IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 30 May 2023, accepted 24 June 2023, date of publication 3 July 2023, date of current version 11 July 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3291617

RESEARCH ARTICLE

Physics-Based Finite Element Simulation of the Dynamics of Soft Robots

KEVIN WANDKE[1,2], (Graduate Student Member, IEEE), AND Y Z[1,2,3,4], (Member, IEEE)
[1]Department of Electrical and Computer Engineering, University of Illinois at Urbana–Champaign, Urbana, IL 61801, USA
[2]Beckman Institute for Advanced Science and Technology, University of Illinois at Urbana–Champaign, Urbana, IL 61801, USA
[3]Department of Nuclear, Plasma, and Radiological Engineering, University of Illinois at Urbana–Champaign, Urbana, IL 61801, USA
[4]Department of Nuclear Engineering and Radiological Sciences, University of Michigan, Ann Arbor, MI 48109, USA

Corresponding author: Y Z (yzyz@umich.edu)

ABSTRACT Soft robots grew to prominence in large part because they promised a new and exciting means for engineers to develop robots that are both highly adaptable and safe for direct human interaction. However, despite showing substantial promise in this area, soft robots have not yet seen widespread adoption. Two major factors that have prevented the development of soft robots are the fundamental challenges of both the modeling and control of soft structures. While traditional robots enjoy a myriad of theoretical and computational tools for modeling and control, the options for soft robots are far more limited. In this work, we introduce a physics-based finite element simulation platform, *Kraken*, that can be used to accurately model the dynamic and oscillatory motions of soft robots. After a brief theoretical introduction to hyperelastic modeling and the finite element method, we show the utility of our approach by simulating the oscillations of a 1D hyperelastic actuator, the dynamics of a 2D hyperelastic pendulum, and a 3D spherical hyperelastic pendulum. We then demonstrate the accuracy of our approach by presenting the agreement of the simulated results with those obtained via physical experiments for three materials with different hyperelastic properties, with percent errors as low as 1%. Taken together, these results demonstrate that the aforementioned simulation platform is a critical step towards the fast and accurate simulation, prototyping, and control of soft robots.

INDEX TERMS Dynamical systems, finite element analysis, open-source software, soft robotics.

I. INTRODUCTION

Soft robots composed of highly deformable elastomers and rubbers have gained popularity as a safe, low cost, and simple alternative to more traditional robots. As researchers develop new materials and the processes to work with them, it is likely that soft robots will continue to find new and exciting uses.

In contrast to soft robots, traditional robots are made of relatively rigid, non-deformable materials. This approach provides several advantages such as straightforward computation of forward and inverse kinematics, predictable motions under load, and a relatively small number of degrees of freedom. These factors are all highly beneficial when completing simple tasks in structured environments, such as in manufacturing or pick and place operations [1], [2]. However, these qualities become disadvantageous when facing unpredictable, unstructured, or dynamic environments.

Robots made of rigid materials are readily modeled using rigid body assumptions as well as design and simulation software tools [3], [4]. While such tools are excellent for traditional robots, they are often largely ineffective for modeling the compliant nature of the materials found in soft robots. Therefore, one major challenge impacting the development of soft robots is the lack of accurate and widely accessible simulation and modeling software. In addition to having highly non-linear stress-strain behaviors, hyperelastic materials also undergo large deformations and change shape. Furthermore, soft robots are often underdamped and can exhibit substantial oscillatory behavior. The combination of these factors make hyperelastic materials difficult to simulate and slows down the design, testing and optimization of soft robots.

In this work, we will introduce a new open-source finite element method (FEM) package, *Kraken*, which is able to

The associate editor coordinating the review of this manuscript and approving it for publication was Amin Mahmoudi.

simulate the complex dynamics of soft robots. This is a critical contribution to the field of soft robotics as this open-source platform will allow researchers to simulate and predict the dynamic behaviors of soft robots with arbitrary geometries. The work is novel because the software package we have developed is open-source and tailored to simulate soft robots, unlike other existing FEM software.

The remainder of the paper will examine the following topics: Section II will review other similar works, and highlight the unique advantages of our approach. Section III will investigate the theoretical foundations underpinning this work, as well as implementation details for our package. Section IV will examine the first validation case, an actuator oscillating in one dimension. Section V increases the complexity of the simulation by considering two dimensional oscillations. The final test case of three dimensional oscillations is demonstrated in Section VI. Section VII concludes the paper by highlighting the key results of this work, as well as additional improvements we plan to make to *Kraken* in the future.

The code and raw data used for this project can be found in the latest version of *Kraken* on github: https://github.com/Z-Laboratory/Kraken

II. RELATED WORKS

As a result of insufficient simulation tools, many researchers have chosen to model the dynamics of soft robots using analytic approaches such as piecewise constant curvature [5], [6], [7], piecewise constant strain [8], Euler-Bernoulli or Cosserat beam theory [9], [10], modified rigid body approximations [11], [12], machine learning methods [8], [13], or other analytic approaches [14], [15], [16]. Occasionally, these methods also use linearly elastic finite element simulations [17] but usually only operate on a low number of discrete states representing the robot. These approaches have the substantial advantage of being efficiently computed, and lend themselves to the design of controllers for soft robots. However, the primary disadvantage of these approaches is that they are often unable to compensate for complexities like external forces, contact with the environment, or hyperelastic deformations. Therefore, in order to deploy soft robots in more complex and dynamic environments, more robust methods of simulation must be employed.

