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METHODS

Research on Improving Gray Wolf Algorithm Based on Multi-Strategy Fusion

XIAOXIAO YANG AND YIHUI QIU
Xiamen University of Technology, Xiamen 361024, China

Corresponding author: Yihui Qiu (qiuyihui@xmut.edu.cn)

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ABSTRACT To address the shortcomings of the basic Gray Wolf Optimization (GWO) algorithm in solving complex problems, such as relying on the initial population, converging too early, and easily falling into local optimality, a chaotic reverse learning initialization strategy, a nonlinear control parameter convergence strategy, and a dynamic position update strategy are introduced to develop a multi-strategy fusion Improved Gray Wolf Optimization (IGWO) algorithm, and this method is used to solve function optimization problems. First, a chaotic backward learning initialization strategy, based on logistic mapping and backward learning, is adopted to improve the random initialization of the GWO algorithm and enhance the traversal and diversity of the initial population. Second, a nonlinear control parameter for local perturbation is constructed to avoid the problem of premature convergence of the GWO algorithm due to linear convergence and to balance the exploration and exploitation ability of the GWO algorithm. Finally, a location guidance strategy based on dynamic weights and individual memory is proposed to effectively improve the algorithm's optimization accuracy and computational efficiency; meanwhile, the Gaussian-Cauchy mutation strategy of superior selection is introduced to optimize the location update of optimal individual α wolves and improve the ability of the population to jump out of local extremes. Simulation experiments are conducted for 11 classical test functions, and the results show that the proposed improved algorithm IGWO for gray wolves is superior to 10 other standard swarm intelligence optimization algorithms and 4 other improved optimization algorithms in terms of solution accuracy, convergence speed, and algorithm stability. It provides a new optimization algorithm for solving complex optimization problems.

INDEX TERMS Gray wolf optimization algorithm, population intelligence optimization, chaotic mapping, lenticular imaging backward learning, nonlinear control parameters.

I. INTRODUCTION

As an emerging interdisciplinary field, swarm intelligence optimization algorithms have been developed comprehensively and rapidly, both in terms of theoretical exploration and application expansion [1], [2], [3], [4], and they have gradually received attention from local and international scholars due to their simple structure and robustness [5], [6]. For instance, the genetic algorithm (GA) [7], particle swarm optimization (PSO) [8], sparrow search algorithm (SSA), Moth-Flame Optimization(MFO) [9], Sailed Fish Optimizer (SFO) [10], and so on. In addition, several scholars use swarm intelligence optimization algorithms to solve practical problems. Lu et al. put forward a hybrid ensemble algorithm combining AdaBoost and GA for cancer classification with gene expression data [11]. Wei et al. proposed a novel self-adjusted particle swarm optimization algorithm (SAPSO) is proposed for selecting the optimal feature subset for classification datasets [12]. Wu et al. propose a novel greedy genetic sparrow search algorithm based on a sine and cosine search strategy (GGSC-SSA) and apply this algorithm to solve the problem of easily falling into local optima in the travel salesman problem [13]. Khurma proposes an effective wrapper approach by integrating the Levy flight and evolutionary

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selection operators into the MFO algorithm and applying this method to solve the feature selection (FS) problem in medical applications [14].

As a new algorithm of bionic swarm intelligence, the Grey Wolf Optimizer (GWO) was proposed by Mirjalili et al [15] in 2014. It was inspired by the gray wolf pack hierarchy, hunting behavior, and swarm intelligence, and it makes this method easy to understand and helps some scientists engaged in different research fields to quickly learn algorithms and apply them to their problems Compared with some other swarm intelligence algorithms, the GWO has stronger optimization capabilities and faster search efficiency, good stability, and strong robustness.Moreover, this method has good application prospects and academic value, and this method is suitable for solving highly nonlinear, and multimodal function optimization problems. [16], [17], [18].

It has been shown that the GWO algorithm has good performance when finding the best function optimization [19]. However, this method still has the shortcomings of (1) relying on the initial population and (2) easily falling into the local optimum, which makes the algorithm suffer from slow convergence speed and low convergence accuracy when finding the best solution. For this reason, several scholars have proposed corresponding hybrid strategies to integrate the advantages of other algorithms into the GWO algorithm, fully utilizing their respective advantages for collaborative search, and enhancing the overall optimization ability of GWO. Their work mainly includes population initialization, designing mutation strategies, and combining other optimization algorithms. For example, Abdel et al [20] used sigmoid function and two-phase mutation to improve the GWO algorithm, significantly improving algorithm mining ability and classification accuracy. Zhang and Zhou [21] Zhang proposed a method combining the GWO algorithm with lateral inhibition to effectively reduce computational costs in solving image extraction problems. Cheng et al. [22] used a contribution-based strategy to initialize the population, a reinforcement learning method to determine the global and the local search parameters, and a two-level variable neighborhood and four replacement strategies to improve the local search capability. Zhang and Zhou [23] integrated the ranking based mutation operator into the GWO to accelerate the convergence speed, and thus enhance the performance. Li and Shen [24] used the good point set method to generate the initial population of the gray wolf individuals; combined with the teaching and learning algorithm and the PSO algorithm, the proposed method optimizes the original position update formula in order to improve the algorithm's search performance. Moreover, Duan and Yu [25] used the sine function to adjust the control parameters nonlinearly and dynamically, and used the Sine Cosine Algorithm (SCA) was used to update the position of the alpha wolf in the GWO algorithm; moreover, the weight-based individual position update and its combination with the individual best were used to improve the search ability of the algorithm. Kewen & Li [26] introduced the Gaussian-Cauchy mutation operator in the alpha wolf search in the GWO algorithm, and retained the outstanding gray wolf individuals through the greedy selection mechanism; thus, this method increased the alpha wolf population diversity and the global search capability of the algorithm. Finally, Singh and Bansal [27] combined four different strategies with a new update search mechanism; therefore, this methodology improved the control parameters, a mutation-driven scheme and a greedy approach to fully expand the optimization performance of the GWO algorithm, and achieved better global search capability and convergence. Nadimi Shahraki et al [28] proposes a novel GWO algorithm based on a dimension learning hunting (DLH) search strategy, which uses different methods to construct neighbors for each wolf, enabling information sharing among wolves, enhancing the search balance between individual gray wolves and the population in the GWO algorithm, and ensuring population diversity. Ma et al. [29] proposed an improved Grey Wolf algorithm based on the Aquila Optimizer (AO), which expands the search range to improve global search ability, reduce the likelihood of falling into local optima, and balance the exploration and development stages.

In summary, most of the proposed methods can improve the performance of one aspect of the GWO algorithm, but it is still difficult to strike a balance in terms of algorithm convergence speed, search accuracy, and ease of falling into local optimum. Thus, these potential defects need to be taken seriously and addressed, this paper proposes an improved gray wolf algorithm with multi-strategy fusion using (1) chaotic backward learning initialization strategy, (2) nonlinear control parameter convergence strategy, (3) position update strategy with dynamic weights and individual memory, and (4) Gaussian-Cauchy variation strategy based on merit selection. On 11 groups of standard test functions, IGWO is compared with some swarm intelligent optimization algorithms and improved GWO algorithms. The experimental results show that the IGWO algorithm can effectively improve convergence accuracy and significantly improve the performance of optimal value search, while avoiding premature convergence and enhancing local and global search capabilities

The innovative points and main contributions of this paper are as follows.

1) Chaotic reverse learning strategy to expand population diversity: The chaotic reverse learning strategy, based on logistic mapping strategy, combined with the lens imaging reverse learning strategy that changes with iteration, is proposed to improve the method of randomly generating the initial population in the original GWO algorithm in order to expand the population diversity, expand the range of initial optional solutions, and realize the quality improvement of the initial population individuals;

2) Sinusoidal law-change convergence strategy to improve convergence speed and solution accuracy, and balance the exploration and exploitation ability of the GWO algorithm:

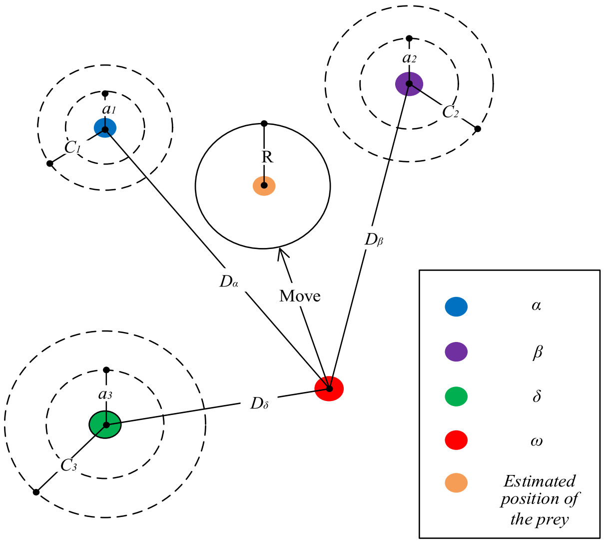


FIGURE 1. Schematic diagram of the update mechanism of GWO.

