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RESEARCH ARTICLE

LMI-Based Luenberger Observer Design for Uncertain Nonlinear Systems With External Disturbances and Time-Delays

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ABSTRACT This paper investigates the simultaneous design of a controller and Luenberger state observer for systems with time-delays, external disturbances, uncertainties, modeling errors, and unknown nonlinear perturbations. The state-feedback control approach and state-observer existence conditions are formulated using the Linear Matrix Inequalities (LMIs). By defining the estimation error, the equations of the closed-loop system are rewritten. External disturbances, uncertainties, unknown nonlinear perturbations, and constant time-delays are considered in system modeling. By using LMI techniques, the estimation error is converged to zero. Therefore, the time-delays, uncertainties, and external disturbance effects on the system output, which have not been considered simultaneously before, are minimized, and the closed-loop system is stabilized. The performance of the proposed approach is verified by simulation of two examples, Flexible-Link Manipulator (FLM) dynamics, and Continuous Stirred Tank Reactor (CSTR) system. These examples illustrate the reliability of the suggested method.

INDEX TERMS Linear matrix inequality, controller design, state-observer, external disturbance, time-delay, flexible-link manipulator.

I. INTRODUCTION

A. BACKGROUND AND MOTIVATION

Disturbances can cause significant disruptions and unwanted effects in the control process. In recent decades, researchers have suggested various methods to reduce the effects of external disturbances on the control system. It is generally impossible to eliminate external disturbances completely; however, many papers have been trying to reduce the effects of disturbances by applying various methods [1], [2], [3], [4], [5], [6]. Significant efforts have been made for the robust stability of linear systems with input disturbances. Another problem in controller design is the lack of accurate or complete information about the states of the system [7], [8]. Therefore, the uncertainty has bad effects on control performances, and it can reduce the accuracy of the designed controller [9]. In many systems, the state-feedback control cannot ensure system stability due to the unavailability of all control system states. For this reason, in feedback control, it is essential to design a state observer [10], [11]. In real industrial processes, there is a time-delay in addition to external disturbances and uncertainties [12], [13]. The time-delay makes non-minimum phase behavior in the system [11], [12]. Therefore, this term should be considered in system modeling. Therefore, many methods have been designed to overcome the time-delay effects [14], [15], [16]. Some research studies

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have been done to decrease the impacts of disturbance, uncertainty, and nonlinear function, but their controller is designed for special systems [14], [15], [17]. In [14], the perturbation observer-based control is designed for voltage converter systems. The impact of disturbance, uncertainty, nonlinear function, and state estimation are considered. The output feedback controller is designed to compensate for the impacts of perturbations; however, time-delay is not considered in [14]. In [18], the observer-based controller is designed for nonlinear systems with unknown time-delay, but the effect of uncertainty has not been considered. In [19], the combination of high gain state observer and LMI is investigated for nonlinear systems. The estimation error stability has been proved. However, time-delay, uncertainty, nonlinear perturbation, output disturbances effects have not been considered.

The main task in controller design is to check the stability of the system. One of the mathematical tools is linear matrix inequalities (LMIs), which can prove system stability [53], [54]. In recent research, the LMIs method is considered an effective tool to help researchers control design and is widely used in different applications [18], [20], [21], [22], [23], [24]. This approach is used to convert the considered problem to an optimization problem. The techniques of convex or quasi-convex optimization problems, involving LMIs, are used to construct Lyapunov stability function, Linear quadratic regulator, optimal system realization, obtain state-feedback gain and Luenberger observer gain via Yalmip and other solvers, numerically. These solvers use effective algorithms to fulfill inequalities' conditions and convex constraints [25], [26]. The fundamental theory in designing controllers by LMIs is the Lyapunov stability theorem, which is used to prove the asymptotical stability of the closed-loop system. Using LMIs can reduce the constraints of system conditions. In the last research, many algorithms have solved LMIs; many papers apply LMIs techniques to various control theories [27], [28]. The design of the controller for electromechanical systems can be formulated as an LMIs problem. Because the optimal values for controller and observer gains are obtained from LMIs, therefore, they can improve system's behavior such as tracking performance, steady state response, etc.

B. LITERATURE REVIEW

In [29], the simultaneous design of the observer and controller is presented in the presence of a nonlinear term using LMIs. In order to design the state-feedback control law, the observer is introduced then the controller is designed. Nevertheless, it does not guarantee system stability in the existence of external disturbances [30]. In [31], the authors present a solution for linear systems observer-based stabilization in the presence of uncertainty. Less restrictive LMIs condition is the result of proper use of the Young relation. In [16] and [32], the sliding mode controller (SMC) and observer are designed, and then the validity of the proposed SMC is proved by using LMIs. The observer-based controllers are useful to stabilize various classes of systems and improve the system's performance [16]. The state observer is used because some states of the system may not be available in real systems. In most research studies, the state observer structure is usually designed with Luenberger form. In [20], the state feedback controller and the state observer combination are designed for systems with stochastic noise and polytopic parameters. An iterative LMIs approach is suggested for solving nonlinear matrix inequalities when the separation principle is not valid. This approach has been able to reduce noise effects, but they haven't been eliminated. In [21], a robust nonlinear controller is designed for uncertain nonlinear discrete time-invariant systems. The fault-tolerant control law is designed by using LMIs toolbox and iterative process. The amount of external disturbances is not clearly expressed in this paper, and the delay effects are not considered and tested by the suggested approach. In [22], an observer-based H∞ output feedback controller is designed for uncertain interconnected nonlinear systems. The gain matrixes of observer and controller are obtained by LMIs procedure. The proposed approach has the output tracking well, but the system can't consider the delay effects. In [27], the observer-based controller is designed for time-varying delay systems. The stabilization of the close loop system is proved via LMIs method. In this paper, the effects of uncertainty and disturbances aren't considered. In [28], the extended state-observer-based control is investigated for systems with locally Lipschitz uncertainties and exponential stability is proved. The effects of time-delay, output disturbances, and uncertainties are not considered. In [29], it has been attempted to prove asymptotical stability using the H∞ -based observer using LMIs. The recommended gains are calculated with the help of H_∞; then, H_∞ is transformed into LMIs. After solving LMIs, the presented H_∞ controller and observer can guarantee the robustness of systems with uncertainties and disturbances. The effects of time-delay and output disturbances have not been considered in this research. In [4], an adaptive control strategy with full error constraints is designed for nonlinear systems. In this method, the adaptive back-stepping control scheme is combined with the nonlinear filter. Like all of previous papers mentioned in this manuscript, the simultaneous effects of time-delay, uncertainty, and nonlinear perturbation have not been considered in the nonlinear system model of [4]. In [33], a fixed-time fuzzy controller is designed for nonlinear multi-agent systems with unmeasurable states and unknown dynamics, and the linear state-observer and fuzzy logic systems are utilized to identify the unknown internal dynamics. It can control the unmanned vehicle well, but the unknown nonlinear perturbations and time-delays are not considered in the system equations. In [34], a finite-time fuzzy adaptive prescribed performance control technique is proposed for non-strict-feedback nonlinear MIMO systems, and a dynamic surface controller is suggested by combining the adaptive back-stepping control algorithm and the nonlinear filters. The proposed method can deal with the computational complexity and improve the control performance. The Luenberger observer is one of appropriate tools to estimate the information of the internal system variables, noise and disturbances which are unknown, and they have bad effects on performance of system such as lower tracking accuracy. Therefore, the Luenberger observer gain should be designed carefully. Using LMI approach can be useful to improve accuracy of system performance. The Luenberger observer is one of the most applicable observers in practice, because of using continues function, clear structure, simple implementation, and excellent steady-state performance; but it can make the system unstable in high gains [35]. Considering the above-mentioned researches, to the best authors' knowledge, the design of the LMI-based Luenberger observer for uncertain nonlinear systems with external disturbances and time-delays is still an open problem. No research study has been done suggesting an LMI-based Luenberger observer in the presence of interval time-delays, unknown nonlinear perturbation, and minimizing effects of disturbances.

C. CONTRIBUTION

Motivated by the above discussion, our goal is to design an observer-based controller to stabilize the linearizable systems in the presence of disturbances, unknown nonlinear perturbation, and time-delay. The main contributions of this paper are as follows:

- An approach that enables the simultaneous design of the observer and controller gains;
- A design that combines the estimation properties of state observers with the optimization properties of linear matrix inequalities for systems in the presence of uncertainties, disturbances, unknown nonlinear perturbation, and time-delay, the effects of these parameters have not been solved by LMI approaches before.
- An approach is suggested for controlling and stabilizing systems with interval time-delays, unknown nonlinear perturbation, and minimizing effects of disturbances;
- The state observer is designed to estimate unmeasured states and guarantee the stability of the closed-loop system with and without parametric uncertainties.
- The exponential stability of a closed-loop system in the presence of uncertainties, disturbances, unknown nonlinear perturbation, and time-delay is proved.
- New Lyapunov functions are considered for the stability analysis of time-delay systems.

