IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 12 June 2023, accepted 26 June 2023, date of publication 30 June 2023, date of current version 7 July 2023.

*Digital Object Identifier 10.1109/ACCESS.2023.3290963*

RESEARCH ARTICLE

# Incomplete Dominance-Based Intuitionistic Fuzzy Rough Sets and Their Application in Estimation of Inflation Rates in the Least Developed Countries

**IFTIKHAR UL HAQ[1], TANZEELA SHAHEEN[1], HAMZA TOOR[2], TAPAN SENAPATI[3,4], AND SARBAST MOSLEM[5]**

[1]Department of Mathematics, Air University, Islamabad 44230, Pakistan
[2]Department of Biomedical Engineering, Riphah International University, Islamabad 45320, Pakistan
[3]Department of Mathematics and Statistics, Southwest University, Beibei, Chongqing 400715, China
[4]Department of Mathematics, Padima Janakalyan Banipith, Kukrakhupi, Jhargram 721517, India
[5]School of Architecture Planning and Environmental Policy, University College of Dublin, Belfield, Dublin 4, D04 V1W8 Ireland

Corresponding authors: Tapan Senapati (math.tapan@gmail.com) and Sarbast Moslem (sarbast.moslem@ucd.ie)

This work was supported by the European Commission through the SENATOR Project [Research and Innovation Actions (RIA)] under Project H2020MG-2018-2020 and Project 861540.

**ABSTRACT** Although the concepts of imprecision are captured by both rough sets and intuitionistic fuzzy sets, investigations combining these two ideas and their applications in incompletely ordered information systems are scarce. The semantics of many kinds of missing information still lacks consensus on a global level. Rule extraction is also an important task in a sort of decision system in which condition attributes are treated as intuitionistic fuzzy values and decision attributes are crisp ones. The main goal of this paper is to address semantic issues related to incomplete information. This paper contributes to the following aspects: First, four types of incomplete information are classified (i.e., ''do-not-care value'', ''partially-known value'', ''class-specific value'' and ''non-applicable value''), and then a complete information system is introduced using novel semantics, followed by a ranking approach to create each object's neighborhood using intuitionistic fuzzy values for condition attributes. Further, a dominance-based intuitionistic fuzzy decision table is proposed. Second, the lower and upper approximation sets of an object and crisp classes validated by decision attributes are determined by comparing their relationships. Third, the rule extraction approach is developed using the discernibility matrix and discernibility function to collect knowledge from existing dominance intuitionistic fuzzy decision tables. Finally, the provided approach is used for the estimation of inflation rates in LDCs with inadequate data.

**INDEX TERMS** Incomplete ordered information, possible-world semantics, intuitionistic fuzzy set, dominance-based rough set approach, dominance relation, discernibility matrix, rule extraction.

## LIST OF ABBREVIATIONS

| *Symbols* | *Description* |
|---|---|
| RST | Rough Set Theory |
| DRSA | Dominance-based Rough Set Approach |
| IFIS | Intuitionistic Fuzzy Information System |
| DIFIS | Dominance-based Intuitionistic Fuzzy Information System |
| OISs | Ordered Information Systems |
| DIFDT | Dominance-based Intuitionistic Fuzzy-valued Decision Table |
| DIFRSA | Dominance-based Intuitionistic Fuzzy Rough Set Approach |
| LDCs | Least Developed Countries |

The associate editor coordinating the review of this manuscript and approving it for publication was Francisco J. Garcia-Penalvo.

## I. INTRODUCTION

Rough set theory (RST) was first introduced by Pawlak [1] as a formal mathematical tool for addressing vagueness and discrepancy in information systems [2]. The main benefit of the rough set approach is that it does not require any preliminary or supplementary data, such as probability distributions in probability theory or grades of membership in fuzzy set theory. This theory is based on the observation that objects with the same description cannot be distinguished based on the information that is currently available about them [3], [4]. The equivalence classes form a partition of the universe of discourse and constitute the basic granules of knowledge. Fundamental ideas of RST include the lower and upper approximations of sets that are created by these equivalence classes. The RST has been successfully used in a variety of fields, including decision analysis, expert systems, machine learning, pattern recognition, and knowledge discovery [5], [6], [7], [8], [9], [10], [11].

However, the rough set theory is not able to discover and process inconsistencies coming from consideration of criteria, that is, attributes with preference-ordered domains (scales), such as test scores, university rankings, and house pricing.

To solve this issue, Greco et al. [12], [13] suggested the dominance-based rough set approach (DRSA), an extension of RST that takes into consideration the ordering patterns of criteria. This invention is primarily based on the substitution of the indiscernibility (or equivalence) connection by a dominance relation, which allows for ordered set approximations in multi-criteria decision-making and multi-criteria sorting issues. The information to be approximated in DRSA is a collection of upward and downward unions of decision classes, and the dominance classes are sets of objects defined by utilizing a dominance relation. Furthermore, the DRSA considers monotonic connections between object descriptions based on condition criteria and their class labels. DRSA has been expanded since its beginnings to deal with knowledge acquisition in many types of ordered information systems (OISs) [14], [15], [16], [17], [18], [19], [20], [21], [22].

Rough set approximations have lately been integrated into intuitionistic fuzzy sets [23], [24], [25], [26], [27], [28]. Based on the notion of fuzzy rough sets provided by Nanda and Majumdar [29], Chakrabarty et al. [27], and Jena and Ghosh [30], intuitionistic fuzzy rough sets were presented, in which the lower and higher approximations are both intuitionistic fuzzy sets. Samanta and Mondal [31] also proposed this concept, which they call a rough intuitionistic fuzzy set. From the perspective of Nanda and Majumdar, this fuzzy set with the membership and non-membership functions is no longer a fuzzy set but a fuzzy rough set. Zhou and Wu [32], [33] investigated fuzzy rough approximation operators. Huang et al. [34], [35] explored and applied dominance-based (interval-valued) intuitionistic fuzzy rough set models.

Pawlak's rough set analysis assumes that an object has only one value for each attribute and that we know what that value is. However, in many cases, accessible information about some objects is inadequate, and we may not know their real values for some attributes. Moreover, two types of values may be considered: "applicable value" and "non-applicable value." The real values must exist for the category of applicable values, although we may not know the values or just know a range of possibilities. For non-applicable values, some attributes do not apply to certain objects; as a result, their values cannot be specified. It might be considered a subtype of missing value. Since we may not be aware of the precise descriptions of certain objects in these situations of incomplete information, the idea of equivalence relations is no longer applicable. Many authors propose and investigate different types of non-equivalence relations to model similarity, including tolerance relations [36], similarity relations [4], conditional tolerance relations [37], and characteristic relations [38], [39], [40], [41], [42], [43], [44]. Indiscernibility is a special type of similarity. An indiscernibility relation is required for deriving rules with complete information, but a similarity relation is required for generating rules with limited knowledge. Various types of similarity relations models are based on various semantics of incomplete information. However, there is no conceptual framework for investigating incomplete information from the perspective of semantics.

Kryszkiewicz [36] deals with incomplete information as a "do-not-care value" that may be changed with any recognized values of an attribute. Stefanowski and Tsoukiàs [4] consider two types of incomplete information: "missing value" and "absent value". The "missing value" semantics allows comparison operations on a missing value. The "absent value" semantics does not allow any comparison. Grzymala-Busse [39], [41] recognize two types of incomplete information: "do-not-care value" and "lost value". He further divides "do-not-care value" into three categories according to their comparison ranges: "do-not-care value", "restricted do-not-care value", and "attribute-concept value". For a "do-not-care value", it can be substituted by any known attribute value. For a "restricted do-not-care value", It can only be replaced by any known attribute values, except "lost values". For an "attribute-concept value," any known values that are restricted to the same concept may be used in their place. For a "lost value," the original value already existed but is no longer there for a variety of reasons.

Based on the existing studies of different semantics of incomplete information, we generalized four types of semantics for incomplete ordered information systems (IOISs):

- ➢ (D) "Do-not-care value" denoted by "$*$":
- ➢ (P) "Partially-known value" denoted by "$\dagger$":
- ➢ (C) "Class-specific value" denoted by "$\nabla$":
- ➢ (N) "Non-applicable value" denoted by "NA":

Despite the previous research initiatives, intuitionistic fuzzy and rough set hybrid models are rarely generated. In both traditional and generalized rough set theory [45], [46], [47], [48], [49], [50], [51], [52], knowledge reduction and rule extraction are crucial tasks. However, in the domain of intuitionistic fuzzy circumstances, these problems have seldom ever been addressed. For this shortcoming, the current work focuses on the development of intuitionistic fuzzy-rough models based on dominance and the simplification

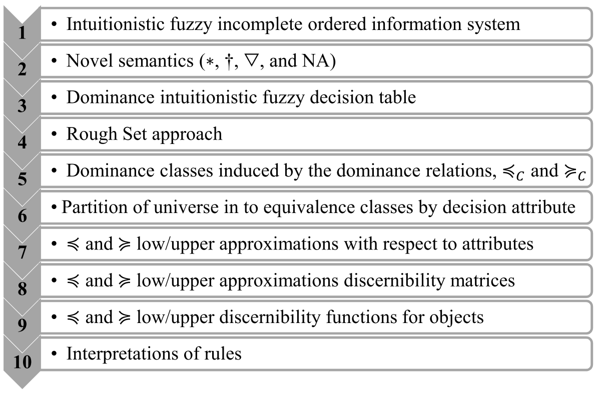


**FIGURE 1. Ten-steps flowchart for DIFRSA in an IOIS by using novel semantics.**

of decision rules in intuitionistic fuzzy information systems. This approach combines the strengths of rough set theory, incomplete dominance, and intuitionistic fuzzy sets to handle uncertainty, enhance discrimination power, and generate interpretable rules. This generalization gives better results then other fuzzy generalizations. First, we use novel semantics to turn an intuitionistic fuzzy-valued incomplete ordered information system into an intuitionistic fuzzy-valued decision table, and then combine it with a dominance relation to create a notion known as dominance intuitionistic fuzzy-valued decision tables (DIFDT). Second, we develop a dominance-based rough set model based principally on the replacement of the indiscernibility relation in conventional rough set theory with a dominance-based relation. Then, from discernibility matrices, we suggest rule extraction methodologies. Third, the suggested intuitionistic rough models and rule extraction procedures are used to estimate inflation rates in low-income countries with inadequate data.

The rest of this paper is organized as follows: In Section II, the basic concepts of IFSs, information systems, intuitionistic fuzzy information systems, dominance-based intuitionistic fuzzy rough set approach to ordered information systems are briefly reviewed. Section III introduces four types of novel semantics for intuitionistic fuzzy incomplete ordered information systems. In Section IV, we discussed the application of novel semantics in a Dominance-based Intuitionistic Fuzzy Rough Set Approach (DIFRSA) for rule extraction from incompletely ordered information systems. In Section V, an approach is illustrated by a numerical example. Finally, Section VI concludes this article and declaration of interests with Section VI.

## II. PRELIMINARIES

This section sums up some basic ideas and properties related to intuitionistic fuzzy values, recalling some notions related with dominance-based intuitionistic fuzzy information systems that will be used in the next section.

