Received 24 May 2023, accepted 19 June 2023, date of publication 26 June 2023, date of current version 29 June 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3289316

RESEARCH ARTICLE

Robust Containment Control of Second-Order Fixed Topology Multi-Agent Systems

YONGXIN BAN AND YONGQIANG YE, (Senior Member, IEEE)
School of Automation, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
Corresponding author: Yongxin Ban (banyongxin@126.com)

ABSTRACT In the study of multi-agent distributed control, the distributed containment control is an important problem, which aims to make all the following agents enter the convex hull formed by the leader agents. It has a wide application prospect in UAV information collection and robust control. In the practical application of distributed containment control, agents usually experience unknown or uncertain working environment and are needed to suppress or even eliminate the adverse effects of external disturbances. In this paper, we study the robust containment control problem for second-order multi-agent systems with fixed topology. Based on the second-order dynamics model, the distributed algorithm is designed, and introduce the model transformation factor, and then transform the model. Based on Lyapunov stability theory, we analyze the stability of multi-agent system by solving differential equations combined with the comparison theorem. Then we minimize the maximum of the output multi-agent system under the external disturbance, and improve the anti-interference performance of the system. Finally, we implement a simulation example to illustrate theoretical results.

INDEX TERMS Containment control, fixed topology, second-order multi-agent systems, $L_2 - L_\infty$ performance index.

I. INTRODUCTION

Distributed consistency is an important research direction in multi-agent systems, which aims to achieving consensus among all agents in a certain state or all states through information exchange between agents. As an important extension, the distributed containment control has important practical applications. Its main goal is to design a distributed control algorithm that makes all follower agents to enter the convex hull formed by the leader agents. In general, there are three types of consensus problems depending on the number of leaders, namely consensus without leader [1], consensus-based tracking with a separated leader [2], and consensus-based containment with multiple leaders [3], [4].

The distributed containment control is an important extension of the consensus problem. In [3] a stop-go control strategy was proposed for a first-order system and a fixed undirected topology, which allows some follower agents to move in an orderly manner into the convex hull formed by the leader agents of mobile robots. In [5] and [6], they studied distributed containment control algorithms for a second-order multi-agent system with a fixed directed topology based on position measurement, and the containment control problem of a first-order multi-agent system with a fixed directed topology based on the sampling data protocol and gain parameter respectively. Over a fixed directed graph, algorithms were given in [6] to achieve containment control for multi-agent systems with integrator dynamics. For the higher-order multi-agent systems with a directed graph, an observer-type dynamical output containment control protocol was developed in [7]. In [8] the adaptive containment control was investigated for a class of fractional-order multi-agent systems with time-varying parameters and disturbances; and in [9] they studied the asymptotic stability of the containment control problem of high-order time-delayed linear stationary systems with switching topologies and provided necessary and sufficient conditions for the implement of containment control. In [10] an observer was designed to estimate the relative state of the agents and in [11] a double-state Markov model was proposed for the first-order discrete multi-agent

The associate editor coordinating the review of this manuscript and approving it for publication was Dongxiao Yu.

containment control problem, which used convexity and LaSalle invariance principle to analyze the specific process of implementing containment control. In [12] they studied the containment control problem of a class of follower agents converging to the convex hull formed by the leader agent's symmetric motion trajectories.

Although researchers have systematically studied the containment control problem, further research is needed in case that external disturbances and parameter uncertainties exist. Moreover, there are very few studies on the robust containment control problems. In [13], [14], and [15] they studied the second-order and high-order distributed robust H_∞ consensus problem for the containment control problem, and [13] provided sufficient conditions for all agents to achieve consensus in a first-order multi-agent system under expected robust H_∞ performance. In [16] and [17] the robust H_∞ consensus control was studied for multi-agent systems with time delays or switching topologies. And the robust containment control problem was considered in [18] for a group of agents with linear dynamics over Markovian switching communication topologies.

In [19], a finite-time sliding-mode estimator was proposed to obtain accurate estimation of the weighted average of the accelerations, velocities, and positions of the leaders.Although there have been significant progresses in the field of containment control and robust control, however external disturbances always exist in practical engineering applications. Thus to make the research on containment control of multi-agent systems more practical, it is necessary to use the $L_2 - L_\infty$ robust control method to study the performance changes under the influence of external disturbance factors, and improve the robustness of the containment control. In [19], a finite-time sliding-mode estimator was proposed to obtain accurate estimation of the weighted average of the accelerations, velocities, and positions of the leaders. But the designed algorithm cannot be directly applied in robust $L_2 - L_\infty$ containment control. In [20] they formulated and solved the asynchronous tracking control problem of multi-agent systems with input uncertainties on switching signed digraphs. In [21] the containment control issue of discrete-time general linear MASs was proposed and solved under the asynchronous setting, in which the network topology can be arbitrary. However, these two articles above both studied the tracking control of asynchronous multiple agents on the basis of the discrete control system research, which only proposed the corresponding discrete control algorithm, but did not investigate the system. Compared with these results, this paper mainly studies the continuous control system, proposes the corresponding control algorithm, and focuses on the suppression of the disturbance of the system.

Differ from [19], [20], and [21], in this paper the containment control of multi-agent systems is further studied, and the changes of system performance are analyzed under the influence of external disturbances and uncertainty factors. The paper focuses on the robust $L_2 - L_\infty$ containment control problem for a second-order system with fixed topology. The valuable contributions of this paper are as follows:

1) Based on existing robust control methods, this paper studies the impact of external disturbances on the system and designs a distributed control algorithm. The negative velocity feedback, the information interaction term between agents, and the projection distance term are all considered in the algorithm.
2) A model transformation factor is introduced to transform the original model. The nonlinear and coupling terms are eliminated by transforming the model, which facilitates the analysis of the system stability.
3) Based on the Lyapunov stability method and robust control theory, the motion trajectory trend between agents is analyzed. As shown by numerical simulations, the maximum of the external disturbances output is minimized for the multi-agent system. Thus designing suitable distributed control algorithm can make all follower agents enter the convex hull formed by the leader agent cluster and satisfy the $L_2 - L_\infty$ performance index.

II. MODEL AND PROBLEM STATEMENT

The multi-agent system studied in this paper is composed by $n + m$ second-order continuous-time multi-agents, i.e., n follower agents together with m leader agents. Each agent is represented by a node in the graph G, and $N_i = \{j \in V(G)|(j, i) \in E(G)\}$ is denoted as the neighbor set of the node i in G, i.e., all the agents where the i-th agent can receive information from. All leader agents are assumed static, and each agent updates its current states based on the information it receives from its neighbor agents denoted by the neighbor set described above. The follower and leader agent clusters are denoted by $F = \{1, 2, \cdots, n\}$ and $L = \{n + 1, n + 2, \cdots, n + m\}$. Then all follower agents and leader agents can be expressed by $f_1, f_2, \cdots, f_n$ and $l_{n+1}, l_{n+2}, \cdots, l_{n+m}$ respectively.

Consider the dynamics model of each second-order agent in the following:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t) + w_i(t), \end{aligned} \tag{1}$$

where the follower agent $i \in F = \{1, 2, \cdots, n\}$, and $x_i(t), v_i(t), u_i(t)$ are denoted as the position state, velocity state and control input of the i-th agent at time t respectively. Moreover, $w_i(t)$ is denoted as the external perturbation corresponding to the i-th agent at time t, where $w_i(t) \in L_2[0, \infty)$.

