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RESEARCH ARTICLE

Frequency-Domain Based Iterative Learning Control for 2-D Discrete Systems

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ABSTRACT In the existing of traditional iterative learning control (ILC) results for two-dimensional (2-D) discrete systems with time-domain based analysis approach, fixed boundary states do not affect the complete convergence of P-type ILC law. However, it does affect the ILC convergence properties in the frequency domain. This paper first investigates the frequency-domain ILC tracking problem for 2-D discrete systems with different boundary states. An extended P-type ILC law is designed and a sufficient convergence condition of which can be derived through a rigorous mathematical proof. A simulation example is given to verify the effectiveness and validation of the proposed extended P-type ILC law. Finally, some comparison results on traditional P-type ILC law and D-type ILC law are presented.

INDEX TERMS Frequency-domain iterative learning control (ILC), two-dimensional (2-D) discrete systems, an extended P-type ILC law.

I. INTRODUCTION

Iterative learning control [1], [2], [3] is capable to addressing the trajectory tracking tasks repetitively over a finite time interval and shows excellent characteristic for 2-D discrete systems, such as heater exchanger [4], [5], multi-function robotics [6], and 2-D ladder circuits [7]. To date, there has been some fruitful ILC results on 2-D discrete systems in [8], [9], and [10]. In [8], a high-order internal model (HOIM) ILC law for 2-D linear discrete systems is designed to achieve the precise tracking on 2-D HOIM-based reference trajectory. In [9], a two-gain ILC law is presented to deal with the prefect tracking for 2-D linear discrete systems with fixed boundary states. The literature [10] investigates an adaptive ILC algorithm for 2-D nonlinear discrete systems with nonuniform trial lengths. It is worth noting that the previously mentioned achievements use the time-domain based analysis approach, such as the lifting technique. From an engineering perspective, the frequency-domain based ILC techniques are sometimes favoured because they exhibit superior spectral characteristics of system signals and provide the lower computation burden for convolution and lifting operation of time-domain signals. However, compared with the fruitful ILC results for 2-D discrete systems in time domain, the frequency-domain based ILC designs are not yet available.

Recently, frequency-domain based ILC achievements for 1-D systems have been extensively reported in [11], [12], [13], [14], and [15]. In [11], the convergence characteristics of the first-order and second-order PD-type ILC schemes for linear time-invariant systems in discrete spectrum is investigated. In [13], frequency domain analysis and design of anticipatory-type ILC were addressed for SISO linear systems by providing an engineering design procedure and a guideline for self-tuning for anticipatory-type ILC. In [15], the ILC problem for linear time-invariant systems with input delay is investigated in the frequency domain and three different ILC schemes are proposed to guarantee the zero tracking error. To our knowledge, the frequency domain

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analysis method plays an important role in the application of ILC. The frequency analysis plays a crucial role in ILC applications, mainly due to relaxation on the convergence condition from the infinite frequency bandwidth to a finite frequency bandwidth. Also, the tracking control problem of 2-D systems is more complex than that of 1-D systems, especially in frequency domain [16]. Correspondingly, a frequency domain-based spatial ILC in [28] has been used to a practical additive manufacturing (AM) systems utilizing a raster trajectory.

Motivated by these interesting observations, this paper first investigates the frequency-domain based ILC tracking problem of 2-D linear discrete systems. Under iteration-dependent boundary states, a frequency-domain based convergence condition can be obtained by using the traditional P-type ILC law and the selection guideline for the learning gain is given theoretically. It is proved that the final ILC tracking error is bounded, the bound of which continuously depends on the boundary states. Specifically, under fixed boundary states, the traditional P-type ILC law is very difficult to achieve zero tracking error. Therefore, the extended P-type ILC law is designed to achieve the precise tracking on 2-D reference trajectory. Simulation tests are provided.

The structure of this paper is shown in the following. Section II presents the problem formulation. Robust convergence analysis on the P-type ILC law (3) in the frequency domain is provided in section III. Simulation example is exhibited in Section IV. The corresponding conclusion is displayed in Section V.

Notations: In this paper, let $\{h_1\}_0^{H_1-1} = \{0, 1, 2, \cdots, H_1 - 1\}$ and $\{h_1\}_0^{\infty} = \{0, 1, 2, \cdots\}$. $\mathbb{R}^m$ and $\mathbb{C}$, respectively, represent the m-dimensional Euclidean space and complex space. $\mathbb{R}^{m\times n}$ denotes real matrices with $m \times n$ dimension. $\|\cdot\|$ stands for any compatible matrix/vector norm. $|\cdot|$ indicates the magnitude of the frequency domain response.

II. PROBLEM FORMULATION

Consider the following 2-D discrete systems [4], which are required to repetitively perform tracking tasks over a finite region $\{h_1\}_0^{H_1-1}$ and $\{h_2\}_0^{H_2-1}$:

$$x_m(h_1+1, h_2+1) = A_1 x_m(h_1+1, h_2) + A_2 x_m(h_1, h_2) + A_3 x_m(h_1, h_2+1) + B u_m(h_1, h_2) \quad (1)$$

$$y_m(h_1, h_2) = C x_m(h_1, h_2) \quad (2)$$

where $\{m\}_0^{\infty}$ denotes the iteration number. $u_m(h_1, h_2) \in \mathbb{R}$, $x_m(h_1, h_2) \in \mathbb{R}^n$, and $y_m(h_1, h_2) \in \mathbb{R}$, respectively, denote control input, state, and output; A_1, A_2, A_3, B, and C are real matrices to be estimated. Many practical systems can be described as the 2-D systems (1)-(2), such as thermal process [17], target echoes collected by a radar [18], and servo systems [19].