The convergence strategy, based on the normal random strategy and the adaptation-related sinusoidal law-change convergence strategy, is adopted to improve the linear convergence method in the original GWO algorithm. Therefore, a nonlinear and dynamic change with the increase of the number of iterations, which can fully improve the convergence speed and solution accuracy of the algorithm, and guide it to avoid local optimum, will be obtained;

3) Proportional weighting strategy and individual historical optimal memory strategy to avoid local optimum: The dynamic proportional weight, based on the Euclidean distance of the step size, is used to adjust the position formula of GWO as well as the individual history optimal memory strategy in PSO is introduced to improve the method of population position update in the original GWO, expand the search range, and avoid the algorithm to fall into local optimum to some extent;

4) Gaussian-Cauchy mutation strategy, based on superiority and inferiority, to improve the search efficiency of the algorithm: In order to further improve the ability of the algorithm to get out of local optimum and fall into early maturity, it adopts the strategy of determining the winners and losers based on the idea of "greedy" selection. The algorithm performs Gaussian-Cauchy mutation on the optimal solution obtained from the original GWO with a certain probability to determine whether to accept the new location of the optimal gray wolf individual after the mutation; moreover, the improved algorithm enables the population to evolve towards the optimal solution faster.

II. DESCRIPTION OF THE IMPROVED GREY WOLF ALGORITHM BASED ON MULTI-STRATEGY FUSION

Gray wolves live in packs and have a strict social hierarchy, as shown in Figure 1. The leader of the pack is called the α wolf, and he is responsible for making decisions such as hunting, food distribution, and resting place. β wolves of the second rank are called wolves and are mainly responsible for assisting in decision making. The wolves of the third level are called δ wolves and they are mainly responsible for scouting and sentry duty. The ω wolves at the lowest end of the hierarchy are mainly responsible for maintaining the balance of relationships within the population.

Suppose that the solution space dimension of the GWO algorithm to solve the optimization problem is d, and the size of the gray wolf population is N. The position of the first gray wolf individual is denoted as follows:

$$X_i = \left\{X_i^1, \quad X_i^2, \ldots.., X_i^d\right\}, i = 1, 2, \ldots, N \tag{1}$$

The optimal solution, sub-optimal solution and 3$^{\text{rd}}$ optimal solution in the gray wolf population are noted as α, β, and δ, respectively, and the rest of the solutions are noted as ω. In order to find the optimal solution or the optimal position, ω will continuously update the position based on the position of α, β, and δ to reach the optimum, and the position update formula of the gray wolf is shown in Eq. (2)

$$X(t+1) = X_p(t) - A * |C * X_p(t) - X(t)| \tag{2}$$

where t is the number of iterations, $X_p(t)$ represents the position of the prey, and $X(t)$ indicates the position of the gray wolf after the t^{th} iteration. Moreover, A and C are matrix coefficients where A is the convergence factor, and C is the oscillation factor. Both factors are defined in Eqs. (3) and (4)

$$A = 2a * r_1 - a \tag{3}$$
$$C = 2r_2 \tag{4}$$

where r_1 and r_2 denote, respectively, two random variables varying between [0,1], and a is the distance control parameter whose value decreases linearly from 2 to 0 with the increase of the number of iterations to gradually approach the optimal solution, and the expression is shown in Eq. (5) aw follows:

$$a(t) = 2(1 - t/T_{\max}) \tag{5}$$

where t is the number of iterations, T_{max} denotes the maximum allowed number of iterations.

After encircling the target prey, β and δ wolves pursue the prey under the leadership of α wolf. Furthermore, in this process, the position of the individual gray wolf population will change due to the escape of the prey; thus, the gray wolf population can update the gray wolf position based on the position X_α, X_β, X_δ of α, β, δ, and the expression is shown in Eq. (7).

$$\begin{cases} X_1(t+1) = X_\alpha(t) - A_1 \cdot |C_1 \cdot X_\alpha(t) - X(t)| \\ X_2(t+1) = X_\beta(t) - A_2 \cdot |C_2 \cdot X_\beta(t) - X(t)| \\ X_3(t+1) = X_\delta(t) - A_3 \cdot |C_3 \cdot X_\delta(t) - X(t)| \end{cases} \tag{6}$$

$$X(t+1) = \sum_{j=1,2,3} W_j X_j(t+1) \tag{7}$$

where t is the number of iterations, $X(t+1)$ denotes the final updated position where the gray wolf is, $X_\alpha X_\beta$, X_δ represents the position of wolves α, β, and δ in turn $X_1X_2X_3$ denotes the estimated prey position based on the position of α, β, and δ, respectively and, finally, $W_j(j = \alpha, \beta, \delta)$ denotes the weight coefficient of α, β, and δ, as shown in Eq. (8):

$$W_j = \frac{||X_j||}{||X_1|| + ||X_2|| + ||X_3||} \tag{8}$$

where $||X_j||$ represents the Euclidean distance of the gray wolf population individuals from locations α, β, δ.

To sum up, the mechanism for updating the position of the GWO algorithm is shown in Figure 1.

A. POPULATION INITIALIZATION STRATEGY BASED ON CHAOTIC BACKWARD LEARNING

The GWO algorithm initializes the population randomly, which limits the gray wolf population individuals in the initial stage. In order to distribute the population of individuals X_i^t as evenly as possible, this paper uses the logistic chaotic mapping and lensing imaging backward learning strategy to improve the population initialization; therefore, the convergence speed of the algorithm will accelerate.

1) LOGISTIC CHAOS MAPPING STRATEGY

The logistic chaos mapping was proposed by Robert May in 1976 to model the way birds and insect populations change their complex behavior over time [30], [31]. The proposed method is ergodic, regular, and stochastic; moreover, it is suitable to be combined with swarm intelligence optimization algorithms and it is commonly used to adjust the initialization of populations with good determinism, and convergence, and it is particularly sensitive to initial values. It has been demonstrated, in the literature, that the logistic chaotic mapping is less computationally intensive compared to other chaotic mappings [30] and can effectively help the algorithm to increase the population diversity and reduce the operational complexity, whose mathematical expression is shown in Eq. (9).

$$X_i^{t+1} = u_1 X_i^t (1 - X_i^t) \tag{9}$$

where, $X_i^t \in rand(0, 1)$ the population size is i=*1,2,..,N*, t is the number of current iterations, and the chaos parameter is $u_1 \in rand(0, 4)$.

However, the logistic mapping based on a simple linear relationship suffers from the defect of uneven distribution of sequence values with blank windows. Therefore, in this paper, an improved logistic mapping strategy, using a sinusoidal variation-based mapping is proposed to make the initial solutions X_i^t uniformly distributed and enhance the diversity of individuals. As for its mathematical expression, it is expressed as shown in Eq. (10).

$$X_i^{t+1} = r \cdot \sin(3\pi \cdot (X_i^t(1 - X_i^t))) + (1 - r) \cdot u_2 X_i^t (1 - X_i^t) \tag{10}$$

where $r \in rand(0, 4)$, $X_i^t \in rand(0, 1)$, and the chaos parameters are $u_2 \in rand(0,4)$.

2) LENS IMAGING REVERSE LEARNING STRATEGY

This strategy considers both the solution and its opposite. By expanding the scope of bidirectional search in the search space to find the optimal solution X_i, the combination of the swarm intelligence algorithm and Reverse learning strategy can significantly improve the optimization performance of the algorithm [32]. Therefore, this paper adopts the lens imaging reverse learning strategy, whose mathematical expression is represented in Eq. (11)

$$X_i^{d*} = \frac{X_{\min}^d + X_{\max}^d}{2} + \frac{X_{\min}^d + X_{\max}^d}{2k} - \frac{X_i^d}{k} \tag{11}$$

X_i^d where $X_{\min}^d$ and $X_{\max}^d$ denote the minimum and maximum values of the d^{th} dimension vector in all initial solutions, respectively; X_i^{d*} is the lensing inverse solution of and k represents the scaling factor of the lens, and its mathematical expression is shown in Eq. (12) as follows:

$$k(t) = k_{\max} - \frac{(k_{\max} - k_{\min})t}{T_{\max}} \tag{12}$$

where the maximum scaling factor of the lens is k_{max} =1 and the minimum scaling factor of the lens is k_{min} =0.

According to the above definition, the steps to initialize the population for the chaotic backward learning strategy of the IGWO algorithm are as follows:

1) The logistic chaotic sequences are used to generate N initial solutions X_i, and the inverse point X_i^* of each individual position x_i is computed using the lens imaging backward learning.

