimated values.

ALGO	a1	a2	b	Avg	Std
LFGWO	-1.4000	0.4900	1.0000	0.0000	7.9008E-30
PSO	-1.4024	0.4925	0.9706	4.0035E-05	1.3970E-05
ABC	-1.2138	0.6850	0.2736	3.5840E-01	1.9870E-01
EM	-1.0301	0.4802	1.0091	3.9648E-05	8.7077E-05
CS	-1.400	0.4900	1.0000	0.0000E+00	0.0000E+00
FPA	-1.400	0.4900	1.0000	4.6246E-32	2.7360E-31

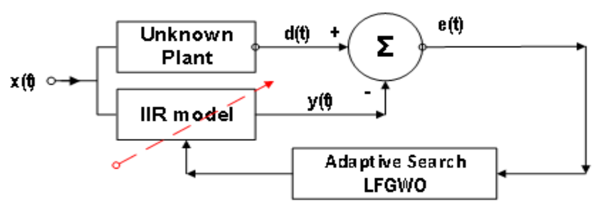


FIGURE 15. Adaptive IIR model using LFGWO for system identification.

TABLE 22. Best estimated values.

ALGO	a1	a2	a3	a4	b0	b1	b2	b3	b4	Avg	Std
LFGWO	-0.0458	-0.3523	0.0526	-0.4615	0.9867	-0.0459	-0.0035	0.0331	-0.1426	0.0019	1.73E-03
FPA	0.0685	-0.2483	-0.0679	-0.6365	0.8338	0.0829	0.3782	0.0650	-0.5203	0.0279	9.20E-03
GSA	-0.2429	0.2219	0.0374	0.0756	0.4117	0.0083	0.1369	-0.0752	0.4704	0.0577	1.65E-02
BA	-0.0937	0.7387	-0.1249	0.2936	0.8062	-0.3099	0.8258	-0.6063	0.5713	0.2684	1.74E-01
ABC	0.5472	0.1691	0.8251	0.2746	-1.1319	0.7325	0.9195	-0.8865	0.0573	0.0464	1.92E-02
MFO	0.0099	-0.0539	-0.0105	-0.8122	0.9986	0.0072	0.2959	0.0024	-0.3606	0.0321	1.35E-02
PSOGSA	-0.1537	-0.1086	0.1114	-0.7520	0.8669	-0.1706	0.2597	0.0288	-0.2067	0.0251	2.58E-02
WOA	0.0144	-0.9032	0.0319	0.1160	0.8809	-0.0660	-0.5753	0.1036	0.2871	0.0303	2.09E-02
LWOA	0.1354	0.0080	0.0469	-0.4933	0.8194	0.2119	0.2902	0.1299	0.0714	0.0211	9.06E-03

In the real world, many optimization problems can be considered as black box challenges. Basic block diagram for system identification configuration using IIR model is shown in Fig. 15, the unknown plant of transfer function is to be identified with the IIR model in such a way so that the outputs from both the systems match closely for the same given input x(t). The main task of the system identification in this work is iteratively using the adaptive LFGWO to search for the adaptive parameters of IIR model until its input/output relationship matches closely to that of the unknown plant.

A recursive IIR system is described by the z transform of the impulse response of the system, the input-output relationship is governed by the following transfer function:

$$G(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}} \quad (20)$$

In (20), Y(z) and X(z) are the system output and system input in the z domain, respectively. Also m and n are the number of numerator and denominator coefficients of the transfer function, respectively, and $n \geq m$ represents the order of the filter. and are the pole and zero parameters of the IIR model (i=1, . . ., n, j=1, . . ., m), respectively. The differential equation of the above relation can be described as an output error adaptive IIR filter structure and the recursive difference equation given in (21).

$$y(t) = \sum_{i=1}^{n} a^i y(t-1) + \sum_{j=0}^{m} b^j x(t-j) \quad (21)$$

Therefore, the set of unknown parameters that models the IIR system is represented by $\theta = a_1, \ldots, a_n, b_0, \ldots, b_m$. θ is the unknown constant vector that should be adjusted by LFGWO adaptive algorithm. Considering that the number of unknown parameters of θ is (n+m+1), the search space S of feasible values for θ is $\Re^{(n+m+1)}$.

The output difference between the actual system and its model yields the error as shown in (22)

$$e(t) = d(t) - y(t) \quad (22)$$

According to the block diagram of Figure 15, the output of the plant is d(t): whereas the output of the IIR filter is y(t). In the system identification mean square error (MSE) of time samples is considered as the objective function, also known as error fitness function and expressed in (23) [75].

$$MSE = f(\theta) = \frac{1}{W} \sum_{t=1}^{W} (d(t) - y(t))^2 \quad (23)$$

In IIR model identification based on output error method, the W is the number of samples used in the simulation.

The main objective of IIR model identification is to minimize the value of the error fitness MSE with proper adjustment of coefficient vector θ of the transfer function (20) of the adaptive filter iteratively, so that output responses of the filter and the unknown plant match closely and hence the error is minimized as follows.

$$\theta^* = \underset{\theta \in S}{argmin}(f(\theta)) \quad (24)$$

In the comparison study, a comprehensive set of experiments has been used to test the performance of the LFGWO. In our experimental work, three widely used challenge IIR identification cases are carefully selected with same and reduced order to assure compatibility between similar works reported in the literature.

1) A PLANT WITH A SECOND-ORDER SYSTEM AND A FIRST-ORDER IIR MODEL

In this experiment, the test case is taken from [76]. The adaptive LFGWO is applied to identify a second-order plant through a first Order IIR model. Under such context, the unknown plant of transfer function is to be identified with the following transfer function.

$$H_P(z^{-1}) = \frac{0.05 - 0.4z^{-1}}{1 - 1.1314z^{-1} + 0.25z^{-2}} \quad (25)$$

And the model to be generated is a reduced order IIR model H_M that hold the transfer functions (26). Since a reduced order model is employed to identify a plant of a superior order, the $f(\theta)$ is not unimodal but multimodal. In the simulations, it has been considered a uniform white sequence of 100 samples ($W = 100$) for the input $x(t)$.

$$H_M(z^{-1}) = \frac{b}{1 - az^{-1}} \quad (26)$$

Owing to the random nature of the LFGWO, in order to eliminate the influence of randomness in the experiment, 30 independent trials/runs with randomly chosen initial positions in [-1.2, 1.2] (a bounded search space for better stability) are performed for each case. The results of the LFGWO to solve IIR system identification and compare to other algorithms are reported in Table 20, which reports the best parameter values (a, b), and both the average minimum

TABLE 23. Twenty-three mathematical benchmark functions.