Remark 1: Actually, some ILC results for 2-D systems (1)-(2) have already been emerged in [4] and [5]. The main result on convergence analysis of [4, Theorem 1 with $H(q^{-1}) = 1$, 5, Theorem 1 with $K = 0$] is summarized and there is the following proposition.

Proposition 1: Consider the 2-D systems (1)-(2) under boundary states $x_m(h_1, 0) = x_0(h_1, 0)$, $\{h_1\}_0^{H_1}$ and $x_m(0, h_2) = x_0(0, h_2)$, $\{h_2\}_1^{H_2}$, and let the P-type ILC law be given as

$$u_{m+1}(h_1, h_2) = u_m(h_1, h_2) + \Gamma e_m(h_1+1, h_2+1). \quad (3)$$

If the learning gain Γ is satisfied as $\rho(I_p - CB\Gamma) < 1$, then, the ILC tracking error converges to zero.

In Proposition 1, the ILC results are obtained by using the lifting-technique based analysis approach in the time-domain. Under the same condition, if the frequency-domain analysis method is used, zero ILC tracking error is difficult to obtain, which is investigated in next section.

For ease of analysis the ILC tracking problem in the frequency domain, the following Definition 1 and Assumptions 1-2 are given.

Definition 1 ([17]): For a discrete 2-D function $f(h_1, h_2)$ satisfying $f(h_1, h_2) = 0$ for $h_1 < 0$ or $h_2 < 0$, its 2-D Z-transform $F(z_1, z_2)$ is defined by

$$F(z_1, z_2) = Z[f(h_1, h_2)] = \sum_{h_1=0}^{\infty}\sum_{h_2=0}^{\infty} f(h_1, h_2) z_1^{-h_1} z_2^{-h_2}. \quad (4)$$

Similarly, there is

$$F(z_1+1, z_2+1) = z_1 z_2 F(z_1, z_2) - z_1 z_2 \sum_{h_2=1}^{H_2} f(0, h_2) z_2^{-h_2} - z_1 z_2 \sum_{h_1=0}^{H_1} f(h_1, 0) z_1^{-h_1} - z_1 z_2 f(0, 0) \quad (5)$$

The derivation process of which is shown in the Appendix. According to the Definition 1, the 2-D systems (1)-(2) is reformulated as the form of

$$z_1 z_2 X_m(z_1, z_2) - z_1 z_2 X_m(0, z_2) - z_1 z_2 X_m(z_1, 0) + z_1 z_2 x_m(0,0) = [z_1 A_1 + A_2 + z_2 A_3] X_m(z_1, z_2) + B U_m(z_1, z_2) - z_1 A_1 x_m(0, h_2) - z_2 A_3 x_m(h_1, 0) \quad (6)$$

$$Y_m(z_1, z_2) = C X_m(z_1, z_2). \quad (7)$$

Rearranging (6) and (7), it generates

$$Y_m(z_1, z_2) = G_p(z_1, z_2) U_m(z_1, z_2) - \hat{G}_p(z_1, z_2) z_1 A_1 x_m(0, h_2) - \hat{G}_p(z_1, z_2) z_2 A_3 x_m(h_1, 0) - \hat{G}_p(z_1, z_2) z_1 z_2 x_m(0,0) + \hat{G}_p(z_1, z_2) z_1 z_2 X_m(0, z_2) + \hat{G}_p(z_1, z_2) z_1 z_2 X_m(z_1, 0) \quad (8)$$

where $G_p(z_1, z_2) = C(z_1 z_2 I_n - z_1 A_1 - A_2 - z_2 A_3)^{-1} B$ and $\hat{G}_p(z_1, z_2) = C(z_1 z_2 I_n - z_1 A_1 - A_2 - z_2 A_3)^{-1}$. Its frequency response is expressed as $G_p(e^{j\omega_h}, e^{j\omega_v}) =$

$\left|G_p(e^{j\omega_h}, e^{j\omega_v})\right| e^{j\angle G_p(e^{j\omega_h}, e^{j\omega_v})}$, where $\left|G_p(e^{j\omega_h}, e^{j\omega_v})\right|$ and $\angle G_p(e^{j\omega_h}, e^{j\omega_v})$ denote the magnitude and phase characteristics, respectively.

For an achievable reference trajectory $y_d(h_1, h_2)$, $\{h_1\}_0^{H_1}$, $\{h_2\}_0^{H_2}$, assume that there exists a unique input $u_d(h_1, h_2)$, $\{h_1\}_0^{H_1-1}$, $\{h_2\}_0^{H_2-1}$ such that

$$y_d(h_1, h_2) = g_p(h_1, h_2)u_d(h_1, h_2) \tag{9}$$

where $g_p(h_1, h_2)$ denotes the impulse response. Accordingly, let the tracking error $e_m(h_1, h_2)$ be given as

$$e_m(h_1, h_2) = y_d(h_1, h_2) - y_m(h_1, h_2). \tag{10}$$

Taking the 2-D Z-transform on (10), it yields

$$E_m(z_1, z_2) = Y_d(z_1, z_2) - Y_m(z_1, z_2) \tag{11}$$

where $Y_d(z_1, z_2)$ is a Z-transform function of $y_d(h_1, h_2)$.