In order to resolve these challenges, many researchers have turned to the finite element method (FEM) to simulate soft robots. Commercial FEM packages such as COMSOL, ABAQUS, and ANSYS have all demonstrated the capability to model soft robots [18], [19], [20], [21]. In recent years, these tools have been used to optimize the designs of pneumatic actuators [22], model and control tendon driven actuators [23], and model more esoteric actuation methods such as magnetic actuation [24]. However, these software packages are closed-source, opaque to the users, and difficult or impossible to extend or customize. Many of the open-source developments in the soft robotics community have focused on hardware that can be easily fabricated using desktop prototyping technologies [25]. However, there is an open-source tool that can be used to simulate soft robots, SOFA. SOFA is able to perform simulations of soft materials via its soft robotics plugin, but lacks support for non-tetrahedral hyperelastic meshes, makes limited use of hyperelasticity in other situations, and is designed to run high speed interactive simulations rather than high accuracy offline ones [26], [27], [28], [29], [30].

Due to the lack of an open-source FEM simulaiton platform designed specifically to simulate soft robots, we have developed our own multiphysics simulation platform, *Kraken*, built on top of Idaho National Laboratory's (INL) Multiphysics Object Oriented Simulation Environment (MOOSE) framework [31]. We demonstrated its capability of simulating the static configuration of a soft actuator [32], as well as the ability to efficiently model contact mechanics [33]. In this paper, we will build on our previous work with *Kraken* by extending the platform's capability to simulate the dynamic motion of soft robots using FEM. This result represents a step towards developing more sophisticated control and simulation platforms for soft robots based on FEM data, rather than reduced dimensional analytic methods. After establishing a theoretical framework for our approach, we built physical actuators out of three different soft materials and used an Optitrack motion capture system (MCS) to record the actuators undergoing different types of motions. After extracting damping parameters from the physical experiments, we run simulations in our new framework using the same parameters. We then demonstrated the agreement between the two results to ensure the accuracy of our approach.

III. THEORETICAL FOUNDATIONS

Previously we have developed a MOOSE application *Kraken* that can simulate the static displacements of hyperelastic materials in response to external loads [32]. MOOSE was originally designed for multiphysics simulation of complex systems, namely nuclear reactors. However, it contains much of the base machinery required to simulate a soft robot including support for statics, dynamics, and contact mechanics. Additionally, MOOSE is modular and open-source, making it possible to modify and customize the framework as needed. In our previous work, we have implemented a neo-Hookean model to accurately describe the nonlinear stress-strain relationship present in hyperelastic materials [32]. To simulate the dynamic motion of soft robots, several additions needed to be made to the existing framework. First, the internal stresses on the material need to be calculated as a function of the inertial, damping, spring, and external forces, rather than directly from the displacements. Second, a method for adding damping to the system must be implemented and validated. Third, numerical integration scheme must be employed. Lastly, we need to establish a means of reliably comparing experimental and simulated data to validate our approach.

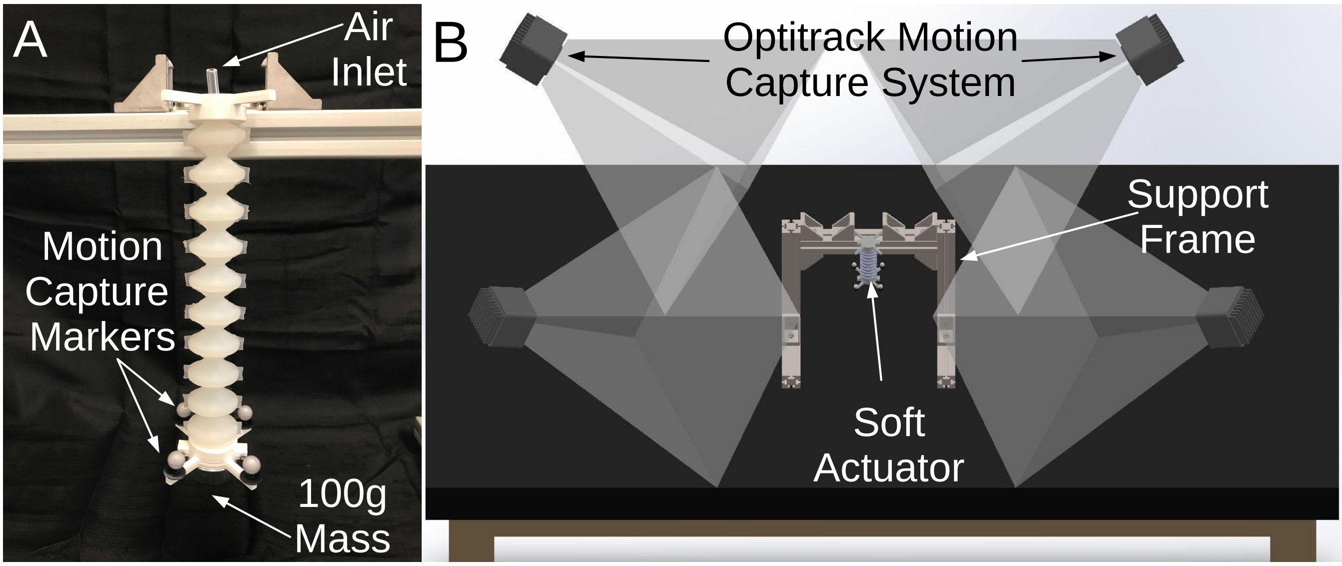

FIGURE 1. **Photo and schematic diagram of the experimental setup. A: Photo of one of the soft actuators used in the experimental tests, featuring retro-reflective infrared markers and a 100 g mass attached to the bottom. The top end of the actuator is fixed in place with an air inlet. B: Schematic diagram of the full experimental setup, featuring the soft actuator, infrared markers, and a 4-camera Optitrack motion capture system recording data at 240 Hz.**

The first task was to implement a solver that would account for the variety of forces present in our system. In all MOOSE based mechanics simulations, the displacement is calculated such that the divergence of the stress is minimized [31]. Thus, solutions for the displacements can be directly computed from stress according to

$$\nabla_x \sigma(x) = 0 \tag{1}$$

where x is the vector of deformation on a material element, and σ is the stress on that element. In the case of a neo-Hookean hyperelastic material σ can be easily computed as seen in [34]

$$\sigma = 2D_1(J-1)I + 2C_1 B \tag{2}$$

where D_1 is a Lagrange multiplier used to enforce incompressibility, J is a normalized measure of the volume of an element, C_1 is a material constant proportional to the shear modulus of the material, and B is the left Cauchy-Green deformation tensor, a measure of the local deformation.