Given a convex hull $Y \subseteq \mathbf{R}^n$ as the linear combination of the position states $x_{l_{n+1}}, x_{l_{n+2}}, \cdots, x_{l_{n+m}}$ of leader agents, which can be expressed as below:

$$Y = \left\{ \sum_{i=n+1}^{n+m} \alpha_i x_{l_i} \middle| \alpha_i \geq 0 \cup \sum_{i=n+1}^{n+m} \alpha_i = 1 \right\}. \tag{2}$$

Construct the following output function $z(t)$:

$$z(t) = c_1 \max_i \left|\left|\rho_i(t) - P_Y(\rho_i(t))\right|\right|, \tag{3}$$

which is a nonlinear expression and can be used to describe the maximal distance from the position or velocity status of the i-th agent in the multi-agent system to the nonempty closed convex set Y. Here c_1 is a given positive number and $P_Y(\rho)$ is the minimum projection of the vector ρ on the nonempty closed convex set Y, i.e.,

$$P_Y(\rho) = \arg\min_{y\in Y} \left|\left|\rho - y\right|\right|.$$

While all follower agents enter the convex hull $Y \subseteq \mathbf{R}^n$ formed by the leader agent cluster, the multi-agent system need to satisfy the following $L_2 - L_\infty$ performance indices:

$$\left|\left|T_{zw}(t)\right|\right|_{L_2-L_\infty} = \sup_{0\neq w(t)\in L_2[0,\infty)} \frac{||z(t)||_\infty}{||w(t)||_2} < \gamma, \tag{4}$$

where γ is a given positive number, $||z(t)||_\infty$ and $||w(t)||_2 = \left(\int_0^\infty |w(t)|^2\right)^{1/2}$ are the supremum of the input function $z(t)$ and the Euclidean norm of the vector $w(t)$ respectively.

Our target in this paper is to design a distributed algorithm based on information interaction between agents such that the agents enter into the given convex hull $Y \subseteq \mathbf{R}^n$ in both position and velocity states, moreover this realizes the containment control satisfying the $L_2 - L_\infty$ performance indices above. Thus the distributed algorithm designed in this paper is in the following:

$$\begin{aligned} u_i(t) = -p_i v_i(t) &+ \sum_{j\in N_i(t)} a_{ij}(t)(x_j(t) - x_i(t)) \\ &- b_i(t)[x_i(t) - P_Y(x_i(t))], \end{aligned} \tag{5}$$

where the agent $i \in F = \{1, \cdots, n\}$, $a_{ij}(t)$ denotes the weight between the agent i and its neighbor agent $j \in N_i(t)$, and $b_i(t)$ denotes the projection weight between the follower agent i and the target area.

For the convenience of the following analysis, the model transformation factor is introduced to transform the original system. The detailed form of the model transformation factor is given in the following:

$$\overline{v}_i(t) = x_i(t) + \frac{v_i(t)}{k_i}, \tag{6}$$

where $k_i = b_i + d_i$ and d_i denotes the maximum of $\sum_{j\in F} a_{ij}(t)$ which represents all possible weights of the agent i with respect to other agent j. Thus the multi-agent system (1) can be transformed to the following:

$$\begin{aligned} \dot{x}_i(t) &= k_i\overline{v}_i(t) - k_i x_i(t), \\ \dot{\overline{v}}_i(t) &= (k_i - p_i)\overline{v}_i(t) \\ &\quad + \left(-k_i + p_i - \sum_{j\in N_i(t)} \frac{a_{ij}(t)}{k_i} - \frac{b_i(t)}{k_i}\right) x_i(t) \\ &\quad + \sum_{j\in N_i(t)} \frac{a_{ij}(t)}{k_i} x_j(t) + \frac{b_i(t)}{k_i} P_Y(x_i(t)) + \frac{w_i(t)}{k_i}. \end{aligned} \tag{7}$$

Thus in the following analysis, we will move to the distributed algorithm (5) which makes the system (7) enter the convex hull formed by the leader agent cluster and realizes the containment control, moreover we will discuss whether the system satisfies the $L_2 - L_\infty$ performance indices.

Set the proportional factor $q_i(t) = \frac{P_Y(x_i(t))}{x_i(t)}$ then $q_i(t) \in [0, 1]$. Then the system (7) can be transformed to the following form:

$$\begin{aligned} \dot{x}_i(t) &= k_i\overline{v}_i(t) - k_i x_i(t), \\ \dot{\overline{v}}_i(t) &= (k_i - p_i)\overline{v}_i(t) \\ &\quad + \left(-k_i + p_i - \frac{b_i(t)}{k_i} + \frac{b_i(t)}{k_i} q_i(t)\right) x_i(t) \\ &\quad + \sum_{j\in N_i(t)} \frac{a_{ij}(t)}{k_i}(x_j(t) - x_i(t)) + \frac{w_i(t)}{k_i} \end{aligned} \tag{8}$$

Set

$$\lambda(t) = \begin{pmatrix} x_1(t) \\ \vdots \\ x_n(t) \\ \overline{v}_1(t) \\ \vdots \\ \overline{v}_n(t) \end{pmatrix}, w(t) = \begin{pmatrix} w_1(t) \\ \vdots \\ w_n(t) \end{pmatrix}$$

For the convenience, we define following invariants:

$$\begin{aligned} \Lambda_1 &= diag(k_1, \cdots, k_n), \\ \Lambda_2 &= diag(k_1 - p_1, \cdots, k_n - p_n), \\ \Lambda_3 &= diag(-\frac{b_1(t)}{k_1} + \frac{b_1(t)}{k_1} q_1(t), \cdots, -\frac{b_n(t)}{k_n} + \frac{b_n(t)}{k_n} q_n(t)), \\ \Lambda_4 &= diag(1/k_1, \cdots, 1/k_n). \end{aligned} \tag{9}$$

By the invariants defined above, the system (8) can be transformed to the following form:

$$\dot{\lambda}(t) = \begin{bmatrix} -\Lambda_1 & \Lambda_1 \\ \Lambda_3 - L\Lambda_4 - \Lambda_2 & \Lambda_2 \end{bmatrix} \lambda(t) + \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_4 \end{bmatrix} \begin{bmatrix} 0 \\ w(t) \end{bmatrix}. \tag{10}$$

Recall that the proportional factor $q_i(t) = \frac{P_Y(x_i(t))}{x_i(t)}$ which implies that $x_i(t) - P_Y(x_i(t)) = (1 - q_i(t))x_i(t)$, combined with the general form (3) of the output function, in fact we can see that the output function $z(t)$ can be expressed as the following:

$$z(t) = \begin{bmatrix} c_{1x}I_n & 0 \\ 0 & c_{1v}I_n \end{bmatrix} \begin{bmatrix} I_n - Q(t) & 0 \\ 0 & I_n \end{bmatrix} \lambda(t), \tag{11}$$

where $\begin{bmatrix} c_{1x}I_n & 0 \\ 0 & c_{1v}I_n \end{bmatrix} \in \mathbf{R}^{2n\times 2n}$ with positive constants c_{1x}, c_{1v} and $Q(t) = diag(q_1(t), \cdots, q_n(t))$.