Assumption 1: Let the 2-D transfer function $G_p(z_1, z_2)$ in (8) be open-loop stable, minimum-phase and its relative degree is one.

Assumption 2: Assume that

$$\|x_m(h_1, 0)\| \leq b_{x1}, \ \{h_1\}_0^{H_1}, \ \|x_m(0, h_2)\| \leq b_{x2}, \ \{h_2\}_1^{H_2}$$

where $b_{x1} \geq 0$ and $b_{x2} \geq 0$ are unknown constants. From Assumption 2, we know

$$\begin{aligned}\|e_m(h_1, 0)\| &= \|y_d(h_1, 0) - y_m(h_1, 0)\| \\ &= \|y_d(h_1, 0) - Cx_m(h_1, 0)\| \leq b_{e1} \\ \|e_m(0, h_2)\| &= \|y_d(0, h_2) - y_m(0, h_2)\| \\ &= \|y_d(0, h_2) - Cx_m(0, h_2)\| \leq b_{e2}\end{aligned}$$

where $b_{e1} \geq 0$ and $b_{e2} \geq 0$ are unknown constants.

Remark 2: Assumption 1 requires that all the zeros and poles of $G_p(z_1, z_2)$ lie in the region $|z_1| < 1$ and $|z_2| < 1$, and can be widely found in [15], [23], and [24]. Additionally, Assumption 2, as a fundamental and reasonable assumption in robustness ILC analysis, shows the boundedness of boundary states, which is presented in [10].

Lemma 1: Give two nonnegative functions $E_m(z_1, z_2) \in \mathbb{C}$ and $b_m(z_1, z_2) \in \mathbb{C}$ over a finite frequency bandwidth $z_1 \in \mathbb{C}$ and $z_2 \in \mathbb{C}$ satisfying:

$$|E_{m+1}(z_1, z_2)| \leq \gamma |E_m(z_1, z_2)| + |b_m(z_1, z_2)|.$$

Under $\limsup_{m\to\infty} |b_m(z_1, z_2)| \leq b'$, if $0 \leq \gamma < 1$ holds, there is

$$\limsup_{m\to\infty} |E_m(z_1, z_2)| \leq \frac{b'}{1-\gamma}.$$

Particularly, when $\lim_{m\to\infty} |b_m(z_1, z_2)| = 0$, it implies that

$$\lim_{m\to\infty} |E_m(z_1, z_2)| = 0.$$

The proof process of Lemma 1 can be referred to [25].

To our knowledge, if the following traditional P-type ILC laws

$$\begin{aligned}u_{m+1}(h_1, h_2) &= u_m(h_1, h_2) + \Gamma e_m(h_1 + 1, h_2) \\ u_{m+1}(h_1, h_2) &= u_m(h_1, h_2) + \Gamma e_m(h_1, h_2 + 1) \\ u_{m+1}(h_1, h_2) &= u_m(h_1, h_2) + \Gamma e_m(h_1, h_2)\end{aligned}$$

are applied to the 2-D systems (1)-(2), the complete tracking on 2-D reference trajectory cannot be met. Since the relative degrees of the 2-D systems (1)-(2) in the horizontal direction h_1 and vertical direction h_2 are equal to be one in [21], respectively. To this end, the P-type ILC law (3) is used in this paper and its Z-transform form is given as

$$\begin{aligned}U_{m+1}(z_1, z_2) &= U_m(z_1, z_2) + \Gamma z_1 z_2 E_m(z_1, z_2) - \Gamma z_1 z_2 \\ &\quad \times e_m(0, 0) - \Gamma z_1 z_2 \sum_{h_2=1}^{H_2-1} e_m(0, h_2) z_2^{-h_2} \\ &\quad - \Gamma z_1 z_2 \sum_{h_1=1}^{H_1-1} e_m(h_1, 0) z_1^{-h_1}.\end{aligned} \tag{12}$$

Remark 3: From (3), it can be seen that these uncontrollable boundary errors $e_m(h_1, 0)$ and $e_m(0, h_2)$ are not affect the ILC convergence characteristics in the time domain (see more details in [8] and [9]). However, they can have an impact on trajectory tracking in the frequency domain, which is explained in (12) and the subsequent proof.

III. ROBUST CONVERGENCE ANALYSIS ON THE P-TYPE ILC LAW (3) IN THE FREQUENCY DOMAIN

Next, we will investigate the robustness property of the P-type ILC law (3) for 2-D systems (1)-(2) in the frequency domain. The following Theorem 1 is presented.

Theorem 1: For the 2-D systems (1)-(2), under Assumptions 1 and 2, the P-type ILC law (3) is used. If there exists the learning gain Γ to make

$$|1 - \Gamma G_p(z_1, z_2) z_1 z_2| < 1, \tag{13}$$

where $G_p(z_1, z_2)$ is given in (8), then, the tracking error $e_m(h_1, h_2)$ is bounded related to b_{x1}, b_{x2}, b_{e1}, and b_{e2} described in Assumption 2.