### A. IFSs

*Definition 1 [53]:* Let $(\mu, \nu)$ be an ordered pair, where $0 \leq \mu, \nu \leq 1$ and $0 \leq \mu + \nu \leq 1$. Then we call $(\mu, \nu)$ an intuitionistic fuzzy value.

*Definition 2 [54]:* Let $\alpha_i = (\mu_i, \nu_i), (1 \leq i \leq 2)$ be two intuitionistic fuzzy values, then

1. $\alpha_1 = \alpha_2 \Leftrightarrow \mu_1 = \mu_2 \wedge \nu_1 = \nu_2$;
2. $\alpha_1 \cap \alpha_2 = ((min\{\mu_1, \mu_2\}, max\{\nu_1, \nu_2\})$;
3. $\alpha_1 \cup \alpha_2 = (max\{\mu_1, \mu_2\}, min\{\nu_1, \nu_2\})$;

Obviously, if $\alpha_1$ and $\alpha_2$ are two intuitionistic fuzzy values, then so are $\alpha_1 \cap \alpha_2$ and $\alpha_1 \cup \alpha_2$. From the intersection and union of two intuitionistic fuzzy values, it is easy to generalize those of $n$ intuitionistic fuzzy values as follows.

*Definition 3 [54]:* Let $\alpha_i = (\mu_i, \nu_i), (1 \leq i \leq n)$ be $n$ intuitionistic fuzzy values, on which we can define the intersection $\bigcap_{1 \leq i \leq n} \alpha_i$ and union, $\bigcup_{1 \leq i \leq n} \alpha_i$ of $\alpha_i (1 \leq i \leq n)$ as follows:

$$\bigcap_{1 \leq i \leq n} \alpha_i = \left( \min_{1 \leq i \leq n} \mu_i, \max_{1 \leq i \leq n} \nu_i \right),$$
$$\bigcup_{1 \leq i \leq n} \alpha_i = \left( \max_{1 \leq i \leq n} \mu_i, \min_{1 \leq i \leq n} \nu_i \right).$$

*Definition 4 [54]:* Let $U$ be a universe of discourse. An intuitionistic fuzzy set '$A$' in $U$ is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) >: x \in U\}$, where $\mu_A : U \rightarrow [0, 1]$ and $\nu_A : U \rightarrow [0, 1]$ satisfy $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in U$, and $\mu_A(x)$ *and* $\nu_A(x)$ are, respectively, called the degree of membership and the degree of non-membership of the element $x \in U$ to $U$.

Obviously, every fuzzy set $A = \{< x, \mu_A(x) >: x \in U\}$ can be identified with the intuitionistic fuzzy set of the form $\{< x, \mu_A(x), 1 - \mu_A(x) >: x \in U\}$ and is thus an intuitionistic fuzzy set.

*Definition 5 [55]:* Let $\alpha_1 = (\mu_1, \nu_1)$ and $\alpha_2 = (\mu_2, \nu_2)$ be two intuitionistic fuzzy values, $S(\alpha_1) = \mu_1 - \nu_1$ and $S(\alpha_2) = \mu_2 - \nu_2$ be the score of $\alpha_1$ and $\alpha_2$, respectively; and $h(\alpha_1) = \mu_1 + \nu_1$ and $h(\alpha_2) = \mu_2 + \nu_2$ be the precisions of $\alpha_1$ and $\alpha_2$, respectively, then

1. If $S(\alpha_1) < S(\alpha_2)$ then $\alpha_1 \prec \alpha_1$;
2. If $S(\alpha_1) = S(\alpha_2)$
   a) and $h(\alpha_1) = h(\alpha_2)$, then $\alpha_1 = \alpha_2$;
   b) and $h(\alpha_1) < h(\alpha_2)$, then $\alpha_1 \prec \alpha_2$;
   c) and $h(\alpha_1) > h(\alpha_2)$, then $\alpha_1 \succ \alpha_2$;

### B. INFORMATION SYSTEM

*Definition 6:* An information system is a quadruple $S = (U, AT, V, f)$, where $U$ is a finite non-empty set of objects, $AT$ is a finite nonempty set of attributes, $V = \bigcup_{a \in AT} V_a$ and $V_a$ is the domain of attribute $a$, and $f : U \times AT \rightarrow V$ is a total function such that $f(x, a) \in V_a$ for every $a \in AT, x \in U$, called an information function.

*Definition 7 [56]:* An information system is called an ordered information system if all attributes are criteria.

*Definition 8:* An information system with incomplete information is called an incomplete information system. There are different types of semantics for incomplete information system in the literature [36], [41], [57], some of them are:

(D) "Do-not-care value" denoted by "$*$":
(P) "Partially-known value" denoted by "†":

(C) "Class-specific value" denoted by "∇":

(N) "Non-applicable value" denoted by "NA":

"Do-not-care value" denoted by "∗". It does not affect the final result of an incomplete information system. For example, most covid 19 diseases do not act by age. This disease can be found at any age period. So this missing value can be replaced by any value of same attribute domain for given information.

"Partially-known value" denoted by "†". This value lies in some given range of missing value. For example if someone asks about a university student who does not know his discipline or department, this is partial information. This missing value can be replaced with one of the value from subset of same attribute domain.

"Class-specific value" denoted by "∇". It depends on another special attribute of the same object. For example, the government job salary employees depends on their pay scale. This missing value can be replaced with one of the value of the objects with the same value on this special attribute.

"Non-applicable value" denoted by "NA". This attribute value is not applicable for that object. For example, some birds can't fly. Thus, the attribute fly in same birds is non-applicable for that type of bird. So, that type of missing value will not replace with any other value, this treated as NA.

*Definition 9:* An incomplete information system is a tuple $IIS = (U, AT, V', F)$, where $U$ is a finite non-empty set of objects called the universe, $AT$ is a finite nonempty set of attributes, $V' = V \cup \{*\} \cup \{\dagger\} \cup \{\nabla\} \cup \{NA\}$, where $V = \bigcup_{a\in AT} V_a$, $V_a$ is the domain of attribute $a$. ∗, †, ∇, and NA are special symbols for do-not-care value, partially-known value, class-specific value, and non-applicable value, respectively. $F = \{f_a | a \in AT\}, f_a : U \to V'_a$ is an information function such that $f_a(x) \in V'_a$, $x\in U$, $V'_a = V_a \cup\{*\}\cup\{\dagger\}\cup\{\nabla\}\cup\{NA\}$.

Two objects are similar with respect to an attribute if they have at least one same value for their possible attribute values on the attribute. If we denote $P_a(x)$ as the set of all possible attribute values of object $x$ with respect to an attribute $a$, where:

(K) "Known value": If $f_a(x) \in V_a$, then $P_a(x) = \{f_a(x)\}$;

(D) "Do-not-care value": If $f_a(x) = *$, then $P_a(x) = V_a$;

(P) "Partially known value": If $f_a(x) = \dagger^x_a$, then $P_a(x) = f_a(x)$;

(C) "Class-specific value": If $f_a(x) = \nabla^x_{(a,b)}$, then

$$\begin{aligned} P_a(x) &= V_a^{\nabla_b}(x) \\ &= \{f_a(y) | f_b(x) = f_b(y) \wedge f_a(y) \in V_a\}, b \in AT; \end{aligned}$$

(N) "Non-applicable value": If $f_a(x) = NA$, then $P_a(x) = \{NA\}$;

Based on these different semantics of incomplete information system, we can define the complete information system that replaces the incomplete information with its possible values.

### C. INTUITIONISTIC FUZZY INFORMATION SYSTEM

*Definition 10 [34]:* An intuitionistic fuzzy information system (IFIS) is a quadruple $S = (U, C \cup D, V, f)$, where $U$ is a finite non-empty set of objects called the universe, $C$ is a finite non-empty set of conditional attributes, $D = \{d\}$ is a singleton of decision attribute $d$, and $C \cap D = \emptyset$. $V$ is the set of all intuitionistic fuzzy values, and $V = V_1 \cup V_2$, where $V_1$ and $V_2$ are domains of condition and decision attributes, respectively. The information function $f$ is a map from $U \times C \cup D$ onto $V$, such that $f(x, c) \in V_c$ for all $c \in C$, $V_c \subseteq V_1$, and $f(x, d) \in V_2$ for $D = \{d\}$, where $f(x, c)$ and $f(x, d)$ are intuitionistic fuzzy values, denoted by $f(x, c) = (\mu_c(x), \nu_c(x))$ and $f(x, d) = (\mu_d(x), \nu_d(x))$.

We call $f(x, c)$ the intuitionistic fuzzy value of object $x$, under the condition attribute $c$, $f(x, d)$ the intuitionistic fuzzy value of $x$ under the decision attribute $d$. In particular, $f(x, c)$ and $f(x, d)$ would degenerate into fuzzy value if $\mu_c(x) = 1 - \nu_c(x)$ and $\mu_d(x) = 1 - \nu_d(x)$ for every $x \in U$. Under this consideration, we regard a fuzzy information system as a special form of intuitionistic fuzzy information systems.

When doing a practical examination of a decision-making process, we always take into account a binary dominance connection between objects that might be dominant in terms of the values of a set of characteristics in an intuitionistic fuzzy information system. A decision-maker often takes into account both growing and decreasing preferences. An attribute is a criterion if the domain of the attribute is arranged in descending or ascending order of preference. We restrict our analysis to dominant intuitionistic fuzzy information systems without losing any generality.

Comparing and ranking objects using condition attributes with intuitionistic fuzzy values is a crucial task in dominant intuitionistic fuzzy information systems. For this, we use the score function and accuracy function for the ranking mechanism of two intuitionistic fuzzy values.

*Definition 11 [34]:* Let $DIFIS = (U, C \cup D, V, f)$, and $B \subseteq C$, for $x, y \in U$ denoted by $x \preccurlyeq_B y \Leftrightarrow f(x, b) \preccurlyeq f(y, b) \Leftrightarrow f(y, b) \prec f(y, b) \vee f(y, b) = f(y, b), \forall b \in B$. Obviously, $\preccurlyeq_B$ is a binary relation in $U$, that is

$$\preccurlyeq_B = \{(x, y) \in U \times U \mid f(y, b) \preccurlyeq f(y, b), \forall b \in B\}.$$

We call the binary relation defined above a dominance relation in DIFIS.

If we adopt definition 11 to compare two intuitionistic fuzzy values, it is possible that the two intuitionistic fuzzy values may not be comparable. For example, $\alpha_1 = (0.2, 0.3)$ and $\alpha_2 = (0.3, 0.4)$ are intuitionistic fuzzy values, there are $\alpha_1 \npreceq \alpha_2$ and $\alpha_2 \npreceq \alpha_1$ but $\alpha_1 \prec \alpha_2$. Despite this definition's rigor, another simple and useful ranking method for objects in $U$ should be introduced.

*Definition 12:* Let $DIFIS = (U, C \cup D, V, f)$ and $B \subseteq C$, for $x, y \in U$, $x \preccurlyeq_B y \Leftrightarrow f(x, b) < f(y, b) \vee f(x, b) = f(y, b)\ \forall b \in B$.

Obviously, $\preccurlyeq_B$ is also a dominance binary relation in $U$, that is

$$\preccurlyeq_B = \{(x, y) \in U \times U \mid f(y, b) < f(y, b) \vee f(x, b) = f(y, b), \forall b \in B\}.$$

We call $\preccurlyeq_B$ a rigorous dominance relation in terms of $B$ in DIFIS.

