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RESEARCH ARTICLE

Evaluation of Electric Motor Cars Based Frank Power Aggregation Operators Under Picture Fuzzy Information and a Multi-Attribute Group Decision-Making Process

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ABSTRACT In this article, we expose the notion of power operator to reduce the impact of negative information on the decision-making (DM) process. The power aggregation tools are also robust mathematical aggregation operators (AOs) which allow input arguments to support each other in the DM process. The Frank aggregation expressions are reliable and updated versions of triangular norms which are used to handle complex and complicated information in a decision-making process. The picture fuzzy (PF) set (PFS) is an extended version of the fuzzy sets (FSs) and intuitionistic FSs (IFSs). A PFS has four terms of an object simultaneously such as positive grade (PG), Abstained grade (AG), negative grade (NG) and refusal grade (RG). By using basic operations of Frank aggregation expressions, we propose a list of new appropriate methodologies under consideration of PF information, including "picture fuzzy frank power average" (PFFPA), and "picture fuzzy frank power geometric" (PFFPG) operators. We also present some new approaches to PFSs based on Frank aggregation tools such as "picture fuzzy frank power weighted average" (PFFPWA) and "picture fuzzy frank power weighted geometric" (PFFPWG) operators. Some appropriate properties and special cases of our currently proposed approaches are also studied. Moreover, to ratify the intensity and reliability of our derived strategies, we illustrated an algorithm of the multi-attribute group decision-making (MAGDM) technique under a PF environment. Furthermore, we illustrated a practical case study to evaluate a suitable optimal option by considering our proposed approaches and analyzed the performance of our currently derived approaches by comparing the results of existing methodologies.

INDEX TERMS Frank aggregation tools, picture fuzzy numbers, power aggregation operates, multi-attribute group decision-making process.

I. INTRODUCTION

The main principle of the multi-attribute decision-making (MADM) method, which is a modified form of the simple decision-making scenario, is a remarkable and dominant strategy for illuminating the valuable preference from the collection of preferences. We evaluate or encounter various decision-making issues in our daily lives, and one of the most important things we can do is learn how to make good decisions. The expert frequently presents traditional knowledge without analyzing its level of ambiguity and uncertainty. We saw many scholars work on

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the decision-making process and multi-criteria decision-making (MCDM) process refer to [1], [2], [3], and [4]. Nonetheless, one of the most useful and deserving hypotheses for expressing the positive degree (PD) of the information whose range is in the unit interval [0, 1] known as fuzzy sets (FSs) [5] to treat the ambiguity in the information. Advanced speculative theory of the FS known as intuitionistic FSs given by Atanassov [6], an IFS contains two terms of any object like negative degree (ND) and PD; the sum of PD and ND lies on [0, 1]. Yager [7] extended the theory of IFSs by relaxing the condition of the PD and ND. Yager [8] illustrated a robust concept of q-rung orthopair FSs (q-ROFSs) with conditions on PD and ND in such a way that the sum of PD and ND lies on [0, 1]. The IFSs and FSs applied in several fields of life, including medical diagnosis, clustering points, networking, and supplier system. But there are some shortcomings in above discussed theories. These theories cannot handle such situations of human opinions which contain more than two aspects of membership terms. In the electoral process, different possibilities are yes, no, abstain or neutral and refusal value. To handle such scenarios, Cuong [9]generalized the concepts of IFS and FSs with PD, abstain degree (AD), and ND. Above discussed theories are used to solve many applications in different fuzzy environments [10], [11], [12].

The AOs are utilized to evaluate human opinions under the consideration of PF information by numerous research scholars in different fuzzy circumstances. The theoretical concepts of weighted average and weighted geometric operators were discovered by Xu [13]. Li et al. [14] stated some appropriate mathematical approaches of Dombi aggregation tools based on IFSs. They also established an experimental case study to find the supremacy and superiority of our derived approaches under the MADM technique. Wei and Gao [15] extended the theoretical concepts of similarity measures and found the correlation among different input arguments under the consideration of our current derived approaches. Garg [16] elaborated on trigonometric function with some particular properties of derived methodologies and analyzed the supremacy and superiority of invented approaches based on the MADM technique. Haktanır and Kahraman [17] illustrated the characteristics of PFSs and presented a novel approach to the decision-making process to assess advanced health technologies. A novel approach to reducing the effects of traditional vehicles and analyzed advanced technology of electric motor cars by He and Wang [18]. Verma and Rohtagi [19]explored the concepts of similarity measures under a system of PF information and examined an application of medical diagnosis based on the decision-making process. Hussain et al. [20]also presented some well-known aggregation approaches to illustrate the technique of vendor management systems. Li et al. [21]also showed the robustness of proposed mathematical aggregation approaches under consideration of a MADM problem. Xing et al. [22] explored the theory of Hesitant FSs and developed a series of new approaches simultaneously. Tang et al. [23] genialized the theory of Hesitant FSs to overcome the impact of different attributes under consideration of Frank aggregation expressions. Xing [24] utilized the notion of power aggregation tools to reduce the impact of negative information based on Frank aggregation expressions in the decision-making process. A large number of scientists proposed several aggregation techniques and algorithms under consideration of Frank aggregation expressions seen in references [25], [26], and [27].

Hussain et al. [28] stated some prominent approaches by utilizing Hamy mean (HM) aggregation models under consideration of complex IFSs. Liu and Gao [29] interpreted the theory of Dempster Shafer based on IFSs and developed some AOs of power Bonferroni mean models to evaluate human opinions based on multicriteria decision-making. Ullah [30] elaborated the theory of PFSs and derived new approaches to solving a MADM technique. Wu et al. [31] examined the theoretical concepts of 2-tuple linguistic neutrosophic sets and presented new approaches of Hamy mean models to assess risk management in the construction industry. Akram and Shahzadi [32] utilized some appropriate properties of Yager aggregating tools to provide a list of new approaches based on q-ROFSs. Riaz et al. [33] anticipated a list of new methodologies by using theoretical concepts of Aczel Alsina aggregation tools based on Spherical FSs. A list of some particular approaches based on power aggregation tools gave an application under consideration of Pythagorean 2-tuple linguistic information by Wei and Gao [34]. Wei [35] evaluated unpredictable human information by using Hamacher aggregation tools under PF information and established applications based on multi-criteria decision-making problems. Ahmmad et al. [36] listed some new appropriate approaches for Spherical FSs by generalizing the theory of weighted average and weighted geometric to evaluate an MCDM problem. An algorithm under consideration of T-Spherical fuzzy information and derived a list of new AOs to evaluate a MADM problem by Garg et al. [37].

The theoretical concepts of triangular norms in statistical metric space with some specific properties are given by Menger [38]. Several scientists explored the potential of triangular norms and their characteristics by introducing advanced mathematical tools, such as algebraic products. The probabilistic algebraic sum of triangular norms in the fuzzy system is given by Lee [39]. Schweizer and Sklar [40] enlarged the theoretical concepts of triangular norms in topological metric space. Garg [41] provided a list of new approaches using Hamacher aggregation tools to overcome the loss of information during the aggregation process under the consideration of IFSs. Some appropriate aggregation tools based on complex T-Spherical FSs developed by Ali et al. [42], Mahmood [43] established a list of new approaches to the complex bipolar fuzzy set and gave an application to solve multi-attribute decision-making (MADM) problems. Hussain et al. [44] proposed an algorithm of a MAGDM technique to evaluate unreliable and unpredictable information under consideration of an interval-valued PyF system. Li et al. [14] proposed a list of particular approaches to accommodate unpredictable situations of human opinions based on different attributes under consideration of IFSs. Akram et al. [45] elaborated on the speculative theory of Dombi aggregation tools. They stated some prominent methodologies to evaluate other optimal options based on different criteria under the system of PyF information. Ali Khan et al. [46] proposed new approaches using the theory of Einstein's prioritized aggregation tools and evaluated a real-life problem based on the MAGDM technique. Some robust aggregation tools use the speculative hypothesis of an Aczel Alsina aggregation model under the consideration of PyF environments given by Hussain et al. [47]. The theory of Frank aggregation operators is generalized as interval-valued IFSs by Zhang [48]. Yahya et al. [49] exposed some new aggregation models using the concepts of Hesitant Fuzzy information. Mahmood et al. [50] elaborated the theory of Frank aggregation tools and investigated the analytic hierarchy process under the consideration of interval-valued PF information. Wei [51] also exposed the speculative theory of cosine similarity measures and observed the correlation among several arguments.

Road transportation contributes significantly to carbon dioxide, nitrogen oxides, and particulate matter emissions, making it a substantial source of both greenhouse gases and air pollutants. Many researchers have different ideas in different environments to overcome these complicated situations. Recently, Bessa and Matos [52] gave an appropriate mechanism to enhance any country's economic growth by developing transportation infrastructure. Jia et al. [53] presented a novel approach and evaluated different strategies to overcome environmental pollution. Loschan et al. [54] illustrated various ways to reduce transmission congestion and studied the flexibility of different electric cars by using aggregation tools.

After evaluating all existing research work, we observed that numerous research scientists have introduced different mathematical strategies and advanced approaches. These strategies producing by different triangular norms such as algebraic product and probabilistic sum, Dombi aggregation expressions, Einstein aggregation expressions, and Aczel Alsina aggregation expressions. All discussed basic operations of triangular norms are utilized to aggregate imprecision information under several fuzzy environments in the decision-making process. The Frank aggregation operations are more reliable than existing ones and provide a smooth approximation during the aggregation process. We concluded sometimes theory of FSs and IFSs cannot deal with complex and complicated challenges. For this situation, the theory of PFS is the extended version of IFSs and FSs and has a great capability to deal with vague information. Keeping in mind the significance and effectiveness of PFSs, we exposed some basic operations of PFSs under consideration Frank mathematical expressions. The Power aggregation models are utilized to reduce the impact of attributes and permitted to input of arguments to support each other in the decision-making process. The aim of this article is proposed as follows:

a) Firstly, we expressed some necessary operations of Frank aggregation tools under consideration of PF information. Which are used to obtained a smooth approximation during aggregation process.
b) Robust concepts of power average and geometric operators were also studied.
c) Some appropriate methodologies of PF information are also derived such as PFFPA, PFFPG, PFFPWA and PFFPWG operators. We also exposed some prominent characteristics and exceptional cases of our derived approaches.
d) To ratify the intensity and validity of the currently discussed approaches, we illustrate a MAGDM technique and try to evaluate the consideration of the proposed algorithm.
e) An experimental case study is also analyzed by using our derived approaches under the decision-making process based on a MAGDM technique.
f) To reveal the supremacy and superiority of our derived approaches, make an extensive comparative study to compare existing results with the results of currently proposed approaches.

The structure of this article is maintained as follows: In section II, a brief discussion about PFSs and their related basic operations under the consideration of PF information. The generalization of triangular norms in section III exposes some appropriate mathematical models like Frank aggregation tools. In section IV, we anticipated some appropriate approaches for PFSs, such as PFFPA and PFFOPG operators. The AOs of the PFFPWA and PFFPWG operators based on PF information are derived in section V. In section VI, we ratify the intensity and validity of our derived approaches and construct a MAGDM technique to evaluate PF information. In section VII, a comparative study is also exposed to find the supremacy and superiority of the currently discussed methodologies. In the end, a summary of this article is also present in this article.

II. PRELIMINARIES

Firstly, a brief discussion about PFSs, related to their fundamental operations and comparison among input arguments based on PF information.

Definition 1 [9]: Suppose $\mathcal{M}$ be a universal set and a PFS Γ is defined as follows:

$$\Gamma = \{\langle \mathfrak{J}, \alpha_{\Gamma}(\mathfrak{J}), \beta_{\Gamma}(\mathfrak{J}), \gamma_{\Gamma}(\mathfrak{J}) | \mathfrak{J} \epsilon \mathcal{M} \rangle\}$$

where $\alpha_{\Gamma}: \mathfrak{M} \to [0,1]$, $\beta_{\Gamma}(\mathfrak{I}) : \mathfrak{M} \to [0,1]$, and $\gamma_{\Gamma}(\mathfrak{I}) : \mathfrak{M} \to [0,1]$ represents the positive degree (PD), abstinence degree (AD), and negative degree (ND), respectively. A PFS satisfies the condition:

$$0 \le \alpha_{\Gamma}(\mathfrak{I}) + \beta_{\Gamma}(\mathfrak{I}) + \gamma_{\Gamma}(\mathfrak{I}) \le 1,\ \forall,\ \mathfrak{I} \in \mathfrak{M}$$

The refusal degree of a PFS is denoted by $H_{\Gamma}(\mathfrak{I}) = 1-\alpha_{\Gamma}(\mathfrak{I})-\beta_{\Gamma}(\mathfrak{I})-\gamma_{\Gamma}(\mathfrak{I})$ and a PF value (PFV) is denoted by the $\Gamma = (\alpha_{\Gamma}, \beta_{\Gamma}, \gamma_{\Gamma})$.