2) The set of the initial solutions X_i and inverse solutions X_i^* as well as their fitness values are calculated separately, and they are sorted in ascending order (to find the minimum value).

3) The top N better solutions are selected as the final initial population according to the ranking result of the fitness value.

B. IMPROVED NONLINEAR DISTANCE CONTROL PARAMETER STRATEGY

In the GWO algorithm, when $|A| \geq 1$, individuals of the gray wolf population will expand their search for the optimal solution to perform global search; however, when $|A| < 1$, these individuals will narrow their search to refine the search for the optimal solution. Referring to Eq. (6), the value of A depends on the distance control parameter a. That is, the value of a plays a decisive role in the global and local search of the balancing algorithm. While a is linearly decreasing from 2 to 0 with the number of iterations, the early convergence speed is too fast leading to a smaller search range and poorer population diversity, whereas the late convergence speed is too slow solving efficiency is low, which cannot really reflect the actual nonlinear search process, leading to get algorithm falling into local optimum.

Therefore, this paper adopts a convergence strategy based on sinusoidal variation; in a such way, the distance control parameter a can change nonlinearly and dynamically with the number of iterations. In order to ensure population variety and increase the algorithm's capacity to search globally, the value of a is improved in the early iterations to retain a higher value for a longer length of time with a smaller change in magnitude and speed. In the late iteration, the distance control parameter a is improved to keep a smaller value for a longer period of time and the magnitude and speed of change is also smaller, which makes the gray wolf strengthen the local search ability of the algorithm and improve its solution efficiency; at the same time, the random perturbation term *randn*(), with normal distribution, is combined so that a can be dynamically adjusted using the perturbation term to reduce the probability of the algorithm falling into a local optimum. Therefore, the specific formula is shown in Eq. (13).

$$a(t) = a_{initial} - \frac{(a_{initial} - a_{finial})(f_i - f_{\min})}{f_{avg} - f_{\min}} \cdot \sin(\frac{2}{\pi} \cdot \frac{t}{T_{\max}}) + randn() \tag{13}$$

where and are the initial and termination values of the distance control parameter a, respectively.

C. INDIVIDUAL HISTORY OPTIMAL MEMORY STRATEGIES

In the GWO algorithm, individual gray wolves learn from the optimal position of the group to achieve position update; however, the influence of individual gray wolves' own experience on its position update is not considered in this process. Therefore, this paper introduces the individual memory function of the PSO algorithm, based on Eq. (7), so that the population individuals X_i learn from the population optimal position X_α and also learn from the individual historical optimal position. Thus, by adjusting the values of b_1 and b_2, the influence of the group communication and the individual memory on the search can be balanced, and the ability of the algorithm to jump out of the local optimum is enhanced. To sum up, the specific formula is shown in Eq. (14).

$$X(t+1) = b_1 \sum_{j=1,2,3} W_j X_j(t+1) + b_2 r_3 (pbest_i(t) - X(t)) \tag{14}$$

where b_1 and b_2 denote the group communication coefficient and the individual memory coefficient, usually constants between [0,1]. By adjusting these two values, the impact of group communication and individual memory on search can be balanced. Therefore, this article makes the algorithm's position update formula half affected by group communication and half affected by individual memory, fully avoiding falling into local optima during the position update process, i.e. b_1=b_2=0.5, r_3 is a random number uniformly distributed varying between [0,1], and $pbest_i(t)$ denotes the best individual historical position of the i^{th} gray wolf individual during the t^{th} iteration process.

D. MUTATION STRATEGIES FOR ELITE INDIVIDUALS

To further improve the ability of the algorithm to jump out of the local optimum in the process of finding the best solution, while maintaining the population diversity of

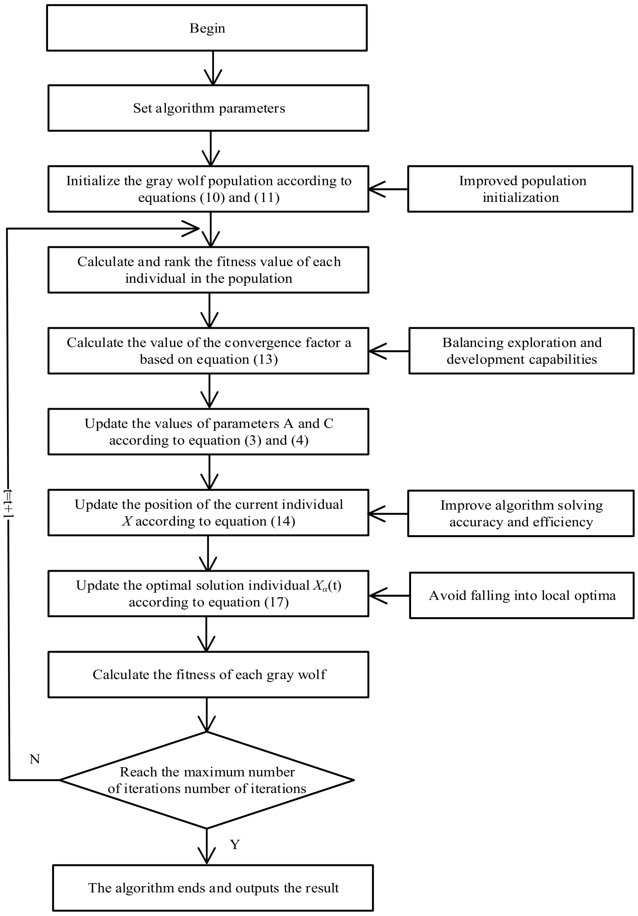


FIGURE 2. The flowchart of the IGWO.

the algorithm at the late stage of convergence, a Gaussian mutation strategy, based on superior selection, is introduced so that the algorithm can mutate the current optimal solution $X_\alpha(t)$ with a certain probability. Therefore, the specific expression of the Gaussian mutation operator is shown in Eq. (15).

$$X_{best}(t+1) = X_\alpha(t)(1 + Gaussion(\sigma)) \tag{15}$$

where $X_{best}(t+1)$ denotes the location of the individual after the mutation and $Gaussion(\sigma)$ represents a random variable following the Gaussian distribution.

However, the Gaussian mutation has a strong local exploitation ability, but it has a slightly weaker perturbation ability, while the Cauchy mutation has a stronger perturbation ability and a stronger global exploration ability [31]. Therefore, in this paper, we propose Eq. (16) that is an upgrade of Eq. (15), which combines the advantages of Gaussian and Corsi variants as follows:

$$X_{best}(t+1) = X_\alpha(t)(1 + r_4 \cdot Gaussion(\sigma) + r_5 \cdot cauchy(0,1)) \tag{16}$$

In order to determine whether to accept the optimal gray wolf individual position X_α after the mutation based on

TABLE 1. Standard test functions.

Func	Expressions	Rang	$f_{\min}$
F_1	$F_1=\sum_{i=1}^{n}x_i^2$	[-100,100]	0
F_2	$F_2=\sum_{i=1}^{n}\lvert x_i\rvert+\prod_{i=1}^{n}\lvert x_i\rvert$	[-10,10]	0
F_3	$F_3=\max\{\lvert x_i\rvert,1\le i\le n\}$	[-100,100]	0
F_4	$F_4=\sum_{i=1}^{n}ix_i^4+random[0,1]$	[-1.28,1.28]	0
F_5	$F_5=\sum_{i=1}^{n}\lvert x_i\rvert^{i+1}$	[-1,1]	0
F_6	$F_6=\sum_{i=1}^{n}ix^2$	[-5.12,5.12]	0
F_7	$F_7=\sum_{i=1}^{n}ix^4$	[-10,10]	0
F_8	$F_8=\sum_{i=1}^{n}\lvert x_i\sin x_i+0.1x_i\rvert$	[-10,10]	0
F_9	$F_9=-20\exp\left(0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right)+20+e$	[-32,32]	0
F_{10}	$F_{10}=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	[-600,600]	0
F_{11}	$F_{11}=\sum_{i=1}^{n}x_i^2+\left(\sum_{i=1}^{n}0.5ix_i\right)^2+\left(\sum_{i=1}^{n}0.5ix_i\right)^4$	[-5,10]	0

the superior selection probability p, the following update is shown in Eq. (17).

$$X_\alpha(t+1)=\begin{cases} X_{best}(t+1), & other \\ X_\alpha(t), & f(X_{best}(t+1))>f(X_\alpha(t)) \\ and & rand<p \end{cases} \tag{17}$$

where *rand()* represents a random number uniformly distributed varying between [0,1]; $f(\cdot)$ is the fitness value of the individual gray wolf population; and the two weighting factors take the values $r_4=r_5=0.5$.