Therefore, after realizing the containment control, we will discuss whether the algorithm (5) designed above can make the system (11) satisfy the $L_2 - L_\infty$ performance indices.

III. MAIN RESULTS

Assumption 1: There exists a positive constant p_i such that it holds that $p_i k_i \geq k_i^2 + \sum_{j \in N_i(t)} a_{ij}(t) + b_i(t)$ for all agents i and time $t \geq 0$.

Assumption 2: In the fixed topological set G, each follower agent i_f can always receive information from each leader agent i_l.

Lemma 1: ([22]) There exists a nonempty closed convex set $Y \subset \mathbf{R}^m$ such that for any vector set $K_i \subset \mathbf{R}^m, i \in \{1, \cdots, n\}$, if $\sum_{i=1}^{n} a_i = 1$ and $a_i \in [0, 1)$, it holds that

$$\Big|\Big|\sum_{i=1}^{n} a_i K_i - P_Y\big(\sum_{i=1}^{n} a_i K_i\big)\Big|\Big| \leq \sum_{i=1}^{n} a_i \Big|\Big|K_i - P_Y(K_i)\Big|\Big|.$$

Lemma 2: ([23]) Given the symmetric matrix $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$, where $X_{11} \in \mathbf{R}^{p\times p}, X_{12} = X_{21}^T \in \mathbf{R}^{p\times(q-p)}, X_{22} \in \mathbf{R}^{(q-p)\times(q-p)}$. The following three situations are equivalent:

$$\begin{aligned} &(a)\ X < 0; \\ &(b)\ X_{11} < 0, X_{22} - X_{12}^T X_{11}^{-1} X_{12} < 0; \\ &(c)\ X_{22} < 0, X_{11} - X_{12} X_{22}^{-1} X_{12}^T < 0. \end{aligned} \tag{12}$$

Note: For a symmetric matrix $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$, when X_{22} is an invertible matrix, the Schur complement of X_{22} in the symmetric matrix X is $X_{11} - X_{12} X_{22}^{-1} X_{21}$. Since it holds that $X_{12} = X_{21}^T$ as the matrix X is symmetric, the Schur complement of X_{22} in the symmetric matrix X can be reduced to $X_{11} - X_{12} X_{22}^{-1} X_{12}^T$. If X_{11} is a invertible matrix, then the Schur complement of X_{11} in the symmetric matrix X is $X_{22} - X_{12}^T X_{11}^{-1} X_{12}$.

Theorem 1: In the fixed topological graph G, by designing suitable control algorithm (5) to the dynamics equation (1) of the agent, if there exists a positive definite matrix M satisfying that

$$H = \begin{bmatrix} H_{\lambda\lambda} & H_{\lambda w} \\ H_{w\lambda} & -\gamma I_n \end{bmatrix} < 0,$$

$$\begin{bmatrix} c_{1x} I_n & 0 \\ 0 & c_{1v} I_n \end{bmatrix}^T \begin{bmatrix} c_{1x} I_n & 0 \\ 0 & c_{1v} I_n \end{bmatrix} < \gamma M,$$

where

$$\begin{aligned} H_{\lambda\lambda} =& \begin{bmatrix} -\Lambda_1 & \Lambda_1 \\ \Lambda_3 - L\Lambda_4 - \Lambda_2 & \Lambda_2 \end{bmatrix}^T M \\ &+ M \begin{bmatrix} -\Lambda_1 & \Lambda_1 \\ \Lambda_3 - L\Lambda_4 - \Lambda_2 & \Lambda_2 \end{bmatrix}, \\ H_{\lambda w} =& Q \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_4 \end{bmatrix}, H_{w\lambda} = H_{\lambda w}^T, \end{aligned}$$

then all follower agents can enter the closed convex hull Y formed by all leader agents and satisfy the $L_2 - L_\infty$ performance indices

$$\Big|\Big|T_{zw}(t)\Big|\Big|_{L_2 - L_\infty} = \sup_{0 \neq w(t) \in L_2[0,\infty)} \frac{||z(t)||_\infty}{||w(t)||_2} < \gamma,$$

where c_{1x}, c_{1v} are positive constants and $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4$ depend on the coefficients of the control algorithm (5).

Proof: We will show the proof of this theorem in two different situations that $w_i(t) = 0$ and $w_i(t) \neq 0$. The details are in the following:

(a)For the system model (1), we set $w_i(t) = 0$ and construct the Lyapunov function as following:

$$V_1(t) = \max_k \Big|\Big|\xi_k(t) - P_Y(\xi_k(t))\Big|\Big|, \tag{13}$$

where

$$\xi(t) = \begin{pmatrix} x_1(t) \\ \bar{v}_1(t) \\ \vdots \\ x_n(t) \\ \bar{v}_n(t) \end{pmatrix}, k \in \{1, 2, \cdots, 2n\}.$$

By this definition the Lyapunov function $V_1(t) \geq 0$.

By the system (7) after the model transformation and the property of the derivatives, it holds that

$$\dot{x}_i(t) = \lim_{\Delta t \to 0} \frac{x_i(t + \Delta t) - x_i(t)}{\Delta t} = k_i \bar{v}_i(t) - k_i x_i(t), \tag{14}$$

i.e., $\lim_{\Delta t \to 0} x_i(t+\Delta t) = \lim_{\Delta t \to 0} (1 - k_i \Delta t) x_i(t) + \lim_{\Delta t \to 0} (k_i \Delta t) \bar{v}_i(t)$, where $\Delta t = t^+ - t$ is a sufficiently small time interval which tends to 0. As $k_i = b_i + d_i$ is positive where d_i denotes the maximum of $\sum_{j \in F} a_{ij}(t)$ which represents all possible weights of the agent i with respect to other agent j thus $k_i \Delta_t \in (0, 1)$. By (14) combined with Lemma 1 we can derive the position state properties of the system as following:

$$\begin{aligned} &\Big|\Big|x_i(t + \Delta t) - P_Y(x_i(t + \Delta t))\Big|\Big| \\ &\quad \leq (1 - k_i \Delta_t)\Big|\Big|x_i(t) - P_Y(x_i(t))\Big|\Big| \\ &\quad + (k_i \Delta_t)\Big|\Big|\bar{v}_i(t) - P_Y(\bar{v}_i(t))\Big|\Big| \end{aligned} \tag{15}$$

Combined with the definition of the derivative, we have

$$\begin{aligned} &\lim_{\Delta t \to 0} \frac{\Big|\Big|x_i(t + \Delta t) - P_Y(x_i(t + \Delta t))\Big|\Big| - \Big|\Big|x_i(t) - P_Y(x_i(t))\Big|\Big|}{\Delta t} \\ &\leq -k_i\Big|\Big|x_i(t) - P_Y(x_i(t))\Big|\Big| + k_i\Big|\Big|\bar{v}_i(t) - P_Y(\bar{v}_i(t))\Big|\Big| \end{aligned} \tag{16}$$

Again by the system (7) after the model transformation and the property of the derivatives, it holds that

$$\begin{aligned} &\lim_{\Delta t \to 0} \frac{\bar{v}_i(t + \Delta t) - \bar{v}_i(t)}{\Delta t} \\ &=(k_i - p_i) + \left(-k_i + p_i - \sum_{j \in N_i(t)} \frac{a_{ij}(t)}{k_i} - \frac{b_i(t)}{k_i}\right) x_i(t) \\ &\quad + \sum_{j \in N_i(t)} \frac{a_{ij}(t)}{k_i} x_j(t) + \frac{b_i(t)}{k_i} P_Y(x_i(t)) + 0. \end{aligned} \tag{17}$$

