Received 8 May 2023, accepted 4 June 2023, date of publication 13 June 2023, date of current version 3 November 2023.
Digital Object Identifier 10.1109/ACCESS.2023.3285763

RESEARCH ARTICLE

A Divide-and-Conquer Information Entropy Algorithm for Dependency Matrix Processing

JIASHUO ZHANG, DERONG CHEN, AND PENG GAO
School of Mechatronical Engineering, Beijing Institute of Technology, Beijing 100081, China
Corresponding author: Peng Gao (gaopeng@bit.edu.cn)

ABSTRACT Processing the system's dependency matrix is a core procedure for system diagnosis. However, the present algorithms for this job always try to maximize its fault isolation capability, which is thus not only unnecessary for the system that cannot be repaired in the field like weapons, but also generates low-efficient test sequence. To this end, the present work proposes a new processing algorithm for testability D-matrix, named Divide-and-Conquer Information Gain (DIG), targeting to those systems without strong fault isolation requirement. It combines the advantage of the classic algorithm Information Gain (IG) and Weight index for Fault Detection (WFD) by introducing a new entropy computing method considering the weight index for fault detection. To verify the advantage, generality and flexibility of the new algorithm, a D-matrix from a real system and random D-Matrixes are tested in the experiments, and measured by test sequence length, actual test cost and expected test cost. The result shows that DIG algorithm is 15.7% and 14.3% better than that of IG and WFD algorithm on the expected test cost metric, respectively.

INDEX TERMS System diagnosis, D-matrix, information gain, weight index for fault isolation, heuristic function.

I. INTRODUCTION

Diagnosing a man-made system to find out its availability or where is fault, is becoming more and more important, since such systems have been widely used in human society, becoming more and more complicated, and some of them have close relations to human life or safety of major property.

As to researches in system diagnosis, one of the core issues is how to generate the test items sequence from the fault-test dependency matrix (D-matrix) which represents the dependency between tests and the system's faults. As to this issue, Simpson et al. [1] gave a first comprehensive discussion. Then, Shakeri [2] discussed system fault modeling for more complex systems. Recently, Li et al. [3] reviewed the dependency matrix and its application in fault diagnosis, including the basics of dependency matrix generation, test selection methods, and fault diagnosis. In all, the popular D-matrix processing algorithms can be classified into three basic categories, that is Weight index for Fault Isolation (WFI) [4], Information Gain (IG) [5], [6], and Weight index for Fault Detection (WFD) algorithm [4]. Among those methods, the IG algorithm now is the most widely used one due to its simplicity and adaptability.

However, those researches always target to the general system diagnosis, which thus prefers to provide a higher isolation rate or suppose full fault isolation. But in practice, this is not true, at least not necessary. For example, the weapon system testing before firing, or some black-box system to the user, those systems cannot be repaired in the field, or their only diagnosis requirement is to find their availability. Therefore, for those systems without requirement of fault isolation, the advantages of present algorithms cannot get guaranteed anymore, neither does the optimality of the generated test sequences, since the precondition is changed [7]. To be specific, it means that considering fault isolation for the system without such a requirement will consume more test costs and diagnostic times. Apart from the impact of various test requirement, when the tested target is special, the present algorithms may also perform poorly. For example, when a system's fault probability distributes very unbalanced,

The associate editor coordinating the review of this manuscript and approving it for publication was Zhaojun Steven Li.

the above-mentioned algorithms may produce the sequence where the fault with a large probability appears in the rear, thus lengthening the test time.

As analyzed above, general D-matrix processing algorithms may not be appropriate for diagnosing special systems or for special-purpose tests. To solve the problem, numerous researches had been carried out. The most common solution to this issue was to modify the D-matrix processing algorithm. For example, Tsai et al [8] combined both fault detection weight and fault isolation weight to decide which test should be selected in D-matrix processing. Fu et al [9] studied the D-matrix of an aircraft ignition control system and proposed an improved discrete IG algorithm. Liu and Chao [10] used the backtracking method to improve the IG algorithm performance, so as to decrease test time and cost. Tian [11] proposed two novel algorithms, whose key ideas were to weighted mix different exist D-matrix algorithms as a new one, and adjust the weights according to different D-matrix density. Tian et al. [12] proposed a quasi-information entropy processing algorithm, which is the most related work to this paper. Its key idea was to use global information entropy (sometimes it is also shorted for information entropy) and local information entropy together to decide which test should be selected, and it indirectly used WDF weight as fault possibilities in local entropy calculation. Although the two works both have a sub-dividing operation, however, they still have significant differences. First, our work targets to the system that all faults have different probabilities, while Tian et al's research was still built on the opposite supposing, where the fault possibility was not utilized at all. Second, the entropy definitions in the two works are different. Third, our work did not use local entropy and it kept the unpassed part as usual. The latest different D-matrix processing algorithm was to consider the matrix as key-value pairs and use a searching method to find the test sequence, as Cui et al. [13] stated. In a word, the D-matrix processing algorithm has been well studied for general and special systems, but for the system having unbalanced fault distribution and without fault isolation requirement, it has still not got enough attention, which is just the concern of the present work.

