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RESEARCH ARTICLE

Exploring Potential Applications of Ising Machines for Power System Operations

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ABSTRACT Recent advancements in Ising machines, both via quantum and quantum-inspired annealing, have shown promising results in solving difficult optimization problems. Owing to the potential of these dedicated machines, there is increasing interest in applying Ising machines to power system operations. However, despite Ising machines exhibiting computational advantages over conventional computational resources, modeling capability of Ising machines limits their potential applications. Hence, this study explores these potential applications in two steps. First, using the known characteristics of Ising machines and theoretical derivation, this study constructed an Ising machine applicability table for optimization problems. Then, using the constructed table, this study assessed the suitability of well-known optimization problems in power system operations to identify potential applications. Furthermore, to understand the performance of applying Ising machines, this study solved a phasor measurement unit placement problem using a quantum-inspired Ising machine. The results show the potential of solving large-scale problems that are unsolvable by conventional methods. The Ising machine usage guideline for power system operations summarizes the findings of this study.

INDEX TERMS Ising model, power system analysis, optimization.

NOMENCLATURE

H	Energy function of the Hamiltonian in the Ising problem.
σ_i	ith spin under the magnetic field h_i.
$J_{i,j}$	Interaction strength between ith and jth spin.
H^{QUBO}	Hamiltonian for QUBO problem.
$\boldsymbol{Q}$	Matrix used in the QUBO problem.
$\boldsymbol{x}$	Array of optimization variables in the QUBO problem.
x_n	nth variable in the QUBO problem.
H_{obj}	Main objective Ising problem.
H_{const}	Ising problem(s) for constraints.
$\boldsymbol{A_{eq}}$,b_{eq}	Matrix and value used for equality constraint(s).
a_n	Coefficient within equality constraint $\boldsymbol{A_{eq}}$.
λ	Penalty factor used for constraints.
H_{eq}^{QUBO}	QUBO model for equality constraint.
H_{ineq}^{QUBO}	QUBO model for inequality constraint.
$\boldsymbol{c}^T$	Relative weight of variables in the integer linear programming problem.
$\boldsymbol{y}$	The optimization variables used in Section III.
$\boldsymbol{A}$,$\boldsymbol{b}$	The matrix and array for equality constraints as used in Section III.
$\underline{\boldsymbol{y}}$, $\bar{\boldsymbol{y}}$	Lower and upper bounds of optimization variables.
$H_{int,1}^{QUBO}$	Example QUBO model for expressing integer variables (0 to integer value).
$H_{int,2}^{QUBO}$	Example QUBO model for expressing integer variables (integer value to integer value).
γ	An integer for creating a bound included QUBO model.
$H_{int,noeq}^{QUBO}$	General form of integer linear programming problem with lower and upper bounds.

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I	Number of integer optimization variables.
$H_{int,i}^{QUBO}$	Bound included QUBO model for one integer variable.
$H_{eqconst,k}^{QUBO}$	kth equality constraint for QUBO.
H_{int}^{QUBO}	General form of integer linear programming problem with lower and upper bounds, and equality constraints.
EQ	Total number of equality constraints.
$\boldsymbol{Q_q}$,$\boldsymbol{R_q}$	Variables used in the quadratic programming problem.
H_{Quad}^{QUBO}	QUBO model for integer quadratic programming problem.
$q_{i,l}$	Coefficient from $\boldsymbol{Q_q}$ (quadratic programming problem).
$\boldsymbol{\theta}$	Phase angles at each bus.
$\boldsymbol{B}$	Susceptance matrix.
$\boldsymbol{P}$	Vector of active power.
$\underline{\boldsymbol{P}}$, $\overline{\boldsymbol{P}}$	Bounds for active power.
$\boldsymbol{Q_{dc}}$,$\boldsymbol{k_{dc}}$	Matrix and vector for objective function of DC optimal power flow.
$f(P_g^t)$	Cost of generation.
$\underline{P_g}$, $\overline{P_g}$	Bounds for generators.
R_g	Generator ramp rate for generator g.
P_g^{t+1}	Generator power of generator g at time $t+1$.
P_g^t	Generator power of generator g at time t.
P_d^t	Demand at time t.
$P_{shed,i}$	Amount of load shed at i.
x_i^{shed}	Selection of feeder for optimal load shedding problem.($x \in \{0, 1\}$).
$\underline{P}_{shed}$	Minimum load to shed.
c_i	Cost of installing the ith PMU.
x_i^{place}	Placement of the ith PMU ($x \in \{0, 1\}$).
N	The number of buses.
L	The number of lines.
Q_c	Diagonal matrix in QUBO for Section VI.
$x_{s,l}$	PMU placement decision (as a binary variable) on sending end.
$x_{r,l}$	PMU placement decision (as a binary variable) on receiving end.
$H_{line,l}^{QUBO}$	Observability constraint written as a QUBO problem.
$\sigma_{s,l}$	PMU placement decision (as a spin) on sending end.
$\sigma_{r,l}$	PMU placement decision (as a spin) on receiving end.

I. INTRODUCTION

Due to the complexity of the power system, optimization is a long-lasting crucial topic within the power system field. Undeniably, optimization applications have allowed power systems to operate, evolve, and accommodate changing environments in the field. In recent years, carbon reduction goals set forth by various nations have accelerated the changing composition of power systems including the addition of inverter-based resources (IBR), high voltage direct current (HVDC) lines, and electric vehicles (EVs). The physical changes to the power system brought along with these resources add variability, uncertainty, and flexibility to the power system, thus driving interest in allocating these resources effectively using optimization.

Advancement in mathematical optimization and optimization modeling have shown a path for effectively utilizing the changing composition of the power system. With Benders' decomposition becoming a well-known tactic in the field [1], [2], [3], [4], researchers have tackled stochastic optimization problems, such as those relating to optimal scheduling of resources. This allowed solving large-scale scenario-based problems, granting access to a more robust solution to combat uncertainty. Furthermore, the recent revisiting of optimal power flow (OPF), as a convex optimization problem with certain assumptions has also opened new opportunities for using OPFs [5], [6], [7]. As for computational performance of optimization problems, [8], [9], [10], [11] studied the use of graphics processing units (GPUs).

Until GPU-assisted computing, algorithmic advancements have led optimization research in power systems as evidenced in reviews such as [12] and [13]. Because computing via central processing units (CPUs) offers limited computing capabilities with flexibility, these algorithmic improvements were crucial in optimizing power systems to accommodate various constraints.

Within this landscape of power system optimization, there is an increasing opportunity in the field of quantum computing. Recent research related to quantum computing, specifically those that employ an Ising problem, which this research will refer to as "Ising machine(s)", have shown promising results. A notable example is the modern use of quantum annealing as introduced by [14]. According to a recent advancement to quantum annealing, [15] explains that "about 200 seconds to sample one instance of a quantum circuit a million times–our benchmarks currently indicate that the equivalent task for a state-of-the-art classical supercomputer would take approximately 10,000 years." Similarly, previous research has also proposed other accessible variations (quantum-inspired Ising machine): CMOS implemented annealing [16] and GPU implemented momentum annealing [17]. Both types of Ising machines, whether quantum or quantum-inspired, exhibit far greater computing capability for optimization beyond the considered baseline in the field.

Noticing the capabilities of Ising machines, researchers in the field of power systems have started to experiment with Ising machines. In this early stage of Ising machine adoption in the power system field, [19] examined an application in EV charging station placement and [20] have proposed an application for combinatorial optimal power flow. Each study shows promising results that open new possibilities in the field of power systems.