It immediately follows that

$$\lim_{\Delta t\to 0}\bar{v}_i(t+\Delta t)=(1+(k_i-p_i)\Delta t)\bar{v}_i(t)$$
$$+\left(-k_i+p_i-\sum_{j\in N_i(t)}\frac{a_{ij}(t)}{k_i}-\frac{b_i(t)}{k_i}\right)x_i(t)\Delta t$$
$$+\sum_{j\in N_i(t)}\frac{a_{ij}(t)}{k_i}x_j(t)\Delta t+\frac{b_i(t)}{k_i}P_Y(x_i(t))\Delta t. \tag{18}$$

As Δt is a sufficiently small time interval, note that $k_i > 0$, $\sum_{j\in N_i(t)} a_{ij}(t) > 0, b_i(t) > 0$ we can set the following notations and derive that

$$\alpha_{1i}=1+(k_i-p_i)\Delta t>0,$$
$$\alpha_{2i}=\left(-k_i+p_i-\sum_{j\in N_i(t)}\frac{a_{ij}(t)}{k_i}-\frac{b_i(t)}{k_i}\right)\Delta t>0,$$
$$\alpha_{3ij}=\frac{a_{ij}(t)}{k_i}\Delta t>0,\ \alpha_{4i}=\frac{b_i(t)}{k_i}\Delta t>0. \tag{19}$$

The Assumption 1 gives that $p_ik_i\geq k_i^2+\sum_{j\in N_i(t)}a_{ij}(t)+b_i(t)$ and $k_i=b_i+d_i$ is a positive number thus $k_i<p_i-1$ where d_i denotes as above. Thus the terms in (19) satisfy that

$$-k_i+p_i-\sum_{j\in N_i(t)}\frac{a_{ij}(t)}{k_i}-\frac{b_i(t)}{k_i}>0,$$
$$k_i<p_i,\ \sum_{j\in N_i(t)}\frac{a_{ij}(t)}{k_i}<1,\ \frac{b_i(t)}{k_i}<1. \tag{20}$$

Then it follows that

$$\alpha_{1i}=1+(k_i-p_i)\Delta t\in(0,1],$$
$$\alpha_{2i}=\left(-k_i+p_i-\sum_{j\in N_i(t)}\frac{a_{ij}(t)}{k_i}-\frac{b_i(t)}{k_i}\right)\Delta t\in(0,1],$$
$$\alpha_{3ij}=\frac{a_{ij}(t)}{k_i}\Delta t\in(0,1],\ \alpha_{4i}=\frac{b_i(t)}{k_i}\Delta t\in(0,1]. \tag{21}$$

Moreover we observe in (21) that

$$\alpha_{1i}+\alpha_{2i}+\sum_{j\in N_i(t)}\alpha_{3ij}+\alpha_{4i}=1. \tag{22}$$

As $P_Y(x_i(t))\in Y$, by the property of the projection operator that the first and second projections of the vector $x_i(t)$ on the nonempty closed convex set Y are the same, i.e., $P_Y(x_i(t))=P_Y(P_Y(x_i(t)))$, according to (17)(20)(21) and combined with Lemma 1, we can derive the velocity state property of the system as below:

$$\left|\left|\bar{v}_i(t+\Delta t)-P_Y\left(\bar{v}_i(t+\Delta t)\right)\right|\right|$$
$$\leq\alpha_{1i}\left|\left|\bar{v}_i(t)-P_Y\left(\bar{v}_i(t)\right)\right|\right|+\alpha_{2i}\left|\left|x_i(t)-P_Y(x_i(t))\right|\right|$$
$$+\sum_{j\in N_i(t)}\alpha_{3ij}\left|\left|x_j(t)-P_Y(x_j(t))\right|\right|$$
$$+\alpha_{4i}\left|\left|P_Y(x_i(t))-P_Y(P_Y(x_i(t)))\right|\right|$$
$$=\alpha_{1i}\left|\left|\bar{v}_i(t)-P_Y\left(\bar{v}_i(t)\right)\right|\right|+\alpha_{2i}\left|\left|x_i(t)-P_Y(x_i(t))\right|\right|$$
$$+\sum_{j\in N_i(t)}\alpha_{3ij}\left|\left|x_j(t)-P_Y(x_j(t))\right|\right|. \tag{23}$$

Based on the properties of the position state (15) and the velocity state (23) we have that

$$\left|\left|\xi_k(t+\Delta t)-P_Y(\xi_k(t+\Delta t))\right|\right|\leq\max_k\left|\left|\xi_k(t)-P_Y(\xi_k(t))\right|\right|, \tag{24}$$

which implies that $V_1(t+\Delta t)\leq V_1(t)$ thus the Lyapunov function $V_1(t)$ is non-increasing with respect to time t. Therefore on the time interval $[t_0,t_1)$ it holds that $0\leq V_1(t)\leq V_1(t_0)$ thus $V_1(t)$ is bounded.

On the other hand, it follows from (23) and the definition of derivatives that

$$\lim_{\Delta t\to 0}\frac{1}{\Delta t}\Big(\left|\left|\bar{v}_i(t+\Delta t)-P_Y\left(\bar{v}_i(t+\Delta t)\right)\right|\right|$$
$$-\left|\left|\bar{v}_i(t)-P_Y\left(\bar{v}_i(t)\right)\right|\right|\Big)$$
$$\leq\beta_{1i}\left|\left|\bar{v}_i(t)-P_Y\left(\bar{v}_i(t)\right)\right|\right|+\beta_{2i}\left|\left|x_i(t)-P_Y(x_i(t))\right|\right|$$
$$+\sum_{j\in N_i(t)}\beta_{3ij}\left|\left|x_j(t)-P_Y(x_j(t))\right|\right|, \tag{25}$$

where

$$\beta_{1i}=k_i-p_i,\ \beta_{2i}=\left(-k_i+p_i-\sum_{j\in N_i(t)}\frac{a_{ij}(t)}{k_i}-\frac{b_i(t)}{k_i}\right),$$
$$\beta_{3ij}=\frac{a_{ij}(t)}{k_i},$$

which satisfies that $\beta_{1i}+\beta_{2i}+\sum_{j\in N_i(t)}\beta_{3ij}=-\frac{b_i(t)}{k_i}$. By Assumption 2, each follow agent i_f can always receive information from the leader agent i_l, then the coefficient $b_i(t)=b_i\neq 0$. Combined this with (16) (25), the convergence of the multi-agent system can be analyzed. It follows from (25) and Lemma 1 that

$$\frac{d\left|\left|\bar{v}_{i_f}(t)-P_Y\left(\bar{v}_{i_f}(t)\right)\right|\right|}{dt}$$
$$\leq\beta_{1i_f}\left|\left|\bar{v}_{i_f}(t)-P_Y\left(\bar{v}_{i_f}(t)\right)\right|\right|+(-\beta_{1i_f}-\frac{b_{i_f}}{k_{i_f}})V_1(t)$$
$$\leq(k_{i_f}-p_{i_f})\left|\left|\bar{v}_{i_f}(t)-P_Y\left(\bar{v}_{i_f}(t)\right)\right|\right|$$
$$+(-k_{i_f}+p_{i_f}-\frac{b_{i_f}}{k_{i_f}})V_1(t_0), \tag{26}$$