Function	Dimensions(N, T, dim)	Range	F_{best}
Unimodal Test Functions			
F1:Sphere Function $F_1(x)=\sum_{i=1}^{dim} x_i^2$	100,100,30	[-100, 100]	0
F2:Schwefel's Problem 2.22 $F_2(x)=\sum_{i=1}^{dim}\lvert x_i\rvert+\prod_{i=1}^{dim}\lvert x_i\rvert$	100,100,30	[-10, 10]	0
F3:Shifted Schwefel's Problem 1.2 $F_3(x)=\sum_{i=1}^{dim}\sum_{j=1}^{i} x_j^2$	100,100,30	[-100, 100]	0
F4:Schwefel's Problem 2.21 $F_4(x)=max_i\lvert x_i\rvert, 1\le i\le dim$	100,100,30	[-100, 100]	0
F5:Generalized Rosenbrock's Function $F_5(x)=\sum_{i=1}^{dim-1}[100(x_{i-1}-x_i^2)+(x_i-1)^2$	100,100,30	[-30, 30]	0
F6:Step Function $F_6(x)=\sum_{i=1}^{dim}(\lvert x_i+0.5\rvert)^2$	100,100,30	[-100, 100]	0
F7:Quartic Function i.e. Noise $F_7(x)=\sum_{i=1}^{dim} iX_i{}^4+random[0,1]$	100,100,30	[-1.28, 1.28]	0
Multimodal Test Functions			
F8:Generalized Schwefel's Problem 2.26 $F_8(x)=\sum_{i=1}^{dim}\lvert x_i\rvert-x_i\sin(\sqrt{\lvert x_i\rvert})$	100,100,30	[-500, 500]	-418.9829*dim
F9:Generalized Rastrigin's Function $F_9(x)=\sum_{i=1}^{dim}[x_i^2-10\cos(2\pi x_i)+10]$	100,100,30	[-5.12, 5.12]	0
F10:Ackley's Function $F_{10}(x)=-20\exp\left(-0.2\sqrt{\frac{1}{dim}\sum_{i=1}^{dim}x_i^2}\right)-\exp\left(\frac{1}{dim}\sum_{i=1}^{dim}cos(2\pi x_i)\right)+20+e$	100,100,30	[-32, 32]	0
F11:Generalized Griewank's Function $F_{11}(x)=\frac{1}{4000}\sum_{i=1}^{dim}x_i^2-\prod_{i=1}^{dim}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	100,100,30	[-600, 600]	0
F12:Generalized Penalized Function 1 $F_{12}(x)=\frac{\pi}{dim}10sin^2(\pi y_1)+\sum_{i=1}^{n-1}(y_i-1)^2[1+\sin^2(\pi y_{i+1})]+(y_{dim}-1)^2+\sum_{i=1}^{n}Ufun(x_i,10,100,4)$	100,100,30	[-50, 50]	0
F13:Generalized Penalized Function 2 $F_{13}(x)=0.1\left\{\sin^2(3\pi x_1)+\sum_{i=1}^{dim}(x_i-1)^2[1+\sin^2(3\pi x_{i+1})]+(x_{dim}-1)^2[1+\sin^2(2\pi x_{dim})]+\sum_{i=1}^{dim}Ufun(x_i,5,100,4)\right\}$ where $Ufun(x_i,5,100,4)=\begin{cases}k(x_i-a)^m & x_i>a\\ 0-a & -a<x_i<a\\ k(-x_i-a)^m & x_i<-a\end{cases}$ $y_i=1+\frac{x_i+1}{4}$	100,100,30	[-50, 50]	0
Fixed-dimension multimodal Test Functions			
F14:Shekel's Foxholes Function $F_{14}(x_1,x_2)=(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6})^{-1}$ $a_{1j}=\{-32;-16;0;16;32;-32;-16;0;16;32;-32;-16;0;16;32;-32;-16;0;16;32;-32;-16;0;16;32\}$ $a_{2j}=\{-32;-32;-32;-32;-32;-16;-16;-16;-16;-16;0;0;0;0;0;16;16;16;16;16;32;32;32;32;32\}$	100,100,2	[-65.5360, 65.5360]	0.9980
F15:Kowalik's Function $F_{15}(x)=\sum_{k=1}^{11}[a_k-\frac{x_1(b_k^2+b_kx_2)}{b_k^2+b_kx_3+x_4}]^2$ i a_i b_i^{-1} 1 0.1957 0.25 2 0.1947 0.5 3 0.1735 1 4 0.16 2 5 0.0844 4 6 0.0627 6 7 0.0456 8 8 0.0342 10 9 0.0323 12 10 0.0235 14 11 0.0246 16	100,100,4	[-5, 5]	0.0003075
F16:Six-Hump Camel-Back Function $F_{16}(x_1,x_2)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	100,100,2	[-5, 5]	-1.0316285
F17:Branin Function $F_{17}(x_1,x_2)=(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6)^2+10\left(1-\frac{1}{8\pi}\right)cosx_1+10$	100,100,2	[-5, 0, 10, 15]	0.397887
F18:Goldstein-Price Functionn $F_{18}(x_1,x_2)=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)]\times[30+(2x_1-3x_2)^2-3x_2]^2\times(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	100,100,2	[-5, 5]	3
F19 and F20:Hartman's Family			
F19: Hartman's Family $F_{19}(x)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2)$	100,100,3	[0,1]	-3.863
F20: Hartman's Family $F_{20}(x)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2)$ Where, in this exercise: i a_{ij} c_i p_{ij} 1 3.0 10.0 30.0 1.0 0.689 0.1170 0.2673 2 0.1 10.0 35.0 1.2 0.4699 0.4387 0.7470 3 3.0 10.0 30.0 3.0 0.1091 0.8732 0.5547 4 0.1 10.0 35.0 3.2 0.0381 0.5743 0.8828	100,100,6	[0,1]	-3.322
F21 and F22 and F23:Shekel's Familyl $F_{ij}(x)=-\sum_{i=1}^{5}\sum_{j=1}^{dim}[(x_j-a_{ij})^2+c_j]^{-1}$			
F21:Shekel's 5 Family $F_{21}(x)=-\sum_{i=1}^{5}[(x-a_i)(x-a_i)^T+c_i]^{-1}$ Where, in this exercise: $a=\begin{bmatrix}4&4&4&4\\1&1&1&1\\8&8&8&8\\6&6&6&6\\3&7&3&7\end{bmatrix}$ $c=\begin{bmatrix}0.1\\0.2\\0.2\\0.4\\0.6\end{bmatrix}$	100,100,4	[0,10]	-5.0551
F22:Shekel's 7 Family $F_{22}(x)=-\sum_{i=1}^{7}[(x-a_i)(x-a_i)^T+c_i]^{-1}$ Where, in this exercise: $a=\begin{bmatrix}4&4&4&4\\1&1&1&1\\8&8&8&8\\6&6&6&6\\3&7&3&7\\2&9&2&9\\5&5&3&3\end{bmatrix}$ $c=\begin{bmatrix}0.1\\0.2\\0.2\\0.4\\0.4\\0.6\\0.3\end{bmatrix}$	100,100,4	[0, 10]	-5.088
F23:Shekel's 10 Family $F_{23}(x)=-\sum_{i=1}^{10}[(x-a_i)(x-a_i)^T+c_i]^{-1}$ Where, in this exercise: $a=\begin{bmatrix}4&4&4&4\\1&1&1&1\\8&8&8&8\\6&6&6&6\\3&7&3&7\\2&9&2&9\\5&5&3&3\\8&1&8&1\\6&2&6&2\\7&3.6&7&3.6\end{bmatrix}$ $c=\begin{bmatrix}0.1\\0.2\\0.2\\0.4\\0.4\\0.6\\0.3\\0.7\\0.5\\0.5\end{bmatrix}$	100,100,4	[0, 10]	-5.128