Proof: Using (7), it generates

$$\begin{aligned}&Y_{m+1}(z_1, z_2) - Y_m(z_1, z_2) \\ &= G_p(z_1, z_2)U_{m+1}(z_1, z_2) + \hat{G}_p(z_1, z_2) z_1 A_1 x_{m+1}(0, h_2) \\ &\quad + \hat{G}_p(z_1, z_2) z_2 A_3 x_{m+1}(h_1, 0) + \hat{G}_p(z_1, z_2) z_1 z_2 \\ &\quad \times x_{m+1}(0, 0) + \hat{G}_p(z_1, z_2) z_1 z_2 \sum_{h_2=1}^{H_2} x_{m+1}(0, h_2) z_2^{-h_2} \\ &\quad - G_p(z_1, z_2)U_m(z_1, z_2) - \hat{G}_p(z_1, z_2) z_1 A_1 x_m(0, h_2) \\ &\quad - \hat{G}_p(z_1, z_2) z_2 A_3 x_m(h_1, 0) - \hat{G}_p(z_1, z_2) z_1 z_2 x_m(0, 0) \\ &\quad - \hat{G}_p(z_1, z_2) z_1 z_2 \sum_{h_2=1}^{H_2} x_m(0, h_2) z_2^{-h_2} \\ &= G_p(z_1, z_2)[U_{m+1}(z_1, z_2) - U_m(z_1, z_2)]\hat{G}_p(z_1, z_2) z_1 A_1 \\ &\quad \times [x_{m+1}(0, h_2) - x_m(0, h_2)] + \hat{G}_p(z_1, z_2) z_2 A_3 \\ &\quad \times [x_{m+1}(h_1, 0) - x_m(h_1, 0)] + \hat{G}_p(z_1, z_2) z_1 z_2\end{aligned}$$

$$\times [x_{m+1}(0,0) - x_m(0,0)] + \hat{G}_p(z_1,z_2)z_1z_2 \sum_{h_2=1}^{H_2} \times [x_{m+1}(0,h_2) - x_m(0,h_2)]z_2^{-h_2} + \hat{G}_p(z_1,z_2)z_1z_2 \times \sum_{h_1=1}^{H_1}[x_{m+1}(h_1,0) - x_m(h_1,0)]z_1^{-h_1}. \quad (14)$$

Substituting the ILC law (3) into (14), we have

$$\begin{aligned}
&Y_{m+1}(z_1,z_2) - Y_m(z_1,z_2) \\
&= G_p(z_1,z_2)\Gamma z_1z_2E_m(z_1,z_2) + \hat{G}_p(z_1,z_2)z_1A_1[x_{m+1}(0,h_2) \\
&\quad - x_m(0,h_2)] + \hat{G}_p(z_1,z_2)z_2A_3[x_{m+1}(h_1,0) - x_m(h_1,0)] \\
&\quad + \hat{G}_p(z_1,z_2)z_1z_2[x_{m+1}(0,0) - x_m(0,0)] + \hat{G}_p(z_1,z_2)z_1z_2 \\
&\quad \times \sum_{h_2=1}^{H_2}[x_{m+1}(0,h_2) - x_m(0,h_2)]z_2^{-h_2} + \hat{G}_p(z_1,z_2)z_1z_2 \\
&\quad \times \sum_{h_1=1}^{H_1}[x_{m+1}(h_1,0) - x_m(h_1,0)]z_1^{-h_1} - \hat{G}_p(z_1,z_2)\Gamma z_1z_2 \\
&\quad \times e_m(0,0) - \hat{G}_p(z_1,z_2)\Gamma z_1z_2 \sum_{h_2=1}^{H_2-1} e_m(0,h_2)z_2^{-h_2} \\
&\quad - \hat{G}_p(z_1,z_2)\Gamma z_1z_2 \sum_{h_1=1}^{H_1-1} e_m(h_1,0)z_1^{-h_1}. \qquad (15)
\end{aligned}$$

On the other hand, using (11), it follows that

$$\begin{aligned}
&E_{m+1}(z_1,z_2) - E_m(z_1,z_2) \\
&\quad = Y_d(z_1,z_2) - Y_{m+1}(z_1,z_2) - Y_d(z_1,z_2) + Y_m(z_1,z_2) \\
&\quad = -Y_{m+1}(z_1,z_2) + Y_m(z_1,z_2). \qquad (16)
\end{aligned}$$

Inserting (15) into (16), we obtain

$$\begin{aligned}
&E_{m+1}(z_1,z_2) - E_m(z_1,z_2) \\
&= -G_p(z_1,z_2)\Gamma z_1z_2E_m(z_1,z_2) - \hat{G}_p(z_1,z_2)z_1A_1 \\
&\quad \times [x_{m+1}(0,h_2) - x_m(0,h_2)] - \hat{G}_p(z_1,z_2)z_2A_3 \\
&\quad \times [x_{m+1}(h_1,0) - x_m(h_1,0)] - \hat{G}_p(z_1,z_2)z_1z_2 \\
&\quad \times [x_{m+1}(0,0) - x_m(0,0)] - \hat{G}_p(z_1,z_2)z_1z_2 \\
&\quad \times \sum_{h_2=1}^{H_2}[x_{m+1}(0,h_2) - x_m(0,h_2)]z_2^{-h_2} - \hat{G}_p(z_1,z_2)z_1z_2 \\
&\quad \times \sum_{h_1=1}^{H_1}[x_{m+1}(h_1,0) - x_m(h_1,0)]z_1^{-h_1} \\
&\quad + G_p(z_1,z_2)\Gamma z_1z_2e_m(0,0) + G_p(z_1,z_2)\Gamma z_1z_2 \\
&\quad \times \sum_{h_2=1}^{H_2-1} e_m(0,h_2)z_2^{-h_2} + G_p(z_1,z_2)\Gamma z_1z_2 \\
&\quad \times \sum_{h_1=1}^{H_1-1} e_m(h_1,0)z_1^{-h_1}. \qquad (17)
\end{aligned}$$

