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RESEARCH ARTICLE

Stabilization of Linear Time-Invariant Systems With Unbounded Disturbances via DE-Based Control Method

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ABSTRACT This paper investigates the stabilization problem of linear time-invariant (LTI) systems with unbounded disturbances. Firstly, three suitable filters are designed to asymptotically estimate the corresponding external disturbances: $w(t) = p\cos(3t) + q$, $w(t) = pe^{0.1t}$, and $(p\cos(3t) + q)e^{0.1t}$, where p, q are unknown constants. Secondly, a disturbance estimator (DE)-based control strategy is proposed by combining the linear feedback control method with the obtained filters, and thus the stabilization of such systems is realized. It is the first time to suppress the unbounded disturbances by designing suitable filters. Thus, the presented conclusions have some advantages over the existing ones. Finally, illustrative examples with computer simulation verify the effectiveness and correctness of the proposed results.

INDEX TERMS Stabilization, disturbance, suppression, estimator, unbounded, periodically.

I. INTRODUCTION

In the field of systems and control science, linear system is the basic research object. It has made many results and important progress in the past decades and its application is very wide, see Refs. [1], [2], and [3]. However, in practical engineering, the control system is inevitably affected by various external disturbances, and the performance of the control system will be weakened obviously if it is not dealt with in the design of the control system, especially for the linear systems. So far, there have been a number of methods to suppress these external disturbances. For instance, the Hamilton-based method [4], H_∞ control method [5], sliding-mode control [6], [7], disturbance-observer-based control (DOBC) method [8], [9], and so on. But the above methods more or less have some shortcomings. Such as, some of these methods will produce chattering, some are not easy to realize and optimize, and some are needed to assume that the disturbance is bounded and solved by linear matrix inequalities (LMIS), which leads to overly conservative conclusions.

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Recently, the uncertainty and disturbance estimator (UDE)-based control method [10], [11], [12], [13], [14], [15], [16], [17], [18], [19] can deal with model uncertainty and external disturbance effectively. Because of its strong anti-disturbance performance this method has been widely used in various systems. The UDE-based control method is introduced first in the next.

Consider the controlled nonlinear system with uncertainty and disturbance:

$$\dot{y} = h(y) + u_d + bu \tag{1}$$

where $y \in R^n$ is the state, $h(y) \in R^n$ is a continuous vector function, $b \in R^n$ is a constant matrix, u is the controller to be designed, $u_d = \Delta h(y) + d(t)$ is the whole of uncertainty $\Delta h(y)$ and the external disturbance $d(t)$. The core idea of the UDE-based control method is to design a suitable filter $g_f'(t)$ to achieve the performance:

$$\hat{u}_d(t) = u_d(t) * g_f'(t) \to u_d(t), \quad t \to +\infty \tag{2}$$

where "$*$" represents the convolution of two functions. Therefore, the design of filter is very important. Although the authors in Refs. [14] and [15] have proposed some filter design schemes, the obtained results are only robust, lack rigorous proof of asymptotical stability, and do not achieve good asymptotical estimation of the whole uncertainty and disturbance $u_d(t)$. As a matter of fact,

$$\begin{aligned}\hat{u}_d(t) &= u_d(t) * g_f'(t) = (\Delta h(y) + d(t)) * g_f'(t) \\ &= \Delta h(y) * g_f'(t) + d(t) * g_f'(t) \\ &\to \Delta h(y) * g_f'(t) + \hat{d}(t), \quad t \to +\infty \end{aligned} \tag{3}$$

i.e., the filter $g_f'(t)$ in Eq. (3) only asymptotically estimate the disturbance $d(t)$, and $\Delta h(y)$ weakens the performance of $g_f'(t)$. Thus, $\hat{u}_d(t)$ does not asymptotically estimate u_d. In conclusion, the suitable filter which can asymptotically estimate the whole uncertainty and disturbance (u_d) has been not presented so far. Therefore, the existing methods do not solve the control problem of linear systems with external disturbances well, especially in the face of unbounded external disturbances. In practical applications, the modeling process of the system is often faced with a more complex external environment, so it is necessary to study how to suppress unbounded disturbances.

Although the UDE-based control method has strong anti-disturbance performance, it only obtains robust practical results due to the influence of uncertainties. A natural idea arises, the control problem of the system only with disturbances can be solved by designing suitable filters. In the latest work [20], we have proposed a suitable filter which can be used to asymptotically estimate the periodic external disturbance $p\sin(\omega t)+q$, where p and q are unknown constants, ω is known in advance. In addition, in the face of exponential growth or even more complex unbounded disturbances, the traditional low-pass filter has not to meet the requirements, so it is necessary and urgent to design suitable filters to deal with such unbounded external disturbances.

