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RESEARCH ARTICLE

Accounting for Azimuthal Coupling in Long-Range Ocean Acoustics Calculations

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ABSTRACT The parabolic equation method is an accurate and efficient approach for solving nonseparable problems in ocean acoustics in which there are horizontal variations in the environmental parameters. Many range-dependent problems may be solved using 2-D parabolic equation models that ignore coupling of energy between planes of constant azimuth. When azimuthal coupling must be taken into account, the splitting method may be used to efficiently solve a 3-D parabolic equation that handles the depth operator to higher order but handles the azimuth operator only to leading order. Despite the fact that this approximation provides a favorable combination of accuracy and efficiency for 3-D problems, run times have generally been regarded as prohibitive for the long-range problems that are often of interest in ocean acoustics. It is demonstrated here that, when propagation paths from source to receiver are confined to a relatively narrow neighborhood of the vertical plane containing the source and receiver, it is practical to solve 3-D problems out to long ranges by using nonuniform azimuthal sampling, with fine sampling near the vertical plane and extremely coarse sampling elsewhere.

INDEX TERMS Ocean acoustics, range dependence, azimuthal coupling, 3-D effects, parabolic equation method, splitting method, nonuniform grids.

I. INTRODUCTION

An ocean acoustics problem is referred to as 'range dependent' if there are horizontal variations in the bathymetry, sound speed, and/or other properties of the environment. Range-dependent problems are nonseparable, but they may often be solved accurately and efficiently with the parabolic equation method [1], which is based on factoring the operator in the frequency-domain wave equation into a product of operators that correspond to outgoing and incoming energy, assuming that outgoing energy dominates, and ignoring incoming energy that arises from backscattering. It is practical to solve range-dependent problems out to ranges of many thousands of wavelengths with 2-D parabolic equation models, which neglect the term in the wave equation that allows energy to couple between planes of constant azimuth. After the first 3-D parabolic equation models were developed, it was discovered that azimuthal coupling is negligible for many (if not most) problems [2]. If it were always necessary to account for azimuthal coupling, much of the progress that has been made in ocean acoustics during the past several decades would not have been possible. For this reason, the uncoupled azimuth approximation is arguably the most important approximation in ocean acoustics. Azimuthal coupling must be taken into account for some problems, however, such as when variations in bathymetry guide energy out of planes of constant azimuth [3].

Parabolic equation models are based on approximations of the square root of the transverse (depth and azimuth) component of the operator in the wave equation. For 2-D problems, the transverse operator is a depth operator, and it is possible to achieve a high level of accuracy by using higher-order expansions in the approximation of the square root [4], [5]. A formidable challenge arises for the 3-D case when attempting to implement expansions that are of higher-order accuracy in both of the transverse directions [6]; the splitting method is not applicable for such higher-order expansions, and run times are prohibitive for the long-range problems

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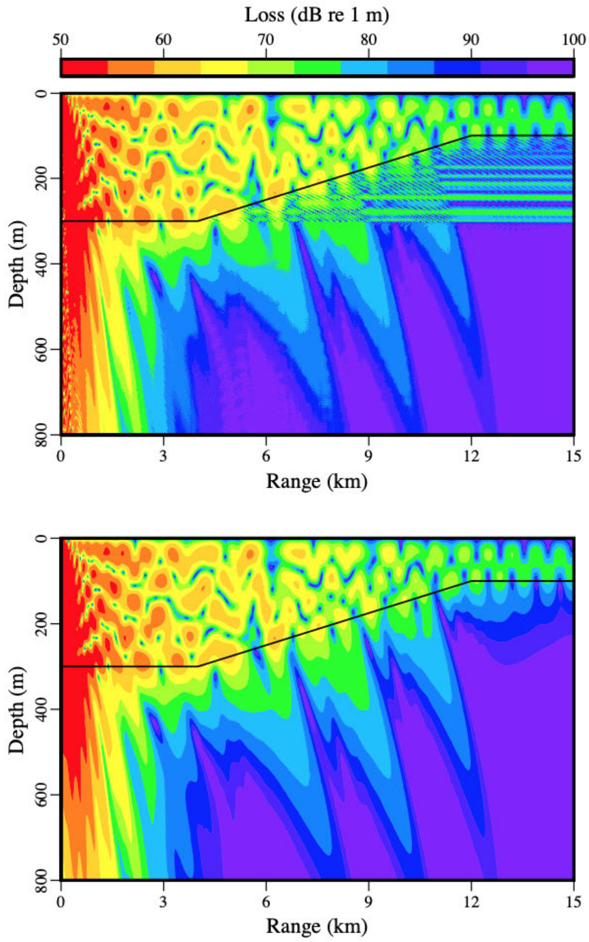