*Proposition 1:* Let DIFIS $= (U, C \cup D, V, f)$ and $E \subseteq B \subseteq C$, then

1. $\preccurlyeq_B$ *and* $\leq_B$ are reflexive, transitive and anti-symmetric.
2. $\preccurlyeq_B = \cap \preccurlyeq_{b \in B} \preccurlyeq_{\{b\}}$ *and* $\leq_B = \cap_{b \in B} \leq_{\{b\}}$.
3. $\preccurlyeq_E \subseteq \preccurlyeq_B$ *and* $\leq_E \subseteq \leq_B$.
4. $\leq_B \subseteq \preccurlyeq_B$.

The dominance class induced by the dominance relation, $\preccurlyeq_B$, in terms of $B \subseteq C$, is the set of objects dominating $x$, *ie* $[x]_B^{\preccurlyeq} = \{y \in U \mid (x, y) \in \preccurlyeq_B\}$ where $[x]_B^{\preccurlyeq}$ describes the set of objects that may dominate x in terms of $B \subseteq C$ and is called the $B$-dominating set with respect to $x \in U$. Meanwhile, the $B$-dominated set with respect to $x \in U$ can be defined as $[x]_B^{\succcurlyeq} = \{y \in U \mid (y, x) \in \preccurlyeq_B\}$. Similarly, $[x]_B^{\leq} = \{y \in U \mid (x, y) \in \leq_B\}$ and $[x]_B^{\geq} = \{y \in U \mid (y, x) \in \leq_B\}$.

### D. DOMINANCE-BASED INTUITIONISTIC FUZZY ROUGH SET APPROACH TO ORDERED INFORMATION SYSTEM

In this section, we examine at the set approximation challenges in connection to the dominance intuitionistic fuzzy relation in the dominance intuitionistic fuzzy information system (DIFIS).

*Definition 13:* Let DIFIS $= (U, C \cup \{d\}, V, f)$, and $B \subseteq C$.the universe, $U$, is partioned into $m$ equivalence classes by the decision attribute $d$ that is, $U/\{d\} = \{U_1, U_2, \ldots, U_m\}$, where $U_1 \prec U_2 \prec \ldots \prec U_m$ and $U_i \prec U_j$ denotes that $\forall x \in U_i, y \in U_j$ implies $f(x, d) < f(y, d)$. Denote $U_k = \bigcup_{1 \leq j \leq k} U_j (1 \leq k \leq m)$, $U_k^{\succcurlyeq} = \bigcup_{k \leq j \leq m} U_j (1 \leq k \leq m)$ and let:

$$\underline{App_B}\left(U_k^{\succcurlyeq}\right) = \left\{x | [x]_B^{\succcurlyeq} \subseteq U_k^{\succcurlyeq}\right\} (1 \leq k \leq m);$$
$$\overline{App_B}\left(U_k^{\succcurlyeq}\right) = \left\{x | \, [x]_B^{\preccurlyeq} \cap U_k^{\succcurlyeq} \neq \emptyset\right\} = \bigcup_{x \in U_k^{\succcurlyeq}} [x]_B^{\preccurlyeq} (1 \leq k \leq m);$$
$$BND_B\left(U_k^{\succcurlyeq}\right) = \overline{App_B}\left(U_k^{\succcurlyeq}\right) - \underline{App_B}\left(U_k^{\succcurlyeq}\right);$$
$$\underline{App_B}\left(U_k^{\preccurlyeq}\right) = \left\{x | [x]_B^{\preccurlyeq} \subseteq U_k^{\preccurlyeq}\right\} (1 \leq k \leq m);$$
$$\overline{App_B}\left(U_k^{\preccurlyeq}\right) = \left\{x | \, [x]_B^{\succcurlyeq} \cap U_k^{\preccurlyeq} \neq \emptyset\right\} = \bigcup_{x \in U_k^{\preccurlyeq}} [x]_B^{\succcurlyeq} (1 \leq k \leq m);$$
$$BND_B\left(U_k^{\preccurlyeq}\right) = \overline{App_B}\left(U_k^{\preccurlyeq}\right) - \underline{App_B}\left(U_k^{\preccurlyeq}\right);$$
$$\underline{App}_B^{\succcurlyeq}(x) = \bigcap_{[x]_B^{\succcurlyeq} \cap U_k^{\preccurlyeq} \neq \phi} U_k^{\preccurlyeq};$$
$$\overline{App}_B^{\succcurlyeq}(x) = \bigcap_{[x]_B^{\succcurlyeq} \subseteq U_k^{\preccurlyeq}} U_k^{\succcurlyeq};$$
$$BND_B^{\succcurlyeq}(x) = \overline{App}_B^{\succcurlyeq}(x) - \underline{App}_B^{\succcurlyeq}(x);$$
$$\underline{App}_B^{\preccurlyeq}(x) = \bigcap_{[x]_B^{\preccurlyeq} \cap U_k^{\succcurlyeq} \neq \phi} U_k^{\succcurlyeq};$$
$$\overline{App}_B(x) = \bigcap_{[x]_B^{\preccurlyeq} \subseteq U_k^{\succcurlyeq}} U_k^{\succcurlyeq};$$
$$BND_B^{\preccurlyeq}(x) = \overline{App}_B^{\preccurlyeq}(x) - \underline{App}_B^{\preccurlyeq}(x);$$

Then, $\underline{App_B}\left(U_k^{\succcurlyeq}\right)$, $\overline{App_B}\left(U_k^{\succcurlyeq}\right)$ and $BND_B\left(U_k^{\succcurlyeq}\right)$ are called the lower approximation, upper approximation and boundary of the dominated class, $U_k^{\succcurlyeq}$ with respect to condition attribute subset $B$; $\underline{App_B}\left(U_k^{\preccurlyeq}\right)$, $\overline{App_B}\left(U_k^{\preccurlyeq}\right)$ and $BND_B\left(U_k^{\preccurlyeq}\right)$ are the lower approximation, upper approximation and boundary of the dominating class, $U_k^{\preccurlyeq}$ with respect to $B$; In terms of $B$, $\underline{App}_B^{\succcurlyeq}(x)$, $\overline{App}_B^{\succcurlyeq}(x)$ *and* $BND_B^{\succcurlyeq}(x)$ are the $\succcurlyeq$ lower approximation, upper approximation and boundary of $x$. $\underline{App}_B^{\preccurlyeq}(x)$, $\overline{App}_B^{\preccurlyeq}(x)$ *and* $BND_B^{\preccurlyeq}(x)$ are the $\preccurlyeq$ lower approximation, upper approximation, and boundary of $x$.

Susmaga et al. [45] introduced a discernibility matrix to dominance-based decision tables and addressed the computation of dominance-based reducts using the dominance information table in DRSA. And investigate an attribute reduction approach for the $\preccurlyeq$ and $\succcurlyeq$ lower approximations in DIFDT. Moreover, it can be easily generalized to the $\preccurlyeq$ and $\succcurlyeq$ upper approximations in DIFDT.

*Definition 14:* Let DIFIS $= (U, C \cup \{d\}, V, f)$, and $U = \{x_1, x_2, \ldots, x_n\}$. Define $\preccurlyeq$ and $\succcurlyeq$ lower approximations discernibility matrices of DIFIS as $\underline{M}^{\preccurlyeq} = \left(\underline{d}_{ij}^{\preccurlyeq}\right)_{n \times n}$ and $\underline{M}^{\succcurlyeq} = \left(\underline{d}_{ij}^{\succcurlyeq}\right)_{n \times n}$ where

$$\underline{d}_{ij}^{\preccurlyeq} = \left\{ \begin{array}{ll} \{c \in C | f(x_j, c) \succ f(x_i, c)\}, f(x_j, d) > \left|\underline{App}_c^{\preccurlyeq}(x_i)\right|, \\ C \quad \textit{otherwise} \end{array} \right\};$$

$$\underline{d}_{ij}^{\succcurlyeq} = \left\{ \begin{array}{ll} \{c \in C | f(x_j, c) \prec f(x_i, c)\}, f(x_j, d) < \left|\underline{App}_c^{\succcurlyeq}(x_i)\right|, \\ C \quad \textit{otherwise} \end{array} \right\}.$$

*Definition 15:* Let DIFIS $= (U, C \cup \{d\}, V, f)$, and $U = \{x_1, x_2, \ldots, x_n\}$. We define $\preccurlyeq$ and $\succcurlyeq$ lower discernibility functions of $x_i$ as

$$\underline{f}^{\preccurlyeq}(x_i) = \bigwedge_j (\bigvee \underline{d}_{ij}^{\preccurlyeq}) \text{ and } \underline{f}^{\succcurlyeq}(x_i) = \bigwedge_j (\bigvee \underline{d}_{ij}^{\succcurlyeq}).$$

## III. THE NOVEL SEMANTICS FOR INTUITIONISTIC FUZZY INCOMPLETE ORDERED INFORMATION SYSTEM

Now we generalize different semantics. Do-not-care value "$*$", Partially-known value "†" Class-specific value "∇" and Non-applicable value "NA" for the intuitionistic fuzzy incomplete ordered information system.

In an intuitionistic fuzzy incomplete ordered information system IFIOIS $= (U, C \cup D, V \cup \{*\} \cup \{\dagger\} \cup \{\nabla\}, f)$. If we denote $p(x, c)$ as the set of all possible attribute values of object $x$ with respect to attribute $c$. Then:

The semantics of the applicable values are as follows.

- **Do-not-care value**

"Do-not-care value" denoted by "$*$": For $f(x, c) = *$, although the value of an object $x \in U$ on an attribute $c \in C$ is missing, we do not care what is its actual value. In other words, if we replace $*$ by any value in $V_c$ or take average of values from same attribute domain, we will obtain the same result independent of the choice of $c$ value in $V_c$. Then,

$$f(x,c) = * = \left(\overline{\mu_i(x)}, \overline{\nu_i(x)}\right), p(x,c) = (\mu_i(x), \nu_i(x)) \quad \forall x \in U, c \in C, \mu_i(x) \in [0,1], \nu_i(x) \in [0,1] \ with\ 0 \le \mu_i(x) + \nu_i(x) \ge 1.$$

- **Partially-known value**

"Partially-known value" denoted by "†": For $f(x,c) = \dagger_c^x$, the value of an object $x \in U$ on an attribute $c \in C$ is missing, we know that the actual value is in a subset $p(x,c)$ of $V_c$ such that

$$f(x,c) = \dagger_a^x = \left(\widehat{\mu_i}(x), \widehat{\nu_i}(x)\right), p(x,c) = (\mu_i(x), \nu_i(x)) \forall x \in U, c \in C, \widehat{\mu_i}(x) \in \left[min\, \mu_i(x), max\, \mu_i(x)\right] \wedge \widehat{\nu_i}(x) \in [min\, \nu_i(x), max\, \nu_i(x)], \mu_i(x) \in [0,1], \nu_i(x) \in [0,1] \ with\ 0 \le \mu_i(x) + \nu_i(x) \ge 1.$$

- **Class-specific value**

"Class-specific value" denoted by "∇": For $f(x,c) = \nabla_{(c,c')}^x$, the value of an object $x \in U$ on an attribute $c \in C$ is missing, we know that the value depends on the attribute value of a special attribute $c' \in C$. The class specific value may be the average of the values of the objects with the same value on this special attribute. In other word, $\nabla_{(c,c')}^x$ will be any value in

$$f(x,c) = \nabla_{(c,c')}^x = \left(\overline{\mu_i(y)}, \overline{\nu_i(y)}\right), p(y,c) = (\mu_i(y), \nu_i(y)) | f(x,c') = f(y,c') \wedge f(y,c') = p(y,c) \in V_a \quad \forall c, c' \in C \ and\ x, y \in U, \mu_i(x) \in [0,1], \nu_i(x) \in [0,1] \ with\ 0 \le \mu_i(x) + \nu_i(x) \ge 1.$$

The semantics of the non-applicable values is as follows.