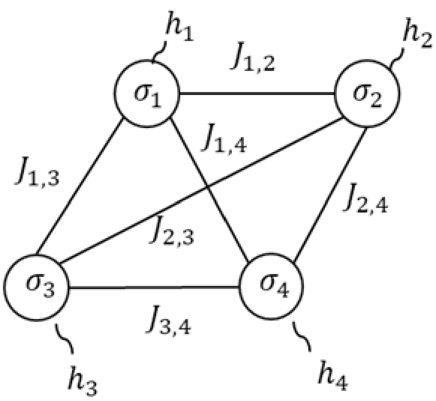


FIGURE 1. Conceptual diagram of the Ising model.

However, unlike CPU-based computing, Ising machines are limited in structure and not applicable to all optimization problems. Ising machines have the specific task of solving the Ising problem; thus, they are not suitable for every optimization problem. Hence, this study focused on the following research questions:

1) What type of optimization problems are Ising machines suitable for?
2) What are potential applications of Ising machines in power system operations?
3) What kind is the expected performance from using Ising machines?

Answering these three questions would provide guidance to accelerate the adoption of Ising machines in power system operations. Therefore, to answer the three fundamental research questions, this study makes the following contributions:

1) The Ising machine applicability table for optimization problems (Table 2) constructed using known characteristics of Ising machines (Section II) and theoretical derivation (Section III).
2) The Ising machine suitability table for notable power system optimization problems (Table 3) related to power system operations constructed using the Ising machine applicability table for optimization problems. (Section IV).
3) Example problem: a phasor measurement unit placement problem, solved on a quantum-inspired Ising machine (Section V).
4) Usage guideline for Ising machines in the power system based on contributions 1) to 3) (Section VI).

II. ISING MACHINE

This section overviews the known characteristics of Ising machines.

A. ISING MODEL, ISING PROBLEM, AND ISING MACHINE

At the time of writing, "Ising machine" was an unfamiliar term in the power system field. Hence, it is crucial to define the terms related to Ising machines. Here we explain the Ising model, the Ising problem, and the Ising machine.

The Ising model is one of the most fundamental models for magnets in statistical physics. The variables of this model are the state of spins, which represent the freedom of electrons inducing magnetization in materials. The Hamiltonian (energy function) of the Ising model, which is commonly reproduced in quantum and quantum-inspired computing for solving binary optimization, is

$$H = \sum_{i \neq j} J_{i,j}\sigma_i\sigma_j + \sum_i h_i\sigma_i | \boldsymbol{\sigma} \in \{-1, 1\}, \quad (1)$$

where σ_i indicates the ith spin under magnetic field h_i and $J_{i,j}$ is the interaction strength between ith and jth spin. Fig. 1 presents a conceptual diagram of the Ising model. Furthermore, solving the Ising problem refers to finding a set of $\boldsymbol{\sigma}$ values when H becomes minimum (i.e., the lowest energy state of the Ising model). The Ising machine, as used in the context of this study, refers to any computing resource capable of solving the Ising problem. Therefore, modeling a problem as an Ising problem allows access to an Ising machine.

The following subsections discuss the properties of the Ising problem and the Ising machines to lay the foundation for later sections.

B. PROPERTIES OF THE ISING PROBLEM

The Ising problem has an isomorphic property that allows for easy conversion to a quadratic binary unconstrained optimization (QUBO) problem. One way to express the QUBO problem is as follows:

$$H^{QUBO} = \boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x} | \boldsymbol{x} \in \{0, 1\}. \quad (2)$$

The objective of the QUBO problem is to determine a set of $\boldsymbol{x} = x_1, x_2, \ldots, x_n$ that minimizes the objective function. The difference between the Ising problem and the QUBO problem lies in the optimization variables. Unlike Ising problems, QUBO problems use $\boldsymbol{x} \in \{0, 1\}$. Substituting $x = (\sigma + 1)/2$ and removing any offsets (constants) converts the QUBO problem into an Ising problem (i.e., $H^{QUBO} \mapsto H$). This commonly known isomorphic property between Ising problems and QUBO problems is useful for assessing the applicability of Ising machines.

Because Ising problems are too simple to model explicit constraints, a common tactic to build a constrained optimization problem is to combine two Ising problems such that,

$$H = H_{obj} + H_{const}, \quad (3)$$

where H_{obj} is the Ising problem for the main objective and H_{const} is the Ising problem for constraints. If the user designs H_{const} such that H_{const} reaches a minimum when all constraints are satisfied, then the solution to H theoretically minimizes the objective and satisfies the constraints, given that constraint satisfaction is plausible. Usually, the user must size the potential penalty of H_{const} to be greater than H_{obj} to prioritize constraint satisfaction.

Several methods are known for building these constraints. Ising models for constraints penalizes the objective function when constraints are not satisfied (such as in barrier or interior-point methods). The simplest form of constraint written in QUBO is

$$H^{QUBO} = \lambda x, \tag{4}$$

or in Ising form,

$$H = \lambda \sigma. \tag{5}$$

This formulation implies that for the model to minimize the total energy, $x \neq 1$ or $\sigma \neq 1$. Given an equality constraint $\boldsymbol{A_{eq}x} = b_{eq}$, extending the same concept to more complicated constraints, we can write the equality constraints as,

$$H_{eq}^{QUBO} = \lambda(b_{eq} - \sum_{n=0}^{N} a_n x_n)^2, \tag{6}$$

where a_n is a coefficient in $\boldsymbol{A_{eq}}$. The model takes a nonzero value when the constraint is unsatisfactory. A similar technique is known in the field, where we can express inequality constraints as follows:

$$H_{ineq}^{QUBO} = \alpha + \beta, \tag{7}$$

where,

$$\alpha = \lambda(1 - \sum_{a=1}^{bineq} x_a)^2,$$
$$\beta = \lambda(\sum_{a=1}^{bineq} a x_a - \sum_{n=0}^{N} x_n)^2. \tag{8}$$

This formulation requires the use of ancillary variables/spins to function. Various optimization problems are compatible with Ising machines by utilizing variations in the concept.

C. CHARACTERISTICS OF ISING MACHINES

One must first understand the characteristics of Ising machines to understand the applicable range. Building an Ising problem ensures that the Ising machine is compatible; however, if the application does not align with the characteristics of Ising machines, the application may be unsuitable.

As Ising machines encompass a wide variety of machines, this study generalizes the common traits of these machines based on the observations of [15], [16], [17], and [18].

1) COMPUTATION TIME

The computation time of Ising machines, quantum or quantum-inspired, is faster than that of traditional CPU-based computing resources, given that the problem is binary or integer optimization. The improvement in computation time with quantum-inspired Ising computing resources as reported in [16] and [17] is up to two orders of magnitude better than that of conventional computing resources. For quantum, these are several orders higher [15], [18].

2) REPRODUCIBILITY

The reproducibility of the results depends on the operating principle of the Ising machine. Usually, Ising machines cannot guarantee the reproducibility of the results because solving is usually a heuristic process. However, certain Ising machines such as the one shown in [17] may be able to reproduce results (given the Ising machine uses a seed of random numbers). The quantum variants of Ising machines cannot guarantee the same.

3) SCALABILITY

The scalability of Ising machines depends on the computing resources that are used. The quantum-based approach had limited hardware scalability of a few hundred variables [15]. However, recent hardware [18] has been improved to 20,000 for dense problems (Ising problems with many interactions between variables). For the quantum-inspired approach, sufficient scalability exists for both binary and integer optimization problems as reported in [16] and [17].

4) OPTIMALITY

Unlike classical optimization which is within convex optimization, Ising machines cannot guarantee optimality because the Ising problem is non-continuous and non-convex. However, the evolution of the energy function may be sufficient to determine whether the solution is close to optimal.