Definition 2 [55]: Consider $A = (\alpha_A, \beta_A, \gamma_A)$ and $B = (\alpha_B, \beta_B, \gamma_B)$ are two PFVs over the universal set X and $\tau > 0$ be any real number. Then we have:

a) $A \le B$ if $\alpha_A \le \alpha_B$, $\beta_A \le \beta_B$ and $\gamma_A \ge \gamma_B$
b) $A \vee B = (max\{\alpha_A, \alpha_B\},\ min\{\beta_A, \beta_B\},\ min\{\gamma_A, \gamma_B\})$
c) $A \wedge B = (min\{\alpha_A, \alpha_B\}, max\{\beta_A, \beta_B\}, max\{\gamma_A, \gamma_B\})$
d) $A^{\complement} = (\gamma_A, \beta_A, \alpha_A)$
e) $A \oplus B = (\alpha_A + \alpha_B - \alpha_A\alpha_B, \beta_A\beta_B, \gamma_A\gamma_B)$
f) $A \otimes B = \begin{pmatrix} \alpha_A\alpha_B, \\ \beta_A + \beta_B - \beta_A\beta_B, \gamma_A + \gamma_B - \gamma_A\gamma_B \end{pmatrix}$
g) $\tau A = (1-(1-\alpha_A)^{\tau}, {\beta_A}^{\tau}, {\gamma_A}^{\tau})$
h) $A^{\tau} = ({\alpha_A}^{\tau},\ 1-(1-\beta_A)^{\tau}, 1-(1-\gamma_A)^{\tau})$

Definition 3 [12]: Suppose that $\Gamma = (\alpha_{\Gamma}, \beta_{\Gamma}, \gamma_{\Gamma})$ be a PFV. Then, the score function is defined by:

$$\Delta(\Gamma) = \alpha_A - \beta_A - \gamma_A,\ \Delta(\Gamma) \in [-1, 1] \tag{1}$$

And accuracy function of a PFV $\Gamma = (\alpha_{\Gamma}, \beta_{\Gamma}, \gamma_{\Gamma})$ is defined by:

$$\psi(\Gamma) = \alpha_{\Gamma} + \gamma_{\Gamma},\ \psi(\Gamma) \in [0, 1] \tag{2}$$

Definition 4: Consider $A = (\alpha_A, \beta_A, \gamma_A)$ and $B = (\alpha_B, \beta_B, \gamma_B)$ are two PFVs. Then we compare these two PFVs given as:

i. If $\Delta(A) > \Delta(B)$, then $A > B$
ii. If $\Delta(A) < \Delta(B)$, then $A < B$
iii. If $\Delta(A) = \Delta(B)$, then:
   a) If $\psi(A) > \psi(B)$, then $A > B$
   b) If $\psi(A) < \psi(B)$, then $A < B$
   c) If $\psi(A) = \psi(B)$, then $A = B$

Definition 5 [56]: Consider $\Gamma_{\varsigma} = (\alpha_{\Gamma_{\varsigma}}, \beta_{\Gamma_{\varsigma}}, \gamma_{\Gamma_{\varsigma}})$, $\varsigma = 1, 2, 3, \ldots, þ$ be the collection of PFVs. Then, the PF weighted averaging (PFWA) operators are given as:

$$PFWA(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ) = \overset{þ}{\underset{\varsigma=1}{\oplus}} (\omega_{\varsigma}\Gamma_{\varsigma}) = \begin{pmatrix} 1 - \prod_{\varsigma=1}^{þ}(1-\alpha_{\Gamma})^{\omega_{\varsigma}}, \\ \prod_{\varsigma=1}^{þ}{\beta_{\Gamma}}^{\omega_{\varsigma}},\ \prod_{\varsigma=1}^{þ}\gamma_{\Gamma}^{\omega_{\varsigma}} \end{pmatrix}$$

where $\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_þ)$ be the associated weight vector of Γ_{ς} such that $\omega_{\varsigma} \in [0, 1]$ and $\sum_{\varsigma=1}^{þ} \omega_{\varsigma} = 1$.

III. NECESSARY OPERATIONS OF FRANK AGGREGATION MODELS

Using the speculative theory of Frank aggregation tools, we exposed basic operational laws under the consideration of PF information.

Definition 6 [57]: Suppose that c and d are two real numbers. Then, Frank t-norm and Frank t-conorm are described as follows:

$$Fra(c,d) = log_{\kappa}\left(1 + \frac{(\kappa^c - 1)(\kappa^d - 1)}{\kappa - 1}\right) \text{and } Fra'(c,d) = 1 - log_{\kappa}\left(1 + \frac{(\kappa^{1-c} - 1)(\kappa^{1-d} - 1)}{\kappa - 1}\right)$$

where $(c, d) \in [0, 1] \times [0, 1]$ and $\kappa \neq 1$.

i. If $r \to 1$, then $Fra'(c,d) \to c + d - cd$ and $Fra(c,d) \to cd$. Therefore, if r tends to 1, then the Frank sum and Frank product are converted into probabilistic sum and product.
ii. If $r \to \infty$ then $Fra'(c,d) \to m\varsigma þ\{c+d, 1\}$ and $Fra(c,d) \to max\{0, c+d-1\}$ for r tends to infinitely the Frank sum and Frank product reduce to the Lukasiewicz sum and Lukasiewicz product.

Example 1: Suppose $a = 0.29$, $b = 0.56$ and $r = 4$. Then we have:

$$Fra(0.29,\ 0.56) = log_4\left(1 + \frac{(4^{1-0.29} - 1)(4^{1-0.56} - 1)}{4 - 1}\right) = 0.1276$$

$$Fra'(0.29, 0.56) = 1 - log_4(1 + \frac{(4^{1-0.29} - 1)(4^{1-0.56} - 1)}{4 - 1}) = 0.8723$$

Definition 7 [57]: Suppose $\Gamma = (\alpha_{\Gamma}, \beta_{\Gamma}, \gamma_{\Gamma})$, $\Gamma_1 = (\alpha_{\Gamma 1}, \beta_{\Gamma 1}, \gamma_{\Gamma 1})$ and $\Gamma_2 = (\alpha_{\Gamma 2}, \beta_{\Gamma 2}, \gamma_{\Gamma 2})$ are the three PFVs and any real numbers $\kappa > 1, \aleph > 0$. Some necessary operations of PFVs are given:

i $\Gamma_1 \oplus \Gamma_2$

$$= \begin{pmatrix} 1 - log_{\kappa}\left(1 + \frac{(\kappa^{1-\alpha_{\Gamma 1}} - 1)(\kappa^{1-\alpha_{\Gamma 2}} - 1)}{\kappa - 1}\right), \\ log_{\kappa}\left(1 + \frac{(\kappa^{\beta_{\Gamma 1}} - 1)(\kappa^{\beta_{\Gamma 2}} - 1)}{\kappa - 1}\right), \\ log_k\left(1 + \frac{(\kappa^{\gamma_{\Gamma 1}} - 1)(\kappa^{\gamma_{\Gamma 2}} - 1)}{\kappa - 1}\right) \end{pmatrix}$$

ii $\Gamma_1 \otimes \Gamma_2$

$$= \begin{pmatrix} \log_\kappa\left(1+\frac{(\kappa^{\alpha_{\Gamma 1}}-1)(\kappa^{\alpha_{\Gamma 2}}-1)}{\kappa-1}\right), \\ 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\beta_{\Gamma 1}}-1\right)\left(\kappa^{1-\beta_{\Gamma 2}}-1\right)}{\kappa-1}\right), \\ 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\gamma_{\Gamma 1}}-1\right)\left(\kappa^{1-\gamma_{\Gamma 2}}-1\right)}{\kappa-1}\right) \end{pmatrix}$$

iii $\aleph\Gamma$

$$= \begin{pmatrix} 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\alpha_\Gamma}-1\right)^{\aleph}}{(\kappa-1)^{\aleph-1}}\right), \\ \log_\kappa\left(1+\frac{\left(\kappa^{\beta_\Gamma}-1\right)^{\aleph}}{(\kappa-1)^{\aleph-1}}\right), \\ \log_\kappa\left(1+\frac{(\kappa^{\gamma_\Gamma}-1)^{\aleph}}{(\kappa-1)^{\aleph-1}}\right) \end{pmatrix}$$

iv $\Gamma^{\aleph}$

$$= \begin{pmatrix} \log_\kappa\left(1+\frac{(\kappa^{\alpha_\Gamma}-1)^{\aleph}}{(\kappa-1)^{\aleph-1}}\right), \\ 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\beta_\Gamma}-1\right)^{\aleph}}{(\kappa-1)^{\aleph-1}}\right), \\ 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\gamma_\Gamma}-1\right)^{\aleph}}{(\kappa-1)^{\aleph-1}}\right) \end{pmatrix}$$

Theorem 1: Consider $\Gamma = (\alpha_\Gamma, \beta_\Gamma, \gamma_\Gamma)$, $\Gamma_1 = (\alpha_{\Gamma 1}, \beta_{\Gamma 1}, \gamma_{\Gamma 1})$ and $\Gamma_2 = (\alpha_{\Gamma 2}, \beta_{\Gamma 2}, \gamma_{\Gamma 2})$ are three PFVs and $\kappa > 1$, $\aleph, \aleph_1, \aleph_2 > 0$ are any real numbers. Then we have:

i. $\Gamma_1 \oplus \Gamma_2 = \Gamma_2 \oplus \Gamma_1$
ii. $\Gamma_1 \otimes \Gamma_2 = \Gamma_2 \otimes \Gamma_1$
iii. $\aleph(\Gamma_1 \oplus \Gamma_2) = \aleph\Gamma_1 \oplus \aleph\Gamma_2$
iv. $\aleph_1\Gamma \oplus \aleph_2\Gamma = (\aleph_1 + \aleph_2)\Gamma$
v. $(\Gamma_1 \otimes \Gamma_2)^{\aleph} = \Gamma_1^{\aleph} \otimes \Gamma_2^{\aleph}$
vi. $\Gamma^{\aleph_1} \otimes \Gamma^{\aleph_2} = \Gamma^{\aleph_1+\aleph_2}$

Proof: Consider $\Gamma = (\alpha_\Gamma, \beta_\Gamma, \gamma_\Gamma)$, $\Gamma_1 = (\alpha_{\Gamma 1}, \beta_{\Gamma 1}, \gamma_{\Gamma 1})$ and $\Gamma_2 = (\alpha_{\Gamma 2}, \beta_{\Gamma 2}, \gamma_{\Gamma 2})$ are three PFVs and for any $\aleph, \aleph_1, \aleph_2 > 0$. Then,

i $\Gamma_1 \oplus \Gamma_2$

$$= \begin{pmatrix} 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma 1}}-1\right)\left(\kappa^{1-\alpha_{\Gamma 2}}-1\right)}{\kappa-1}\right), \\ \log_\kappa\left(1+\frac{\left(\kappa^{\beta_{\Gamma 1}}-1\right)\left(\kappa^{\beta_{\Gamma 2}}-1\right)}{\kappa-1}\right), \\ \log_\kappa\left(1+\frac{(\kappa^{\gamma_{\Gamma 1}}-1)(\kappa^{\gamma_{\Gamma 2}}-1)}{\kappa-1}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma 2}}-1\right)\left(\kappa^{1-\alpha_{\Gamma 1}}-1\right)}{\kappa-1}\right), \\ \log_\kappa\left(1+\frac{\left(\kappa^{\beta_{\Gamma 2}}-1\right)\left(\kappa^{\beta_{\Gamma 1}}-1\right)}{\kappa-1}\right), \\ \log_\kappa\left(1+\frac{(\kappa^{\gamma_{\Gamma 2}}-1)(\kappa^{\gamma_{\Gamma 1}}-1)}{\kappa-1}\right) \end{pmatrix}$$

$$= \Gamma_2 \oplus \Gamma_1$$

ii $\Gamma_1 \otimes \Gamma_2$

$$= \begin{pmatrix} \log_\kappa\left(1+\frac{(\kappa^{\alpha_{\Gamma 1}}-1)(\kappa^{\alpha_{\Gamma 2}}-1)}{\kappa-1}\right), \\ 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\beta_{\Gamma 1}}-1\right)\left(\kappa^{1-\beta_{\Gamma 2}}-1\right)}{\kappa-1}\right), \\ 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\gamma_{\Gamma 1}}-1\right)\left(\kappa^{1-\gamma_{\Gamma 2}}-1\right)}{\kappa-1}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \log_\kappa\left(1+\frac{(\kappa^{\alpha_{\Gamma 2}}-1)(\kappa^{\alpha_{\Gamma 1}}-1)}{\kappa-1}\right), \\ 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\beta_{\Gamma 2}}-1\right)\left(\kappa^{1-\beta_{\Gamma 1}}-1\right)}{\kappa-1}\right), \\ 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\gamma_{\Gamma 2}}-1\right)\left(\kappa^{1-\gamma_{\Gamma 1}}-1\right)}{\kappa-1}\right) \end{pmatrix}$$

$$= \Gamma_2 \otimes \Gamma_1$$

iii $\aleph(\Gamma_1 \oplus \Gamma_2)$

$$= \aleph \begin{pmatrix} 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma 1}}-1\right)\left(\kappa^{1-\alpha_{\Gamma 2}}-1\right)}{\kappa-1}\right), \\ \log_\kappa\left(1+\frac{\left(\kappa^{\beta_{\Gamma 1}}-1\right)\left(\kappa^{\beta_{\Gamma 2}}-1\right)}{\kappa-1}\right), \\ \log_\kappa\left(1+\frac{(\kappa^{\gamma_{\Gamma 1}}-1)(\kappa^{\gamma_{\Gamma 2}}-1)}{\kappa-1}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma 1}}-1\right)^{\aleph}\left(\left(\kappa^{1-\alpha_{\Gamma 2}}-1\right)^{\aleph}\right)}{(\kappa-1)^{2\aleph-1}}\right), \\ \log_\kappa\left(1+\frac{\left(\kappa^{1-\beta_{\Gamma 1}}-1\right)^{\aleph}\left(\left(\kappa^{1-\beta_{\Gamma 2}}-1\right)^{\aleph}\right)}{(\kappa-1)^{2\aleph-1}}\right), \\ \log_\kappa\left(1+\frac{\left(\kappa^{1-\gamma_{\Gamma 1}}-1\right)^{\aleph}\left(\left(\kappa^{1-\gamma_{\Gamma 2}}-1\right)^{\aleph}\right)}{(\kappa-1)^{2\aleph-1}}\right) \end{pmatrix}$$