- **Non-applicable value**

"Non-applicable value" denoted by "NA": For $f(x,c) =$ NA, we know that the value of an object $x \in U$ on an attribute $c \in C$ does not exist. Although the "Non-applicable value" is a type of incomplete information, we regard it as a special known value.

*Example 1:* Table 1 represents an incomplete ordered information system of heart patients of age 60 plus, whose values are intuitionistic fuzzy values. There are four different types of missing attribute values. The attributes are:

$a$ = Age, $b$ = Residential space (above 2000 square feet), $c$ = Salary (above 500 $) and $d$ = Socially active scale (0-5).

In Table 1, there are 6 objects and 4 attributes. Some attribute domain values are missing. The age of Michael is missing. We replace this by "do-not-care value" sign because we know that, all heart patients are of age 60 plus. So its value is any value in $V_c$ or take average of values from same attribute domain. The salary domain value of Richard is also missing. We replace this by the "non-applicable" value sign because Richard is not doing a job. So, the attribute "salary" is non-applicable for him. The salary attribute value for Katherine is also missing. We replace this by the sign of class-specific value because generally salary depends on residential space of the people. So, the attribute "salary" depends on the attribute "residential place". Lastly, the attribute value of the attribute "socially active scale" for Victoria is missing. We replace this by partially-known value because Victoria is a female and in our information system Elizabeth and Katherine are also females. Their social active scale is batter then males socially active scale. So, we have some information about that missing attribute of Victoria.

## IV. APPLICATION OF NOVEL SEMANTICS IN DOMINANCE BASED INTUITIONISTIC FUZZY ROUGH SET APPROACH FOR RULES EXTRACTION TO INCOMPLETE ORDERED INFORMATION SYSTEM

In this section, we use novel semantics in Dominance-based Intuitionistic Fuzzy Rough Set Approach to Incomplete Ordered Information System. Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universe set of "$n$" objects. Each is described by "$m$" attributes. The condition attributes set "$C$" is $C = \{c_1, c_2, \ldots, c_m\}$ with incomplete information. The decision attributes $D = \{d\}$ is singleton set of $d$. For intuitionistic fuzzy incomplete information system IFIIS $= (U, C \cup D, V \cup \{*\} \cup \{\dagger\} \cup \{\nabla\}, f)$. Where $f(x,c)$ is intuitionistic fuzzy value, denoted by $f(x,c) = (\mu_c(x), \nu_c(x))$ and $f(x,d)$ is a crisp value.

**Step 1.** Find the complete ordered information system. $(U, C \cup D, V, f)$ through semantics do-not-care value "$*$", partially known value "$\dagger_c^x$" and Class-specific value $\nabla_{(c,\acute{c})}^x$:

The complete ordered information system is

$$(U, C \cup D, V, f) = \langle f(x_i, c_j), f(x_i, d) \ for\ (i = 1, 2, \ldots, n)\ and\ (j - 1, 2, \ldots, m)\rangle$$

**Step 2.** Determine dominance classes induced by the dominance relation, $\preccurlyeq_C$ and $\succcurlyeq_C$:

$$[x]_C^{\preccurlyeq} = \{y \in U \mid (x,y) \in \preccurlyeq_C\}, \quad [x]_C^{\succcurlyeq} = \{y \in U \mid (y,x) \in \preccurlyeq_C\}$$

**Step 3.** Partition universe $U$ in to m equivalence classes by decision attribute $d$:

$$U/\{d\} = \{U_1, U_2, \ldots, U_m\}, U_1 \prec U_2 \prec \ldots \prec U_m$$

Denote $U_k^{\preccurlyeq} = \bigcup_{1 \le j \le k} U_j (1 \le k \le m)$, $U_k^{\preccurlyeq} = \bigcup_{k \le j \le m} U_j (1 \le k \le m)$.

**Step 4.** Determine $\preccurlyeq$ and $\succcurlyeq$ low/upper approximations with respect to $C$:

$$\underline{App_B}\left(U_k^{\preccurlyeq}\right) = \left\{x | [x]_B^{\preccurlyeq} \subseteq U_k^{\preccurlyeq}\right\} (1 \le k \le m);$$

$$\overline{App_B}\left(U_k^{\preccurlyeq}\right) = \left\{x | [x]_B^{\succcurlyeq} \cap U_k^{\preccurlyeq} \ne \emptyset\right\} = \bigcup_{x \in U_k^{\preccurlyeq}} [x]_B^{\succcurlyeq} (1 \le k \le m);$$

$$\underline{App_B}\left(U_k^{\succcurlyeq}\right) = \left\{x | [x]_B^{\succcurlyeq} \subseteq U_k^{\succcurlyeq}\right\} (1 \le k \le m);$$

**TABLE 1.** Incomplete ordered information system for heart patients of age 60 plus.

| Objects /Attributes | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| Charles | (0.7, 0.2) | (0.6, 0.4) | (0.6, 0.2) | (0.4, 0.3) |
| Michael | $*$ | (0.6, 0.3) | (0.5, 0.3) | (0.5, 0.2) |
| Richard | (0.8, 0.1) | (0.9, 0.1) | NA | (0.5, 0.1) |
| Elizabeth | (0.9, 0.1) | (0.7, 0.3) | (0.7, 0.2) | (0.8, 0.1) |
| Katherine | (0.7, 0.3) | (0.7, 0.2) | $\nabla^{K}_{(c,b)}$ | (0.9, 0.1) |
| Victoria | (0.6, 0.2) | (0.8, 0.1) | (0.8, 0.1) | † |

$$\overline{App_B}\left(U_k^{\succcurlyeq}\right) = \left\{x \mid [x]_B^{\preccurlyeq} \cap U_k^{\succcurlyeq} \neq \emptyset\right\} = \bigcup\nolimits_{x \in U_k^{\succcurlyeq}} [x]_B^{\preccurlyeq} \ (1 \leq k \leq m);$$

**Step 5.** Determine the and $\preccurlyeq$ lower $\succcurlyeq$ approximations discernibility matrices:

**(a)** $\underline{M}^{\preccurlyeq} = \left(\underline{d}_{ij}^{\preccurlyeq}\right)_{n\times n}$ Where,

$$\underline{d}_{ij}^{\preccurlyeq} = \begin{cases} \{c \in C | f(x_j, c) \succ f(x_i, c)\}, f\left(x_j, d\right) > \left|\underline{App}_c^{\preccurlyeq}(x_i)\right|, \\ C \quad \text{otherwise} \end{cases};$$

**(b)** $\underline{M}^{\succcurlyeq} = \left(\underline{d}_{ij}^{\succcurlyeq}\right)_{n\times n}$ Where,

$$\underline{d}_{ij}^{\succcurlyeq} = \begin{cases} \{c \in C | f(x_j, c) \prec f(x_i, c)\}, f\left(x_j, d\right) < \left|\underline{App}_c^{\succcurlyeq}(x_i)\right|, \\ C \quad \text{otherwise} \end{cases}.$$

**Step 6.** Determine the $\preccurlyeq$ and $\succcurlyeq$ lower discernibility functions for $x_i$:

$$\underline{f}^{\preccurlyeq}(x_i) = \bigwedge_j (\bigvee \underline{d}_{ij}^{\preccurlyeq}), \quad \underline{f}^{\succcurlyeq}(x_i) = \bigwedge_j (\bigvee \underline{d}_{ij}^{\succcurlyeq}).$$

**Step 7.** Interpretation of these rules.

## V. APPLICATION EXAMPLE

The intuitionistic fuzzy approach has been successfully used to execute multi-criteria decision-making, group decision-making, and grey relational analysis as a beneficial tool that manages incomplete imperfect data and information, as well as incomplete imprecise knowledge. [26], [58], [59]. The suggested DIFRSA can likewise be similarly used in these fields. We provide its application to illustrate its potential. We apply our method to extract the lower and upper bound intuitionistic fuzzy rules in their simplest form.

*Example 2:* Inflation rate assessment in the least developed countries with incomplete information.

A multi-criteria decision-making problem concerns NGOs, which want to assess the inflation rates in the least developed countries. The ten least developed countries in the world are considered. They are denoted by $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$, as object set. Each is described by six inflation attributes: ($c_1$)"Demand-pull inflation", ($c_2$)"Cost-push inflation", ($c_3$)"Increased money supply", ($c_4$)"Devaluation", ($c_5$)"Rising wages", and ($c_6$)"Policies and regulations" with incomplete information. So the condition attributes set "$C$" is $C = \{c_1, c_2, c_3, c_4, c_5, c_6\}$. Every value to which a condition attribute applies has a unique actual meaning. For example, $f(x_1, c_6) = (0.7, 0.2)$ means that the membership degree of policies and regulations is 0.7, and the non-membership degree of policies and regulations is 0.2. The decision attribute set, "$d$" is the inflation level. The domain of $d = \{1, 2, 3, 4, 5\}$, where 1 means "Inflation level is (2.5 to 7.9)%", 2 means "Inflation level is (8.0 to 10.5)%", 3 means "Inflation level is (10.6 to 12.5)%", 4 means "Inflation level is (12.6 to 14.5)%", and 5 means "Inflation level is (14.6 to 20.0)%".

**For "do-not-care value"** $f(x, c) = *$**:**

$$f_{c_1}(x_7) = (0.38, 0.50), \quad f_{c_1}(x_9) = (0.38, 0.50),$$
$$f_{c_2}(x_3) = (0.42, 0.47), \quad f_{c_3}(x_5) = (0.73, 0.20),$$
$$f_{c_3}(x_{10}) = (0.73, 0.20), \quad f_{c_4}(x_6) = (0.50, 0.43),$$
$$f_{c_5}(x_2) = (0.59, 0.36), \quad f_{c_5}(x_4) = (0.59, 0.36),$$
$$f_{c_6}(x_8) = (0.67, 0.21).$$

**For "partially known value"** $f(x, c) = \dagger_c^x$**:**

$\dagger_{c_1}^{x_1} = (0.47, 0.45)$, $\dagger_{c_1}^{x_4} = (0.39, 0.32)$, $\dagger_{c_2}^{x_8} = (0.41, 0.32)$, $\dagger_{c_3}^{x_2} = (0.62, 0.34)$, $\dagger_{c_4}^{x_7} = (0.51, 0.35)$, $\dagger_{c_6}^{x_3} = (0.65, 0.29)$, $\dagger_{c_6}^{x_{10}} = (0.66, 0.28)$.