III. ISING MACHINE APPLICABILITY TABLE FOR OPTIMIZATION PROBLEMS

According to [21], Ising problems can express Karp's 21 NP-complete problems. Hence, power system-related problems that are within these bounds are "programmable" into Ising problems, such as partitioning problems and binary integer linear programming. However, beyond the problems described in [21], Ising problems can model other types of problems. This section explores the modeling capability of the Ising problem using theoretical derivations.

A. INTEGER LINEAR PROGRAMMING PROBLEM

Take for instance, an integer linear programming problem of the form,

$$\min \boldsymbol{c}^T \boldsymbol{y} \tag{9a}$$
$$\text{subject to } A\boldsymbol{y} = \boldsymbol{b} \tag{9b}$$
$$\underline{\boldsymbol{y}} \leq \boldsymbol{y} \leq \bar{\boldsymbol{y}}, \tag{9c}$$

where $\boldsymbol{c}^T$ is the relative weight of variables, $\boldsymbol{y} \in \mathbb{Z}$ are the optimization variables, $\boldsymbol{A}$ is a matrix for equality constraints, $\boldsymbol{b}$ is a vector for equality constraints, and $\underline{\boldsymbol{y}}, \bar{\boldsymbol{y}}$ are lower and upper bounds of variables. As the Ising and QUBO problems are binary, one may overlook the possibility of converting the integer linear programming problem into the Ising or QUBO form. However, using several spins to express $y_i \in \boldsymbol{y}$ opens the possibility of converting this form of integer linear programming problem into an Ising/QUBO problem.

TABLE 1. Example truth table for the QUBO model in (13).

$x_{1,1}$	$x_{1,2}$	$x_{2,1}$	$x_{3,1}$	Value
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	2
0	1	0	0	2
0	1	0	1	3
0	1	1	0	3
0	1	1	1	4
1	0	0	0	1
1	0	0	1	2
1	0	1	0	2
1	0	1	1	3
1	1	0	0	3
1	1	0	1	4
1	1	1	0	4
1	1	1	1	5

Consider the ith variable in the integer linear programming problem denoted by y_i. From the definition alone, $\underline{y_i} \leq y_i \leq \overline{y_i}$. First, we assume $\underline{y}_i = 0$ and $\overline{y_i} \in 2^N - 1|N \in \mathbb{Z}$. In such a case, all the values that y_i can take, are exactly equal to the space of the following QUBO model:

$$H^{QUBO}_{int,1} = \sum_{n=0}^{N} 2^n x_n | x_n \in \{0, 1\}. \tag{10}$$

In an alternative case, where $\underline{y}_i \neq 0$, but $(\overline{y_i} - \underline{y}_i) \in 2^N - 1|N \in \mathbb{Z}$ holds true, the QUBO model, with offset, is as follows:

$$H^{QUBO}_{int,2} = \underline{y_i} + \sum_{n=0}^{N} 2^n x_n | x_n \in \{0, 1\}. \tag{11}$$

If $(\overline{y_i} - \underline{y}_i) \notin 2^N - 1|N \in \mathbb{Z}$, for example $\overline{y_i} - \underline{y}_i = 5$, the direct application of the above logic is difficult. To overcome this issue, a simple mathematical technique is employed. Given an integer $\gamma \notin 2^N - 1|N \in \mathbb{Z}$, the weighted sum of multiples of 2 can express any γ. This implies that

$$\{0, 1, \ldots, \gamma\} = n_1 x_1 + n_2 x_2 + \ldots + n_j x_j, \tag{12}$$

holds true given $n_1, n_2, \ldots, n_j \in 2^N | N \in \mathbb{Z}$. For instance, when $\gamma = 5$, the QUBO model that satisfies this is

$$\{0, 1, \ldots, \gamma\} = x_{1,1} + 2x_{1,2} + x_{2,1} + x_{3,1}, \tag{13}$$

which requires 4 separate variables. From the equation alone, the maximum value that this QUBO model can attain is 5 and the smallest value that this model can attain is 0. Table 1 verifies this using a truth-table-like structure.

Notice that the value of the QUBO will overlap on certain combinations such as [0,0,1,1] and [1,0,0,1]. However, this property is acceptable, as the ability of the QUBO to express these distinct integer values is of concern. In practice, the coefficients necessary for this integer substitution can be pre-solved or solved accordingly. Therefore, the following expression is the QUBO equivalent to the integer linear programming problem from earlier with lower and upper bounds:

$$H^{QUBO}_{int,noeq} = \sum_{i=1}^{I} c_i H^{QUBO}_{int,i}, \tag{14}$$

where I is the number of integer optimization variables, and $H^{QUBO}_{int,i}$ is the bound-included QUBO model written as

$$H^{QUBO}_{int,i} = \underline{y_i} + n_1 x_1 + n_2 x_2 + \ldots + n_j x_j. \tag{15}$$

Utilizing the equality constraint format from earlier, the kth equality constraint ($A_k x = b_k$) is,

$$H^{QUBO}_{eqconst,k} = \lambda(b_k - A_{k,i} \sum_{i=1}^{I} H^{QUBO}_{int,i})^2, \tag{16}$$

where the model assumes λ to be sufficiently large. Therefore, the full QUBO equivalent to the integer linear programming problem is

$$H^{QUBO}_{int} = \sum_{i=1}^{I} c_i H^{QUBO}_{int,i} + \sum_{k=1}^{EQ} H^{QUBO}_{eqconst,k}, \tag{17}$$

where EQ denotes the total number of equality constraints. Recall that QUBO problems are isomorphic to Ising problems; thus, they can be converted to the Ising form. Because integer programming problems can be written in standard form, most integer programming problems are expressible as Ising problems. In practice, within the field of engineering, it is rare to encounter an integer linear programming problem without the lower and upper bounds of the variables.

B. INTEGER QUADRATIC PROGRAMMING PROBLEM

Further inspection of $\sum_{i=1}^{I} c_i H^{QUBO}_{int,i}$ in (17) reveals that $\sum_{i=1}^{I} c_i H^{QUBO}_{int,i} \in \mathbb{B}^1$, whereas the QUBO and Ising models are $H^{QUBO}, H \in \mathbb{B}^2$. Therefore, the QUBO and Ising problems are also applicable to quadratic integer programming problems in the following form:

$$\min \frac{1}{2} \boldsymbol{y}^T \boldsymbol{Q_q} \boldsymbol{y} + \boldsymbol{R_q} \boldsymbol{y} \tag{18a}$$

$$\text{subject to} \quad \boldsymbol{A}\boldsymbol{y} = \boldsymbol{b} \tag{18b}$$

$$\underline{\boldsymbol{y}} \leq \boldsymbol{y} \leq \overline{\boldsymbol{y}}, \tag{18c}$$

by utilizing a formulation such that,

$$H^{QUBO}_{Quad} = \sum_{l=1}^{L} \sum_{i=1}^{I} \frac{1}{2} q_{i,l} H^{QUBO}_{int,i} H^{QUBO}_{int,j} + \sum_{i=1}^{I} c_i H^{QUBO}_{int,i} + \sum_{k=1}^{EQ} H^{QUBO}_{eqconst,k}, \tag{19}$$

where $q_{i,l}$ is a coefficient from $\boldsymbol{Q_q}$.

C. LINEAR PROGRAMMING PROBLEM AND QUADRATIC PROGRAMMING PROBLEM

Although Ising machines are suitable for solving NP-hard problems, scaling (15) also allows us to realize that both

TABLE 2. Ising machine applicability table for optimization problems.

