Received 11 June 2023, accepted 24 June 2023, date of publication 27 June 2023, date of current version 6 July 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3290046

SURVEY

Dynamic Neural Network Models for Time-Varying Problem Solving: A Survey on Model Structures

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This work was supported in part by the National Natural Science Foundation of China under Grant 62066015 and Grant 62006095.

ABSTRACT In recent years, neural networks have become a common practice in academia for handling complex problems. Numerous studies have indicated that complex problems can generally be formulated as a single or a set of time-varying equations. Dynamic neural networks, as powerful tools for processing time-varying problems, play an essential role in their online solution. This paper reviews recent advances in real-valued, complex-valued, and noise-tolerant dynamic neural networks for solving various time-varying problems, discusses the finite-time convergence, fixed/varying parameters, and noise tolerance properties of dynamic neural network models. This review is highly relevant for researchers and novices interested in using dynamic neural networks to solve time-varying problems.

INDEX TERMS Dynamic neural networks, zeroing neural network (ZNN), time-varying problems, activation function, noise-tolerant.

I. INTRODUCTION

Time-varying problems are a common feature of many real-world applications [1], [2], [3], [4], [5]. These problems are characterized by their dynamic nature, which means that they change over time, making it challenging to develop a single solution that can handle all possible scenarios. Time-varying problems can arise in many fields [6], [7], [8], [9], [10], from finance and economics to engineering and robotics [11], [12], [13], [14], [15], [16], and they often require real-time decision-making. The dynamic nature of time-varying problems presents several challenges for their solution models. These challenges include: (1) Temporal dependency: the dynamic nature of time-varying problems requires modeling methods that can accurately capture and explain these dynamic variations; (2) System complexity: the dynamic behavior of continuous-time systems can be highly complex, and developing models capable of effectively representing and capturing these complex behaviors presents a challenge; (3) Stability and convergence: due to the dynamic variations, ensuring the stability and convergence of solution methods for time-varying problems can be challenging. Despite the existence of these challenges, time-varying problems remain an essential medium for solving practical problems through modeling. Solving these problems can lead to significant improvements in fields such as finance [17], [18], [19], healthcare [20], and transportation [21], etc. As such, there is a growing interest in developing advanced solutions that can handle time-varying problems.

Machine learning techniques have gained significant attention in recent years, particularly neural networks. Neural networks are a class of machine learning models that are inspired by the structure and function of the human brain. They can learn patterns from large amounts of data and use this knowledge to make predictions or decisions. The neural network approach has shown great success in a variety of

The associate editor coordinating the review of this manuscript and approving it for publication was Mauro Gaggero.

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applications, including image recognition [22], natural language processing [23], and recommendation systems [24]. However, traditional neural networks are designed to solve static problems (which do not change over time), and their effectiveness in handling time-varying problems is not ideal. To solve time-varying problems accurately, Zhang et al. proposed a class of dynamic neural networks specifically designed for this purpose [25], called zeroing neural networks (ZNNs). These neural networks use the time derivative of time-varying parameters to track time-varying solutions perfectly. In the past 20 years, various modifications have been presented considering various external and internal factors. In particular, significant progress has been made in stability research after introducing nonlinear activation functions (AFs) into the neural network model. Currently, this dynamic neural network model has achieved good results for numerous time-varying problems and derived many dynamic models for specific time-varying problems, including linear systems [5], [26], [27], [28], nonlinear equations [4], [29], [30], matrix square root finding [31], [32], [33], quadratic programming (QP) and quadratic minimization (QM) problems [34], [35], [36], etc. Additionally, this dynamic neural network model has been widely applied in many popular fields such as robot manipulator [9], [37], [38], image processing [39], [40], [41], chaotic control [42], [43], [44], etc. Therefore, a detailed survey and summary of the development of this dynamic neural network model is necessary.

This work provides a comprehensive review of relevant studies on zeroing neural networks for solving time-varying problems from a model structure perspective. However, it should be noted that dynamic neural network models for handling time-varying problems encompass not only zeroing neural networks. Gradient neural networks and direct-derivation methods are also widely employed in addressing time-varying problems and have demonstrated exceptional performance in domains such as robotics [9], [45], dynamic moore-penrose inversion [6], and time-varying quadratic programming [46]. Furthermore, considering the widespread occurrence of time delays, delay neural networks have also garnered significant attention in the field of neural networks and have achieved remarkable outcomes [47], [48], [49]. These models, equipped with delay units, enable the capture of temporal dependencies and effective modeling of continuous data. As research progresses, advancements in model design and optimization techniques will facilitate the resolution of an increasing number of time-varying/dynamic problems.

In this review paper, recent work on this dynamic neural network model for time-varying problem-solving will be reviewed and summarized from the perspective of model structure. The remaining contents will be presented in four sections. Section II summarizes the structure of various types of real-valued dynamic neural network models and gives several common activation functions. Section III briefly reviews various types of complex-valued dynamic neural network models for dealing with complex time-varying problems. Section IV presents works related to noise-tolerant dynamic neural network models from the viewpoint of single-integral and double-integral model structures. Finally, Section V provides a summary of this paper.

II. REAL-VALUED DYNAMIC NEURAL NETWORKS

Real-valued dynamic neural network models are powerful tools for modeling complex systems that vary over time, and have been extensively researched and applied across diverse fields [50], [51], [52]. These models possess the ability to capture the dynamic behavior of a broad range of systems, ranging from physical processes such as climate change to social phenomena such as stock markets. Dynamic neural networks are specialized models capable of processing time-varying inputs and outputs, as opposed to static neural networks that are designed to handle fixed input-output mappings. These models are constructed with a network of interconnected neurons that can adapt their behavior over time, enabling them to effectively capture the intricate dynamics of complex systems. The capacity of dynamic neural networks to model dynamic behavior makes them a valuable tool for analyzing systems that exhibit such behavior. In summary, real-valued dynamic neural networks have several properties, including:

- Flexibility: Real-valued dynamic neural network models are highly flexible and can be used to model a wide range of complex systems. This flexibility is due to the ability to handle continuous inputs and outputs, as well as the ability to learn complex nonlinear relationships.
- Dynamic: Real-valued dynamic neural network models are particularly suitable for modeling time-varying problems. They can capture the time dependence in complex problems and give efficient results.
- Improved generalization: Real-valued dynamic neural network models can improve generalization performance by capturing the underlying dynamics of the system being modeled. This allows them to make accurate predictions even in the presence of noise and uncertainty.
- Ability to handle multiple inputs and outputs: Real-valued dynamic neural network models can handle multiple inputs and outputs simultaneously. This makes them suitable for modeling complex systems with multiple variables.

In the interest of readability, we will review the real-valued dynamic neural network model by categorizing it based on the presence or absence of an activation function (AF), as well as its finite-time convergence properties.

A. GENERAL DYNAMIC NEURAL NETWORK MODEL

Firstly, the dynamic neural network model without activation function (or consider using linear AF) is considered and discussed in detail from the perspective of two applications: dynamic optimization and dynamic control.

1) DYNAMIC OPTIMIZATION

Dynamic optimization problems necessitate making decisions at various stages or time points, with each decision impacting subsequent ones. Thus, the impact of future decisions must be considered when making decisions. Dynamic neural network algorithms offer a more efficient solution process for such problems than traditional dynamic programming approaches. This efficiency stems from the ability of dynamic neural networks to make decisions based on learned state representations and adapt to changing environments. Numerous dynamic neural network models have been developed and studied for diverse dynamic optimization problems. For instance, in [53], a dynamic optimization scheme for bi-criteria minimization (BCM) has been proposed to address time-varying quadratic programming problems, and its efficacy has been demonstrated through its application to control and motion planning of robot manipulators. The objective of the model is to ensure the final velocity of joint motion approximates zero. An alternative approach to this dynamic optimization problem, referred to as the pseudoinverse-type BCM scheme, was proposed in [54]. In [55], the authors presented a reformulated dynamic optimization scheme for robot control as a QP problem with both equality and inequality constraints. They proposed a dynamic neural network model for solving this problem, which, as indicated by the authors, demonstrated favorable performance in addressing the dynamic optimization problem. In [56], the authors discussed a dynamic optimization scheme utilizing dynamic neural networks for coordinated path tracking of dual robot manipulators. The primary objectives of this scheme were to prevent high joint velocity and eliminate joint-angle drift by incorporating two criteria, namely, the minimum velocity norm and repetitive motion. Additionally, a dynamic neural network model for online time-varying nonlinear optimization (OTVNO) represented as

$$\dot{\boldsymbol{\chi}}(t) = -P^{-1}(\boldsymbol{\chi}(t), t)\left(\zeta\hbar(\frac{\partial\hbar(\boldsymbol{\chi}(t), t)}{\partial t} + \boldsymbol{\chi}(t), t)\right)$$

was introduced and its theoretical properties were investigated by the authors, see [57]. The results suggested that the model exhibited global exponential convergence. Drawing on these results, the authors arrived at the conclusion that the dynamic neural network model was better suited for solving OTVNO problems compared to traditional methods.