Rearranging (17), there is

$$\begin{aligned}
&E_{m+1}(z_1,z_2) \\
&= [1 - G_p(z_1,z_2)\Gamma z_1z_2]E_m(z_1,z_2) - \hat{G}_p(z_1,z_2)z_1A_1 \\
&\quad \times [x_{m+1}(0,h_2) - x_m(0,h_2)] - \hat{G}_p(z_1,z_2)z_2A_3 \\
&\quad \times [x_{m+1}(h_1,0) - x_m(h_1,0)] - \hat{G}_p(z_1,z_2)z_1z_2 \\
&\quad \times [x_{m+1}(0,0) - x_m(0,0)] - \hat{G}_p(z_1,z_2)z_1z_2 \\
&\quad \times \sum_{h_2=1}^{H_2}[x_{m+1}(0,h_2) - x_m(0,h_2)]z_2^{-h_2} - \hat{G}_p(z_1,z_2)z_1z_2 \\
&\quad \times \sum_{h_1=1}^{H_1}[x_{m+1}(h_1,0) - x_m(h_1,0)]z_1^{-h_1} + G_p(z_1,z_2)\Gamma z_1 \\
&\quad \times z_2e_m(0,0) + G_p(z_1,z_2)\Gamma z_1z_2 \sum_{h_2=1}^{H_2-1} e_m(0,h_2)z_2^{-h_2} \\
&\quad + G_p(z_1,z_2)\Gamma z_1z_2 \sum_{h_1=1}^{H_1-1} e_m(h_1,0)z_1^{-h_1}. \qquad (18)
\end{aligned}$$

Taking the norm or magnitude operation on two sides of (18), we have

$$\begin{aligned}
&|E_{m+1}(z_1,z_2)| \\
&\leq |1 - G_p(z_1,z_2)\Gamma z_1z_2||E_m(z_1,z_2)| + |\hat{G}_p(z_1,z_2)z_1| \\
&\quad \times \|A_1\|\|x_{m+1}(0,h_2) - x_m(0,h_2)\| + |\hat{G}_p(z_1,z_2)z_2| \\
&\quad \times \|A_3\|\|x_{m+1}(h_1,0) - x_m(h_1,0)\| + |\hat{G}_p(z_1,z_2)z_1z_2| \\
&\quad \times \|x_{m+1}(0,0) - x_m(0,0)\| + |\hat{G}_p(z_1,z_2)z_1z_2| \\
&\quad \times \sum_{h_2=1}^{H_2} \|x_{m+1}(0,h_2) - x_m(0,h_2)\||z_2|^{-h_2} \\
&\quad + |\hat{G}_p(z_1,z_2)z_1z_2| \sum_{h_1=1}^{H_1} \|x_{m+1}(h_1,0) - x_m(h_1,0)\| \\
&\quad \times |z_1|^{-h_1} + |G_p(z_1,z_2)\Gamma z_1z_2||e_m(0,0)| \\
&\quad + |G_p(z_1,z_2)\Gamma z_1z_2| \sum_{h_2=1}^{H_2-1} |e_m(0,h_2)||z_2|^{-h_2} \\
&\quad + |G_p(z_1,z_2)\Gamma z_1z_2| \sum_{h_1=1}^{H_1-1} |e_m(h_1,0)||z_1|^{-h_1}. \qquad (19)
\end{aligned}$$

In (19), we know from Assumption 2 that $|e_m(0,h_2+1)|$, $|e_m(h_1+1,0)|$, $\|x_{m+1}(0,h_2) - x_m(0,h_2)\|$, and $\|x_{m+1}(h_1,0) - x_m(h_1,0)\|$ are bounded. Applying Lemma 1 to (19), if the learning gain Γ is selected to satisfy (13), we get

$$\limsup_{m\to\infty} |E_m(z_1,z_2)| \leq \frac{\bar{b}}{1 - |1 - G_p(z_1,z_2)\Gamma z_1z_2|},$$

where $\bar{b} > 0$ is relevant to b_{x1}, b_{x2}, b_{e1} and b_{e2} given in Assumption 2. According to inverse Z-transform, we obtain that the ILC tracking error $e_m(h_1,h_2)$ is bounded for $\{h_1\}_1^{H_1}$ and $\{h_2\}_1^{H_2}$.

This completes the proof of Theorem 1.