To this end, the present work proposes a new D-matrix processing algorithm, named Divide-and-Conquer IG (DIG). Its key idea is built on the following facts: The information entropy (IG) algorithm mainly considers the uncertainty variation of the failed part of the D-matrix before/after performing a test, while the fault detection weight (WFD) algorithm mainly concerns how to detect as many faults as possible during each test turn. Therefore, a natural idea about processing the D-matrix which represents the system with different prior probabilities and no isolation requirements, is to combine the two methods together. To be specific, the algorithm divides the information entropy calculation process into several parts: each element of x_{jf} and the whole x_{jp}, then computing the sum of each element's entropy in x_{jf} and the entropy of x_{jp} set. In this manner, the fault probabilities and fault detection weight (number of elements in x_{jf}) are both introduced and fused with information entropy into a new heuristic function, which scores each available test so as to select the best one in each test turn.

II. DIVIDE-AND-CONQUER INFORMATION GAIN (DIG) ALGORITHM

A. BASIC KNOWLEDGE

To formally illustrate the DIG algorithm, a set of five elements (f, p, t, c, D) is first introduced as follows [14],

$$
D_{mn} = \begin{array}{c} \\ f_1 \\ \vdots \\ f_m \\ \\ \end{array}
\begin{array}{c}
\begin{array}{ccc} t_1 & \cdots & t_n \end{array} \\
\begin{bmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{m1} & \cdots & d_{mn} \end{bmatrix} \\
\begin{bmatrix} c_1 & \cdots & c_n \end{bmatrix}
\end{array}
\begin{array}{c} \\ \begin{bmatrix} p(f_1) \\ \vdots \\ p(f_m) \end{bmatrix} \\ \\ \end{array}
$$

FIGURE 1. Five elements for describing a test.

where, f_i $(1 \leq i \leq m)$ represents the i-th fault state of the system. $p(f_i)$ $(1 \leq i \leq m)$ represents the prior probability of f_i. $t_i (1 \leq i \leq n)$ indicates an independent test for the system and c_i $(1 \leq i \leq n)$ is the cost corresponding to the test t_i. $D = \left[d_{ij}\right]_{m \times n}$ is a two-dimensional binary matrix that represents the dependency between the fault and test for the tested system. It means that if the test t_i can find the fault f_i, then the element d_{ij} is set to 1, otherwise it is 0.

Processing the D-matrix always requires several turns of the test, each of which starts from using a heuristic function to calculate a score for each current available test t_j, so as to find which test is the best and should be chosen in this turn. After that, as the test is chosen, according to its dependency with faults, the D-matrix can be divided into two parts: the lines whose corresponding fault can be detected by the test, called the pass group, denoted by x_{jp}, and those cannot, called the failed group, denoted by x_{jf}. If no requirement to fault isolation, the x_{jf} set is used as the next D-matrix and this procedure is repeated until the D-matrix size is 0 or all tests have been used. Then, more definition is given as follows:

$$p(x) = \sum_{f_z \in x} p(f_z) \tag{1}$$

$$p\left(x_{jp}\right) = \sum_{f_j \in x_{jp}} p\left(f_j\right) \Big/ p(x) \tag{2}$$

$$p\left(x_{jf}\right) = \sum_{f_j \in x_{jf}} p\left(f_j\right) \Big/ p(x) \tag{3}$$

where $p\left(x_{jp}\right)$ and $p\left(x_{jf}\right)$ denote the conditional probabilities of the subset x_{jp} and x_{jf}, respectively.

Then we begin to define the heuristic function, which is the core of the D-matrix processing because it controls the test selection. There are two heuristic functions directly related to the present work. The first one uses the weight index of fault detection (WFD). Its definition of j-th test is defined in equation (4):

$$W_{FDj} = \sum_{i=1}^{m} d_{ij} \Big/ c_j \tag{4}$$

where m represents the number of fault states for all sub-matrices divided by the previous test. The second one, named IG, is based on the information entropy. It uses equation (5) to score each available test and select the one with highest score for current turn.

$$I(x, t_j) = -\left\{p(x_{jp})\log_2(p(x_{jp})) + p(x_{jf})\log_2(p(x_{jf}))\right\}/c_j \tag{5}$$

B. FORMALIZATION DESCRIPTION OF THE DIG

In the present work, a new information entropy of test-passed elements in x_{jp} definition is given in equation (6):

$$I'(x_{jp}) = \langle D_j, I_j \rangle = \sum_{i=1}^{l} d_{ij} I(x_{jp}, i) \Big/ c_j \tag{6}$$

where the j-th test of the i-th fault state in the set of elements that pass the test is shown in equation (7):

$$I(x_{jp}, i) = -p(f_i)/p(x)\log_2(p(f_i)/p(x)), \quad f_i \in x_{jp} \tag{7}$$

D_j denotes the vectors that passed the j-th test, where $D_j = [d_{1j}, d_{2j}, \ldots, d_{lj}]$.

