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RESEARCH ARTICLE

Warranty and Bundling Strategies for Pricing Decisions in a Duopoly

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ABSTRACT This study examines pricing decisions by considering two manufacturers' warranty and bundling strategies using spatial competition in a duopolistic market. With the development of advanced technology and evolving societal needs, manufacturers now sell a product with its complement—a warranty—using either a bundled base warranty or an unbundled extended warranty strategy. To investigate the impact of the bundling strategy on warranty and pricing models, we develop two scenarios for the pure bundling and mixed bundling scenarios. By constructing a Nash equilibrium model and assessing customer preferences, we determine the optimal warranty length and level of customer demand corresponding to a manufacturer's pricing and strategic decisions. The analysis demonstrates that one manufacturer's strategic decisions are interdependent with those of its competitor. Moreover, a longer warranty period in the unbundled extended warranty strategy not only attracts customers and increases demand but also imposes additional costs on manufacturers. As a result, while providing more personalized features and flexibility, the unbundled extended warranty strategy is not always more profitable. Thus, a manufacturer must carefully evaluate its warranty and bundling strategies when making pricing decisions to gain a competitive advantage. Finally, numerical and sensitivity analyzes confirm the results of the theoretical analysis and provide managerial insights.

INDEX TERMS Customer preferences, duopolistic market, extended warranty, game theory, Nash equilibrium, spatial competition.

I. INTRODUCTION

Industry 5.0 is characterized by integrating advanced technologies such as artificial intelligence, automation, and the Internet of Things into the manufacturing process. This aims to create a more efficient, sustainable, and personalized manufacturing system to better meet customer expectations and needs [1]. However, this has significantly impacted several management areas, including warranty management. As products and services become more customized to individual preferences and requirements, they inevitably become more complex and technologically advanced [2], [3]. Firms may enhance customer satisfaction, improve brand loyalty, and increase customer retention by offering customized warranty services [4], [5]. Base and extended warranties are product warranties that protect customers against defects and malfunctions in their purchases [6], [7]. They are typically provided for products that are durable and costly to repair or replace such as automobiles, electronics, and appliances [8]. A base warranty includes a product's purchase price and provides coverage for a limited period. Meanwhile, extended warranties are optional, separately sold, and provide coverage beyond the base warranty period for a fee [9]. Therefore, firms should consider potential challenges and adapt their warranty policies to meet the evolving needs of today's customers.

Simultaneously, two pricing models have been developed from analysing two different types of warranties, one in which the price is included and another in which it is charged separately. In marketing theory, "bundling strategy" is the term used to describe when two or more complementary or interrelated products or services are sold as a bundle. This

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strategy usually offers lower prices than when each product is purchased separately or unbundled [10], [11]. In terms of warranty strategies, bundling product pricing with a warranty can increase perceived value and customer satisfaction. Unbundling warranty pricing, however, provides greater price transparency and customization potential to meet customers' needs and preferences. Thus, warranty and bundling strategies influence pricing decisions and are an effective marketing strategy to increase customer value and satisfaction [11], [12], [13]. By exploiting warranty and bundling strategies, firms have different pricing options and a greater ability to differentiate themselves from competitors and therefore increase a competitive advantage.

Nonetheless, market competition occurs when multiple firms offer similar products or services to the same customer base and attempt to gain a larger market share. There are different types of market competition, one of which is duopolistic competition, in which two businesses dominate a particular market and compete for market share and profit maximization [13], [14], [15]. Interestingly, in a duopoly, each firm encounters spatial competition and should therefore consider the actions and reactions of its rival when making decisions [16], [17]. This interdependence generates complex strategic interactions between the two firms. Apple Inc. is an example of a manufacturer that operates in a duopolistic market, which is dominated by two major players—Apple and Samsung—that have a combined market share greater than 50% [18], [19]. Furthermore, Apple provides a base warranty and offers an extended warranty after its expiration [7]. Instead of providing additional warranty coverage and services, Apple should consider its rival's strategy, particularly when making pricing decisions, to win over customers.