Remark 4: It is worth noting that the convergence condition (13) of Theorem 1 is given in the form of

$$|1 - \Gamma G_p(e^{j\omega_h}, e^{j\omega_v}))e^{j\omega_h}e^{j\omega_v}|$$

$$
\begin{aligned}
&= |1 - \Gamma|G_p(e^{j\omega_h}, e^{j\omega_v})| e^{j\angle G_p(e^{j\omega_h}, e^{j\omega_v})} e^{j\omega_h} e^{j\omega_v}| \\
&= |1 - \Gamma|G_p(e^{j\omega_h}, e^{j\omega_v})| e^{j\phi(e^{j\omega_h}, e^{j\omega_v})}|,
\end{aligned}
$$

where $\phi(e^{j\omega_h}, e^{j\omega_v}) = \angle G_p(e^{j\omega_h}, e^{j\omega_v}) + \omega_h + \omega_v$. According to the Euler Theorem, we have

$$
\begin{aligned}
&|1 - \Gamma|G_p(e^{j\omega_h}, e^{j\omega_v})| e^{j\phi(e^{j\omega_h}, e^{j\omega_v})}| \\
&\quad = |1 - \Gamma|G_p(e^{j\omega_h}, e^{j\omega_v})| [\cos(\phi(e^{j\omega_h}, e^{j\omega_v})) \\
&\qquad + j \sin(\phi(e^{j\omega_h}, e^{j\omega_v}))] \\
&\quad = |1 - \Gamma|G_p(e^{j\omega_h}, e^{j\omega_v})| \cos(\phi(e^{j\omega_h}, e^{j\omega_v})) \\
&\qquad - j\Gamma|G_p(e^{j\omega_h}, e^{j\omega_v})| \sin(\phi(e^{j\omega_h}, e^{j\omega_v}))| < 1. \qquad (20)
\end{aligned}
$$

Taking the square on both sides of (20), the above inequality is equivalent to

$$\Gamma^2 |G_p(e^{j\omega_h}, e^{j\omega_v})| < 2\Gamma \cos(\phi(e^{j\omega_h}, e^{j\omega_v})).$$

To guarantee the error convergence, if there exists the learning gain $\Gamma > 0$, the following condition should be satisfied:

$$\Gamma |G_p(e^{j\omega_h}, e^{j\omega_v})| < 2 \cos(\phi(e^{j\omega_h}, e^{j\omega_v})). \qquad (21)$$

Remark 5: To satisfy (13), it is necessary that for all $\omega_h \in [0, \infty)$ and $\omega_v \in [0, \infty)$, there is

$$-\frac{\pi}{2} < \angle G_p(e^{j\omega_h}, e^{j\omega_v}) + \omega_h + \omega_v < \frac{\pi}{2}. \qquad (22)$$

To our knowledge, for most of 2-D systems, the conditions (21) and (22) are difficult to be guaranteed for all frequencies $\omega_h \in [0, \infty)$ and $\omega_v \in [0, \infty)$. For example, when $\omega_h \to \infty$ and $\omega_v \to \infty$, the inequality $\cos(\angle G_p(e^{j\omega_h}, e^{j\omega_v}) + \omega_h + \omega_v) > 0$ no longer holds. Hence, the frequency region ω_h and ω_v needs to be reduced into a learnable band [13] and [15]. Therefore, the learnable band is required to satisfy $\omega_h \in [\omega_h^{\min}, \omega_h^{\max}]$ and $\omega_v \in [\omega_v^{\min}, \omega_v^{\max}]$.

We can see from Theorem 1 that the bounded ILC tracking objective can be achieved by depending on boundary states and errors. Under the desired boundary states and errors, the complete ILC tracking on 2-D reference trajectory can be obtained. There is the following Corollary 1.

Corollary 1: For the 2-D systems (1)-(2) with Assumption 1, and boundary states $x_m(h_1, 0) = x_d(h_1, 0)$ and $x_m(0, h_2) = x_d(0, h_2)$, the P-type ILC law (3) is used. If the learning gain Γ is selected to satisfy (13), then, the tracking error $e_m(h_1, h_2)$ is convergent progressively, i.e., $\lim_{m\to\infty} |e_m(h_1, h_2)| = 0$, $\{h_1\}_1^{H_1}$, $\{h_2\}_1^{H_2}$.

Remark 6: In Corollary 1, with a fixed boundary states $x_m(h_1, 0) = x_0(h_1, 0)$ and $x_m(0, h_2) = x_0(0, h_2)$, we still cannot get the precise tracking. This is in contrast to traditional time-domain based ILC analysis for 2-D discrete systems in [8] and [9]. To this end, the following extended P-type ILC law is presented as

$$
\begin{aligned}
&u_{m+1}(h_1, h_2) \\
&= u_m(h_1, h_2) + \Gamma e_m(h_1 + 1, h_2 + 1) + \Gamma z_1 z_2 \frac{1 - z_1{}^{-1}}{1 - z_1{}^{-H_1}} \\
&\quad \times e_m(0, h_2) + \Gamma z_1 z_2 \frac{1 - z_2{}^{-1}}{1 - z_2{}^{-H_2}} e_m(h_1, 0) \\
&\quad + \Gamma z_1 z_2 \frac{1 - z_1{}^{-1}}{1 - z_1{}^{-H_1}} \frac{1 - z_2{}^{-1}}{1 - z_2{}^{-H_2}} e_m(0, 0) \qquad (23)
\end{aligned}
$$

where $\{h_1\}_0^{H_1 - 1}$ and $\{h_2\}_0^{H_2 - 1}$. Taking the Z-transform on (23), there is

$$U_{m+1}(z_1, z_2) = U_m(z_1, z_2) + \Gamma z_1 z_2 E_m(z_1, z_2). \qquad (24)$$

Theorem 2: For the 2-D systems (1)-(2) under Assumption 1, boundary states $x_m(h_1, 0) = x_0(h_1, 0)$ and $x_m(0, h_2) = x_0(0, h_2)$, the extended P-type ILC law (23) is used. If the learning gain Γ is chosen to make (13) satisfied, then, the tracking error $e_m(h_1, h_2)$ is convergent progressively, i.e., $\lim_{m\to\infty} |e_m(h_1, h_2)| = 0$, $\{h_1\}_1^{H_1}$, $\{h_2\}_1^{H_2}$.