Based on the above discussion, firstly, this paper proposes three suitable filters to asymptotically estimate periodic and unbounded external disturbances, and then a DE-based control strategy is designed to solve the stabilization problem of LTI systems with external disturbances. And the design of the controller takes into account the matching conditions, which means that it can not only achieve the stabilization of single-input systems, but also achieve the stabilization of multi-input systems. It is worth noting that this is the first time to achieve the asymptotic estimation of unbounded disturbances with suitable filters. Different from fault diagnosis, fault tolerant control and traditional control methods based on UDE, the proposed filter is only targeted at the external disturbance of the system, and achieves the objective of asymptotic control rather than just robust control, which is also an important advantage of this paper. Thus, the conclusions presented in this paper have more advantages than the existing results. Illustrative examples with computer simulation verify the effectiveness and correctness of the proposed results.

The organization of this paper is presented as follows. Problem formation is raised in Section II. Main results of this paper are put forward in Section III. Section IV provides two illustrative examples with numerical simulation, Section V gives the conclusions.

Some notations used in this paper are presented before ending this section. I_n denotes the $n \times n$ identity matrix, $\Lambda = \{1, 2, \cdots, n\}$ is a index set, Laplace transformation is expressed by "ℓ", "ℓ^{-1}" stands for the inverse Laplace transformation, and "$*$" represents the convolution of two functions, i.e.,

$$F(s) = \ell[f(t)] = \int_0^{+\infty} f(t)e^{-st}dt,$$

where s is complex variable with $\mathrm{Re}(s) > 0, f(t)$ is a function which meets some appropriate conditions, $f(t) = \ell^{-1}[F(s)]$.

And "$*$" represents the convolution of two functions, that is,

$$f(t) * g(t) = \int_0^t f(t-\tau)g(\tau)d\tau,$$

where $g(t)$ is a function which satisfies certain conditions.

II. PROBLEM FORMATION

A. PRELIMINARIES

Definition 1: Consider the following controlled LTI system

$$\dot{x} = Ax + Bu_s \tag{4}$$

where $x \in R^n$ is the state, $A \in R^{n\times n}$ and $B \in R^r$ are constant matrices, $u_s \in R^r$ is the controller to be designed, $r \geq 1$. If $\lim_{t\to\infty} \|x(t)\| = 0$, we call the system (4) is stabilized by the controller u.

The system (4) is usually called the nominal system. It is well known that there are many methods to design the controller u_s. For simplicity, the linear feedback control method is used in this paper to stabilize the above system.

Lemma 1: Consider the system (4). If (A, B) is controllable, then the linear feedback controller u_s is designed as

$$u_s = -Kx \tag{5}$$

where K is a feedback gain matrix which is got by the pole assignment algorithm.

B. PROBLEM FORMATION

Consider the following LTI system

$$\dot{x} = Ax + Bu + w(t) \tag{6}$$

where $x \in R^n$ is the state, $A \in R^{n\times n}$ and $B \in R^r$ are constant matrices, $w(t) = (w_1(t), \cdots, w_n(t))^T \in R^n$ are the unpredictable external disturbances, $u \in R^r$ is the controller to be designed, $r \geq 1$.

With the development of this paper, two assumptions are presented as follows.

Assumption 1: (A, B) is controllable, and B has full column rank.

Assumption 2: The control matrix B and the disturbance $w(t)$ meet the following condition

$$rank\ (B,\ w(t)) = rank(B). \tag{7}$$

Remark 1: Assumption 2 ensures that the following equation

$$Bu_w = w(t) \tag{8}$$

has an unique solution, where $u_w \in R^r$ is the desired DE-based controller. The equation (7) is usually called the matching condition.

If $B \in R$, then the equation (7) becomes

$$w(t) = w_i(t)B, \tag{9}$$

where $w_i(t)$ is the external disturbance, $i \in \Lambda$.

Remark 2: The most significant difference from the existing results is that the disturbances $w_i(t)$ may be bounded or unbounded. Such as, the bounded periodic disturbance: $w_i(t) = p_i\cos(3t) + q_i$, the unbounded exponentially increasing disturbance: $w_i(t) = p_i e^{a_i t}$, or the unbounded disturbance: $w_i(t) = (p_i\cos(3t) + q_i)e^{0.1t}$, where p_i, q_i are unknown constants, $a_i > 0$ and $\omega_i > 0$ are known constants in advance.

The main goal of this article is to design an appropriate controller u to implement the following performance:

$$\lim_{t\to\infty} \|x(t)\| = 0.$$

i.e., make the system (6) realize stabilization.