value of $f(\theta)$, and the standard deviation (Std) of MSE. According to Table 20, the LFGWO maintains a considerable precision (the lowest Avg value) than the other five algorithms, and provides similar robustness (Std value) with CS and FPA.

2) A PLANT WITH SECOND-ORDER SYSTEM AND SECOND-ORDER IIR MODEL

In this example, the test case is taken from [77], the performance for each algorithm is evaluated at the identification of a second-order plant through a second-order IIR model under the system identification configuration. Since the order of the model H_M is equal to the order of the to be identified system H_P, local minima problem does not occur and only one global minimum exists in $f(\theta)$. The unknown plant transfer function H_P is shown in (27).

$$H_P(z^{-1}) = \frac{1}{1 - 1.4z^{-1} + 0.49z^{-2}} \tag{27}$$

The transfer function of the IIR model H_M is assumed by (28). It is repeatedly adjusted by modeling process using the adaptive LFGWO and finally gets the most optimal and stable filter (with optimal coefficients, the closest functional similarity to the main IIR plant).

$$H_M(z^{-1}) = \frac{b}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{28}$$

The results of the LFGWO solve IIR system identification and comparison to other algorithms is reported in Table 21.

The results in Table 21 show that CS, FPA, and LFGWO have similar values in their performance. The evidence shows that the adaptive LFGWO maintain a similar average performance when they face unimodal low-dimensional functions. In particular, the test remarks that the small difference in performance is directly related to a better exploitation mechanism included in CS, FPA, and LFGWO.

3) A SUPERIOR-ORDER PLANT AND A HIGH-ORDER MODEL

Finally, the experiment case is taken from [75], the performance for each algorithm is evaluated at the identification of a superior-order plant through a high-order IIR model. Since the plant is a sixth-order system and the IIR model a fourth-order system, in this regard, the error surface is multimodal just as it is in the first experiment. And also the control parameter values employed in this example were the same as in the first examples. Therefore, the unknown plant with a sixth-order system and hold the following functions.

$$H_P(z^{-1}) = \frac{1 - 0.4z^{-2} - 0.65z^{-4} + 0.26z^{-6}}{1 - 0.77z^{-2} - 0.8498z^{-4} + 0.6486z^{-6}} \tag{29}$$

The IIR model H_M with a fourth-order system and hold the following functions.

$$H_M(z^{-1}) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}} \tag{30}$$

TABLE 24. 10 composition functions of CEC 2019.

No.	Functions	Dim	Search Range	f_{min}
CEC01	Storn's Chebyshev Polynomial Fitting Problem	9	[-8192, 8192]	1
CEC02	Inverse Hilbert Matrix Problem	16	[-16384, 16384]	1
CEC03	Lennard-Joes Minimum Energy Cluster	18	[-4, 4]	1
CEC04	Rastrigin's Function	10	[-100, 100]	1
CEC05	Griewangk's Function	10		1
CEC06	Weierstrass Function	10	[-100, 100]	1
CEC07	Modified Schwefel's Function	10	[-100, 100]	1
CEC08	Expand Schaffer's F6 function	10	[-100, 100]	1
CEC09	Happy Cat Function	10	[-100, 100]	1
CEC10	Ackley Function	10	[-100, 100]	1

Only the effective amount of iterations was selected to perform the initial and complete maneuver of the algorithm to extract the global minimum. Therefore, relying on heavy iterations to obtain a delayed possible optimal solution has been avoided. The results of the LFGWO to solve IIR system identification and compare to other algorithms are reported in Table 22.

It should be noted that the performance of an algorithm is often determined by its ability to model a known order plant using a reduced order model (especially for the structure without additive noise).

In a dynamic search space or IIR system identification is very challenging and requires special consideration, because of the global optimum changes over time, to verify the performance of our proposed LFGWO for designing optimal IIR systems, an extensive comparison of results selected from the relevant literature [74] is presented in Table 22. After a comprehensive analysing and consideration of the results which involved with several uncertainties involved in inputs, outputs, objective functions and constraints, a conclusion has been drawn that the LFGWO gives the best precision (Avg value) and outperforms the other competitive algorithms. Table 22 says in terms of IIR system identification that the robustness (Std valve) of the LFGWO also surpasses all competitor algorithms.