While previous studies have explored the impact of warranty and bundling strategies on pricing decisions, there is a gap in the literature regarding how various warranty and bundling strategies interact in pricing decisions. Specifically, previous studies fail to examine how these strategies may be applied in a duopolistically competitive market. Despite interactions with the bundling strategy, research has primarily focused on either base or extended warranties. Therefore, this study aims to fill this research gap by exploring how firms can utilize the interaction of warranty and bundling strategies on pricing decisions to maximize profits and gain a competitive advantage in a duopolistic market, focusing on the optimal combination of bundled base and unbundled extended warranty strategies.

In this study, we construct a supply chain of two competing manufacturers offering products and warranty services to the same customer segments to represent a duopolistic market. Using game theoretical models, we investigate the following research questions: (1) How do manufacturers determine optimal pricing decisions under different warranty and bundling strategies to maximize profits in a duopolistic market? (2) How do the bundled base and unbundled extended warranty strategies affect the pricing decisions and profitability of manufacturers in a duopolistic market? (3) How do customer preferences and warranty length affect pricing decisions under warranty and bundling strategies in a duopolistic market? (4) How should each manufacturer implement warranty and bundling strategies to maximize profit in a duopolistic market? By addressing all of that, our conclusions present theoretical and managerial implications for manufacturers' decision-making regarding warranty and bundling strategies in terms of pricing.

To investigate the impact of various warranty and bundling strategies for two competing manufacturers in a duopolistic market, we propose models based on two scenarios: One in which (1) both manufacturers offer a bundled base warranty with their product (pure bundle scenario); and (2) both offer a bundled base warranty but one enhances its service by offering an unbundled extended warranty as well (i.e. the mixed bundled scenario). We develop these analytical models to determine each manufacturer's optimal decisions. We then analyze the properties of their optimal decisions and compare manufacturers' profits under different scenarios.

Our study builds upon existing studies on warranty, bundling, and pricing strategies and makes three significant contributions to the operations and supply chain management literature. First, this study suggests that offering an extended warranty does not always increase a manufacturer's profit, especially in highly competitive markets. Providing a base warranty might be sufficient to satisfy customers when they are susceptible to the extended warranty price or customers' willingness to purchase from the competitor is low. Second, this study provides insights into bundling strategies' impact on pricing decisions. The results suggest that offering bundled products increases manufacturers' profits under certain conditions. Third, this study significantly contributes to the broader literature on the complexity of pricing decisions by considering warranty and bundling strategies in a duopolistically competitive supply chain. Previous studies have mainly focused on either warranty or bundling strategies, but not both. In contrast, this study examines the joint effects of the two strategies, providing a more comprehensive analysis. Accordingly, it may help scholars and managers develop more effective pricing models and make strategic decisions about investment in warranty and bundling programmes.

The remainder of this article is organized as follows. The next section reviews the related literature, particularly on the influence of warranty and bundling strategies on pricing decisions. Section III describes the problem and this study's assumptions. We develop model scenarios in Section IV and perform a numerical illustration and sensitivity analysis to verify the optimal strategy in Section V. We also provide managerial insights in Section V. Finally, our conclusions and possibilities for extending this study in the future are presented in Section VI.

II. LITERATURE REVIEW

In this paper, we investigate the joint effect of various warranty and bundling strategies on pricing decisions in the operations management and economics literature. Therefore, this section reviews the literature on pricing decisions from two research streams: warranty services and bundling strategy.

A. WARRANTY SERVICES IN PRICING DECISIONS

The first stream of the literature investigates the design and management of warranty services in terms of pricing. A warranty service has recently emerged as an essential competitive strategy for manufacturers in devising pricing. In such a case, Li et al. [20] explored the effect of quality perception and customer valuation on a manufacturer's optimal decisions regarding warranty and price for a new product in a monopolistic market. Liu et al. [21] investigated a monopolistic manufacturer's optimal pricing and production strategies with a non-renewing free replacement warranty. In a competitive duopolistic market, Fang [15] established a rational, noncooperative game theoretic model for optimal price and warranty decision-making strategies for substitutable durable products. Hosseini-Motlagh et al. [22] discussed warranty and sales service decisions for pricing strategy in a competitive dual-channel retailing system.