D. PAPER ORGANIZATION

The remainder of the paper is organized as follows. Section I-A, the problem formulation, includes system description, assumptions, and preliminaries. The proposed controller/observer approaches are derived in section I-B, main results, which include exponential stability analysis, design of controller design for systems without uncertainty, and in another subsection with uncertainty, unknown nonlinear perturbations, and disturbances. Their performance is assessed by implementing them to two examples in Section I-C. Some concluding remarks are finally drawn in section I-D.

II. PROBLEM FORMULATION

Consider a class of nonlinear state space systems with uncertainties, unknown nonlinear perturbations, time-delays, and disturbances as

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A(t))\,x(t)+(A_d+\Delta A_d(t))\,x(t-d)\\ &\quad +(B+\Delta B(t))\,u(t)+f_1(x(t))+f_2(x(t-d))\\ &\quad +B_\omega \omega(t)\\ y(t) &= Cx(t)+D_\omega \omega(t)\end{aligned} \tag{1}$$

where $x(t)\epsilon R^n$, $u(t)$, $y(t)$, and $\omega(t)$ are the states of the system, the input signal, the output, and the input disturbance of the system including noise, respectively. d is time-delay value, $f_1(x(t))$, and $f_2(x(t-d))$ are unknown nonlinear perturbations. The constants of A, A_d, B, C, D_ω and B_ω are matrices with proper dimensions. $\Delta A(t), \Delta A_d(t)$, and $\Delta B(t)$ are uncertainties assumed to be norm-bounded with appropriate dimensions satisfying the following condition:

$$\begin{aligned}\Delta A(t) &= EF(t)H_1,\\ \Delta A_d(t) &= EF(t)H_2,\\ \Delta B(t) &= EF(t)H_3\end{aligned} \tag{2}$$

where E and H_i, $i = 1, 2, 3$ are the constant matrices with appropriate dimensions and $F(t)$ is the unknown continuous time-varying matrix function, satisfying

$$F^T(t)\,F(t) \leq I. \tag{3}$$

Equation (3) is utilized to obtain the following results [10], [30]:

$$\Delta^T(t)\,\Delta(t) \leq \Gamma^T \mathrm{MM}^T \Gamma, \tag{4}$$

where

$$\begin{aligned}\Delta(t) = F(t)\,[&(H_1+H_3K_1)x(t)+(H_2+H_3K_2)\,x(t-d)\\ &-(H_3K_1)e(t)-(H_3K_2)e(t-d)]\end{aligned}$$

and

$$\Gamma = \begin{bmatrix} x(t)\\ x(t-d)\\ e(t)\\ e(t-d)\end{bmatrix}, \mathrm{M} = \begin{bmatrix}(H_1+H_3K_1)^T\\ (H_2+H_3K_2)^T\\ -(H_3K_1)^T\\ -(H_3K_2)^T\end{bmatrix}$$

The model (1) can be an electromechanical system such as a robot, which has low accuracy and delay in the modeling of its state variables. It is also affected by environmental disturbances such as damage to actuators and low accuracy of sensors. For the Lyapunov functions and stability analysis, it is important to carefully consider the limitations and validity of the obtained results. This requires taking into account the specific assumptions made in the analysis, as well as the computational requirements of the chosen Lyapunov function.

Assumption 1 ([30]): The nonlinear functions $f_1(x(t))$, and $f_2(x(t-d))$ are unknown perturbations that satisfy $f_1(t,0)=0, f_2(t,0)=0$, and

$$f_1^T(x(t))f_1(x(t)) \leq \beta_1^2 x^T(t)\,x(t), \tag{5}$$

$$f_2^T\left(x\left(t-d\right)\right)f_2\left(x\left(t-d\right)\right) \le \beta_2^2 x^T\left(t-d\right)x\left(t-d\right), \quad (6)$$

where $\beta_1 \ge 0$, and $\beta_2 \ge 0$, are constants [30].

Assumption 2: The nonlinear functions $f_1(x)$ and $f_2(x(t-d))$ are called Lipschitz functions if the constants $\Omega_1, \Omega_2 > 0$ exist and satisfy

$$||f_1(x) - f_1(\hat{x})|| \le ||\Omega_1(x-\hat{x})|| \quad (7)$$

$$\left\|f_2\left(x\left(t-d\right)\right) - f_2\left(\hat{x}\left(t-d\right)\right)\right\| \le ||\Omega_2(x(t-d) - \hat{x}(t-d))|| \quad (8)$$

This assumption is used in many papers such as [36] and [37].

Lemma 1: [10], [38]: Consider $M_1(x)$ and $M_2(x)$ two quadratic matrix functions over R^n, and $M_2(x) \le 0$ for all $x \in \mathrm{R}^n - \{0\}$. Then, $M_1(x) < 0$ holds for all $x \in \mathrm{R}^n - \{0\}$ if and only if the constant $\varepsilon \ge 0$ exists such that

$$M_1(x) - \varepsilon M_2(x) < 0, \forall x \in \mathrm{R}^n - \{0\}. \quad (9)$$

The state observer and the state feedback controller are considered as

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + A_d\hat{x}(t-d) + Bu(t) \\ &\quad + L\left(y(t) - \hat{y}(t)\right) + f_1\left(\hat{x}(t)\right) + f_2\left(\hat{x}(t-d)\right) \\ \hat{y}(t) &= C\hat{x}(t) \\ u(t) &= K_1\hat{x}(t) + K_2\hat{x}(t-d) \end{aligned} \quad (10)$$

where $\hat{x}(t)$ is the estimation of $x(t)$, L is the state observer gain, $\hat{y}(t)$ is the estimation of $y(t)$, and K_1 and K_2 are the gains of the controller. The error of estimation is defined as $e(t) = x(t) - \hat{x}(t)$, where using (1) and (10), the following Equation is attained:

$$\begin{aligned}\dot{e}(t) &= (A - LC)e + A_d e(t-d) + B_\omega \omega(t) \\ &\quad + \left(f_1(x(t)) - f_1(\hat{x})\right) \\ &\quad + \left(f_2(x(t-d)) - f_2(\hat{x}(t-d))\right) \\ &\quad + \Delta A(t)x(t) + \Delta A_d x(t-d) \\ &\quad + \Delta B(t)\begin{pmatrix} K_1(x(t) - e(t)) \\ + K_2(x(t-d) - e(t-d)) \end{pmatrix} \end{aligned} \quad (11)$$

Substituting $u(t)$ from Equation (10) into Equation (1), and considering equation (2), we will have:

$$\begin{aligned}\dot{x}(t) &= (A + BK_1)x(t) + (A_d + BK_2)x(t-d) \\ &\quad - BK_1 e(t) - BK_2 e(t-d) \\ &\quad + f_1(x(t)) + f_2(x(t-d)) \\ &\quad + B_\omega \omega(t) + E\Delta(t) \end{aligned} \quad (12)$$

where $\Delta(t)$ is defined in equation (4). From Equations (11) and (12), the closed-loop system is represented as:

$$\begin{aligned}\begin{bmatrix}\dot{x}(t) \\ \dot{e}(t)\end{bmatrix} &= \begin{bmatrix} A + BK_1 & -BK_1 \\ 0 & A - LC \end{bmatrix}\begin{bmatrix} x(t) \\ e(t)\end{bmatrix} + \begin{bmatrix} A_d + BK_2 & -BK_2 \\ 0 & A_d \end{bmatrix}\begin{bmatrix} x(t-d) \\ e(t-d)\end{bmatrix} + \begin{bmatrix} B_\omega \\ B_\omega \end{bmatrix}\omega(t) \\ &\quad + \begin{bmatrix} f_1(x(t)) + f_2(x(t-d)) \\ \left(f_1(x(t)) - f_1(\hat{x})\right) + \left(f_2(x(t-d)) - f_2(\hat{x}(t-d))\right) \end{bmatrix} \\ &\quad + \begin{bmatrix} E \\ E \end{bmatrix}\Delta(t) \end{aligned} \quad (13)$$

It should be noted that Equations (7) and (8) can be rewritten as follows:

$$\begin{aligned}(f_1(x) - f_1(\hat{x}))^T I (f_1(x) - f_1(\hat{x})) &\le e^T(t)\Omega_1^T\Omega_1 e(t) \\ (f_2(x(t-d)) - f_2(\hat{x}(t-d)))^T I (f_2(x(t-d)) \\ - f_2(\hat{x}(t-d))) &\le e^T(t-d)\Omega_2^T\Omega_2 e(t-d) \end{aligned} \quad (14)$$

Lemma 2 (Schur Complement [38], [39]): For a Hermitian matrix M, the following inequalities are established:

$$\begin{aligned} M := \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} &< 0 \\ M_{11} \prec 0, M_{22} - M_{12}^T M_{11}^{-1} M_{12} &< 0 \\ M_{22} \prec 0, M_{11} - M_{12} M_{22}^{-1} M_{12}^T &< 0 \end{aligned} \quad (15)$$

III. MAIN RESULTS

A. EXPONENTIAL STABILITY ANALYSIS

In this part, the sufficient conditions for exponential stability of the following system with unknown nonlinear perturbations, time-delay, and disturbances are considered.