2) DYNAMIC CONTROL

Dynamic control refers to the intricate process of regulating and managing systems that exhibit fluctuations and variability over time. It involves an extensive analysis of the system's behavior, identification of significant factors that affect the behavior, and implementation of suitable strategies to achieve desirable changes. Dynamic control is a fundamental concept in various fields of science, including engineering [58], [59], [60], physics [61], [62], biology [63], [64], and economics [15], [65]. It is leveraged to attain optimal performance, minimize errors, and prevent system failures. To ensure effective dynamic control, a thorough comprehension of the system's underlying principles and the ability to monitor and respond to real-time changes are critical. In this regard, dynamic neural networks have garnered significant attention and research as powerful tools for addressing dynamic control problems. In [13], a novel approach called enhanced repetitive motion planning scheme via dynamic neural networks (ERMPS-DNN) was proposed to solve repetitive motion planning problems in robotic systems. The ERMPS-DNN model is based on quadratic programming and employs a time-varying unified constraint to improve its performance. Through simulations conducted on a UR10 robot manipulator, the authors demonstrated the superiority of ERMPS-DNN over traditional schemes in terms of practicality, validity, and completeness. In [66], a dynamic neural network-based approach was proposed for the dynamic control of mobile manipulator robotic systems (MMRSs) to perform periodic tasks and return to their initial state. The proposed approach employs a QP-based repetitive motion planning and feedback control (RMPFC) scheme, which considers the physical limitations of the system and can mix motion planning and reactive control. The authors validated the efficacy of the RMPFC scheme using gradient dynamics analysis and designed, modeled, and analyzed a kinematically redundant MMRS to demonstrate the effectiveness of the dynamic neural network-based approach. In [67], a dynamic control methodology was proposed for fractional-order uncertain systems, which utilized a dynamic neural network and constructed an augmented fractional-order system to transform the optimal output tracking problem into a linear quadratic regulator design problem. The authors demonstrated the effectiveness of the proposed approach in controlling and tracking the output trajectory of fractional-order systems and suggested that it represented a promising alternative to traditional control methods. In [68], the authors proposed two dynamic gradient controllers for the dynamic control of an inverted pendulum system. The proposed controllers were evaluated through three experimental cases, and their superior performance in solving singularity problems without any switch strategy was demonstrated. Additionally, the robustness of the controllers was analyzed under the presence of time delay and disturbance.

Recurrent neural networks (RNNs) are a type of dynamic neural network model that possesses distinct characteristics compared to other dynamic neural network models. Unlike traditional feedforward neural networks, RNNs introduce recurrent connections, enabling the continuous propagation and processing of information within the network. This recurrent connection enables RNNs to find extensive applications in fields such as robotics [69], quadratic programming [70], and optimization control [71]. For example, in study [72], an accelerated RNN was applied to visual servo control of a physically constrained robotic flexible endoscope. All theoretical, simulation, and physical experimental results demonstrate that the proposed RNN solution is effective in achieving visual servoing and simultaneously handling the physical constraints of the robotic endoscope.

With the rapid development of numerical computing and the extensive demand for numerical computation in practical applications, numerous discrete-time models/algorithms for time-varying problem processing have been widely proposed for better computational or simulation purposes in optimization control. For example, a noise-perturbed discrete-time advanced zeroing neurodynamic (NP-DTAZN) algorithm, capable of simultaneously suppressing various noise sources and providing real-time solutions to future equality-constrained nonlinear optimization problems, has been proposed in [73]. The algorithm has been applied to a Kinova JACO^2 robot manipulator to further substantiate the effectiveness and superiority of the NP-DTAZN algorithm. Moreover, based on the velocity level weighted multicriteria optimization scheme and the six-step extrapolation-backward discretization rule, a novel discretized zeroing neural network model has been further proposed for robot manipulator control with multiple constraints, demonstrating higher solution accuracy compared to existing discretized models, see [74].

B. NONLINEARLY ACTIVATED DYNAMIC NEURAL NETWORK MODEL

Nonlinear activation functions (AFs) are an essential component of dynamic neural networks since they introduce nonlinearity, thereby enabling complex data representations and avoiding the vanishing gradient problem. The output of dynamic neural networks is influenced by all previous inputs, and nonlinear AFs enable the network to learn and model more intricate patterns in the data. If nonlinear AFs were not used, the neural network's ability to model the relationship between inputs and outputs would be limited to linear functions. In summary, the incorporation of nonlinear AFs plays a crucial role in enhancing the modeling capability of dynamic neural networks in dealing with intricate and time-varying systems [75]. Next, we summarize and analyze some dynamic model studies using general nonlinear AFs.

In study [76], a dynamic neural network model under the nonlinear activation function for addressing underdetermined linear systems that have double bound limits on residual errors and state variables. The model transforms the bound-limited underdetermined linear system into a time-varying system consisting of both linear and nonlinear formulas by constructing a nonnegative time-varying variable. Study [77] proposed an adaptive fuzzy-type dynamic neural network (AFT-DNN) model to address time-varying QP problems. The model uses multiple nonlinear activation functions and investigates its characteristics through different membership functions and fuzzy control values. Additionally, in [78], the authors proposed a fuzzy adaptive nonlinearly activated dynamic neural network model, represented as

$$A\dot{\boldsymbol{\chi}}(t) = -(\zeta + \gamma)(AA^{\mathrm{T}} + I)\mathcal{F}(A\boldsymbol{\chi}(t) - I).$$

This model combines the advantages of gradient neural network and zeroing neural network for computing time-varying matrices, and has achieved promising results. The authors conclude that the proposed model demonstrates significant improvement over existing techniques in this field. Nonlinearly activated dynamic neural networks have been extensively investigated and applied in robot control. For instance, a variable gain nonlinearly activated dynamic neural network (VG-NADNN) model was proposed by the authors in [79] for online solution of time-varying matrices, with the structural

$$\dot{\boldsymbol{\chi}}(t) = -\zeta \exp(\lambda t)\mathcal{F}(\boldsymbol{\chi}(t)).$$

Later, in [80], the VG-NADNN model was extended to solve the joint-angular drift problem in redundant robot manipulators. The authors conducted computer simulations and physical experiments on a six-degree-of-freedom Kinova Jaco2 robot to assess the effectiveness, accuracy, safety, and practicality of the VG-NADNN model, and confirmed its efficacy. In another study reported in [81], a DNN model with nonlinear AF was applied to solve the inverse kinematics problem of mobile manipulators. The authors demonstrated that the model could globally and exponentially converge to the solution of the time-varying inverse kinematics problem, while also coordinating the wheels and manipulator. Unlike previous dynamic neural network models that employed a single type of activation function, a new model with a combined AF was proposed in [69] for kinematic control of redundant robot manipulators. The authors demonstrated that this model exhibited improved convergence performance.

C. FINITE-TIME CONVERGENT DYNAMIC NEURAL NETWORK MODEL

With the continuous advancement of computer science and hardware technology, the pursuit of efficient and stable system operation has become a hot topic. Certainly, as the main focus of this paper, dynamic neural networks are no exception. Addressing some of the shortcomings of traditional neural network models, such as long training times, high computational resource consumption, and poor performance in adapting to dynamic environmental changes [82], the proposed class of finite-time-convergence dynamic neural networks undoubtedly provides an effective solution to these issues. In general, finite-time convergent dynamic neural networks have the following two advantages:

- Convergence: The finite-time convergent dynamic neural network models are capable of quickly adapting to new data and environmental changes within a limited time frame by utilizing new AFs or techniques, achieving faster and more accurate convergence. This enables these models to better cope with real-time and dynamic application scenarios.
- Robustness: The finite-time convergent dynamic neural network models exhibit higher robustness and reliability in practical applications, as these models can adaptively adjust their structures and parameters to accommodate different data and environmental conditions.