Problem type	Expressible?	Scalable?
Karp's 21 NP-complete problems (as in [21])	YES	YES
Integer Linear Programming (ILP) Problem w/ bounds	YES	YES
Integer Quadratic Programming Problem (IQP) w/ bounds	YES	YES
Linear Programming (LP) Problem w/ bounds	Partially*	NO
Quadratic Programming (QP) Problem w/ bounds	Partially*	NO
Mixed Integer Linear Programming (MILP) Problem w/ bounds	YES	Partially**
Mixed Integer Quadratic Programming (MIQP) Problem w/ bounds	YES	Partially**
Other Continuous Non-Linear Problem	Partially	Unknown

*With discretization

**Assuming there are more integer variables than continuous variables

linear and quadratic programming problems can be ''approximated'' as QUBO/Ising problems. In conventional wisdom, the approximation of linear programming (LP) and quadratic programming (QP) problems with integer variants is unintuitive and redundant. However, if the Ising machine is sufficiently powerful, approximating into the integer form may prove to be more advantageous than solving directly on conventional machines using known algorithms such as simplex and interior-point. Although expressible as an Ising problem, these may be bound to scalability issues, as physical hardware may not accommodate extra ancillary variables.

D. MIXED INTEGER LINEAR PROGRAMMING PROBLEM AND MIXED INTEGER QUADRATIC PROGRAMMING PROBLEM

Because integer linear programming problem with bounds and linear programming problems with bounds are both expressible in the QUBO/Ising form, a mixed integer linear programming (MILP) problems with bounds are also expressible. The same can be deduced for mixed integer quadratic programming (MIQP) problems. Although not as drastic as linear programming problems and quadratic programming problems, they may run the risk of hardware limitations.

E. OTHER CONTINUOUS NON-LINEAR PROBLEMS

Beyond the common forms of optimization problems, other continuous non-linear programs may also be expressible. As of this study, these are not completely understood. The sine and cosine terms commonly found in AC power flow analysis of a power system may not be completely expressible in the QUBO/Ising problem.

F. SCALABILITY OF EACH PROBLEM

Scalability, as used here, is not related to computational scalability, but rather to the scalability of the problem size with respect to hardware. As linear and quadratic programming problems require many variables (spins in the Ising problem) per optimization variable of the original problem, the problem may have scalability issues unless there is an unlimited number of variables in the Ising machine. Mixed integer linear programming problems may have similar issues but may have limited effects if there are more integer variables than continuous variables.

G. SUMMARY OF SECTION

Based on the observations in the earlier subsection, Table 2 summarizes the general applicability of the Ising problem. The next section uses this reference table to analyze its applicability in the power system.

IV. SUITABILITY OF THE ISING MACHINE TO PROBLEMS IN THE POWER SYSTEM

This section examines the suitability of Ising machines for various optimization problems in power system operations. We used Table 2 to formulate the Ising machine suitability table for notable power system optimization problems.

A. AC POWER FLOW AND VARIANTS

Although the full AC power flow calculation is the most widely used ''optimization problem'' in the field, the network equations, namely the sine and cosine components, classify AC power flow problems as non-linear optimization. This makes programming into the Ising problem difficult. Hence, the evolution of the problem, such as optimal power flow using AC network equations, is unlikely. Thus, we conclude that the AC power flow and its variants are unsuitable for Ising machines.

B. DC POWER FLOW AND VARIANTS

The DC power flow problem is a well-used alternative (or approximation) to the AC power flow problem used in the industry which neglects the resistance of the network, assumes a constant voltage, and approximates the sine terms of the network equations with the angle. The objective of the DC power flow is simple, where given networks injections P at each bus, determine the phase angles at each bus θ by using

$$\boldsymbol{\theta} = \boldsymbol{B}^{-1}\boldsymbol{P}, \tag{20}$$

where $\boldsymbol{B}$ denotes the susceptance matrix. One variant of this is a QP problem, but simple observation also tells us that this alone does not require an Ising problem, especially because the susceptance matrix is sparse and has myriad methods, such as L-U decomposition, to efficiently solve. However, the application of DC power flows, such as DC OPF, can potentially benefit from the use of Ising machines. As indicated in [22], a simple DC OPF is a semi-definite quadratic

TABLE 3. Ising machine suitability table for notable power system optimization problems.

Power System Problem	Problem Type	Applicability	Benefit	Suitability
AC Power Flow	Non-linear Optimization	Unlikely	-	Unsuitable
AC Optimal Power Flow	Non-linear Optimization	Unlikely	-	Unsuitable
DC Power Flow	QP Problem w/ Bounds	Applicable	Unlikely	Unsuitable
DC Optimal Power Flow w/o Switching Components	QP Problem w/ Bounds	Applicable	Unlikely	Unsuitable
DC Optimal Power Flow w/ Switching Components	QP Problem w/ Bounds	Applicable	Likely	Suitable
State Estimation	Non-linear optimization	Unlikely	Unlikely	Unsuitable
State Estimation	LP Problem w/ Bounds	Applicable	Unlikely	Unsuitable
State Estimation	MILP Problem w/ Bounds	Applicable	Unlikely	Unsuitable
Economic Dispatch as LP	LP Problem	Applicable	Unlikely	Unsuitable
Economic Dispatch as QP	QP Problem	Applicable	Unlikely	Unsuitable
Unit Commitment	MILP Problem	Applicable w/ limitation	Likely	Suitable
Optimal Load Shedding	Knapsack (Karp's 21 NP Problem)	Applicable	Likely	Suitable
PMU/Sensor Placement	ILP or Vertex Cover (Karp's 21 NP Problem)	Applicable w/ limitation	Likely	Suitable

programming problem in the following form:

$$\min \ \frac{1}{2}\boldsymbol{P}^T\boldsymbol{Q}_{dc}\boldsymbol{P} + \boldsymbol{k}_{dc}\boldsymbol{P} \tag{21a}$$

$$\text{subject to} \quad \boldsymbol{AP} = \boldsymbol{b} \tag{21b}$$

$$\underline{\boldsymbol{P}} \leq \boldsymbol{P} \leq \overline{\boldsymbol{P}}, \tag{21c}$$

where $\boldsymbol{P}$ is a vector of active power and $\underline{\boldsymbol{P}}$ and $\overline{\boldsymbol{P}}$ are the bounds. Reference [20] shows the variation in this DC optimal power flow as a combinatorial optimization problem.

Hence, according to Table 2, a simple DC OPF is programmable (applicable) as an Ising problem. The addition of ramping constraints, which is also a linear inequality constraint, for multi-period optimization may also be possible. Given that there are integer or binary variables, such as switching of network components or generation/load, additional complexity may make Ising machines suitable for this application.

C. STATE ESTIMATION

State estimation covers the lack of full coverage of measurements in the power system–usually solved via weighted least squares regression or least absolute value (LAV) optimization. Owing to the recent introduction of phasor measurement units (PMUs), state estimation has regained popularity as shown in [23], [24], and [25]. Similar to power flow, state estimation via weighted least squares regression usually involves an iterative process that requires calculation of the power flow Jacobian at each step. In such a case, formulating an Ising problem may be possible, but may not be beneficial.

With the advancement of linear programming solvers, the recent implementation of state estimation has also been experimented with using least absolute value optimization [25]. Such formulations are known to be more robust than the weighted least squares method. Again, according to Table 2, because linear programming problem is part of the applicable problems, the LAV-based state estimation is expressible (with discretization) as an Ising problem. The ability of the Ising problem to consider binary variables may also render the problem applicable for topology estimation.