FIGURE 1. Results for example A. Transmission loss for a 2-D problem that was generated without (top) and with (bottom) stability constraints.

that are often of interest in ocean acoustics. An expansion to higher order in the depth operator but to only leading order in the azimuth operator [3], [7], [8] may be solved efficiently with the splitting method and is sufficiently accurate for many problems. Although this approach provides an unmatched combination of accuracy and efficiency among existing approaches for solving 3-D problems, it is usually regarded as practical only for calculations out to moderate ranges. Some progress has been made in improving accuracy [9], [10], but finding a practical approach for solving 3-D problems with higher-order expansions remains an elusive goal.

We demonstrate here that, when propagation paths from source to receiver are confined to a relatively narrow neighborhood of the vertical plane containing the source and receiver, it is practical to solve 3-D problems out to long ranges by using variable azimuthal grid spacing in the implementation of the splitting method. The key is to use fine azimuthal sampling near the vertical plane and extremely coarse azimuthal sampling elsewhere. Additional gains in efficiency may be achieved by using variable depth grid spacing in the sediment to prevent nonphysical reflections from the bottom of the computational grid [11].

II. THE PARABOLIC EQUATION AND ITS SOLUTION

We begin with the following outgoing wave equation (which corresponds to Eq. 2.64 in [1]) in the farfield:

$$\frac{\partial p}{\partial r} = ik_0(1+X+Y)^{1/2}p, \tag{1}$$

$$X = \frac{\rho}{k_0^2}\frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial}{\partial z} + \frac{k^2-k_0^2}{k_0^2}, \tag{2}$$

$$Y = \frac{1}{k_0^2 r^2}\frac{\partial^2}{\partial \theta^2}, \tag{3}$$

where the cylindrical spreading factor $r^{-1/2}$ has been removed from the complex pressure p, k is the wave number, the constant k_0 is a reference wave number, and ρ is the density. A parabolic equation that is accurate to higher order in the depth operator and leading order in the azimuth operator is based on the approximation,

$$(1+X+Y)^{1/2} = (1+X)^{1/2} + \tfrac{1}{2}Y + O\left(XY, Y^2\right), \tag{4}$$

which is correct to all orders in X and leading order in Y but neglects all cross terms. Substituting the approximation into Eq. (1), we obtain

$$\frac{\partial p}{\partial r} = ik_0(1+X)^{1/2}p + \frac{i}{2k_0 r^2}\frac{\partial^2 p}{\partial \theta^2}. \tag{5}$$

The splitting method [12] may be used to solve a partial differential equation in the form of Eq. (5), which has a first derivative on the left side and a sum of differential operators on the right side. This approach may provide enormous gains in efficiency when the operators on the right side involve partial derivatives with respect to different variables, as they do in this case. As discussed in [12], the splitting method involves two steps, which require numerical methods for solving the equations,

$$\frac{\partial p}{\partial r} = ik_0(1+X)^{1/2}p, \tag{6}$$

$$\frac{\partial p}{\partial r} = \frac{i}{2k_0 r^2}\frac{\partial^2 p}{\partial \theta^2}, \tag{7}$$

each of which is much easier to solve than Eq. (5). In the first step, Eq. (6) is integrated over (i.e., solved over) the range step Δr to obtain an intermediate solution, which is the initial condition for the second step, in which Eq. (7) is integrated over Δr.