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_d x(t-d) + Bu(t) + f_1(x(t)) \\ &\quad + f_2(x(t-d)) + B_\omega \omega(t) \\ y(t) &= Cx(t) + D_\omega \omega(t) \\ x(t) &= \varphi(t), \forall t \in [-d, 0], \end{aligned} \quad (16)$$

where $\varphi(t)$ is a continuous function for the initial state of the system.

Theorem 1: For the model of the system with unknown nonlinear perturbations and disturbances in Equation (16), if the following LMI is true

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} < 0 \quad (17)$$

where

$$A_1 = \begin{bmatrix} \alpha P + A^T P + PA + Q + \beta_1^2 I & PA_d & PB_\omega \\ * & -e^{-\alpha d}Q + \beta_2^2 I & 0 \\ * & * & -\gamma^{-1}I \end{bmatrix},$$

$$B_1 = \begin{bmatrix} P & P & C^T \\ 0 & 0 & 0 \\ 0 & 0 & D_\omega^T \end{bmatrix}, C_1 = \begin{bmatrix} P^T & 0 & 0 \\ P^T & 0 & 0 \\ C & 0 & D_\omega \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -I & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -\gamma^{-1}I \end{bmatrix},$$

for $\alpha > 0$, the symmetric matrices $P > 0$ and $Q > 0$ exist, then the system is exponentially stable, and the disturbance effect on the system output is minimized as follows:

$$\sup_{\omega(t)\neq 0} \frac{\|y(t)\|_{L2}}{\|\omega(t)\|_{L2}} \le \gamma^{-1} \quad (18)$$

Proof: From Equation (16), we have:

$$\begin{aligned}&\underset{\omega(t)\neq 0}{sup}\frac{\|y(t)\|_{L2}}{\|\omega(t)\|_{L2}}\leq\gamma^{-1}\\&\equiv\|y(t)\|_{L_2}\prec\gamma^{-1}\|\omega(t)\|_{L_2}\\&\equiv\|y(t)\|_{L_2}^2\prec\gamma^{-2}\|\omega(t)\|_{L_2}^2\\&\equiv\gamma\|y(t)\|_{L_2}^2\\&\prec\gamma^{-1}\|\omega(t)\|_{L_2}^2\\&\equiv\int_0^{t\to\infty}\\&\times(\gamma y^T(\tau)y(\tau)-\gamma^{-1}\omega^T(\tau)\omega(\tau))d\tau\prec 0\end{aligned}\tag{19}$$

The Lyapunov function is considered as

$$V(t)\triangleq x^T(t)Px(t)+\int_{t-d}^{t}e^{\alpha(s-t)}x^T(s)Qx^T ds\tag{20}$$

Now, we want to show that the following inequality is correct:

$$\begin{aligned}&\dot V(x(t))+\alpha V(t)+\gamma y^T(t)y(t)\\&-\gamma^{-1}\omega^T(t)\omega(t)<0.\end{aligned}\tag{21}$$

Therefore, by comparing with Equation (19), the inequality (21) is true. By substituting Equation (16) into (21), the following relation is obtained:

$$\begin{bmatrix}x(t)\\x(t-d)\\\omega(t)\\f_1(x(t))\\f_2(x(t-d))\end{bmatrix}^T H\begin{bmatrix}x(t)\\x(t-d)\\\omega(t)\\f_1(x(t))\\f_2(x(t-d))\end{bmatrix}<0\tag{22}$$

where

$$H=\begin{bmatrix}\Lambda_1 & PA_d & PB_\omega+\gamma C^TD_\omega & P & P\\ * & -e^{-\alpha d}Q & 0 & 0 & 0\\ * & * & \gamma D^TD-\gamma^{-1}I & 0 & 0\\ * & * & * & 0 & 0\\ * & * & * & * & 0\end{bmatrix},$$

$$\Lambda_1=\alpha P+A^TP+PA+Q+\gamma C^TC.$$

Due to the presence of zero on the main diagonal of the matrix H, it is impossible to prove that H is a negative definite matrix. From Assumptions 1 and 2, the equations (5) and (6) can be rewritten as below:

$$\begin{aligned}&f_1^T(x(t))f_1(x(t))-\beta_1^2x^T(t)x(t)\leq 0,\\&f_2^T(x(t-d))f_2(x(t-d))-\beta_2^2x^T(t-d)x(t-d)\leq 0,\end{aligned}\tag{23}$$

where $\beta_1$andβ_2 are two positive constants. From Equations (22) and (23), we will have:

$$\begin{bmatrix}x(t)\\x(t-d)\\\omega(t)\\f_1(x(t))\\f_2(x(t-d))\end{bmatrix}^T \bar H\begin{bmatrix}x(t)\\x(t-d)\\\omega(t)\\f_1(x(t))\\f_2(x(t-d))\end{bmatrix}<0\tag{24}$$

where

$$\bar H=\begin{bmatrix}\bar\Lambda_1 & PA_d & PB_\omega+\gamma C^TD_\omega & P & P\\ * & -e^{-\alpha d}Q+\beta_2^2I & 0 & 0 & 0\\ * & * & \gamma D^TD-\gamma^{-1}I & 0 & 0\\ * & * & * & -I & 0\\ * & * & * & * & -I\end{bmatrix},$$

$$\bar\Lambda_1=\alpha P+A^TP+PA+Q+\gamma C^TC+\beta_1^2I.\tag{25}$$

If the matrix $\bar H$ is negative-definite, then Equation (24) is satisfied. Using Schur complement lemma on $\bar H<0$, it yields Equation (17).

By defining $\gamma^{-1}=\vartheta$ in (25) and D_1 matrix of Equation (17), the final LMI is obtained as

$$\begin{cases}\begin{bmatrix}A_1 & B_1\\ C_1 & D_1\end{bmatrix}<0\\ Q>0\end{cases}\equiv LMI(\vartheta,Q)\tag{26}$$

After solving the LMI (26), the optimal values of ϑ^*, P^* and Q^* are obtained and γ^* is calculated as $\gamma^*=\vartheta^{*-1}$. Therefore, the effect of external disturbance on the output of the system is reduced.

B. ROBUST STABILIZATION BASED ON OBSERVER FEEDBACK CONTROLLER

In this subsection, we will design an observer-based feedback controller for the nonlinear system (1) under two conditions: (A) regardless of uncertainty, (B) considering disturbance, unknown nonlinear perturbations, and uncertainty.

C. WITHOUT UNCERTAINTY

The state-space model of a nonlinear system with unknown nonlinear perturbations, time-delay, and disturbance is considered as

$$\begin{aligned}\dot x(t)&=Ax(t)+A_dx(t-d)+Bu(t)+f_1(x(t))\\&\quad+f_2(x(t-d))+B_\omega\omega(t)\\y(t)&=Cx(t)+D_\omega\omega(t)\\x(t)&=\varphi(t),\forall t\in[-d,0],\end{aligned}\tag{27}$$

The following theorem supplies the stabilization of the system (27) by the observer-based control.

Theorem 2: Consider the system model (16) and the state observer (10). If there exist symmetric matrices $P>0$, $Q>0$, and constants $\alpha,\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4,\beta_1,\beta_2>0$ such that fulfilled LMIs (29), then the system is stabled asymptotically and disturbance effects on system output are minimized as follows:

$$\underset{\omega(t)\neq 0}{sup}\frac{\|y(t)\|_{L2}}{\|\omega(t)\|_{L2}}\leq\gamma^{-1}\tag{28}$$

The following LMIs should be fulfilled:

$$\begin{cases} \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix} < 0, \\ P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0, \\ Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} > 0, \end{cases} \tag{29}$$

where

$$\Xi_{11} = \begin{bmatrix} \Phi_{11} & A_d + BK_2 & -BK_2 & -BK_2 & N_1 \\ * & -e^{-\alpha d} Q_1 & 0 & 0 & 0 \\ * & * & \Phi_{33} & P_2 A_d & 0 \\ * & * & * & \Phi_{44} & 0 \\ * & * & * & * & -N_2 \end{bmatrix}$$

$$\Xi_{12} = \begin{bmatrix} B_\omega & I & I & 0 & 0 & N_1(C^T + C_d^T) & N_1 C^T & \varepsilon_1 N_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_d^T & C_d^T & 0 & \varepsilon_2 I \\ P_2 B_\omega & 0 & 0 & P_2 & P_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_{22} = diag \begin{pmatrix} -\gamma^{-1} I, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_4 I, -\gamma^{-1} I, \\ -\gamma^{-1} I, -\varepsilon_1 \beta_1^{-2} I, -\varepsilon_2 \beta_2^{-2} I \end{pmatrix}$$

$$\Phi_{11} = \alpha N_1 + N_1 A^T + A N_1 + X^T B^T + BX,$$

$$\Phi_{33} = \alpha P_2 + A^T P_2 + P_2 A - C^T Y^T - YC + Q_2 + \varepsilon_3 \Omega_1^T \Omega_1$$

$$\Phi_{44} = -e^{-\alpha d} Q_2 + \varepsilon_4 \Omega_2^T \Omega_2$$

Proof: From equation (28), we have $\sup_{\omega(t)\neq 0} \frac{\|y(t)\|_{L2}}{\|\omega(t)\|_{L2}} \leq \gamma^{-1}$, or equivalently $\|y(t)\|_{L_2} \prec \gamma^{-1} \|\omega(t)\|_{L_2}$. If both sides of this expression are squared, then we obtain $\|y(t)\|_{L_2}^2 \prec \gamma^{-2} \|\omega(t)\|_{L_2}^2$. Then, multiplying both sides of the last term by γ yields $\gamma \|y(t)\|_{L_2}^2 \prec \gamma^{-1} \|\omega(t)\|_{L_2}^2$. Finally, we can find

$$\int_0^{t\to\infty} (\gamma y^T(\tau) y(\tau) - \gamma^{-1} \omega^T(\tau)\omega(\tau)) d\tau \prec 0 \tag{30}$$