However, when the system is in motion, the stresses can no longer be directly computed from the displacements. In the dynamic case, we have

$$M\ddot{x} + C\dot{x} + \sigma(x) = F_{\text{ext}}(x) \tag{3}$$

where M is the mass matrix, C is the damping matrix, and F_{ext} is total external force. This means that in order to determine σ and solve Eq. (1), the other terms of Eq. (3) must be computed.

Using Moose's ''Inertial Force'' and ''Dynamic Stress Divergence'' Kernels, we can determine the first (mass) and third (stress) terms of the left side of Eq. (3). The external forces on the right can be supplied by a boundary condition constraint. This leaves only the damping term on the left side of the equation. To avoid having to compute the damping matrix, we can use a simplified form of Rayleigh damping and assume the value of the damping C is proportional to the stiffness K.

$$C = \zeta K \tag{4}$$

Furthermore, from this equation one can readily compute the damping ratio to be

$$\xi = \frac{\zeta}{2}\omega \tag{5}$$

This type of damping can be implemented and reduces the number of parameters a user needs to tune when compared to standard Raleigh damping that requires an additional constant to relate the relationship between the damping and the mass.

With a way to solve for all the variables, MOOSE can now iteratively solve for the displacements that minimize the gradient of the stress, while also respecting the other constraints on the problem imposed by Eq. (3).

Moose has already implemented the Hilber-Hughes-Taylor (HHT) integration scheme [35], [36], so we use it to compute the velocity and acceleration terms in our dynamical equation. In the scheme, $\alpha = -0.25$ is used for linear interpolation between the current and previous time step, $\beta = 0.4$ is used in numerically computing velocity and $\gamma = 0.75$ is used in numerically computing the acceleration. These constants were chosen to make the system stable and second order accurate [37]. This gave us the following equations Eq. (6) for the balance of forces, Eq. (7) for the acceleration, and Eq. (8) for the velocity:

$$M\ddot{x}(t+\delta t) + C[(1+\alpha)\dot{x}(t+\delta t)\dot{x} - \alpha\dot{x}(t)]$$

$$+ (1+\alpha)\sigma(x(t+\delta t)) - \alpha\sigma(x(t)) = F_{\text{ext}}(t + (1+\alpha)\delta t) \tag{6}$$

$$\ddot{x}(t+\delta t) = \frac{x(t+\delta t) - x(t)}{\beta \delta t^2} - \frac{\dot{x}(t)}{\beta \delta t} + \frac{\beta - 0.5}{\beta}\ddot{x}(t) \tag{7}$$

$$\dot{x}(t+\delta t) = \dot{x} + (1-\gamma)\delta t \ddot{x}(t) + \gamma t \ddot{x}(t+\delta t) \tag{8}$$

Implementing the HHT time integration scheme now allows us to solve for all of the terms in Eq. (3), and move forward in time. This means that we can now simulate dynamic systems accurately.

The last task to be completed prior to running our simulations was to determine a validation process. One difficulty in comparing experimental and simulated data is that ensuing that the simulated and experimental tests have identical initial conditions is quite challenging. Thus, exact comparisons of time-series trajectories would not be useful for quantitatively assessing the accuracy of the simulation. Instead, we decided to compare the resonant frequencies and damping ratios of the simulated and experimental datasets. The frequencies could be identified by computing the Fourier transform of the datasets. From the frequency domain data we identified the resonant frequency by picking the peak frequency. After this, we were able to choose between several methods for computing the damping ratio from the raw data. In this work we saw that the damping ratio could be computed from the quality factor, Q, using the half power bandwidth method [38].

$$\xi = \frac{1}{2Q} \tag{9}$$

$$Q = \frac{\omega_{\text{peak}}}{\text{half power bandwidth}} \tag{10}$$

The half power bandwidth can be computed by identifying the bandwidth of the Fourier transform of the data 3 dB below the maximum.

With this approach we could reliably perform quantitative comparisons of various simulated and experimental displacement datasets.

IV. 1D OSCILLATIONS OF A HYPERELASTIC ACTUATOR

First, we simulated the elongation process of a soft actuator under gravity. We selected this test as it was a good representation of the motion of an actuator in free space without any additional external forces acting on it. A trio of soft actuators were fabricated out of different silicone rubbers produced by Smooth-On: Ecoflex 00-50 (EF 00-50) with a Shore 00 hardness of 50, Dragonskin 20 (DS 20) with a Shore A hardness of 20, and Dragonskin 30 (DS 30) with a Shore A hardness of 30. To induce oscillations, a 100 gram mass was attached to the end of each actuator. 3D printed parts were attached to the tops and bottoms of the actuators, allowing the tops to be fixed to a rigid scaffold and for retro reflective infrared markers for the motion capture system to be attached to the bottom as seen in Figure 1A. The actuator compressed to its minimum length, and the input sealed, resulting in a soft vacuum on the inside of the actuator. The seal was then opened, allowing the internal pressure to equalise. The result was that the actuator experienced a step input in terms of pressure. The location of 3D printed part fixed to the bottom of the actuator was then recorded at 240 Hz using a 4-camera motion capture system from Optitrack, as shown in Figure 1B.