**For "Class-specific value"** $f(x, c) = \nabla_{(c,\acute{c})}^x$**:**

$$\nabla_{(c_2,c_4)}^{x_5} = (0.41, 0.48), \quad \nabla_{(c_2,c_5)}^{x_6} = (0.50, 0.47),$$
$$\nabla_{(c_4,c_5)}^{x_1} = (0.49, 0.30), \quad \nabla_{(c_5,c_6)}^{x_9} = (0.60, 0.34).$$

For $U/\{d\} = \{U_1, U_2, U_3, U_4, U_5\}$, where $U_1 = \{x_3, x_4\}$, $U_2 = \{x_1, x_2, x_5\}$, $U_3 = \{x_6\}$, $U_4 = \{x_7, x_8\}$, $U_5 = \{x_9, x_{10}\}$, then $U_1^{\preccurlyeq} = U_1 = \{x_3, x_4\}$, $U_2^{\preccurlyeq} = U_1 \cup U_2 = \{x_1, x_2, x_3, x_4, x_5\}$, $U_3^{\preccurlyeq} = U_1 \cup U_2 \cup U_3 = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $U_4^{\preccurlyeq} = U_1 \cup U_2 \cup U_3 \cup U_4 = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, $U_5^{\preccurlyeq} = U_1 \cup U_2 \cup U_3 \cup U_4 \cup U_5 = U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$.

$U_1^{\succcurlyeq} = U_1 \cup U_2 \cup U_3 \cup U_4 \cup U_5 = U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$, $U_2^{\succcurlyeq} = U_2 \cup U_3 \cup U_4 \cup U_5 = \{x_1, x_2, x_5, x_6, x_7, x_8, x_9, x_{10}\}$, $U_3^{\succcurlyeq} = U_3 \cup U_4 \cup U_5 = \{x_6, x_7, x_8, x_9, x_{10}\}$, $U_4^{\succcurlyeq} = U_4 \cup U_5 = \{x_7, x_8, x_9, x_{10}\}$, $U_5^{\succcurlyeq} = U_5 = \{x_9, x_{10}\}$.

**TABLE 2. An incomplete ordered information system inflation rate assessment decision table.**

| U/C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $d$ |
|---|---|---|---|---|---|---|---|
| $x_1$ | $\dagger_{c_1}^{x_1}$ | (0.3, 0.4) | (0.8, 0.2) | $\square_{(c_4,c_5)}^{x_1}$ | (0.7, 0.2) | (0.7, 0.2) | 2 |
| $x_2$ | (0.3, 0.5) | (0.4, 0.5) | $\dagger_{c_3}^{x_2}$ | (0.4, 0.5) | $*$ | (0.6, 0.3) | 2 |
| $x_3$ | (0.3, 0.5) | $*$ | (0.8, 0.1) | (0.4, 0.5) | (0.7, 0.3) | $\dagger_{c_6}^{x_3}$ | 1 |
| $x_4$ | $\dagger_{c_1}^{x_4}$ | (0.1, 0.8) | (0.4, 0.5) | (0.6, 0.4) | $*$ | (0.8, 0.2) | 1 |
| $x_5$ | (0.4, 0.5) | $\square_{(c_2,c_4)}^{x_5}$ | $*$ | (0.4, 0.5) | (0.7, 0.3) | (0.6, 0.1) | 2 |
| $x_6$ | (0.5, 0.4) | $\square_{(c_2,c_5)}^{x_6}$ | (0.7, 0.2) | $*$ | (0.4, 0.5) | (0.6, 0.3) | 3 |
| $x_7$ | $*$ | (0.4, 0.5) | (0.6, 0.2) | $\dagger_{c_4}^{x_7}$ | (0.4, 0.5) | (0.7, 0.2) | 4 |
| $x_8$ | (0.4, 0.6) | $\dagger_{c_2}^{x_8}$ | (0.9, 0.1) | (0.7, 0.3) | (0.8, 0.2) | $*$ | 4 |
| $x_9$ | $*$ | (0.7, 0.3) | (0.9, 0.1) | (0.5, 0.4) | $\square_{(c_5,c_6)}^{x_9}$ | (0.7, 0.2) | 5 |
| $x_{10}$ | (0.4, 0.5) | (0.6, 0.3) | $*$ | (0.5, 0.4) | (0.4, 0.5) | $\dagger_{c_6}^{x_{10}}$ | 5 |

**TABLE 3. A complete ordered information system inflation rate assessment decision table.**

| U/C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $d$ |
|---|---|---|---|---|---|---|---|
| $x_1$ | (0.47, 0.45) | (0.3, 0.4) | (0.8, 0.2) | (0.49, 0.30) | (0.7, 0.2) | (0.7, 0.2) | 2 |
| $x_2$ | (0.3, 0.5) | (0.4, 0.5) | (0.62, 0.34) | (0.4, 0.5) | (0.59, 0.36) | (0.6, 0.3) | 2 |
| $x_3$ | (0.3, 0.5) | (0.42, 0.47) | (0.8, 0.1) | (0.4, 0.5) | (0.7, 0.3) | (0.65, 0.29) | 1 |
| $x_4$ | (0.39, 0.32) | (0.1, 0.8) | (0.4, 0.5) | (0.6, 0.4) | (0.59, 0.36) | (0.8, 0.2) | 1 |
| $x_5$ | (0.4, 0.5) | (0.41, 0.48) | (0.73, 0.20) | (0.4, 0.5) | (0.7, 0.3) | (0.6, 0.1) | 2 |
| $x_6$ | (0.5, 0.4) | (0.50, 0.47) | (0.7, 0.2) | (0.50, 0.43) | (0.4, 0.5) | (0.6, 0.3) | 3 |
| $x_7$ | (0.38, 0.50) | (0.4, 0.5) | (0.6, 0.2) | (0.51, 0.35) | (0.4, 0.5) | (0.7, 0.2) | 4 |
| $x_8$ | (0.4, 0.6) | (0.41, 0.32) | (0.9, 0.1) | (0.7, 0.3) | (0.8, 0.2) | (0.67, 0.21) | 4 |
| $x_9$ | (0.38, 0.50) | (0.7, 0.3) | (0.9, 0.1) | (0.5, 0.4) | (0.60, 0.34) | (0.7, 0.2) | 5 |
| $x_{10}$ | (0.4, 0.5) | (0.6, 0.3) | (0.73, 0.20) | (0.5, 0.4) | (0.4, 0.5) | (0.66, 0.28) | 5 |

**TABLE 4. Dominating and dominated classes induced by $\preccurlyeq_C$ and $\succcurlyeq_C$.**

| U | $[x]_C^{\preccurlyeq}(x)$ | $[x]_C^{\succcurlyeq}(x)$ |
|---|---|---|
| $x_1$ | $\{x_1, x_2, x_7\}$ | $\{x_1\}$ |
| $x_2$ | $\{x_2\}$ | $\{x_1, x_2, x_3, x_8, x_9\}$ |
| $x_3$ | $\{x_2, x_3\}$ | $\{x_3, x_8\}$ |
| $x_4$ | $\{x_4\}$ | $\{x_4\}$ |
| $x_5$ | $\{x_2, x_5\}$ | $\{x_5\}$ |
| $x_6$ | $\{x_6\}$ | $\{x_6\}$ |
| $x_7$ | $\{x_7\}$ | $\{x_1, x_7\}$ |
| $x_8$ | $\{x_2, x_3, x_8\}$ | $\{x_8\}$ |
| $x_9$ | $\{x_2, x_9\}$ | $\{x_9\}$ |
| $x_{10}$ | $\{x_{10}\}$ | $\{x_{10}\}$ |

The $\preccurlyeq$ lower approximations discernibility matrix, as shown in the equation at the bottom of the next page.

The $\succcurlyeq$ lower approximations discernibility matrix, as shown in the equation at the bottom of the next page.

The $\preccurlyeq$ lower discernibility functions of $x_i (6 \le i \ge 10)$ are:

$$\underline{f}^{\preccurlyeq}(x_6) = c_4,$$

$$\underline{f}^{\preccurlyeq}(x_7) = (c_2 \vee c_3 \vee c_5) \wedge (c_1 \vee c_2 \vee c_3)$$

**TABLE 5.** $\preccurlyeq$ and $\succcurlyeq$ low/upper approximations with respect to $C$.

| U | $\underline{App}_C^{\preccurlyeq}(x)$ | $\overline{App}_C^{\preccurlyeq}(x)$ | $\underline{App}_C^{\succcurlyeq}(x)$ | $\overline{App}_C^{\succcurlyeq}(x)$ |
|---|---|---|---|---|
| $x_1$ | $U_4^{\succcurlyeq}$ | $U_2^{\succcurlyeq}$ | $U_2^{\preccurlyeq}$ | $U_2^{\preccurlyeq}$ |
| $x_2$ | $U_2^{\succcurlyeq}$ | $U_2^{\succcurlyeq}$ | $U_1^{\preccurlyeq}$ | $U_5^{\preccurlyeq}$ |
| $x_3$ | $U_2^{\succcurlyeq}$ | $U_1^{\succcurlyeq}$ | $U_1^{\preccurlyeq}$ | $U_4^{\preccurlyeq}$ |
| $x_4$ | $U_1^{\succcurlyeq}$ | $U_1^{\succcurlyeq}$ | $U_1^{\preccurlyeq}$ | $U_1^{\preccurlyeq}$ |
| $x_5$ | $U_2^{\succcurlyeq}$ | $U_2^{\succcurlyeq}$ | $U_2^{\preccurlyeq}$ | $U_2^{\preccurlyeq}$ |
| $x_6$ | $U_3^{\succcurlyeq}$ | $U_3^{\succcurlyeq}$ | $U_3^{\preccurlyeq}$ | $U_3^{\preccurlyeq}$ |
| $x_7$ | $U_4^{\succcurlyeq}$ | $U_4^{\succcurlyeq}$ | $U_2^{\preccurlyeq}$ | $U_4^{\preccurlyeq}$ |
| $x_8$ | $U_4^{\succcurlyeq}$ | $U_1^{\succcurlyeq}$ | $U_4^{\preccurlyeq}$ | $U_4^{\preccurlyeq}$ |
| $x_9$ | $U_5^{\succcurlyeq}$ | $U_2^{\succcurlyeq}$ | $U_5^{\preccurlyeq}$ | $U_5^{\preccurlyeq}$ |
| $x_{10}$ | $U_5^{\succcurlyeq}$ | $U_5^{\succcurlyeq}$ | $U_5^{\preccurlyeq}$ | $U_5^{\preccurlyeq}$ |

$$= c_2 \bigvee c_3 \bigvee (c_1 \bigwedge c_5),$$

$$\underline{f}^{\preccurlyeq}(x_8) = (c_1 \bigvee c_2 \bigvee c_6) \bigwedge (c_1 \bigvee c_2) = c_1 \bigvee c_2,$$

$$\underline{f}^{\preccurlyeq}(x_9) = (c_1 \bigvee c_4) \bigwedge (c_4) \bigwedge (c_4 \bigvee c_5) \bigwedge (c_1) = (c_1 \bigwedge c_4),$$

$$\begin{aligned}\underline{f}^{\preccurlyeq}(x_{10}) &= (c_1) \bigwedge (c_4 \bigvee c_6) \bigwedge (c_3 \bigvee c_4 \bigvee c_5 \bigvee c_6) \\ &\quad \bigwedge (c_2 \bigvee c_3 \bigvee c_5 \bigvee c_6) \\ &= (c_1 \bigwedge c_6) \bigvee (c_1 \bigwedge c_2 \bigwedge c_4) \bigvee (c_1 \bigwedge c_3 \bigwedge c_4) \\ &\quad \bigvee (c_1 \bigwedge c_4 \bigwedge c_5).\end{aligned}$$