where the last inequality follows from the fact that $0 \leq V_1(t) \leq V_1(t_0)$ for $t \in [t_0, t_1)$. By the solution to the differential equation, combined with the comparison theorem, we can solve (26) to derive that

$$\begin{aligned}
&\left|\left|\overline{v}_{i_f}(t) - P_Y\left(\overline{v}_{i_f}(t)\right)\right|\right| \\
&\leq e^{(k_{i_f} - p_{i_f})(t-t_0)}\left|\left|\overline{v}_{i_f}(t_0) - P_Y\left(\overline{v}_{i_f}(t_0)\right)\right|\right| \\
&+ \frac{1 - e^{(k_{i_f} - p_{i_f})(t-t_0)}}{p_{i_f} - k_{i_f}}(-k_{i_f} + p_{i_f} - \frac{b_{i_f}}{k_{i_f}})V_1(t_0) \\
&\leq \left(1 - \frac{1 - e^{(k_{i_f} - p_{i_f})(t-t_0)}}{p_{i_f} - k_{i_f}}\frac{b_{i_f}}{k_{i_f}}\right)V_1(t_0). \qquad (27)
\end{aligned}$$

Thus it follows that for any time $t \in [\frac{t_0+t_1}{2}, t_1)$ it holds that

$$\left|\left|\overline{v}_{i_f}(t) - P_Y\left(\overline{v}_{i_f}(t)\right)\right|\right| \leq \zeta_1 V_1(t_0), \qquad (28)$$

where $\zeta_1 = 1 - \frac{1-e^{(k_{i_f} - p_{i_f})(t-t_0)}}{p_{i_f} - k_{i_f}}\frac{b_{i_f}}{k_{i_f}}$. It follows from Assumption 1 that $p_{i_f} - k_{i_f} \geq \frac{b_{i_f}}{k_{i_f}}$ thus we can easily have that $\zeta_1 \in (0, 1)$. Similarly by the position state (16) we have

$$\begin{aligned}
&\frac{d\left|\left|x_{i_f}(t) - P_Y(x_{i_f}(t))\right|\right|}{dt} \\
&\leq -k_{i_f}\left|\left|x_{i_f}(t) - P_Y(x_{i_f}(t))\right|\right| + k_{i_f}\left|\left|\overline{v}_{i_f}(t) - P_Y(\overline{v}_{i_f}(t))\right|\right| \\
&\leq -k_{i_f}\left|\left|x_{i_f}(t) - P_Y(x_{i_f}(t))\right|\right| + k_{i_f}\zeta_1\left|\left|V_1(t_0)\right|\right|. \qquad (29)
\end{aligned}$$

Then for time $t \in [\frac{t_0+t_1}{2}, t_1)$, by solving (29), combined with comparison theorem, the following inequality can be easily derived:

$$\begin{aligned}
&\left|\left|x_{i_f}(t) - P_Y(x_{i_f}(t))\right|\right| \\
&\leq e^{-k_{i_f}(t-\frac{t_0+t_1}{2})}\left|\left|x_{i_f}(\frac{t_0+t_1}{2}) - P_Y(x_{i_f}(\frac{t_0+t_1}{2}))\right|\right| \\
&+ \zeta_1(1 - e^{-k_{i_f}(t-\frac{t_0+t_1}{2})})V_1(t_0) \\
&\leq (\zeta_1 + (1-\zeta_1)e^{-k_{i_f}(t-\frac{t_0+t_1}{2})})V_1(t_0). \qquad (30)
\end{aligned}$$

As k_{i_f} is positive, let us set $\zeta_2 = \zeta_1 + (1-\zeta_1)e^{-k_{i_f}(t-\frac{t_0+t_1}{2})}$ thus we have $0 < \zeta_2 < \zeta_1 + (1-\zeta_1) = 1$ for $t \in [\frac{t_0+t_1}{2}, t_1)$. Then on this time interval we have that

$$\left|\left|x_{i_f}(t) - P_Y\left(x_{i_f}(t)\right)\right|\right| \leq \zeta_2 V_1(t_0). \qquad (31)$$

Combine (24) (28) (31), it can be derived that for time $t \in [t_0, t_1)$, when the follower agent i_f can receive information from all leaders i_l, the follow agent i_f will converge to the convex hull Y.

(b)When $w_i(t) \neq 0$, let us analyze the performance indices of the multi-agent system to find conditions satisfying the $L_2 - L_\infty$ performance indices. Set a positive definite matrix M and construct the Lyapunov function as following:

$$V_2(t) = \lambda(t)^T M\lambda(t), \qquad (32)$$

where $\lambda(t)$ and its derivative were defined in (10). □

Combined with systems (10) (11), take the derivative of the Lyapunov function $V_2(t)$ in (32), it follows that:

$$\begin{aligned}
\dot{V}_2(t) &= \dot{\lambda}(t)^T M\lambda(t) + \lambda(t)^T M\dot{\lambda}(t) \\
&= \left[\begin{pmatrix} -\Lambda_1 & \Lambda_1 \\ \Lambda_3 - L\Lambda_4 - \Lambda_2 & \Lambda_2 \end{pmatrix}\lambda(t)\right. \\
&\left.+ \begin{pmatrix} 0 & 0 \\ 0 & \Lambda_4 \end{pmatrix}\begin{pmatrix} 0 \\ w(t) \end{pmatrix}\right]^T M\lambda(t) \\
&+ \lambda(t)^T M\left[\begin{pmatrix} -\Lambda_1 & \Lambda_1 \\ \Lambda_3 - L\Lambda_4 - \Lambda_2 & \Lambda_2 \end{pmatrix}\lambda(t)\right. \\
&\left.+ \begin{pmatrix} 0 & 0 \\ 0 & \Lambda_4 \end{pmatrix}\begin{pmatrix} 0 \\ w(t) \end{pmatrix}\right] \\
&= \lambda(t)^T\left[\begin{pmatrix} -\Lambda_1 & \Lambda_1 \\ \Lambda_3 - L\Lambda_4 - \Lambda_2 & \Lambda_2 \end{pmatrix}^T M\right. \\
&\left.+ M\begin{pmatrix} -\Lambda_1 & \Lambda_1 \\ \Lambda_3 - L\Lambda_4 - \Lambda_2 & \Lambda_2 \end{pmatrix}\right]\lambda(t) \\
&+ \begin{pmatrix} 0 & w(t)^T \end{pmatrix}\begin{bmatrix} 0 & 0 \\ 0 & \Lambda_4 \end{bmatrix}^T M\lambda(t) \\
&+ \lambda(t)^T M\begin{bmatrix} 0 & 0 \\ 0 & \Lambda_4 \end{bmatrix}\begin{pmatrix} 0 \\ w(t) \end{pmatrix} \qquad (33)
\end{aligned}$$