As time goes by, scholars began to focus on model design and supply chain optimization for extended warranties. Bian et al. [7] analyzed the pricing decision-making regarding trade-in service on an extended warranty versus offering a free repair and replacement service. Panda et al. [23] investigated the effect of extended warranty provision on price competition in a retail–e-tail channel supply chain. Nevertheless, with a glimpse of the extended warranty potential as an essential profit source, manufacturers have increased extended warranty offerings lately. For instance, He et al. [24] designed an extended warranty price decision-making model considering customers' purchasing decisions based on maintenance level and purchase time. Liu et al. [6] examined emerging policy trends in complimentary extended warranties by developing a warranty pricing model for heterogeneous customers.

From the above analysis, the first stream of the literature mainly explores warranty issues in pricing models and competition in a supply chain. However, existing studies have not yet investigated the interaction between warranty and bundling strategy issues in depth, although some have considered whether a warranty is offered in a bundle or vice versa. As noted earlier, bundling strategy can be an essential tool to attract customers. Therefore, it is worth studying along with warranty strategy.

B. BUNDLING STRATEGY IN PRICING DECISIONS

The second stream of the literature explores the issues related to bundling strategies. Failing to consider bundling a strategic marketing variable might decrease a firm's profit. Thus, scholars have recently examined decision-making concerns in bundle pricing from various viewpoints in the supply chain management literature. Vamosiu [13] developed a differentiated products model to examine compatibility and bundling for a duopolist, namely, product bundling and price bundling under competition. Wei and Chen [25] examined the effect of pricing strategy and channel choice on a dual-channel supply chain profit model under different market-dominant powers and bundling strategies. Cao et al. [26] demonstrated that manufacturer-initiated bundling decisions generate greater supply chain profits than retailer-initiated ones. In a duopolistic supply chain network, Jena and Ghadge [27] examined bundling decisions and advertising efforts under various power structures and price competition. Hemmati et al. [10] developed two sales strategies—bundling and separate sales—for pricing complementary products using participating customer bidding.

In practise, manufacturers in the same business may handle bundling differently depending on the specific situation. Scholars have developed product bundling strategies, and they are typically categorized into the following three types: pure bundling, unbundling, and mixed bundling strategies [28], [29]. In each scenario, two or more products are sold in one bundled package, separately, or together. Vamosiu [29] analyzed the profitability for a two-product seller of differentiated products competing with a one-product rival using pure and mixed bundling strategies. Kopczewski et al. [11] developed an integrated simulation model to demonstrate the impact of dependence, complementarity, and economic level on profitability with various pricing strategies: pure components, pure bundling, and mixed bundling. Zhou et al. [14] analyzed the effect of a bundling strategy on products and prices in two competing firms by developing pure bundling, mixed bundling, and unbundling game theoretic scenarios. In a dual-channel supply chain, Chen et al. [30] examined the optimality of two bundling strategies—fully and partially mixed bundling—for an online retailer.

From the aforementioned studies, we have learned that a proper bundling strategy may help increase profits for all participants on an entire supply chain. However, current products have become increasingly sophisticated and heightened after-sales risk has resulted in the recent growth of after-sales services, particularly warranties. Recent articles have explored product bundling strategies in terms of how bundling influences warranty issues. Dan et al. [31] explored the product unbundling and bundling strategies of extended warranty services that a manufacturer or a retailer might provide. Wang et al. [5] offered the option of extended warranty menus of different lengths of time and various prices to maximize the expected warranty benefits. Zhang and Gao [32] investigated the manufacturer's optimal pricing decisions and transferability of an extended warranty that may be bundled or sold separately. Lee and Venkataraman [33] theoretically explained why customers buy unbundled extended warranties even though they infrequently claim them and may already have purchased a bundled base with the product. Cui et al. [34] developed discriminated pricing strategies for recycling-product-related unbundled extended warranty services offered by an e-commerce platform in a closed-loop supply chain.