In the next the desired controller $u = u_s + u_w$ is proposed by two steps. In the first step, the stabilization controller $u_s = -Kx$ for the nominal system (4) is presented by the pole assignment algorithm, where K is a constant matrix with appropriate dimension. The DE-based controller u_w is designed in the second step. For several external disturbances $w_i(t)$ mentioned above, the corresponding controllers u_{wi} are designed by designing suitable filters $G_{fi}(s)$ which can asymptotically estimate such disturbances, $i \in \Lambda$.

III. MAIN RESULTS

First, some preliminary results are presented as follows.

A. STABILITY OF LTI SYSTEMS WITH ASYMPTOTICALLY STABLE DISTURBANCE

Theorem 1: About the linear system $\dot{x} = \bar{A}x + \tilde{w}(t)$. If $\bar{A}$ is a Hurwitz matrix and $\|\tilde{w}(t)\| \le L_1 e^{\lambda_1 t}$, where $L_1 > 0$, $\lambda_1 < 0$, then $\lim_{t\to\infty} \|x(t)\| = 0$.

Proof: Since $\bar{A}$ is a Hurwitz matrix, there exist two constants $L_2 > 0$, $\lambda_2 < 0$ such that $\|e^{\bar{A}t}\| \le L_2 e^{\lambda_2 t}$.

The solution of the system $\dot{x} = \bar{A}x + \tilde{w}(t)$ is obtained as follows

$$x(t) = e^{\bar{A}t}C + \int_0^t e^{\bar{A}(t-s)}w(s)ds \tag{10}$$

$C \in R^n$ is an arbitrary constant matrix.

It results in

$$\begin{aligned}\|x(t)\| &\le \|e^{\bar{A}t}C\| + \int_0^t \|e^{\bar{A}(t-s)}w(s)\|ds \\ &\le \|C\|\|e^{\bar{A}t}\| + \int_0^t \|e^{\bar{A}(t-s)}\|\|w(s)\|ds \\ &\le \|C\|L_2 e^{\lambda_2 t} + \int_0^t L_2 e^{\lambda_2(t-s)} L_1 e^{\lambda_1 s}ds \\ &\le \|C\|L_2 e^{\lambda_2 t} + L_1 L_2 e^{\lambda_2 t}\int_0^t e^{(\lambda_1-\lambda_2)s}ds \end{aligned} \tag{11}$$

The following two cases are considered.

If $\lambda_1 = \lambda_2$, then the equation (11) is obtained as follows

$$\|x(t)\| \le \|C\|L_2 e^{\lambda_2 t} + L_1 L_2 e^{\lambda_2 t} t \to 0.$$

If $\lambda_1 \ne \lambda_2$, then the equation (11) is got as follows

$$\|x(t)\| \le \|C\|L_2 e^{\lambda_2 t} + \frac{L_1 L_2}{\lambda_1 - \lambda_2}\left(e^{\lambda_1 t} - e^{\lambda_2 t}\right) \to 0.$$

In conclusion, the results are derived, which completes the proof.

B. A SUITABLE FILTER IS DESIGNED FOR THE BOUNDED PERIODIC DISTURBANCE

Theorem 2: For the bounded periodic disturbance: $w_i(t) = p_i\cos(3t) + q_i$, where $p_i \ne 0$, q_i are unknown constants, a suitable filter is designed as

$$G_{fi}(s) = \frac{20s^4 + 920s^3 + 11960s^2 + 31200s + 73620}{s^5 + 63s^4 + 1398s^3 + 12347s^2 + 35421s + 73620} \tag{12}$$

and meets the following performance:

$$\hat{w}_i(t) = w_i(t) * \ell^{-1}\left[G_{fi}(s)\right] = w_i(t) * g_{fi}(t) \to w_i(t), \quad \text{as } t \to \infty, \quad i \in \Lambda. \tag{13}$$

Proof: Since

$$\begin{aligned} g_{fi}(t) &= \ell^{-1}[G_{fi}(s)] \\ &= 20e^{-20t}[52319689 - 6560502\cos(3t) \\ &\quad - 6e^{\frac{37}{2}t}(185793\cos(\tfrac{3\sqrt{3}\,t}{2}) + 86930\sqrt{3}\sin(\tfrac{3\sqrt{3}\,t}{2})) \\ &\quad + 52301140\sin(3t)] \Big/ 44644429 \end{aligned} \tag{14}$$

it concludes

$$\begin{aligned} &(p_i\cos(3t) + q_i) * g_{fi}(t) \\ &= \int_0^t (p_i\cos(3\tau) + q_i)g_{fi}(t-\tau)d\tau \\ &= p_i\cos(3t) + q_i - e^{-20t}\Bigg[\frac{9\,(289p_i + 400q_i)}{127921}(\cos(3t) \\ &\quad + \frac{20\,(7313p_i + 7466q_i)}{9\,(289p_i + 400q_i)}\sin(3t)) + \frac{400p_i + 409q_i}{349}\Bigg] \\ &\quad + e^{-\frac{3}{2}t}\left(\frac{60\,(123862p_i + 148861q_i)}{44644429}\right)(\cos(\frac{3\sqrt{3}\,t}{2}) \\ &\quad + \frac{\sqrt{3}\,(173860p_i - 98863q_i)}{371586p_i + 446583q_i}\sin(\frac{3\sqrt{3}\,t}{2})) \\ &\to p_i\cos(3t) + q_i, \text{ as } t \to \infty \end{aligned} \tag{15}$$