I_j denotes the vectors of information entropy passing the test of item j, where $I_j = [I(x_{jp}, 1), I(x_{jp}, 2), \ldots, I(x_{jp}, l)]$. l denotes the number of fault states that passed the test in this turn. The information entropy of the failed elements in x_{jf} is list in equation (8), same to IG algorithm:

$$I'(x_{jf}) = -\left(\sum_{f_i \in x_{jf}} p(f_i) \Big/ p(x)\right) \times \log_2\left(\sum_{f_i \in x_{jf}} p(f_i) \Big/ p(x)\right) \Big/ c_j \tag{8}$$

So, the total information gain of the system under test t_j can be defined as (9), which acts as the heuristic function to score each available test during each turn of testing in DIG algorithm.

$$I'(x, t_j) = I'(x_{jp}) + I'(x_{jf}) \tag{9}$$

FIGURE 2 gives an example to clearly distinguish the difference between the DIG algorithm and the existing IG algorithm, where j-th test is chosen.

As FIGURE 2 shows, the existing IG algorithm only divides the information entropy of the D-matrix into two parts, representing for the elements that passed the test and those failed. While DIG algorithm divides the information entropy of the object matrix into more parts: the $I(x_{1f})$, as it was defined, the $I(x_{1p}, 1)$, and $I(x_{1p}, 2)$($l = 2$). The latter two definitions represent that each element's entropy will be counted individually (there are only two elements here), rather than only giving a subset's whole entropy, that is, we use the sum of entropies to replace the entropy of the sum.

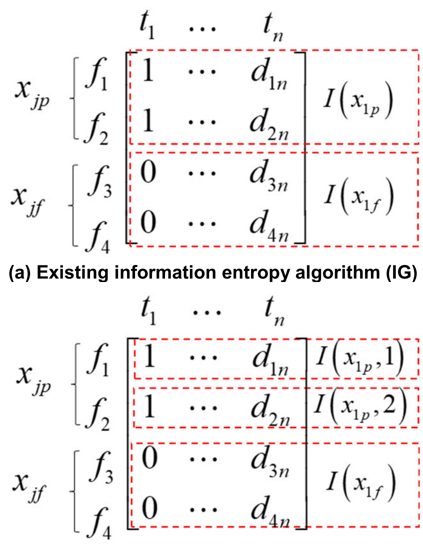
(a) Existing information entropy algorithm (IG)
(b) information entropy of DIG

FIGURE 2. Differences between DIG algorithm and existing information entropy algorithm.

So, in this example, the information entropy corresponding to the elements passed the test is expended as follows:

$$\begin{aligned} I(x_{1p}) &= \sum_{k=1} I(x_{1p}, k) = I(x_{1p}, 1) + I(x_{1p}, 2) \\ &= \sum_{i=1} (p(f_i)/p(x))\log_2(p(f_i)/p(x)) \\ &= (p(f_1)/p(x))\log_2(p(f_1)/p(x)) \\ &\quad + (p(f_2)/p(x))\log_2(p(f_2)/p(x)) \end{aligned}$$

In this manner, the weight of each sub item of x_{jp} can be considered in the DIG algorithm.

C. COMPARISON OF IG, WFD AND DIG ALGORITHM

To furtherly clarify the difference and advantage of DIG to existing algorithms IG and WFD, we place a detailed analysis and comparison of those algorithms in the following.

IG algorithm targets to lower the uncertainty of each test turn by using entropy from faults' possibility distribution. During the calculation process in the IG algorithm, as equation (5) shows, the possibilities of all faults are summed up as the set's possibility before calculating the entropy, rather than using each fault's possibility individually. In this manner, the IG algorithm brings a better fault isolation rate, but it also comes at the cost of a relatively longer test sequence since there is no connection between the test covering more faults and the test set with large entropy. Large number domination is another shortage of the IG algorithm, that is, when summing fault possibilities, the faults with large

TABLE 1. D-Matrix of the ESA.

Fault states	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	P A priori probability Note: (Prior probabilities are normalized for calculation)
Tests Cost (C)	1.612	1.297	1.078	1.748	1.021	1.619	1.68	1.979	1.252	1.697	1.435	
f_{10}	1	0	0	1	1	1	1	1	1	1	1	0.38
f_{20}	0	1	0	0	0	0	0	1	1	1	1	0.629
f_{30}	0	0	1	0	0	0	0	1	1	1	1	0.403
f_{40}	0	0	0	1	1	1	1	1	1	1	1	0.952
f_{50}	0	0	0	0	1	1	1	1	1	1	1	0.829
f_{60}	0	0	0	0	1	0	0	1	0	0	0	0.181
f_{61}	0	0	0	0	0	0	0	0	1	0	0	0.56
f_{70}	0	0	0	0	0	1	0	0	0	0	0	0.12
f_{71}	0	0	0	0	0	0	0	0	0	1	0	0.799
f_{80}	0	0	0	0	0	0	1	0	0	0	0	0.232
f_{81}	0	0	0	0	0	0	0	0	0	0	1	0.528
f_{90}	0	0	0	0	0	0	0	1	0	0	0	0.965

possibility values dominate the result, those ones with small possibilities cannot be distinguished, and thus be blended into the result set. For example, when a fault's possibility is very close to zero, then this fault still has a chance to be picked up to the next x_{jp} if other non-zero possibility faults could contribute enough possibilities to the sum.

Another popular D-matrix processing algorithm WFD only concerns which test is able to cover more faults in each test turn, and it doesn't use the faults' possibility at all. Therefore, a fault with high possibility but covered by few tests has a high possibility to be placed in the rear of the test sequence, which brings a larger test cost.