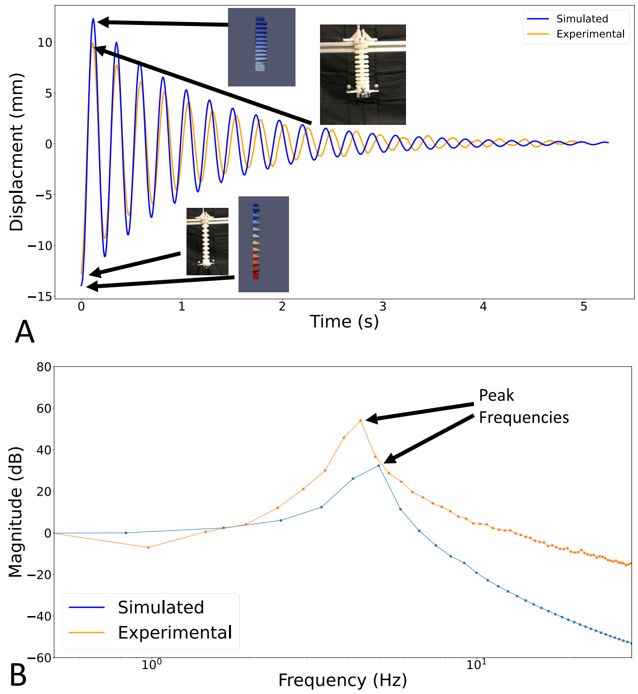

FIGURE 2. Collection of results relating to the 1D oscillations of a hyperelastic actuator. A: Plot of simulated and experimental displacements as a function of time for the actuator composed of DS 20. B: Plot of the discrete Fourier transform of the same data, with both the magnitude and frequency being plotted in logarithmic scales.

From this data, we were able to compute the damping ratios and oscillation frequencies of all three actuators as seen in Table 1. With the damping ratios calculated, we were able to prepare our simulations of these same actuators. All three of the actuators were modeled as being composed of homogeneous neo-Hookean materials. The material parameters were obtained from best fit parameters of a database of soft material uniaxial tension tests [39]. From the database we modeled EF 00-50 with C1 = 0.0224 MPa, DS 20 with C1 = 0.116 MPa, and DS 30 with C1 = 0.1916 MPa. Additionally for all three actuators the parameter D1 was selected to be two orders of magnitude larger than C1. This was found to ensure near incompressibility while still retaining attractive model convergence properties. To decrease computation time, radial symmetry was employed and only one quarter of the actuator was simulated. The simulation was run with maximum time steps of dt = .01 seconds on a coarse mesh of 1743 elements. Each timestep was required to reach an absolute error of 1×10^{-13} prior to convergence. Newton's method was used in conjunction with a PETSC preconditioner and linear solver to generate a solution to the stem at each timestep. Each simulation took an average of approximately 1800 seconds to simulate approximately 500 timesteps using 4 threads on an Intel Core i7 105100 CPU. At the end of

TABLE 1. 1D Oscillations of hyperelastic actuator comparison.

Parameter	EF 00-50	DS 20	DS 30
ω(Hz) (experimental)	2.05	4.20	5.57
ω(Hz) (simulated)	2.06	4.33	5.44
ξ (experimental)	0.041	0.021	0.035
ξ (simulated)	0.040	0.023	0.035

each timestep, the position of the bottom of the weight was recorded. For reference, results of the DS 20 simulation and experiment in both the time and frequency domains can be seen in Figure 2.

Given that the system is oscillatory in nature, we determined that measuring the average error in the position between the simulated and physical experiment would be dominated by the small frequency error, and would not be a useful measure of the system's accuracy. Instead, we elected to measure the peak frequencies and damping ratios of the simulated and physical actuators. The excellent agreement between the simulated and experimental data sets demonstrates that our approach is an appropriate method of simulating the dynamics of soft actuators. In the case of the EF 00-50, an overall elongation of over 150% was accurately modeled. DS 20 and DS 30 also were modeled accurately, although the stiffer materials experienced lower overall strains. The small differences between the experimental and simulated data likely arise from unmodeled factors such as, imperfections in the physical actuators, errors in the material constants, the materials not being perfectly described by the neo-Hookean model, and the small amount of the experimental actuator that is unable to stretch due to being clamped in the 3D printed fixture. However, despite these potential sources of discrepancies, our approach is able to produce excellent agreement with the experiment.

V. DYNAMICS OF 2D HYPERELASTIC PENDULUM

After demonstrating that we could accurately model the 1D oscillatory dynamics of a soft actuator, we moved onto the more complex case of the 2D oscillations of a hyperelastic pendulum. The ability to model this system is especially valuable because similar oscillations often occur in soft robots, when two non-parallel forces are acting on the same body. For example, such behavior could emerge if a linear soft actuator is actuated in a direction other than vertical. In this case, the force of gravity and the force of the actuator are not aligned, and these two-dimensional oscillations will occur. Similar two-dimensional oscillation scenarios arise with actuators that are designed to bend, or if an actuator collides with an obstacle. In this test, the actuator is both stretching and rotating, so the system is similar to the elastic pendulum [40] but more complicated because of the hyperelastic materials the soft actuator is made of. This system is nonlinear and exhibits chaotic behavior.