Proof: Using (23) and considering $x_m(h_1, 0) = x_0(h_1, 0)$ and $x_m(0, h_2) = x_0(0, h_2)$, (18) can be reformulated as

$$E_{m+1}(z_1, z_2) = [1 - G_p(z_1, z_2)\Gamma z_1 z_2] E_m(z_1, z_2). \qquad (25)$$

Taking the norm or magnitude operations on two sides of (25), we obtain

$$|E_{m+1}(z_1, z_2)| \le |1 - G_p(z_1, z_2)\Gamma z_1 z_2| |E_m(z_1, z_2)|. \qquad (26)$$

For (26), applying Lemma 1, if the learning gain Γ is selected to meet (13), it can be concluded that $\lim_{m\to\infty} |e_m(h_1, h_2)| = 0$, $\{h_1\}_1^{H_1}$, $\{h_2\}_1^{H_2}$.

The proof of Theorem 2 is completed.

IV. ILLUSTRATIVE EXAMPLE

This section gives some simulation results to illustrate the effectiveness of the extended P-type ILC law (23). The 2-D transfer function of (1)-(2) in [4] and [23] is given as

$$G_p(z_1, z_2) = \frac{0.8}{z_1 z_2 - 0.1 z_1 + 0.03 - 0.3 z_2}.$$

The poles of $G_p(z_1, z_2)$ are computed as $z_1 = 0.3$ and $z_2 = 0.1$, which satisfy the Assumption 1. According to Remark 5, we select the horizontal interval frequency and vertical interval frequency $\omega_h \in [0, \frac{\pi}{4}]$ and $\omega_v \in [0, \frac{\pi}{4}]$, and the horizontal sampling rate and vertical sampling rate $\frac{\pi}{80}$. The magnitude characteristics of $G_p(e^{j\omega_h}, e^{j\omega_v})$ and phase characteristics of $\angle G_p(e^{j\omega_h}, e^{j\omega_v}) + \omega_h + \omega_v$ are presented in Figs. 1 and 2, respectively. Let the 2-D desired reference trajectory $y_r(h_1, h_2)$ be given as

$$y_r(h_1, h_2) = \cos(0.2\pi h_1) + \cos(0.2\pi h_2), \{h_1\}_0^{20}, \ \{h_2\}_0^{20}$$

which is shown in Fig 3. Let the boundary outputs be described as $y_m(h_1, 0) = 0.5 \sin(0.2\pi h_1)$, $\{h_1\}_0^{20}$ and $y_m(0, h_2) = \sin(0.2\pi h_2)$, $\{h_2\}_1^{20}$. Under the initial control input $u_0(h_1, h_2) = 0$, $\{h_1\}_0^{19}$, $\{h_2\}_0^{19}$ in the extended P-type ILC law (23), we select the learning gain $\Gamma = 0.3$, which satisfies the convergence condition (13). The sum of tracking error index EE_m is used to evaluate the accuracy of ILC tracking:

$$EE_m = \sum_{h_1=1}^{20} \sum_{h_2=1}^{20} |y_r(h_1, h_2) - y_m(h_1, h_2)|$$

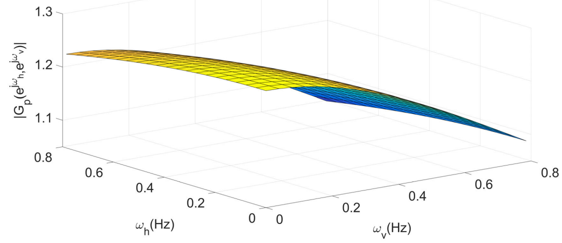

FIGURE 1. The magnitude characteristics of $G_p(e^{jw_h}, e^{jw_v})$.

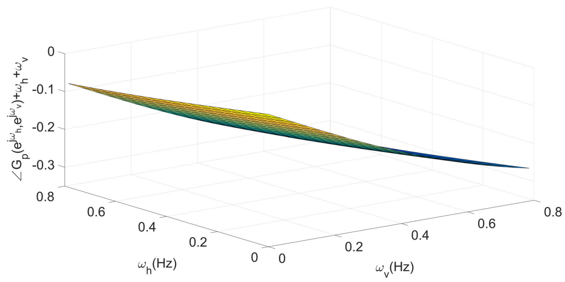

FIGURE 2. The phase characteristics of $\angle G_p(e^{j\omega_h}, e^{j\omega_v}) + \omega_h + \omega_v$.

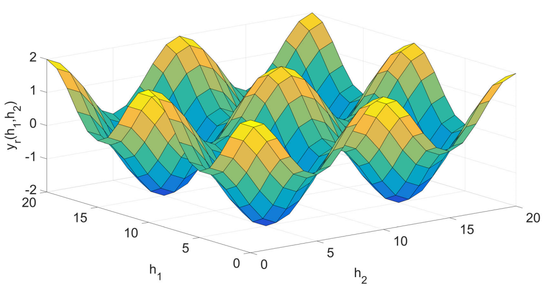

FIGURE 3. The 2-D reference trajectory $y_r(h_1, h_2)$, $\{h_1\}_0^{20}$, $\{h_2\}_0^{20}$.