As contrast, each fault in x_{jp} will be considered individually in DIG. A fault with a small possibility contributes almost nothing to the entropy, therefore, it is less possible to be picked up to the next x_{jp}. Besides, as equation (6) shows, the fault weights used by WFD, are now merged into the entropy calculation in DIG, which could make contribution to a shorter test sequence.

III. EXPERIMENTS

This section uses a D-matrix of Electronic Safety and Arming (ESA) system and a random binary matrix together to verify the universality and stability of the DIG algorithm.

The widely used ESA device is responsible for safety and fire function in the weapon system. It basically cannot be repaired in the field and its different parts always have different fault possibilities. So, the present work uses ESA described in works [15], [16] to verify our algorithm. While using a random binary matrix is a commonly used way to verify the universality of the test algorithm in this area.

A. PROCESSING THE TESTABILITY D-MATRIX OF THE ESA

TABLE 1 shows the D-matrix of the ESA. $F = \{f_{00}, f_{10}, f_{11}, \ldots, f_{90}\}$ indicates the faults in the system under test, where f_{i0} and f_{i1} indicate general faults and functional faults of the tested unit, respectively. $T = \{t_1, t_2, \ldots, t_{11}\}$ denotes the set of available test items. $P = \{p(f_{00}), p(f_{10}), \ldots, p(f_{90})\}$ denotes the prior fault probability. $C = \{c_1, c_2, \ldots, c_{11}\}$ denotes the cost of the test.

1) DIANOSIS PROCEDURE AND COMPARISON

At first, taking t_5 as an example to calculate the information entropy of the set of elements passing the test t_j in TABLE 1, by using equations (6) to (7):

$$\begin{aligned} I'(x_{5p}, t_5) &= -\left(\frac{0.38}{7.318}\log_2\frac{0.38}{7.318} + \frac{0.952}{7.318}\log_2\frac{0.952}{7.318}\right. \\ &\quad \left. + \frac{0.829}{7.318}\log_2\frac{0.829}{7.318} + \frac{0.181}{7.318}\log_2\frac{0.181}{7.318}\right) \Big/ 1.021 \\ &= 1.06984 \end{aligned}$$

Then using equation (8) to calculate the information entropy of the set of elements that failed in the test t_5.

$$I'(x_{5f}, t_5) = -\left(\frac{4.976}{7.318}\log_2\frac{4.976}{7.318}\right) \Big/ 1.021 = 0.37059$$

Finally, the DIG entropy $I'(x, t_5)$ of t_5 is got by adding two values $I'(x_{5p}, t_5)$ and $I'(x_{5f}, t_5)$ as (9).

$$I'(x, t_5) = 1.06984 + 0.37059 = 1.44044$$

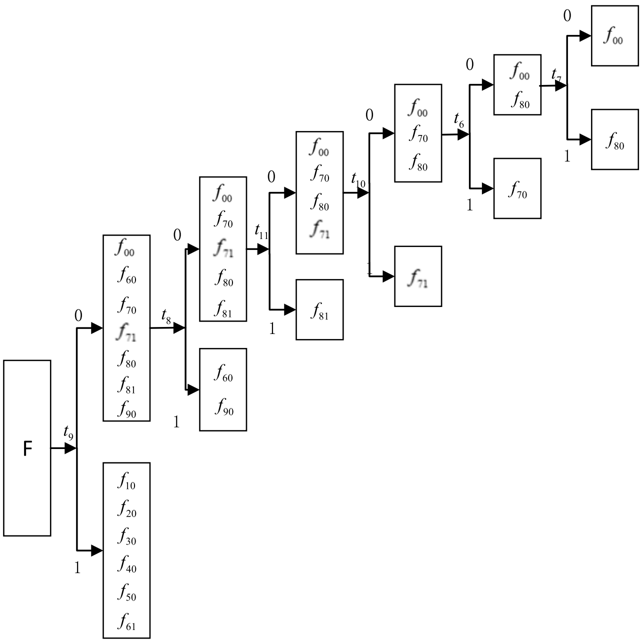

FIGURE 3. ESA diagnostic trees based on DIG algorithm.

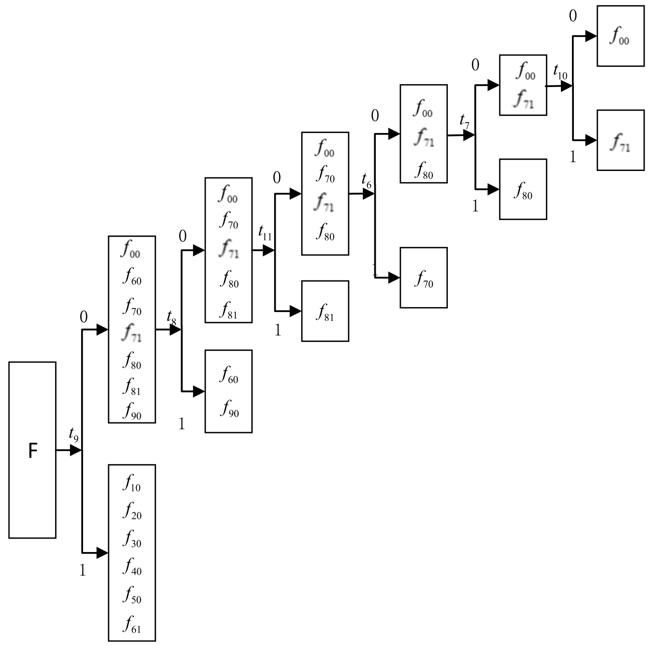

FIGURE 4. ESA diagnostic trees based on WFD algorithm.