Now

$\aleph\Gamma_1 \oplus \aleph\Gamma_2$

$$= \left(\begin{pmatrix} 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma 1}}-1\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right), \\ \log_\kappa\left(1+\frac{\left(\kappa^{\beta_{\Gamma 1}}-1\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right), \\ \log_\kappa\left(1+\frac{(\kappa^{\gamma_{\Gamma 1}}-1)^{\aleph}}{(\kappa-1)^{\aleph}}\right) \end{pmatrix} \oplus \begin{pmatrix} 1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma 2}}-1\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right), \\ \log_\kappa\left(1+\frac{\left(\kappa^{\beta_{\Gamma 2}}-1\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right), \\ \log_\kappa\left(1+\frac{(\kappa^{\gamma_{\Gamma 2}}-1)^{\aleph}}{(\kappa-1)^{\aleph}}\right) \end{pmatrix}\right)$$

$$
=\begin{pmatrix}
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma 1}}-1\right)^{\aleph}\left(\left(\kappa^{1-\alpha_{\Gamma 2}}-1\right)^{\aleph}\right)}{(\kappa-1)^{2\aleph-1}}\right),\\
log_{\kappa}\left(1+\frac{\left(\kappa^{1-\beta_{\Gamma 1}}-1\right)^{\aleph}\left(\left(\kappa^{1-\beta_{\Gamma 2}}-1\right)^{\aleph}\right)}{(\kappa-1)^{2\aleph-1}}\right),\\
log_{\kappa}\left(1+\frac{\left(\kappa^{1-\gamma_{\Gamma 1}}-1\right)^{\aleph}\left(\left(\kappa^{1-\gamma_{\Gamma 2}}-1\right)^{\aleph}\right)}{(\kappa-1)^{2\aleph-1}}\right),
\end{pmatrix}
$$

$\aleph\left(\Gamma_1\oplus\Gamma_2\right)$

$=\aleph\Gamma_1\oplus\aleph\Gamma_2$

iv $\aleph_1\Gamma\oplus\aleph_2\Gamma$

$$
=\begin{pmatrix}
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma}}-1\right)^{\aleph_1}}{(\kappa-1)^{\aleph_1}}\right),\\
log_{\kappa}\left(1+\frac{\left(\kappa^{\beta_{\Gamma}}-1\right)^{\aleph_1}}{(\kappa-1)^{\aleph_1}}\right),\\
log_{\kappa}\left(1+\frac{\left(\kappa^{\gamma_{\Gamma}}-1\right)^{\aleph_1}}{(\kappa-1)^{\aleph_1}}\right)
\end{pmatrix}
$$

$$
\oplus\begin{pmatrix}
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma}}-1\right)^{\aleph_2}}{(\kappa-1)^{\aleph_2}}\right),\\
log_{\kappa}\left(1+\frac{\left(\kappa^{\beta_{\Gamma}}-1\right)^{\aleph_2}}{(\kappa-1)^{\aleph_2}}\right),\\
log_{\kappa}\left(1+\frac{\left(\kappa^{\gamma_{\Gamma}}-1\right)^{\aleph_2}}{(\kappa-1)^{\aleph_2}}\right)
\end{pmatrix}
$$

$$
=\begin{pmatrix}
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\alpha_{\Gamma}}-1\right)^{\aleph_1+\aleph_2}}{(\kappa-1)^{\aleph_1+\aleph_2}}\right),\\
log_{\kappa}\left(1+\frac{\left(\kappa^{\beta_{\Gamma}}-1\right)^{\aleph_1+\aleph_2}}{(\kappa-1)^{\aleph_1+\aleph_2}}\right),\\
log_{\kappa}\left(1+\frac{\left(\kappa^{\gamma_{\Gamma}}-1\right)^{\aleph_1+\aleph_2}}{(\kappa-1)^{\aleph_1+\aleph_2}}\right)
\end{pmatrix}
$$

$=\left(\aleph_1+\aleph_2\right)\Gamma$

v. $\left(\Gamma_1\otimes\Gamma_2\right)^{\aleph}$

$$
=\begin{pmatrix}
log_{\kappa}\left(1+\frac{\left(\kappa^{\alpha_{\Gamma 1}}-1\right)\left(\kappa^{\alpha_{\Gamma 2}}-1\right)}{\kappa-1}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\beta_{\Gamma 1}}-1\right)\left(\kappa^{1-\beta_{\Gamma 2}}-1\right)}{\kappa-1}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\gamma_{\Gamma 1}}-1\right)\left(\kappa^{1-\gamma_{\Gamma 2}}-1\right)}{\kappa-1}\right)
\end{pmatrix}^{\aleph}
$$

$$
=\begin{pmatrix}
log_{\kappa}\left(1+\frac{\left(\left(\kappa^{\alpha_{\Gamma 1}}-1\right)\left(\kappa^{\alpha_{\Gamma 2}}-1\right)\right)^{\aleph}}{(\kappa-1)^{2\aleph-1}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\left(\kappa^{1-\beta_{\Gamma 1}}-1\right)\left(\kappa^{1-\beta_{\Gamma 2}}-1\right)\right)^{\aleph}}{(\kappa-1)^{2\aleph-1}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\left(\kappa^{1-\gamma_{\Gamma 1}}-1\right)\left(\kappa^{1-\gamma_{\Gamma 2}}-1\right)\right)^{\aleph}}{(\kappa-1)^{2\aleph-1}}\right)
\end{pmatrix}
$$

$$
=\begin{pmatrix}
log_{\kappa}\left(1+\frac{\left(\left(\kappa^{\alpha_{\Gamma 1}}-1\right)\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\left(\kappa^{1-\beta_{\Gamma 1}}-1\right)\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\left(\kappa^{1-\gamma_{\Gamma 1}}-1\right)\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right)
\end{pmatrix}
$$

$$
\otimes\begin{pmatrix}
log_{\kappa}\left(1+\frac{\left(\left(\kappa^{\alpha_{\Gamma 2}}-1\right)\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\left(\kappa^{1-\beta_{\Gamma 2}}-1\right)\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\left(\kappa^{1-\gamma_{\Gamma 2}}-1\right)\right)^{\aleph}}{(\kappa-1)^{\aleph}}\right)
\end{pmatrix}
$$

$=\Gamma_1{}^{\aleph}\otimes\Gamma_2{}^{\aleph}$

vi. $\Gamma^{\aleph_1}\otimes\Gamma^{\aleph_2}$

$$
=\begin{pmatrix}
log_{\kappa}\left(1+\frac{\left(\kappa^{\alpha_{\Gamma}}-1\right)^{\aleph_1}}{(\kappa-1)^{\aleph_1-1}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\beta_{\Gamma}}-1\right)^{\aleph_1}}{(\kappa-1)^{\aleph_1-1}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\gamma_{\Gamma}}-1\right)^{\aleph_1}}{(\kappa-1)^{\aleph_1-1}}\right)
\end{pmatrix}
$$

$$
\otimes\begin{pmatrix}
log_{\kappa}\left(1+\frac{\left(\kappa^{\alpha_{\Gamma}}-1\right)^{\aleph_2}}{(\kappa-1)^{\aleph_2-1}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\beta_{\Gamma}}-1\right)^{\aleph_2}}{(\kappa-1)^{\aleph_2-1}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\gamma_{\Gamma}}-1\right)^{\aleph_2}}{(\kappa-1)^{\aleph_2-1}}\right)
\end{pmatrix}
$$

$$
=\begin{pmatrix}
log_{\kappa}\left(1+\frac{\left(\kappa^{\alpha_{\Gamma}}-1\right)^{\aleph_1+\aleph_2}}{(\kappa-1)^{\aleph_1+\aleph_2-1}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\beta_{\Gamma}}-1\right)^{\aleph_1+\aleph_2}}{(\kappa-1)^{\aleph_1+\aleph_2-1}}\right),\\
1-log_{\kappa}\left(1+\frac{\left(\kappa^{1-\gamma_{\Gamma}}-1\right)^{\aleph_1+\aleph_2}}{(\kappa-1)^{\aleph_1+\aleph_2-1}}\right)
\end{pmatrix}
$$

$=\Gamma^{\aleph_1+\aleph_2}$

IV. PICTURE FUZZY FRANK POWER AGGREGATION OPERATORS

In this section, we developed appropriate aggregating models using the theory of Frank aggregation tools like PFFPA and PFFPG operators. We also stated some unique properties of our derived approaches.

Throughout this article, we use support degrees by the $\text{ƕ}_\varsigma = \frac{1+T(\Gamma_\varsigma)}{\sum_{\varsigma=1}^{\text{þ}}(1+T(\Gamma_\varsigma))}$, where $T(\Gamma_\varsigma) = \sum_{\substack{\varsigma, j=1 \\ \varsigma \neq j}}^{\text{þ}} Sup(\Gamma_\varsigma, \Gamma_j)$.

Definition 8: Suppose that $\Gamma_\varsigma = (\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma})$, $\varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of PFVs. Then PFFPA operator is given by:

$$PFFPA(\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}) = \overset{\text{þ}}{\underset{\varsigma=1}{\oplus}} (\text{ƕ}_\varsigma \Gamma_\varsigma) \tag{3}$$

Theorem 2: Suppose that $\Gamma_\varsigma = (\alpha\Gamma_\varsigma, \beta\Gamma_\varsigma, \gamma\Gamma_\varsigma)$, $\varsigma = 1, 2, \ldots, \text{þ}$ be the assemblage of PFVs. Then, the integrated value of the PFFPA operator is still a PFV, so we have:

$$PFFPA(\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}) = \begin{pmatrix} 1 - \log_\kappa\left(1 + \prod_{\varsigma=1}^{\text{þ}} \left(\kappa^{1-\alpha_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}\right), \\ \log_\kappa\left(1 + \prod_{\varsigma=1}^{\text{þ}} \left(\kappa^{\beta_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}\right), \\ \log_\kappa\left(1 + \prod_{\varsigma=1}^{\text{þ}} \left(\kappa^{\gamma_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}\right) \end{pmatrix} \tag{4}$$

Proof: Since $\Gamma_\varsigma = (\alpha\Gamma_\varsigma, \beta\Gamma_\varsigma, \gamma\Gamma_\varsigma)$ $(\varsigma = 1, 2, \ldots, \text{þ})$ be the set of PFVs. We prove the above theorem by using the mathematical induction technique.