E. IGWO ALGORITHM PSEUDO-CODE

To sum up, this paper proposes a multi-strategy fusion improved gray wolf algorithm (IGWO), whose features make the algorithm avoid falling into local optimum and improve its convergence speed and accuracy. The flowchart of the algorithm is shown in Figure 2, and the algorithm steps are shown as follows:

Inputs: population size N, search space dimension d, the maximum number of iterations T_{max}, initial value $a_{initial}$ and termination value a_{finial} of the distance control parameter a, population communication coefficient b_1 and individual memory coefficient b_2, and, finally, maximum value k_{max} and minimum value k_{min} of the lens scaling factor.

Output: X_α.

Algorithm description:

1) Initialization using Eqs. (10) and (11) to generate gray wolf populations $X_i=\{X_i^1, X_i^2, \ldots, X_i^d\}$, where $i=1,2,\ldots,N$.

2) Calculate the fitness value of individuals in the population $f(X_i)=\{f(X_1), f(X_2), \ldots, f(X_N)\}$ and record the current optimal individual α, the next optimal individual β and the third optimal individual δ, and their corresponding positions X_α, $X_\beta X_\delta$.

3)*while ($t<T_{max}$) do*

4) *for* $i=1$ *to N do*

5) Calculate the value of distance control parameter a according to Eq. (13);

6) Calculate the values of parameters A and C according to Eqs. (3) and (4);

7) Update the position of individuals X according to Eq. (14);

8) Update the position of the individual optimal solution X_α(t) according to Eq. (17).

9) *end for*

10) Calculate the fitness values $f(X_i)$ of individuals in the population and update the optimal individual α, the second optimal individual β and the third optimal individual δ as well as their corresponding positions X_α, X_β, X_δ and the historical optimal position *pbesti(t)*.

11) *Increase t*

12) *end while*

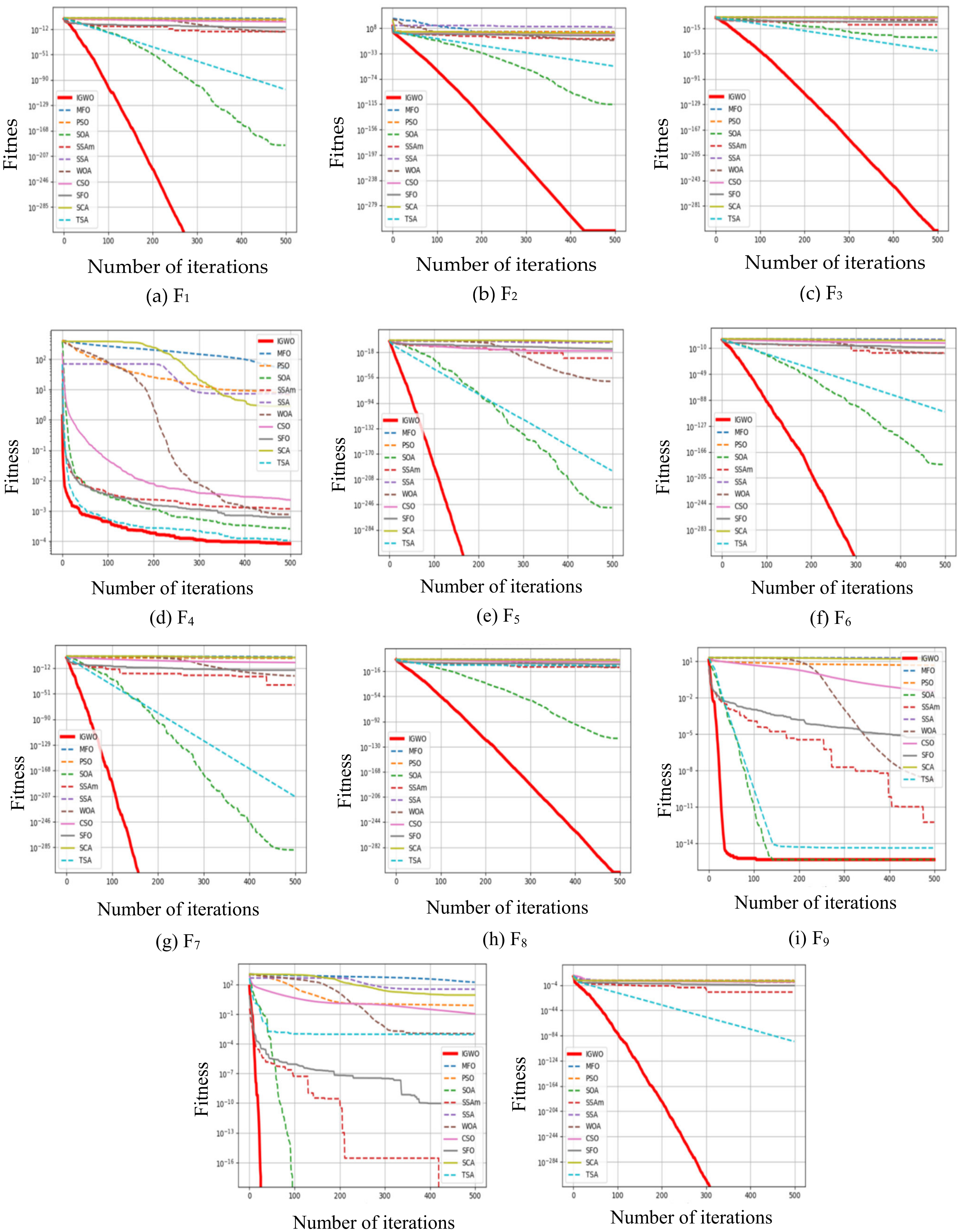
TABLE 2. Comparison of the results of IGWO and GWO algorithms.

Func	Dim	IGWO		GWO	
		Avg	Std	Avg	Std
F_1	30	0	0	2.29e-27	9.07e-27
	50	0	0	5.23e-20	4.43e-20
	100	0	0	1.94e-12	1.78e-12
F_2	30	3.44e-321	0	1.04e-16	8.76e-17
	50	2.47e-320	0	2.22e-12	1.26e-12
	100	1.26e-318	0	4.24e-08	1.48e-08
F_3	30	4.56e-321	0	7.66e-07	8.29e-07
	50	1.02e-319	0	4.68e-04	7.78e-04
	100	4.12e-316	0	7.00e-01	5.27e-01
F_4	30	4.77e-05	4.89e-05	1.95e-03	7.79e-04
	50	4.77e-05	5.28e-05	2.74e-03	1.42e-03
	100	6.49e-05	5.64e-05	6.68e-03	2.79e-03
F_5	30	0	0	5.54e-96	2.92e-95
	50	0	0	5.53e-85	1.85e-84
	100	0	0	7.98e-63	4.28e-62
F_6	30	0	0	4.04e-29	1.03e-28
	50	0	0	3.83e-21	4.39e-21
	100	0	0	1.29e-13	6.94e-14
F_7	30	0	0	6.69e-48	2.31e-47
	50	0	0	2.98e-34	7.29e-34
	100	0	0	5.51e-22	7.69e-22
F_8	30	2.27e-322	0	6.29e-04	7.68e-04
	50	3.94e-321	0	9.63e-04	1.17e-03
	100	3.12e-318	0	3.11e-03	2.70e-03
F_9	30	4.44e-16	0	1.03e-13	1.55e-14
	50	4.44e-16	0	3.98e-11	2.82e-11
	100	4.44e-16	0	1.34e-07	4.93e-08
F_{10}	30	0	0	3.17e-03	7.49e-03
	50	0	0	3.99e-03	8.30e-03
	100	0	0	4.17e-03	9.65e-03
F_{11}	30	0	0	3.16e-07	5.76e-07
	50	0	0	6.20e-02	8.10e-02
	100	0	0	1.23+02	5.42+01

III. SIMULATION RESULTS AND ANALYSIS

A. STANDARD TEST FUNCTIONS

In order to test the optimization performance and effectiveness of the proposed IGWO algorithm, 11 widely used global benchmark test functions were selected for simulation and experimental validation in this paper [18]. Thus, Table 1 gives the expressions, search intervals (Rang), and theoretical optimal values ($f_{\min}$) of the benchmark test functions, where $F_1\sim F_7$ represent the unimodal functions used to test the convergence accuracy and information mining ability of



(a) F_1 (b) F_2 (c) F_3 (d) F_4 (e) F_5 (f) F_6 (g) F_7 (h) F_8 (i) F_9 (j) F_{10} (k) F_{11}

FIGURE 3. Convergence curves of IGWO algorithm and other swarm intelligence. algorithms (d = 50).

TABLE 3. Comparison of the results of IGWO with other population intelligence optimization algorithms.