Similar to the above calculation, the information entropy of each column is calculated and compared, resulting in $\max I'\left(x, t_j\right) = I'\left(x, t_9\right) = 1.8235$. Therefore, the test t_9 is chosen as the first test.

The calculation process for the second and subsequent test turns is as same as the first one. Finally, the diagnostic tree for ESA produced by the DIG algorithm is shown in FIGURE 3. As a comparison, the diagnostic tree generated using the WFD/IG algorithm is respectively shown in FIGURE 4 and FIGURE 5. Note that f_{00} indicates a fault-free system state.

It can be learned from FIGURE 3 to FIGURE 5 that the length of the diagnosis tree generated by DIG, is equal to the WFD algorithm and better than the IG algorithm. The diagnosis tree generated by DIG and WFD is very similar, the only difference lies in the 4^{th} test, DIG and WFD algorithm respectively chooses f_{71} and f_{70} as pass set. Although the length of diagnosis tree generated by the DIG and WFD algorithm are the same, the difference between the two algorithms will be obvious in terms of the cost of testing due to the different test chosen.

2) ANALYSIS AND COMPARISON OF RESULTS

In this section, the diagnosis trees shown in FIGURE 3 to FIGURE 5 are quantitatively analyzed to calculate the fault detection rate FDR, the average number of diagnosis steps N_D, the number of test items T_N, the average diagnosis time *Times* and the expectation of test cost (ETC) to evaluate the IG, WFD and DIG algorithm [4], [9]:

$$\mathrm{FDR} = U_{FD} / U_T \times 100\%, \tag{10}$$

$$N_D = \sum_{i=0}^{m-1} |T_i| / m, \tag{11}$$

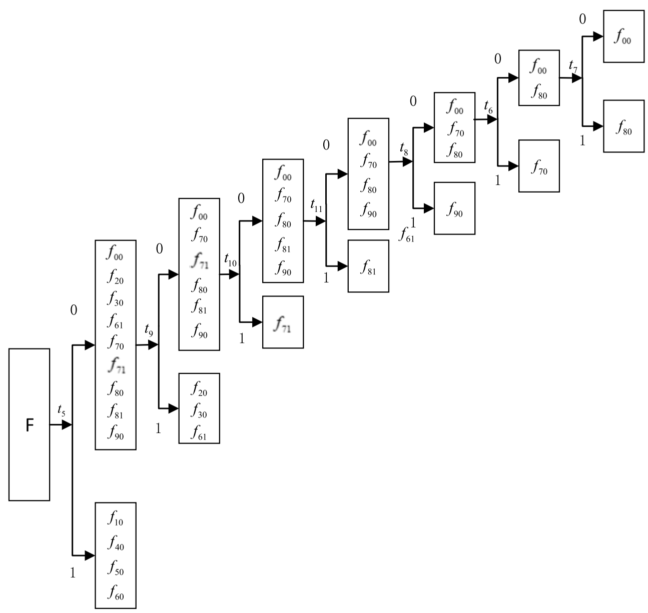

FIGURE 5. ESA diagnostic trees based on IG algorithm.

$$T_N = \left| \bigcup_{i=0}^{m-1} T_i \right|, \tag{12}$$

$$\mathrm{ETC} = \sum_{i=0}^{m-1} p\left(f_i\right) \sum_{t_j \in T_i} c_j \tag{13}$$

where U_{FD} is the number of Units Under Tests(UUTs) that can be detected by the test; U_T is the total number of UUTs; m is the number of Failure states;T_i is the optimal test sequence for f_i; $|T_i|$ is the length of T_i;$P\left(f_i\right)$ is the probability off_i; c_j is the corresponding test cost.

Besides of using length, another intuitive way to measure the performance of a test sequence is to use the sum of the actual cost of each test in the sequence. So, we introduce this index and name it as the Actual Test Cost (ATC), which definition is as equation (14) shows:

$$\mathrm{ATC} = \sum_{t_j \in T_i} c_j, \tag{14}$$

For the D-matrix of the ESA system described in TABLE 1, the comparison of each index is shown in TABLE 2.

TABLE 2. Comparison between existing algorithm and DIG algorithm used for ESA.

Algorithm	FDR /100%	N_D	T_N	ATC	ETC
IG	100	3.5	7	10.6855	22.75
WFD	100	2.83	6	**9.6643**	21.93
DIG	**100**	**2.83**	**6**	**9.6643**	**19.18**

As it can be seen from TABLE, compared to the IG algorithm, DIG reduces the average number of diagnostic steps by 0.67(19.1%), the number of test items by 1.0(14.3%), the average diagnostic time by 0.04 (14%) and the ATC by 1.0212 (9.2%). Compared to the IG algorithm, DIG reduces the ETC by 3.57(15.6%). compared to the WFD algorithm, DIG reduces the ETC by 2.75(12.5%).