We solve Eq. (6) by approximating the square root of the operator using a rational function (e.g., see [4], [13]), applying Galerkin's method with nonuniform spacing to discretize the depth operator [11], and using an approach for approximately conserving energy to accurately handle sloping bathymetry [14]. The basis for selecting the number of terms in the rational approximation includes the desired accuracy in X and stability issues, which can arise when employing the energy-conservation approximation; many problems

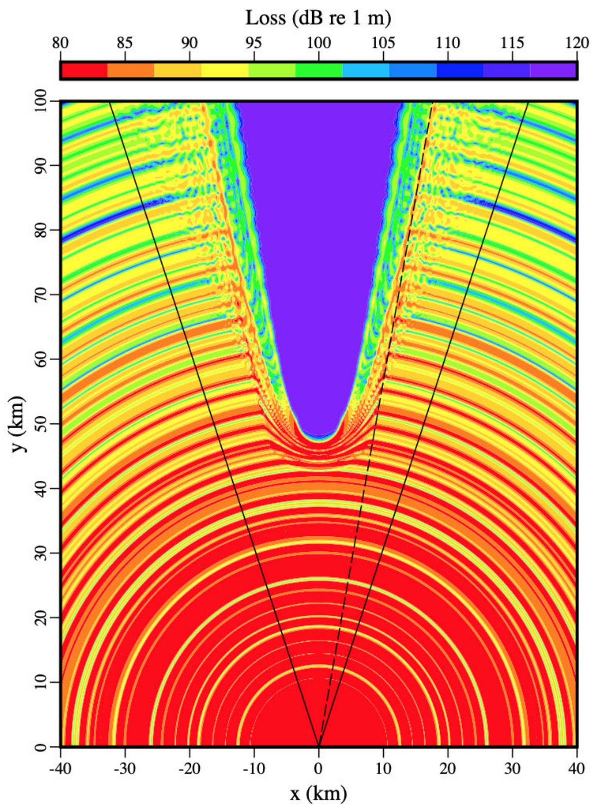

FIGURE 2. Results for example B. Transmission loss at $z = 30$ m that was generated with the 3-D parabolic equation. Fine azimuthal sampling was used in the 36° sector between the solid lines. Quantitative comparisons along the dashed line appear in Fig. 4.

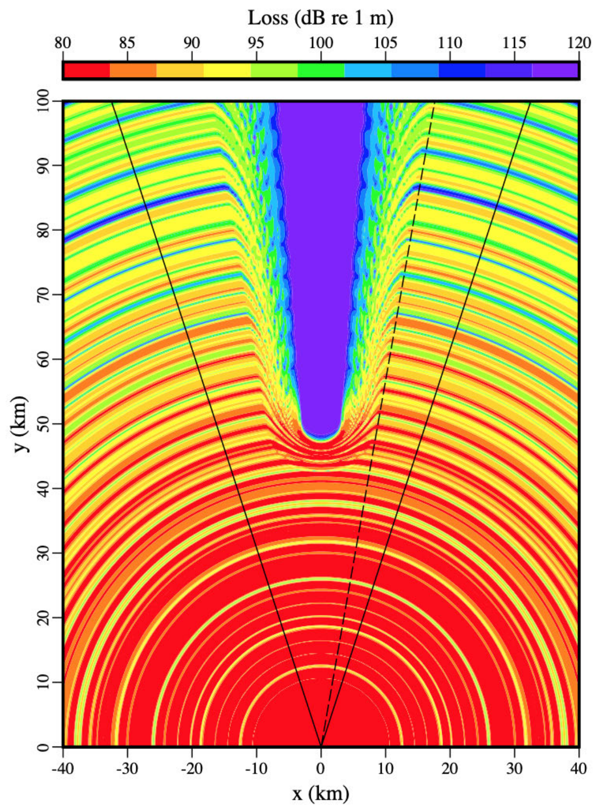

FIGURE 3. Results for example B. Transmission loss at $z = 30$ m that was generated with the 2-D parabolic equation. In generating a 3-D solution, fine azimuthal sampling was used in the 36° sector between the solid lines. Quantitative comparisons along the dashed line appear in Fig. 4.