Func	Dim	IGWO		PSO		SCA		WOA		MFO		SOA		SSAm		SSA		CSO		SFO		TSA	
		Avg	Std	Avg	Std	Avg	Std	Avg	Std	Avg	Std	Avg	Std	Avg	Std	Avg	Std	Avg	Std	Avg	Std	Avg	Std
F_1	30	**0**	**0**	2.90e+00	9.77e-01	3.32e+01	1.21e+02	3.08e-21	6.22e-21	3.54e+03	5.84e+03	3.56e-187	0	4.77e-28	2.56e-27	5.99e+01	7.33e+01	2.81e-03	1.08e-02	4.89e-10	2.31e-09	1.82e-110	8.84e-111
	50	**0**	**0**	2.26e+01	4.88e+00	9.39e+02	1.09e+03	5.31e-17	1.40e-16	1.96e+04	8.59e+03	4.09e-191	0	1.77e-16	9.55e-16	3.56e+03	1.61e+03	2.46e-01	4.31e-01	2.75e-10	9.80e-10	2.03e-105	1.64e-105
	100	**0**	**0**	2.09e+02	3.38e+01	1.21e+04	7.28e+03	1.63e-13	6.27e-13	1.01e+05	1.40e+04	4.20e-181	0	2.59e-18	1.38e-17	5.63e+04	1.27e+04	1.74e+01	8.51e+00	3.76e-10	1.11e-09	2.96e-100	1.76e-100
F_2	30	**3.44e-321**	**0**	3.12e+01	2.84e+01	1.51e-02	2.15e-02	6.07e-15	1.11e-14	3.90e+01	2.27e+01	1.07e-114	5.81e-114	4.26e-12	2.29e-11	1.22e+03	5.23e+03	2.57e-04	2.74e-04	6.74e-05	1.85e-04	4.72e-57	2.34e-57
	50	**2.47e-320**	**0**	9.17e+01	4.95e+01	1.01e+00	1.20e+00	1.73e-12	1.92e-12	1.05e+02	3.67e+01	4.66e-116	2.47e-115	3.29e-10	1.65e-09	2.87e+09	1.05e+10	4.11e-02	3.07e-02	7.02e-05	1.14e-04	2.18e-54	7.22e-55
	100	**1.26e-318**	**0**	1.30e+07	6.95e+07	6.84e+00	4.91e+00	4.01e-10	6.25e-10	3.27e+02	4.80e+01	4.27e-116	1.60e-115	5.97e-12	3.05e-11	4.14e+25	1.32e+26	1.69e+00	4.49e+01	4.13e-04	1.73e-03	1.81e-51	3.68e-52
F_3	30	**4.56e-321**	**0**	4.11e+00	1.44e+00	3.77e+01	1.12e+01	1.90e-04	3.34e-04	6.89e+01	8.23e+00	3.96e-23	2.10e-22	3.77e-23	1.93e-22	4.39e+01	6.74e+00	1.00e+00	1.09e+00	2.87e-06	1.03e-05	4.22e-51	2.32e-51
	50	**1.02e-319**	**0**	8.64e+00	1.67e+00	6.91e+01	7.63e+00	1.71e-03	1.98e-03	8.40e+01	3.18e+00	6.12e+00	3.13e+00	3.41e-10	1.84e-09	6.11e+01	4.90e+00	8.55e-01	5.92e-01	7.86e-06	3.35e-05	2.27e-49	1.04e-49
	100	**4.12e-316**	**0**	1.54e+01	1.91e+00	9.00e+01	2.65e+00	1.97e-02	2.87e-02	9.31e+01	1.80e+00	4.98e-25	2.67e-24	2.74e-15	1.47e-14	6.75e+01	3.01e+00	2.48e+00	6.73e-01	1.45e-06	1.74e-06	1.18e-47	4.08e-48
F_4	30	**4.77e-05**	**4.89e-05**	1.26e+00	7.64e-01	9.12e-02	1.04e-01	3.76e-04	3.44e-04	7.51e+00	1.26e+01	1.98e-04	1.69e-04	1.13e-03	1.04e-03	1.01e+00	4.48e-01	1.20e-03	6.27e-04	7.70e-04	5.70e-04	9.83e-05	9.32e-05
	50	**4.77e-05**	**5.28e-05**	6.84e+00	3.79e+00	2.80e+00	3.41e+00	7.59e-04	8.64e-04	4.81e+01	3.74e+01	2.54e-04	2.28e-04	1.13e-03	9.55e-04	7.24e+00	4.27e+00	2.27e-03	1.01e-03	6.08e-03	4.64e-03	1.03e-04	1.05e-04
	100	**6.49e-05**	**5.64e-05**	7.63e+01	4.84e+01	1.43e+02	8.47e+01	6.73e-04	6.37e-04	4.82e+02	1.49e+02	2.10e-04	2.14e-04	1.29e-03	9.33e-04	1.50e+02	4.85e+01	8.26e-03	4.61e-03	9.95e-04	7.42e-04	9.11e-05	7.05e-05
F_5	30	**0**	**0**	2.04e-03	1.67e-03	3.33e-04	9.6e-04	6.08e-65	2.98e-64	1.89e-04	7.50e-04	3.30e-239	0	5.26e-38	2.83e-37	2.49e-05	1.62e-05	9.39e-15	3.75e-14	2.71e-13	4.42e-13	1.88-197	0
	50	**0**	**0**	8.37e-03	1.24e-02	1.48e-02	2.31e-02	1.90e-62	1.02e-61	1.56e-03	3.03e-03	2.13e-251	0	2.14e-27	1.14e-26	3.86e-04	1.30e-03	1.86e-16	6.87e-16	1.72e-13	3.48e-13	1.02e-195	0
	100	**0**	**0**	4.29e-02	5.28e-02	2.46e-01	2.15e-01	1.17e-56	5.91e-56	5.33e-02	8.47e-02	6.07e-241	0	5.05e-27	2.72e-26	1.58e-02	2.46e-02	6.13e-20	1.23e-19	2.04e-13	5.22e-13	3.01e-194	0
F_6	30	**0**	**0**	2.24e+01	1.31e+01	1.91e-01	2.08e-01	7.79e-23	2.69e-22	3.13e+02	3.71e+01	8.61e-195	0	8.55e-28	4.60e-27	1.92e+01	1.19e+01	1.18e-05	2.65e-05	3.79e-08	1.44e-07	5.57-112	5.21e-112
	50	**0**	**0**	1.35e+02	8.79e+01	2.94e+01	3.40e+01	1.65e-18	3.58e-18	1.26e+03	5.48e+02	1.38e-185	0	6.75e-18	3.63e-17	3.36e+02	1.41e+02	4.45e-03	8.54e-03	2.57e-09	6.11e-09	1.43e-106	8.49e-107
	100	**0**	**0**	1.50e+03	8.61e+02	1.11e+03	7.27e+02	6.79e-15	1.45e-14	130e+04	2.31e+03	1.65e-191	0	9.47e-24	5.10e-23	5.73e+03	1.23e+03	1.39e+00	6.93e-01	4.28e-09	6.91e-09	4.25e-101	2.62e-101
F_7	30	**0**	**0**	1.86e+01	1.07e+01	4.79e+01	1.10e+02	2.23e-30	9.87e-30	1.30e+04	2.17e+04	5.98e-304	0	6.24e-43	3.36e-42	9.73e+01	2.04e+02	4.11e-06	1.07e-05	9.14e-21	2.45e-20	3.01e-217	0
	50	**0**	**0**	5.14e+02	2.86e+02	1.16e+04	1.53e+04	1.52e-24	8.14e-24	1.02e+05	1.03e+05	1.60e-289	0	1.85e-38	9.97e-38	9.38e+03	7.25e+03	2.44e-04	6.46e-04	7.97e-16	3.05e-15	4.94e-208	0
	100	**0**	**0**	4.50e+04	7.59e+04	5.30e+05	2.65e+05	4.45e-20	1.19e-19	1.89e+06	4.85e+05	1.80e-309	0	2.81e-50	1.51e-49	5.70e+05	2.01e+05	1.60e-01	2.65e-01	1.90e-20	4.06e-20	2.16e-198	0
F_8	30	**2.27e-322**	**0**	1.44e+01	5.62e+00	1.38e+00	2.42e+00	9.05e-14	2.56e-13	8.50e+00	5.67e+00	1.69e-120	6.93e-120	9.04e-18	4.69e-17	1.57e+01	3.01e+00	6.4e-04	2.41e-03	9.28e-06	3.89e-05	3.22e-08	1.73e-07
	50	**3.94e-321**	**0**	3.47e+01	8.03e+00	6.96e+00	5.80e+00	1.97e-10	9.86e-10	2.40e+01	9.45e+00	3.35e-118	1.71e-117	6.49e-11	3.50e-10	4.14e+01	6.93e+00	1.75e-02	1.34e-02	4.92e-06	5.17e-06	2.98e-08	1.60e-07
	100	**3.12e-318**	**0**	9.58e+01	1.34e+01	2.76e+01	1.08e+01	8.03e-06	4.32e-05	9.53e+01	1.26e+01	5.00e-116	2.68e-115	2.20e-13	1.18e-12	1.26e+02	1.71e+01	1.90e-01	1.23e-01	3.09e-05	9.11e-05	2.08e-52	5.72e-53
F_9	30	**4.44e-16**	**0**	3.59e+00	4.48e-01	1.53e+01	7.82e+00	1.70e-11	2.82e-11	1.92e+01	1.77e+00	**4.44e-16**	**0**	6.08e-14	3.02e-13	1.38e+01	4.51e+00	2.59e-03	2.59e-03	6.23e-06	8.23e-06	3.99e-15	0
	50	**4.44e-16**	**0**	4.81e+00	5.16e-01	1.63e+01	6.75e+00	1.80e-09	3.90e-09	1.98e+01	2.44e-01	**4.44e-16**	**0**	5.46e-13	2.80e-12	1.83e+01	1.07e+00	3.30e-02	4.01e-02	5.21e-06	5.07e-06	4.11e-15	6.37e-16
	100	**4.44e-16**	**0**	7.62e+00	4.52e-01	1.95e+01	3.10e+00	5.42e-08	9.17e-08	1.99e+01	5.93e-02	**4.44e-16**	**0**	2.61e-11	1.40e-10	1.91e+01	2.26e-01	8.79e-01	3.60e-01	9.71e-06	2.79e-05	5.53e-15	1.76e-15
F_{10}	30	**0**	**0**	3.55e-01	7.48e-02	8.74e-01	3.16e-01	6.66e-17	3.18e-16	5.47e+01	5.92e+01	**0**	**0**	**0**	**0**	1.42e+00	7.95e+01	7.15e+03	1.82e+02	1.34e-11	3.25e-11	5.80e-04	3.12e-03
	50	**0**	**0**	7.53e-01	7.70e-02	8.38e+00	1.03e+01	1.06e-03	5.74e-03	1.65e+02	9.06e+01	**0**	**0**	**0**	**0**	3.30e+01	1.53e+01	1.06e-01	1.41e-01	8.65e-11	3.91e-10	8.53e-04	3.19e-03
	100	**0**	**0**	1.11e+00	1.74e-02	1.12e+02	5.09e+01	8.76e-13	3.43e-12	9.40e+02	1.30e+02	**0**	**0**	**0**	**0**	4.68e+02	8.85e+02	1.12e+00	8.00e-02	1.18e-10	4.93e-10	**0**	**0**
F_{11}	30	**0**	**0**	1.00e+03	5.76e+02	2.83e+01	1.74e+01	5.83e-02	1.98e-01	5.20e+02	7.43e+01	2.58e+02	1.22e+02	2.49e-18	1.34e-17	2.90e+02	9.28e+01	8.01e-02	2.18e-01	7.47e-06	3.21e-05	2.68e-100	4.58e-100
	50	**0**	**0**	4.15e+03	2.85e+03	1.70e+02	5.34e+01	3.09e-00	5.64e-00	1.23e+03	1.23e+02	4.52e+02	2.26e+02	8.36e-16	4.50e-15	7.38e+02	1.58e+02	9.04e+00	6.73e+00	2.03e-05	9.60e-05	4.22e-94	6.38e-94
	100	**0**	**0**	2.18e+04	1.27e+04	6.91e+02	1.55e+02	2.16e+01	2.07e+01	3.20e+03	2.52e+02	8.60e+02	5.28e+02	4.96e-18	2.63e-17	1.75e+03	1.58e+02	2.68e+02	7.48e+01	7.99e-04	4.13e-03	1.31e-83	5.43e-83