TABLE 1. Literature comparisons.

Author(s)	Year	Market Structure	Competition	Customer Preference	Pricing Discrimination	Warranty Strategy		Bundling strategy	
						Base	Extended	Bundled	Unbundled
Vamosiu	2018	Duopoly	√		√			√	
Dan et al.	2018	Monopoly					√	√	√
Fang	2020	Duopoly	√		√	√			
Liu et al.	2020	Monopoly		√			√	√	
Zhou et al.	2020	Duopoly	√		√			√	√
Wang et al.	2020	Monopoly		√	√	√	√		
Wei and Chen	2021	Duopoly	√		√			√	√
Zhang and Gao	2021	Monopoly					√	√	√
Jena and Ghadge	2022	Duopoly	√		√			√	√
Lee and Venkataraman	2022	Monopoly		√		√	√	√	√
Cui et al.	2023	Monopoly			√		√		√
This study	2023	Duopoly	√	√	√	√	√	√	√

Despite most of the above studies using the game theory method in developing a model, to the best of our knowledge, these studies have yet to discuss the joint effect of different bundling and warranty strategies in a competitive environment. In addition, existing studies only highlight one kind of warranty without considering potential combinations of different warranty types. Therefore, our paper investigates different warranty and bundling strategies scenarios in a duopolistic market with two competing manufacturers to fill this gap in the literature. Our study contributes to this stream of research and observes parallels between the results of our analysis and the extant literature in this area. To position this paper intuitively, we compare previous related studies to ours in Table 1.

III. PROBLEM DESCRIPTION AND STUDY ASSUMPTIONS

We consider a duopolistic market including customers and two spatially competing manufacturers, $i = 1, 2$. Both manufacturers offer different bundle pricing strategies for warranty services. Generally speaking, there are two types of warranty services: a base warranty and an extended warranty [6], [7]. A base warranty is typically bundled with the product sold, while an extended warranty is provided separately for a particular period after the base warranty is over. In addition, there are two price bundle strategies (i.e. bundled and unbundled pricing strategies) [11], [35]. With bundled pricing or a bundling strategy, we examine manufacturer i, which sells a product and base warranty simultaneously to customers at price p_i. Manufacturer i can also sell the product and an extended warranty separately to customers using unbundled pricing or an unbundling strategy. As we aim to investigate the joint effect of different warranty and bundling strategies in a duopoly, we develop two model scenarios—pure bundling scenario (b) and mixed bundling scenario (m)—as illustrated in Fig. 1.

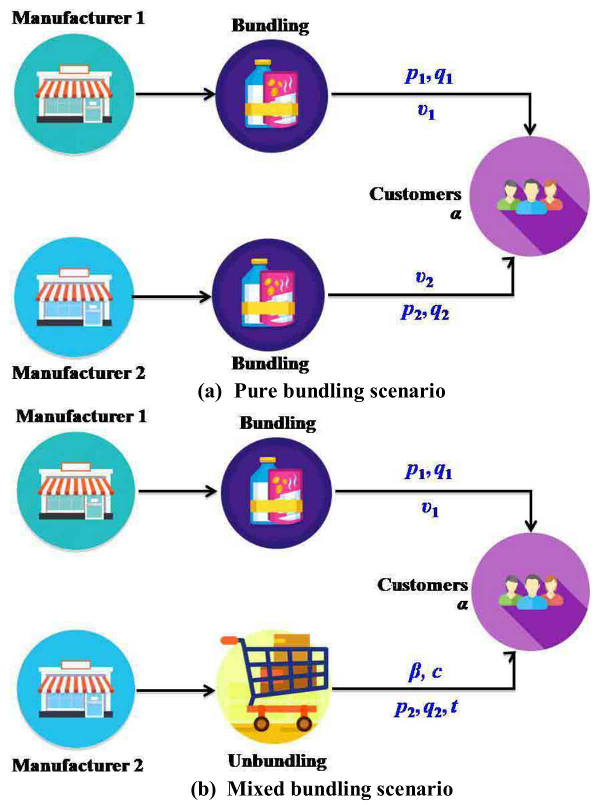


FIGURE 1. The structure of two bundling scenarios in a duopolistic market.