Therefore, the proposed finite-time convergent dynamic neural network models are aimed at better meeting the needs of practical application scenarios and have been widely applied and validated in multiple fields, such as time-varying Sylvester equation solution [83], dynamic linear system solution [76], [84], dynamic quaternion matrix inversion [41] and vision control of surgical robots [85]. In this subsection, we will categorize and summarize finite-time convergent dynamic neural network models based on the types of parameters (fixed parameters or variable/adapted parameters).

1) FIXED PARAMETER

Real-time solutions of dynamic linear matrix equations (DLMEs), including time-varying matrix inversion which can be viewed as a specific type of DLMEs, have garnered considerable attention from researchers as a fundamental problem in various scientific and engineering domains [86]. In study [87], a dynamic neural network model was introduced by the authors, with the objective of solving time-varying matrix equations, and expressed as

$$A(t)\dot{\boldsymbol{\chi}}(t)B(t) = -\dot{A}(t)\boldsymbol{\chi}(t)B(t) - A(t)\boldsymbol{\chi}(t)\dot{B}(t) + \dot{C}(t) - \zeta\mathcal{F}(\boldsymbol{\chi}(t)B(t) - C(t)), \quad (1)$$

which achieves finite-time convergence for computing dynamic matrix equations. The finite-time convergence of this model was theoretically analyzed and its excellent performance was demonstrated through experiments. Additionally, by designing two new nonlinear activation functions for model (1), the authors investigated another type of dynamic neural network model [88] with faster convergence and lower convergence bound. According to the description in this paper, the convergence bound of this model is

$$t_{\text{up}} \leq \max\{t_{\text{up}}^{-}, t_{\text{up}}^{+}\} \leq \max\left\{\frac{|\eta^{-}(0)|^{1-\zeta}}{\lambda(1-\zeta)}, \frac{|\eta^{+}(0)|^{1-\zeta}}{\lambda(1-\zeta)}\right\}.$$

Similarly, the authors in [89] introduced two finite-time dynamic neural network (DNN) models, namely DNN-I and DNN-II, for matrix inversion, which is a special case of linear matrix equations. They designed novel error functions to enhance the performance of these models. Theoretical analysis indicates that the proposed models demonstrate superior stability and finite-time convergence properties. The paper also presented three simulation examples, which demonstrated the effectiveness of DNN-I and DNN-II in finding dynamic matrix inversion and verified the correctness of the corresponding theorems. In [90], the authors extended the solution of a single linear matrix equation to the solution of linear matrix equation systems, and presented a DNN model activated by a continuous SBPAF (7). The authors pointed out that compared to the DNN model with a non-continuous activation function, the dynamic neural network model with continuous AF eliminates the problem of equilibrium point oscillation. Similarly, in [26], a new dynamic system, called the finite-time convergent dynamic system, is proposed for solving online simultaneous linear equations, expressed as

$$A(t)\dot{\boldsymbol{\chi}}(t) = -\zeta(\text{sgn}^{\kappa}(A(t)\boldsymbol{\chi}(t) - e) + \text{sgn}^{1/\kappa}(A(t)\boldsymbol{\chi}(t) - e)),$$

where $\kappa \in (0, 1)$. Compared with existing gradient-based dynamic systems, this system has better convergence performance, and the upper bound of convergence time and theoretical zero-error bound are analytically derived. The authors have verified the effectiveness of this dynamic system via experiments.

Nonlinear equations are frequently used to describe a variety of phenomena in physics and other fields. When it comes to understanding the physical mechanisms underlying natural phenomena, exploring real-time and accurate solutions to nonlinear equations becomes an inevitable problem. In [30], a novel method was investigated that utilized nonlinearly activated neural dynamics for real-time solutions of dynamic nonlinear equations. Unlike most existing neural dynamics, the proposed method was shown to converge in finite time, and the upper bound of the convergence time was estimated through theoretical analysis. In [91], a specially-constructed AF was studied for dynamic neural networks. The authors pointed out that compared with traditional dynamic neural network models, this model was presented in the form of implicit dynamics, which has better consistency with actual situations and stronger ability in representing dynamical systems. At the same time, the authors expanded the application of this dynamic model to deal with the systems of nonlinear equations and utilized it in the motion tracking of robots [37].

Quadratic optimization is a common problem encountered in various scientific and engineering fields, such as tracking control [92], image processing [41], obstacle avoidance [93] and communication processing. In addition, by transforming the initial problem into a quadratic optimization problem subject to equality constraints, various real-world problems can be solved. Numerous dynamic neural network models have been designed and proposed to tackle QP problems in real-time. In study [94], the authors introduced a novel neurodynamic model for the real-time resolution of equality-constrained QP problems. The article derived the upper bound of the finite convergence time of the model via Lyapunov theory analysis. As the conclusion gives, the proposed model has superior convergence performance compared with existing optimization models. Similarly, The study in [36] presented a fixed-parameter dynamic neural network model with finite-time convergence to address time-varying QM problems, and the model is mathematically formulated as

$$Q(t)\dot{\boldsymbol{\chi}}(t) = -\dot{Q}(t)\boldsymbol{\chi}(t) - \zeta\,\text{sgn}^{1/\xi}(Q(t)\boldsymbol{\chi}(t) + s(t)) - \dot{s}(t) - \zeta\,\text{sgn}^{\xi}(Q(t)\boldsymbol{\chi}(t) + s(t)).$$

The finite-time convergent dynamic neural network models have been further applied in various fields. One of these applications includes the solution of time-varying Sylvester matrix equations [95], [96]. Additionally, these models have been utilized for computing dynamic Lyapunov matrix equations [97] and online finding dynamic matrix square root [98].

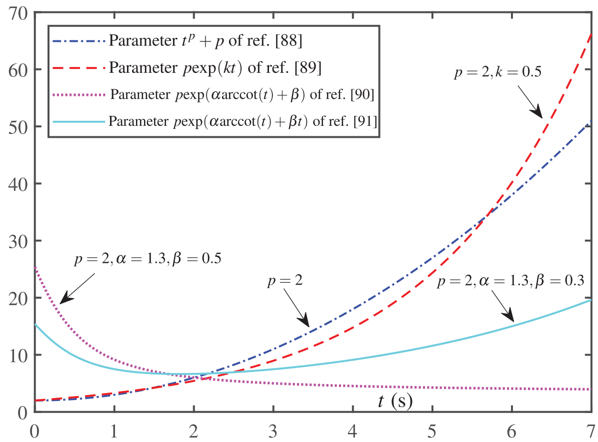

FIGURE 1. Profiles for time-varying parameters commonly used in dynamic neural network models.

These applications have demonstrated the effectiveness of finite-time convergent dynamic neural network models in handling complex mathematical problems with time-varying parameters and provided new directions for future research.

2) VARIABLE/ADAPTED PARAMETER

In [99], the authors proposed a varying parameter dynamic neural network (VPDNN) model for the online inversion of time-varying matrices, which differs from the traditional fixed parameter dynamic neural network (FPDNN) model. The authors highlighted that the proposed model exhibits a fast increase in parameter values with time iterations, which renders it well-suited for hardware implementations. The finite-time convergence and global exponential convergence of this model have been demonstrated via theoretical analysis. The proposed VPDNN model outperforms the traditional FPDNN model in solving time-varying matrix inversion problems, as concluded by the authors. In [100], the authors investigated a DNN model with adaptive parameters for the solution of the dynamic Sylvester equation with the structure of

$$A(t)\dot{\chi}(t) - \dot{\chi}(t)B(t) = \dot{\chi}(t) - \dot{C}(t) + \dot{A}(t)\chi(t) - \zeta\mathcal{F}(C(t)) - \zeta\mathcal{F}(A(t)\chi(t) - \chi(t)B(t)).$$

Detailed theoretical proofs have been provided in the paper for the stability and convergence verification of the model. Similarly, in [101], a dynamic neural network model with varying parameters was proposed for the dynamic Sylvester equation solution. Three types of self-adaption parameters for activation functions have been devised and proposed in this paper, enabling the construction of three distinct models of dynamic neural networks with adaptive coefficients. As summarized by the authors, under the linear-type AF, the upper bound of error function for this dynamic model is:

$$\|\chi(t) - \chi^*(t)\|_F \leq \frac{\lambda_1(xy + \sqrt{xy})\gamma_2}{2(\mu(t^\varsigma + P) - \lambda_1\lambda_1)}.$$

TABLE 1. Details of various linear and nonlinear activation functions.