Based on the equality (33), by use of the $L_2 - L_\infty$ performance indices, the performance index function $J(w)$ can be constructed to analyze the stability of the multi-agent system. Now construct the performance index function $J(w)$ which is used to indicate the energy of the system as the following:

$$J(w) = V_2(t) - \gamma\int_0^t w(s)^T w(s)ds, \qquad (34)$$

where the external turbulence $w(t) = [w_1(t), \cdots, w_n(t)]^T \in L_2[0, \infty)$. Due to the zero initial condition, we have that $V_2(0) = 0$. Thus combined with (33), the function $J(w)$ in (34) can be transformed to the following form:

$$\begin{aligned}
J(w) &= V_2(t) - V_2(0) - \gamma\int_0^t w(s)^T w(s)ds \\
&= \int_0^t [\dot{V}_2(s) - \gamma w(s)^T w(s)]ds \\
&= \int_0^t [\lambda(s)^T, w(s)^T]H\begin{bmatrix} \lambda(s) \\ w(s) \end{bmatrix}ds, \qquad (35)
\end{aligned}$$

where $H = \begin{bmatrix} H_{\lambda\lambda} & H_{\lambda w} \\ H_{w\lambda} & -\gamma I_n \end{bmatrix}$ with $H_{\lambda\lambda}, H_{\lambda w}, H_{w\lambda}$ defined in Theorem 1, and then H is a symmetric matrix. By Lemma

2 we can derive the condition satisfying $H < 0$ and when $H < 0$ it follows that $J(w) < 0$ which implies that

$$\gamma V_2(t) < \gamma^2 \int_0^t w(s)^T w(s) ds. \tag{36}$$

Meanwhile, when $\begin{bmatrix} c_{1x}I_n & 0 \\ 0 & c_{1v}I_n \end{bmatrix}^T \begin{bmatrix} c_{1x}I_n & 0 \\ 0 & c_{1v}I_n \end{bmatrix} < \gamma M$, as $1 - q_i(t) \in [0, 1]$, it follows that

$$\begin{aligned} z(t)^T z(t) = \lambda(t)^T & \begin{bmatrix} I_n - Q(t) & 0 \\ 0 & I_n \end{bmatrix}^T \begin{bmatrix} c_{1x}I_n & 0 \\ 0 & c_{1v}I_n \end{bmatrix}^T \\ & \begin{bmatrix} c_{1x}I_n & 0 \\ 0 & c_{1v}I_n \end{bmatrix} \begin{bmatrix} I_n - Q(t) & 0 \\ 0 & I_n \end{bmatrix} \lambda(t) \\ & < \gamma \lambda(t)^T M \lambda(t) = \gamma V_2(t). \end{aligned} \tag{37}$$

Combined with (36) it follows that

$$z(t)^T z(t) < \gamma V_2(t) < \gamma^2 \int_0^t w(s)^T w(s) ds. \tag{38}$$

Therefore for any external turbulence $w(t) \in L_2[0, \infty)$, it holds that

$$||T_{zw}(t)||_{L_2 - L_\infty} = \sup_{0 \neq w(t) \in L_2[0,\infty)} \frac{||z(t)||_\infty}{||w(t)||_2} < \gamma, \tag{39}$$

which satisfies the $L_2 - L_\infty$ performance indices.

To summarize, the distributed algorithm designed in (5) can make the follower agents according to the dynamics equation (1) enter into the convex hull formed by the leader agents, which also satisfies the $L_2 - L_\infty$ performance indices, i.e., $||T_{zw}(t)||_{L_2 - L_\infty} = \sup_{0 \neq w(t) \in L_2[0,\infty)} \frac{||z(t)||_\infty}{||w(t)||_2} < \gamma$.

IV. NUMERICAL EXAMPLE

In this paper, the simulation of a multi-agent system formed by six follower agents is considered to verify the theoretical analysis we did above. In particular, we need to use the simulation results to verify that the distributed algorithm designed in (5) can make the follower agents according to the dynamics equation (1) enter into the convex hull formed by the leader agents, which also satisfies the $L_2 - L_\infty$ performance indices.

Based on the theoretical results above, the coefficients in the distributed algorithm (5) is set as the following:

$$b_i(t) = 0.5, p_i = 5,$$

and the initial position and velocity states as the following:

$$\begin{aligned} & x_1(0) = [-3.0, 2.0]^T, x_2(0) = [-3.0, -1.0]^T, \\ & x_3(0) = [0, -3.5]^T, x_4(0) = [2.5, -1]^T, \\ & x_5(0) = [2.5, 3.0]^T, x_6(0) = [0, 3.5]^T, \\ & v_1(0) = [1.0, 1.0]^T, v_2(0) = [1.0, 1.0]^T, \\ & v_3(0) = [1.0, 1.0]^T, v_4(0) = [1.0, 1.0]^T, \\ & v_5(0) = [1.0, 1.0]^T, v_6(0) = [1.0, 1.0]^T. \end{aligned}$$

Meanwhile, the position states of all static leader agents are in the following:

$$\begin{aligned} & y_1(0) = [-2.0, 2.0]^T, y_2(0) = [2.0, 2.0]^T, \\ & y_3(0) = [2.0, -2.0]^T, y_4(0) = [-2.0, -2.0]^T, \end{aligned}$$

The robust $L_2 - L_\infty$ containment control problem in this paper only need states of a small portion of neighbor agents. Then by designing a suitable distributed algorithm (5), the position and velocity states of each agent can be modified according to the states of its neighbor agents.According to the dynamics equation (1), the follower agents enter into the convex hull formed by the leader agents. Thus the communication topological graph set G describes the communication among agents in the following:

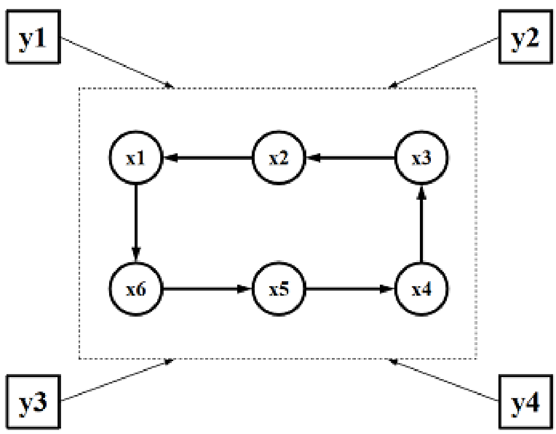

FIGURE 1. Communication topological graph.

By this communication topological figure 1, we can set the weights of all edges in this graph to be $a_{ij} = 0.8$, and then calculate the Laplacian matrix set L corresponding to the communication topological graph of the multi-agent system.

To verify whether the distributed algorithm (5) acting on the dynamics system (1) designed above satisfies the $L_2 - L_\infty$ performance indices, we assume the external turbulence is the pulse signal which has finite energy and appear at certain time intervals. This external turbulence $w_i(t)$ is given in the graph below: When the assumed external turbulence $w_i(t)$ acting on the second-order multi-agent system, the simulation results of the position, velocity states and the $L_2 - L_\infty$ performance indices are given in Graph 3, 4, 5, which show the control effects of the distributed algorithm (5) designed above acting on the multi-agent system formed by six agents satisfying the dynamics equation (1). In Graph 3, 4, the position and velocity states of all the follower agents enter into the convex hull formed by the leader agents, which solves the containment control problem of the multi-agent system. Meanwhile in Graph 5 we can see that $z(t)^T z(t)$ is decreasing along the time which satisfies the $L_2 - L_\infty$

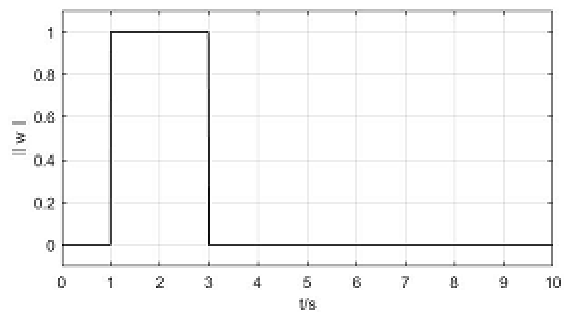

FIGURE 2. External turbulence (pulse signal).