In the pure bundling scenario, both manufacturers perform a bundled pricing strategy to sell a product and a base warranty. Manufacturers 1 and 2 market homogenous products at prices p_1 and p_2, respectively. Thus, customers with initial market demand α have two preferences for purchasing decisions. If we suppose that the base warranty cost is w ($w > 0$), we may assume that the proportion of customers to manufacturer 1 is θ ($0 < \theta < 1$). Consequently, the preference percentage of customers to manufacturer 2 under spatial competition is $(1 - \theta)$, a fact that has been widely applied in operations and supply chain management studies [9], [23], [36]. Customers who purchase products from manufacturers 1 and 2 obtain utility U_1 and U_2, respectively. Meanwhile, the customer's willingness to purchase a product and a base warranty with the bundling strategy provided by manufacturers 1 and 2 is denoted as υ_1 and υ_2. As a result, we determine customer's utilities from both manufacturers as follows:

$$U_1 = \alpha + \upsilon_1 - p_1 - w\theta$$
$$U_2 = \alpha + \upsilon_2 - p_2 - w(1-\theta)$$

Note that customers purchase a product from a manufacturer if and if only if they will acquire a positive utility (i.e. $U_1 \geq 0$ and $U_2 \geq 0$).

Similarly, a customer purchases a product from the manufacturer 1 when $\alpha \leq \upsilon_1 - p_1 - w\theta$, whereas they purchase it from manufacturer 2 when $\alpha \leq \upsilon_2 - p_2 - w(1-\theta)$. Following Cai et al. [37], we may calculate the customer demand for manufacturers' q_1 and q_2 as follows:

$$q_1 = \int_0^\theta d\alpha = \upsilon_1 - p_1 - w\theta$$
$$q_2 = \int_\theta^1 d\alpha = \upsilon_2 - p_2 - w(1-\theta)$$

Then, solving the above equations yields the following:

$$q_{b1} = \frac{\upsilon_1 - \upsilon_2 - p_1 + p_2 + w}{2w} \quad (1)$$
$$q_{b2} = \frac{\upsilon_2 - \upsilon_1 - p_2 + p_1 + w}{2w} \quad (2)$$

In the mixed bundling scenario, the manufacturer 1 sticks with the strategy of bundling a product and a base warranty. The manufacturer 2 enhances its service offering using the unbundling strategy and provides an extended warranty as a separate product item. Therefore, we determine the extended warranty length t ($t > 0$) and customer sensitivity to that service β ($\beta > 0$). Customers will obtain longer product protection coverage when the extended warranty length t is longer and vice versa. Accordingly, extended warranty length t also represents a customer's willingness to purchase the extended warranty service [4], [38]. In the real-world market, prices in the unbundling strategy are typically higher than those in the bundling strategy due to a particular discount. According to Hemmati et al. [10] and Cao et al. [26], we consider the marginal cost in unbundling strategy c. The manufacturer 2 incurs extended warranty cost $T = 1/2\ kt^2$, where k is the extended warranty cost coefficient [20].

From these settings, we determine the utility of manufacturers, U_1 and U_2, in the mixed bundling strategy. The customer's utility derived from purchasing a product for each manufacturer is defined as follows:

$$U_1 = \alpha + \upsilon_1 - p_1 - w\theta$$
$$U_2 = \alpha - p_2 - w(1-\theta) + \beta t - c$$

Afterwards, by solving the demand above with $\alpha \in [0, 1]$, we obtain the demand for manufacturers' q_1 and q_2 as follows:

$$q_{m1} = \frac{\upsilon_1 - \beta t + c - p_1 + p_2 + w}{2w} \quad (3)$$
$$q_{m2} = \frac{\beta t - c - \upsilon_1 - p_2 + p_1 + w}{2w} \quad (4)$$

TABLE 2. Notations and descriptions.