AFs	Type	Finite-time convergence	Reference
LAF (2)	Linear	No	[51], [53], [55]–[57], [67], [110], [111]
PAF (3)	Non-linear	No	[76], [77], [79]
BPAF (4)	Non-linear	No	[69], [76], [81], [112]
HSAF (5)	Non-linear	No	[87], [101], [113]
PSAF (6)	Non-linear	No	[35], [69], [79]
SBPAF (7)	Non-linear	Yes	[87]–[89], [113]–[115]
TSBPAF (8)	Non-linear	Yes	[26], [35], [36], [87], [88], [114]–[119]

The dynamic Lyapunov equation, as a specialized form of the time-dependent Sylvester equation, holds significant importance in the field of control theory. Therefore, the online solution of dynamic Lyapunov equation is a hot topic. In [102], an accelerated convergence dynamic neural network model with varying parameters has been proposed for finding the theoretical solution of the dynamic Lyapunov equation online, and the excellence of the model has been verified by theoretical analysis and numerical experiments. Studies [103] and [104] proposed a class of variable parameter dynamic neural network models for time-varying matrix inversion. The proposed variable parametric dynamic neural network model demonstrates a better convergence property and higher solution efficiency compared to conventional dynamic neural network models with fixed parameters and gradient-based properties, as reported by the authors. Furthermore, this type of variable parametric dynamic neural network model has been successfully applied for online finding of square roots of matrices [31], [98], [105], solving nonlinear non-convex optimization problems [94], [106], matrix 4th root finding [107], and angle of arrival (AOA) kinematic positioning [108]. A comprehensive review of various types of variable-parameter dynamic neural networks was also conducted by the study [109], and the study presented the profiles of four common types of varying parameters in Fig. 1.

D. VARIOUS TYPES OF ACTIVATION FUNCTIONS

For the reader's convenience, some of the activation functions commonly used in dynamic neural network design are listed in Table 1 and the details are given below.

1) general linear AF:

$$\mathcal{F}(\chi) = \chi. \tag{2}$$

2) power AF:

$$\mathcal{F}(\chi) = \chi^\kappa, \tag{3}$$

where κ is an odd integer and $\kappa > 3$.

3) bipolar sigmoid AF:

$$\mathcal{F}(\chi) = (1 - \exp(-\kappa\chi))/(1 + \exp(-\kappa\chi)) \text{ with } \kappa > 1. \tag{4}$$

4) hyperbolic sine AF:

$$\mathcal{F}(\chi) = (\exp(\kappa\chi) - \exp(-\kappa\chi))/2 \text{ with } \kappa > 1. \tag{5}$$

5) power-sigmoid AF:

$$\mathcal{F}(\chi)=\begin{cases}\chi^{\kappa}, & \text{if } |\chi|\geq 1,\\ \dfrac{1+\exp(-\kappa)}{1-\exp(-\kappa)}\cdot\dfrac{1-\exp(-\kappa\chi)}{1+\exp(-\kappa\chi)}, & \text{otherwise.}\end{cases} \tag{6}$$

6) sign-bi-power AF:

$$\mathcal{F}(\chi)=(|\chi|^{\kappa}+|\chi|^{1/\kappa})\operatorname{sgn}(\chi)/2, \tag{7}$$

where $0<\kappa<1$ and

$$\operatorname{sgn}(\chi)=\begin{cases}1, & \text{for } \chi>0,\\ 0, & \text{for } \chi=0,\\ -1, & \text{for } \chi<0.\end{cases}$$

7) tunable sign-bi-power AF:

$$\mathcal{F}(\chi)=\frac{1}{2}\varrho_1|\chi|^{\kappa}\operatorname{sgn}(\chi)+\frac{1}{2}\varrho_2\chi+\frac{1}{2}\varrho_3|\chi|^{1/\kappa}\operatorname{sgn}(\chi), \tag{8}$$

where $\kappa\in(0,1)$, ϱ_1, ϱ_2 and ϱ_3 are greater than 1.

8) nonlinear AF 1 (NF1-AF):

$$\mathcal{F}(\chi)=\operatorname{sgn}^{\kappa}(\chi) \text{ with } 0<\kappa<1, \tag{9}$$

9) nonlinear AF 2 (NF2-AF):

$$\mathcal{F}(\chi)=\zeta\operatorname{sgn}^{\kappa}(\chi)+\lambda\chi, \tag{10}$$

where $0<\kappa<1$, $\zeta>0$, and $\lambda>0$.

III. COMPLEX-VALUED DYNAMIC NEURAL NETWORKS

In recent years, various machine learning methods based on neural network models have been widely applied in the real world [60], [65], [120], [121], [122], [123], [124]. In the field of neural networks, real-valued weights and activations are commonly used. However, for certain tasks, complex-valued neural networks employing complex-valued weights and activations can offer improved expressive power and wider applicability. Several situations and reasons for using complex-valued neural networks are reviewed below:

- The ability to handle phase information: In some applications, such as signal processing and image processing, the phase information is crucial. Complex-valued neural networks can effectively handle such information, as complex numbers consist of a magnitude and a phase component.
- Increased robustness to noise: Complex-valued neural networks are more robust to noise than their real-valued counterparts due to their inherent redundancy in the representation.
- Enhanced representation power: Complex-valued neural networks can represent a wider range of functions than real-valued neural networks, which can be beneficial in certain applications.

In summary, complex-valued neural networks have shown potential benefits in a variety of applications where handling phase information, robustness to noise, enhanced representation power, or efficient computations are desired [125]. In this section, we will review various dynamic neural network models with complex-valued activation functions, specifically with regards to their ability to solve time-varying problems, and their finite-time convergence properties.

A. GENERAL COMPLEX-VALUED DYNAMIC NEURAL NETWORK MODEL

Online solutions of complex-valued linear matrix equations (CVLMEs) are often found in many important scientific and engineering applications, such as neuro-fuzzy inference systems [126], [127], human action recognition [122], [128], [129], [130], and blind signal extraction [131], [132]. Complex-valued dynamic neural network models have received extensive attention and research as a powerful tool for dealing with complex time-varying problems. In [133], a fully complex dynamic neural network was studied for computing CVLMEs in the complex domain. It was emphasized that the proposed model has advantages over real-valued neural networks in reducing unnecessary complexities in real-time computation and theoretical analysis. The paper also includes numerical experiments and convergence analysis to demonstrate the superiority of the DNN model for online solving of CVLMEs. Furthermore, in study [110], the authors expanded the complex-valued dynamic model to solve a set of CVLMEs and demonstrated its effectiveness and convergence through theoretical analysis and simulation examples. The dynamic neural network model proposed in this study was ultimately applied to the task of motion tracking for a mobile manipulator, and the experiment results provided empirical evidence for the feasibility of this approach in robotic applications.

Regarding the solution of time-varying complex generalized inverse (TVCGI) in the complex domain, the authors introduced five distinct error functions and correspondingly constructed five complex-valued dynamic models in study [134]. The mathematical representations of the five error functions are given as

$$E_1(t)=A(t)B(t)B^H(t)-B^H(t)\in\mathbb{C}^{n\times m} \tag{11a}$$
$$E_2(t)=B^H(t)B(t)A(t)-B^H(t)\in\mathbb{C}^{n\times m} \tag{11b}$$
$$E_3(t)=B(t)A(t)-I\in\mathbb{C}^{m\times m} \tag{11c}$$
$$E_4(t)=A(t)B(t)-I\in\mathbb{C}^{n\times n} \tag{11d}$$
$$E_5(t)=B(t)-A^{+}(t)\in\mathbb{C}^{m\times n} \tag{11e}$$

where $A^{+}(t)$ represents the generalized inverse of the time-varying complex matrix $A(t)$. Additionally, this paper revealed the relationship between the proposed complex-valued dynamic models and the Getz-Marsden dynamic system in the complex domain and validated the effectiveness of the five dynamic models for solving the problem of TVCGI through theoretical analysis and numerical experiments. In [135], an error function was constructed for processing dynamic complex-valued outer inverses, which was utilized in the design of complex-valued dynamic models. The authors emphasized that the results of Moore-Penrose [136], [137] and Drazin inverses [138] could be simply derived as special representations of the proposed model, avoiding restrictions on the spectrum and requirements for certain matrix nonsingularity.