To summarize, the LFGWO shows desirable performance for IIR system identification. By combining suitable IIR model, it will play an important role in IIR system identification. Thus, in general, the LFGWO is a potential candidate for identification of IIR plant compared to the other optimal algorithms.

VI. CONCLUSION

In this paper, the overall performances of the LFGWO algorithm were tested and verified using the standard 23 benchmark functions and 10 composition functions of CEC 2019 compared with the other eight state-of-art algorithms. There are 17 out of 23 best values obtained by LFGWO, which are more than those obtained by the other eight optimization algorithms. The 28 out of 33 average and 27 out of 33 standard deviation values obtained by LFGWO are all less than those obtained by the other eight optimization algorithms, which verified and demonstrated the performance, stability, and robustness of the LFGWO. The extensibility test with different scales of dimensions 50,

TABLE 25. The setup of the parameters (t : current iteration, T: the maximal iteration).

Algorithm	Parameters	Value
AHA	Migration coefficient	$2n$
AO	quality function used to equilibrium the search strategies QF the slope from the first location (1) to the last location (t)	$G_1 = 2 \times rand - 1$, $G_2 = 2 \times (1 - \frac{t}{T})$, $QF(t) = t^{\frac{2 \times rand - 1}{(1-T)^2}}$
DA	Inertia weight w	$w = max + t \times \left(\frac{max - min}{T}\right)$, $min = 0.4,\ max = 0.9$
	separation weight s	$s = 2 \times rand \times [0.1 - t(\frac{0.1}{T/2})]$
	alignment weight a	$a = 2 \times rand \times [0.1 - t(\frac{0.1}{T/2})]$
	the cohesion weight c	$c = 2 \times rand \times [0.1 - t(\frac{0.1}{T/2})]$
	food factor f	$f = 2 \times rand$
	enemy factor e	$e = 0.1 - t(\frac{0.1}{T/2})$
DMOA	Convergence constant a	$a = (1 - \frac{t}{T})^{2 \times \frac{t}{T}}$
GBO	β the most significant parameter in the GBO to balance the exploration and exploitation searching processes	$\beta = \beta_{min} + (\beta_{max} - \beta_{min}) \times (1 - (\frac{t}{T})^3)^2$ $\beta_{min} = 0.2, \beta_{max} = 1.2,\ pr = 0.5$
HGS	Convergence constant a	$a = 2 \times (1 - \frac{t}{T})$
HHO	E_0 is the initial state of its energy, E indicates the the escaping energy of the prey.	$E = 2E_0 \times (1 - \frac{t}{T})$
LFGWO	Convergence constant a	$a = 2 \times (1 - \frac{t}{T})$
MVO	Travelling distance rate (TDR) $\in [0.6\ 1]$	$TDR = 1 - \frac{t^{1/p}}{T^{1/p}},\ p = 6$

100, 300, and 500, is conducted by comparing LFGWO with the GWO and IGWO to assess the dimensional influence on consistency and optimization quality. The statistical results of three tests, Wilcoxon rank-sum, Friedman and post-hoc Nemenyi test prove that the LFGWO is superior to other compared algorithms for most test functions, showing that the LFGWO are successful. In addition, the results of five engineering design problems, one infinite impulse response (IIR) system identification problem, which demonstrated the applicability of the LFGWO.

THE ADVANTAGES OF THE LFGWO

Firstly, the position of all wolves initialized with Levy flight, so that the position of the candidate solutions is random, ergodic, avoid stagnant local optima and find the global optimal, thereby avoiding premature convergence effectively, which increase the diversity of the population to a certain extent and lays the foundation for the LFGWO's global search. The main reason the LFGWO is superior to the other competitive algorithms on these composition functions is hidden in the unique structure of this algorithm. The LFGWO inherits some advantages from the GWO, such as having three guide wolves of Alpha, Beta, and Delta, which, in turn, helps the diversity of the solutions in the search space to be considerably preserved. The other characteristic of the GWO which the LFGWO benefits from is the high exploitation capability of this algorithm. These characteristics are strengthened in LFGWO by adding the Levy flight mechanisms into the structure of the GWO to enable the LFGWO to further preserve diversity and avoid missing the good candidate solutions in the search space. In addition, the Levy flight mechanisms can boost the ability of the LFGWO to both explore and exploit the promising regions in the search space and to get rid of the local optimal solution. Finally, the Levy flight mechanisms can intensify the convergence to the optimal point of the problems and enhance the exploitation capability of the LFGWO.

THE LIMITATIONS OF THE LFGWO

Needless to say, even the performance of the LFGWO with greatly improved by the GWO embedded the Levy flight mechanism, however, the lack of adaptivity and dynamic behavior in Levy flight are still inevitably exist. Therefore, the LFGWO with adaptivity and dynamic behavior of Levy flight is worth pondering. Meanwhile, it will be difficult to guarantee for the LFGWO getting an accurate approximation of the global optimum when the LFGWO face composition test functions with high dimensions. However, experiments indicate in Table 1 that LFGWO rank 9 in cec05 in average values, in Table 2 that LFGWO rank 9 in cec03, cec05, and cec10 in Std. are all more than those obtained by the other eight optimization algorithms, caused by the problems' high dimension. As the old saying goes, every coin has twofold. The main resean is using levy flight also makes more interruption in the search space due to the implementation of big steps. It must be noted that the LFGWO also provides very competitive results on the remaining benchmarks.

For future work, the LFGWO employed to tackle constrained non-line optimization problems of specific fields is an alternative way. The LFGWO hybrid other meta-heuristic algorithms, such as brain storm optimization algorithm [78] is worthy of further study to maximize the tremendous potential of LFGWO. Additionally, the LFGWO to apply to multi-objective tasks, and dynamic optimization problems of science and engineering fields in the upcoming years are also attractive and interesting topics.

APPENDIX

See Tables 23–25.