We used the same physical actuators as in the case of the simple one-dimensional dynamic tests. However, rather

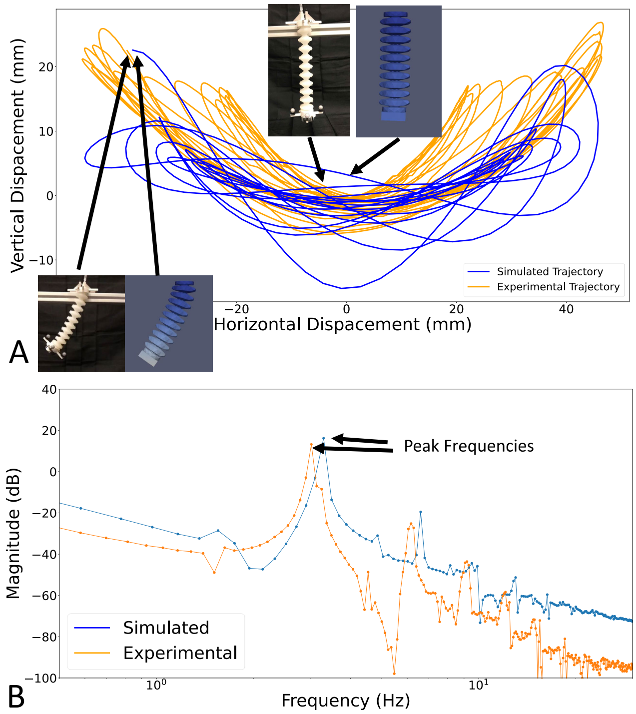

FIGURE 3. Collection of results relating to the 2D hyperelastic pendulum. A: Plot of simulated and experimental trajectories for an actuator composed of DS 20. B: Plot of the discrete Fourier transformation of the magnitude of the displacement data.

than simply displacing the actuator in the vertical direction, a horizontal component was added to the displacement as well. After release, the actuator was allowed to oscillate for several seconds while the position of the markers fixed to the bottom of the actuator was recorded. In the vertical oscillator case, extracting the vertical displacement was trivial as it was easy to align the z-axis of the motion capture system with the direction of motion. However, when performing the hyperelastic pendulum test, it was not possible to ensure that the direction of the horizontal motion alighted perfectly with the y-axis of the motion capture system. To resolve this problem, a best-fit plane for the displacement data was computed, subject to the constraint that the plane had to be parallel to the motion capture system's z-axis. Next, all the data was projected onto this plane. This forced the displacement data to only have 2 dimensions, making it easier to replicate in simulation. After running these experiments, we were able to obtain the frequencies and damping ratios for all 3 actuators in both the vertical and horizontal directions. These results can be found in Table 2. Additionally, the results of the DS 20 simulation and experiment in both the time and frequency domains can be seen in Figure 3.

In a manner similar to the vertical oscillator, the damping parameters were extracted and passed to the simulation. The simulations were then run with the same parameters as the vertical simulation. However, the newly extracted damping parameters were applied to damp the motion in the vertical

TABLE 2. 2D Hyperelastic pendulum comparison.

Vertical Data			
Parameter	EF 00-50	DS 20	DS 30
ω(Hz) (experimental)	2.01	2.88	3.07
ω(Hz) (sim)	1.85	3.14	3.25
ξ (experimental)	0.029	0.011	0.013
ξ (simulated)	0.038	0.019	0.018
Horizontal Data			
Parameter	EF 00-50	DS 20	DS 30
ω(Hz) (experimental)	1.04	1.44	1.53
ω(Hz) (sim)	1.30	1.58	1.62
ξ (experimental)	0.035	0.025	0.022
ξ (simulated)	0.040	0.037	0.040

and horizontal directions independently. Another notable difference was that because the actuator was now moving in two directions, the quarter actuator was no longer valid. Instead, we modeled half of an actuator with a coarse mesh that had 3670 elements. This increased computation time of approximately 600 timesteps to 5000 seconds.

It should also be noted that while there is still strong agreement between simulated and experimental frequencies, the agreement between the damping ratios is somewhat weaker. This is likely because in the case of the 2D oscillator in the horizontal case most of the damping comes from air resistance, whereas in the vertical case, most of the energy is dissipated by the stretching of the material. In the 1D case these effects were combined and their total impact on the system was easily measured. In the 2D case, these factors are no longer operating along the same axis, which may give rise to additional error. To better model these effects, it may be useful to implement a damping scheme that contains a term proportional to the total deformation or strain rate, in addition to a term proportional to the velocity. However, despite these factors, our approach is still able to accurately model the deformations, frequencies, and damping of these actuators undergoing 2D oscillations.

VI. DYNAMICS OF 3D SPHERICAL HYPERELASTIC PENDULUM

As a final demonstration of *Kraken* we decided to consider the case of a full 3D hyperelastic spherical pendulum. This case minimcs the behavior of an actuator that is subjected to several external forces that are not parallel. In this test we used the same test setup as the 2D case, with one major change. In the spherical hyperelastic pendulum test, the actuator was given an initial velocity orthogonal to the plane that the hyperelastic pendulum moved in. The data was not projected onto a best-fit plane and instead the entire 3D trajectory was recorded. These types of complex motions often occur in soft robots where there may be multiple actuators working together, resulting in coupled 3D motions. As before, the Fourier transformations of the displacements in all 3 dimensions were computed and used to identify the resonant frequencies and damping ratios. Then, using the computed damping parameters, the same motion was simulated. In this