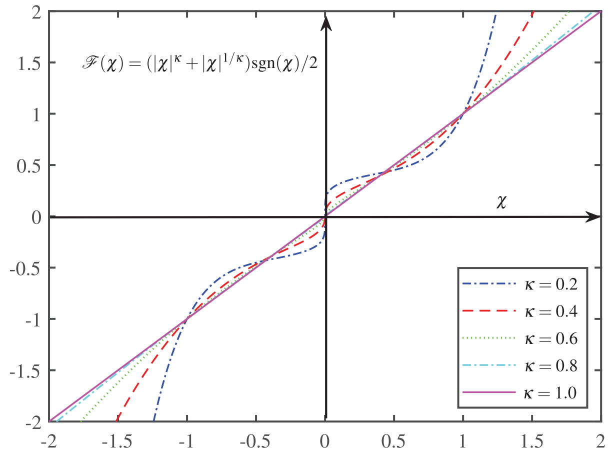

FIGURE 2. Sign-bi-power activation function (7) at different κ.

Quadratic programming (QP) problem is often found in scientific and engineering applications, such as nonlinear control, image processing and communication systems. Many DNN models have been designed and developed for the purpose of resolving QP problems. In [139], the authors proposed two complex-valued dynamic neural network models by defining two distinct complex error functions. These models were investigated for computing complex-valued dynamic QP (CVDQP) subject to complex linear equality constraints. The paper showed that both of these complex dynamic models can globally and exponentially converge to the theoretical optimal solution of the CVDQP, and their performance is superior to that of complex traditional gradient neural network models extended from real domain. In [111], a novel dynamic model named CVDQP decomposition-based linear matrix equation (CVDQP-LME) model was proposed to address the complex time-varying linear matrix equation problems frequently encountered in science and engineering. The proposed model exhibits notable benefits in its capability to handle linear systems with square or rectangular coefficient matrices for both matrices and vectors. Its efficacy has been confirmed through diverse numerical simulations as well as practical applications such as robot motion tracking and reaching angle positioning.

B. NONLINEARLY ACTIVATED COMPLEX-VALUED DYNAMIC NEURAL NETWORK MODEL

The nonlinearly activated complex-valued dynamical neural network (CDNN) is a distinctive neural network category that utilizes complex numbers as input and output values of the network's neurons. The nonlinear activation function of the CDNN is specifically designed for the domain of complex numbers. This neural network model has been widely applied in addressing complex challenges in speech and image recognition, natural language processing and machine learning. Compared to the nonlinearly activated real-valued dynamic neural network, the CDNN exhibits greater expressive capability and improved potential for generalization.

In [140], a class of nonlinearly activated CDNN models was proposed to address the complex time-varying matrix inversion problem. Considering the unavoidable noise in practical applications, the authors verified the convergence and stability of the proposed dynamic model under noisy environments. Additionally, in study [141], a class of CDNN models with general nonlinear AFs and noise tolerance capability was proposed for solving the time-varying matrix pseudoinverse problem. For the online solution of complex linear equations, a class of CDNN models with a novel nonlinear AF was proposed in [116], and its model structure is

$$\dot{\boldsymbol{\chi}}(t) = -\zeta\,\mathrm{sign}(\boldsymbol{\chi}(t))(\rho_1|\boldsymbol{\chi}(t)|^{\kappa} + \rho_2|\boldsymbol{\chi}(t)|^{1/\kappa} - \rho_3|\boldsymbol{\chi}(t)|),$$

where $\kappa \in (0, 1)$. In this work, both theoretical derivations and numerical examples have supported the effectiveness of this model. Moreover, the authors pointed out that the nonlinear AF used in this study is superior to the linear AF and adjustable AF used in previous dynamic neural networks, and has higher convergence efficiency. Building upon this, in [112], this model was further extended for robot manipulator control, and simulation results confirmed the effectiveness and superiority of the proposed complex-valued dynamic neural network model.

In [138], the authors investigated and proposed two CDNN models for computing the Drazin inverse of arbitrary complex-valued dynamic matrices. These two dynamic neural networks were designed based on the corresponding matrix-valued error functions generated by the limit representation of the Drazin inverse and using two different AFs. The paper also provided theoretical results for convergence analysis to demonstrate the desirable properties of the CDNN models. Finally, numerical results were presented to verify the effectiveness of the proposed models. In [142], a CDNN model for computing the dynamic complex matrix Moore-Penrose inverse was proposed. In constructing this model, the authors defined a special type of saturating activation function that relaxes the convex constraint on the AF, i.e.,

$$\mathcal{F}(\chi) = \begin{cases} \chi, & \|\chi\|_{\mathrm{F}} \leq \varphi, \\ \varphi\dfrac{\chi}{\|\chi\|_{\mathrm{F}}}, & \|\chi\|_{\mathrm{F}} > \varphi, \end{cases} \tag{12}$$

where $\varphi > 0$. Furthermore, various types of nonlinear activation CDNN models have also been widely designed for time-varying QP problems [11], [143], time-varying equality and inequality constraints [1], [144], etc.

C. FINITE-TIME CONVERGENT COMPLEX-VALUED DYNAMIC NEURAL NETWORK MODEL

A crucial feature of dynamic neural network models is finite-time convergence, which ensures that the steady-state solution can be attained in a specified amount of time. This property is particularly significant in modeling and predicting complex systems, as it offers numerous benefits such as:

- Practicality: Dynamic neural network models with finite-time convergence and complex-valued outputs

TABLE 2. Various types of dynamic neural network models for time-varying problem solving.

Time-varying problems	Dynamic neural network models	Ref.
Matrix inversion $V(t)\boldsymbol{\chi}(t) = I$	$V(t)\dot{\boldsymbol{\chi}}(t) = -\zeta\mathcal{F}(V(t)\boldsymbol{\chi}(t) - I) - \dot{V}(t)\boldsymbol{\chi}(t)$	[99]
Matrix square root finding $\boldsymbol{\chi}^2(t) = V(t)$	$\dot{\boldsymbol{\chi}}(t)\boldsymbol{\chi}(t) + \boldsymbol{\chi}(t)\dot{\boldsymbol{\chi}}(t) = -\zeta\mathcal{F}(\boldsymbol{\chi}^2(t) - V(t)) - \dot{V}(t)$	[31]
Linear system $V(t)\boldsymbol{\chi}(t) = \boldsymbol{\mu}(t)$	$V(t)\dot{\boldsymbol{\chi}}(t) = -\dot{V}(t)\boldsymbol{\chi}(t) + \dot{\boldsymbol{\mu}}(t) - \zeta\mathcal{F}(V(t)\boldsymbol{\chi}(t) - \boldsymbol{\mu}(t))$	[84]
Nonlinear equations $f(\boldsymbol{\chi}(t), t) = 0$	$\dot{\boldsymbol{\chi}}(t) = -J^{-1}(\boldsymbol{\chi}(t), t)\big(\zeta\mathcal{F}(f(\boldsymbol{\chi}(t), t)) + \frac{\partial f(\boldsymbol{\chi}(t),t)}{\partial t}\big)$	[30]
4th root finding $\boldsymbol{\chi}^4(t) = \nu(t)$	$\dot{\boldsymbol{\chi}}(t) = \dfrac{\dot{\nu}(t) - \zeta\mathcal{F}(\boldsymbol{\chi}^4(t) - \nu(t))}{4\boldsymbol{\chi}^3(t)}$	[107]
Matrix pseudoinverse $V(t)\boldsymbol{\chi}(t)V(t) = V(t)$	$\dot{\boldsymbol{\chi}}(t)V(t)V^{\mathrm{H}}(t) = \dot{V}^{H}(t) - \boldsymbol{\chi}(t)\big(\dot{V}(t)V^{\mathrm{H}}(t) + V(t)\dot{V}^{\mathrm{H}}(t)\big) - \zeta\big(\boldsymbol{\chi}(t)V(t)V^{\mathrm{H}}(t) - V^{\mathrm{H}}(t)\big)$	[134]
Quadratic optimization min. $\boldsymbol{\mu}^{\mathrm{T}}(t)V(t)\boldsymbol{\mu}(t)/2 + \boldsymbol{\omega}(t)\boldsymbol{\mu}(t)$ s.t. $A(t)\boldsymbol{\mu}(t) + d(t) = 0$	$S(t)\dot{\boldsymbol{\chi}}(t) = -\dot{S}(t)\boldsymbol{\chi}(t) + \dot{\boldsymbol{v}}(t) - \zeta\mathcal{F}(S(t)\boldsymbol{\chi}(t) - \boldsymbol{v}(t))$	[75]

have practical advantages for prediction and control tasks. By reaching a steady state within a limited time, such models can facilitate efficient and effective real-time decision-making, making them applicable in various domains, such as control systems, robotics, and signal processing.