REFERENCES

[1] A. M. Dalavi, A. Gomes, and A. J. Husain, "Bibliometric analysis of nature inspired optimization techniques," *Comput. Ind. Eng.*, vol. 169, Jul. 2022, Art. no. 108161, doi: 10.1016/j.cie.2022.108161. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0360835222002315

[2] W. Zhao, Z. Zhang, and L. Wang, "Manta ray foraging optimization: An effective bio-inspired optimizer for engineering applications," *Eng. Appl. Artif. Intell.*, vol. 87, Jan. 2020, Art. no. 103300, doi: 10.1016/j.engappai.2019.103300. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0952197619302593

[3] A. Tang, H. Zhou, T. Han, and L. Xie, "A modified manta ray foraging optimization for global optimization problems," *IEEE Access*, vol. 9, pp. 128702–128721, 2021, doi: 10.1109/ACCESS.2021.3113323.

[4] M. A. Elaziz, L. Abualigah, A. A. Ewees, M. A. Al-qaness, R. R. Mostafa, D. Yousri, and R. A. Ibrahim, "Triangular mutation-based manta-ray foraging optimization and orthogonal learning for global optimization and engineering problems," *Appl. Intell.*, vol. 53, no. 7, pp. 7788–7817, Apr. 2023, doi: 10.1007/s10489-022-03899-1.

[5] D. Yousri, A. M. Abdelaty, M. A. A. Al-Qaness, A. A. Ewees, A. G. Radwan, and M. A. Elaziz, "Discrete fractional-order caputo method to overcome trapping in local optima: Manta ray foraging optimizer as a case study," *Expert Syst. Appl.*, vol. 192, Apr. 2022, Art. no. 116355, doi: 10.1016/j.eswa.2021.116355. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0957417421016523

[6] S. S. Band, S. Ardabili, A. Seyed Danesh, Z. Mansor, I. AlShourbaji, and A. Mosavi, "Colonial competitive evolutionary rao algorithm for optimal engineering design," *Alexandria Eng. J.*, vol. 61, no. 12, pp. 11537–11563, Dec. 2022, doi: 10.1016/j.aej.2022.05.018. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1110016822003301

[7] A. Faramarzi, M. Heidarinejad, S. Mirjalili, and A. H. Gandomi, "Marine predators algorithm: A nature-inspired metaheuristic," *Expert Syst. Appl.*, vol. 152, Aug. 2020, Art. no. 113377, doi: 10.1016/j.eswa.2020.113377. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0957417420302025

[8] G. Hua, "MCSA: Multi-strategy boosted chameleon-inspired optimization algorithm for engineering applications," *Comput. Methods Appl. Mech. Eng.*, vol. 403, Jan. 2023, Art. no. 115676, doi: 10.1016/j.cma.2022.115676.

[9] C. Ma, H. Huang, Q. Fan, J. Wei, Y. Du, and W. Gao, "Grey wolf optimizer based on Aquila exploration method," *Expert Syst. Appl.*, vol. 205, Nov. 2022, Art. no. 117629, doi: 10.1016/j.eswa.2022.117629. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S095741742200937X

[10] A. Saxena, R. Kumar, and S. Mirjalili, "A harmonic estimator design with evolutionary operators equipped grey wolf optimizer," *Expert Syst. Appl.*, vol. 145, May 2020, Art. no. 113125, doi: 10.1016/j.eswa.2019.113125. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0957417419308425

[11] S. Dhargupta, M. Ghosh, S. Mirjalili, and R. Sarkar, "Selective opposition based grey wolf optimization," *Expert Syst. Appl.*, vol. 151, Aug. 2020, Art. no. 113389.

[12] R. Rajakumar, K. Sekaran, C.-H. Hsu, and S. Kadry, "Accelerated grey wolf optimization for global optimization problems," *Technolog. Forecasting Social Change*, vol. 169, Aug. 2021, Art. no. 120824, doi: 10.1016/j.techfore.2021.120824. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0040162521002560

[13] M. H. Nadimi-Shahraki, S. Taghian, and S. Mirjalili, "An improved grey wolf optimizer for solving engineering problems," *Expert Syst. Appl.*, vol. 166, Mar. 2021, Art. no. 113917.

[14] F. Rezaei, H. R. Safavi, M. A. Elaziz, S. H. A. El-Sappagh, M. A. Al-Betar, and T. Abuhmed, "An enhanced grey wolf optimizer with a velocity-aided global search mechanism," *Mathematics*, vol. 10, no. 3, p. 351, Jan. 2022.

[15] J. Adhikary and S. Acharyya, "Randomized balanced grey wolf optimizer (RBGWO) for solving real life optimization problems," *Appl. Soft Comput.*, vol. 117, Mar. 2022, Art. no. 108429.

[16] S. Singh and J. C. Bansal, "Mutation-driven grey wolf optimizer with modified search mechanism," *Expert Syst. Appl.*, vol. 194, May 2022, Art. no. 116450.

[17] J. Hu, H. Chen, A. A. Heidari, M. Wang, X. Zhang, Y. Chen, and Z. Pan, "Orthogonal learning covariance matrix for defects of grey wolf optimizer: Insights, balance, diversity, and feature selection," *Knowl.-Based Syst.*, vol. 213, Feb. 2021, Art. no. 106684.

[18] H. N. Soloklo and N. Bigdeli, "Fast-dynamic grey wolf optimizer for solving model order reduction of bilinear systems based on multi-moment matching technique," *Appl. Soft Comput.*, vol. 130, Nov. 2022, Art. no. 109730, doi: 10.1016/j.asoc.2022.109730. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1568494622007797

[19] J. Jiang, Z. Zhao, Y. Liu, W. Li, and H. Wang, "DSGWO: An improved grey wolf optimizer with diversity enhanced strategy based on group-stage competition and balance mechanisms," *Knowl.-Based Syst.*, vol. 250, Aug. 2022, Art. no. 109100, doi: 10.1016/j.knosys.2022.109100. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0950705122005421

[20] M. H. Nadimi-Shahraki, S. Taghian, S. Mirjalili, H. Zamani, and A. Bahreininejad, "GGWO: Gaze cues learning-based grey wolf optimizer and its applications for solving engineering problems," *J. Comput. Sci.*, vol. 61, May 2022, Art. no. 101636.