**The simplified $\preccurlyeq$ upper bound rules for information system inflation rate assessment are:**

$$f(x, c_4) \preccurlyeq (0.2, 0.6) \Longrightarrow f(x, d) \leq 3 \ (\textit{supported by } x_6),$$
$$f(x, c_2) \preccurlyeq (0.4, 0.5) \Longrightarrow f(x, d) \leq 4 \ (\textit{supported by } x_7),$$
$$f(x, c_3) \preccurlyeq (0.6, 0.2) \Longrightarrow f(x, d) \leq 4 \ (\textit{supported by } x_7),$$
$$f(x, c_1) \preccurlyeq (0.38, 0.50) \bigwedge f(x, c_5) \preccurlyeq (0.4, 0.5) \Longrightarrow f(x, d) \leq 4 \ (\textit{supported by } x_7),$$
$$f(x, c_1) \preccurlyeq (0.4, 0.6) \Longrightarrow f(x, d) \leq 4 \ (\textit{supported by } x_8),$$
$$f(x, c_2) \preccurlyeq (0.41, 0.32) \Longrightarrow f(x, d) \leq 4 \ (\textit{supported by } x_8),$$
$$f(x, c_1) \preccurlyeq (0.38, 0.50) \bigwedge f(x, c_4) \preccurlyeq (0.5, 0.4) \Longrightarrow f(x, d) \leq 5 \ (\textit{supported by } x_9),$$
$$f(x, c_1) \preccurlyeq (0.4, 0.5) \bigwedge f(x, c_6) \preccurlyeq (0.66, 0.28) \Longrightarrow f(x, d) \leq 5 \ (\textit{supported by } x_{10}),$$
$$f(x, c_1) \preccurlyeq (0.4, 0.5) \bigwedge f(x, c_2) \preccurlyeq (0.6, 0.3) \bigwedge f(x, c_4) \preccurlyeq (0.5, 0.4) \Longrightarrow f(x, d) \leq 5 \ (\textit{supported by } x_{10}),$$
$$f(x, c_1) \preccurlyeq (0.4, 0.5) \bigwedge f(x, c_3) \preccurlyeq (0.73, 0.20) \bigwedge f(x, c_4) \preccurlyeq (0.5, 0.4)$$

$$\underline{M}^{\preccurlyeq} = \begin{pmatrix} C & C & C & C & C & C & C & C & C & C \\ C & C & C & C & C & C & C & C & C & C \\ C & C & C & C & C & C & C & C & C & C \\ C & C & C & C & C & C & C & C & C & C \\ C & C & C & C & C & C & C & C & C & C \\ C & C & C & C & c_4 & C & C & C & C & C \\ C & C & C & C & C & C & C & C & c_2c_3c_5 & c_1c_2c_3 \\ C & C & C & C & C & C & C & C & c_1c_2c_6 & c_1c_2 \\ C & C & C & C & C & c_1c_4 & c_4 & c_4c_5 & C & c_1 \\ C & C & C & C & C & c_1 & c_4c_6 & c_3c_4c_5c_6 & c_2c_3c_5c_6 & C \end{pmatrix}.$$

$$\underline{M}^{\succcurlyeq} = \begin{pmatrix} C & C & C & C & C & C & C & C & C & C \\ C & C & C & C & C & C & C & C & C & C \\ c_2c_3 & c_2c_3c_5c_6 & C & c_2c_3c_5 & c_2c_3 & c_3c_5c_6 & c_2c_3c_5 & C & c_5 & c_3c_5 \\ c_1c_4c_6 & c_1c_4c_6 & c_1c_4c_6 & C & c_1c_4c_6 & c_4c_5c_6 & c_1c_4c_5c_6 & c_1c_6 & c_1c_4c_6 & c_1c_4c_5c_6 \\ c_2 & c_1c_2c_3c_5c_6 & c_1c_6 & c_2c_3c_5 & C & c_3c_5c_6 & c_1c_2c_3c_5 & c_1c_6 & c_1c_5 & c_5c_6 \\ c_1c_2 & c_1c_2c_3c_4 & c_1c_2c_4 & c_1c_2c_3 & c_1c_2c_4 & C & c_1c_2c_3 & c_1 & c_1 & c_1c_3 \\ C & c_1c_3c_4c_6 & c_1c_4c_6 & c_2c_3 & c_4 & c_4c_6 & C & c_1c_6 & c_4 & c_3c_4c_6 \\ c_2c_3c_4c_5 & c_2c_3c_4c_5c_6 & c_2c_3c_4c_5c_6 & c_2c_3c_4c_5 & c_2c_3c_4c_5 & c_2c_3c_4c_5c_6 & c_2c_3c_4c_5 & C & c_4c_5 & c_3c_4c_5c_6 \\ c_2c_3 & c_1c_2c_3c_4c_5c_6 & c_1c_2c_3c_5c_6 & c_2c_3c_5 & c_2c_3c_4 & c_2c_3c_5c_6 & c_2c_3c_5 & c_1c_2c_6 & C & c_2c_3c_5c_6 \\ c_2 & c_1c_2c_3c_4c_6 & c_1c_2c_4c_6 & c_2c_3 & c_2c_4 & c_2c_3c_4 & c_1c_2c_3 & c_1c_2 & c_1 & C \end{pmatrix}.$$

$$\Longrightarrow f(x,d) \leq 5 \ (supported\ by\ x_{10}),$$
$$f(x,c_1) \preccurlyeq (0.4,0.5) \bigwedge f(x,c_4) \preccurlyeq (0.5,0.4) \bigwedge f(x,c_5) \preccurlyeq (0.4,0.5) \Longrightarrow f(x,d) \leq 5 \ (supported\ by\ x_{10}).$$

**The first rule can be interpreted as:**

**If "Devaluation" is less than (0.2, 0.6), then the Inflation level is at most** (10.6 *to* 12.5)%.

**The lower discernibility functions of $x_i(3 \leq i \geq 10)$ are:**

$$\underline{f}^{\succcurlyeq}(x_3) = (c_2 \bigvee c_3) \bigwedge (c_2 \bigvee c_3 \bigvee c_5 \bigvee c_6) \bigwedge (c_2 \bigvee c_3 \bigvee c_5) \bigwedge (c_3 \bigvee c_5 \bigvee c_6) \bigwedge (c_5) \bigwedge (c_3 \bigvee c_5) = (c_2 \bigwedge c_5) \bigvee (c_3 \bigwedge c_5),$$
$$\underline{f}^{\succcurlyeq}(x_4) = (c_1 \bigvee c_4 \bigvee c_6) \bigwedge (c_4 \bigvee c_5 \bigvee c_6) \bigwedge (c_1 \bigvee c_4 \bigvee c_5 \bigvee c_6) \bigwedge (c_1 \bigvee c_6) = c_6 \bigvee (c_1 \bigwedge c_4) \bigvee (c_1 \bigwedge c_5),$$
$$\underline{f}^{\succcurlyeq}(x_5) = (c_2) \bigwedge (c_1 \bigvee c_2 \bigvee c_3 \bigvee c_5 \bigvee c_6) \bigwedge (c_1 \bigvee c_6) \bigwedge (c_2 \bigvee c_3 \bigvee c_5) \bigwedge (c_3 \bigvee c_5 \bigvee c_6) \bigwedge (c_1 \bigvee c_2 \bigvee c_3 \bigvee c_5) \bigwedge (c_1 \bigvee c_5) \bigwedge (c_5 \bigvee c_6) = (c_1 \bigwedge c_2 \bigwedge c_5) \bigvee (c_1 \bigwedge c_2 \bigwedge c_6) \bigvee (c_2 \bigwedge c_5 \bigwedge c_6) \bigvee (c_1 \bigwedge c_2 \bigwedge c_5 \bigwedge c_6),$$
$$\underline{f}^{\succcurlyeq}(x_6) = (c_1 \bigvee c_2) \bigwedge (c_1 \bigvee c_2 \bigvee c_3 \bigvee c_4) \bigwedge (c_1 \bigvee c_2 \bigvee c_4) \bigwedge (c_1 \bigvee c_2 \bigvee c_3) \bigwedge (c_1) \bigwedge (c_1 \bigvee c_3) = c_1,$$
$$\underline{f}^{\succcurlyeq}(x_7) = (c_1 \bigvee c_3 \bigvee c_4 \bigvee c_6) \bigwedge (c_1 \bigvee c_4 \bigvee c_6) \bigwedge (c_2 \bigvee c_3) \bigwedge (c_4) \bigwedge (c_4 \bigvee c_6) \bigwedge (c_1 \bigvee c_6) \bigwedge (c_3 \bigvee c_4 \bigvee c_6) = (c_1 \bigwedge c_2 \bigwedge c_4) \bigvee (c_1 \bigwedge c_3 \bigwedge c_4) \bigvee (c_2 \bigwedge c_4 \bigwedge c_6) \bigvee (c_3 \bigwedge c_4 \bigwedge c_6),$$
$$\underline{f}^{\succcurlyeq}(x_8) = (c_2 \bigvee c_3 \bigvee c_4 \bigvee c_5) \bigwedge (c_2 \bigvee c_3 \bigvee c_4 \bigvee c_5 \bigvee c_6) \bigwedge (c_4 \bigvee c_5) \bigwedge (c_3 \bigvee c_4 \bigvee c_5 \bigvee c_6) = c_4 \bigvee c_5,$$
$$\underline{f}^{\succcurlyeq}(x_9) = (c_2 \bigvee c_3) \bigwedge (c_1 \bigvee c_2 \bigvee c_3 \bigvee c_4 \bigvee c_5 \bigvee c_6) \bigwedge (c_1 \bigvee c_2 \bigvee c_3 \bigvee c_5 \bigvee c_6) \bigwedge (c_2 \bigvee c_3 \bigvee c_5) \bigwedge (c_2 \bigvee c_3 \bigvee c_4) \bigwedge (c_2 \bigvee c_3 \bigvee c_5 \bigvee c_6) \bigwedge (c_1 \bigvee c_2 \bigvee c_6) = c_2 \bigvee (c_1 \bigwedge c_3) \bigvee (c_3 \bigwedge c_6),$$
$$\underline{f}^{\succcurlyeq}(x_{10}) = (c_2) \bigwedge (c_1 \bigvee c_2 \bigvee c_3 \bigvee c_4 \bigvee c_6) \bigwedge (c_1 \bigvee c_2 \bigvee c_4 \bigvee c_6) \bigwedge (c_2 \bigvee c_3) \bigwedge (c_2 \bigvee c_4) \bigwedge (c_2 \bigvee c_3 \bigvee c_4) \bigwedge (c_1 \bigvee c_2 \bigvee c_3) \bigwedge (c_1 \bigvee c_2) \bigwedge (c_1) = (c_1 \bigwedge c_2).$$