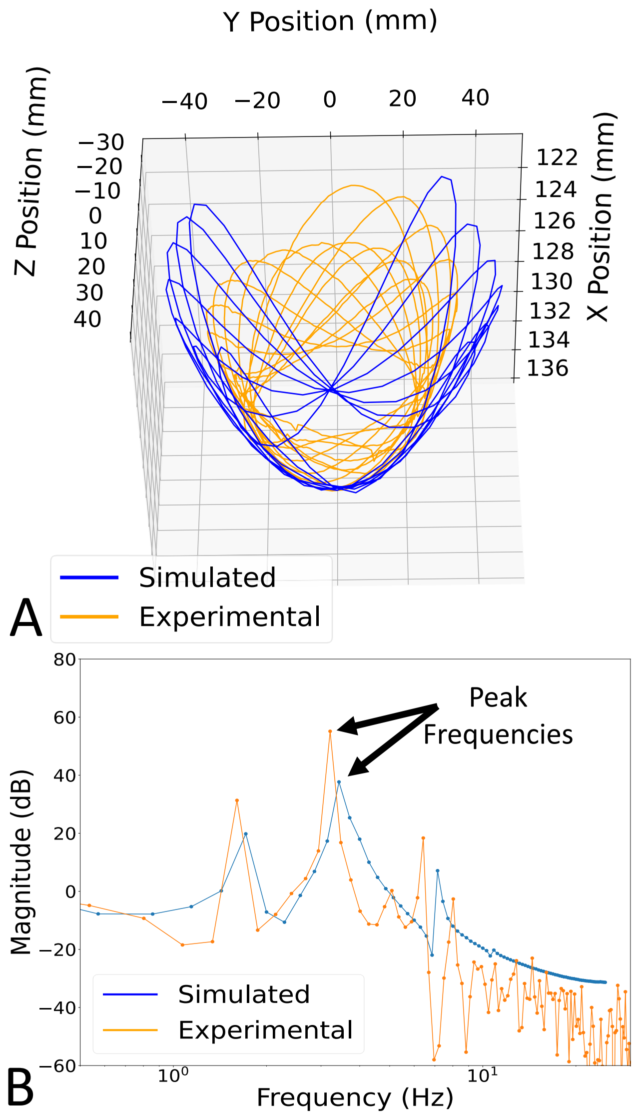

FIGURE 4. Collection of results relating to the 3D spherical hyperelastic pendulum. A: Plot of simulated and experimental trajectories for an actuator composed of DS 20. B: Plot of the discrete Fourier transformation of the magnitude of the displacement data.

case there was no easily identifiable planes of symmetry that could be used to simplify the mesh. As a result, the entire mesh needed to be simulated with 4178 elements. For these simulations, approximately 200 timesteps were run, which took about 2200 seconds.

After running each of the simulations, the relevant parameters were computed and then recorded in Table 3. These results were slightly less accurate with the simulations overestimating the resonant frequencies and damping ratios by an average of between 10% and 30%. These discrepancies likely further arose from the differences between the actual material properties and those taken from best fit lines. Further error could have emerged from self contact and the simplified damping scheme. Nevertheless, these results demonstrate that

TABLE 3. Hyperelastic spherical pendulum system information.

Vertical Data			
Parameter	**EF 00-50**	**DS 20**	**DS 30**
ω(Hz) (experimental)	1.91	2.96	3.22
ω(Hz) (sim)	2.82	3.31	3.52
ξ (experimental)	0.034	0.015	0.022
ξ (simulated)	0.054	0.021	0.026
Horizontal Data			
Parameter	**EF 00-50**	**DS 20**	**DS 30**
ω(Hz) (experimental)	1.10	1.57	1.61
ω(Hz) (sim)	1.45	1.69	1.78
ξ (experimental)	0.053	0.024	0.043
ξ (simulated)	0.088	0.046	0.054
Depth Data			
Parameter	**EF 00-50**	**DS 20**	**DS 30**
ω(Hz) (experimental)	1.10	1.57	1.61
ω(Hz) (sim)	1.45	1.77	1.87
ξ (experimental)	0.059	0.024	0.051
ξ (simulated)	0.095	0.036	0.043

Kraken is able to predict the complex 3D motions of real world systems.

VII. CONCLUSION

In this paper we presented the development, implementation and testing of a novel open-source FEM application, *Kraken*, which enables researchers to accurately simulate the complex dynamics of soft robotic systems. This work is novel as this is the first open-source FEM tool that has been explicitly designed to simulate the complex hyperelastic behaviors of soft robots. We began by laying out the theoretical foundations that are needed to implement the dynamic simulations. Then we demonstrated the accuracy of the approach by accurately modeling soft actuators composed of 3 different materials undergoing one dimensional vertical oscillations due to gravity. We compared the frequency and damping ratio as a quantitative way to compare the simulation and experiment. In the case of the 1D oscillator, excellent agreement was achieved in both the resonant frequencies and damping ratios. We then moved on to the more complicated case of weakly coupled 2D hyperelastic pendulum. In that case the experimental and simulated resonant frequencies were quite close, but there was some error in the damping ratios. Lastly, we demonstrated the accuracy of the approach by showing agreement between experimental and simulated bending angle results for a 3D spherical hyperelastic pendulum. Given the several loading conditions and materials present in the systems studied here, we can conclude that the methodology presented here is a valid way to simulate realistic dynamics of soft robots using a physics-based approach. The released application will enable soft robotic researchers, to accurately simulate the time-dependent deformations of soft robotic systems.

In the future we plan to further improve this work via two different directions. First, we plan to add fluid modeling capabilities to *Kraken*. This will allow for greater realism in our simulations of soft robotic arms as we can simulate the motion of fluids within and around the actuator to better model the damping and actuation of the system. Another future direction is to incorporate uncertainty quantification (QU) into our framework as a means of predicting the impacts that defects and imperfections will have on the behaviors of actuators.

REFERENCES

[1] D. A. Bristow and A.-G. Alleyne, "A high precision motion control system with application to microscale robotic deposition," *IEEE Trans. Control Syst. Technol.*, vol. 14, no. 6, pp. 1008–1020, Nov. 2006.

[2] M. E. Moran, "Evolution of robotic arms," *J. Robot. Surg.*, vol. 1, no. 2, pp. 103–111, Jun. 2007.

[3] R. Sam, K. Arrifin, and N. Buniyamin, "Simulation of pick and place robotics system using solidworks softmotion," in *Proc. Int. Conf. Syst. Eng. Technol. (ICSET)*, Sep. 2012, pp. 1–6.

[4] W. Qian, Z. Xia, J. Xiong, Y. Gan, Y. Guo, S. Weng, H. Deng, Y. Hu, and J. Zhang, "Manipulation task simulation using ROS and gazebo," in *Proc. IEEE Int. Conf. Robot. Biomimetics (ROBIO)*, Dec. 2014, pp. 2594–2598.