Notation	Description
Indexes	
$i = 1, 2$	Subscript, index for manufacturer i
$j = b, m$	Subscript, b for pure bundling scenario and m for mixed bundling market scenario
Given parameters	
α	Initial market demand, $0 \leq \alpha \leq 1$
w	Base warranty cost, $w > 0$
θ	The demand percentage of the manufacturer 1
β	Customer sensitivity to extended warranty service, $\beta > 0$
c	Marginal cost of the unbundled extended warranty strategy
k	Warranty cost coefficient in the unbundled extended warranty strategy, $k > 0$
U_i	Customer's utility from purchasing a product from manufacturer i
υ_i	Customer's willingness to purchase a product from manufacturer i, $\upsilon_i > 0$
Decision variables	
t	Warranty length in the unbundled extended warranty strategy, $t > 0$
q_i	Customer demand of manufacturer i
p_i	Product price of manufacturer i
Objective functions	
π_{ji}	Profit of manufacturer i in bundling market scenario j

In decision-making problems, risks can occur due to uncertainty, conflicting decision variables, and different preferences [39], [40], [41]. However, according to related studies [28], [42], we consider there is no information asymmetry for both manufacturers. Furthermore, we assume that both manufacturers are self-interested, neutral, and rational decision-makers who seek to maximize their profit π_i, as follows:

$$\pi_{ji} = p_{ji}q_{ji} - T \quad (5)$$

where

$$T = \begin{cases} \frac{1}{2}kt^2, & t > 0 \\ 0, & t = 0 \end{cases}$$

The primary purpose of this study is to maximize each manufacturer's profit π_i by considering different warranty and bundling strategies. Without loss of generality, we assume that the production cost is constant and normalized to zero [7], [9]. To this end, we construct two models in the next section. We describe the notation in Table 2.

IV. MODEL STRATEGY DEVELOPMENT AND ANALYSIS

In this section, we develop two scenario models using a Nash equilibrium game theoretic model to examine the equilibrium of different warranty and bundle pricing strategies in a supply chain with a duopolistic competition. We use the Wolfram Mathematica to solve the game theory problem and define the optimal decisions. To maximize decentralized profits, two spatially competing manufacturers are considered. The manufacturer 1 performs a bundled base warranty strategy, while the manufacturer 2 performs two strategies, either a bundled base warranty or an unbundled extended warranty strategy. We use the superscript * to indicate an optimal decision.

A. PURE BUNDLING SCENARIO

In the pure bundling scenario, both manufacturers provide bundled base warranty service at cost w. Manufacturer i sells the product at price p_i, and with customers' willingness to purchase υ_i. Substituting (1) and (2) into (5) yields a profit model for manufacturers' π_{b1} and π_{b2} as follows:

$$\pi_{b1} = p_1\left(\frac{\upsilon_1 - \upsilon_2 - p_1 + p_2 + w}{2w}\right) \quad (6)$$

$$\pi_{b2} = p_2\left(\frac{\upsilon_2 - \upsilon_1 - p_2 + p_1 + w}{2w}\right) \quad (7)$$

In the following theorem, we determine the optimal decisions for each manufacturer to maximize profit.

Theorem 1: When $-3w \le \upsilon_2 - \upsilon_1 \le 3w$ is satisfied, the optimal price p^*_{bi}, demand q^*_{bi}, and profit π^*_{bi} for each manufacturer in the pure bundling scenario are presented as:

a. Manufacturer 1: $p^*_{b1} = \frac{\upsilon_1-\upsilon_2+3w}{3}$, $q^*_{b1} = \frac{\upsilon_1-\upsilon_2+3w}{6w}$, $\pi^*_{b1} = \frac{(\upsilon_1-\upsilon_2+3w)^2}{18w}$.

b. Manufacturer 2: $p^*_{b2} = \frac{\upsilon_2-\upsilon_1+3w}{3}$, $q^*_{b2} = \frac{\upsilon_2-\upsilon_1+3w}{6w}$, $\pi^*_{b2} = \frac{(\upsilon_2-\upsilon_1+3w)^2}{18w}$.

Proof: See the Appendix.