- Accuracy: Models that exhibit finite-time convergence have the advantage of achieving stable states within a shorter duration of time, leading to higher accuracy in prediction and control tasks. Consequently, these models are less prone to prediction and control errors than their counterparts lacking finite-time convergence.
- Stability: The stability of the system is a critical aspect in the prediction and control of complex systems. CDNN models with finite-time convergence can attain a stable state in a shorter period of time, and once in a stable state, the output of the model will remain largely unchanged. This characteristic of limited variation in the output is particularly valuable in ensuring the stability of the system.

Therefore, CDNN models with finite-time convergence have significant importance for modeling and predicting complex systems, and can provide more accurate and reliable prediction and control methods for practical applications.

Nonlinear AFs, as an important component of dynamic neural network models, are of great significance in enhancing the expressive power, robustness and generalization ability of the models. Various nonlinear activation functions have been designed and proposed for complex-valued dynamic neural network models aimed at solving various time-varying problems to achieve convergence within a finite time. In [114], a class of complex-valued composite DNN models were proposed for the inverse of dynamic matrices. The sign-bi-power function was used as the AF in this model, and the theoretical upper bound of the convergence time of the model was derived as $(\zeta + \eta)\lambda_0^{1-\kappa}/\zeta\eta(1 - \kappa)$. Fig. 2 shows the contours of the sign-bi-power function (7) at different κ. In [145], a variable-parameter finite-time convergent dynamic neural network model was proposed for the same problem of time-varying matrix inversion, and was applied in manipulator trajectory tracking. Similarly, in the studies presented in [146] and [147], finite-time dynamic neural network models were proposed for the problems of time-varying matrix pseudoinverse and complex time-varying matrix inversion, respectively. Combined with the advantages of fuzzy logic systems for computing uncertainty and dynamic neural network models with parallel processing properties, two complex fuzzy dynamic neural network models were established in [148]. The authors concluded that the proposed complex fuzzy dynamic neural network not only had the inherent ability of finite-time convergence and robustness to noise, but also had faster adaptive convergence efficiency in noisy environments. Meanwhile, for the solution of complex linear dynamic equation systems, multiple CDNN models with finite-time convergence capability have been proposed, see [117] and [149]. The Sylvester equation, as a famous matrix equation, has a wide range of applications in control theory. Solving time-varying Sylvester equations (TVSEs) is a major problem in multiple engineering fields and digital applications, such as image processing, robot applications, and eigenvalue assignment. In [150], a class of complex-valued dynamic neural network models with finite-time convergence property was proposed for online solving TVSEs, and the finite-time convergence properties of the DNN were supported by theoretical and numerical results. In [151], an improved DNN model was designed for solving the TVSE. The authors proposed a new sign-multi-power activation function and provided a theoretical convergence upper bound for the model in this work. Additionally, a class of variable-parameter arctan-type DNN models with accelerated convergence capability was proposed for online solving complex TVSEs [152]. The convergence factor of this model continuously increases over time, and the proportion factor approaches a constant when the model converges, which ensures fast convergence speed and avoids waste of computing resources. Table 2 presents the structure of the dynamic neural network model used for different time-varying problems.

D. COMPLEX-VALUED NONLINEAR ACTIVATION FUNCTIONS

The property of finite-time convergence in CDNN models is closely linked to the complex nonlinear AF. This function extends the nonlinear AF from the real domain to the complex domain, thereby enabling more efficient processing of complex inputs and outputs. Compared to their counterparts in the real domain, complex nonlinear AFs have greater efficacy in handling the nonlinearity of complex data, as they can operate on complex inputs and outputs.

In [153], the authors studied a new approach to extend nonlinear AFs from the real domain to the complex domain. They demonstrated the convergence of the CDNN model using complex-valued non-linear AFs through theoretical analysis. Specifically, the authors proposed two methods to extend real-valued AFs to complex-valued AFs, which are as follows.

- Extending Method 1:

$$\mathcal{A}(a + ib) = \mathcal{F}(a) + i\mathcal{F}(b). \tag{13}$$

This method involves applying the real-valued AF to the real and imaginary parts of the complex-valued input, respectively. Here, $\mathcal{A}(\cdot)$ in equation (13) represents the complex-valued AF.

- Extending Method 2:

$$\mathcal{A}(a + ib) = \mathcal{F}(\Gamma) \circledast \exp(i\Omega). \tag{14}$$

This method constructs a complex-valued AF from the perspective of the modulus $\Gamma \in \mathbb{R}$ and argument $\Omega \in (-\pi, \pi]$ of the complex number $a+ib$, using real-valued AF. In equation (14), the operator $\circledast$ represents the element-wise multiplication between two matrices or vectors. i.e., $C \circledast D = [c_{xy}d_{xy}]$, where c_{xy} and d_{xy} denote the xy-th sub-elements of the real-valued matrices C and D, respectively.

IV. NOISE-TOLERANT DYNAMIC NEURAL NETWORKS

Dynamic neural networks are used to model complex and dynamic systems that change over time. They are able to adapt and learn from changing input data, and can handle unexpected changes in the environment. However, real-world data is often noisy and contains variability (noise may arise from inaccuracy of the sensor, interference in signal transmission, error in signal sampling and rounding errors in calculations, etc.), which can affect the performance of these networks.

Noise-tolerant dynamic neural networks are designed to address this challenge by incorporating mechanisms for robustness and adaptability in their architecture [154]. They are able to handle noisy and dynamic data by incorporating feedback loops, recurrent connections, and dynamic state variables that allow them to learn and update their representations over time [155]. The key characteristics of noise-tolerant dynamic neural networks include their ability to handle noisy and dynamic data, their adaptability and robustness, and their suitability for real-world applications. They are well-suited for applications such as image processing and natural language understanding, where noise and variability are common.

Overall, the use of noise-tolerant dynamic neural networks represents an important advancement in the field of neural networks, as they are able to handle real-world data and improve the accuracy and robustness of models in practical applications. This section aims to present an overview of noise-tolerant dynamic neural network models from the perspective of activation function type, single-integral model structure or double-integral model structure.

A. SINGLE-INTEGRAL-STRUCTURE DYNAMIC NEURAL NETWORK MODEL

Solving time-varying equations is essentially similar to controlling dynamic systems: the residuals are required to be reduced to an acceptably small value as soon as possible. Study [156] proposed a dynamic neural network design formula, its structure can be mathematically expressed as,

$$\dot{\chi}(t) = -J^{-1}(\chi(t), t)\Big(\zeta f(\chi(t), t) + \lambda \int_0^t f(\chi(\tau), \tau)\mathrm{d}\tau + \frac{\partial f(\chi(t), t)}{\partial t}\Big), \tag{15}$$

from the perspective of control theory, based on the essential similarity between solving time-varying equations and controlling dynamic systems, aiming to handle the convergence, stability, and robustness issues of continuous (and discrete) time models. The design formula can be regarded as a control theory framework, which provides valuable tools for these problems. The authors concluded that this study was the first to extend existing research results from a control theory viewpoint for time-varying problems processing, building upon the previous work that addressed static problems. The design formula can serve as a foundation for further research on solving time-varying problems based on the internal model principle and can be extended to more complex formulas, opening a door for research on solving time-varying problems with noise. The basic framework of the dynamic neural network model (16) is shown in Fig. 3.

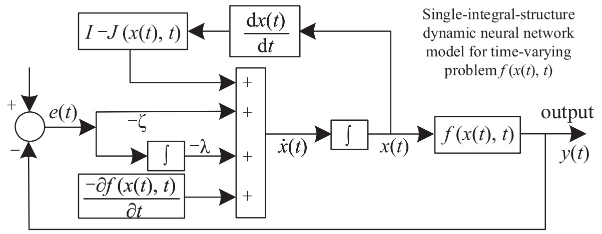

FIGURE 3. Basic framework of dynamic neural network model (16).