can be handled accurately using only one or two terms, but additional terms may be included to improve stability without any additional gain in accuracy. We solve Eq. (7) using Crank-Nicolson integration in range and the following difference formula on a nonuniform azimuth grid [11]:

$$\gamma_i \frac{\partial^2 p}{\partial \theta^2} \cong \frac{1}{h_i} p_{i-1} - \left(\frac{1}{h_i} + \frac{1}{h_{i+1}} \right) p_i + \frac{1}{h_{i+1}} p_{i+1}, \tag{8}$$

where the ith grid point is at $\theta = \theta_i$, the ith grid spacing is $h_i = \theta_i - \theta_{i-1}$, and $2\gamma_i = h_i + h_{i+1}$. For the case of uniform spacing, this difference formula reduces to the standard $(1, -2, 1)$ difference formula for the second derivative.

Ocean acoustics problems are often solved by including an absorbing layer deep within the sediment to prevent reflections from the bottom boundary. Waves that penetrate into the sediment without being refracted or reflected back into the water column may be handled efficiently by using a nonuniform depth grid with extremely coarse sampling deep in the sediment and absorbing layer [11]. It is not necessary to handle such waves accurately when the objective is to obtain an accurate solution in the water column. A similar approach is applied here in the solution of Eq. (7). A fine grid is used to accurately handle the field in a narrow region that includes the vertical plane containing the source and receiver. Outside of that region, an extremely coarse grid is used to allow the field to spread in azimuth without producing nonphysical reflections.

III. IMPROVED STABILITY

Range-independent problems may be solved with the method of separation of variables. In the far field ($kr \gg 1$), the normal-mode solution is of the form [1],

$$p(r, z) = r^{-1/2} \sum_j a_j \phi_j(z) \exp\left(ik_j r\right), \tag{9}$$

$$\rho \frac{d}{dz} \left(\frac{1}{\rho} \frac{d\phi_j}{dz} \right) + k^2 \phi_j = k_j^2 \phi_j, \tag{10}$$

where the ϕ_j are the modes, the k_j^2 are the eigenvalues, and the coefficients a_j depend on the source condition. When k is real (no attenuation), $k_j^2 > 0$ for the propagating modes and $k_j^2 < 0$ for the non-propagating modes.

In parabolic equation solutions, the propagating modes may be handled to any level of accuracy with rational approximations that are based only on accuracy constraints (e.g., matching derivatives of the square root function at $X = 0$), but such approximations do not properly handle the non-propagating modes. This issue may not be a concern if non-propagating modes are not excited, but such modes may be excited and give rise to numerical noise when there is range dependence.

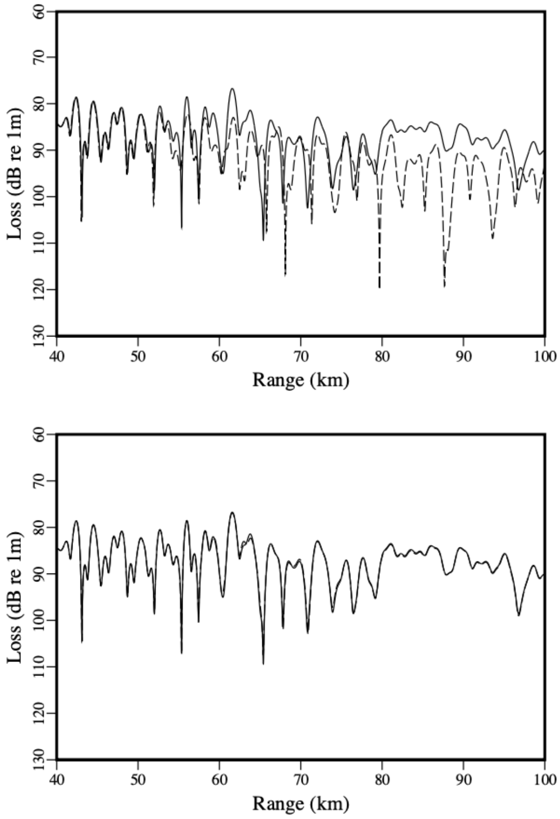

FIGURE 4. Results for example B. Top: Transmission loss at $z = 30$ m that was generated with the 3-D parabolic equation (solid) and the 2-D parabolic equation (dashed). Bottom: Comparison of 3-D solutions generated with the azimuthal grid described in the text (solid) and with double the number of azimuth grid points in the 36° sector (dashed).