Thus, the designed filter $G_{fi}(s)$ meets the requirements, $i \in \Lambda$.

C. SOME SUITABLE FILTERS ARE DESIGNED FOR THE UNBOUNDED DISTURBANCE

Theorem 3: For the unbounded exponentially increasing disturbance: $w_i(t) = m_i e^{a_i t}$, where $a_i > 0$ is known (a_i is usually 0.1), and m_i is unknown, a suitable filter is designed as

$$G_{fi}(s) = \frac{20}{s + (20 - a_i)} \tag{16}$$

and meets the following performance:

$$\hat{w}_i(t) = w_i(t) * \ell^{-1}\left[G_{fi}(s)\right] = w_i(t) * g_{fi}(t) \to w_i(t), \quad \text{as } t \to \infty, \ i \in \Lambda. \tag{17}$$

Proof: Since

$$g_{fi}(t) = \ell^{-1}\left[G_{fi}(t)\right] = \ell^{-1}\left[\frac{20}{s + (20 - a_i)}\right] = 20e^{-(20-a_i)t} \tag{18}$$

it is to get

$$\begin{aligned} m_i e^{a_i t} * \left[20e^{-(20-a_i)t}\right] &= m_i 20 \int_0^t e^{a_i \tau} e^{-(20-a_i)(t-\tau)} ds \\ &= m_i 20 e^{-(20-a_i)t} \int_0^t e^{20\tau} ds \\ &= m_i e^{-(20-a_i)t}\left[e^{20t} - 1\right] \\ &= m_i e^{a_i t}\left[1 - e^{-20t}\right] \\ &\to m_i e^{a_i t}, \ \text{as } t \to \infty \end{aligned} \tag{19}$$

So the presented filter $G_{fi}(s)$ meets the requirements.

Theorem 4: For the unbounded disturbance: $w_i(t) = [p_i \cos(3t) + q_i]\, e^{0.1t}$, where $p_i \neq 0$, q_i are unknown constants, a suitable filter is designed as

$$G_{fi}(s) = \frac{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \tag{20}$$

where $a_4 = 2000000, a_3 = 91200000, a_2 = 1168520000, a_1 = 2883552000, a_0 = 7061868200; b_5 = 100000, b_4 = 6250000, b_3 = 137290000, b_2 = 1193137000, b_1 = 3299328850, b_0 = 7019997829$.

and meets the following performance:

$$\hat{w}_i(t) = w_i(t) * \ell^{-1}\left[G_{fi}(s)\right] = w_i(t) * g_{fi}(t) \to w_i(t), \quad \text{as } t \to \infty, \ i \in \Lambda. \tag{21}$$

Proof: Since

$$\begin{aligned} g_{fi}(t) &= \ell^{-1}[G_{fi}(s)] \\ &= 20e^{-\frac{199}{10}t}[52319689 - 6560502\cos(3t) \\ &\quad - 6e^{\frac{37}{2}t}(185793\cos(\tfrac{3\sqrt{3}\,t}{2}) + 86930\sqrt{3}\sin(\tfrac{3\sqrt{3}\,t}{2})) \\ &\quad + 52301140\sin(3t)] \Big/ 44644429 \end{aligned} \tag{22}$$

in the next

$$\begin{aligned} &[p_i \cos(3t) + q_i]\, e^{0.1t} * g_{fi}(t) \\ &= \int_0^t [p_i \cos(3\tau) + q_i]\, e^{0.1\tau} g_{fi}(t-\tau) d\tau \\ &= [p_i \cos(3t) + q_i]\, e^{0.1t} - e^{-\frac{199}{10}t}\left[\frac{9\,(289p_i + 400q_i)}{127921}(\cos(3t)\right. \\ &\quad \left. + \frac{20\,(7313p_i + 7466q_i)}{9\,(289p_i + 400q_i)}\sin(3t)) + \frac{400p_i + 409q_i}{349}\right] \\ &\quad + e^{-\frac{7}{5}t}\left(\frac{60\,(123862p_i + 148861q_i)}{44644429}\right)(\cos(\frac{3\sqrt{3}\,t}{2}) \\ &\quad + \frac{\sqrt{3}\,(173860p_i - 98863q_i)}{371586p_i + 446583q_i}\sin(\frac{3\sqrt{3}\,t}{2})) \\ &\to [p_i \cos(3t) + q_i]\, e^{0.1t}, \ \text{as } t \to \infty \end{aligned} \tag{23}$$

Therefore, the designed filter $G_{fi}(s)$ meets the requirements.