**The simplified $\succcurlyeq$ lower bound rules for information system inflation rate assessment are:**

$$f(x,c_2) \succcurlyeq (0.42,0.47) \bigwedge f(x,c_5) \succcurlyeq (0.7,0.3) \Longrightarrow f(x,d) \geq 1 \ (supported\ by\ x_3),$$
$$f(x,c_3) \succcurlyeq (0.8,0.1) \bigwedge f(x,c_5) \succcurlyeq (0.7,0.3) \Longrightarrow f(x,d) \geq 1 \ (supported\ by\ x_3),$$
$$f(x,c_6) \succcurlyeq (0.8,0.2) \bigvee f(x,c_1) \succcurlyeq (0.39,0.32) \bigwedge f(x,c_4) \succcurlyeq (0.6,0.4) \Longrightarrow f(x,d) \geq 1 \ (supported\ by\ x_4),$$
$$f(x,c_1) \succcurlyeq (0.39,0.32) \bigwedge f(x,c_5) \succcurlyeq (0.59,0.36) \Longrightarrow f(x,d) \geq 1 \ (supported\ by\ x_4),$$
$$f(x,c_1) \succcurlyeq (0.4,0.5) \bigwedge f(x,c_2) \succcurlyeq (0.41,0.48) \bigwedge f(x,c_5) \succcurlyeq (0.7,0.3) \Longrightarrow f(x,d) \geq 2 \ (supported\ by\ x_5),$$
$$f(x,c_1) \succcurlyeq (0.4,0.5) \bigwedge f(x,c_2) \succcurlyeq (0.41,0.48) \bigwedge f(x,c_6) \succcurlyeq (0.6,0.1) \Longrightarrow f(x,d) \geq 2 \ (supported\ by\ x_5),$$
$$f(x,c_2) \succcurlyeq (0.41,0.48) \bigwedge f(x,c_5) \succcurlyeq (0.7,0.3) \bigwedge f(x,c_6) \succcurlyeq (0.6,0.1) \Longrightarrow f(x,d) \geq 2 \ (supported\ by\ x_5),$$
$$f(x,c_1) \succcurlyeq (0.4,0.5) \bigwedge f(x,c_2) \succcurlyeq (0.41,0.48) \bigwedge f(x,c_5) \succcurlyeq (0.7,0.3) \bigwedge f(x,c_6)\,(0.6,0.1) \Longrightarrow f(x,d) \geq 2 \ (supported\ by\ x_5),$$
$$f(x,c_1) \succcurlyeq (0.5,0.4) \Longrightarrow f(x,d) \geq 3 \ (supported\ by\ x_6),$$
$$f(x,c_1) \succcurlyeq (0.38,0.50) \bigwedge f(x,c_2) \succcurlyeq (0.4,0.5) \bigwedge f(x,c_4) \succcurlyeq (0.51,0.35) \Longrightarrow f(x,d) \geq 4 \ (supported\ by\ x_7),$$
$$f(x,c_1) \succcurlyeq (0.38,0.50) \bigwedge f(x,c_3) \succcurlyeq (0.6,0.2) \bigwedge f(x,c_4) \succcurlyeq (0.51,0.35) \Longrightarrow f(x,d) \geq 4 \ (supported\ by\ x_7),$$
$$f(x,c_2) \succcurlyeq (0.4,0.5) \bigwedge f(x,c_4) \succcurlyeq (0.51,0.35) \bigwedge f(x,c_6) \succcurlyeq (0.7,0.2) \Longrightarrow f(x,d) \geq 4 \ (supported\ by\ x_7),$$
$$f(x,c_3) \succcurlyeq (0.6,0.2) \bigwedge f(x,c_4) \succcurlyeq (0.51,0.35) \bigwedge f(x,c_6) \succcurlyeq (0.7,0.2) \Longrightarrow f(x,d) \geq 4 \ (supported\ by\ x_7),$$
$$f(x,c_4) \succcurlyeq (0.7,0.3) \bigvee f(x,c_5) \succcurlyeq (0.8,0.2) \Longrightarrow f(x,d) \geq 4 \ (supported\ by\ x_8),$$
$$f(x,c_2) \succcurlyeq (0.7,0.3) \bigvee f(x,c_1) \succcurlyeq (0.38,0.50) \bigwedge f(x,c_3) \succcurlyeq (0.9,0.1) \Longrightarrow f(x,d) \geq 5 \ (supported\ by\ x_9),$$
$$f(x,c_3) \succcurlyeq (0.9,0.1) \bigwedge f(x,c_6) \succcurlyeq (0.7,0.2) \Longrightarrow f(x,d) \geq 5 \ (supported\ by\ x_9),$$
$$f(x,c_1) \succcurlyeq (0.4,0.5) \bigwedge f(x,c_2) \succcurlyeq (0.6,0.3) \Longrightarrow f(x,d) \geq 5 \ (supported\ by\ x_{10}).$$

The first rule can be interpreted as:

If "Cost-push inflation" is greater than (0.42, 0.47) and "Rising wages" is greater than (0.7, 0.3), then the Inflation level is at least (2.5 *to* 7.9)%.

## VI. CONCLUSION AND FUTURE WORK

The dominance-based rough set approach generalizes rough set theory by using dominance relations rather than equivalence relations. Due to several factors, incompleteness is a typical feature of information systems. This paper's main goal is to present several interpretations of missing information based on various semantics. We review four forms of incomplete information semantics (i.e., "do-not-care value", "partially-known value", "class-specific value" and "non-applicable value") and provide a generic description of an incomplete information table. However, when we consider ranking fuzzy-valued objects rather than classifying them, conventional rough set theory is unable to solve these problems. One of the extensions of the classic rough set approach is the dominance fuzzy-valued rough set approach. The intuitionistic fuzzy decision table is an extended version of fuzzy-valued information systems and a fundamental type of data. We concentrate on developing a fuzzy-rough set model and rule extraction in DIFDT that support decision-making in intuitionistic fuzzy contexts based on dominance. First, we introduce innovative semantics for incomplete data to describe the concept of DIFDT. Second, we established a fuzzy-rough set strategy in DIFDT based on the dominance-based relation. Third, we employed the discernibility matrices to derive the simplest dominant intuitionistic fuzzy lower and upper bound rules. Finally, we applied these approaches to incomplete information systems for the estimation of inflation rates in LDCs with inadequate data. The application example yielded valuable rules. Moreover, the resulting rules can aid in knowledge acquisition. Our main contributions of these novel semantics are also applicable to other vague complete or incomplete ordered information systems. In our future work, we will extend our developments to IHF, PF, FF, q-ROP, SF, and TSF environments. That will be more effective for DM issues.

## DECLARATION OF INTERESTS

The authors affirm that they have no known financial or interpersonal conflicts that would have seemed to have an impact on the research described in this publication. Additionally, the authors affirm that there are no competing interests in the publishing of this paper.

## REFERENCES


[1] Z. Pawlak, "Rough sets," *Int. J. Comput. Inf. Sci.*, vol. 11, no. 5, pp. 341–356, Oct. 1982.

[2] Z. Pawlak, *Rough Sets: Theoretical Aspects of Reasoning About Data*, vol. 9. Cham, Switzerland: Springer, 1991.

[3] Z. Pawlak and A. Skowron, "Rudiments of rough sets," *Inf. Sci.*, vol. 177, no. 1, pp. 3–27, Jan. 2007.

[4] J. Stefanowski and A. Tsoukias, "Incomplete information tables and rough classification," *Comput. Intell.*, vol. 17, no. 3, pp. 545–566, Aug. 2001.

[5] J. Bazan, J. F. Peters, A. Skowron, N. Hung Son, and M. Szczuka, "Rough set approach to pattern extraction from classifiers," *Electron. Notes Theor. Comput. Sci.*, vol. 82, no. 4, pp. 20–29, Mar. 2003.

[6] R. Jensen and Q. Shen, "Fuzzy–rough attribute reduction with application to web categorization," *Fuzzy Sets Syst.*, vol. 141, no. 3, pp. 469–485, Feb. 2004.

[7] G. Jeon, D. Kim, and J. Jeong, "Rough sets attributes reduction based expert system in interlaced video sequences," *IEEE Trans. Consum. Electron.*, vol. 52, no. 4, pp. 1348–1355, Nov. 2006.

[8] Z. Pawlak, "Rough sets and intelligent data analysis," *Inf. Sci.*, vol. 147, nos. 1–4, pp. 1–12, Nov. 2002.

[9] R. W. Swiniarski and L. Hargis, "Rough sets as a front end of neural-networks texture classifiers," *Neurocomputing*, vol. 36, nos. 1–4, pp. 85–102, Feb. 2001.

[10] R. W. Swiniarski and A. Skowron, "Rough set methods in feature selection and recognition," *Pattern Recognit. Lett.*, vol. 24, no. 6, pp. 833–849, Mar. 2003.

[11] S. Tsumoto, "Automated extraction of medical expert system rules from clinical databases based on rough set theory," *Inf. Sci.*, vol. 112, nos. 1–4, pp. 67–84, Dec. 1998.

[12] S. Greco, B. Matarazzo, and R. Slowinski, "Rough sets theory for multicriteria decision analysis," *Eur. J. Oper. Res.*, vol. 129, no. 1, pp. 1–47, Feb. 2001.

[13] S. Greco, B. Matarazzo, and R. Slowinski, "Rough approximation by dominance relations," *Int. J. Intell. Syst.*, vol. 17, no. 2, pp. 153–171, Feb. 2002.

[14] J. Błaszczyński, S. Greco, R. Słowiński, and M. Szelg, "Monotonic variable consistency rough set approaches," *Int. J. Approx. Reasoning*, vol. 50, no. 7, pp. 979–999, Jul. 2009.

[15] J. Chai and J. N. K. Liu, "Dominance-based decision rule induction for multicriteria ranking," *Int. J. Mach. Learn. Cybern.*, vol. 4, no. 5, pp. 427–444, Oct. 2013.

[16] W. S. Du and B. Q. Hu, "Approximate distribution reducts in inconsistent interval-valued ordered decision tables," *Inf. Sci.*, vol. 271, pp. 93–114, Jul. 2014.

[17] B. Huang, "Graded dominance interval-based fuzzy objective information systems," *Knowl.-Based Syst.*, vol. 24, no. 7, pp. 1004–1012, Oct. 2011.

[18] C. Luo, T. Li, H. Chen, and L. Lu, "Fast algorithms for computing rough approximations in set-valued decision systems while updating criteria values," *Inf. Sci.*, vol. 299, pp. 221–242, Apr. 2015.

[19] R. Susmaga, "Reducts and constructs in classic and dominance-based rough sets approach," *Inf. Sci.*, vol. 271, pp. 45–64, Jul. 2014.

[20] W.-H. Xu, X.-Y. Zhang, J.-M. Zhong, and W.-X. Zhang, "Attribute reduction in ordered information systems based on evidence theory," *Knowl. Inf. Syst.*, vol. 25, no. 1, pp. 169–184, Oct. 2010.

[21] W. Xu, Y. Li, and X. Liao, "Approaches to attribute reductions based on rough set and matrix computation in inconsistent ordered information systems," *Knowl.-Based Syst.*, vol. 27, pp. 78–91, Mar. 2012.