intelligent algorithms; $F_8\sim F_{11}$ represent the multimodal functions used to test the ability and exploration ability of intelligent algorithms to solve complex optimization problems.

B. COMPARISON WITH GWO ALGORITHM

In order to compare the performance of the IGWO algorithm with the GWO algorithm to find the best performance, the two algorithms are used to solve for the 11 standard test functions listed in Table 1. For the comparison and evaluation of benchmark function test results of different algorithms, the results are presented in the form of average accuracy (Avg) and standard deviation (Std). The average will indicate the ability of algorithms in avoiding local solutions, and the standard deviations show the variation in the results and stability of algorithms in avoiding local solutions. When the average value of the optimized function is closer to the theoretical optimal value in Table 1, it indicates that the optimization effect of the model is good; When the standard deviation of the optimized function approaches 0, it indicates that the model has strong robustness. To ensure the fairness of the simulation experiment, the same parameter settings are used for both algorithms, where the population size $N = 30$ and the maximum number of iterations are $T_{max} = 500$. In the IGWO algorithm, the maximum value of the scaling factor K of the lens is $K_{max} = 1$ and the minimum value is $K_{min} = 0$ whereas the initial value of the control parameter is $a_{initial} = 2$ and its termination value is $a_{finial} = 0$. The two algorithms are executed 30 times independently under dimensions $d = 30$, $d = 50$, and $d = 100$. The average accuracy (Avg) and the standard deviation (Std) of the two algorithms for finding the best of the 11 test functions are recorded.

Note that, in this paper, the used CPU is an I5 processor with a memory of 16GB, and where the system is Win10 64-bit operating system, and the program is implemented using Python2018. The solution results are shown in Table 2, and the bolded numbers indicate the best experimental results.

Referring to Table 2, the IGWO algorithm achieves better search results than the GWO algorithm for all 11 test functions. where the IGWO algorithm is able to converge to the optimal result, i.e., the theoretical optimal value of 0, when solving the function F_1, F_5, F_6, F_7, F_{10}, F_{11} in different dimensions. As for the functions F_2, F_3, F_7, F_8, F_9, they did not converge to the optimum, but the IGWO algorithm achieves better convergence accuracy than the GWO algorithm. Finally, for the function F_4, both algorithms appear to converge prematurely and fall into the local optimum, and similar convergence results are obtained. In addition, the standard deviation of the IGWO algorithm is 0 except for the function F_4, yielding in a strong robustness for the IGWO algorithm. In summary, the IGWO algorithm has better convergence accuracy and stability than the original GWO algorithm, which verifies the effectiveness of the IGWO algorithm improvement strategy.

C. COMPARISON WITH OTHER SWARM INTELLIGENCE ALGORITHMS

To further verify the effectiveness of the proposed IGWO algorithm in this paper, it is compared with other swarm intelligence optimization algorithms, and the compared algorithms are the PSO algorithm [8], SCA algorithm [34], Whale Optimization Algorithm (WOA) [35], MFO [9], Seagull Optimization Algorithm (SOA) [36], SSA [37], Salp Swarm Algorithm (SSA) [38], Crisscross Optimization algorithm (CSO) [39], SFO [10], and the Tunicate Swarm Algorithm (TSA) [40] (the sparrow algorithm is labeled as SSAm in order to distinguish between the sparrow algorithm SSA and the bottle sea squirt algorithm SSA). In order to test the performance of the proposed IGWO algorithm and the fairness of the simulation experiments, the population size ($N = 30$) and the maximum number of iterations ($T_{max} = 500$) were set equally for all algorithms. It should be noted that any adjustment of control parameters for each problem can effectively improve the performance of the algorithm. Generally speaking, the selection of parameters requires some experimentation. In this article, the parameters in the above literature are referenced to set these models. In addition, the parameter settings of the IGWO algorithm are as described in the previous section of the experiment. Thus, Table 3 contains the results of the comparative analysis of the above 11 algorithms for the functions shown in Table 1, after being executed 30 times under the condition of dimensions $d = 30$, $d = 50$, and $d = 100$, respectively.

Referring to Table 3, compared with the other 10 swarm intelligence optimization algorithms, the IGWO algorithm in all three dimensions can fetch better solution results for all 11 functions and has better convergence performance. Among them, except for functions F_9 and F_{10}, the IGWO algorithm has a better convergence effect compared to the other ten swarm intelligence optimization algorithms, and the search result for functions F_1, F_5, F_6, F_7, F_{10}, F_{11} finds the theoretical optimal value 0 in all three dimensions. In addition, when the dimension d increases from 30 to 50 and reaches 100, the convergence accuracy and stability of each algorithm tend to decrease; however, the IGWO algorithm can still maintain a significant advantage in its search results. In summary, IGWO has the fastest convergence speed in finding global minima and is significantly superior to all other algorithms. The standard deviation test of the algorithm also indicates that IGWO has strong robustness. In order to compare the convergence performance of several algorithms more clearly, the adaptation of the 11 functions shown in Table 1 is visualized and compared. Referring to Figure 3, among the 11 functions, IGWO achieves faster convergence speed and higher convergence accuracy compared to the other 10 swarm intelligence algorithms, which is related to the effective improvement of the local development ability of

TABLE 4. Comparison of the results of IGWO and improved GWO algorithms.