D. THE DE-BASED CONTROLLER DESIGNED

Based on the above discussion, a conclusion is got in the next.

Theorem 5: About the system (6). If the suitable filters $G_{fi}(s)$ are proposed, $i \in \Lambda$, then this system is stabilized by the following controller:

$$u = u_s + u_w \tag{24}$$

where

$$u_s = -Kx \tag{25}$$

K is a feedback gain matrix which is got by the pole assignment algorithm, and

$$\begin{aligned} u_w = B^+ \Big\{ &\ell^{-1}\left[(I_n - G_f(s))^{-1} G_f(s)\right] * \left[\bar{A}x(t)\right] \\ &- \ell^{-1}\left[(I_n - G_f(s))^{-1}(sG_f(s))\right] * x(t) \Big\} \end{aligned} \tag{26}$$

where $B^+ = (B^T B)^{-1} B^T$, $G_f(s) = \ell\left[g_f(t)\right]$, $\bar{A} = A - BK$, and

$$G_f(s) = \begin{pmatrix} G_{f1}(s) & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & G_{fn-1}(s) & 0 \\ 0 & 0 & \cdots & 0 & G_{fn}(s) \end{pmatrix} \tag{27}$$

$$g_f(t) = \begin{pmatrix} g_{f1}(t) & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & g_{fn-1}(t) & 0 \\ 0 & 0 & \cdots & 0 & g_{fn}(t) \end{pmatrix} \tag{28}$$

Proof: Since (A, B) is controllable, thus $\bar{A}$ is a Hurwitz matrix according to the pole assignment theory of linear systems.

Substituting the controller presented in Eq. (25) into the system (6), it gets

$$\dot{x} = \bar{A}x + Bu_w + w(t) \tag{29}$$

Let

$$Bu_w = -\hat{w}(t), \tag{30}$$

and

$$\tilde{w}(t) = w(t) - \hat{w}(t). \tag{31}$$

According to Theorem 2, Theorem 3 and Theorem 4,

$$\tilde{w}(t) = w(t) - \hat{w}(t) \to 0, \text{ as } t \to \infty \tag{32}$$

Therefore, system (29) can be rewritten as

$$\dot{x} = \bar{A}x + \tilde{w}(t) \tag{33}$$

According to Lemma 1, it is noted that the system (33) is asymptotically stable.

Making Laplace transformation of the two sides of the Eq. (30) with zero initial condition

$$Bu_w = -\hat{w}(t) = -g_f(t) * (\dot{x} - \bar{A}x - Bu_w), \tag{34}$$

it yields

$$Bu_w(s) = -G_f(s)\left[sX(s) - \bar{A}X(s) - Bu_w(s)\right], \tag{35}$$

where $X(s) = \ell[x(t)]$.

Simplifying the above equation (35), it results in

$$u_w(s) = B^+ \left\{ \left[(I_n - G_f(s))^{-1} G_f(s)\right]\left[\bar{A}X(s)\right] - \left[(I_n - G_f(s))^{-1}(sG_f(s))\right] X(s) \right\}. \tag{36}$$

Thus, the controller u_w given in Eq. (26) is obtained by making inverse Laplace transformation of the equation (36).

Remark 3: If $B \in R$, the DE-based controller u_w is presented as follows

$$u_w = B^+ \left\{ \ell^{-1}\left[\frac{G_f(s)}{1 - G_f(s)}\right] * \left[\bar{A}x(t)\right] - \ell^{-1}\left[\frac{sG_f(s)}{1 - G_f(s)}\right] * x(t) \right\}. \tag{37}$$

IV. ILLUSTRATIVE EXAMPLES WITH NUMERICAL SIMULATION

Illustrative examples are taken as examples and the corresponding numerical simulations are carried out to verify the above obtained theoretical results in this section.