The estimation error is defined with the following Equation:

$$e(t) = x(t) - \hat{x}(t). \tag{31}$$

Using (30), (27), and (31), we will obtain:

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} &= \begin{bmatrix} A + BK_1 & -BK_1 \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \\ &+ \begin{bmatrix} A_d + BK_2 & -BK_2 \\ 0 & A_d \end{bmatrix} \begin{bmatrix} x(t-d) \\ e(t-d) \end{bmatrix} \\ &+ \begin{bmatrix} B_\omega \\ 0 \end{bmatrix} \omega(t) \\ &+ \begin{bmatrix} f_1(x(t)) + f_2(x(t-d)) \\ (f_1(x(t)) - f_1(\hat{x})) + \begin{pmatrix} f_2(x(t-d)) \\ -f_2(\hat{x}(t-d)) \end{pmatrix} \end{bmatrix} \end{aligned} \tag{32}$$

The Lyapunov function is considered as

$$\begin{aligned} V(t) &\triangleq \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix} P \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \\ &+ \int_{t-d}^{t} e^{\alpha(s-t)} \begin{bmatrix} x^T(s) & e^T(s) \end{bmatrix} Q \begin{bmatrix} x(s) \\ e(s) \end{bmatrix} ds \end{aligned} \tag{33}$$

where $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0$, and $Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} > 0$ are positive definite matrix which should be found.

Now, we want to show that the following inequality is correct:

$$\dot{V}(x(t)) + \alpha V(t) + \gamma y^T(t) y(t) - \gamma^{-1} \omega^T(t) \omega(t) < 0. \tag{34}$$

Therefore, by comparing with Equation (30), the inequality (34) is true. By substituting Equation (27) into (34), the following relation is obtained:

$$\begin{bmatrix} x(t) \\ x(t-d) \\ e(t) \\ e(t-d) \\ \omega(t) \\ f_1(x(t)) \\ f_2(x(t-d)) \\ f_1(x(t)) - f_1(\hat{x}(t)) \\ f_2(x(t-d)) - f_2(\hat{x}(t-d)) \end{bmatrix}^T \times \Pi \begin{bmatrix} x(t) \\ x(t-d) \\ e(t) \\ e(t-d) \\ \omega(t) \\ f_1(x(t)) \\ f_2(x(t-d)) \\ f_1(x(t)) - f_1(\hat{x}(t)) \\ f_2(x(t-d)) - f_2(\hat{x}(t-d)) \end{bmatrix} < 0 \tag{35}$$

where as in (36), shown at the bottom of the next page.

Due to the presence of zero on the main diagonal of the matrix Π, it is impossible to prove that Π is a negative definite matrix. Equations (5)-(8) can be rewritten as below:

$$f_1^T(x(t)) f_1(x(t)) - \beta_1^2 x^T(t) x(t) \leq 0, \tag{37}$$

$$f_2^T(x(t-d)) f_2(x(t-d)) - \beta_2^2 x^T(t-d) x(t-d) \leq 0, \tag{38}$$

$$(f_1(x) - f_1(\hat{x}))^T I (f_1(x) - f_1(\hat{x})) - e^T(t) \Omega_1^T \Omega_1 e(t) \leq 0 \tag{39}$$

$$\begin{aligned} &(f_2(x(t-d)) - f_2(\hat{x}(t-d)))^T \\ &\times I (f_2(x(t-d)) - f_2(\hat{x}(t-d))) \\ &- e^T(t-d) \Omega_2^T \Omega_2 e(t-d) \leq 0 \end{aligned} \tag{40}$$

From (35), and (37)-(40), Lemma 1 , and Assumptions 1 and 2, we will have:

$$\begin{bmatrix} x(t) \\ x(t-d) \\ e(t) \\ e(t-d) \\ \omega(t) \\ f_1(x(t)) \\ f_2(x(t-d)) \\ f_1(x(t)) - f_1(\hat{x}(t)) \\ f_2(x(t-d)) - f_2(\hat{x}(t-d)) \end{bmatrix}^T \bar{\Pi} \begin{bmatrix} x(t) \\ x(t-d) \\ e(t) \\ e(t-d) \\ \omega(t) \\ f_1(x(t)) \\ f_2(x(t-d)) \\ f_1(x(t)) - f_1(\hat{x}(t)) \\ f_2(x(t-d)) - f_2(\hat{x}(t-d)) \end{bmatrix} < 0 \qquad (41)$$

where as in (42), shown at the bottom of the next page.

If the matrix $\bar{\Pi}$ is negative-definite, then Equation (34) is satisfied. Using Schur complement lemma on $\bar{\Pi} < 0$, it yields as in (43), shown at the bottom of page 9, where

$$\bar{\bar{\Pi}}_1 = \alpha P_1 + A^T P_1 + P_1 A + Q_1 + K_1^T B^T P_1 + P_1 B K_1$$

By pre-and-post multiplying both sides of Equation (43) by $diag(P_1^{-1}, I, I, I, I, I, I, I, I, I, I, I, I)$, we have

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ * & \Gamma_{22} \end{bmatrix}, \qquad (44)$$

where

$$\Gamma_{11} = \begin{bmatrix} \theta_{11} & A_d + BK_2 & -BK_2 & -BK_2 \\ * & -e^{-\alpha d} Q_1 & 0 & 0 \\ * & * & \theta_{33} & P_2 A_d \\ * & * & * & \theta_{44} \end{bmatrix},$$

$$\Gamma_{12} = \begin{bmatrix} B_\omega & I & I & 0 & 0 & P_1^{-1} C^T & P_1^{-1} C^T & \varepsilon_1 P_1^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2 I \\ P_2 B_\omega & 0 & 0 & P_2 & P_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{22} = diag\begin{pmatrix} -\gamma^{-1} I, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_4 I, -\gamma^{-1} I, \\ -\gamma^{-1} I, -\varepsilon_1 \beta_1^{-2} I, -\varepsilon_2 \beta_2^{-2} I \end{pmatrix},$$

$$\theta_{11} = \alpha P_1^{-1} + P_1^{-1} A^T + A P_1^{-1} + P_1^{-1} Q_1 P_1^{-1} + P_1^{-1} K_1^T B^T + B K_1 P_1^{-1},$$

$$\theta_{33} = \alpha P_2 + (A - LC)^T P_2 + P_2 (A - LC) + Q_2 + \varepsilon_3 \Omega_1^T \Omega_1,$$

$$\theta_{44} = -e^{-\alpha d} Q_2 + \varepsilon_4 \Omega_2^T \Omega_2$$

Using Schur complement lemma and defining $P_1^{-1} = N_1$, $Q_1^{-1} = N_2$, $X = K_1 P_1^{-1}$, and $P_2 L = Y$ for linearization purpose, the LMI condition (29) is obtained. Therefore, the effect

$$\Pi = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} < 0,$$

$$A_2 = \begin{bmatrix} \Pi_1 & P_1 A_d + P_1 B K_2 & -P_1 B K_2 & -P_1 B K_2 \\ (P_1 A_d + P_1 B K_2)^T & -e^{-\alpha d} Q_1 & 0 & 0 \\ (-P_1 B K_2)^T & 0 & \Pi_3 & P_2 A_d \\ (-P_1 B K_2)^T & 0 & (P_2 A_d)^T & -e^{-\alpha d} Q_2 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \chi & P_1 & P_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ P_2 B_\omega & 0 & 0 & P_2 & P_2 \\ 0 & 0 & 0 & 0 & 0 \\ -\gamma^{-1} I + \gamma D_\omega^T D_\omega & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\chi = P_1 B_\omega + \gamma C^T D_\omega$$

$$C_2 = \begin{bmatrix} \chi^T & 0 & (P_2 B_\omega)^T & 0 & (-\gamma^{-1} I + \gamma D_\omega^T D_\omega)^T \\ P_1^T & 0 & 0 & 0 & 0 \\ P_1^T & 0 & 0 & 0 & 0 \\ 0 & 0 & P_2^T & 0 & 0 \\ 0 & 0 & P_2^T & 0 & 0 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_1 = \alpha P_1 + A^T P_1 + P_1 A + Q_1 + K_1^T B^T P_1 + P_1 B K_1 + \gamma C^T C$$

$$\Pi_3 = \alpha P_2 + (A - LC)^T P_2 + P_2 (A - LC) + Q_2 \qquad (36)$$

of external disturbances, time-delay, and unknown nonlinear perturbations on the system's output is reduced. □

D. WITH UNCERTAINTY, UNKNOWN NONLINEAR PERTURBATIONS, AND DISTURBANCES

The following theorem considers designing an observer-based feedback controller regarding the set of LMIs with uncertainty, unknown nonlinear perturbations, time-delay, and disturbances.