[22] X. Yang, Y. Qi, D.-J. Yu, H. Yu, and J. Yang, "$\alpha$-dominance relation and rough sets in interval-valued information systems," *Inf. Sci.*, vol. 294, pp. 334–347, Feb. 2015.

[23] A. M. Radzikowska and E. E. Kerre, "A comparative study of fuzzy rough sets," *Fuzzy Sets Syst.*, vol. 126, no. 2, pp. 137–155, Mar. 2002.

[24] I. K. Vlachos and G. D. Sergiadis, "Intuitionistic fuzzy information—Applications to pattern recognition," *Pattern Recognit. Lett.*, vol. 28, no. 2, pp. 197–206, Jan. 2007.

[25] R. H. Golan and W. Ziarko, "A methodology for stock market analysis utilizing rough set theory," in *Proc. Conf. Comput. Intell. Financial Eng.*, 1995.

[26] C. Cornelis, M. De Cock, and E. E. Kerre, "Intuitionistic fuzzy rough sets: At the crossroads of imperfect knowledge," *Expert Syst.*, vol. 20, no. 5, pp. 260–270, Nov. 2003.

[27] K. Chakrabarty and T. L. G. Koczy, "Intuitionistic fuzzy rough set," in *Proc. 4th Joint Conf. Inf. Sci. (JCIS)*, Durham, NC, USA, 1998.

[28] A. M. Radzikowska, "Rough approximation operations based on IF sets," in *Proc. Int. Conf. Artif. Intell. Soft Comput.*, Berlin, Germany: Springer, Jun. 2006.

[29] L. Lin, X.-H. Yuan, and Z.-Q. Xia, "Multicriteria fuzzy decision-making methods based on intuitionistic fuzzy sets," *J. Comput. Syst. Sci.*, vol. 73, no. 1, pp. 84–88, Feb. 2007.

[30] S. P. Jena, S. K. Ghosh, and B. K. Tripathy, "Intuitionistic fuzzy rough sets," *Notes IFS*, vol. 8, no. 1, pp. 1–18, 2002.

[31] S. K. Samanta and T. K. Mondal, "Intuitionistic fuzzy rough sets and rough intuitionistic fuzzy sets," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 561–582, 2001.

[32] L. Zhou and W. Wu, "On generalized intuitionistic fuzzy rough approximation operators," *Inf. Sci.*, vol. 178, no. 11, pp. 2448–2465, Feb. 2008.

[33] L. Zhou, W.-Z. Wu, and W.-X. Zhang, "On characterization of intuitionistic fuzzy rough sets based on intuitionistic fuzzy implicators," *Inf. Sci.*, vol. 179, no. 7, pp. 883–898, Mar. 2009.

[34] B. Huang, H.-X. Li, and D.-K. Wei, "Dominance-based rough set model in intuitionistic fuzzy information systems," *Knowledge-Based Syst.*, vol. 28, pp. 115–123, Apr. 2012.

[35] B. Huang, D.-K. Wei, H.-X. Li, and Y.-L. Zhuang, "Using a rough set model to extract rules in dominance-based interval-valued intuitionistic fuzzy information systems," *Inf. Sci.*, vol. 221, pp. 215–229, Feb. 2013.

[36] M. Kryszkiewicz, "Rough set approach to incomplete information systems," *Inf. Sci.*, vol. 112, nos. 1–4, pp. 39–49, Dec. 1998.

[37] G. Wang, "Extension of rough set under incomplete information systems," in *Proc. IEEE World Congr. Comput. Intell., IEEE Int. Conf. Fuzzy Syst.*, Mar. 2002.

[38] J. W. Grzymala-Busse and M. Hu, "A comparison of several approaches to missing attribute values in data mining," in *Proc. 2nd Int. Conf. Rough Sets Current Trends Comput.*, Oct. 2000.

[39] J. W. Grzymała-Busse, "Characteristic relations for incomplete data: A generalization of the indiscernibility relation," in *Proc. 4th Int. Conf. Rough Sets Current Trends Comput.*, Uppsala, Sweden, Jun. 2004.

[40] Y. Leung, J.-M. Ma, W.-X. Zhang, and T.-J. Li, "Dependence-space-based attribute reductions in inconsistent decision information systems," *Int. J. Approx. Reasoning*, vol. 49, no. 3, pp. 623–630, Nov. 2008.

[41] J. W. Grzymala-Busse, "Incomplete data and generalization of indiscernibility relation, definability, and approximations," in *Proc. 10th Int. Conf. Rough Sets Current Trends Comput.*, 2005.

[42] W. Grzymala-Busse, "Rough set strategies to data with missing attribute values," in *Foundations and Novel Approaches in Data Mining*. Berlin, Heidelberg: Springer, 2005, pp. 197–212.

[43] Y. Leung, "Maximal consistent block technique for rule acquisition in incomplete information systems," *Inf. Sci.*, vol. 153, pp. 85–106, Jul. 2003.

[44] J. W. Grzymala-Busse, P. G. Clark, and M. Kuehnhausen, "Generalized probabilistic approximations of incomplete data," *Int. J. Approx. Reasoning*, vol. 55, no. 1, pp. 180–196, Jan. 2014.

[45] R. Susmaga, "Generation of reducts and rules in multi-attribute and multicriteria classification," *Control Cybern.*, vol. 29, no. 4, pp. 969–988, 2000.

[46] F. F. Xu, D. Q. Miao, and L. Wei, "Fuzzy-rough attribute reduction via mutual information with an application to cancer classification," *Comput. Math. Appl.*, vol. 57, no. 6, pp. 1010–1017, Mar. 2009.

[47] Y. Yao and Y. Zhao, "Discernibility matrix simplification for constructing attribute reducts," *Inf. Sci.*, vol. 179, no. 7, pp. 867–882, Mar. 2009.

[48] Y. Yin, G. Gong, and L. Han, "Control approach to rough set reduction," *Comput. Math. Appl.*, vol. 57, no. 1, pp. 117–126, Jan. 2009.

[49] Y. Chen, D. Miao, R. Wang, and K. Wu, "A rough set approach to feature selection based on power set tree," *Knowl.-Based Syst.*, vol. 24, no. 2, pp. 275–281, Mar. 2011.

[50] Y. Cheng and D. Miao, "Rule extraction based on granulation order in interval-valued fuzzy information system," *Expert Syst. Appl.*, vol. 38, no. 10, pp. 12249–12261, Sep. 2011.

[51] T.-F. Fan, C.-J. Liau, and D.-R. Liu, "A relational perspective of attribute reduction in rough set-based data analysis," *Eur. J. Oper. Res.*, vol. 213, no. 1, pp. 270–278, Aug. 2011.

[52] C. Cornelis, R. Jensen, G. Hurtado, and D. Śle, "Attribute selection with fuzzy decision reducts," *Inf. Sci.*, vol. 180, no. 2, pp. 209–224, Jan. 2010.

[53] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, Aug. 1986.

[54] K. T. Atanassov, *Intuitionistic Fuzzy Sets*. Heidelberg, Germany: Physica-Verlag HD, 1999.

[55] Z. Xu, "Intuitionistic preference relations and their application in group decision making," *Inf. Sci.*, vol. 177, no. 11, pp. 2363–2379, Jun. 2007.

[56] M.-W. Shao and W.-X. Zhang, "Dominance relation and rules in an incomplete ordered information system," *Int. J. Intell. Syst.*, vol. 20, no. 1, pp. 13–27, Jan. 2005.

[57] W. Lipski, "On semantic issues connected with incomplete information databases," *ACM Trans. Database Syst.*, vol. 4, no. 3, pp. 262–296, Sep. 1979.

[58] J. Ashayeri, G. Tuzkaya, and U. R. Tuzkaya, "Supply chain partners and configuration selection: An intuitionistic fuzzy Choquet integral operator based approach," *Expert Syst. Appl.* vol. 39, no. 3, pp. 3642–3649, 2012.

[59] S. F. Zhang and S. Y. Liu, "A GRA-based intuitionistic fuzzy multi-criteria group decision making method for personnel selection," *Expert Syst. Appl.*, vol. 38, pp. 1–12, Jan. 2011.


**IFTIKHAR UL HAQ** received the M.Sc. degree from Punjab University, Pakistan, and the M.Phil. degree in mathematics from Riphah International University, Islamabad, Pakistan. He is currently pursuing the Ph.D. degree in mathematics with the Department of Mathematics, Air University, Islamabad, under the supervision of Dr. Tanzeela Shaheen. He is also a Senior Lecturer with the OPF Boys College, Islamabad. His research interests include fuzzy aggregation operators, multi-attribute decision making, fuzzy graph theory, and fuzzy rough set theory. He has more than three international publications to his credit.

**TANZEELA SHAHEEN** received the M.Phil. and Ph.D. degrees in mathematics from Quaid-i-Azam University, Islamabad, in 2012 and 2017, respectively. She is currently an Assistant Professor with Air University, Islamabad. As a Guest Scholar, she also visited the School of Computing, University of Leeds, in 2015. Her research interests include fuzzy sets, rough sets, and decision-making.

**HAMZA TOOR** received the M.S. degree from Riphah International University, Islamabad, in 2017, where he is currently pursuing the Ph.D. degree. He is also a Bio-Medical Engineer with Riphah International University. His research interests include the applications of artificial intelligence in bio-medical engineering, bio-medical signal processing, and bio-mechanics. He has worked on detection of Alzheimer disease using EEG signals.

**TAPAN SENAPATI** received the B.Sc., M.Sc., and Ph.D. degrees in mathematics from Vidyasagar University, India, in 2006, 2008, and 2013, respectively. Currently, he is an Assistant Teacher in mathematics with the Government of West Bengal, India, and a Postdoctoral Fellow with the School of Mathematics and Statistics, Southwest University, Chongqing, China. He has published three books and more than 90 articles in reputed international journals. His research results have been published in *Fuzzy Sets and Systems*, *Information Science*, IEEE TRANSACTIONS ON FUZZY SYSTEMS, *Expert Systems with Applications*, *Applied Soft Computing*, *Engineering Applications of Artificial Intelligence*, *International Journal of Intelligent Systems*, *International Journal of General Systems*, and among others. His research interests include fuzzy sets, fuzzy optimization, soft computing, multi-attribute decision making, and aggregation operators. He is also an Editor of the book titled *Real Life Applications of Multiple Criteria Decision Making Techniques in Fuzzy Domain* (Springer). Recently, his name has been listed in the "World's Top 2% Scientists' list" with Stanford University, from 2020 to 2021. He is a reviewer of several international journals and an Academic Editor of *Computational Intelligence and Neuroscience* (SCIE, Q1), *Discrete Dynamics in Nature and Society* (SCIE), and *Mathematical Problems in Engineering* (SCIE).

**SARBAST MOSLEM** received the Ph.D. degree in transportation engineering with a focus on decision making policy from the Budapest University of Technology and Economics, in 2020. He is currently a Postdoctoral Research Fellow with the School of Architecture Planning and Environmental Policy, University College Dublin (UCD), Ireland. His research interests include decision policy, fuzzy set theory, transport planning; traffic engineering, logistics, supply chain management, soft computing, sustainability, and citizens science. He has been involved in several research projects on national and international level, the author and the coauthor of several conference papers and journals. He worked in several EU projects. He is also a Coordinator and the Principal Investigator for a National Project (VOTE-TRA).