Example 1: Consider the following single input LTI system with the bounded periodic disturbance

$$\dot{x} = Ax + Bu + w(t), \tag{38}$$

where $x \in R^3$ is the state, and

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \tag{39}$$

$$w(t) = \begin{pmatrix} 0 \\ 2\cos(3t) + 3 \\ 0 \end{pmatrix}.$$

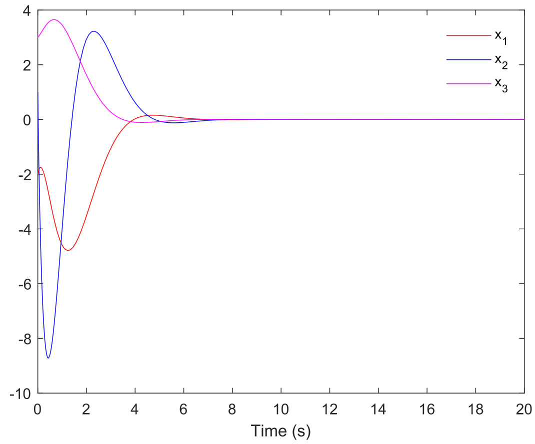

FIGURE 1. The system (38) is asymptotically stable.

It is clear that

$$w(t) = \begin{pmatrix} 0 \\ 2\cos(3t) + 3 \\ 0 \end{pmatrix} = w_1(t)B, \tag{40}$$

where $w_1(t) = 2\cos(3t) + 3$, i.e., the match condition (9) is met.

Let the eigenvalues of the matrix $\bar{A} = A - BK$ be $-2, \; -1 \pm j$. According to Theorem 5, the desired controller u is expressed as

$$u = u_s + u_w, \tag{41}$$

where

$$u_s = -Kx = -\begin{pmatrix} 22 & 7 & 29 \end{pmatrix} x = -22x_1 - 7x_2 - 29x_3, \tag{42}$$

$$G_{fi}(s) = \frac{20s^4 + 920s^3 + 11960s^2 + 31200s + 73620}{s^5 + 63s^4 + 1398s^3 + 12347s^2 + 35421s + 73620}, \tag{43}$$

and

$$u_w = \ell^{-1}\left[\frac{G_f(s)}{1 - G_f(s)}\right] * (-19x_1 - 6x_2 - 29x_3) - \ell^{-1}\left[(I_n - G_f(s))^{-1}(sG_f(s))\right] * x_2. \tag{44}$$

Next, numerical simulation is implemented with the initial condition: $x_0 = [-2, 1, 3]^T$. It can be seen from Figure 1 that the system (38) is asymptotically stable, and Figure 2 shows that the disturbance estimator $\hat{w}_1(t)$ tends to the disturbance $w_1(t)$ as $t \to \infty$.

Remark 4: For the disturbance $w_2(t)$, the proposed filter in Ref. [15] is given as follows

$$G_f(s) = \frac{a_1 s + (a_2 - w^2)}{s^2 + a_1 s + a_2} \tag{45}$$

where $w = 4\pi$, $a_1 = 10w$, $a_2 = 100w^2$.

Let $\hat{w}_2 = w_2(t) * g_f(t) = w_2(t)\ell^{-1}[G_f(s)]$ and $e = \hat{w}_2 - w_2(t)$, numerical simulation is carried out with $t = 0 : 0.01 : 10$, the state of e is shown below

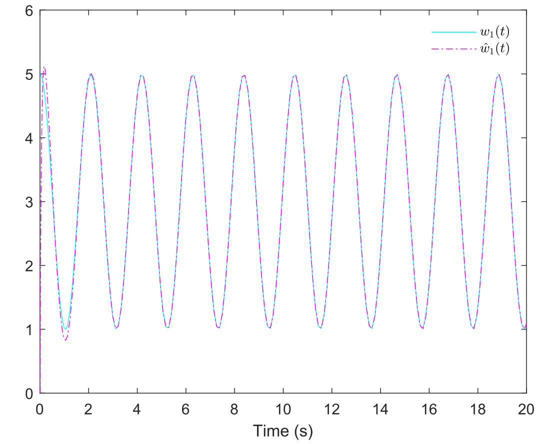

FIGURE 2. $\hat{w}_1(t)$ **tends to** $w_1(t)$.

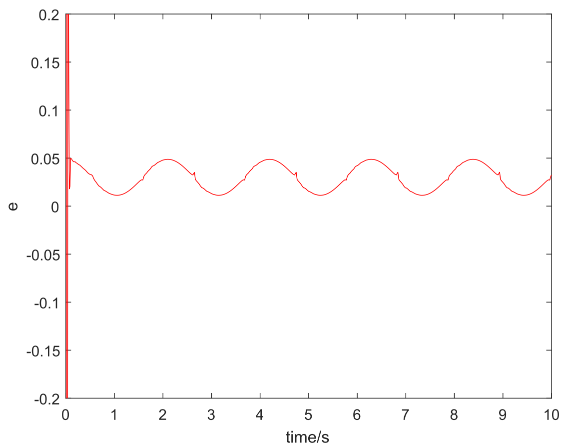

FIGURE 3. **The state of** e.