Func	Dim	IGWO		EGWO		HGWO		WGWO		MGWO	
		Avg	Std	Avg	Std	Avg	Std	Avg	Std	Avg	Std
F_1	30	**0**	**0**	**0**	**0**	**0**	**0**	7.72e-30	1.54e-29	1.85e-38	3.92e-38
	50	**0**	**0**	**0**	**0**	**0**	**0**	1.52e-21	2.09e-21	1.18e-27	1.53e-27
	100	**0**	**0**	**0**	**0**	**0**	**0**	1.92e-13	1.53e-13	5.69e-18	6.69e-18
F_2	30	**3.44e-321**	**0**	1.08e-190	0	2.14e-217	0	2.19e-19	2.23e-19	1.71e-23	1.10e-23
	50	**2.47e-320**	**0**	2.64e-184	0	7.36e-213	0	2.25e-14	2.34e-14	1.96e-17	1.05e-17
	100	**1.26e-318**	**0**	1.79e-178	0	1.58e-208	0	1.13e-09	6.19e-10	1.14e-11	5.79e-12
F_3	30	**4.56e-321**	**0**	9.95e-176	0	1.31e-207	0	5.96e-07	5.85e-07	2.13e-09	2.89e-09
	50	**1.02e-319**	**0**	1.61e-172	0	5.65e-205	0	6.33e-04	1.23e-03	5.51e-06	5.03e-06
	100	**4.12e-316**	**0**	2.18e-169	0	1.22e-202	0	7.84e-01	7.37e-01	2.89e-01	5.42e-01
F_4	30	**4.77e-05**	**4.89e-05**	9.51e-05	4.96e-05	6.08e-05	6.27e-05	5.56e+00	3.16e+00	7.32e-02	2.05e-01
	50	**4.77e-05**	**5.28e-05**	7.39e-05	6.34e-05	8.84e-05	7.70e-05	2.99e+01	1.25e+01	2.29e+00	1.58e+00
	100	**6.49e-05**	**5.64e-05**	1.32e-04	1.17e-04	8.28e-05	7.57e-05	2.75+02	1.00+02	3.49+01	2.25+01
F_5	30	**0**	**0**	**0**	**0**	**0**	**0**	5.39e-98	2.53e-97	4.60e-127	1.88e-126
	50	**0**	**0**	**0**	**0**	**0**	**0**	3.32e-85	1.73e-84	7.05e-111	3.79e-110
	100	**0**	**0**	**0**	**0**	**0**	**0**	9.92e-63	5.32e-62	1.07e-76	4.99e-76
F_6	30	**0**	**0**	**0**	**0**	**0**	**0**	6.13e+01	3.79e+01	6.45e-3	9.11e-3
	50	**0**	**0**	**0**	**0**	**0**	**0**	6.31e+02	2.21e+02	4.12e-01	5.01e-01
	100	**0**	**0**	**0**	**0**	**0**	**0**	6.75e+03	1.84e+03	4.43e+01	4.05e+01
F_7	30	**0**	**0**	**0**	**0**	**0**	**0**	2.85e+03	2.93e+02	1.29e-02	6.76e-02
	50	**0**	**0**	**0**	**0**	**0**	**0**	5.75e+04	3.39e+04	2.72e+02	5.72e+02
	100	**0**	**0**	**0**	**0**	**0**	**0**	1.10e+06	3.64e+05	3.80e+04	4.12e+04
F_8	30	**2.27e-322**	**0**	1.62e-04	4.15e-04	3.26e-218	0	9.20e-04	1.22e-03	3.23e-04	4.78e-04
	50	**3.94e-321**	**0**	7.49e-05	1.36e-4	7.01e-214	0	9.11e-04	1.26e-03	2.62e-04	4.93e-04
	100	**3.12e-318**	**0**	5.71e-05	1.28e-4	1.45e-209	0	3.51e-03	2.21e-03	8.38e-04	1.33e-03
F_9	30	**4.44e-16**	**0**	3.99e-15	0	4.35e-15	1.07e-15	7.46e+00	9.79e+00	2.37e-14	4.83e-15
	50	**4.44e-16**	**0**	3.99e-15	0	5.41e-15	1.740e-15	1.09e+01	1.01e+01	7.16e-14	1.13e-14
	100	**4.44e-16**	**0**	3.99e-15	0	4.58e-15	1.61e-15	1.16e+01	1.02e+01	2.52e-10	1.12e-10
F_{10}	30	**0**	**0**	5.17e-03	7.52e-03	**0**	**0**	1.93e-03	5.06e-03	1.96e-03	5.11e-03
	50	**0**	**0**	2.75e-03	6.11e-03	**0**	**0**	3.74e-03	7.88e-03	4.54e-03	9.98e-03
	100	**0**	**0**	2.02e-03	5.54e-03	**0**	**0**	7.30e-03	1.26e-02	1.93e-03	5.88e-03
F_{11}	30	**0**	**0**	0	0	**0**	**0**	5.84e-08	2.75e-07	7.09e-12	2.98e-11
	50	**0**	**0**	4.17e-304	0	**0**	**0**	7.29e-03	1.14e-02	2.11e-04	7.39e-04
	100	**0**	**0**	9.59e-248	0	**0**	**0**	7.17e+01	7.86e+01	1.74e+01	2.40e+01

the algorithm by nonlinear perturbation at the end of the iteration.

D. COMPARISON WITH GWO ALGORITHM

To further verify the effectiveness of the IGWO algorithm, it is compared with four improved GWO algorithms (noted as the EGWO algorithm [41], HGWO algorithm [24], WGWO algorithm [18], and MGWO algorithm [42]). Similar to the previous simulations, the five algorithms were executed 30 times with the same parameters in dimensions $d = 30, d = 50$, and $d = 100$, respectively, and the specific comparative results are shown in Table 4.

Referring to Table 4, compared to the other four improved GWO algorithms, the IGWO algorithm in all three dimensions can fetch better solution results, and the search results for the function F_1, F_5, F_6, F_7, F_{10}, F_{11} can reach the theoretical optimum value 0 in all three dimensions, and this result indicates that IGWO has overcome local optima and achieved the ability to achieve global optima. From the standard deviation (std), it can be seen that IGWO is still the most robust method, with more obvious advantages.

Moreover, Figure 4 shows the convergence curves obtained by solving the five improved algorithms in $d = 50$ dimensions, and it can be intuitively seen, from the convergence plots, that the IGWO algorithm can jump out of the local optimum faster than other improved GWO algorithms, and it has a faster convergence speed and a higher convergence accuracy. In summary, compared to the above four different improved GWO algorithms, the IGWO algorithm has good convergence and a good performance in finding the optimum, resulting in certain advantages for solving the optimal solution of the function.