Case 1: For $\text{þ} = 2$

$$PFFPA(\Gamma_1, \Gamma_2) = \overset{2}{\underset{\varsigma=1}{\oplus}} (\text{ƕ}_\varsigma \Gamma_\varsigma) = \text{ƕ}_1\Gamma_1 \oplus \text{ƕ}_2\Gamma_2$$

$$= \begin{pmatrix} 1 - log_k\left(1 + \frac{\left(k^{1-\alpha_{\Gamma_1}} - 1\right)^{\text{ƕ}_1}}{(k-1)^{\text{ƕ}_1 - 1}}\right), \\ log_k\left(1 + \frac{\left(k^{\beta_{\Gamma_1}} - 1\right)^{\text{ƕ}_1}}{(k-1)^{\text{ƕ}_1 - 1}}\right), \\ log_k\left(1 + \frac{\left(k^{\gamma_{\Gamma_1}} - 1\right)^{\text{ƕ}_1}}{(k-1)^{\text{ƕ}_1 - 1}}\right), \end{pmatrix} \oplus \begin{pmatrix} 1 - log_k\left(1 + \frac{\left(k^{1-\alpha_{\Gamma_2}} - 1\right)^{\text{ƕ}_2}}{(k-1)^{\text{ƕ}_2 - 1}}\right), \\ log_k\left(1 + \frac{\left(k^{\beta_{\Gamma_2}} - 1\right)^{\text{ƕ}_2}}{(k-1)^{\text{ƕ}_2 - 1}}\right), \\ log_k\left(1 + \frac{\left(k^{\gamma_{\Gamma_2}} - 1\right)^{\text{ƕ}_2}}{(k-1)^{\text{ƕ}_2 - 1}}\right), \end{pmatrix}$$

$$= \begin{pmatrix} 1 - log_\kappa\left(1 + \prod_{\varsigma=1}^{2} \left(\kappa^{1-\alpha_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}\right), \\ log_\kappa\left(1 + \prod_{\varsigma=1}^{2} \left(\kappa^{\beta_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}\right), \\ log_\kappa\left(1 + \prod_{\varsigma=1}^{2} \left(\kappa^{\gamma_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}\right) \end{pmatrix}$$

Hence the result is true for $\text{þ} = 2$

Next, suppose that the given result is true for $\text{þ} = t$ so we have

$$PFFPA(\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}) = \overset{t}{\underset{\varsigma=1}{\oplus}} (\text{ƕ}_\varsigma \Gamma_\varsigma) = \begin{pmatrix} 1 - log_\kappa\left(1 + \prod_{\varsigma=1}^{t} \left(\kappa^{1-\alpha_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}\right), \\ log_\kappa\left(1 + \prod_{\varsigma=1}^{t} \left(\kappa^{\beta_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}\right), \\ log_\kappa\left(1 + \prod_{\varsigma=1}^{t} \left(\kappa^{\gamma_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}\right) \end{pmatrix}$$

Now, we have to prove Eq. 4 is true for $\text{þ} = t + 1$.

$$PFFPA(\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}) = \overset{t+1}{\underset{\varsigma=1}{\oplus}} (\text{ƕ}_\varsigma \Gamma_\varsigma) = \overset{t}{\underset{\varsigma=1}{\oplus}} (\text{ƕ}_\varsigma \Gamma_\varsigma) \oplus \text{ƕ}_{t+1}\Gamma_{t+1}$$

$$= \begin{pmatrix} 1 - log_k\left(1 + \frac{\prod_{\varsigma=1}^{t}\left(\kappa^{1-\alpha_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}}{(k-1)^{\sum_{\varsigma=1}^{t}\text{ƕ}_\varsigma - 1}}\right), \\ log_k\left(1 + \frac{\prod_{\varsigma=1}^{t}\left(\kappa^{\beta_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}}{(k-1)^{\sum_{\varsigma=1}^{t}\text{ƕ}_\varsigma - 1}}\right), \\ log_k\left(1 + \frac{\prod_{\varsigma=1}^{t}\left(\kappa^{\gamma_{\Gamma_\varsigma}} - 1\right)^{\text{ƕ}_\varsigma}}{(k-1)^{\sum_{\varsigma=1}^{t}\text{ƕ}_\varsigma - 1}}\right), \end{pmatrix} \oplus \begin{pmatrix} 1 - log_k\left(1 + \frac{\left(k^{1-\alpha_{\Gamma_{t+1}}} - 1\right)^{\text{ƕ}_{t+1}}}{(k-1)^{\text{ƕ}_{t+1} - 1}}\right), \\ log_k\left(1 + \frac{\left(k^{\beta_{\Gamma_{t+1}}} - 1\right)^{\text{ƕ}_{t+1}}}{(k-1)^{\text{ƕ}_{t+1} - 1}}\right), \\ log_k\left(1 + \frac{\left(k^{\gamma_{\Gamma_{t+1}}} - 1\right)^{\text{ƕ}_{t+1}}}{(k-1)^{\text{ƕ}_{t+1} - 1}}\right), \end{pmatrix}$$

$$= \begin{pmatrix} 1 - log_{\kappa}\left(1 + \prod_{\varsigma=1}^{t+1}\left(\kappa^{1-\alpha_{\Gamma_{\varsigma}}} - 1\right)^{\text{ƕ}_{\varsigma}}\right), \\ log_{\kappa}\left(1 + \prod_{\varsigma=1}^{t+1}\left(\kappa^{\beta_{\Gamma_{\varsigma}}} - 1\right)^{\text{ƕ}_{\varsigma}}\right), \\ log_{\kappa}\left(1 + \prod_{\varsigma=1}^{t+1}\left(\kappa^{\gamma_{\Gamma_{\varsigma}}} - 1\right)^{\text{ƕ}_{\varsigma}}\right) \end{pmatrix}$$

Hence, the above result is true for $\text{þ} = t + 1$.

Theorem 3: Suppose that $\Gamma_{\varsigma} = \left(\alpha_{\Gamma_{\varsigma}}, \beta_{\Gamma_{\varsigma}}, \gamma_{\Gamma_{\varsigma}}\right)$, $\varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of an identical PFVs such that $\Gamma_{\varsigma} = \Gamma$. Then we have:

$$PFFPA\left(\Gamma_1, \ \Gamma_2, \ldots, \Gamma_{\text{þ}}\right) = \Gamma$$

Proof: Since $\Gamma_{\varsigma} = \left(\alpha_{\Gamma_{\varsigma}}, \beta_{\Gamma_{\varsigma}}, \gamma_{\Gamma_{\varsigma}}\right), \varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of an identical PFVs, so we have:

$$PFFPA\left(\Gamma_1, \ \Gamma_2, \ldots, \Gamma_{\text{þ}}\right)$$

$$= \begin{pmatrix} 1 - log_{\kappa}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(\kappa^{1-\alpha_{\Gamma_{\varsigma}}} - 1\right)^{\text{ƕ}_{\varsigma}}\right), \\ log_{\kappa}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(\kappa^{\beta_{\Gamma_{\varsigma}}} - 1\right)^{\text{ƕ}_{\varsigma}}\right), \\ log_{\kappa}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(\kappa^{\gamma_{\Gamma_{\varsigma}}} - 1\right)^{\text{ƕ}_{\varsigma}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - log_{\kappa}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(\kappa^{1-\alpha_{\Gamma}} - 1\right)^{\text{ƕ}_{\varsigma}}\right), \\ log_{\kappa}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(\kappa^{\beta_{\Gamma}} - 1\right)^{\text{ƕ}_{\varsigma}}\right), \\ log_{\kappa}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(\kappa^{\gamma_{\Gamma}} - 1\right)^{\text{ƕ}_{\varsigma}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - log_{k}\left(1 + \left(k^{1-\alpha_{\Gamma}} - 1\right)^{\sum_{\varsigma=1}^{\text{þ}}\text{ƕ}_{\varsigma}}\right), \\ log_{k}\left(1 + \left(k^{\beta_{\Gamma}} - 1\right)^{\sum_{\varsigma=1}^{\text{þ}}\text{ƕ}_{\varsigma}}\right), \\ log_{k}\left(1 + \left(k^{\gamma_{\Gamma}} - 1\right)^{\sum_{\varsigma=1}^{\text{þ}}\text{ƕ}_{\varsigma}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - log_{k}\left(1 + \left(k^{1-\alpha_{\Gamma}} - 1\right)\right), \\ log_{k}\left(1 + \left(k^{\beta_{\Gamma}} - 1\right)\right), \\ log_{k}\left(1 + \left(k^{\gamma_{\Gamma}} - 1\right)\right) \end{pmatrix}$$

$$= \left(\alpha_{\Gamma}, \beta_{\Gamma}, \gamma_{\Gamma}\right) = \Gamma$$

Which is the required result.

Theorem 4: Consider $\Gamma_{\varsigma} = \left(\alpha_{\Gamma_{\varsigma}}, \beta_{\Gamma_{\varsigma}}, \gamma_{\Gamma_{\varsigma}}\right)$, $\varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of PFVs. Let $\Gamma^{-} = min\left\{\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}\right\} = \left(min\left\{\alpha_{\Gamma_{\varsigma}}\right\}, max\left\{\beta_{\Gamma_{\varsigma}}\right\}, max\left\{\gamma_{\Gamma_{\varsigma}}\right\}\right)$ and $\Gamma^{+} = max\left\{\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}\right\} = \left(max\left\{\alpha_{\Gamma_{\varsigma}}\right\}, min\left\{\beta_{\Gamma_{\varsigma}}\right\}, min\left\{\gamma_{\Gamma_{\varsigma}}\right\}\right)$. Then we have:

$$\Gamma^{-} \leq PFFPA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}\right) \leq \Gamma^{+}$$

Proof: Consider $\Gamma_{\varsigma} = \left(\alpha_{\Gamma_{\varsigma}}, \beta_{\Gamma_{\varsigma}}, \gamma_{\Gamma_{\varsigma}}\right)$, $\varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of PFVs. Let $\Gamma^{-} = min\left\{\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}\right\} = \left(min\left\{\alpha_{\Gamma_{\varsigma}}\right\}, max\left\{\beta_{\Gamma_{\varsigma}}\right\}, max\left\{\gamma_{\Gamma_{\varsigma}}\right\}\right)$ *and* $\Gamma^{+} = max\left\{\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}\right\} = \left(max\left\{\alpha_{\Gamma_{\varsigma}}\right\}, min\left\{\beta_{\Gamma_{\varsigma}}\right\}, min\left\{\gamma_{\Gamma_{\varsigma}}\right\}\right)$.

Then we have:

$$\alpha^{-} = min_{\varsigma}\left\{\alpha_{\Gamma_{\varsigma}}\right\}, \quad \beta^{-} = max_{\varsigma}\left\{\beta_{\Gamma_{\varsigma}}\right\},$$
$$\gamma^{-} = max_{\varsigma}\left\{\gamma_{\Gamma_{\varsigma}}\right\}, \text{ and } \alpha^{+} = max_{\varsigma}\left\{\alpha_{\Gamma_{\varsigma}}\right\},$$
$$\beta^{+} = min_{\varsigma}\left\{\beta_{\Gamma_{\varsigma}}\right\}, \text{ and } \gamma^{+} = min_{\varsigma}\left\{\gamma_{\Gamma_{\varsigma}}\right\}.$$

Now,

$$1 - log_{k}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{1-\alpha^{-}} - 1\right)^{\text{ƕ}_{\varsigma}}\right) \leq 1$$
$$- log_{k}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{1-\alpha_{\Gamma_{\varsigma}}} - 1\right)^{\text{ƕ}_{\varsigma}}\right) \leq 1$$
$$- log_{k}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{1-\alpha^{+}} - 1\right)^{\text{ƕ}_{\varsigma}}\right),$$
$$log_{k}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{\beta^{+}} - 1\right)^{\text{ƕ}_{\varsigma}}\right)$$
$$\leq log_{k}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{\beta_{\Gamma_{\varsigma}}} - 1\right)^{\text{ƕ}_{\varsigma}}\right)$$
$$\leq log_{k}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{\beta^{-}} - 1\right)^{\text{ƕ}_{\varsigma}}\right),$$
$$log_{k}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{\gamma^{+}} - 1\right)^{\text{ƕ}_{\varsigma}}\right)$$
$$\leq log_{k}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{\gamma_{\Gamma_{\varsigma}}} - 1\right)^{\text{ƕ}_{\varsigma}}\right)$$
$$\leq log_{k}\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{\gamma^{-}} - 1\right)^{\text{ƕ}_{\varsigma}}\right),$$

From this, we concluded:

$$\Gamma^{-} \leq PFFPA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}\right) \leq \Gamma^{+}$$

Theorem 5: Consider $\Gamma_{\varsigma} = \left(\alpha_{\Gamma_{\varsigma}}, \beta_{\Gamma_{\varsigma}}, \gamma_{\Gamma_{\varsigma}}\right)$ and $\Gamma_j' = \left(\alpha_{\Gamma_j}', \beta_{\Gamma_j}', \gamma_{\Gamma_j}'\right)$, $\varsigma, j = 1, 2, \ldots, \text{þ}$ be any two sets of PFVs. If $\Gamma_{\varsigma} \leq \Gamma_j'$ such that $\alpha_{\Gamma_{\varsigma}} \leq \alpha_{\Gamma_{\varsigma}}'$, $\beta_{\Gamma_{\varsigma}} \geq \beta_{\Gamma_{\varsigma}}'$ and $\gamma_{\Gamma_{\varsigma}} \geq \gamma_{\Gamma_{\varsigma}}'$. Then we have:

$$PFFPA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_{\text{þ}}\right) \leq PFFPA\left(\Gamma_1', \Gamma_2', \ldots, \Gamma_{\text{þ}}'\right)$$