1) GENERAL NOISE-TOLERANT DYNAMIC NEURAL NETWORK MODEL

Building upon the work [156], the study [157] introduces a novel dynamic neural network model for computing the matrix outer inverse subject to null space and specific range constraints under the various noise. Theoretical analysis shows that the proposed model converges globally and exponentially to the theoretical solution, and simulation results show that it performs well in the presence of various noises. Furthermore, in [158], the authors further extended the single-integral-structure dynamic neural network model (15) to the online solution of time-varying Lyapunov equations. In [159], A hybrid enhanced dynamic neural network model was proposed based on the previous dynamic neural network model. This work studied the convergence and robustness of the proposed hybrid enhanced dynamic model both theoretically and numerically, and compared it with the standard dynamic model. Based on the aforementioned control theory framework, study [160] proposed a novel dynamic neural network for redundant manipulator kinematic control that can handle input disturbances and physical constraints such as joint angle and velocity constraints, while optimizing a general quadratic performance index. The neural network is suitable for both regulation and tracking tasks, and theoretical analysis shows that it can achieve asymptotic convergence to zero for the tracking and regulation errors of the end effector, even in the presence of input disturbances and two constraints.

2) NONLINEARLY ACTIVATED NOISE-TOLERANT DYNAMIC NEURAL NETWORK MODEL

Based on the single-integral structure dynamic neural network model (15) and nonlinear AFs (9) and (10), two complex-valued dynamic neural network models were designed in [113] to solve complex time-varying linear equations (TVLEs). The convergence analysis results of the paper showed that the stable error upper bound of the model's convergence can be successfully obtained, demonstrating its superior robustness. The authors pointed out that the two proposed nonlinear activation functions have better performance compared to Linear (2), Power (3), Bipolar sigmoid power-sigmoid (4), Power-sigmoid (6) and Sign-bi-power (7) AFs. Similarly, for the problem of solving TVLEs defined in the real domain, a dynamic neural network model with nonlinearity and robustness was proposed [27]. The authors pointed out that this dynamic model can converge to the theoretical solution of the time-varying matrix equation regardless of the type of activation function used. In [32], a new type of nonlinear activation dynamic neural network model was proposed for finding the time-varying and static matrix square roots with the model structure of

$$\dot{\boldsymbol{\chi}}(t) = -\zeta_1 \mathcal{F}\big(\boldsymbol{\chi}(t)\big) - \zeta_2 \mathcal{G}\Big(\boldsymbol{\chi}(t) + \zeta_1 \int_0^t \mathcal{F}\big(\boldsymbol{\chi}(t)\big)\mathrm{d}t\Big), \tag{16}$$

where $\boldsymbol{\chi}(t) = A^2(t) - B(t) \in \mathbb{R}^{n\times n}$, and $B(t)$ is a time-varying positive definite matrix, $B(t)$ is the time-varying unknown matrix to be solved. The authors pointed out that the proposed model considered the noise interference that exists in the hardware implementation process. Even in the face of large noise errors, the state solution of this model can still converge to the theoretical square root of the given matrix. In study [35], the nonlinear activation and noise-resistant dynamic neural network model were further extended for online solving time-dependent QP and time-dependent QM problems. In this work, the authors also provided two practical application cases (robot tracking and risk investment), further verifying the effectiveness, accuracy, and wide applicability of the proposed model. Taking into account the interference of noise, the nonlinearly activated noise-tolerant dynamic neural network model has been widely used in popular fields such as redundant mechanical arm motion planning [161] and distributed network coordinated motion of robots [38], [162].

TABLE 3. Various types of noise-tolerant dynamic neural network models for time-varying problem solving.

Time-varying problems	Noise-tolerant dynamic neural network models	Ref.
Matrix inversion $V(t)\boldsymbol{\chi}(t) = I$	$V(t)\dot{\boldsymbol{\chi}}(t) = -\dot{V}(t)\boldsymbol{\chi}(t) - \zeta_1(V(t)\boldsymbol{\chi}(t) - I)$ $- \zeta_2 \int_0^t (V(\tau)\boldsymbol{\chi}(\tau) - I)\mathrm{d}\tau$	[163]
Matrix square root finding $\boldsymbol{\chi}^2(t) = V(t)$	$\boldsymbol{\chi}(t)\dot{\boldsymbol{\chi}}(t) + \dot{\boldsymbol{\chi}}(t)\boldsymbol{\chi}(t) = -\zeta_1(\boldsymbol{\chi}^2(t) - V(t)) - \dot{V}(t)$ $- \zeta_2 \int_0^t (\boldsymbol{\chi}^2(\tau) - V(\tau))\mathrm{d}\tau$	[32]
Linear system $V(t)\boldsymbol{\chi}(t) = \boldsymbol{\mu}(t)$	$V(t)\dot{\boldsymbol{\chi}}(t) = -\dot{V}(t)\boldsymbol{\chi}(t) + \dot{\boldsymbol{\mu}}(t) - \zeta_1(V(t)\boldsymbol{\chi}(t) - \boldsymbol{\mu}(t))$ $- \zeta_2 \int_0^t (V(\tau)\boldsymbol{\chi}(\tau) - \boldsymbol{\mu}(\tau))\mathrm{d}\tau$	[113]
Nonlinear equations $f(\boldsymbol{\chi}(t), t) = 0$	$\dot{\boldsymbol{\chi}}(t) = -J^{-1}(\boldsymbol{\chi}(t), t)\big(\zeta_1(f(\boldsymbol{\chi}(t), t)) + \frac{\partial f(\boldsymbol{\chi}(t),t)}{\partial t}$ $- \zeta_2 \int_0^t (f(\boldsymbol{\chi}(\tau), \tau))\mathrm{d}\tau\big)$	[118]
4th root finding $\boldsymbol{\chi}^4(t) = \nu(t)$	$\dot{\boldsymbol{\chi}}(t) = \frac{\dot{\nu}(t) - \zeta_1(\boldsymbol{\chi}^4(t) - \nu(t)) - \zeta_2 \int_0^t (f(\boldsymbol{\chi}(\tau), \tau))\mathrm{d}\tau}{4\boldsymbol{\chi}^3(t)}$	[164]
Matrix pseudoinverse $V(t)\boldsymbol{\chi}(t)V(t) = V(t)$	$\dot{\boldsymbol{\chi}}(t)V(t)V^{\mathrm{H}}(t) = \dot{V}^H(t) - \boldsymbol{\chi}(t)\big(\dot{V}(t)V^{\mathrm{H}}(t) + V(t)\dot{V}^{\mathrm{H}}(t)\big)$ $- \zeta_1\big(\boldsymbol{\chi}(t)V(t)V^{\mathrm{H}}(t) - V^{\mathrm{H}}(t)\big)$ $- \zeta_2 \int_0^t (f(\boldsymbol{\chi}(\tau), \tau))\mathrm{d}\tau$	[165]
Quadratic optimization min. $\boldsymbol{\mu}^{\mathrm{T}}(t)V(t)\boldsymbol{\mu}(t)/2 + \boldsymbol{\omega}(t)\boldsymbol{\mu}(t)$ s.t. $A(t)\boldsymbol{\mu}(t) + d(t) = 0$	$S(t)\dot{\boldsymbol{\chi}}(t) = -\dot{S}(t)\boldsymbol{\chi}(t) + \dot{\boldsymbol{\upsilon}}(t) - \zeta_1(S(t)\boldsymbol{\chi}(t) - \boldsymbol{\upsilon}(t))$ $- \zeta_2 \int_0^t (S(\tau)\boldsymbol{\chi}(\tau) - \boldsymbol{\upsilon}(\tau))\mathrm{d}\tau$	[35]

3) FINITE-TIME CONVERGENT NOISE-TOLERANT DYNAMIC NEURAL NETWORK MODEL

In practical applications, dynamic neural networks often face problems such as model instability, convergence difficulties, and sensitivity to noise. To address these issues, various dynamic neural network with finite-time convergent capability and noise tolerant have been proposed in recent years [166], [167]. Dynamic neural networks with finite-time convergence capability can solve time-varying problems in a limited time, thus facilitating optimization and control more effectively [115]. The noise-tolerant ability of the model enables it to deal with time-varying tasks with noise and reduces instability and errors caused by noise [29], [33]. In this subsection, we will review the finite time convergent and noise tolerant dynamic neural networks from the perspective of dynamic optimization control and dynamic linear system solving.