[21] Preeti and K. Deep, "A random walk grey wolf optimizer based on dispersion factor for feature selection on chronic disease prediction," *Expert Syst. Appl.*, vol. 206, Nov. 2022, Art. no. 117864, doi: 10.1016/j.eswa.2022.117864. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0957417422011198

[22] J. Wang, D. Lin, Y. Zhang, and S. Huang, "An adaptively balanced grey wolf optimization algorithm for feature selection on high-dimensional classification," *Eng. Appl. Artif. Intell.*, vol. 114, Sep. 2022, Art. no. 105088, doi: 10.1016/j.engappai.2022.105088. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0952197622002378

[23] P. Hu, J.-S. Pan, and S.-C. Chu, "Improved binary grey wolf optimizer and its application for feature selection," *Knowl.-Based Syst.*, vol. 195, May 2020, Art. no. 105746, doi: 10.1016/j.knosys.2020.105746. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S095070512030160X

[24] T. V. Geetha and A. J. Deepa, "A FKPCA-GWO WDBiLSTM classifier for intrusion detection system in cloud environments," *Knowl.-Based Syst.*, vol. 253, Oct. 2022, Art. no. 109557, doi: 10.1016/j.knosys.2022.109557. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0950705122007857

[25] S. Arora, H. Singh, M. Sharma, S. Sharma, and P. Anand, "A new hybrid algorithm based on grey wolf optimization and crow search algorithm for unconstrained function optimization and feature selection," *IEEE Access*, vol. 7, pp. 26343–26361, 2019, doi: 10.1109/ACCESS.2019.2897325.

[26] X. Meng, J. Jiang, and H. Wang, "AGWO: Advanced GWO in multi-layer perception optimization," *Expert Syst. Appl.*, vol. 173, Jul. 2021, Art. no. 114676, doi: 10.1016/j.eswa.2021.114676. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0957417421001172

[27] R. Mohakud and R. Dash, "Designing a grey wolf optimization based hyper-parameter optimized convolutional neural network classifier for skin cancer detection," *J. King Saud Univ. Comput. Inf. Sci.*, vol. 34, no. 8, pp. 6280–6291, Sep. 2022, doi: 10.1016/j.jksuci.2021.05.012. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1319157821001270

[28] A. A. Heidari, R. A. Abbaspour, and H. Chen, "Efficient boosted grey wolf optimizers for global search and kernel extreme learning machine training," *Appl. Soft Comput.*, vol. 81, Aug. 2019, Art. no. 105521, doi: 10.1016/j.asoc.2019.105521. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1568494619302911

[29] S. Kumar and M. Singh, "Breast cancer detection based on feature selection using enhanced grey wolf optimizer and support vector machine algorithms," *Vietnam J. Comput. Sci.*, vol. 8, no. 2, pp. 177–197, May 2021.

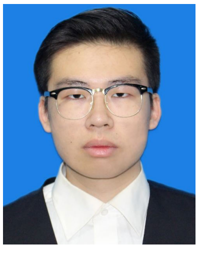
[30] E. Badr, S. Almotairi, M. A. Salam, and H. Ahmed, "New sequential and parallel support vector machine with grey wolf optimizer for breast cancer diagnosis," *Alexandria Eng. J.*, vol. 61, no. 3, pp. 2520–2534, Mar. 2022, doi: 10.1016/j.aej.2021.07.024. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1110016821004890