Afterwards, we investigate the effect of customers' willingness to purchase υ_i on the optimal manufacturer profit π^*_{bi} in Proposition 1. In pure bundling scenario, the customers' willingness to purchase υ_i is for purchasing products and base warranties with the bundling strategy provided by both manufacturers. ■

Proposition 1: In the pure bundling scenario, we have $\frac{\partial\pi^*_{b1}}{\partial\upsilon_1} > 0$, $\frac{\partial\pi^*_{b1}}{\partial\upsilon_2} < 0$, $\frac{\partial\pi^*_{b2}}{\partial\upsilon_1} < 0$, and $\frac{\partial\pi^*_{b2}}{\partial\upsilon_2} > 0$.

Proof: See the Appendix.

Proposition 1 points out that, for manufacturer 1, an increase of υ_1 and a decrease of υ_2 generate an increase in the optimal profit π^*_{b1}. Similar situations also occur for manufacturer 2. Obviously, the optimal profit for manufacturer i increases when the corresponding customers' willingness to purchase υ_i increases, while the customers' willingness to purchase from the other manufacturer decreases in the pure bundling scenario. ■

B. MIXED BUNDLING SCENARIO

In mixed bundling scenario, both manufacturers perform different warranty and bundling strategies. With the bundling strategy, the manufacturer 1 sells the product and base warranty simultaneously at price p_1 with warranty cost w. In addition to providing a bundled base warranty service, the manufacturer 2 offers an unbundled extended warranty service. If the customer's willingness to purchase products and a warranty with a bundling strategy from manufacturer 1 is υ_1, then for manufacturer 2, the extended warranty length t and marginal cost c become the concern or willingness of customers to purchase the product. Substituting (3) and (4) into (5) yields a profit model for manufacturers' π_{m1} and π_{m2} as follows:

$$\pi_{m1} = p_1\left(\frac{\upsilon_1 - \beta t + c - p_1 + p_2 + w}{2w}\right) \quad (8)$$

$$\pi_{m2} = p_2\left(\frac{\beta t - c - \upsilon_1 - p_2 + p_1 + w}{2w}\right) - \tfrac{1}{2}kt^2 \quad (9)$$

In the following theorem, we define the optimal decisions to maximize profit for each manufacturer.

Theorem 2: When $\upsilon_1 + c \le 3w$ and $\beta^2 \le k(3w + \upsilon_1 + c)$ are satisfied, optimal price p^*_{mi}, extended warranty length t^*, demand q^*_{mi}, and profit π^*_{mi} for each manufacturer in the mixed bundling scenario are presented as follows:

a. Manufacturer 1: $p^*_{m1} = \frac{2w((\upsilon_1+c+3w)k-\beta^2)}{6kw-\beta^2}$, $q^*_{m1} = \frac{(\upsilon_1+c+3w)k-\beta^2}{6kw-\beta^2}$, $\pi^*_{m1} = \frac{2w((\upsilon_1+c+3w)k-\beta^2)^2}{(6kw-\beta^2)^2}$.

b. Manufacturer 2: $p^*_{m2} = \frac{2kw(3w-\upsilon_1-c)}{6kw-\beta^2}$, $t^* = \frac{\beta(3w-\upsilon_1-c)}{6kw-\beta^2}$, $q^*_{m2} = \frac{k(3w-\upsilon_1-c)}{6kw-\beta^2}$, $\pi^*_{m2} = \frac{k(3w-\upsilon_1-c)^2(4kw-\beta^2)}{2(6kw-\beta^2)^2}$.

Proof: See the Appendix.

We now investigate the impact of several key parameters associated with the unbundled extended warranty strategy in the mixed bundling scenario on the manufacturer's optimal decision variables. ■

Proposition 2: Considering the optimal profit of manufacturer π^*_{m2}, which provides an unbundled extended warranty service, we have $\frac{\partial\pi_{m2}*}{\partial\upsilon_1} < 0$ and $\frac{\partial\pi_{m2}*}{\partial c} < 0$ when $3w - \upsilon_1 + c > 0$ and $4kw - \beta^2 > 0$. Moreover, if $\beta \in \left(0, \sqrt{2kw}\right)$ and $\beta \in \left(\sqrt{2kw}, 2\sqrt{kw}\right)$ exist, we have $\frac{\partial\pi^*_{m2}}{\partial\beta} > 0$ and $\frac{\partial\pi^*_{m2}}{\partial\beta} < 0$, respectively.