In a word, the DIG performs better than the control group in every index for the system without fault isolation requirement and meanwhile having unbalanced fault possibility distribution.

B. EXPERIMENTAL ANALYSES FOR RANDOM D-MATRIX

In this section, the DIG algorithm is applied to random D-matrices of different densities (the ratio of elements in the matrix with value 1 to the number of all elements) and sizes, to verify its universality and stability. The control group uses the classic algorithm IG, WFD, and the latest related algorithm MIX1 [11]. For a fair comparison, the parameters keep the same to the original literature.

The experimental setup is as follows: three D-matrix groups of dimension 21×20, 41×40 and 61×60 are generated. Each group is consisted of 19 random matrices with densities ranging from 0.05, 0.1…0.95, as work [17] states. The IG, WFD, DIG, and MIX1 algorithms are then applied to these matrices. Each of the experiments will repeated 100 times. Each time a new matrix, as well as test cost and prior probability are regenerated. Finally, the algorithm is evaluated in terms of the mean value ETC, which is as equation (15) shows,

$$\mathrm{avg\_ETC} = \sum_{i=1}^{Q} \mathrm{ETC_i}/Q \tag{15}$$

where $\mathrm{ETC_i}$ denotes the ETC of the i-th experiment and Q is the total number.

TABLE 3. Comparison of avg_ETC indicators for test sequences generated by DIG, IG, and WFD algorithms (21×20).

Density of D-matrix	avg_ETC for WFD	avg_ETC for IG	avg_ETC for MIX	avg_ETC for DIG
0.08	4.44	3.87	4.24	3.89
0.11	3.49	3.16	3.41	3.16
0.16	2.89	2.68	2.83	2.66
0.20	2.41	2.32	2.38	2.25
0.25	2.08	2.10	2.09	1.96
0.30	1.85	2.01	1.95	1.76
0.35	1.67	1.98	1.74	1.61
0.40	1.53	1.96	1.85	1.49
0.45	1.47	1.95	1.85	1.42
0.50	1.35	1.95	1.84	1.32
0.55	1.29	1.94	1.86	1.26
0.60	1.22	1.93	1.90	1.22
0.65	1.17	1.93	1.91	1.17
0.70	1.13	1.89	1.88	1.12
0.75	1.08	1.81	1.79	1.08
0.80	1.05	1.69	1.68	1.05
0.85	1.03	1.57	1.56	1.04
0.90	1.01	1.41	1.41	1.03
0.95	1.00	1.25	1.24	1.03

TABLE 4. Comparison of avg_ETC indicators for test sequences generated by DIG, IG, and WFD algorithms (41×40).

Density of D-matrix	avg_ETC for WFD	avg_ETC for IG	avg_ETC for MIX	avg_ETC for DIG
0.06	6.46	5.60	6.17	5.61
0.10	4.26	3.87	4.15	3.88
0.15	3.15	2.93	3.09	2.91
0.20	2.61	2.47	2.57	2.43
0.25	2.24	2.20	2.23	2.11
0.30	1.98	2.04	2.02	1.87
0.35	1.76	1.99	1.82	1.70
0.40	1.62	1.98	1.89	1.56
0.45	1.50	1.97	1.88	1.46
0.50	1.41	1.97	1.87	1.38
0.55	1.35	1.97	1.90	1.31
0.60	1.27	1.97	1.93	1.24
0.65	1.20	1.95	1.93	1.18
0.70	1.15	1.90	1.89	1.14
0.75	1.11	1.79	1.77	1.11
0.80	1.08	1.66	1.64	1.07
0.85	1.04	1.52	1.51	1.04
0.90	1.02	1.37	1.37	1.02
0.95	1.00	1.22	1.22	1.02

The experiment result is listed in FIGURE 6 to FIGURE 8 (and TABLE 3 to TABLE 5). As the figures show, whatever

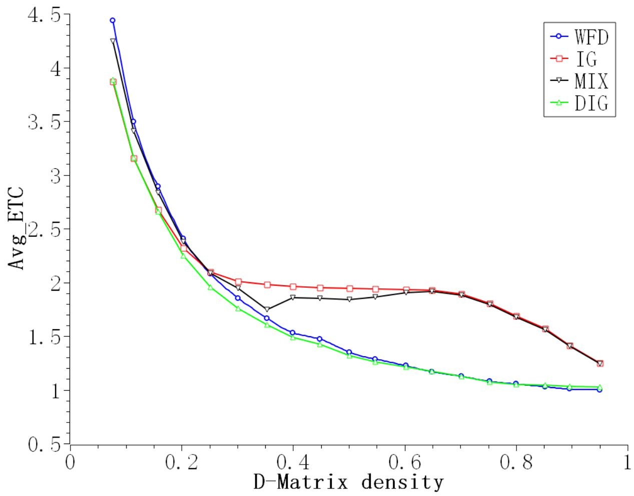

FIGURE 6. **Comparison of** avg_ETC **Indicators for Test Sequences Generated by DIG, IG, and WFD Algorithms (21 × 20).**