Similar to the former, whether Ising machines are suitable for this application depends on the type of variables considered. If the state estimation problem requires non-linear weighted least squares regression, then the problem type becomes a non-linear optimization problem inexpressible in Ising form. The state estimation problem formulated as an LP problem using LAV is applicable according to Table 2 but is unlikely to scale and gain benefits. However, if one considers the state estimation problem as an MILP problem using LAV with switching of topologies, then the problem is applicable and beneficial according to Table 2 but is unlikely to scale and gain benefits.

D. ECONOMIC DISPATCH

Economic dispatch is an elementary form of optimal power flow without the need for network constraints and is usually given in the following form,

$$\min \sum_{t\in T}\sum_{g\in G} f(P_g^t) \tag{22a}$$

$$\text{subject to} \quad s.t. \sum_{g\in G} P_g^t = P_d^t | \forall t \in T \tag{22b}$$

$$|P_g^{t+1} - P_g^t| \leq R_r | \forall t \in T, \tag{22c}$$

$$\underline{P_g} \leq P_g^t \leq \overline{P_g} | \forall t \in T, \tag{22d}$$

Assuming that the objective function is linear or quadratic, an economic dispatch is either a linear programming problem or a quadratic programming problem. Therefore, it is safe to assume that one can formulate the economic dispatch problem as an Ising problem; however, the benefit of formulating this as an Ising problem may be small as convex optimizations are well studied.

E. UNIT COMMITMENT

The unit commitment problem is an extended form of the economic dispatch problem or optimal power flow, which minimizes the cost over a selected interval. Owing to unit commitment problems that are capable of implementing a variety of regulations and considerations, they have many variations. Unlike in previous examples, the unit commitment problem is known to be difficult to solve. Based on Table 2, one can convert a simple unit commitment problem in a mixed integer linear programming problem form found in [26] into

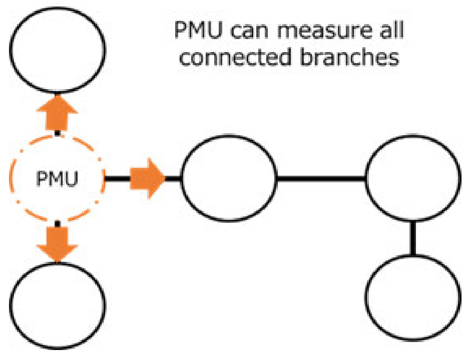


FIGURE 2. PMU visibility.

an Ising problem and is likely to benefit. See the Appendix for an example formulation.

F. OPTIMAL LOAD SHEDDING

One niche application of optimization in the power field is the optimal load shedding problem. Under frequency load shedding relays (UFLS), or under frequency relays (UFRs), shed the load by directly disconnecting the feeders. Although optimal load shedding is a well-researched topic as shown in [27], [28], and [29], the direct disconnection with the UFR is binary (either connected or disconnected), selecting the optimal combination of loads is difficult to solve. A straightforward way of expressing this problem is through the knapsack problem of the following form:

$$\min \sum_{i \in L} P_{shed,i} x_i^{shed} \tag{23a}$$

$$\text{subject to} \sum_{i \in L} P_{shed,i} x_i^{shed} \geq \underline{P}_{shed}. \tag{23b}$$

From the problem alone, it is evident that the problem is a combinatorial optimization problem, where the objective is to minimize the shedding amount. However, the consideration of network constraints is only possible with a DC-based approach.

G. OPTIMAL PMU PLACEMENT PROBLEM

Another crucial problem in power is the optimal PMU placement problem as researched in [30], [31], [32], [33], [34], [35], [36], and [37]. Optimal PMU placement problems cover a large variety of problems that seek to optimize the placement of PMUs for a given network. In the most general form, as introduced in [30], the optimization problem can be formulated as follows:

$$\min f(x) = \sum_{i \in \mathbb{Z}:\, i=[1,N]} c_i x_i^{place}, \tag{24a}$$

$$\text{subject to } \boldsymbol{A}\boldsymbol{x}^{\boldsymbol{place}} \geq 1, \tag{24b}$$

where c_i indicates the cost of installing the ith PMU, x_i^{place} indicates the placement of the ith PMU ($x \in \{0, 1\}$), N indicates the number of buses, $\boldsymbol{A}$ indicates a matrix of size $L \times N$ that indicates the lines observable from a PMU at a bus, and L indicates the number of lines. Similar to $\boldsymbol{x}$, each element of $\boldsymbol{A}$, indicated by $A_{i,j}$ specifies whether a line is observable from the bus ($A_{i,j} = 1$) or not ($A_{i,j} = 0$). The above formulation limits the constraint of measuring at least one voltage phasor of at least one end of each line, as outlined in the earlier sections of [31].

This formulation assumes that the PMU at a bus measures the voltage and current phasors of all connected lines. Given the network model, voltage phasor, and current phasor, the voltage phasor on the other side of the line is observable or solvable. This ensures the full observability of the system while being able to detect any faults that occur in the system. Fig. 2 provides a visual example of the effect of placing a PMU at a location. We can also observe this is a vertex cover problem.

As the formulation is an integer linear programming problem or a vertex cover problem, it is both applicable and beneficial when using Ising machines.

H. SUMMARY OF APPLICATIONS

Table 3 summarizes the suitability of each application. Suitable, as presented here, refers to meeting both the applicability and benefits. As discussed throughout this section, the use of the Ising machine is promising. However, the table is non-exhaustive and there may be additional applications.

V. AN EXAMPLE OF USING ISING MACHINE ON A PHASOR MEASUREMENT UNIT PLACEMENT PROBLEM

This section provides an example of using an Ising machine for a PMU placement problem to understand the actual performance of the problem on a quantum-inspired Ising machine. We selected the PMU placement problem because it is both applicable and likely to bring about benefits. First, we provide the formulation, followed by a numerical example.

A. FORMULATION

As presented earlier, to use an Ising machine, the formulation requires a QUBO or Ising form of (24a) and (24b), respectively.

As outlined earlier, the Ising and QUBO problems are isomorphic. Because the objective function of the original optimal PMU placement problem is both integer and linear, the objective function as a QUBO problem is,

$$H_{obj}^{QUBO}(x) = \boldsymbol{x}^T \boldsymbol{Q_c} \boldsymbol{x} | \, \boldsymbol{x} \in \{0, 1\}, \tag{25}$$

where $\boldsymbol{Q_c}$ is a diagonal matrix such that $\boldsymbol{Q_c} = diag\{c_1, \ldots, c_N\}$. Using the isomorphic property between QUBO and the Ising problem, H_{obj} is,

$$H_{obj}(\sigma) = \sum_{i \in N} \frac{c_i \sigma_i}{2}. \tag{26}$$

TABLE 4. Models used [38] and [39].

Model	# of Buses	# of Branches
IEEE 9 bus	9	9
IEEE 14 bus	14	20
IEEE 24 bus RTS	24	38
IEEE 30 bus	30	41
IEEE 39 bus	39	46
IEEE 57 bus	57	80
85 bus	85	84
case141.m	141	140
IEEE 145 bus	145	453
Synthetic Illinois 200-bus [39]	200	245
IEEE 300 bus	300	411
Synthetic South Carolina 500-bus [39]	500	597
case1888rte	1888	2531
case1951rte	1951	2596
Synthetic Texas 2000-bus [39]	2000	3206
case2383wp	2383	2896
case2737sop.m	2737	3506
case2746wop.m	2746	3514
case2848rte	2848	3776
case2868rte	2868	3808
case3012wp	3012	3572
case3120sp	3120	3693
case3375wp	3374	4161
case6470rte	6470	9005

The observability of a line requires a PMU placement on either side of the line. In QUBO form, this is,

$$H^{QUBO}_{line,\ l}(x) = \lambda(1 - x_{s,l})(1 - x_{r,l}), \tag{27}$$

that penalizes if no PMUs are placed on either side. λ is a penalty factor.