Theorem 3: Consider the state-space model of systems (1) with uncertainties, unknown nonlinear perturbations, time-delay, and disturbances. Suppose $P > 0$, $Q > 0$ and $Y > 0$ are matrices with proper dimensions and $\alpha, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \beta_1, \beta_2 > 0$. If the following LMIs exist:

$$\begin{cases} \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} < 0, \\ P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0, \\ Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} > 0, \end{cases} \tag{45}$$

where

$$\Theta_{11} = \begin{bmatrix} F_{11} & A_d + BK_2 & -BK_2 & -BK_2 & N_1 \\ * & -e^{-\alpha d}Q_1 & 0 & 0 & 0 \\ * & * & F_{33} & P_2A_d & 0 \\ * & * & * & F_{44} & 0 \\ * & * & * & * & -N_2 \end{bmatrix}$$

$$\Theta_{12} = \begin{bmatrix} B_\omega & I & I & 0 & 0 & N_1C^T & N_1C^T & \varepsilon_1 N_1 & 0 & N_1C^T + X^T H_3^T \\ 0 & 0 & 0 & 0 & 0 & C_d^T & C_d^T & 0 & \varepsilon_2 I & (H_2 + H_3K_2)^T \\ P_2B_\omega & 0 & 0 & P_2 & P_2 & 0 & 0 & 0 & 0 & -(H_3K_2)^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(H_3K_2)^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Theta_{22} = diag\begin{pmatrix} -\gamma^{-1}I, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_4 I, -\gamma^{-1}I, \\ -\gamma^{-1}I, -\varepsilon_1\beta_1^{-2}I, -\varepsilon_2\beta_2^{-2}I, -I \end{pmatrix}$$

$$\bar{\Pi} = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} < 0,$$

$$A_3 = \begin{bmatrix} \bar{\Pi}_1 & P_1A_d + P_1BK_2 & -P_1BK_2 & -P_1BK_2 \\ (P_1A_d + P_1BK_2)^T & \bar{\Pi}_2 & 0 & 0 \\ (-P_1BK_2)^T & 0 & \bar{\Pi}_3 & P_2A_d \\ (-P_1BK_2)^T & 0 & (P_2A_d)^T & \bar{\Pi}_4 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} \chi & P_1 & P_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ P_2B_\omega & 0 & 0 & P_2 & P_2 \\ 0 & 0 & 0 & 0 & 0 \\ -\gamma^{-1}I + \gamma D_\omega^T D_\omega & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} (\chi)^T & \left(\gamma C_d^T D_\omega\right)^T & (P_2B_\omega)^T & 0 & \left(-\gamma^{-1}I + \gamma D_\omega^T D_\omega\right)^T \\ P_1^T & 0 & 0 & 0 & 0 \\ P_1^T & 0 & 0 & 0 & 0 \\ 0 & 0 & P_2^T & 0 & 0 \\ 0 & 0 & P_2^T & 0 & 0 \end{bmatrix}$$

$$D_3 = \begin{bmatrix} -\varepsilon_1 I & 0 & 0 & 0 \\ 0 & -\varepsilon_2 I & 0 & 0 \\ 0 & 0 & -\varepsilon_3 I & 0 \\ 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix}$$

$$\begin{aligned} \bar{\Pi}_1 &= \alpha P_1 + A^T P_1 + P_1 A + Q_1 + K_1^T B^T P_1 + P_1 B K_1 + \gamma C^T C + \varepsilon_1 \beta_1^2 I \\ \bar{\Pi}_2 &= -e^{-\alpha d} Q_1 + \varepsilon_2 \beta_2^2 I + \gamma C_d^T C_d \\ \bar{\Pi}_3 &= \alpha P_2 + (A - LC)^T P_2 + P_2 (A - LC) + Q_2 + \varepsilon_3 \Omega_1^T \Omega_1 \\ \bar{\Pi}_4 &= -e^{-\alpha d} Q_2 + \varepsilon_4 \Omega_2^T \Omega_2 \end{aligned} \tag{42}$$

$$F_{11} = \alpha N_1 + N_1 A^T + A N_1 + X^T B^T + BX,$$
$$F_{33} = \alpha P_2 + A^T P_2 + P_2 A - C^T Y^T - YC + Q_2 + \varepsilon_3 \Omega_1^T \Omega_1$$
$$F_{44} = -e^{-\alpha d} Q_2 + \varepsilon_4 \Omega_2^T \Omega_2$$

then, the system (1) is exponentially stable.

Proof: The proof is similar to Theorem 2; the following relation can be written as in (46), shown at the bottom of page 10, where as in (47), shown at the bottom of page 10. Due to the presence of zero on the main diagonal of the matrix Σ, it is impossible to prove that Σ is a negative definite matrix. From Equations (37)-(40), and following inequality

$$\Delta^T(t)\Delta(t) - \Gamma^T.\mathrm{MM}^T.\Gamma \le 0,$$

we will have

$$\begin{bmatrix} x(t) \\ x(t-d) \\ e(t) \\ e(t-d) \\ \omega(t) \\ f_1(x(t)) \\ f_2(x(t-d)) \\ f_1(x(t)) - f_1(\hat{x}(t)) \\ f_2(x(t-d)) - f_2(\hat{x}(t-d)) \\ \Delta(t) \end{bmatrix}^T \bar{\Sigma} \begin{bmatrix} x(t) \\ x(t-d) \\ e(t) \\ e(t-d) \\ \omega(t) \\ f_1(x(t)) \\ f_2(x(t-d)) \\ f_1(x(t)) - f_1(\hat{x}(t)) \\ f_2(x(t-d)) - f_2(\hat{x}(t-d)) \\ \Delta(t) \end{bmatrix} < 0 \quad (48)$$

where, as shown in the equation at the bottom of the next page,

$$\bar{\Sigma}_1 = \alpha P_1 + A^T P_1 + P_1 A + Q_1 + K_1^T B^T P_1 + P_1 B K_1 + \gamma C^T C + \beta_1^2 I$$
$$\bar{\Sigma}_2 = P_1 A_d + P_1 B K_2$$
$$\bar{\Sigma}_5 = P_1 B_\omega + \gamma C^T D_\omega$$
$$\bar{\Sigma}_3 = \alpha P_2 + (A - LC)^T P_2 + P_2 (A - LC) + Q_2 - e^{-\alpha d} Q_2 + \Omega_1^T \Omega_1$$

Using Schur complement lemma on $\bar{\Sigma} < 0$, it yields as in (49), shown at the bottom of page 11, where

$$\bar{\bar{\Sigma}}_1 = \alpha P_1 + A^T P_1 + P_1 A + Q_1 + K_1^T B^T P_1 + P_1 B K_1$$
$$\bar{\bar{\Sigma}}_3 = \alpha P_2 + (A - LC)^T P_2 + P_2 (A - LC) + Q_2 + \varepsilon_3 \Omega_2^T \Omega_2$$
$$\bar{\bar{\Sigma}}_4 = -e^{-\alpha d} Q_2 + \varepsilon_4 \Omega_1^T \Omega_1$$

$$\bar{\bar{\Pi}} = \begin{bmatrix} A_4 & B_4 \\ C_4 & D_4 \end{bmatrix} < 0,$$

$$A_4 = \begin{bmatrix} \bar{\Pi}_1 & P_1 A_d + P_1 B K_2 & -P_1 B K_2 & -P_1 B K_2 & P_1 B_\omega & P_1 \\ (P_1 A_d + P_1 B K_2)^T & -e^{-\alpha d} Q_1 & 0 & 0 & 0 & 0 \\ (-P_1 B K_2)^T & 0 & \bar{\Pi}_3 & P_2 A_d & P_2 B_\omega & 0 \\ (-P_1 B K_2)^T & 0 & (P_2 A_d)^T & \bar{\Pi}_4 & 0 & 0 \\ (P_1 B_\omega)^T & 0 & (P_2 B_\omega)^T & 0 & -\gamma^{-1} I & 0 \\ P_1^T & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix}$$

$$B_4 = \begin{bmatrix} P_1 & 0 & 0 & C^T & C^T & \varepsilon_1 I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2 I \\ 0 & P_2 & P_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} P_1^T & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_2^T & 0 & 0 & 0 \\ 0 & 0 & P_2^T & 0 & 0 & 0 \\ C & 0 & 0 & 0 & D_\omega^T & 0 \\ C & 0 & 0 & 0 & 0 & 0 \\ (\varepsilon_1 I)^T & 0 & 0 & 0 & 0 & 0 \\ 0 & (\varepsilon_2 I)^T & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_4 = \begin{bmatrix} -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varepsilon_4 I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma^{-1} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma^{-1} I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 \beta_1^{-2} I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 \beta_2^{-2} I \end{bmatrix} \quad (43)$$

By pre-and-post multiplying both sides of Equation (49) by $diag(P_1^{-1}, I, I, I, I, I, I, I, I, I, I, I, I)$, we have

$$\begin{bmatrix} \Lambda_1 & \Lambda_2 \\ * & \Lambda_3 \end{bmatrix}, \tag{50}$$

where, as shown in the equation at the bottom of the next page. Using Schur complement lemma and defining $P_1^{-1} = N_1, Q_1^{-1} = N_2z, X = K_1P_1^{-1}$, and $P_2L = Yz$ for linearization purpose, the LMI condition (45) is obtained. Therefore, the effect of uncertainties, external disturbances, time-delay, and unknown nonlinear perturbations on the system's output is reduced.