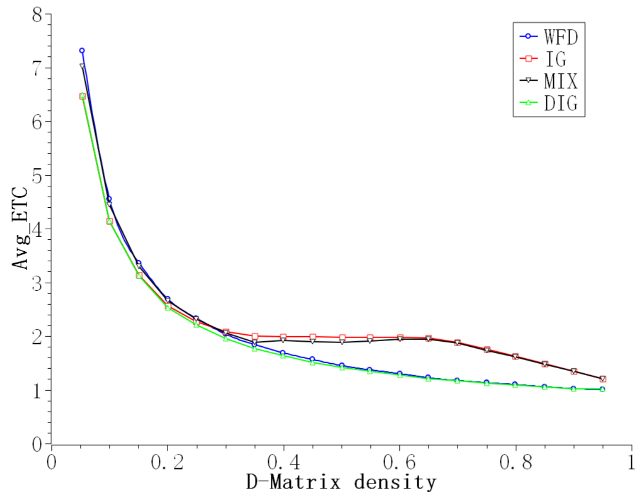

FIGURE 8. **Comparison of** avg_ETC **Indicators for Test Sequences Generated by DIG, IG, and WFD Algorithms (61 × 60).**

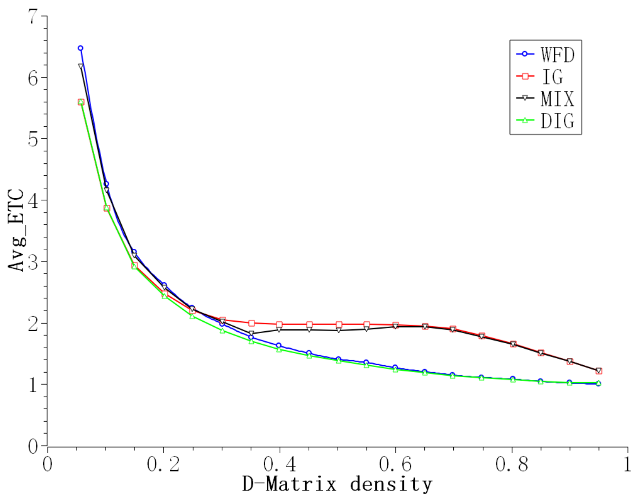

FIGURE 7. **Comparison of** avg_ETC **Indicators for Test Sequences Generated by DIG, IG, and WFD Algorithms (41 × 40).**

TABLE 5. **Comparison of** avg_ETC **indicators for test sequences generated by DIG, IG, and WFD algorithms (61 × 60).**

Density of D-matrix	avg_ETC for WFD	avg_ETC for IG	avg_ETC for MIX	avg_ETC for DIG
0.05	7.31	6.46	7.02	6.49
0.10	4.55	4.14	4.44	4.15
0.15	3.36	3.13	3.30	3.12
0.20	2.69	2.56	2.65	2.53
0.25	2.33	2.27	2.31	2.21
0.30	2.04	2.08	2.06	1.95
0.35	1.84	1.99	1.88	1.76
0.40	1.69	1.99	1.91	1.64
0.45	1.56	1.98	1.90	1.51
0.50	1.45	1.98	1.88	1.41
0.55	1.37	1.97	1.90	1.35
0.60	1.30	1.97	1.94	1.27
0.65	1.22	1.96	1.95	1.21
0.70	1.17	1.89	1.87	1.16
0.75	1.13	1.75	1.74	1.13
0.80	1.10	1.63	1.61	1.09
0.85	1.06	1.49	1.48	1.05
0.90	1.02	1.35	1.34	1.02
0.95	1.00	1.20	1.20	1.01

the size of the D-matrix is, if the density of the matrix is lower than 25% to 30%, performances of DIG and IG algorithms are almost same, better than that of the WFD and MIX1 algorithms. When the matrix density increases from 30% to around 70%, the avg_ETC of IG and MIX1 algorithms are both increasing so rapidly that makes them lose the advantage, but meanwhile, the performance of DIG and WFD algorithms keep the decreasing trend, while DIG is still the best. When the matrix density is getting higher than 70%, which means that the dependency of tests and faults is so tight that only a few tests are enough to finish the diagnosis, the performance gap among all algorithms gradually becomes less discriminative.

So, to sum up, on the avg_ETC index, whatever the size and density of the D-matrix is, the proposed DIG algorithm performs equal to or better than the controlled group including the classic IG, WFD algorithm, and the latest MIX1 algorithm.

IV. CONCLUSION

The present work proposes a new testability D-matrix processing algorithm based on the divide-and-conquer method to the information entropy. The experiment results on certain

D-matrix and random matrixes prove the advantages of the DIG algorithm as well as its wide adaptivity, for the tested system that has unbalanced priori probabilities and without fault isolation requirement.

REFERENCES

[1] W. R. Simpson and J. W. Sheppard, *System Test and Diagnosis*. Boston, MA, USA: Kluwer, 1994, pp. 91–138.

[2] M. Shakeri, ''Advances in system fault modeling and diagnosis,'' Ph.D. dissertation, UConn, Mansfield, CT, USA, 1996.

[3] X. Li, W. Cui, L. Li, and J. Yang, ''A review on dependency matrix and its application in fault diagnosis,'' in *Proc. Global Rel. Prognostics Health Manage. (PHM-Yantai)*, Yantai, China, Oct. 2022, pp. 1–8, doi: 10.1109/PHM-Yantai55411.2022.9941798.