From observations, (27) reaches a minimum value in the following cases: $[x_{s,l}\ \ x_{r,l}] = [1, 1]$, $[x_{s,l}\ \ x_{r,l}] = [1, 0]$, and $[x_{s,l}\ \ x_{r,l}] = [0, 1]$. In contrast, when $[x_{s,l}\ \ x_{r,l}] = [0, 0]$, $H_{line,\ l} = \lambda$. Therefore, a solved QUBO problem avoids $[x_{s,l}\ \ x_{r,l}] = [0, 0]$. Extending the concept to all lines, the QUBO formulation for the constraint part is as follows:

$$\begin{aligned} H^{QUBO}_{const}(x) &= \sum_{l\in[1,L]} H^{QUBO}_{line,\ l}(x) \\ &= \sum_{l\in[1,L]} \lambda(1 - x_{s,l})(1 - x_{r,l}). \end{aligned} \tag{28}$$

where L the number of lines. Again, using the isomorphism between the QUBO and Ising problems, we express the Ising problem without constants as follows:

$$H_{const}(\sigma) = \frac{\lambda}{4} \sum_{l\in[1,L]} (\sigma_{s,l}\sigma_{r,l} - \sigma_{s,l} - \sigma_{r,l}). \tag{29}$$

In summary, using (26) and (29), the full Ising problem formulation for this PMU placement problem is as follows:

$$H(\sigma) = H_{obj}(\sigma) + H_{const}(\sigma). \tag{30}$$

Note that other formulations based on an integer linear programming problem equivalent are possible; however, for ease of understanding, we utilized the formulation presented.

TABLE 5. Parameters for numerical evaluation.

Method	Parameter	Value
Ising	Software	MA* [17].
	Number of Sweeps	1000
	Number of Repetitions	20
	Initial inverse temperature	0.08
	Final inverse temperature	30
	Random seed	13
	Number of replicated Ising models	100
	λ	100
ILP**	Software	Python 3 CVXOPT GLPK
	CPU	Intel i-7 8700K
	RAM	32.0 GB
Common	$c_i \mid i \in [1, N]$	1
	Wall time [min]	30

*Momentum Annealing
**Integer Linear Programming

B. EVALUATION SETUP

To (a) evaluate the validity of the Ising formulation, (b) understand the potential performance increase that can result from a dedicated Ising machine, and (c) understand the limitation of the approach, we solve the Ising problem to the optimal PMU placement on an Ising machine outlined in [17].

We compared the results of the Ising machine by solving an integer linear program (ILP) using GLPK with a wall time of 30 minutes. We selected ILP with GLPK for comparison because modeling in ILP is commonly known in the study of PMU placement (as in [30]) whereas GLPK is an easily accessible solver for ILP. The wall time was set to allow a comparison between the two methods, as large problems may be unsolvable because of the exponential increase in the complexity of the problem.

To conduct the evaluation, we used network models of various sizes. The network models are all from MATPOWER [38] and some added by [39]. Table 4 lists the models used.

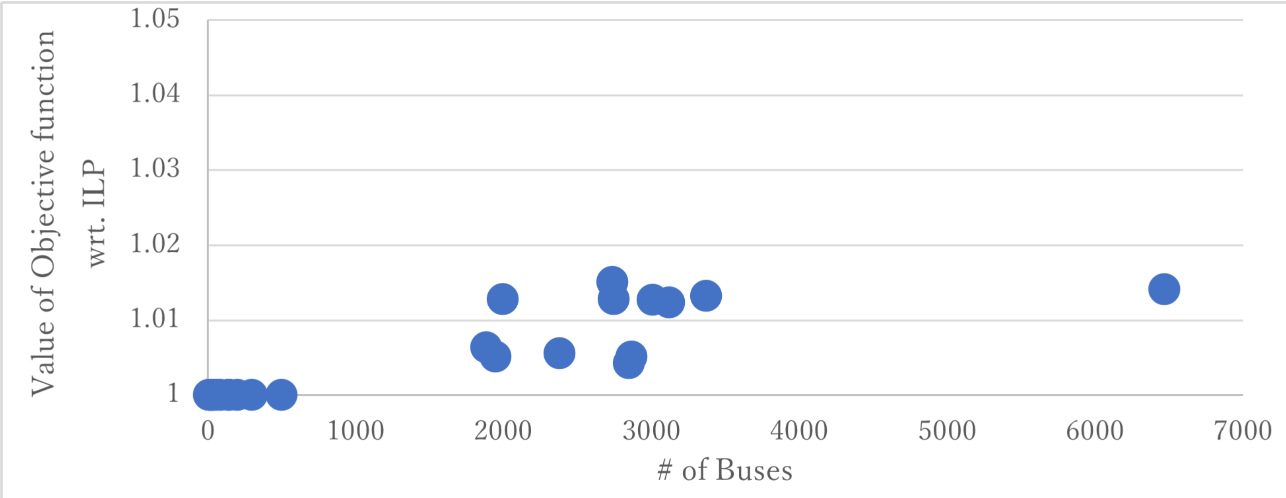
Table 5 overviews the parameters relating to this method. We used the parameters automatically generated via the Ising machine of [17]. The actual parameters, except for λ, are near the values used to evaluate the computational performance in [17].

The cost of installing a PMU is set to 1 ($c_i = 1 | i \in [1, N]$), which implies that there are no differences in installing a PMU between the locations. Furthermore, for the Ising problem, the penalty factor is set to $\lambda = 100$ for all cases. Although not optimal, we did not observe significant performance changes with this parameter; hence, we show the results with $\lambda = 100$.

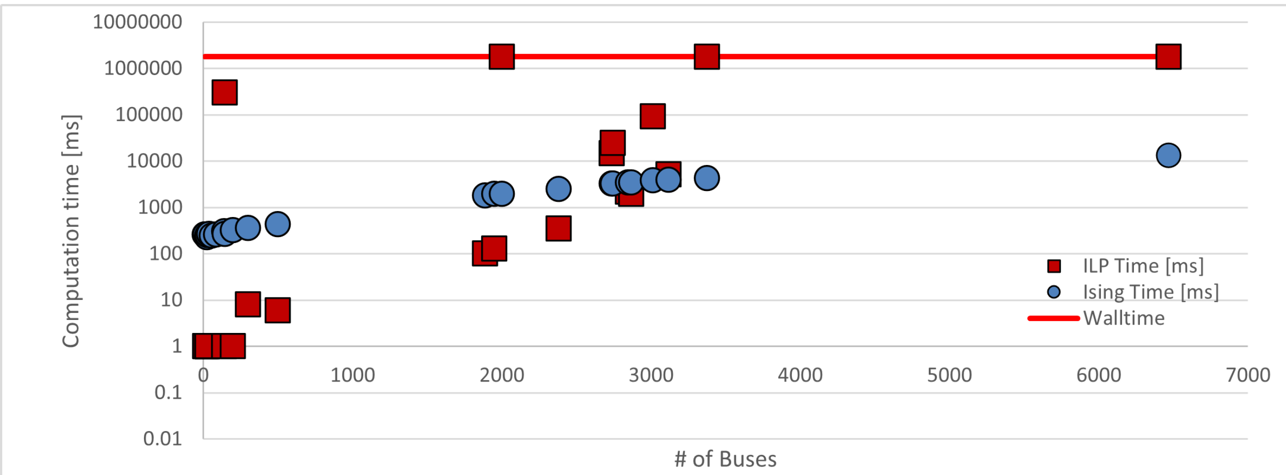
C. DISCUSSION OF RESULTS

Table 6 (solutions values) and Fig. 3 (solution quality and computational times) summarize the results of the PMU placement problem. As shown, the results yielded different observations based on the problem size.