Remark 1: By proving the inequalities (35), (41), (46) and (48) in the mentioned systems, in addition to the stability proof of systems, the observer estimation error will also be converged to zero.

IV. SIMULATION RESULTS

Over the past few decades, there has been tremendous growth in flexible-link manipulator usage in various industrial and medical applications [40], [41]. Conventional robotic arms are designed to achieve a minimum vibration [42]. As a result, these robotic arms require heavy materials, high power drives, and huge parts [43], [44]. However, FLMs have more advantages than rigid arms: low power consumption, faster-operating speed, more excellent mass loading capability, lower arm motion, easier transportation, lower cost, and greater safety for operators. In addition, it is easier to maintain and repair these arms. The most common challenges

$$\begin{bmatrix} x(t) \\ x(t-d) \\ e(t) \\ e(t-d) \\ \omega(t) \\ f_1(x(t)) \\ f_2(x(t-d)) \\ f_1(x(t)) - f_1(\hat{x}(t)) \\ f_2(x(t-d)) - f_2(\hat{x}(t-d)) \\ \Delta(t) \end{bmatrix}^T \Sigma \begin{bmatrix} x(t) \\ x(t-d) \\ e(t) \\ e(t-d) \\ \omega(t) \\ f_1(x(t)) \\ f_2(x(t-d)) \\ f_1(x(t)) - f_1(\hat{x}(t)) \\ f_2(x(t-d)) - f_2(\hat{x}(t-d)) \\ \Delta(t) \end{bmatrix} < 0 \tag{46}$$

$$\Sigma = \begin{bmatrix} \Sigma_1 & P_1A_d + P_1BK_2 & -P_1BK_2 & -P_1BK_2 & P_1B_\omega + \gamma C^T D_\omega & P_1 & P_1 & 0 & 0 & P_1E \\ * & -e^{-\alpha d}Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_2 & P_2A_d & P_2B_\omega & 0 & 0 & P_2 & P_2 & P_2E \\ * & * & * & -e^{-\alpha d}Q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^{-1}I + \gamma D_\omega^T D_\omega & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & * & 0 \end{bmatrix}$$

$$\Sigma_1 = \alpha P_1 + A^T P_1 + P_1 A + Q_1 + K_1^T B^T P_1 + P_1 B K_1 + \gamma C^T C$$
$$\Sigma_2 = \alpha P_2 + (A - LC)^T P_2 + P_2 (A - LC) + Q_2 \tag{47}$$

$$\bar{\bar{\Sigma}} = \begin{bmatrix} \bar{\Sigma}_1 & \bar{\Sigma}_2 & -P_1BK_2 & -P_1BK_2 & \bar{\Sigma}_5 & P_1 & P_1 & 0 & 0 & P_1E & (H_1 + H_3K_1)^T \\ * & -e^{-\alpha d}Q_1 + \beta_2^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (H_2 + H_3K_2)^T \\ * & * & \bar{\Sigma}_3 & P_2A_d & P_2B_\omega & 0 & 0 & P_2 & P_2 & P_2E & -(H_3K_2)^T \\ * & * & * & -e^{-\alpha d}Q_2 + \Omega_2{}^T\Omega_2 & 0 & 0 & 0 & 0 & 0 & 0 & -(H_3K_2)^T \\ * & * & * & * & -\gamma^{-1}I + \gamma D_\omega^T D_\omega & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix}$$

and problems in the control of FLMs are vibration control in the presence of external disturbances [45], [46]. The flexible arm's vibration can affect the end effector's final position and reduce its accuracy. The control of these systems is always subject to various disturbances, including climatic and environmental conditions, data errors, uncertainties, and variations in some system parameters. Therefore, the system dynamic of example 1 in the simulation part is FLM. The second example is the continuous stirred tank reactor system at the presence of time-delay, disturbance, uncertainty, and nonlinear perturbations. This system is used in chemical processes, and it is an irreversible and exothermic reaction. The dynamic model is highly nonlinear with external disturbances and uncertain parameters; this process cannot be controlled robustly by traditional controllers [47], [48]. Therefore, lots of research studies have been done to control this system accurately [49], [50], [51]. In this part, the feasible solutions of parameters of state feedback controller and Luenburger observer K and L are obtained via MATLAB YALMIP toolbox. These gains are found by calculating the values of P, Q and Y in the mentioned theorems via YALMIP solver satisfying LMI of Eq. (45). In Fig.1, the algorithm of this approach is demonstrated.

Example 1 (Without Uncertainty): To assess the performance of the proposed controller/ observer designs, we implement them to the flexible-link manipulator system, which is demonstrated in Fig., and described by the following equations [52]:

$$\begin{aligned}\dot{\theta}_m &= \omega_m\\ \dot{\omega} &= \frac{k}{I_m}(\theta_l-\theta_m)-\frac{C_{vf}}{I_m}\omega_m+\frac{K_\tau}{I_m}u\end{aligned}$$

$$\left[\begin{array}{cccccccccccccc}
\overline{\overline{\Sigma}}_1 & P_1A_d+P_1BK_2 & -P_1BK_2 & -P_1BK_2 & P_1B_\omega & P_1 & P_1 & 0 & 0 & C^T & C^T & \varepsilon_1 I & 0 & (H_1+H_3K_1)^T\\
* & -e^{-\alpha d}Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2 I & (H_2+H_2K_2)^T\\
* & * & \overline{\bar{\Sigma}}_3 & P_2A_d & P_2B_\omega & 0 & 0 & P_2 & P_2 & 0 & 0 & 0 & 0 & -(H_3K_2)^T\\
* & * & * & \overline{\bar{\Sigma}}_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(H_3K_2)^T\\
* & * & * & * & -\gamma^{-1}I & 0 & 0 & 0 & 0 & D_\omega & 0 & 0 & 0 & 0\\
* & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & * & -\gamma^{-1}I & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & * & * & -\gamma^{-1}I & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & * & * & * & -\varepsilon_1\beta_1^{-2}I & 0 & 0\\
* & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2\beta_2^{-2}I & 0\\
* & * & * & * & * & * & * & * & * & * & * & * & * &
\end{array}\right. \tag{49}$$

$$\Lambda_1=\begin{bmatrix}\Lambda_{11} & A_d+BK_2 & -BK_2 & -BK_2\\ * & -e^{-\alpha d}Q_1 & 0 & 0\\ * & * & \Lambda_{33} & P_2A_d\\ * & * & * & \Lambda_{44}\end{bmatrix},$$

$$\Lambda_2=\left[\begin{array}{ccccccc}B_\omega & I & I & 0 & 0 & P_1^{-1}C^T & P_1^{-1}C^T\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ P_2B_\omega & 0 & 0 & P_2 & P_2 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}\right.$$

$$\left.\begin{array}{ccc}\varepsilon_1P_1^{-1} & 0 & P_1^{-1}(H_1+H_3K_1)^T\\ 0 & \varepsilon_2 I & (H_2+H_3K_2)^T\\ 0 & 0 & -(H_3K_2)^T\\ 0 & 0 & -(H_3K_2)^T\end{array}\right],$$

$$\Lambda_3=diag\begin{pmatrix}-\gamma^{-1}I,-\varepsilon_1 I,-\varepsilon_2 I,-\varepsilon_3 I,-\varepsilon_4 I,-\gamma^{-1}I,\\ -\gamma^{-1}I,-\varepsilon_1\beta_1^{-2}I,-\varepsilon_2\beta_2^{-2}I,-I\end{pmatrix}$$

$$\Lambda_{11}=\alpha P_1^{-1}+P_1^{-1}A^T+AP_1^{-1}+P_1^{-1}Q_1P_1^{-1}+P_1^{-1}K_1^TB^T+BK_1P_1^{-1},$$

$$\Lambda_{33}=\alpha P_2+(A-LC)^TP_2+P_2(A-LC)+Q_2+\varepsilon_3\Omega_1^T\Omega_1,$$

$$\Lambda_{44}=-e^{-\alpha d}Q_2+\varepsilon_4\Omega_2^T\Omega_2$$

$$\dot{\theta}_l = \omega_l$$
$$\dot{\omega}_l = -\frac{k}{I_l}(\theta_l - \theta_m) - \frac{mgh}{I_m} sin(\theta_l). \tag{51}$$

TABLE 1. Parameters values [52].