Proof: Since $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$ and $\Gamma_j' = \left(\alpha_{\Gamma_j}', \beta_{\Gamma_j}', \gamma_{\Gamma_j}'\right)$, $\varsigma, j = 1, 2, \ldots, þ$ be any two sets of PFVs. If $\Gamma_\varsigma \leq \Gamma_j'$ such that $\alpha_{\Gamma_\varsigma} \leq \alpha_{\Gamma_\varsigma}'$, $\beta_{\Gamma_\varsigma} \geq \beta_{\Gamma_\varsigma}'$ and $\gamma_{\Gamma_\varsigma} \geq \gamma_{\Gamma_\varsigma}'$. Then we have:

$$\left(k^{1-\alpha_{\Gamma_\varsigma}} - 1\right)^{ƕ_\varsigma}$$

$$\geq \left(k^{1-\alpha_{\Gamma_\varsigma}'} - 1\right)^{ƕ_\varsigma}$$

$$=> log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\alpha_{\Gamma_\varsigma}} - 1\right)^{ƕ_\varsigma}\right)$$

$$\geq log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\alpha_{\Gamma'_\varsigma}} - 1\right)^{ƕ_\varsigma}\right)$$

$$=> 1 - log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\alpha_{\Gamma_\varsigma}} - 1\right)^{ƕ_\varsigma}\right)$$

$$\leq 1 - log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\alpha_{\Gamma'_\varsigma}} - 1\right)^{ƕ_\varsigma}\right)$$

In the same way, we can prove that:

$$log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{\beta_{\Gamma_\varsigma}} - 1\right)^{ƕ_\varsigma}\right) \geq log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\beta_{\Gamma'_\varsigma}} - 1\right)^{ƕ_\varsigma}\right)$$

And,

$$log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{\gamma_{\Gamma_\varsigma}} - 1\right)^{ƕ_\varsigma}\right) \geq log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\gamma_{\Gamma'_\varsigma}} - 1\right)^{ƕ_\varsigma}\right).$$

Hence,

$$PFFPA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\right) \leq PFFPA\left(\Gamma_1', \Gamma_2', \ldots, \Gamma_þ'\right)$$

Definition 9: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, þ$ be the assemblage of PFVs. Then PFFOPA operator is given by:

$$PFFOPA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\right) = \bigoplus_{\varsigma=1}^{þ}\left(ƕ_\varsigma \Gamma_{\rho(\varsigma)}\right) \tag{5}$$

where $(\rho(1), \rho(2), \ldots, \rho(þ))$ be any permutation of Γ_ς, $\varsigma = 1, 2, \ldots, þ$ and $\Gamma_{\rho(\varsigma-1)} \geq \Gamma_{\rho(\varsigma)}$ for all $\varsigma = 1, 2, \ldots, þ$.

Theorem 6: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, \ldots, þ$ be the assemblage of PFVs. Then, the integrated value of the PFFOPA operator is still a PFV, so we have:

$$PFFOPA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\right) = \begin{pmatrix} 1 - log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\alpha_{\Gamma_{\rho(\varsigma)}}} - 1\right)^{ƕ_\varsigma}\right), \\ log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\beta_{\Gamma_{\rho(\varsigma)}}} - 1\right)^{ƕ_\varsigma}\right), \\ log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\gamma_{\Gamma_{\rho(\varsigma)}}} - 1\right)^{ƕ_\varsigma}\right) \end{pmatrix} \tag{6}$$

where $(\rho(1), \rho(2), \ldots, \rho(þ))$ be any permutation of Γ_ς, $\varsigma = 1, 2, \ldots, þ$ and $\Gamma_{\rho(\varsigma-1)} \geq \Gamma_{\rho(\varsigma)}$ for all $\varsigma = 1, 2, \ldots, þ$.

Proof: We can prove theorem 6 by using the PFFOPA operator.

Theorem 7: Consider $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, þ$ be the assemblage of PFVs. Let $\Gamma^- = min\{\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\} = \left(min\left\{\alpha_{\Gamma_\varsigma}\right\}, max\left\{\beta_{\Gamma_\varsigma}\right\}, max\left\{\gamma_{\Gamma_\varsigma}\right\}\right)$ and $\Gamma^+ = max\{\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\} = \left(max\left\{\alpha_{\Gamma_\varsigma}\right\}, min\left\{\beta_{\Gamma_\varsigma}\right\}, min\left\{\gamma_{\Gamma_\varsigma}\right\}\right)$.

Then we have:

$$\Gamma^- \leq PFFOPA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\right) \leq \Gamma^+$$

Theorem 8: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, þ$ be the assemblage of an identical PFVs such that $\Gamma_\varsigma = \Gamma$. Then we have:

$$PFFOPA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\right) = \Gamma$$

Theorem 9: Consider $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$ and $\Gamma_j' = \left(\alpha_{\Gamma_j}', \beta_{\Gamma_j}', \gamma_{\Gamma_j}'\right)$, $\varsigma, j = 1, 2, \ldots, þ$ be any two sets of PFVs. If $\Gamma_\varsigma \leq \Gamma_j'$ such that $\alpha_{\Gamma_\varsigma} \leq \alpha_{\Gamma_\varsigma}'$, $\beta_{\Gamma_\varsigma} \geq \beta_{\Gamma_\varsigma}'$ and $\gamma_{\Gamma_\varsigma} \geq \gamma_{\Gamma_\varsigma}'$. Then we have:

$$PFFOPA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\right) \leq PFFOPA\left(\Gamma_1', \Gamma_2', \ldots, \Gamma_þ'\right)$$

Definition 10: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, þ$ be the assemblage of PFVs. Then PFFPA operator is given by:

$$PFFPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\right) = \bigotimes_{\varsigma=1}^{þ}\left(\Gamma_\varsigma\right)^{ƕ_\varsigma} \tag{7}$$

Theorem 10: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, \ldots, þ$ be the assemblage of PFVs. Then, the integrated value of the PFFPG operator is still a PFV, so we have:

$$PFFPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\right) = \begin{pmatrix} log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{\alpha_{\Gamma_\varsigma}} - 1\right)^{ƕ_\varsigma}\right), \\ 1 - log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\beta_{\Gamma_\varsigma}} - 1\right)^{ƕ_\varsigma}\right), \\ 1 - log_k\left(1 + \prod_{\varsigma=1}^{þ}\left(k^{1-\gamma_{\Gamma_\varsigma}} - 1\right)^{ƕ_\varsigma}\right) \end{pmatrix} \tag{8}$$

Proof: The proof of Theorem 11 is similar to Theorem 2.

Theorem 11: Consider $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of PFVs. Let $\Gamma^- = min\{\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\} = \left(min\left\{\alpha_{\Gamma_\varsigma}\right\}, max\left\{\beta_{\Gamma_\varsigma}\right\}, max\left\{\gamma_{\Gamma_\varsigma}\right\}\right)$ and $\Gamma^+ = max\{\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\} = \left(max\left\{\alpha_{\Gamma_\varsigma}\right\}, min\left\{\beta_{\Gamma_\varsigma}\right\}, min\left\{\gamma_{\Gamma_\varsigma}\right\}\right)$. Then we have:

$$\Gamma^- \leq PFFPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) \leq \Gamma^+$$

Theorem 12: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right), \varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of an identical PFVs such that $\Gamma_\varsigma = \Gamma$. Then we have:

$$PFFPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) = \Gamma$$

Theorem 13: Consider $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$ and $\Gamma_j' = \left(\alpha_{\Gamma_j}', \beta_{\Gamma_j}', \gamma_{\Gamma_j}'\right), \varsigma, j = 1, 2, \ldots, \text{þ}$ be any two sets of PFVs. If $\Gamma_\varsigma \leq \Gamma_j'$ such that $\alpha_{\Gamma_\varsigma} \leq \alpha_{\Gamma_\varsigma}', \beta_{\Gamma_\varsigma} \geq \beta_{\Gamma_\varsigma}'$ and $\gamma_{\Gamma_\varsigma} \geq \gamma_{\Gamma_\varsigma}'$. Then we have:

$$PFFPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) \leq PFFPG\left(\Gamma_1', \Gamma_2', \ldots, \Gamma_\text{þ}'\right)$$

Now, we will be introduced the PFFOPG operator.

Definition 11: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right), \varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of PFVs. Then the PFFPA operator is given by:

$$PFFOPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) = \bigotimes_{\varsigma=1}^{\text{þ}} \left(\Gamma_{\rho(\varsigma)}\right)^{\mathfrak{w}_\varsigma} \tag{9}$$

where $(\rho(1), \rho(2), \ldots, \rho(\text{þ}))$ be any permutation of $\varsigma = 1, 2, \ldots, \text{þ}$ which satisfy the condition $\Gamma_{\rho(\varsigma-1)} \geq \Gamma_{\rho(\varsigma)}$.

Theorem 14: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right), \varsigma = 1, 2, \ldots, \text{þ}$ be the assemblage of PFVs. Then, the integrated value of the PFFOPG operator is still a PFV, so we have:

$$PFFOPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) = \begin{pmatrix} log_k\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{\alpha_{\Gamma_{\rho(\varsigma)}}} - 1\right)^{\mathfrak{w}_\varsigma}\right), \\ 1 - log_k\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{1-\beta_{\Gamma_{\rho(\varsigma)}}} - 1\right)^{\mathfrak{w}_\varsigma}\right), \\ 1 - log_k\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(k^{1-\gamma_{\Gamma_{\rho(\varsigma)}}} - 1\right)^{\mathfrak{w}_\varsigma}\right) \end{pmatrix} \tag{10}$$

where $(\rho(1), \rho(2), \ldots, \rho(\text{þ}))$ be the permutation of $\varsigma = 1, 2, \ldots, \text{þ}$ which satisfy the condition $\Gamma_{\rho(\varsigma-1)} \geq \Gamma_{\rho(\varsigma)}$

Proof: We can prove Theorem 16 by using the PFFOPG operator.

Theorem 15: Consider $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of PFVs. Let $\Gamma^- = min\{\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\} = \left(min\left\{\alpha_{\Gamma_\varsigma}\right\}, max\left\{\beta_{\Gamma_\varsigma}\right\}, max\left\{\gamma_{\Gamma_\varsigma}\right\}\right)$ and $\Gamma^+ = max\{\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\} = \left(max\left\{\alpha_{\Gamma_\varsigma}\right\}, min\left\{\beta_{\Gamma_\varsigma}\right\}, min\left\{\gamma_{\Gamma_\varsigma}\right\}\right)$. Then we have:

$$\Gamma^- \leq PFFOPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) \leq \Gamma^+$$

Theorem 16: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of an identical PFVs such that $\Gamma_\varsigma = \Gamma$. Then we have:

$$PFFOPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) = \Gamma$$

Theorem 17: Consider $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$ and $\Gamma_j' = \left(\alpha_{\Gamma_j}', \beta_{\Gamma_j}', \gamma_{\Gamma_j}'\right), \varsigma, j = 1, 2, \ldots, \text{þ}$ be any two sets of PFVs. If $\Gamma_\varsigma \leq \Gamma_j'$ such that $\alpha_{\Gamma_\varsigma} \leq \alpha_{\Gamma_\varsigma}', \beta_{\Gamma_\varsigma} \geq \beta_{\Gamma_\varsigma}'$ and $\gamma_{\Gamma_\varsigma} \geq \gamma_{\Gamma_\varsigma}'$. Then we have:

$$PFFOPG\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) \leq PFFOPG\left(\Gamma_1', \Gamma_2', \ldots, \Gamma_\text{þ}'\right)$$

V. PICTURE FUZZY FRANK POWER-WEIGHTED AGGREGATION OPERATORS

We derived a list of new approaches in the form of the PFFPWA and PFFPWG operators under the consideration of the PF information.

Definition 12: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of PFVs. Then the PFFPWA operator is given by:

$$PFFPWA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) = \bigoplus_{\varsigma=1}^{\text{þ}} \left(\mathfrak{d}_\varsigma \Gamma_\varsigma\right) \tag{11}$$

where $\mathfrak{d}_\varsigma = \frac{\omega_\varsigma(1+\mathfrak{R}(\Gamma_\varsigma))}{\sum_{\varsigma=1}^{\text{þ}} \omega_\varsigma(1+\mathfrak{R}(\Gamma_\varsigma))}$ and $\mathfrak{R}\left(\Gamma_\varsigma\right) = \sum_{\substack{\varsigma, j = 1 \\ \varsigma \neq j}}^{\text{þ}} \omega_\varsigma Sup\left(\Gamma_\varsigma, \Gamma_j\right)$ and $\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_\text{þ})$ be the corresponding weight vector of Γ_ς such that $\omega_\varsigma > 0$ and $\sum_{\varsigma=1}^{\text{þ}} \omega_\varsigma = 1$.

Theorem 18: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, \ldots, \text{þ}$ be the assemblage of PFVs. Then, the integrated value using the PFFPWA operator on PFV is defined as:

$$\begin{aligned} PFFPWA&\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) \\ &= \bigoplus_{\varsigma=1}^{\text{þ}} \left(\mathfrak{d}_\varsigma \Gamma_\varsigma\right) \\ &= \begin{pmatrix} 1 - \log_\kappa\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(\kappa^{1-\alpha_{\Gamma_\varsigma}} - 1\right)^{\mathfrak{d}_\varsigma}\right), \\ \log_\kappa\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(\kappa^{\beta_{\Gamma_\varsigma}} - 1\right)^{\mathfrak{d}_\varsigma}\right), \\ \log_\kappa\left(1 + \prod_{\varsigma=1}^{\text{þ}}\left(\kappa^{\gamma_{\Gamma_\varsigma}} - 1\right)^{\mathfrak{d}_\varsigma}\right) \end{pmatrix} \end{aligned} \tag{12}$$

Theorem 19: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, \text{þ}$ be the assemblage of an identical PFVs such that $\Gamma_\varsigma = \Gamma$. Then we have:

$$PFFPWA\left(\Gamma_1, \Gamma_2, \ldots, \Gamma_\text{þ}\right) = \Gamma$$

Theorem 20: Consider $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$ and $\Gamma_j' = \left(\alpha_{\Gamma_j}', \beta_{\Gamma_j}', \gamma_{\Gamma_j}'\right), \varsigma, j = 1, 2, \ldots, \text{þ}$ be any two sets of PFVs.