[31] S. R. Kamel, R. YaghoubZadeh, and M. Kheirabadi, "Improving the performance of support-vector machine by selecting the best features by gray wolf algorithm to increase the accuracy of diagnosis of breast cancer," *J. Big Data*, vol. 6, no. 1, p. 90, Dec. 2019, doi: 10.1186/s40537-019-0247-7.
[32] R. Ahmadi, G. Ekbatanifard, and P. Bayat, "A modified grey wolf optimizer based data clustering algorithm," *Appl. Artif. Intell.*, vol. 35, no. 1, pp. 63–79, Jan. 2021, doi: 10.1080/08839514.2020.1842109.
[33] M. Ghorbanvirdi and S. M. Mazinani, "Energy efficient multi-clustering using grey wolf optimizer in wireless sensor network," *Int. J. Inf. Commun. Technol. Res.*, vol. 14, no. 1, pp. 1–12, Mar. 2022. [Online]. Available: https://civilica.com/doc/1452211
[34] X. Zhang, Q. Lin, W. Mao, S. Liu, Z. Dou, and G. Liu, "Hybrid particle swarm and grey wolf optimizer and its application to clustering optimization," *Appl. Soft Comput.*, vol. 101, Mar. 2021, Art. no. 107061, doi: 10.1016/j.asoc.2020.107061. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1568494620309996
[35] R. Purushothaman, S. P. Rajagopalan, and G. Dhandapani, "Hybridizing gray wolf optimization (GWO) with grasshopper optimization algorithm (GOA) for text feature selection and clustering," *Appl. Soft Comput.*, vol. 96, Nov. 2020, Art. no. 106651, doi: 10.1016/j.asoc.2020.106651. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1568494620305895
[36] S. Mirjalili, S. M. Mirjalili, and A. Lewis, "Grey wolf optimizer," *Adv. Eng. Softw.*, vol. 69, pp. 46–61, Mar. 2014, doi: 10.1016/j.advengsoft.2013.12.007.
[37] A. A. Heidari and P. Pahlavani, "An efficient modified grey wolf optimizer with Lévy flight for optimization tasks," *Appl. Soft Comput.*, vol. 60, pp. 115–134, Nov. 2017, doi: 10.1016/j.asoc.2017.06.044.
[38] X. Chen, F. Cheng, C. Liu, L. Cheng, and Y. Mao, "An improved wolf pack algorithm for optimization problems: Design and evaluation," *PLoS ONE*, vol. 16, no. 8, Aug. 2021, Art. no. e0254239, doi: 10.1371/journal.pone.0254239.
[39] B. Bhatt, H. Sharma, K. Arora, G. P. Joshi, and B. Shrestha, "Lévy flight-based improved grey wolf optimization: A solution for various engineering problems," *Mathematics*, vol. 11, no. 7, p. 1745, Apr. 2023, doi: 10.3390/math11071745.
[40] P. Niu, S. Niu, N. Liu, and L. Chang, "The defect of the grey wolf optimization algorithm and its verification method," *Knowledge-Based Syst.*, vol. 171, pp. 37–43, May 2019, doi: 10.1016/j.knosys.2019.01.018. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0950705119300188
[41] E. Akbari, A. Rahimnejad, and S. A. Gadsden, "A greedy non-hierarchical grey wolf optimizer for real-world optimization," *Electron. Lett.*, vol. 57, no. 13, pp. 499–501, Jun. 2021.
[42] Y.-C. Ho and D. L. Pepyne, "Simple explanation of the no-free-lunch theorem and its implications," *J. Optim. Theory Appl.*, vol. 115, pp. 549–570, 2002.
[43] T. Tran, "Global optimization using Lévy flights," 2014, *arXiv:1407.5739*.
[44] E. H. Houssein, M. R. Saad, F. A. Hashim, H. Shaban, and M. Hassaballah, "Lévy flight distribution: A new metaheuristic algorithm for solving engineering optimization problems," *Eng. Appl. Artif. Intell.*, vol. 94, Sep. 2020, Art. no. 103731, doi: 10.1016/j.engappai.2020.103731.
[45] M. Gutowski, "Lévy flights as an underlying mechanism for global optimization algorithms," 2001, *arXiv:math-ph/0106003*.
[46] W. Zhao, L. Wang, and S. Mirjalili, "Artificial hummingbird algorithm: A new bio-inspired optimizer with its engineering applications," *Comput. Methods Appl. Mech. Eng.*, vol. 388, Jan. 2022, Art. no. 114194, doi: 10.1016/j.cma.2021.114194. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0045782521005259
[47] L. Abualigah, D. Yousri, M. A. Elaziz, A. A. Ewees, M. A. A. Al-Qaness, and A. H. Gandomi, "Aquila optimizer: A novel meta-heuristic optimization algorithm," *Comput. Ind. Eng.*, vol. 157, Jul. 2021, Art. no. 107250, doi: 10.1016/j.cie.2021.107250. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0360835221001546
[48] S. Mirjalili, "Dragonfly algorithm: A new meta-heuristic optimization technique for solving single-objective, discrete, and multi-objective problems," *Neural Comput. Appl.*, vol. 27, no. 4, pp. 1053–1073, May 2016, doi: 10.1007/s00521-015-1920-1.
[49] J. O. Agushaka, A. E. Ezugwu, and L. Abualigah, "Dwarf mongoose optimization algorithm," *Comput. Methods Appl. Mech. Eng.*, vol. 391, Mar. 2022, Art. no. 114570, doi: 10.1016/j.cma.2022.114570. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0045782522000019
[50] I. Ahmadianfar, O. Bozorg-Haddad, and X. Chu, "Gradient-based optimizer: A new metaheuristic optimization algorithm," *Inf. Sci.*, vol. 540, pp. 131–159, Nov. 2020, doi: 10.1016/j.ins.2020.06.037. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0020025520306241
[51] Y. Yang, H. Chen, A. A. Heidari, and A. H. Gandomi, "Hunger games search: Visions, conception, implementation, deep analysis, perspectives, and towards performance shifts," *Expert Syst. Appl.*, vol. 177, Sep. 2021, Art. no. 114864, doi: 10.1016/j.eswa.2021.114864. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0957417421003055
[52] A. A. Heidari, "Harris hawks optimization: Algorithm and applications," *Future Gener. Comput. Syst.*, vol. 97, pp. 849–872, Aug. 2019, doi: 10.1016/j.future.2019.02.028. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0167739X18313530
[53] S. Mirjalili, S. M. Mirjalili, and A. Hatamlou, "Multi-verse optimizer: A nature-inspired algorithm for global optimization," *Neural Comput. Appl.*, vol. 27, no. 2, pp. 495–513, Feb. 2016, doi: 10.1007/s00521-015-1870-7.
[54] K. V. Price, *The 100-Digit Challenge: Problem Definitions and Evaluation Criteria for the 100-Digit Challenge Special Session and Competition on Single Objective Numerical Optimization*. Singapore: Nanyang Technological Univ., Nov. 2018.
[55] L. Wu, J. Wu, and T. Wang, "Enhancing grasshopper optimization algorithm (GOA) with Lévy flight for engineering applications," *Sci. Rep.*, vol. 13, no. 1, p. 124, Jan. 2023, doi: 10.1038/s41598-022-27144-4. [Online]. Available: https://www.nature.com/articles/s41598-022-27144-4
[56] J. Derrac, S. García, S. Hui, P. N. Suganthan, and F. Herrera, "Analyzing convergence performance of evolutionary algorithms: A statistical approach," *Inf. Sci.*, vol. 289, pp. 41–58, Dec. 2014, doi: 10.1016/j.ins.2014.06.009. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0020025514006276
[57] Q. Jin, "An improved whale optimization algorithm with random evolution and special reinforcement dual-operation strategy collaboration," *Symmetry*, vol. 13, pp. 1–24, Jan. 2021, doi: 10.3390/sym13020238.
[58] V. Goodarzimehr, S. Talatahari, S. Shojaee, and S. Hamzehei-Javaran, "Special relativity search for applied mechanics and engineering," *Comput. Methods Appl. Mech. Eng.*, vol. 403, Jan. 2023, Art. no. 115734, doi: 10.1016/j.cma.2022.115734. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0045782522006892
[59] H. Tao, N. M. H. Al-Aragi, I. Ahmadianfar, M. H. Naser, R. H. Shehab, J. M. Zain, B. Halder, and Z. M. Yaseen, "Ranked-based mechanism-assisted biogeography optimization: Application of global optimization problems," *Adv. Eng. Softw.*, vol. 174, Dec. 2022, Art. no. 103301, doi: 10.1016/j.advengsoft.2022.103301. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0965997822002022
[60] M. Azizi, S. Talatahari, and A. Giaralis, "Optimization of engineering design problems using atomic orbital search algorithm," *IEEE Access*, vol. 9, pp. 102497–102519, 2021, doi: 10.1109/ACCESS.2021.3096726.
[61] N. Kumar, S. K. Mahato, and A. K. Bhunia, "Design of an efficient hybridized CS-PSO algorithm and its applications for solving constrained and bound constrained structural engineering design problems," *Results Control Optim.*, vol. 5, Dec. 2021, Art. no. 100064, doi: 10.1016/j.rico.2021.100064. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2666720721000370
[62] A. H. Gandomi, "Cuckoo search algorithm: A metaheuristic approach to solve structural optimization problems," *Eng. With Comput.*, vol. 29, pp. 17–35, Jan. 2013, doi: 10.1007/s00366-011-0241-y.
[63] Z.-K. Feng, W.-J. Niu, and S. Liu, "Cooperation search algorithm: A novel metaheuristic evolutionary intelligence algorithm for numerical optimization and engineering optimization problems," *Appl. Soft Comput.*, vol. 98, Jan. 2021, Art. no. 106734, doi: 10.1016/j.asoc.2020.106734. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1568494620306724
[64] X. Xiang, Q. Su, G. Huang, and Z. Hu, "A simplified non-equidistant grey prediction evolution algorithm for global optimization," *Appl. Soft Comput.*, vol. 125, Aug. 2022, Art. no. 109081, doi: 10.1016/j.asoc.2022.109081. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1568494622003726
[65] O. E. Turgut and M. S. Turgut, "Local search enhanced Aquila optimization algorithm ameliorated with an ensemble of wavelet mutation strategies for complex optimization problems," *Math. Comput. Simul.*, vol. 206, pp. 302–374, Apr. 2023, doi: 10.1016/j.matcom.2022.11.020. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0378475422004694