Parameter	Value
Motor inertia, $I_m(kgm^2)$	3.7×10^{-3}
Link inertia, $I_l(kgm^2)$	9.3×10^{-3}
Constant of torsional spring $k(Nm/rad)$	1.8×10^{-1}
mass of pointer, $m(kg)$	2.1×10^{-1}
Coefficient of viscous friction $C_{vf}(Nm/V)$	4.6×10^{-2}
Gain of amplifier $K_\tau(Nm/V)$	8.0×10^{-2}

The parameters of the system are given in Table 1. The sinusoidal disturbance is added to E, and its coefficient matrix is $B_\omega = [0 \ \ 1 \ \ 0.05 \ \ 0]^T$. Using the parameter values provided in Table 1, we can rewrite system (51) is the state-space form (1), with:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.26 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 \ 0 \ 0 \ 0 \\ 0 \ 1 \ 0 \ 0 \end{bmatrix}.$$

The results of Theorem 2 are used to obtain the controller and the observer gains so as to stabilize the flexible-link manipulator system. The initial values of the system are chosen as: $x(0) = \left[1.5 \ -1 \ 2 \ -0.2\right]^T$ and $\hat{x}(0) = \left[1.5 \ 1 \ -2 \ -0.2\right]^T$. The optimal values of controller and observer gains are obtained using MATLAB solver as:

$$L = \begin{bmatrix} 5390 \ -5580 \\ 43.4 \ 43.4 \\ 5460 \ 5510 \\ 58.1 \ 58.1 \end{bmatrix}$$
$$k = \left[-0.2 \ -0.6 \ -0.2 \ -0.02\right]$$

Figs. 3 and 4 depict the dynamics of the system states along with their estimates in 2.5 seconds, it is clear that if the states are inaccessible, the suggested method can estimate states well. Fig. 5 shows the estimation errors converge to zero before 2 seconds, and it shows the good performance of Luenberger observer which its gain is obtained from LMI, and Fig. 6 illustrates control input with appropriate overshoot, respectively. In order to show the validation of this approach, the results are compared with [5]. In [5], an observer-based composite nonlinear feedback controller is designed for systems with uncertainty, nonlinear function, disturbance and time-delay; but, as it is obvious in example 1, it cannot make the system stable.

It is shown in the simulation results that the system states, as well as system state estimates, converge to zero, and the system is asymptotically stable. The method of paper [5] cannot make the system stable and the response of system is oscillating; but, the Luenberger observer estimates the states of the system suitably. To evaluate the performance of suggested method and method of paper [5], the following table contains steady state error E_{ss}, settling time T_s, maximum

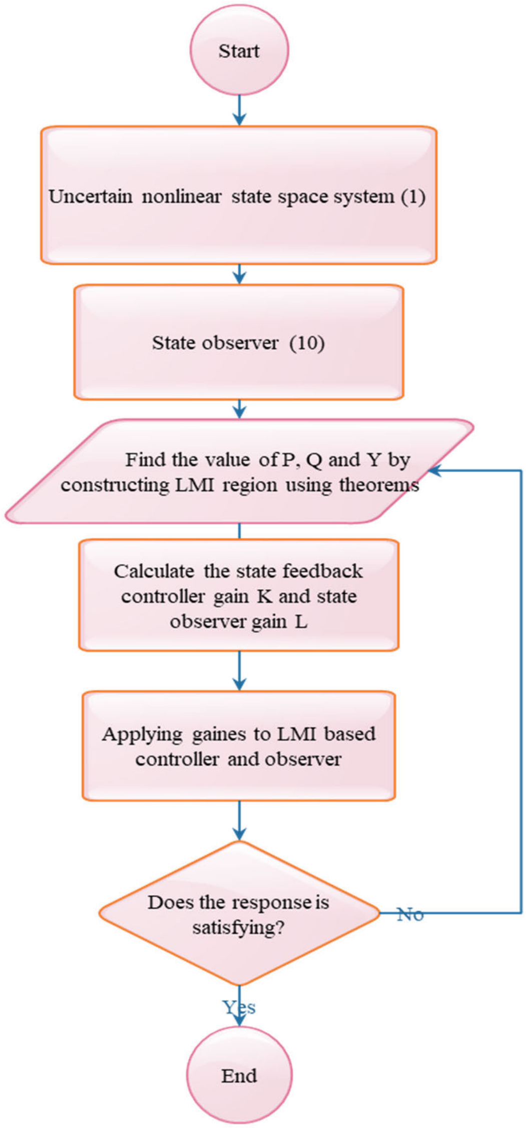

FIGURE 1. LMI-based observer and controller algorithm.

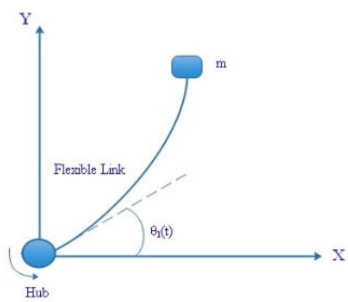

FIGURE 2. Flexible link manipulator.

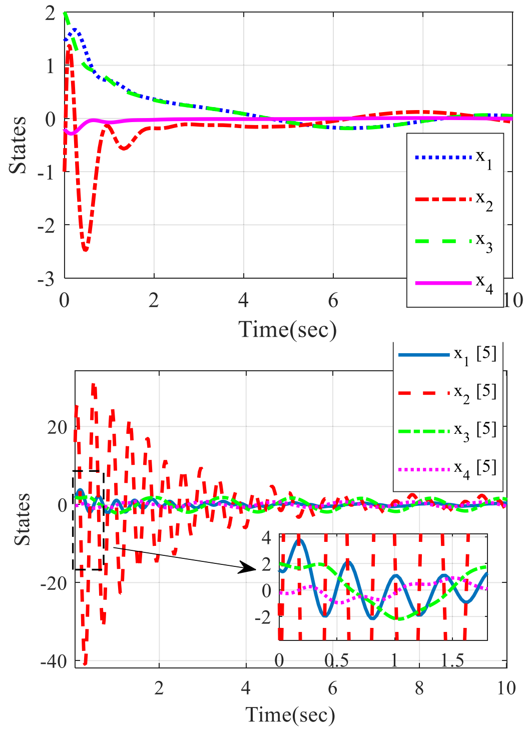

FIGURE 3. Time responses of the system states.

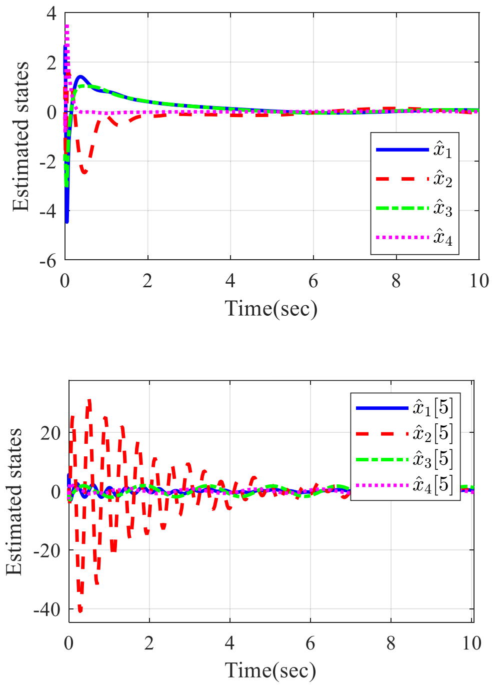

FIGURE 4. Time trajectories of the state's estimates.

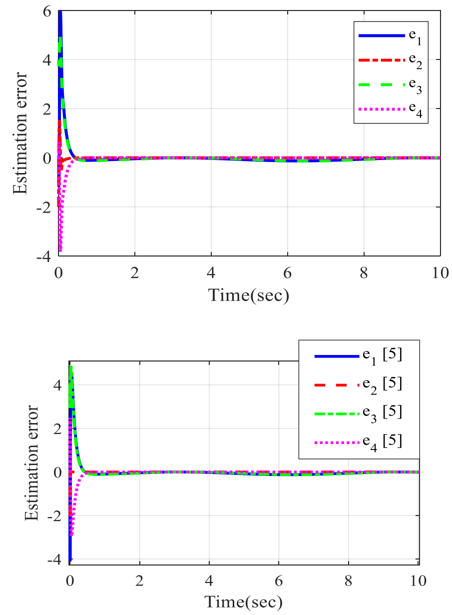

FIGURE 5. Time histories of the estimation errors.

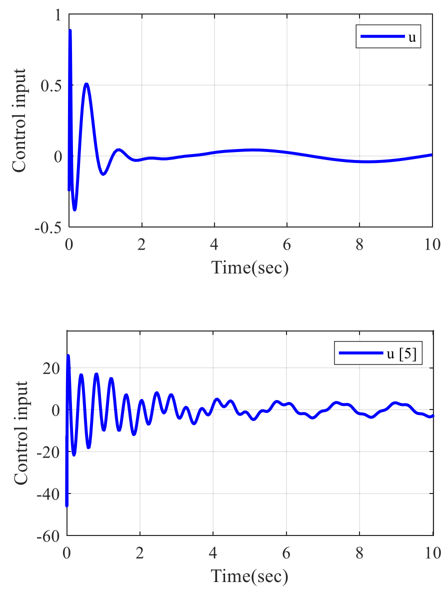

FIGURE 6. Control input.