This problem may be avoided by using rational approximations that are designed to (1) accurately handle the propagating modes and (2) annihilate the non-propagating modes. For practical purposes, it is usually not necessary to annihilate the non-propagating modes at the appropriate rate but rather to simply ensure that their contributions decay with range. We illustrate this issue with example A, which involves a 25 Hz point source at $z = 25$ m in a water column in which the sound speed is 1500 m/s; in the sediment, the sound speed is 1700 m/s, the density is 1.5 times the density of water, and the attenuation is 0.5 decibels per wavelength; the bathymetry (ocean depth) is 300 m for $r < 4$ km, 100 m for $r > 12$ km, and linearly sloping between these values for $4\,\text{km} < r < 12\,\text{km}$. The solutions appearing in Fig. 1 were generated using three-term rotated rational approximations [13] for the square root in Eq. (6). When the rotation angle is zero (stability is neglected), numerical noise is generated along the sloping ocean bottom. When the rotation angle is 20°, the numerical noise is annihilated.

Given that numerical noise can be an issue in the solution of Eq. (6), one would expect that it can also be an issue in the solution of Eq. (7). Both of these equations are parabolic equations, with one in Cartesian coordinates and the other in cylindrical coordinates. In testing the approach described in the previous section, we found that numerical noise can indeed arise in the solution of Eq. (7). In order to annihilate this type of numerical noise, we consider an approach based on the following alternatives to Eqs. (4) and (7):

$$(1 + X + Y)^{1/2} = (1 + X)^{1/2} + (1 + Y)^{1/2} - 1 + O(XY), \tag{11}$$

$$\frac{\partial p}{\partial r} = ik_0(1 + Y)^{1/2}p. \tag{12}$$

For 3-D problems in ocean acoustics, it is unlikely that there is any advantage to replacing the error term in Eq. (4) with the error term in Eq. (11), but non-propagating modes may be annihilated by using a rotated rational approximation for the square root in Eq. (12) as an alternative to the linear approximation in Eq. (7). In the splitting solution, the constant term in Eq. (11) may be taken into account exactly by including the factor $\exp(-ik_0 r)$.

IV. TEST CASES

Testing parabolic equation solutions has often been challenging due to the fact that this approach offers a unique combination of accuracy and efficiency. For many large-scale range-dependent problems, the parabolic equation method may be the only approach that is regarded as practical for obtaining solutions (e.g., run times for calculations with the finite-element method may be regarded as prohibitive for such problems). This is especially true for 3-D problems, but some parabolic equation solutions that handle azimuthal coupling have been tested qualitatively using ray models [15], [16]. The objectives here are to test the accuracy of the solution of Eq. (5) for different azimuthal grids and to illustrate the efficiency of the approach. For both of the test cases, the source frequency, source depth, and environmental parameters are the same as for example A.

For example B, energy is diffracted by an island that is defined by the bathymetry,

$$d(r,\theta) = \begin{cases} 0 \text{ for } R < R_1 \\ 0.05 \times (R - R_1) \text{ for } R_1 < R < R_2 \\ 300\,\text{m for } R > R_2 \end{cases}, \tag{13}$$