[4] J. Y. Shi, *Testability Design Analysis and Verification?* Beijing, China, 2011, pp. 1–50.

[5] K. R. Pattipati and M. G. Alexandridis, ''Application of heuristic search and information theory to sequential fault diagnosis,'' *IEEE Trans. Syst., Man, Cybern.*, vol. 20, no. 4, pp. 872–887, Jul./Aug. 1990, doi: 10.1109/21.105086.

[6] O. E. Kundakcioglu and T. Unluyurt, ''Bottom-up construction of minimum-cost and/or trees for sequential fault diagnosis,'' *IEEE Trans. Syst., Man, Cybern. A, Syst., Humans*, vol. 37, no. 5, pp. 621–629, Sep. 2007.

[7] S. Deb, K. R. Pattipati, V. Raghavan, M. Shakeri, and R. Shrestha, ''Multi-signal flow graphs: A novel approach for system testability analysis and fault diagnosis,'' *IEEE Aerosp. Electron. Syst. Mag.*, vol. 10, no. 5, pp. 14–25, May 1995, doi: 10.1109/62.373993.

[8] Y.-T. Tsai and Y.-Y. Hsu, ''A study of function-based diagnosis strategy and testability analysis for a system,'' *Proc. Inst. Mech. Eng., C, J. Mech. Eng. Sci.*, vol. 226, no. 1, pp. 273–282, Jan. 2012, doi: 10.1177/0954406211412017.

[9] X. Fu, J. Shen, and B. Luo, ''Research on optimal diagnostic strategy based on multi-signal flow graph model and information entropy,'' presented at the 5th Int. Conf. Intell. Comput. Technol. Automat., Zhangjiajie, China, 2012. [Online]. Available: https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6150163

[10] S. Liu and L. Chao, ''Approach of optimal diagnosis test sequence based on improved information entropy,'' *Electron. Meas. Technol.*, vol. 36, no. 12, pp. 28–31, 2013, doi: 10.19651/j.cnki.emt.2013.12.008.

[11] H. Tian, ''Research of fault diagnosis strategy based on testability D-matrix,'' Ph.D. dissertation, Dept. Mech. Manuf. Automat., DUT Univ, Dalian, China, 2019.

[12] H. Tian, Y. Duan, and Y. Sand, ''A novel fault diagnosis algorithm of testability D matrix based on quasi information entropy,'' *Acta Armamentarii*, vol. 37, no. 5, pp. 923–928, 2016, doi: 10.3969/j.issn.1000-1093.2016.05.021.

[13] W. Cui, L. Li, H. Li, F. Guan, D. Hong, and G. Liu, ''Dependency matrix-based fast fault diagnosis using search algorithm,'' *IEEE Trans. Instrum. Meas.*, vol. 72, pp. 1–8, 2023, doi: 10.1109/TIM.2023.3259042.

[14] J. W. Sheppard and S. G. W. Butcher, ''A formal analysis of fault diagnosis with D-matrices,'' *J. Electron. Test.*, vol. 23, no. 4, pp. 309–322, Aug. 2007, doi: 10.1007/s10836-006-0628-7.

[15] W. Jiang et al., ''Test sequence optimization for fire control system of a certain antiaircraft gun,'' *Electron. Opt. Control*, vol. 17, no. 4, pp. 74–78, 2010, doi: 10.3969/j.issn.1671-637X.2010.04.019.

[16] X. Yan et al., ''Study on fault diagnosis strategy of radio fuze,'' *Chin. J. Sci. Instrum.*, vol. 29, no. 4, pp. 63–66. 2008.

[17] H. Tian, F. Duan, L. Fan, and Y. Sang, ''Novel solution for sequential fault diagnosis based on a growing algorithm,'' *Rel. Eng. Syst. Saf.*, vol. 192, Dec. 2019, Art. no. 106174, doi: 10.1016/j.ress.2018.06.002.

JIASHUO ZHANG was born in Gaobeidian, Baoding, Hebei, China. He received the bachelor's and master's degrees from the School of Mechatronical Engineering, Beijing Institute of Technology, in 2016 and 2019, respectively, where he is currently pursuing the Ph.D. degree. From 2016 to 2019, he was mainly engaged in research on electronic safety systems. Since 2019, he has been involved in system testability design and safety system testing. He has participated in several relevant projects.

DERONG CHEN received the bachelor's degree from the Beijing University of Aeronautics and Astronautics, in 1988, and the Ph.D. degree from the Beijing Institute of Technology (BIT), in 2002. After graduating from BIT, she was a Senior Engineer with the Department of Electronics, Tsinghua University, for two years. From 2008 to 2009, she was a Visiting Scholar with Oklahoma State University. Since 2004, she has been a Professor with the School of Mechatronical Engineering, BIT. Her research interests include vision measurement and data compression and system diagnosis.

PENG GAO was born in Huhehaote, Inner Mongolia, China. He received the bachelor's degree from the College of Computer Science and Technology, Zhejiang University, in 2005, and the Ph.D. degree from the College of Computer Science and Technology, Tsinghua University, in 2018. He is currently an Assistant Professor with the School of Mechatronical Engineering, Beijing Institute of Technology. His research interests include testability modeling and anomaly detection techniques.