TABLE 2. Comparison of performance parameters of example 1.

Method	E_{ss}	T_s	u_{max}	u_{min}
Proposed method	0.1	6	0.9	-0.4
Method in [5]	1	6	28	-46

value of control input u_{max}, minimum value of control input u_{min}.

Example 2 (With Uncertainty, Unknown Nonlinear Perturbations and Disturbances): In this part, to demonstrate the validity of the performance of the proposed controller/observer, we implement them to the continuous stirred tank reactor system according to Fig.7, which is described by the following state-space model coefficient [48]:

$$A = \begin{bmatrix} 0.6 & 0 \\ 0 & -0.24 \end{bmatrix},$$

$$E = I_{2\times 2}, F = \cos(0.2t) \times I_{2\times 2},$$
$$H1 = \begin{bmatrix} -0.9444\mu_1 & -0.002\mu_1 \\ -2.3331\mu_1\mu_2 & -0.916\mu_1\mu_2 \end{bmatrix},$$
$$A_d = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.16 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0.8368 \end{bmatrix},$$
$$B_w = \begin{bmatrix} 0.02 \\ 0.4 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C_d = D_w = 0, w = \sin(t + \frac{\pi}{4}), \quad (52)$$

where $\mu_1$andμ_2 are uncertainty parameters, the time-delay is $d = 1$ sec. It should be mentioned that we assume $H_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, H_3 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$.

The results of Theorem 3 are used to obtain the controller and the observer gains so as to stabilize the continuous stirred tank reactor system when the initial values of the system are chosen as $x(0) = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}$ and $\hat{x}(0) = \begin{bmatrix} 0.5 \\ 5 \end{bmatrix}$.The optimal values of controller and observer gains are obtained using MATLAB solver as:

$$L = \begin{bmatrix} 200 & 0 \\ 0 & 200 \end{bmatrix}$$
$$K = \begin{bmatrix} 1.1 & -51 \end{bmatrix}$$

By applying these gains to the system results of simulation are shown in Figs. 8 to 11. Figs. 8 and 9 depict the dynamics of the system states with uncertainty parameters and their estimations, it is clear that the suggested method can estimate states well if the states are inaccessible. Fig. 10 shows the estimation errors in uncertain modes converge to zero in 5 seconds, and it shows the good performance of Luenberger observer which its gain obtaining from LMI and Fig. 11 illustrates control input of uncertain system, respectively.

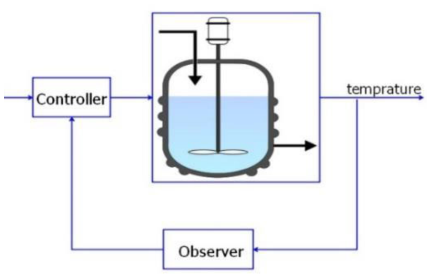

FIGURE 7. Schematic of continuous stirred tank reactor control.

As it is obvious from Figs. 8-11, in [6], the system states converge to zero in 45 *sec*, the response of system is oscillating, the estimation error is not equal to zero, the steady-state error is more than that of our suggested approach, and the amplitude of control signal is more than that of our proposed method. The comparison of performance parameters of example 2 is shown in Table 3.

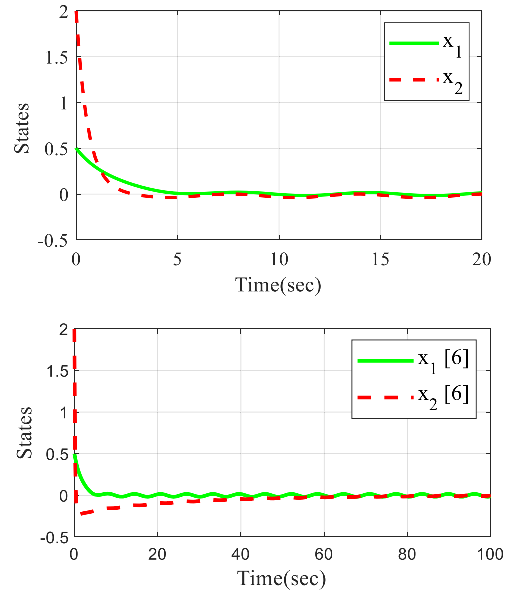

FIGURE 8. Time responses of the system states.

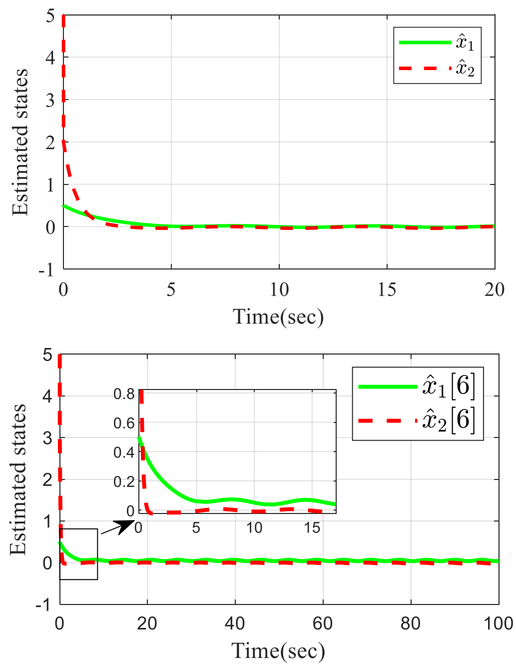

FIGURE 9. Time trajectories of the state's estimates$\hat{x}_1, \hat{x}_2$.

TABLE 3. Comparison of performance parameters of example 2.

Method	E_{ss}	T_s	u_{max}	u_{min}
Proposed method	0.001	5	0	-250
Method in [6]	0.017	45	1.2	-660

According to the two examples mentioned above, when the uncertainties are not considered, the simulation worked well, and in the presence of uncertainty, the theory had a good performance, and in special circumstances $\Delta A_d = \Delta B_d = 0$, this theory is responsive.

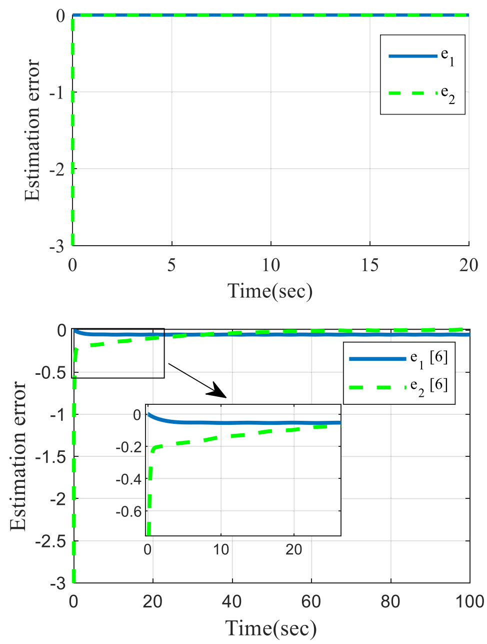

FIGURE 10. Time histories of the estimation errors.

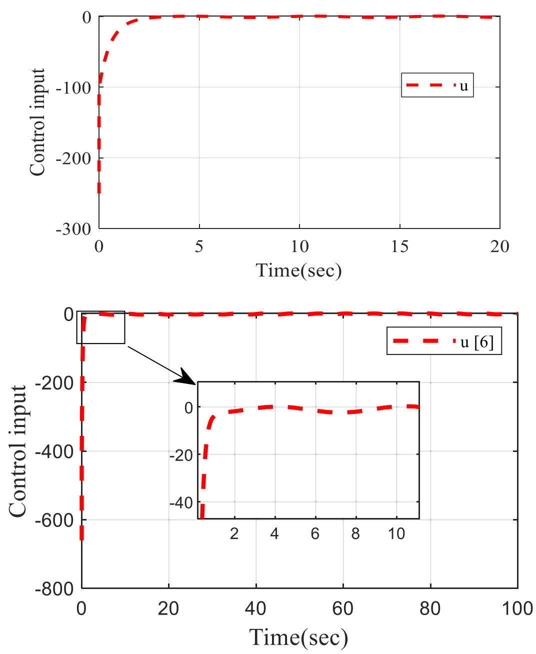

FIGURE 11. Control input.

V. CONCLUSION

The simultaneous design of controller and observer for a class of systems in the presence of uncertainties, unknown nonlinear perturbations, constant time-delays, and disturbances is considered in this paper. The observer-based state-feedback controller is proposed. Using the Lyapunov theory and LMIs techniques, the exponential stability of the close loop system is proved. Both observer and controller gains are calculated. The proposed controller is successfully implemented into two examples. The obtained simulation results showed that the estimation errors and the system states converge to the origin and that system stability was guaranteed. In this study, the effects of input saturation and time varying delay have not been investigated. Thus, future studies could be focused on how to stabilize the nonlinear systems in the presence of uncertainties, unknown nonlinear perturbations, time-delays, disturbances, sensor fault, cyber-attack, input saturation and time varying delay using LMIs.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

CONFLICTS OF INTEREST

The authors declare that they have no conflict of interest.

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