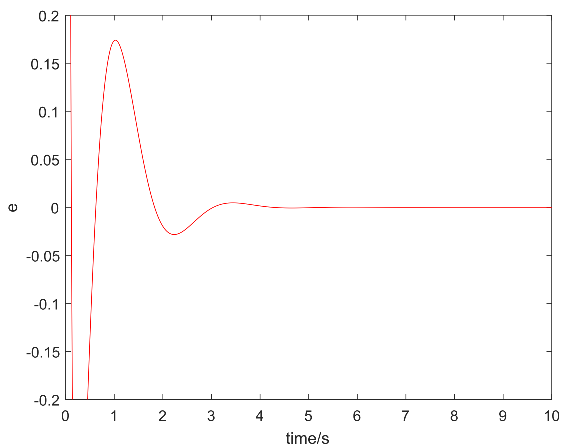

FIGURE 4. **The state of** e.

From Figure 3, it can be seen that the proposed filter in Eq. (45) only realize the robust estimation of $w_2(t)$.

The same simulation result by the designed filter in (20) is shown in the following,

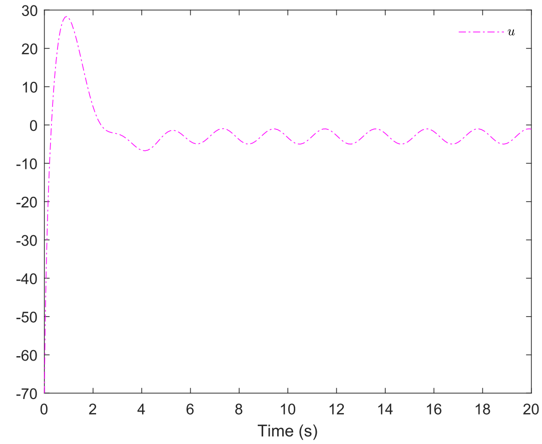

FIGURE 5. **The state of controller** u.

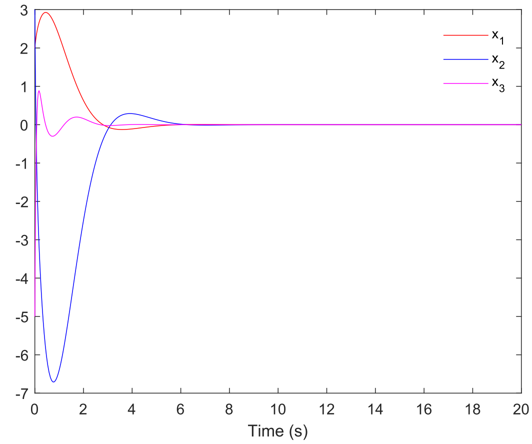

FIGURE 6. **The system (46) is asymptotically stable.**

It can be observed from Figure 4 that the designed filter in (20) can asymptotically estimate the $w_2(t)$. Therefore, the obtained result has advantages over the existing ones.

The control signal u is displayed in the following figure

Example 2: Consider the following multi-input LTI system with the unbounded external disturbances

$$\dot{x} = Ax + Bu + w(t), \tag{46}$$

where $x \in R^3$ is the state, and

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 6 \\ 1 & 2 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$w(t) = \begin{pmatrix} 0 \\ 5e^{0.1t} \\ e^{0.1t}\,[10\cos(3t) + 1] \end{pmatrix}. \tag{47}$$

It is easy to verify that

$$rank(B,\ w(t)) = rank \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 5e^{0.1t} \\ 0 & 1 & e^{0.1t}\,[10\cos(3t) + 1] \end{pmatrix}$$

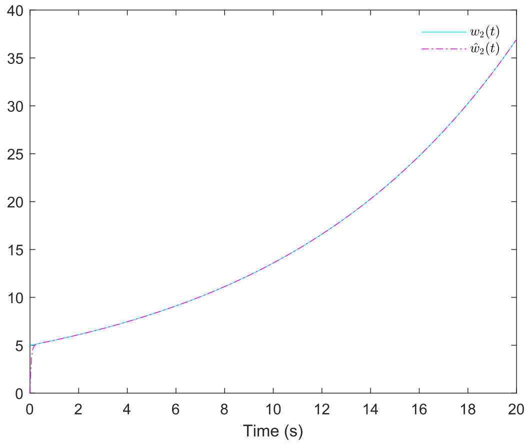

FIGURE 7. $\hat{w}_2(t)$ **converges to** $w_2(t)$.