[5] G. Runge, M. Wiese, L. Günther, and A. Raatz, "A framework for the kinematic modeling of soft material robots combining finite element analysis and piecewise constant curvature kinematics," in *Proc. 3rd Int. Conf. Control, Autom. Robot. (ICCAR)*, Apr. 2017, pp. 7–14.

[6] R. K. Katzschmann, C. D. Santina, Y. Toshimitsu, A. Bicchi, and D. Rus, "Dynamic motion control of multi-segment soft robots using piecewise constant curvature matched with an augmented rigid body model," in *Proc. 2nd IEEE Int. Conf. Soft Robot. (RoboSoft)*, Apr. 2019, pp. 454–461.

[7] C. D. Santina, R. K. Katzschmann, A. Bicchi, and D. Rus, "Model-based dynamic feedback control of a planar soft robot: Trajectory tracking and interaction with the environment," *Int. J. Robot. Res.*, vol. 39, no. 4, pp. 490–513, Mar. 2020.

[8] T. G. Thuruthel, F. Renda, and F. Iida, "First-order dynamic modeling and control of soft robots," *Frontiers Robot. AI*, vol. 7, p. 95, Jul. 2020.

[9] J. Hiller and H. Lipson, "Dynamic simulation of soft multimaterial 3D-printed objects," *Soft Robot.*, vol. 1, no. 1, pp. 88–101, Mar. 2014.

[10] F. Renda, F. Giorgio-Serchi, F. Boyer, C. Laschi, J. Dias, and L. Seneviratne, "A unified multi-soft-body dynamic model for underwater soft robots," *Int. J. Robot. Res.*, vol. 37, no. 6, pp. 648–666, May 2018.

[11] F. Angelini, C. D. Santina, M. Garabini, M. Bianchi, G. M. Gasparri, G. Grioli, M. G. Catalano, and A. Bicchi, "Decentralized trajectory tracking control for soft robots interacting with the environment," *IEEE Trans. Robot.*, vol. 34, no. 4, pp. 924–935, Aug. 2018.

[12] S. Grazioso, V. Sonneville, G. Di Gironimo, O. Bauchau, and B. Siciliano, "A nonlinear finite element formalism for modelling flexible and soft manipulators," in *2016 IEEE Int. Conf. Simulation, Modeling, Program. Auto. Robots (SIMPAR)*, 2016, pp. 185–190.

[13] W. D. Null and J. Menezes, "Development of a modular and submersible soft robotic arm and corresponding learned kinematics models," in *Proc. IEEE Int. Conf. Soft Robot. (RoboSoft)*, Apr. 2023, pp. 1–6.

[14] C. D. Onal and D. Rus, "Autonomous undulatory serpentine locomotion utilizing body dynamics of a fluidic soft robot," *Bioinspiration Biomimetics*, vol. 8, no. 2, Mar. 2013, Art. no. 026003.

[15] Y. Fei and H. Xu, "Modeling and motion control of a soft robot," *IEEE Trans. Ind. Electron.*, vol. 64, no. 2, pp. 1737–1742, Feb. 2017.

[16] W. Huang, X. Huang, C. Majidi, and M. K. Jawed, "Dynamic simulation of articulated soft robots," *Nature Commun.*, vol. 11, no. 1, p. 2233, May 2020.

[17] S. Grazioso, G. Di Gironimo, and B. Siciliano, "A geometrically exact model for soft continuum robots: The finite element deformation space formulation," *Soft Robot.*, vol. 6, no. 6, pp. 790–811, Dec. 2019.

[18] M. Pozzi, C. Gaudeni, Z. Iqbal, D. Prattichizzo, and M. Malvezzi, "Modeling a sensorized soft layer for adding compliance to the environment in robotic manipulation," in *Advances in Italian Mechanism Science*, V. Niola and A. Gasparetto, Eds. Cham, Switzerland: Springer, 2021, pp. 370–377.

[19] C. Tawk and G. Alici, "Finite element modeling in the design process of 3D printed pneumatic soft actuators and sensors," *Robotics*, vol. 9, no. 3, p. 52, Jul. 2020.

[20] J. T. B. Overvelde, Y. Mengüç, P. Polygerinos, Y. Wang, Z. Wang, C. J. Walsh, R. J. Wood, and K. Bertoldi, "Mechanical and electrical numerical analysis of soft liquid-embedded deformation sensors analysis," *Extreme Mech. Lett.*, vol. 1, pp. 42–46, Dec. 2014.