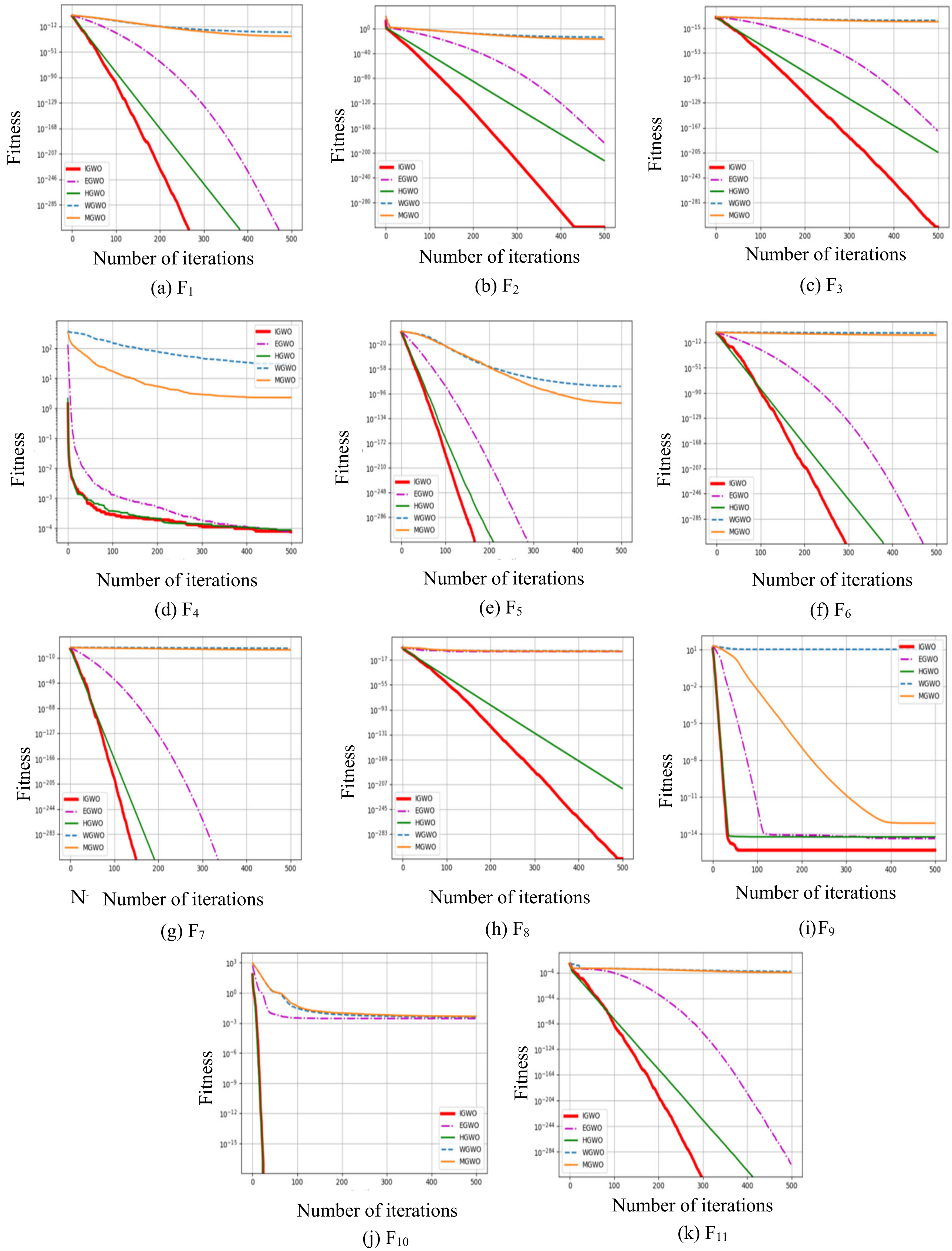


FIGURE 4. **Convergence curves of IGWO algorithm and other improved GWO algorithms (d = 50).**

TABLE 5. The p-values of Wilcoxon rank for IGWO and other population intelligence optimization algorithms.

Func	vs PSO	vs SCA	vs WOA	vs MFO	vs SOA	vs SSAm	vs SSA	vs CSO	SFO	vs TSA
	p-V alue S	p-V alue S	p-V alue S	p-V alue S	p-V alue S	p-V alue S	p-V alue S	p-V alue S	p-V alue S	p-V alue S
F_1	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	6.55e-05+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_2	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	6.56e-05+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_3	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	1.06e-06+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_4	2.87e-11+	2.87e-11+	1.44e-04+	2.87e-11+	1.09e-02+	1.26e-10+	2.87e-11+	2.87e-11+	7.03e-11+	7.33e-01-
F_5	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	1.02e-07+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_6	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	1.02e-07+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_7	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	2.67e-01-	2.51e-05+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_8	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	6.77e-07+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_9	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	N/A	2.67e-01-	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_{10}	2.87e-11+	2.87e-11+	8.24e-01-	2.87e-11+	N/A	N/A	2.87e-11+	2.87e-11+	2.87e-11+	5.05e-01-
F_{11}	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+	1.06e-06-	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+

TABLE 6. The p-value of Wilcoxon rank for IGWO with the IGWO algorithm.

Functions	vs EGWO	vs HGWO	vs WGWO	vs MGWO
	p-V alue S	p-V alue S	p-V alue S	p-V alue S
F_1	N/A	N/A	2.87e-11+	2.87e-11+
F_2	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_3	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_4	7.01e-01-	8.87e-03+	2.87e-11+	2.87e-11+
F_5	N/A	N/A	2.87e-11+	2.87e-11+
F_6	N/A	N/A	2.87e-11+	2.87e-11+
F_7	N/A	N/A	2.87e-11+	2.87e-11+
F_8	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_9	2.87e-11+	2.87e-11+	2.87e-11+	2.87e-11+
F_{10}	7.78e-03+	N/A	1.47e-02+	5.05e-01-
F_{11}	N/A	N/A	2.87e-11+	2.87e-11+

E. WILCOXON RANK AND TESTS

Although the average and standard deviation of 11 reference functions were run independently 30 times to illustrate the optimization performance and accuracy of different algorithms to a certain extent, it could not verify whether IGWO and the 14 algorithms mentioned above had significant differences in solving complex optimization problems. In view of this, the Wilcoxon rank sum test is used to compare the performance test features between algorithms. Wilcoxon rank sum test is a nonparametric null hypothesis test statistical method used to evaluate the fairness and robustness of algorithms. The results of 30 independent runs of IGWO and 14 other algorithms were taken as samples and tested with a confidence level of 0.05. When the p-value of the Wilcoxon rank sum test was less than 0.05, it was 95% likely that the two compared algorithms were significantly different. When the p-value of the Wilcoxon rank sum test is greater than 0.05, it indicates that there is little difference between IGWO and the comparison algorithm in overall optimization results, and Nan indicates that the two groups of samples are basically the same. Specifically, the P values of the Wilcoxon rank sum test are shown in Table 5 and Table 6, where "S" indicates discriminatory discrimination, "+/=/-" indicates that IGWO's performance is better than/equivalent to/inferior to other algorithms, and N/A indicates that the performance of the two algorithms is comparable.

Referring to Tables 5 and 6, most of the p-values of the IGWO algorithm are less than 0.05 and the "+" sign indicates that IGWO outperforms the swarm intelligence optimization algorithms and the improved GWO algorithms. The performance is equivalent in the global optimization to obtain the optimal value. In general, the IGWO algorithm has significant advantages over the other comparative algorithms, and this is evidence that the proposed algorithm has high performance in dealing with unimodal, multimodal benchmark functions.

IV. CONCLUSION

For the shortcomings of the GWO algorithm relying on the initial population, converging too early, and easily falling into local optimum, this paper proposes an improved gray wolf algorithm with multi-strategy fusion. A chaotic backward learning initialization strategy, based on logistic mapping and backward learning, is adopted to improve the traversal and the diversity of the initial population and lay the foundation for improving the efficiency of the algorithm. Moreover, a nonlinear control parameter convergence strategy is proposed to increase the population diversity by slowing down the convergence speed in the early stage and improve the accuracy of the algorithm by speeding it up in the later stage, which comprehensively improves the convergence performance of the algorithm. Finally, a position update strategy based on the dynamic weights and the individual memory, and a Gaussian-Cauchy mutation strategy, based on superior selection, are used to modify the original position update formula in order to optimize the algorithm search mode, avoid the algorithm falling into local optimum, and improve the convergence accuracy and convergence speed of the algorithm. Through simulation experiments on 11 standard test functions, compared with ten standard swarm intelligence optimization algorithms and four improved GWO algorithms proposed in other literature, the experimental results show that the IGWO algorithm, proposed in this paper, has better convergence speed and robustness, with more balanced performance in terms of global search and local exploitation capability.

As for the future work, lots of ideas can be implemented to enhance this work. The IGWO algorithm can be applied to feature selection to solve the problem of big data redundancy, and the performance of the algorithm can be further verified by practical problems.

V. DISCUSSION

To address the shortcomings of the basic gray wolf optimization algorithm in solving complex problems, such as relying on the initial population, converging too early, and easily falling into local optimum, a multi-strategy fusion improved gray wolf optimization algorithm is proposed to be applied in solving function optimization problems. The improvement strategy mainly includes the following three aspects: First, a chaotic backward learning initialization strategy, based on logistic mapping and backward learning, is adopted to improve the random initialization of the GWO algorithm and enhance the traversal and diversity of the initial population. Second, a nonlinear control parameter for local perturbation is constructed to avoid the problem of premature convergence of the GWO algorithm due to linear convergence and to balance the exploration and exploitation ability of the GWO algorithm. Finally, a location guidance strategy based on dynamic weights and individual memory is proposed to effectively improve the algorithm's optimization accuracy and computational efficiency; meanwhile, the Gaussian-Cauchy mutation strategy of superior selection is introduced to optimize the location update of optimal individual α wolves and improve the ability of the population to jump out of local extremes.

From the experimental results we can summarize as follows:

1) Our proposed IGWO method is effective and has high overall performance. It provides better results than comparative algorithms for seeking the optimal solution of functions.
2) The simulation results are statistically validated by the Wilcoxon rank sum test. Under majority conditions, IGWO has stronger robustness compared to other algorithms

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XIAOXIAO YANG is currently pursuing the master's degree in management science and the Master of Engineering degree with the Xiamen University of Technology. Her current research interests include intelligent optimization, nonlinear science, machine learning, and data mining and their practical applications.

YIHUI QIU received the Ph.D. degree in systems engineering from Xiamen University, China. She is currently a Professor with the School of Economics and Management, Xiamen University of Technology. Her research interests include data mining, pattern recognition, deep learning, and intelligent manufacturing and their practical applications.