If $\Gamma_\varsigma \leq \Gamma_j'$ such that $\alpha_{\Gamma_\varsigma} \leq \alpha_{\Gamma_\varsigma}'$, $\beta_{\Gamma_\varsigma} \geq \beta_{\Gamma_\varsigma}'$ and $\gamma_{\Gamma_\varsigma} \geq \gamma_{\Gamma_\varsigma}'$. Then we have:

$$PFFPWA\,(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ) \leq PFFPWA\left(\Gamma_1', \Gamma_2', \ldots, \Gamma_þ'\right)$$

Theorem 21: Consider $\Gamma_\varsigma = (\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma})$, $\varsigma = 1, 2, 3, \ldots, þ$ be the assemblage of PFVs. Let $\Gamma^- = min\,\{\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\} = \left(min\,\{\alpha_{\Gamma_\varsigma}\}, max\,\{\beta_{\Gamma_\varsigma}\}, max\,\{\gamma_{\Gamma_\varsigma}\}\right)$ and $\Gamma^+ = max\,\{\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\} = \left(max\,\{\alpha_{\Gamma_\varsigma}\}, min\,\{\beta_{\Gamma_\varsigma}\}, min\,\{\gamma_{\Gamma_\varsigma}\}\right)$.

Then we have:

$$\Gamma^- \leq PFFPWA\,(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ) \leq \Gamma^+$$

Definition 13: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, þ$ be the assemblage of PFVs. Then the PFFPWG operator is given by:

$$PFFPWG\,(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ) = \bigotimes_{\varsigma=1}^{þ} \left(\Gamma_\varsigma^{\mathfrak{F}_\varsigma}\right) \tag{13}$$

where $\mathfrak{F}_\varsigma = \frac{\omega_\varsigma(1+\Re(\Gamma_\varsigma))}{\sum_{\varsigma=1}^{þ}\omega_\varsigma(1+\Re(\Gamma_\varsigma))}$ and $\Re\left(\Gamma_\varsigma\right) = \sum_{\substack{\varsigma, j=1 \\ \varsigma \neq j}}^{þ} \omega_\varsigma Sup\left(\Gamma_\varsigma, \Gamma_j\right)$ and $\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_þ)$ be the corresponding weight vector of Γ_ς such that $\omega_\varsigma > 0$ and $\sum_{\varsigma=1}^{þ} \omega_\varsigma = 1$.

Theorem 22: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, \ldots, þ$ be the assemblage of PFVs. Then, the integrated value of the PFFPWG operator is still a PFV, so we have:

$$PFFPWG\,(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ) = \left(\begin{array}{c} \log_\kappa\left(1 + \prod_{\varsigma=1}^{þ}\left(\kappa^{\alpha_{\Gamma_\varsigma}} - 1\right)^{\mathfrak{F}_\varsigma}\right) \\ 1 - \log_\kappa\left(1 + \prod_{\varsigma=1}^{þ}\left(\kappa^{1-\beta_{\Gamma_\varsigma}} - 1\right)^{\mathfrak{F}_\varsigma}\right), \\ 1 - \log_\kappa\left(1 + \prod_{\varsigma=1}^{þ}\left(\kappa^{1-\gamma_{\Gamma_\varsigma}} - 1\right)^{\mathfrak{F}_\varsigma}\right), \end{array}\right) \tag{14}$$

Theorem 23: Suppose that $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, þ$ be the assemblage of an identical PFVs such that $\Gamma_\varsigma = \Gamma$. Then we have:

$$PFFPWG\,(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ) = \Gamma$$

Theorem 24: Consider $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$ and $\Gamma_j' = \left(\alpha_{\Gamma_j}', \beta_{\Gamma_j}', \gamma_{\Gamma_j}'\right)$, $\varsigma, j = 1, 2, \ldots, þ$ be any two sets of PFVs. If $\Gamma_\varsigma \leq \Gamma_j'$ such that $\alpha_{\Gamma_\varsigma} \leq \alpha_{\Gamma_\varsigma}'$, $\beta_{\Gamma_\varsigma} \geq \beta_{\Gamma_\varsigma}'$ and $\gamma_{\Gamma_\varsigma} \geq \gamma_{\Gamma_\varsigma}'$.

Then we have:

$$PFFPWG\,(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ) \leq PFFPWG\left(\Gamma_1', \Gamma_2', \ldots, \Gamma_þ'\right)$$

Theorem 25: Consider $\Gamma_\varsigma = \left(\alpha_{\Gamma_\varsigma}, \beta_{\Gamma_\varsigma}, \gamma_{\Gamma_\varsigma}\right)$, $\varsigma = 1, 2, 3, \ldots, þ$ be the assemblage of PFVs. Let $\Gamma^- = min\,\{\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\} = \left(\min\{\alpha_{\Gamma_\varsigma}\}, \max\{\beta_{\Gamma_\varsigma}\}, \max\{\gamma_{\Gamma_\varsigma}\}\right)$ and $\Gamma^+ = max\,\{\Gamma_1, \Gamma_2, \ldots, \Gamma_þ\} = \left(\max\{\alpha_{\Gamma_\varsigma}\}, \min\{\beta_{\Gamma_\varsigma}\}, \min\{\gamma_{\Gamma_\varsigma}\}\right)$. Then we have:

$$\boldsymbol{\Gamma^- \leq PFFPWG\,(\Gamma_1, \Gamma_2, \ldots, \Gamma_þ) \leq \Gamma^+}$$

VI. EVALUATION OF MAGDM TECHNIQUE BASED ON OUR PROPOSED METHODOLOGIES

In this section, we evaluate the MAGDM technique under the consideration of our proposed methodologies of PFFPWA and PFFPWG operators. Let a set of discrete alternatives represented by $\mathbf{T} = \{\boldsymbol{T_1, T_2, \ldots, T_g}\}$ and the set of attributes represented by $\boldsymbol{R} = \{\boldsymbol{R_1, R_2, \ldots, R_q}\}$ with the significance level of the attributes should be indicated by $\boldsymbol{\varpi} = (\boldsymbol{\varpi_1, \varpi_2, \ldots, \varpi q})^{\boldsymbol{T}}$ such that $\boldsymbol{\varpi_j \epsilon\, [0, 1]}$ and $\sum_{j=1}^{q} \boldsymbol{\varpi_p} = \mathbf{1}$. A collection of experts expressed by $\boldsymbol{X} = \{\boldsymbol{X_1, X_2, \ldots, X_z}\}$ who have been asked to provide information regarding the evaluation, and the significance level to the experts is specified by $\boldsymbol{\psi} = (\boldsymbol{\psi_1, \psi_2, \ldots, \psi_z})^{\boldsymbol{T}}$ such that $\boldsymbol{\psi_a \epsilon}\, [0, 1]$, $(\boldsymbol{a} = 1, 2, \ldots, z)$ and $\sum_{\boldsymbol{a}=1}^{z} \boldsymbol{\psi_a} = \mathbf{1}$.The expert $\boldsymbol{X_a}$ calculates each attribute $\boldsymbol{R_a}$ corresponding to each alternative $\boldsymbol{T_\varsigma}$ by the form of PFV $\Omega_{\boldsymbol{\varsigma j}} = \langle\boldsymbol{\alpha_{\varsigma j}, \beta_{\varsigma j}, \gamma_{\varsigma j}}\rangle$, $(\boldsymbol{\varsigma} = 1, 2, \ldots, \boldsymbol{g}; \boldsymbol{j} = 1, 2, \ldots, \boldsymbol{q})$ then the decision matrices $\boldsymbol{DM_a} = \left(\Omega_{\boldsymbol{\varsigma j}}\right)(\boldsymbol{a} = 1, 2, \ldots, z)$ is recognized. The next goal is to evaluate all options using the defined algorithm's steps. The experts collect information randomly about any objects based on alternatives and criteria. The following algorithm of the MAGDM problem is used to evaluate different individuals based on certain criteria.

A. ALGORITHM

Step 1: The experts collect PF information and listed it in different decision matrices $DM_a = \left(\Omega_{\varsigma j}\right)_{g\times q}$.

Step 2: First of all, the given decision matrices $DM_a = \left(\Omega_{\varsigma j}\right)_{g\times q}$ must be transformed into standardized decision matrices. We convert the cost-type attribute into a benefit-type attribute by using the following expression. (15), as shown at the bottom of the next page.

Step 3: Determine the support:

$$\begin{aligned} &sup\left(\Omega_{\varsigma j}^c, \Omega_{\varsigma j}^d\right) \\ &\quad = 1 - D\left(\Omega_{\varsigma j}^c, \Omega_{\varsigma j}^d\right), c, d = 1, 2, \ldots, z,\ j = 1, 2, \ldots, q \\ &\qquad \text{and } \varsigma = 1, 2, \ldots, g \end{aligned} \tag{16}$$

where distance $D\left(\Omega_{\varsigma j}^c, \Omega_{\varsigma j}^d\right)$ represent expressed in the following Eq. 17, so we have:

$$D\left(\Omega_{\varsigma j}^c, \Omega_{\varsigma j}^d\right) = \frac{1}{3}\left(\left|\alpha_{\varsigma j}^c - \alpha_{\varsigma j}^d\right| + \left|\beta_{\varsigma j}^c - \beta_{\varsigma j}^d\right| + \left|\gamma_{\varsigma j}^c - \gamma_{\varsigma j}^d\right|\right) \tag{17}$$

Step 4: Determine the support $\delta_{\varsigma j}^c$ of PFV $\Omega_{\varsigma j}^c$ by other $\Omega_{\varsigma j}^d$ where $c \neq d$ and $c, d = 1, 2, \ldots, z$ and (18), as shown at the bottom of the next page:

Then the importance of the degree of the experts $\psi_c\,(c = 1, 2, \ldots, z)$ of DM X_c, $(c = 1, 2, \ldots, z)$ are used to calculate the importance of a degree of support.

$$\delta_{\varsigma j}^{c}=\frac{\psi_c\left(1+\Re\left(\Omega_{\varsigma j}^{c}\right)\right)}{\sum_{c=1}^{z}\psi_c\left(1+\Re\left(\Omega_{\varsigma j}^{c}\right)\right)};c,d=1,2,\ldots,z,\quad j=1,\ 2,\ \ldots,\ q\text{ and }\varsigma=1,2,\ldots,g \tag{19}$$

where $\delta_{\varsigma j}^{c}\geq 0$ and $\sum_{c=1}^{z}\delta_{\varsigma j}^{c}=1$

Step 5: Utilize the support with the help of PFFPWA and PFFPWG operators expressed by:

$$PFFPWA\left(\Omega_{1j_1}^{c},\Omega_{2j}^{c},\ldots,\Omega_{gj}^{c}\right)=\Omega_{\varsigma j} \tag{20}$$

And

$$PFFPWG\left(\Omega_{1j_1}^{c},\Omega_{2j}^{c},\ldots,\Omega_{gj}^{c}\right)=\Omega_{\varsigma j} \tag{21}$$

Step 6: Determine the support

$$sup\left(\Omega_{\varsigma j},\Omega_{\varsigma l}\right)=1-D\left(\Omega_{\varsigma j},\Omega_{\varsigma l}\right),\varsigma=1,2,\ldots,g;j,l=1,2,\ldots,q \tag{22}$$

Step 7: Determine the support $\Re\left(\Omega_{\varsigma j}\right)$ of the PFV $\Omega_{\varsigma j}\ (\varsigma=1,2,\ldots,g;j=1,2,\ldots,q)$ by the importance of degree ϖ_j of the attributes R_j and the importance degree $\delta_{\varsigma j}$ that are associated with the PFV $\Omega_{\varsigma j}$ by the importance of degree ϖ_j of the attributes R_j:

$$\Re\left(\Omega_{\varsigma j}\right)=\sum_{l=1,l\neq j}^{q}\varpi_j sup\left(\Omega_{\varsigma j},\Omega_{\varsigma l}\right);\quad \varsigma=1,2,\ldots,g;j,l=1,2,\ldots,q$$

$$\mathfrak{C}_{\varsigma j}=\frac{\varpi_j\left(1+\Re\left(\Omega_{\varsigma j}\right)\right)}{\sum_{j=1}^{q}\varpi_j\left(1+\Re\left(\Omega_{\varsigma j}\right)\right)};\quad \varsigma=1,2,\ldots,g;j=1,2,\ldots,q \tag{23}$$

With condition that $\mathfrak{C}_{\varsigma j}\geq 0$ and $\sum_{j=1}^{q}\mathfrak{C}_{\varsigma j}=1$

Step 8: Compute results from our invented approaches of the PFFPWA and PFFPWG operators:

$$PFFPWA\left(\Omega_{1j_1},\Omega_{2j},\ldots,\Omega_{gj}\right)=\Omega_{\varsigma} \tag{24}$$

$$PFFPWG\left(\Omega_{1j_1},\Omega_{2j},\ldots,\Omega_{gj}\right)=\Omega_{\varsigma} \tag{25}$$

Step 9: Determine the score values by using Eq. 1 against each alternative.