a: DYNAMIC LINEAR SYSTEM SOLVING

Time-varying matrices, which are matrices with elements that change over time, are widely used in various fields to describe the dynamic characteristics of systems. The inversion of time-varying matrices is a fundamental mathematical operation that is frequently required. The ability to perform this operation accurately and efficiently in the presence of external disturbances or errors is crucial for the effective functioning of dynamic systems. In [168], a new robust dynamic neural network was proposed for online solving the time-varying matrix inversion problem with disturbances. This dynamic neural network uses a universal activation function and solves the inverse of the time-varying matrix within a predetermined time. In [169], a dynamic model with changing parameters was investigated for dynamic matrix inversion, which has faster convergence compared to traditional dynamic neural networks with fixed parameters. Furthermore, considering the dynamic characteristics of harmonic noise, study [163] proposed a dynamic neural network model that can simultaneously suppress harmonic noise and perform time-varying matrix inversion within a specified time. More generally, the design of noise-tolerant dynamic neural networks for solving time-varying matrix pseudoinverse problems was further discussed in [165] and [170]. The Lyapunov equation is a matrix equation widely used to analyze the stability and performance of linear time-invariant systems. In practical applications, systems are frequently time-varying, making it critical to consider the solution of time-varying Lyapunov equations. As a result, numerous dynamic neural network models have been proposed for this purpose. In [171], a Z-type neural dynamical model was introduced, which employs a nonlinear activation function with an integral term and converges in a finite time while suppressing inherent noise. This model is effective in solving time-varying problems in noisy environments. In [172], two robust nonlinear dynamic models were designed to solve time-varying Lyapunov equations. The models were shown to have a predetermined convergence property and the convergence effect was independent of the initial values. In [173], a dynamic model based on variable scaling factors was proposed to solve more general time-varying Sylvester equations. In [174], a dynamic method with a nonlinear activation was introduced to solve dynamic Sylvester equations, while in [175], dynamic neural networks were used to solve these equations in various noise environments within a finite time.

b: DYNAMIC OPTIMIZATION CONTROL

Dynamic quadratic minimization (DQM) is a mathematical optimization problem, whose objective function is a time-varying quadratic function that can be mathematically described as

$$\begin{aligned} &\min.\ \boldsymbol{\chi}^T(t)K(t)\boldsymbol{\chi}(t)/2+\boldsymbol{\mu}^T(t)\boldsymbol{\chi}(t)\\ &\text{s.t.}\, f(\boldsymbol{\chi}(t),t)=0 \end{aligned} \tag{17}$$

where $K(t) \in \mathbb{R}^{n\times n}$ and $\boldsymbol{\mu}(t) \in \mathbb{R}^n$ are the known time-varying coefficient matrix and vector, respectively; $\boldsymbol{\chi}(t) \in \mathbb{R}^n$ is the unknown variable to be computed, and $f(\boldsymbol{\chi}(t),t)=0$ denotes a set of equation constraints. DQM has been widely applied in optimization [176], [177], collaborative processing [178], [179], [180], control theory [181], and engineering [182], [183]. To achieve fast and accurate solutions to DQM problems, numerous dynamic neural network models with limited time convergence and noise tolerance capabilities have been proposed. A unified framework was proposed in [34] to design dynamic neural network models that exhibit both noise tolerance and predefined time convergence to address the DQM problem (17). This framework was obtained by using AF (8) on the basis of the dynamic model (16). In [184], the authors investigated a new dynamic method for solving the DQM problem considering additive noise. The proposed dynamic method was designed based on a new nonlinear activation integral design formula. In [185], a new piecewise time-varying dynamic model was designed to handle DQM problems. This dynamic model includes an integral term and a nonlinear AF, as well as two specially constructed time-varying piecewise parameters, which enable the model to have both faster convergence speed and better noise resistance. Similar to DQM problems, time-varying nonlinear minimization problems (TVNMPs) have also received widespread attention. In [118], a robust predefined-time DNN model was studied for solving the TVNMP. The authors pointed out that previous models for TVNMPs either had finite-time convergence or noise suppression, but not both. This model was designed to have both finite-time convergence and noise suppression, and theoretical analysis was provided to support its superior performance. In addition, in [119], the authors further extended this finite-time robust neural network model to solve TVNMPs with equality constraints. Table 3 lists the structure of the single-integral-structural noise-tolerant dynamic neural network model for solving different time-varying problems.

B. DOUBLE-INTEGRAL-STRUCTURE DYNAMIC NEURAL NETWORK MODEL

Drawing on the principles of control theory, a dynamic neural network design framework with a single integrator structure was designed and proposed in the previously mentioned study [156]. Numerous dynamic neural network models for solving time-varying problems have been proposed based on this framework [186], which possess features such as finite-time convergence, dual acceleration convergence, various noise tolerances, and adaptive parameters. However, enhancing the noise tolerance ability of dynamic neural network models is an important topic for improving model performance. Therefore, in [187], the authors further extended the single integral framework in [156] to a design formula with double integral structure for solving various types of time-varying problems, its structure can be mathematically expressed as

$$\dot{\boldsymbol{\chi}}(t)=-3\zeta\boldsymbol{\chi}(t)-3\zeta^2\int_0^t\boldsymbol{\chi}(\tau)\mathrm{d}\tau-\zeta^3\int_0^t\int_0^\tau\boldsymbol{\chi}(\sigma)\mathrm{d}\sigma\,\mathrm{d}\tau.$$

Using the aforementioned design formula, the authors investigated a DNN model for computing complex dynamic Lyapunov equations. They established the convergence and robustness of the DNN model via theoretical evaluation and numerical testing. The author concludes that the double-integral structure dynamic neural network is more general compared to the single-integral structure dynamic neural network, and this model has better noise suppression ability (i.e., it can achieve complete suppression of linear noise).

In [42], the authors proposed a similar structured DNN model for solving time-varying matrix inversion problems and verified its noise tolerance performance. Furthermore, this double integral structure DNN model was further used for the control of chaotic systems in controllable permanent magnet synchronous motors, further verifying its excellent noise tolerance performance. In [188], this model was further used to solve time-varying quadratic matrix equations. In [43], the authors proposed an accelerated dynamic neural network model for handling time-varying Sylvester equations via adding a nonlinear AF, mathematically expressed as

$$\dot{\chi}(t) = -2\zeta_1\chi(t) - \zeta_2\mathcal{F}(\chi(t)) - \zeta_1^2\int_0^t \chi(\tau)\mathrm{d}\tau - 2\zeta_1\zeta_2\int_0^t \mathcal{F}(\chi(\tau))\mathrm{d}\tau - \zeta_1^2\zeta_2\int_0^t\int_0^\tau \chi(\sigma)\mathrm{d}\sigma\mathrm{d}\tau.$$

The article concludes that compared with the single integral structure dynamic neural network model (15), this model can achieve complete suppression of linear noise while having a faster convergence rate. Furthermore, the excellent performance of this model was further validated in the control of the sine function memristor chaotic system. At present, the noise-tolerant dynamic neural network model with double-integral structures has done some work on the online solution of the time-varying Lyapunov and Sylvester equations. In the future, the noise-tolerant dynamic neural network model with double-integral structure can be accelerated by adding nonlinear activation functions that can be considered as

$$\mathcal{F}(\chi) = a\exp(|\chi|^q)|\chi|^p\mathrm{sgn}(\chi) \text{ with } a > 0, \; q > 0, \text{ and } p < 1.$$

In addition, it is also a worthwhile research direction to improve the convergence rate of noise-tolerant dynamic neural network models with double-integral structure by designing adaptive parameters.

V. CONCLUSION

This review paper has primarily investigated DNN models for handling various time-varying problems from the perspective of model structure. These structures include general nonlinearly activated, finite-time convergence, varying parameters, single-integral structures with noise tolerance, and double-integral structures with noise tolerance.This work has highlighted the applicability and advantages of DNN models in processing time-varying problems, and has compared their performance. Furthermore, the practical applications of these models, including matrix inversion, Lyapunov equations, quadratic programming, and robot manipulators, have been thoroughly discussed.

DNNs have undergone continuous development and have been widely applied in various practical scenarios. However, there are still many new challenges that need to be addressed. In the future, the following aspects require further efforts: (1) Designing dynamic neural network models with stronger noise tolerance capabilities; (2) Constructing better nonlinear activation functions to accelerate the convergence speed of neural network models, which remains a popular research topic in this field; (3) Expanding the application scenarios is crucial for the practical implementation and advancement of dynamic neural networks and applied mathematics.

In summary, this review paper offers a valuable reference for readers seeking a comprehensive understanding of the utilization of DNN models in addressing time-varying problems. It thoroughly examines the distinctions, benefits, and drawbacks associated with various model structures.

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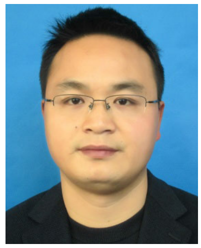

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