[21] M. S. Xavier, A. J. Fleming, and Y. K. Yong, "Finite element modeling of soft fluidic actuators: Overview and recent developments," *Adv. Intell. Syst.*, vol. 3, no. 2, Feb. 2021, Art. no. 2000187.
[22] Y. Yao, L. He, and P. Maiolino, "A simulation-based toolbox to expedite the digital design of bellow soft pneumatic actuators," in *Proc. IEEE 5th Int. Conf. Soft Robot. (RoboSoft)*, Apr. 2022, pp. 29–34.
[23] L. Ding, L. Niu, Y. Su, H. Yang, G. Liu, H. Gao, and Z. Deng, "Dynamic finite element modeling and simulation of soft robots," *Chin. J. Mech. Eng.*, vol. 35, no. 1, p. 24, Apr. 2022, doi: 10.1186/s10033-022-00701-8.
[24] B. Seyidoglu, T. Atakuru, K. Uyanik, and E. Samur, "Finite element analysis of magnetorheological elastomers for stiffness variation in soft robots," in *Proc. IEEE 4th Int. Conf. Soft Robot. (RoboSoft)*, Apr. 2021, pp. 531–534.
[25] B. Deutschmann, J. Reinecke, and A. Dietrich, "Open source tendon-driven continuum mechanism: A platform for research in soft robotics," in *Proc. IEEE 5th Int. Conf. Soft Robot. (RoboSoft)*, Apr. 2022, pp. 54–61.
[26] F. Faure, C. Duriez, H. Delingette, J. Allard, B. Gilles, S. Marchesseau, H. Talbot, H. Courtecuisse, G. Bousquet, I. Peterlik, and S. Cotin, "SOFA: A multi-model framework for interactive physical simulation," in *Soft Tissue Biomechanical Modeling for Computer Assisted Surgery* (Studies in Mechanobiology, Tissue Engineering and Biomaterials), vol. 11, Y. Payan, Ed. Berlin, Germany: Springer, Jun. 2012, pp. 283–321.
[27] F. Largilliere, V. Verona, E. Coevoet, M. Sanz-Lopez, J. Dequidt, and C. Duriez, "Real-time control of soft-robots using asynchronous finite element modeling," in *Proc. IEEE Int. Conf. Robot. Autom. (ICRA)*, May 2015, pp. 2550–2555.
[28] O. Goury and C. Duriez, "Fast, generic, and reliable control and simulation of soft robots using model order reduction," *IEEE Trans. Robot.*, vol. 34, no. 6, pp. 1565–1576, Dec. 2018.
[29] C. Duriez, "Control of elastic soft robots based on real-time finite element method," in *Proc. IEEE Int. Conf. Robot. Autom.*, May 2013, pp. 3982–3987.
[30] P. Schegg and C. Duriez, "Review on generic methods for mechanical modeling, simulation and control of soft robots," *PLoS ONE*, vol. 17, no. 1, pp. 1–14, Jan. 2022, doi: 10.1371/journal.pone.0251059.
[31] C. J. Permann, D. R. Gaston, D. Andrš, R. W. Carlsen, F. Kong, A. D. Lindsay, J. M. Miller, J. W. Peterson, A. E. Slaughter, R. H. Stogner, and R. C. Martineau, "MOOSE: Enabling massively parallel multiphysics simulation," *SoftwareX*, vol. 11, Jan. 2020, Art. no. 100430.
[32] K. Wandke, "MOOSE-based finite element hyperelastic modeling for soft robot simulations," *IEEE Access*, vol. 9, pp. 139627–139635, 2021.
[33] K. Wandke, "An efficient framework for the solution of contact mechanics problems in soft robotics," in *Proc. IEEE Int. Conf. Soft Robot. (RoboSoft)*, Apr. 2023, pp. 1–7.
[34] R. Ogden, *Non-Linear Elastic Deformations* (Dover Civil and Mechanical Engineering). New York, NY, USA: Dover, 2013.
[35] T. J. R. Hughes, *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Mineola, NY, USA, Dover, 2000.
[36] N. M. Newmark, "A method of computation for structural dynamics," *J. Eng. Mech. Division*, vol. 85, no. 3, pp. 67–94, Jul. 1959.
[37] H. P. Gavin. *Numerical Integration in Structural Dynamics, CEE 541, Structural Dynamics*. Accessed: Feb. 8, 2022. [Online]. Available: https://people.duke.edu/~hpgavin/StructuralDynamics/NumericalIntegration.pdf
[38] G. A. Papagiannopoulos and G. D. Hatzigeorgiou, "On the use of the half-power bandwidth method to estimate damping in building structures," *Soil Dyn. Earthq. Eng.*, vol. 31, no. 7, pp. 1075–1079, Jul. 2011.
[39] L. Marechal, P. Balland, L. Lindenroth, F. Petrou, C. Kontovounisios, and F. Bello, "Toward a common framework and database of materials for soft robotics," *Soft Robot.*, vol. 8, no. 3, pp. 284–297, Jun. 2021.
[40] Anurag, B. Mondal, J. K. Bhattacharjee, and S. Chakraborty, "Understanding the order-chaos-order transition in the planar elastic pendulum," *Phys. D, Nonlinear Phenomena*, vol. 402, Jan. 2020, Art. no. 132256. [Online]. Available: https://www.sciencedirect.com/science/article/abs/pii/S0167278919300119

KEVIN WANDKE (Graduate Student Member, IEEE) received the B.S. degree in mechanical science and engineering and the M.S. degree in electrical engineering from the University of Illinois at Urbana–Champaign, in 2019 and 2022, respectively, where he is currently pursuing the Ph.D. degree in electrical and computer engineering. He was a member of the SULI Program, Argonne National Laboratory, and an Intern of the Edison Engineering Program, General Electric's Global Research Center. He was a recipient of the Chancellor's Scholarship, the Olsen Award for Excellence in Undergraduate Teaching, and a Mavis Future Faculty Fellow in the Grainger College of Engineering.

Y Z (Member, IEEE) received the B.S. degree in electrical science and technology from the University of Science and Technology of China, in 2004, and the Ph.D. degree in nuclear science and engineering from the Massachusetts Institute of Technology, in 2010. He was a Clifford G. Shull Fellow with the Oak Ridge National Laboratory, from 2010 to 2012, and a Professor with the University of Illinois at Urbana–Champaign, from 2012 to 2022. He is currently a Professor with the Department of Nuclear Engineering and Radiological Sciences, the Department of Materials Science and Engineering, the Department of Robotics, and the Applied Physics Program, University of Michigan. His current research interests include matter and machines. On the matter side, his group studies far-from-equilibrium physics. They synergistically combine and push the boundaries of statistical and stochastic thermodynamic theories, accelerated molecular simulations, understandable AI/ML/DS methods, and neutron scattering experiments, with the goal of significantly extending our understanding of a wide range of long timescale phenomena and rare events. Particular emphasis is given to the physics and chemistry of liquids and complex fluids, especially at interfaces, driven away from equilibrium, or under extreme conditions. On the machine side, leveraging their expertise in materials and modeling, his group advances the development of soft robots and human-compatible machines, swarm robots and collective intelligence, and robots in extreme environments, which can lead to immediate societal impact.