Step 10: To investigate a more appropriate and optimal solution of an alternative, make ranking and ordering of computed score values from our derived approaches.

B. APPLICATION

Automobiles emit nitrogen dioxide, carbon dioxide, hydrocarbons, Sulphur dioxide, and particulate matter into the atmosphere seen in Fig. 1, as they use gasoline supplied from fossil fuels. These emissions' pollutants have been associated with adverse health consequences for people, particularly when exposed for a prolonged period or at high concentrations, as well as to environmental problems and climate change seen in Fig. 2. Electric automobiles will be an important component of global efforts to stop auto-related air pollution. Undoubtedly, electric motor cars don't emit more harmful gases than traditional cars seen in Fig. 1. However, clean fuels are accessible that can burn with less pollution, and fuel-efficient cars require less gasoline to cover the same distance.

FIGURE 1. Shows impact on the increasing rate of vehicles.

Throughout 95% of the world, according to a 2020 study conducted in 59 different areas and observed environmental pollution see Fig. 2, driving an electric vehicle is more environmentally friendly than doing so in a gasoline-powered vehicle. The good news is that prospective air quality upgrades and reductions in worldwide carbon dioxide emissions for 2020–21 have already been observed. As most

$$\Omega_{\varsigma j}=\begin{cases}\Omega_{\varsigma j}=\left\langle\alpha_{\varsigma j},\beta_{\varsigma j},\gamma_{\varsigma j}\right\rangle\ for\ benefit-attribute\ \Xi_j\ \varsigma=1,2,\ldots,g;j=1,2,\ldots,q\\ \left(\Omega_{\varsigma j}\right)^{c}=\left\langle N_{\varsigma j},A_{\varsigma j},M_{\varsigma j}\right\rangle\ for\ benefit-type\ attribute\ \Xi_j\end{cases} \tag{15}$$

$$\Re\left(\Omega_{\varsigma j}^{C}\right)=\sum_{d=1;c\neq d}^{z}\psi_d sup\left(\Omega_{\varsigma j}^{c},\Omega_{\varsigma j}^{d}\right);c,d=1,2,\ldots,z,\ j=1,\ 2,\ \ldots,\ q\text{ and }\varsigma=1,2,\ldots,g \tag{18}$$

TABLE 1. Decision matrix $\mathcal{R}_1$.

	R_1	R_2	R_3	R_4
$\not{C}_1$	$(0.19, 0.29, 0.39)$	$(0.32, 0.44, 0.12)$	$(0.20, 0.30, 0.35)$	$(0.23, 0.33, 0.32)$
$\not{C}_2$	$(0.23, 0.34, 0.20)$	$(0.27, 0.18, 0.31)$	$(0.24, 0.10, 0.14)$	$(0.66, 0.13, 0.10)$
$\not{C}_3$	$(0.17, 0.24, 0.33)$	$(0.55, 0.11, 0.22)$	$(0.16, 0.17, 0.18)$	$(0.55, 0.15, 0.12)$
$\not{C}_4$	$(0.29, 0.13, 0.10)$	$(0.34, 0.22, 0.14)$	$(0.66, 0.11, 0.12)$	$(0.16, 0.12, 0.37)$
$\not{C}_5$	$(0.23, 0.32, 0.15)$	$(0.33, 0.45, 0.11)$	$(0.30, 0.17, 0.10)$	$(0.23, 0.23, 0.13)$

TABLE 2. Decision matrix $\mathcal{R}_2$.

	R_1	R_2	R_3	R_4
$\not{C}_1$	$(0.22, 0.33, 0.19)$	$(0.19, 0.40, 0.19)$	$(0.16, 0.37, 0.09)$	$(0.28, 0.21, 0.08)$
$\not{C}_2$	$(0.19, 0.29, 0.35)$	$(0.51, 0.11, 0.13)$	$(0.27, 0.37, 0.15)$	$(0.22, 0.20, 0.09)$
$\not{C}_3$	$(0.22, 0.32, 0.42)$	$(0.61, 0.09, 0.12)$	$(0.25, 0.35, 0.15)$	$(0.37, 0.12, 0.18)$
$\not{C}_4$	$(0.21, 0.31, 0.41)$	$(0.71, 0.05, 0.09)$	$(0.33, 0.32, 0.16)$	$(0.11, 0.30, 0.07)$
$\not{C}_5$	$(0.20, 0.30, 0.33)$	$(0.51, 0.11, 0.08)$	$(0.20, 0.30, 0.10)$	$(0.23, 0.11, 0.05)$

TABLE 3. Decision matrix $\mathcal{R}_3$.

	R_1	R_2	R_3	R_4
$\not{C}_1$	$(0.1, 0.18, 0.12)$	$(0.16, 0.47, 0.35)$	$(0.14, 0.45, 0.09)$	$(0.33, 0.41, 0.15)$
$\not{C}_2$	$(0.33, 0.35, 0.19)$	$(0.18, 0.26, 0.63)$	$(0.09, 0.27, 0.31)$	$(0.12, 0.23, 0.15)$
$\not{C}_3$	$(0.17, 0.45, 0.09)$	$(0.21, 0.16, 0.43)$	$(0.32, 0.44, 0.16)$	$(0.32, 0.22, 0.24)$
$\not{C}_4$	$(0.22, 0.19, 0.22)$	$(0.09, 0.56, 0.09)$	$(0.13, 0.15, 0.21)$	$(0.13, 0.32, 0.19)$
$\not{C}_5$	$(0.11, 0.21, 0.35)$	$(0.15, 0.45, 0.32)$	$(0.19, 0.24, 0.19)$	$(0.11, 0.25, 0.21)$

FIGURE 2. Shows environmental pollution or atmosphere.

people in the world were told to stay home and off the roads, CO2 emissions unexpectedly decreased by as much as 26% in some regions of the world and by 17% generally. To handle the above-discussed situation, Dadashnialehi et al. [58]presented a sensorless mechanism to improve electric vehicle manufacturing and maintenance quality. Senapati et al. [59]exposed the significance of electric motor cars by utilizing derived methodologies considering IF information. In the following experimental case study, we express the selection criteria for electric vehicles.

C. NUMERICAL EXAMPLE

In this section, we are preparing to sketch an application to evaluate an experimental case study under PF information. A multinational company wants to buy an electric motor car for their officials from five different electric motor cars $\not{C} = \{\not{C}_1, \not{C}_2, \not{C}_3, \not{C}_4, \not{C}_5\}$, based on some appropriate characteristics such as: R_1 is the storage capacity of an electric car, R_2 denotes the performance and efficiency of the DC motor, R_3 represents the rate of maximal torque and speed, R_4 easy and understandable operating features. To evaluate information about an electric motor car, there are three experts are invited with some specific degree (0.35, 0.25, 0.40). To determine an appropriate electric motor car under the consideration of PF information by the derived approaches with the help of defined weight vectors (0.30. 020, 035, 0.15). Information about electric motor cars is listed in Table 1-3, each triplet contains PF information.

D. EVALUATION PROCESS OF PF INFORMATION

Now we determine illustrated experimental case study about electric motor cars under the consideration of PF information.

Step 1: The experts collect information about five different types of electric cars under the PF environment and list them in different decision matrices of Table 1-3.

Step 2: Given the information in Table 1-3 under the consideration of PF environments, the experts examined all

TABLE 4. Covered aggregated results by the PFFPWA operator.

	R_1	R_2	R_3	R_4
$\mathring{S}_1$	(0.1963, 0.2731, 0.2293)	(0.2274, 0.4323, 0.1991)	(0.1684, 0.3660, 0.1581)	(0.2758, 0.3003, 0.1649)
$\mathring{S}_2$	(0.2407, 0.3229, 0.2511)	(0.3407, 0.1678, 0.3022)	(0.2120, 0.2330, 0.1827)	(0.3493, 0.1813, 0.1078)
$\mathring{S}_3$	(0.1892, 0.3231, 0.2824)	(0.4861, 0.1134, 0.2236)	(0.2377, 0.3037, 0.1627)	(0.4201, 0.1542, 0.1715)
$\mathring{S}_4$	(0.2405, 0.2088, 0.2341)	(0.4265, 0.1965, 0.1061)	(0.3960, 0.1908, 0.1572)	(0.1325, 0.2333, 0.1806)
$\mathring{S}_5$	(0.1864, 0.2813, 0.2653)	(0.3550, 0.2952, 0.1366)	(0.2322, 0.2358, 0.1203)	(0.1986, 0.1821, 0.1071)

TABLE 5. Covered aggregated results by the PFFPWG operator.

	R_1	R_2	R_3	R_4
$\mathring{S}_1$	(0.1957, 0.2764, 0.2415)	(0.2226, 0.4326, 0.2085)	(0.1674, 0.3676, 0.1799)	(0.2747, 0.3047, 0.1823)
$\mathring{S}_2$	(0.2379, 0.3233, 0.2558)	(0.3301, 0.1737, 0.3263)	(0.2021, 0.2496, 0.1890)	(0.3192, 0.1841, 0.1096)
$\mathring{S}_3$	(0.1884, 0.3267, 0.3018)	(0.4744, 0.1156, 0.2380)	(0.2335, 0.3124, 0.1630)	(0.4167, 0.1571, 0.1752)
$\mathring{S}_4$	(0.2393, 0.2163, 0.2533)	(0.3885, 0.2442, 0.1079)	(0.3721, 0.2025, 0.1595)	(0.1314, 0.2435, 0.2056)
$\mathring{S}_5$	(0.1822, 0.2830, 0.2732)	(0.3423, 0.3196, 0.1524)	(0.2298, 0.2390, 0.1239)	(0.1937, 0.1890, 0.1201)

alternatives that are evaluated based on the beneficial type of attributes. So, there is no need to transform decision matrices into normalizer matrices.

Step 3: Determined support by using Eq. 16 and Eq. 17 under consideration of PF information in Tables 1-3. For the support value, we have to investigate the distance among each two different input arguments by using Eq. 17.

Step 4: By using the results of supports values, compute the Results of weighted support of $\delta^c_{\varsigma j}$ by using Eq. 18 and Eq. 19. Results of weighted support $\delta^c_{\varsigma j}$ of $\Omega^c_{\varsigma j}$ shown in δ^1, δ^2 and δ^2.

Step 5: Evaluate all individuals by using computed support of δ^1, δ^2 and δ^3. We applied Eq. 20 and Eq. 21 on PF information depicted in Tables 1-3. Table 4 and Table 5 cover all obtained results from Eq. 20 and Eq. 21.

$$\delta^1 = \delta^1_{\varsigma j} = \begin{pmatrix} 0.3478 & 0.3477 & 0.3442 & 0.3482 \\ 0.3518 & 0.3566 & 0.3493 & 0.3426 \\ 0.3512 & 0.3542 & 0.3458 & 0.3468 \\ 0.3482 & 0.3571 & 0.3459 & 0.3447 \\ 0.3468 & 0.3565 & 0.3476 & 0.3520 \end{pmatrix}$$

$$\delta^2 = \delta^2_{\varsigma j} = \begin{pmatrix} 0.2656 & 0.2662 & 0.2675 & 0.2631 \\ 0.2606 & 0.2577 & 0.2649 & 0.2688 \\ 0.2644 & 0.2644 & 0.2671 & 0.2665 \\ 0.2601 & 0.2576 & 0.2655 & 0.2646 \\ 0.2671 & 0.2560 & 0.2650 & 0.2619 \end{pmatrix}$$

$$\delta^3 = \delta^3_{\varsigma j} = \begin{pmatrix} 0.3866 & 0.3861 & 0.3884 & 0.3887 \\ 0.3876 & 0.3857 & 0.3857 & 0.3886 \\ 0.3844 & 0.3814 & 0.3872 & 0.3867 \\ 0.3917 & 0.3854 & 0.3886 & 0.3908 \\ 0.3862 & 0.3875 & 0.3874 & 0.3861 \end{pmatrix}$$

Step 6: Again, determine support values by applying Eq. 22 to the information shown in Table 4 and Table 5. The degree of weighted support $\mathfrak{C}_{\varsigma j}$ and $\mathfrak{C}_{\varsigma j}{}'$ investigated by using the information from Table 4 and Table 5 respectively.

$$\mathfrak{C}_{\varsigma j} = \begin{pmatrix} 0.2845 & 0.2003 & 0.3607 & 0.1545 \\ 0.2849 & 0.2005 & 0.3613 & 0.1533 \\ 0.2854 & 0.1975 & 0.3626 & 0.1545 \\ 0.2849 & 0.2005 & 0.3611 & 0.1535 \\ 0.2828 & 0.1995 & 0.3631 & 0.1547 \end{pmatrix}$$

$$\mathfrak{C}'_{\varsigma j} = \begin{pmatrix} 0.2842 & 0.2004 & 0.3609 & 0.1544 \\ 0.2857 & 0.1986 & 0.3616 & 0.1540 \\ 0.2855 & 0.1980 & 0.3621 & 0.1544 \\ 0.2849 & 0.2008 & 0.3607 & 0.1536 \\ 0.2830 & 0.1990 & 0.3630 & 0.1549 \end{pmatrix}$$

Step 7: integrate results by all individuals which are listed in Table 4 and Table 5. After applying Eq. 24 and Eq. 25 to the information which is listed in Table 4 and Table 5 respectively. All investigated results by Eq. 24 and Eq. 25 are shown in Table 6.