As shown in Fig. 3(a), the solution quality of the Ising machine is comparable to that of a conventional solver. For cases that were unsolvable within the wall time, the suboptimal solution of the ILP was used for comparison. However, this starts to deviate when a larger problem size



(a) Value of objective function with respect to ILPs



(b) Number of buses vs. computational time. The solid line represents the wall time.

FIGURE 3. Summary of example results.

is encountered. The difference arises from the Ising machine being a heuristic approach; there is no process that guarantees optimality. For the PMU placement problem, this has no major disadvantage, as additional PMUs will not hurt the observability of the system.

However, based on Fig. 3 (b), the Ising machine-based approach is far superior in terms of computing time. The quantum-inspired Ising machine exhibited considerable improvements in computation time. The computational improvement of the Ising machine becomes more significant as the problem size increases. For the considered problem type, baseline computing resources, and Ising machine type used, the change occurred around 3000 buses, a medium number of buses when considering actual network sizes. In the considered problem, the number of buses is the same as the number of optimization variables. Therefore, in a more general case, if there are more than 3000 optimization variables, there may be a good case for using the Ising machine over the traditional means of computing. The critical point where a particular Ising machine is advantageous may vary among Ising machines.

A more valuable result may be in larger network sizes, where the ILP-based approach fails to complete the computation. The termination of the ILP solver yielded a sub-optimal solution at the wall time, whereas the Ising machine-based approach was able to yielded an approximate solution in the order of seconds. The solution quality is better with the ILP-based approach; however, waiting for these suboptimal solutions may not always be necessary in most applications. Utilizing the suboptimal solution from the Ising machine as a starting point for a conventional solver (i.e., a hybrid approach) may yield more desirable results with a lower overhead. Expanding on this approach, solving parts of the problem with the Ising machine and another part with a conventional solver may be a comprehensive form of the hybrid approach.

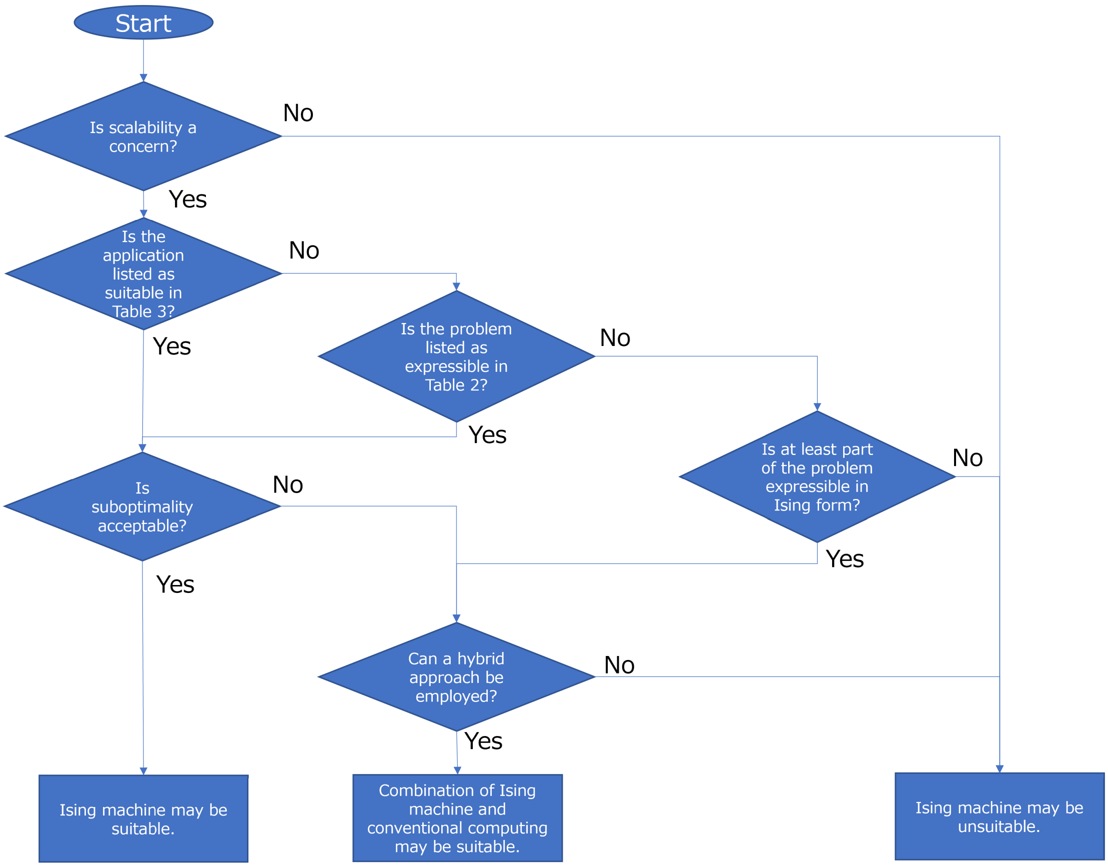


FIGURE 4. Ising machine usage guideline for power system operations.

TABLE 6. Summary of results.

Model name	ILP [#PMUs]	Ising [#PMUs] **
IEEE 9 bus	3	3
IEEE 14 bus	8	8
IEEE 24 bus RTS	13	13
IEEE 30 bus	16	16
IEEE 39 bus	18	18
IEEE 57 bus	30	30
85 bus	36	36
case141.m	62	62
IEEE 145 bus	80	80
Synthetic Illinois 200-bus	76	76
IEEE 300 bus	136	136
Synthetic South Carolina 500-bus	198	198
case1888rte	791	796
case1951rte	786	790
Synthetic Texas 2000-bus	861*	872
case2383wp	1077	1083
case2737sop	1322	1342
case2746wop	1328	1345
case2848rte	1187	1192
case2868rte	1170	1176
case3012wp	1413	1431
case3120sp	1460	1478
case3375wp	1583*	1604
case6470rte	2687*	2725

*Cases that were unsolvable within 30 minutes
**Constraint satisfaction verified

VI. SUMMARY, CONCLUSION, AND FURTHER WORK

This study explored the use of Ising machines in power system operations by first identifying the known characteristics of Ising machines (Section II), identifying the applicability of Ising machines for general optimization problems (Section III), identifying suitable potential applications of Ising machines in power system operations (Section IV), and providing a numerical evaluation of one specific problem (Section V). Generally traversing backward through this study allows us to compile a guideline for Ising machine usage to summarize this study.

Section V revealed that in practice, Ising machines exhibit superior performance relative to a conventional solver if the problem is sufficiently large. Conversely, if the problem is insufficient to cause computational concerns, the use of Ising machines is not beneficial. Hence, the first question to ask is about the scalability need of the application. If this is not an issue, then the Ising machine may be unsuitable.

If the computation time is of concern, it is necessary to check whether the application is theoretically suitable, as shown in Table 3 from Section V. As Table 3 in Section V is non-exhaustive, the optimization problem may still be expressible and scalable, as shown in Table 2. If either of these is true, the user must determine if the suboptimality of the solution, which was theorized in Section II and verified in Section V, is acceptable. If the application satisfies this criterion, the Ising machine may be "practically" suitable. This depends on the user's decision-making process. If the suboptimality is unacceptable, the application may consider a hybrid approach to correct the suboptimality (based on the discussion in Section V). A hybrid approach may also be possible if part of the application is Ising machine suitable.