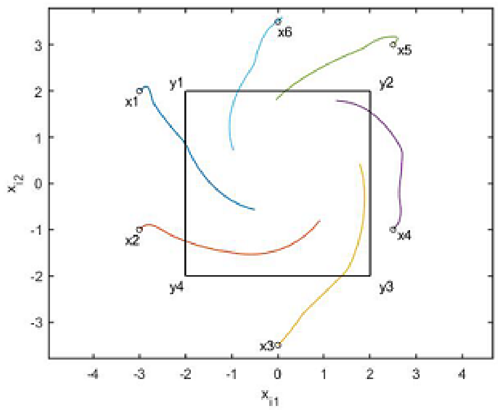

FIGURE 3. Curve of position states (pulse signal).

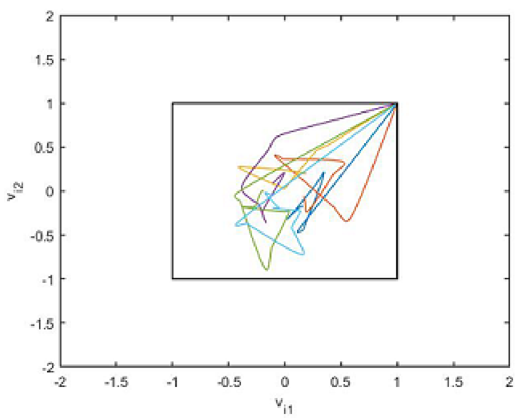

FIGURE 4. Curve of velocity states (pulse signal).

performance indices:

$$||T_{zw}(t)||_{L_2-L_\infty} = \sup_{0\neq w(t)\in L_2[0,\infty)} \frac{||z(t)||_\infty}{||w(t)||_2} < \gamma.$$

For the further verification of the restrain of the system in other forms of disturbance signals, we design a Sine signal whose external turbulence $w_i(t)$ has finite energy, which is given in the graph below: When the assumed external turbulence $w_i(t)$ acting on the second-order multi-agent system, the simulation results of the position, velocity states and the $L_2 - L_\infty$ performance indices are given in Graph 7, 8, 9:

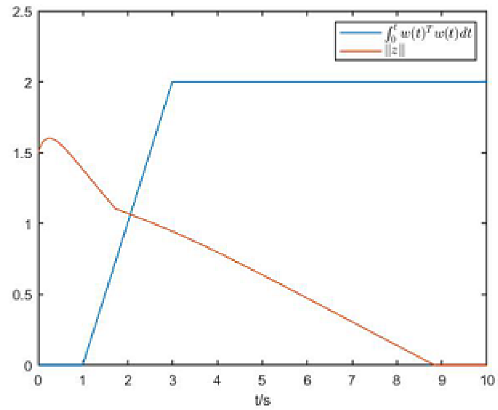

FIGURE 5. Orbit curve of $\int_0^t w(s)^T w(s)ds$ and $z(t)^T z(t)$ (pulse signal).

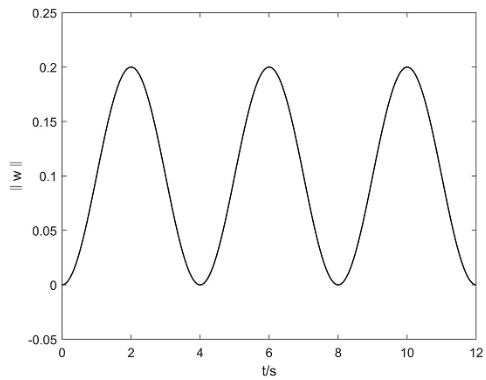

FIGURE 6. External turbulence (Sine signal).

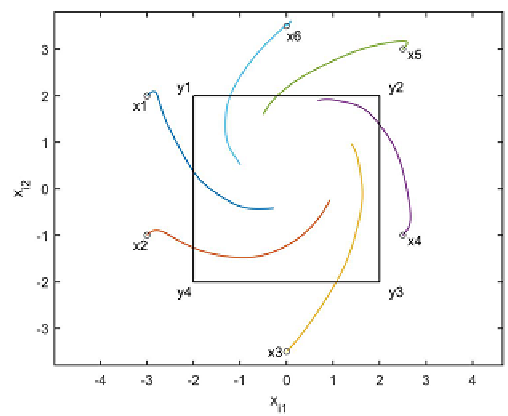

FIGURE 7. Curve of position states (Sine signal).

We find that when the form of external turbulence changes, if the energy of the disturbance signal is finite,it can always achieve almost the same restraint effect. Thus the numerical simulation results based on the MATLAB/SIMULINK simulation platform is consistent with the theoretical results in Theorem 1.

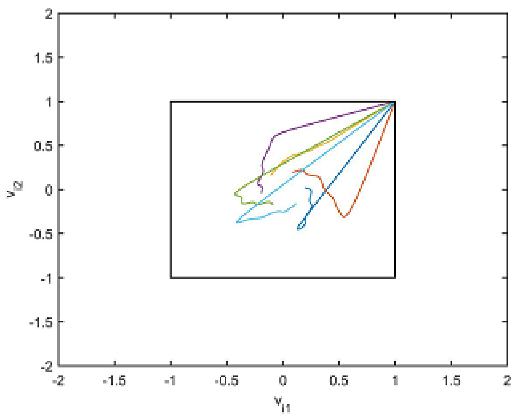

FIGURE 8. **Curve of velocity states (Sine signal).**

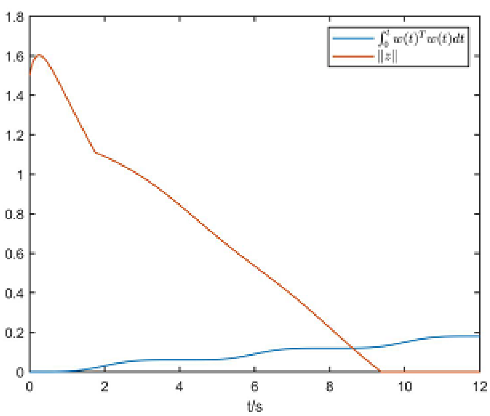

FIGURE 9. **Orbit curve of $\int_0^t w(s)^T w(s)ds$ and $z(t)^T z(t)$ (Sine signal).**

V. CONCLUSION

In this paper, the robust $L_2 - L_\infty$ containment control problem of a second-order continuous-time multi-agent system is investigated under external disturbances in uncertain or even unknown environments. For the second-order dynamic equation (1), we propose a distributed algorithm (5) to make all follower agents enter the non-empty closed convex hull formed by the leader agent cluster. In the process of theoretical analysis and derivation, we design the distributed algorithm, construct the Lyapunov function, and by use of the properties of projection operator, combine the Schur complement lemma to analyze and derive the suitable parameter range that achieves the containment control and satisfies the $L_2 - L_\infty$ performance indices under zero input and zero state conditions. Meanwhile, based on the theoretical analysis results, two simulation examples have been utilized to illustrate the conclusion of Theorem 1. Finally, based on the theoretical analysis results and numerical simulation example results of this paper, we can conclude that even experiencing external disturbances in uncertain or unknown working environments, the multi-agent system can still complete tasks normally when the communication topology among agents satisfies the conditions proposed in Theorem 1 and is strongly connected. Thus this work is of great significance in both theoretical research and engineering and can be applied to practical projects.