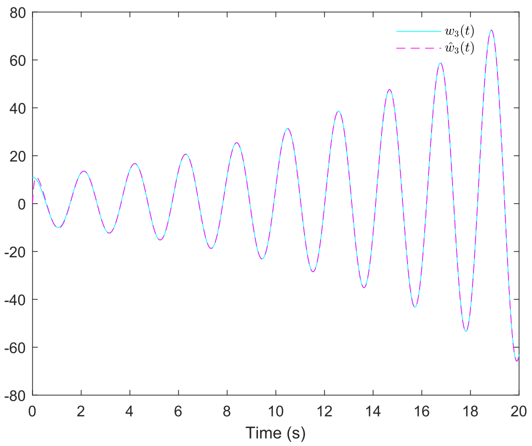

FIGURE 8. $\hat{w}_3(t)$ **converges to** $w_3(t)$.

$$= 2 = rank(B) = rank \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{48}$$

i.e., the match condition (7) is satisfied.

By similar procedure to Example 1, the controller u is designed as

$$u = u_s + u_w, \tag{49}$$

where (50) and (51), as shown at the bottom of the page, and

$$G_{f_2}(s) = \frac{20}{s + 19.9}, \tag{52}$$

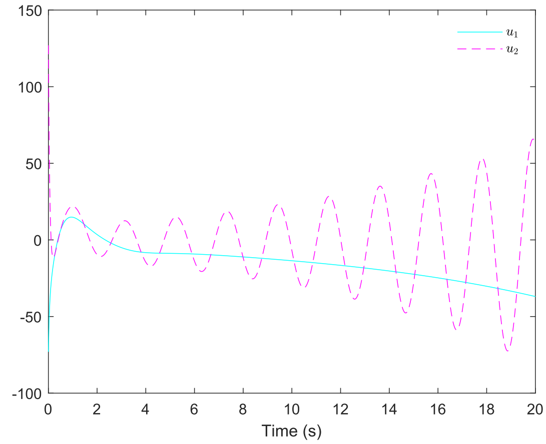

FIGURE 9. **The state of controller** u.

$$G_{f_3}(s) = \frac{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \tag{53}$$

where $a_4 = 2000000, a_3 = 91200000, a_2 = 1168520000, a_1 = 2883552000, a_0 = 7061868200; b_5 = 100000, b_4 = 6250000, b_3 = 137290000, b_2 = 1193137000, b_1 = 3299328850, b_0 = 7019997829$.

With the initial condition: $x_0 = [2, 3, -5]^T$, numerical simulation is carried out. From Figure 6 it is easy to see that the system (46) is asymptotically stable, and Figure 7 shows that $\hat{w}_2(t)$ converges to $w_2(t)$ as $t \to \infty$, Figure 8 shows that $\hat{w}_3(t)$ converges to $w_3(t)$ as $t \to \infty$.

The control signal u is displayed in the following figure

V. CONCLUSION AND PROSPECT

The stabilization of the LTI systems with external disturbances (which are bounded or unbounded) has been investigated by the DE-based control method. Firstly, the stabilization of linear nominal system has been achieved by pole assignment algorithm of linear system. Next, some suitable filters have been proposed to asymptotically estimate the corresponding external disturbances, and it has been the first time to propose suitable filters for unbounded disturbances. Then, the DE-based control method considering matching conditions has been proposed and applied to realize the stabilization of such systems by estimating the disturbance asymptotically and eliminating it exactly. Therefore, the method presented in this paper has advantages over previous results. Finally, numerical examples with

$$u_s = -Kx = -\begin{pmatrix} 11 & 7 & 6 \\ 1 & 2 & 7 \end{pmatrix} x = \begin{pmatrix} -11x_1 - 7x_2 - 6x_3 \\ -x_1 - 2x_2 - 7x_3 \end{pmatrix}, \tag{50}$$

$$u_w = \begin{pmatrix} \ell^{-1}\left[\dfrac{G_{f2}(s)}{1 - G_{f2}(s)}\right] * (-10x_1 - 4x_2) - \ell^{-1}\left[\dfrac{sG_{f2}(s)}{1 - G_{f2}(s)}\right] * x_2 \\ \ell^{-1}\left[\dfrac{G_{f3}(s)}{1 - G_{f3}(s)}\right] * (-2x_3) - \ell^{-1}\left[\dfrac{sG_{f3}(s)}{1 - G_{f3}(s)}\right] * x_3 \end{pmatrix}, \tag{51}$$

computer simulation have been taken to verify the validity and effectiveness of the proposed results.

The limitation of the method proposed in this paper is that only the external disturbances of the system has been investigated without considered the model uncertainty of the system. The model uncertainty of the system is also one of the factors affecting the control performance of the system. Therefore, in the following work, we will investigate the stabilization problem of linear systems with both parametric uncertainty and external disturbance, and we hope to put forward a better method to improve the control performance of the system.

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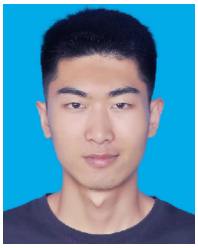

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