$$R^2 = x^2 + (y - 50\,\text{km})^2, \tag{14}$$

where $R_1 = 2.5$ km, $R_2 = 8.5$ km, $x = r\cos\theta$, and $y = r\sin\theta$ (the parabolic equation method was previously applied to other problems involving diffraction by islands in [10] and [17]). The same bottom slope and sediment parameters were used in a 2-D benchmark problem for which the parabolic equation method is known to be accurate [18]. A 3-D solution was generated using fine azimuthal sampling in a 36° sector centered at $\theta = 90°$, in which $\Delta\theta$ is 0.1° for $r < 10$ km, 0.05° for $10\,\text{km} < r < 20\,\text{km}$, 0.025° for $20\,\text{km} < r < 40\,\text{km}$, 0.0125° for $40\,\text{km} < r < 80\,\text{km}$, and 0.00625° for $r > 80$ km; $\Delta\theta$ is 5° outside of the 36° sector. The depth grid spacing Δz is 1 m for $z < 500$ m and 20 m for $500\,\text{m} < z < 1200\,\text{m}$.

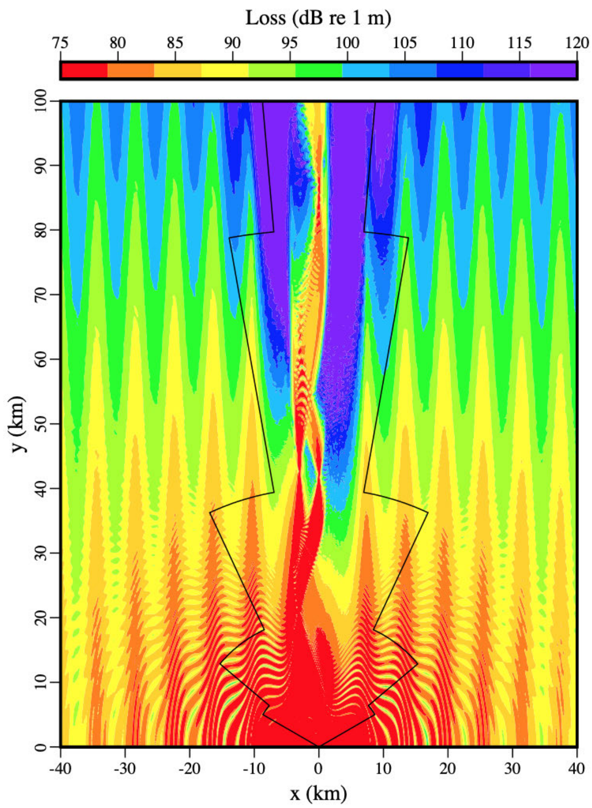

FIGURE 5. Results for example C. Transmission loss at $z = 30$ m that was generated with the 3-D parabolic equation. Fine azimuthal sampling was used in the region between the solid lines.

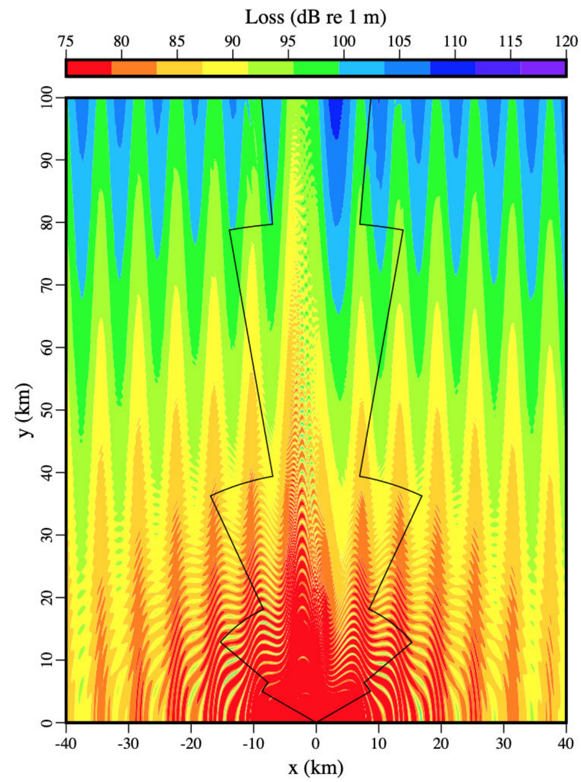

FIGURE 6. Results for example C. Transmission loss at $z = 30$ m that was generated with the 2-D parabolic equation. In generating a 3-D solution, fine azimuthal sampling was used in the region between the solid lines.