[66] Y. Xin-She, "Bat algorithm: A novel approach for global engineering optimization," *Eng. Computations*, vol. 29, no. 5, pp. 464–483, 2012.
[67] H. Eskandar, A. Sadollah, A. Bahreininejad, and M. Hamdi, "Water cycle algorithm—A novel metaheuristic optimization method for solving constrained engineering optimization problems," *Comput. Struct.*, vols. 110–111, pp. 151–166, Nov. 2012, doi: 10.1016/j.compstruc.2012.07.010. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0045794912001770
[68] K. Wang, M. Guo, C. Dai, and Z. Li, "Information-decision searching algorithm: Theory and applications for solving engineering optimization problems," *Inf. Sci.*, vol. 607, pp. 1465–1531, Aug. 2022, doi: 10.1016/j.ins.2022.06.008. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0020025522005898
[69] R. V. Rao, V. J. Savsani, and D. P. Vakharia, "Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems," *Comput.-Aided Des.*, vol. 43, no. 3, pp. 303–315, Mar. 2011, doi: 10.1016/j.cad.2010.12.015. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0010448510002484
[70] G. Hu, J. Zhong, G. Wei, and C.-T. Chang, "DTCSMO: An efficient hybrid starling murmuration optimizer for engineering applications," *Comput. Methods Appl. Mech. Eng.*, vol. 405, Feb. 2023, Art. no. 115878, doi: 10.1016/j.cma.2023.115878. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0045782523000014
[71] M. Dehghani, Z. Montazeri, G. Dhiman, O. P. Malik, R. Morales-Menendez, R. A. Ramirez-Mendoza, A. Dehghani, J. M. Guerrero, and L. Parra-Arroyo, "A spring search algorithm applied to engineering optimization problems," *Appl. Sci.*, vol. 10, no. 18, p. 6173, Sep. 2020, doi: 10.3390/app10186173.
[72] M. Nancy and E. A. Stephen, "Modelling and analysis of the cone coupling problem using optimization," *Math. Model. Eng. Problems*, vol. 9, no. 5, pp. 1385–1392, Dec. 2022, doi: 10.18280/mmep.090529.
[73] A. Anita, "Artificial electric field algorithm for engineering optimization problems," *Expert Syst. With Appl.*, vol. 149, Jul. 2020, Art. no. 113308, doi: 10.1016/j.eswa.2020.113308.
[74] E. Cuevas, J. Gálvez, S. Hinojosa, O. Avalos, D. Zaldívar, and M. Pérez-Cisneros, "A comparison of evolutionary computation techniques for IIR model identification," *J. Appl. Math.*, vol. 2014, pp. 1–9, Dec. 2014. [Online]. Available: http://www.hindawi.com/journals/jam/2014/827206/
[75] Y. Ling, Y. Zhou, and Q. Luo, "Lévy flight trajectory-based whale optimization algorithm for global optimization," *IEEE Access*, vol. 5, pp. 6168–6186, 2017, doi: 10.1109/ACCESS.2017.2695498.
[76] A. Mohammadi, S. H. Zahiri, S. M. Razavi, and P. N. Suganthan, "Design and modeling of adaptive IIR filtering systems using a weighted sum–variable length particle swarm optimization," *Appl. Soft Comput.*, vol. 109, Sep. 2021, Art. no. 107529, doi: 10.1016/j.asoc.2021.107529. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S156849462100452X
[77] P. Upadhyay, R. Kar, D. Mandal, and S. P. Ghoshal, "A new design method based on firefly algorithm for IIR system identification problem," *J. King Saud Univ. Eng. Sci.*, vol. 28, no. 2, pp. 174–198, Jul. 2016, doi: 10.1016/j.jksues.2014.03.001. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1018363914000099 and https://www.sciencedirect.com/science/article/pii/S0957417420301330
[78] S. Cheng, Y. Shi, Q. Qin, T. O. Ting, and R. Bai, "Maintaining population diversity in brain storm optimization algorithm," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Jul. 2014, pp. 3230–3237, doi: 10.1109/CEC.2014.6900255.

WU LEI is currently an Associate Professor in computer science and technology with the North China University of Technology (NCUT). He supervises various the master's and graduation student's projects. He has published different scientific research papers in international journals and conferences. His current research interests include data analytic and data processing, metaheuristic optimization techniques, and artificial intelligence. He is a Senior Member of the Chinese Institute of Electronics.

WU JIAWEI was born in Greenwich, New York City, NY, USA. She is currently pursuing the degree in civil engineering with the Beijing University of Technology (BJUT). She is also studying structure design and the stability of bridges and architecture. Her current research interests include finite element simulation analysis and principle structure of composition. She has committed to several scientific projects, publishing relevant papers.



MENG ZEZHOU was born in 1997. He is currently pursuing the degree with the North China University of Technology. His research interest includes natural language processing. He is dedicated to the study of text similarity algorithms and has published related papers.