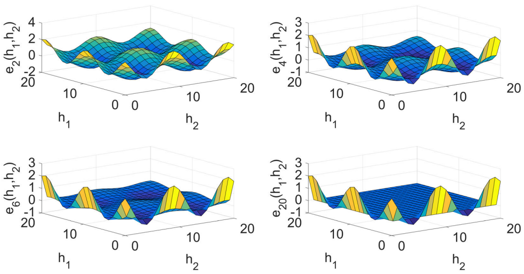

FIGURE 4. Under the extended P-type ILC law (23), the ILC tracking error $e_m(h_1, h_2)$ for $m = 2, 4, 6, 20$.

which does not include the uncontrollable boundary outputs $y_m(0, h_2)$ and $y_m(h_1, 0)$. As a result, Fig. 4 presents the ILC tracking error $e_m(h_1, h_2)$ at $m = 2, 4, 6, 20$. Fig. 5 depicts the profile of ILC tracking index EE_m with iteration number m. Obviously, it can be observed from Figs. 4-5 that the effectiveness of the extended P-type ILC law (23) is validated.

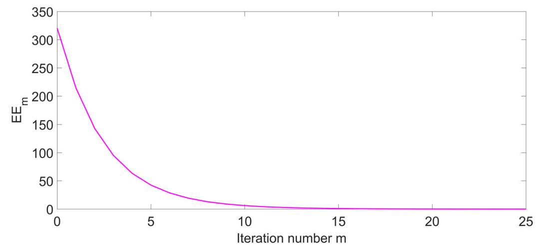

FIGURE 5. Under the extended P-type ILC law (23), the profile of EE_m with m.

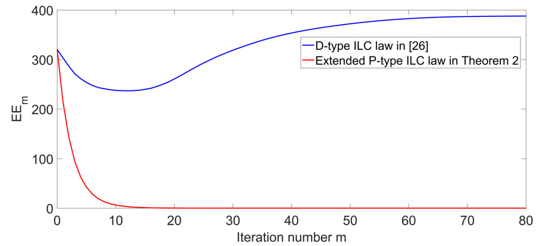

FIGURE 6. Under the D-type ILC law in [20], the profile of EE_m with m.

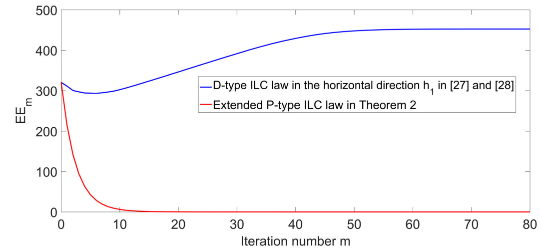

FIGURE 7. Under the D-type ILC law in the horizontal direction h_1 in [26] and [27], the profile of EE_m with m.

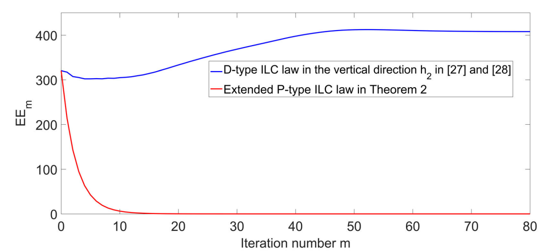

FIGURE 8. Under the D-type ILC law in the vertical direction h_2 in [26] and [27], the profile of EE_m with m.

Discussions: In this section, we will provide some comparison results with D-type ILC laws in [20], [26], and [27], which are given as:

(1) D-type ILC law in [20]:

$$u_{m+1}(h_1, h_2) = u_m(h_1, h_2) + 0.3[e_m(h_1 + 1, h_2 + 1) - e_m(h_1, h_2)], \tag{27}$$

(2) D-type ILC law in the horizontal direction h_1 in [26] and [27]:

$$u_{m+1}(h_1, h_2) = u_m(h_1, h_2) + 0.4[e_m(h_1 + 1, h_2 + 1) - e_m(h_1, h_2 + 1)], \tag{28}$$

(3) D-type ILC law in the vertical direction h_2 in [26] and [27]:

$$u_{m+1}(h_1, h_2) = u_m(h_1, h_2) + 0.4[e_m(h_1+1, h_2+1) - e_m(h_1+1, h_2)]. \quad (29)$$

Fig. 6 show the profile of ILC tracking index EE_m under the D-type ILC law (27), respectively. Apparently, it can be seen from Fig. 6 that D-type ILC law cannot make the ILC tracking error converge to zero. This is mainly due to the fact that the relative degrees of 2-D systems (1)-(2) in the horizontal direction and vertical direction is one. To further illustrate the inadequacy of the D-type ILC law, we continue to use the D-type ILC law in the horizontal direction h_1 and in the vertical direction h_2 in [26] and [27], respectively, to 2-D systems (1)-(2), simulation results on EE_m of which are displayed on Figs. 7-8.

V. CONCLUSION

Compared with D-type and P-type ILC laws, the proposed extended P-type ILC law in this paper can well handle the tracking problem on 2-D reference trajectory in the frequency-domain. This brief investigates the frequency-domain ILC for 2-D linear discrete systems with iteration-dependent boundary states and boundary errors. Different from time-domain based ILC approach for 2-D systems, frequency ILC approach is more sensitive to boundary states and errors. In the future work, frequency-domain ILC analysis will be used to solve the iteration-varying reference trajectory.

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