Step 8: Determined score values by the consequences of the PFFPWA and PFFPWG operator under Eq. 1. Table 7 covered all score values obtained by the PFFPWA and PFFPWG operators.

Step 9: To choose a reasonable alternative, rearrange all computed score values by the PFFPWA and PFFPWG operators and displayed them in Table 7.

Step 10: After analyzing the raking of score values, we concluded that alternative $\mathscr{C}_4$ is more reliable by our currently proposed methodologies. We also study the nature and behavior of score values in Figure 3.

E. INFLUENCE STUDY

To examine the impact of different parametric values on the consequences of the PFFPWA and PFFPWG operators. We applied our invented methodologies of the PFFPWA and PFFPWG operators to assess the results of score values by the setting of different parametric values κ. All results of score

TABLE 6. Shows score values derived by the PFFPWA and PFFPWG operators.

	$\triangle(\not{C}_1)$	$\triangle(\not{C}_2)$	$\triangle(\not{C}_3)$	$\triangle(\not{C}_4)$	$\triangle(\not{C}_5)$	Ranking and Ordering
PFFPWA	-0.3029	-0.1428	-0.1074	-0.0045	-0.1428	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_2 \succ \not{C}_5 \succ \not{C}_1$
PFFPWG	-0.3596	-0.2434	-0.2140	-0.1536	-0.2181	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$

TABLE 7. Shows aggregated results by the PFFPWA and PFFPWG operators.

PFFPWA	PFFPWG
$(0.2061, 0.3348, 0.1741)$	$(0.1989, 0.3488, 0.2096)$
$(0.2784, 0.2240, 0.1971)$	$(0.2392, 0.2500, 0.2325)$
$(0.3153, 0.2255, 0.1972)$	$(0.2724, 0.2610, 0.2254)$
$(0.3448, 0.1895, 0.1597)$	$(0.2634, 0.2287, 0.1883)$
$(0.2439, 0.2414, 0.1472)$	$(0.2222, 0.2646, 0.1757)$

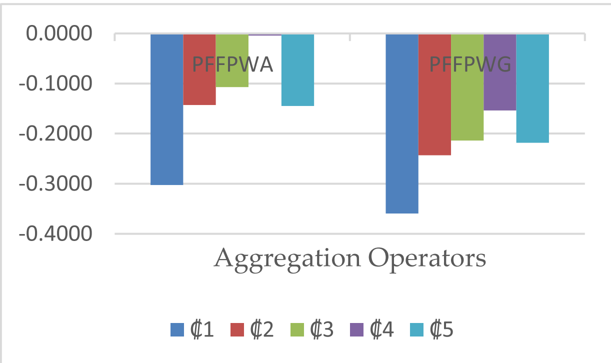

FIGURE 3. Shows the results of PFFPWA and PFFPWG operators.

values computed by the PFFPWA and PFFPWG operators are stated in Table 8 and Table 9 respectively. From Table 8, we can see if the parametric values of κ are increased, the results of score values begging to decrease gradually. However, the appropriate optimal option remains the same $\not{C}_4$. In addition, we also applied the PFFPWA operator to take score values by changing the magnitude of the parametric values of κ. From Table 9, we can observe when the values of κ are increased in the PFFPWG operator, then the results of score values are begging to maximize. However, there is no matter what is the magnitude of the parametric values, after ranking and ordering the score values we examined $\not{C}_4$ is the best optimal alternative for the PFFPWA and PFFPWG operators.

VII. COMPARATIVE STUDY

To ratify the validity and superiority of our proposed methodologies under the considerations of PF information. We applied some existing approaches to evaluate information which are depicted in Tables 1-3. A list of AOs based on a weighted average and weighted geometric operators was given by Wei [56], and Jana et al. [55]invented some

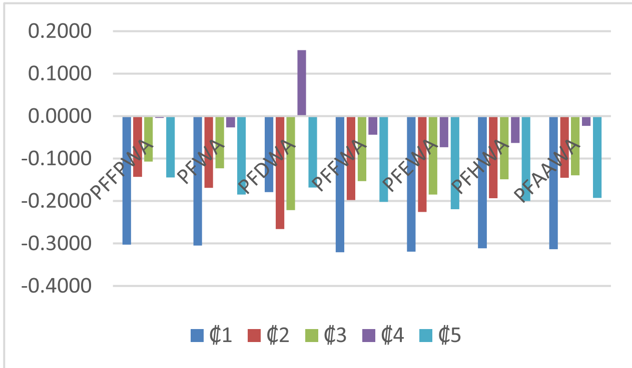

FIGURE 4. Comparison with existing average operators.

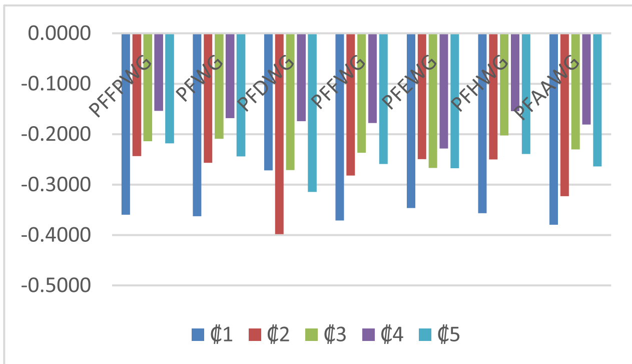

FIGURE 5. Comparison with existing geometric operators.

appropriate AOs of PF Dombi weighted average (PFDWA) and PF Dombi weighted geometric (PFDWG) operators. Seikh and Mandal [57] proposed some AOs based on Frank aggregation tools in the form of PF Frank weighted average (PFFWA) and PF Frank weighted geometric (PFFWG), AOs of PF Einstein weighted average (PFEWA) and PF Einstein weighted geometric (PFEWG) given by the Khan et al. [60]. Wei [35] introduced some AOs of PF Hamacher weighted average (PFHWA) and PF Hamacher weighted geometric (PFHWG) operators, Senapati [61] proposed PF Aczel Alsina weighted average (PFAAWA), Naeem et al. [62]also illustrate some new approaches of PF Aczel Alsina weighted geometric (PFAAWG) and Hussain et al. [20]gave AOs of Complex PFSs under Hamy mean aggregation models. After applying all the above-discussed existing approaches under the consideration of the proposed algorithm, we stated all obtained results in Table 10. Form Table 10, we concluded

TABLE 8. Shows outcomes obtained by the PFFPWA operator at varying κ.

	$\triangle(\not{C}_1)$	$\triangle(\not{C}_2)$	$\triangle(\not{C}_3)$	$\triangle(\not{C}_4)$	$\triangle(\not{C}_5)$	Ranking and Ordering
$\boldsymbol{\kappa=2}$	-0.3029	-0.1428	-0.1070	-0.0045	-0.1448	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_2 \succ \not{C}_5 \succ \not{C}_1$
$\boldsymbol{\kappa=25}$	-0.3138	-0.1626	-0.1296	-0.0350	-0.1581	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=50}$	-0.3163	-0.1667	-0.1341	-0.0408	-0.1610	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=75}$	-0.3177	-0.1689	-0.1365	0.0438	-0.1626	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=100}$	-0.3186	-0.1703	-0.1381	-0.0458	-0.1636	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=130}$	-0.3194	-0.1716	-0.1395	-0.0476	-0.1646	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=155}$	-0.3200	-0.1724	-0.1305	-0.0487	-0.1651	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=175}$	-0.3204	-0.1730	-0.1411	-0.0495	-0.1656	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=205}$	-0.3208	-0.1737	-0.1419	-0.0504	-0.1661	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=265}$	-0.3216	-0.1748	-0.1431	-0.0519	-0.1669	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=300}$	-0.3219	-0.1753	-0.1436	-0.0526	-0.1673	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=400}$	-0.3227	-0.1764	-0.1449	-0.0541	-0.1682	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$

TABLE 9. Shows outcomes obtained by the PFFPWG operator at varying κ.

	$\triangle(\not{C}_1)$	$\triangle(\not{C}_2)$	$\triangle(\not{C}_3)$	$\triangle(\not{C}_4)$	$\triangle(\not{C}_5)$	Ranking and Ordering
$\boldsymbol{\kappa=2}$	-0.3595	-0.2433	-0.2139	-0.1535	-0.2181	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=25}$	-0.3510	-0.2256	-0.1951	-0.1275	-0.2070	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=50}$	-0.3497	-0.2223	-0.1916	-0.1223	-0.2050	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=75}$	-0.3490	-0.2207	-0.1899	-0.1197	-0.2040	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=100}$	-0.3486	-0.2196	-0.1887	-0.1179	-0.2034	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=130}$	-0.3483	-0.2187	-0.1877	-0.1164	-0.2028	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=155}$	-0.3481	-0.2181	-0.1871	-0.1154	-0.2025	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=175}$	-0.3480	-0.2177	-0.1867	-0.1147	-0.2023	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=205}$	-0.3478	-0.2172	-0.1862	-0.1139	-0.2020	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=265}$	-0.3475	-0.2165	-0.1854	-0.1126	-0.2015	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=300}$	-0.3474	-0.2161	-0.1850	-0.1121	-0.2013	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\boldsymbol{\kappa=400}$	-0.3472	-0.2153	-0.1842	-0.1107	-0.2009	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$

TABLE 10. Shows the results of the comparative study.

Aggregation Operators	Ranking and Ordering
PFFPWA	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_2 \succ \not{C}_5 \succ \not{C}_1$
PFFPWG	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
PFWA **[56]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_2 \succ \not{C}_5 \succ \not{C}_1$
PFWG **[56]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
PFDWA **[55]**	$\not{C}_4 \succ \not{C}_5 \succ \not{C}_1 \succ \not{C}_3 \succ \not{C}_2$
PFDWG **[55]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_1 \succ \not{C}_2$
PFFWA **[57]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_2 \succ \not{C}_5 \succ \not{C}_1$
PFFWG **[57]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
PFEWA **[60]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
PFEWG **[60]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
PFHWA **[35]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_2 \succ \not{C}_5 \succ \not{C}_1$
PFHWG **[35]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
PFAAWA **[61]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
PFAAWG **[62]**	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
Hussain et al. **[20]**	Not Applicable

that the ranking of all existing approaches remains same, this shows the robustness and validity of our presented methodologies because currently proposed approaches provide smooth approximation.

We also observed the geometrical behavior of score values by existing weighed average and weighted geometric operators in Fig. 4 and Fig. 5 respectively.

VIII. CONCLUSION

The theoretical concept of PFS is an updated version of an IFSs and FSs, with four terms of any object like PG, AG, NG and RG. A PFS has a restricted condition, such as the sum of all three grades less than or equal to 1. Frank aggregation models are more reliable and updated versions of triangular norms, which are used to handle complex and complicated information about human opinions in the decision-making process. Frank aggregation tools provide a flexible and smooth approximation. Power aggregation tools are robust mathematical AOs, which allow input arguments to support each other of different arguments. The power aggregation model and Frank aggregation tools are more helpful in handling the unpredictable and dubious information during the aggregation process Some effective aspects of our proposed methodologies are given by:

a) We exposed theoretic concepts of Frank aggregation tools and illustrated some necessary operations of Frank triangular norms under consideration of PF information.
b) Robust concepts of power average and geometric operators are also presented.
c) We derived appropriate methodologies under the system of PFSs including PFFPA, PFFPG, PFFPWA and PFFPWG operators. We also exposed some prominent characteristics and special cases of our derived approaches.
d) To ratify the intensity and validity of the currently discussed approaches, we illustrated a MAGDM technique and evaluated a real-life problem based on the proposed algorithm.
e) An experimental case study also gave to evaluate an appropriate electric motor car by using our derived approaches.
f) To observe the effectiveness and superiority of our derived approaches, make an extensive comparative study to compare the results of existing AOs with the results of currently proposed approaches.

Sometimes our derived approaches cannot handle unpredictable and uncertain information about human opinions. To cope with such scenarios, we will enlarge the circle of our research work in different fuzzy frameworks such as Spherical FSs and T-Spherical FSs [63], [64], [65]. We will try to apply our derived approaches in different spaces such as medical diagnosis, networking, clustering, selection supplier and multi-criteria decision-making approaches.

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