Color plots of transmission loss for the 2-D and 3-D solutions appear in Figs. 2 and 3. Since the sloping bottom deflects energy away from the island, there is a broader shadow zone in the 3-D solution. Another difference in the 3-D solution is diffracted energy between the boundary of the shadow zone and the solid lines (which appears as an interference pattern). Transmission loss at $\theta = 80^\circ$ and $z = 30$ m is compared quantitatively in Fig. 4. There are large differences between the 2-D and 3-D solutions in the top part of Fig. 4. Both of the solutions in the bottom part of Fig. 4 account for azimuthal coupling, but one of them was obtained by doubling the sampling in the 36° sector.

When numerically solving the frequency-domain wave equation (which accounts for outgoing and incoming energy), it is necessary to dedicate memory for the solution at every grid point in the domain; this is a serious limiting factor for 3-D problems. Since a parabolic equation is solved by marching in range, memory is required only for the solution at grid points near the current range. The number of grid points is proportional to $N_z \times N_\theta$, where N_z and N_θ are the numbers of grid points in depth and azimuth. The run time is proportional to the integral of $N_z \times N_\theta$ over range. All of the calculations in this study were done on a MacBook Pro with a 2.3 GHz 8-core Intel processor. With variable sampling in azimuth and depth, the run time was 38.5 minutes. On a uniform grid with $\Delta z = 1$ m for all z and $\Delta\theta = 0.00625^\circ$ for all (r, θ), the run time was more than 32 hours. About a factor of ten of the gain in efficiency was due to using fine sampling only in a 36° sector rather than in all directions; the rest of the gain in efficiency was due to refining the sampling with range (as opposed to using the finest sampling at all ranges) and using variable sampling in depth. For the uniform sampling calculation, the dependent variable was stored in a complex array with about 69 million entries. For the non-uniform sampling case, the dimension of the array was reduced by about a factor of twenty in the region with the most dense sampling.

For example C, energy is guided horizontally by a valley in the bathymetry,

$$d(r, \theta) = \left[3 - \sin\left(\frac{2\pi x}{6 \text{ km}}\right)\right] \times 50 \text{ m}. \quad (15)$$

This was the first ocean acoustics problem for which azimuthal coupling was found to be important [3]. For this case, we used fine sampling in a region that was updated at the same ranges as for example B, with the same values of $\Delta\theta$ but with the width of the sector starting at 120° and reducing to 100° at $r = 10$ km, 50° at $r = 20$ km, 20° at $r = 40$ km, and 10° at $r = 80$ km. Color plots of transmission loss for the 2-D and 3-D solutions appear in Figs. 5 and 6. There is strong horizontal ducting of energy along the valley. We also generated a 3-D solution for this case using fine sampling in a wider region and obtained similar results.

V. DISCUSSION

For many ocean acoustics problems, accurate solutions may be obtained by ignoring coupling of energy between planes of constant azimuth. At the next level of complexity, propagation paths from source to receiver are confined to a relatively narrow neighborhood of the vertical plane containing the source and receiver. For such problems, it should be possible to obtain accurate solutions by accounting for the depth operator to second order and accounting for the azimuth operator only to leading order. Although parabolic equations based on this approximation may be solved with the splitting method, it has been conventional wisdom that long-range calculations are often prohibitive. It is demonstrated here that problems involving azimuthal coupling may be solved efficiently out to long ranges by using a non-uniform grid to discretize the azimuth operator, with fine sampling in a narrow window that includes the plane containing the source and receiver and extremely coarse sampling elsewhere. This approach allows energy to propagate out of the narrow window without causing spurious reflections, which would occur if the computational domain were truncated in azimuth. Reductions in run time by more than a factor of ten were achieved for the examples. Even greater gains in efficiency are possible when narrower windows are appropriate.

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