Fig. 4 summarizes the Ising machine usage guideline for power system operations based on the above points. Future research may benefit from following the guideline. To conclude, we would like to suggest to those with access to Ising computing, both in quantum and non-quantum variations, to explore various possibilities of the model, not limited by the theoretical applications presented in this study.

APPENDIX

A. NOMENCLATURE FOR APPENDIX

T	Time periods considered.
G	Total number of generators.
$C_{u,g}$	Start-up cost.
$C_{d,g}$	Shut-down cost.
p_g^t	Power of generator g at time t.
u, g	Unit status.
y_g^t,z_g^t	Variables for unit start-up and shutdown.
D^t	Demand at t.
UR_g	Upward ramp rate of generator g.
DR_g	Downward ramp rate of generator g.
a_g, b_g, c_g	Cost coefficients for generator g.
$H_{p_g^t}^{QUBO}$	Integer approximated QUBO model for power of generator g at time t.
H_{s-d}^{QUBO}	QUBO model for supply-demand balance.
H_{ramp}^{QUBO}	QUBO model for ramp rate.
H_{UR_g,DR_g}^{QUBO}	Supplementary QUBO model for ramp rate.
H_{unit1}^{QUBO}	QUBO model for unit start-up variables.
H_{unit2}^{QUBO}	QUBO model for start-up and shutdown.
H_{minup}^{QUBO}	QUBO model for minimum up times.
$H_{mindown}^{QUBO}$	QUBO model for minimum down times.
H_{output}^{QUBO}	QUBO model to constraint decomitted generators.

B. UNIT COMMITMENT FORMULATION

Although the main text exemplifies the use of Ising machines for the optimal PMU placement problem, other problems can be tested as indicated in the applicability table. Another example is the unit commitment problem, which is formulated as a mixed integer linear programming problem. An example of the unit commitment (UC) problem in [26] is loosely adapted as follows:

$$
\begin{aligned}
min\ & \sum_{t=1}^{T}\sum_{g=1}^{G} f_g(p_g^t)u_g^t + C_{u,g}y_g^t + C_{d,g}z_g^t \\
s.t.\ & \sum_{g=1}^{G} p_g^t = D^t, \forall t \in 1\ldots T \\
& (p_g^{t+1} - p_g^t) \le UR_g,\ \forall t \in 1\ldots T, g \in 1\ldots G \\
& (p_g^{t+1} - p_g^t) \le DR_g,\ \forall t \in 1\ldots T, g \in 1\ldots G \\
& u_g^t - u_g^{t-1} = y_g^t - z_g^t,\ \forall t \in 2\ldots T, g \in 1\ldots G \\
& y_g^t + \sum_{t=1}^{t+U_{tg}-1} z_g^t \le 1, \forall t \in 2\ldots T, g \in 1\ldots G \\
& z_g^t + \sum_{t=1}^{t+D_{tg}-1} y_g^t \le 1, \forall t \in 2\ldots T, g \in 1\ldots G,
\end{aligned} \tag{31}
$$

which is a mixed integer linear programming problem. Hence, an example formulation is as follows.

1) OBJECTIVE FUNCTION

Minimize the total fuel cost, start up cost, and shut down cost.

$$
\begin{aligned}
H_{obj}^{QUBO} &= \sum_{t=1}^{T}\sum_{g=1}^{G} a_g H_{p_g^t}^{QUBO^2} \\
&+ \sum_{t=1}^{T}\sum_{g=1}^{G} b_g H_{p_g^t}^{QUBO} \\
&+ \sum_{t=1}^{T}\sum_{g=1}^{G} c_g + \sum_{t=1}^{T}\sum_{g=1}^{G} C_{u,g}x_{y_g^t} \\
&+ \sum_{t=1}^{T}\sum_{g=1}^{G} C_{d,g}x_{z_g^t},
\end{aligned} \tag{32}
$$

Note that $H_{p_g^t}^{QUBO}$ is an integer variable approximated with a one-dimensional QUBO model that takes values between $\underline{p_g}$ and $\overline{p}^g$ such that,

$$
\begin{aligned}
H_{p_g^t}^{QUBO} := \underline{p_g} + \sum_{n=1}^{N} n_1 x_n + \sum_{n_2=1}^{N_2} n_2 x_{n_2,2}\ldots \\
+ \sum_{nj=1}^{Nj} n_j x_{n_j,j}.
\end{aligned} \tag{33}
$$

2) CONSTRAINTS

(a) The total power of the generators is always equal to demand.

$$
H_{s-d}^{QUBO} = \lambda \sum_{t=1}^{T}(D^t - \sum_{g=1}^{G}(H_{p_g^t}^{QUBO}))^2 \tag{34}
$$

(b) Generators must operate within their minimum and maximum values(pre-included in the objective).

(c) Sufficient ramping rate in both directions.

$$
H_{ramp}^{QUBO} = \lambda \sum_{g=1}^{G}\sum_{t=1}^{T}(H_{p_g^{t+1}}^{QUBO} - H_{p_g^t}^{QUBO} - H_{UR_g,DR_g}^{QUBO})^2. \tag{35}
$$

Note, H_{UR_g,DR_g}^{QUBO} are integer variables approximated with a one-dimensional QUBO model that takes values between $-DR_g$ and UR_g. Note,

$$
H_{UR,DR}^{QUBO} = -DR_g + \sum_{n=1}^{N} n_1 x_n + \sum_{n_2=1}^{N_2} n_2 x_{n_2,2}\ldots
$$

$$+\sum_{nj=1}^{Nj} n_j x_{n_j,j}. \quad (36)$$

(d) Unit start up variables

$$H_{unit1}^{QUBO} = \sum_{g=1}^{G}\sum_{t=1}^{T} \lambda(x_{u_g^t} - x_{u_g^{t-1}} - x_{y_g^t} + x_{z_g^t})^2. \quad (37)$$

(e) Units must not start up and shut down simultaneously

$$H_{unit2}^{QUBO} = \sum_{g=1}^{G}\sum_{t=1}^{T} \lambda x_{y_g^t} x_{z_g^t}. \quad (38)$$

(f) Minimum up and down times.

$$H_{minup}^{QUBO} = \sum_{g=1}^{G}\sum_{t=2}^{T} \lambda(1 - x_{y_g^t} - \sum_{t=1}^{t+U_{ti}-1} x_{z_g^t})^2. \quad (39)$$

$$H_{mindown}^{QUBO} = \sum_{g=1}^{G}\sum_{t=2}^{T} \lambda(1 - x_{z_g^t} - \sum_{t=1}^{t+U_{ti}-1} x_{y_g^t})^2. \quad (40)$$

(g) Constraint to the main objective: (generators cannot have output when decommitted)

$$H_{output}^{QUBO} = \sum_{g=1}^{G}\sum_{t=1}^{T} \lambda(1 - x_{u_g^t}) H_{p_g^t}^{QUBO}. \quad (41)$$

3) FINAL FORMULATION

Adding all the constraints together yields the full QUBO formulation to UC,

$$\begin{aligned} H^{QUBO} &= H_{obj}^{QUBO} + H_{s-d}^{QUBO} + H_{ramp}^{QUBO} \\ &\quad + H_{unit1}^{QUBO} + H_{unit2}^{QUBO} + H_{minup}^{QUBO} \\ &\quad + H_{mindown}^{QUBO} + H_{output}^{QUBO}, \end{aligned} \quad (42)$$

where the QUBO problem is converted to the Ising problem using the isomorphic property from earlier. Future research should conduct tests similar to those described in the main text.

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