REFERENCES

[1] P. Gong, "Distributed consensus of non-linear fractional-order multi-agent systems with directed topologies," *IET Control Theory Appl.*, vol. 10, no. 18, pp. 2515–2525, Dec. 2016.

[2] Z. Meng, Z. Lin, and W. Ren, "Robust cooperative tracking for multiple non-identical second-order nonlinear systems," *Automatica*, vol. 49, no. 8, pp. 2363–2372, Aug. 2013.

[3] M. Ji, G. Ferrari-Trecate, M. Egerstedt, and A. Buffa, "Containment control in mobile networks," *IEEE Trans. Autom. Control*, vol. 53, no. 8, pp. 1972–1975, Sep. 2008.

[4] Z. Meng, W. Ren, and Z. You, "Distributed finite-time attitude containment control for multiple rigid bodies," *Automatica*, vol. 46, no. 12, pp. 2092–2099, Dec. 2010.

[5] J. Li, W. Ren, and S. Xu, "Distributed containment control with multiple dynamic leaders for double-integrator dynamics using only position measurements," *IEEE Trans. Autom. Control*, vol. 57, no. 6, pp. 1553–1559, Jun. 2012.

[6] H. Liu, G. Xie, and L. Wang, "Necessary and sufficient conditions for containment control of networked multi-agent systems," *Automatica*, vol. 48, no. 7, pp. 1415–1422, Jul. 2012.

[7] G. Wen, Y. Zhao, Z. Duan, W. Yu, and G. Chen, "Containment of higher-order multi-leader multi-agent systems: A dynamic output approach," *IEEE Trans. Autom. Control*, vol. 61, no. 4, pp. 1135–1140, Apr. 2016.

[8] Y. Liu, H. Zhang, Y. Wang, and H. Liang, "Adaptive containment control for fractional-order nonlinear multi-agent systems with time-varying parameters," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 9, pp. 1627–1638, Sep. 2022.

[9] X. Dong, Q. Li, Z. Ren, and Y. Zhong, "Formation-containment analysis and design for high-order linear time-invariant swarm systems with time delays," in *Proc. 34th Chin. Control Conf. (CCC)*, Hangzhou, China, Jul. 2015, pp. 6847–6853.

[10] D. Wang, N. Zhang, J. Wang, and W. Wang, "Cooperative containment control of multiagent systems based on follower observers with time delay," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 1, pp. 13–23, Jan. 2017.

[11] Z. Kan, J. M. Shea, and W. E. Dixon, "Leader–follower containment control over directed random graphs," *Automatica*, vol. 66, pp. 56–62, Apr. 2016.

[12] D. Meng, "Bipartite containment tracking of signed networks," *Automatica*, vol. 79, pp. 282–289, May 2017.

[13] P. Lin, Y. Jia, and L. Li, "Distributed robust consensus control in directed networks of agents with time-delay," *Syst. Control Lett.*, vol. 57, no. 8, pp. 643–653, Aug. 2008.

[14] P. Lin and Y. Jia, "Robust H_∞ consensus analysis of a class of second-order multi-agent systems with uncertainty," *IET Control Theory Appl.*, vol. 4, no. 3, pp. 487–498, Mar. 2010.

[15] J. Yu, X. Dong, Q. Li, and Z. Ren, "Robust H_∞ guaranteed cost time-varying formation tracking for high-order multiagent systems with time-varying delays," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 4, pp. 1465–1475, Apr. 2020.

[16] P. Li, K. Qin, and M. Shi, "Distributed robust H_∞ rotating consensus control for directed networks of second-order agents with mixed uncertainties and time-delay," *Neurocomputing*, vol. 148, pp. 332–339, Jan. 2015.

[17] H. Zhang, R. Yang, H. Yan, and F. Yang, "H_∞ consensus of event-based multi-agent systems with switching topology," *Inf. Sci.*, vols. 370–371, pp. 623–635, Nov. 2016.

[18] J. Wang, Z. Duan, and G. Wen, "H_∞ containment control for multi-agent systems over Markovian switching topologies," in *Proc. 33rd Youth Academic Annu. Conf. Chin. Assoc. Autom. (YAC)*, Nanjing, China, May 2018, pp. 18–20, doi: 10.1109/YAC.2018.8406378.

[19] D. Li, Y. Sun, G. Ma, and C. Li, "Distributed containment control for Euler–Lagrange systems with dynamic leaders," in *Proc. 6th Int. Conf. Instrum. Meas., Comput., Commun. Control (IMCCC)*, Harbin, China, Jul. 2016, pp. 73–77.

[20] J. Shao, L. Shi, Y. Cheng, and T. Li, "Asynchronous tracking control of leader–follower multiagent systems with input uncertainties over switching signed digraphs," *IEEE Trans. Cybern.*, vol. 52, no. 7, pp. 6379–6390, Jul. 2022, doi: 10.1109/TCYB.2020.3044627.

[21] L. Shi, Y. Xiao, J. Shao, and W. X. Zheng, "Containment control of asynchronous discrete-time general linear multiagent systems with arbitrary network topology," *IEEE Trans. Cybern.*, vol. 50, no. 6, pp. 2546–2556, Jun. 2020, doi: 10.1109/TCYB.2019.2915941.

[22] P. Lin and W. Ren, "Constrained consensus in unbalanced networks with communication delays," *IEEE Trans. Autom. Control*, vol. 59, no. 3, pp. 775–781, Mar. 2014.

[23] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1987.

YONGXIN BAN received the B.S. degree in automation from the Beijing Institute of Technology, Beijing, China, in 2005, and the M.S. degree in control theory and applications from Beihang University, Beijing, in 2008. He is currently pursuing the Ph.D. degree with the Nanjing University of Aeronautics and Astronautics.

His research interests include cooperative control, distributed optimization theory, and switching control.

YONGQIANG YE (Senior Member, IEEE) was born in Tongxiang, China. He received the B.E. and M.E. degrees in electrical engineering from Zhejiang University, Hangzhou, China, in 1994 and 1997, respectively, and the Ph.D. degree in electrical engineering from Nanyang Technological University, Singapore, in 2004.

For more than four years, he was a Faculty Member of the School of Information, Zhejiang University of Finance and Economics, Hangzhou. He was a Distinguished Professor of Henan province affiliated with the School of Electronic and Information, Zhongyuan University of Technology, Zhengzhou, China. He has authored or coauthored one book and more than 75 journal articles. His research interests include the advanced control of power electronics and electrical machines.