Proof: See the Appendix.

Proposition 2 demonstrates the effect of the customer's willingness to purchase bundled base warranty υ_1, at marginal cost c, and with customer sensitivity to unbundled extended warranty β on the optimal profit of manufacturer π^*_{m2}. In the mixed bundling scenario, increasing customer willingness υ_1 decreases optimal profit π^*_{m2}. This suggests that the customer's willingness to purchase the bundled base warranty service from manufacturer 1 impacts the profit of the manufacturer 2. Furthermore, an increase in marginal cost c affects an increase in the sales price and a decrease in the customer demand and profit of manufacturer π^*_{m2}. Like Hemmati et al. [10] and Cao et al. [26], when the marginal cost c between bundling and unbundling differs significantly, customers prefer to purchase products from manufacturer 1 as it has a lower price. Thus, providing a base warranty and bundling strategy is more profitable for both manufacturers under this circumstance.

Nevertheless, an increase in customer sensitivity to unbundled extended warranty β increases the optimal manufacturer's profit π^*_{m2} when $0 < \beta < \sqrt{2kw}$. The reverse situation occurs when $\sqrt{2kw} < \beta < 2\sqrt{kw}$, the optimal manufacturer's profit π^*_{m2} decreases since customer sensitivity β increases. Manufacturer profit π^*_{m2} increases when customer sensitivity β indicates that the customer's preference for the unbundled extended warranty service is within a smaller interval of warranty cost w. Meanwhile, when customer sensitivity β is in a larger interval of warranty cost w, customer sensitivity to the unbundled extended warranty service becomes higher. In turn, it is more difficult for manufacturer 2 to compete with the bundled price offered by the other manufacturer. Associated with Lin et al. [35], this situation leads to customer dissatisfaction and reduced demand, which in turn lowers manufacturer profits. ■

Proposition 3: When $3w - \upsilon_1 + c > 0$, the effect of customer sensitivity β and cost coefficient k from an extended warranty service on the optimal price p^*_{mi} and demand q^*_{mi} for each manufacturer in the mixed bundling scenario are presented as:

a. Manufacturer 1: $\frac{\partial p_{m1}*}{\partial \beta} < 0$, $\frac{\partial q_{m1}*}{\partial \beta} < 0$, $\frac{\partial p_{m1}*}{\partial k} > 0$, $\frac{\partial q_{m1}*}{\partial k} > 0$.
b. Manufacturer 2: $\frac{\partial p_{m2}*}{\partial \beta} > 0$, $\frac{\partial q_{m2}*}{\partial \beta} > 0$, $\frac{\partial p_{m2}*}{\partial k} < 0$, $\frac{\partial q_{m2}*}{\partial k} < 0$.

Proof: See the Appendix.

Proposition 3 presents that the customer's sensitivity to extended warranty service β has a different effect for manufacturers who perform different warranty and bundling strategies in a mixed bundling scenario. For manufacturer 1, who offers a base warranty and performs a bundling strategy, the increase in customer sensitivity β induces a decrease in optimal price p^*_{m1} and demand q^*_{m1}. Conversely, the increase in customer sensitivity β induces an increase in optimal price p^*_{m2} and demand q^*_{m2} of the manufacturer 2, who offers an extended warranty and performs an unbundling strategy. These results confirm Liu et al. [6] and Panda et al. [23] that the higher the sensitivity β, the more sensitive customers are to the extended warranty service. Thus, they prefer to purchase from manufacturer 2. In turn, the manufacturer 2 obtains higher demand q_{m2} and maximizes its profits by increasing price p_{m2}. Meanwhile, due to decreased demand q_{m1}, the manufacturer 1 will maintain its market share by reducing price p_{m1}.

Similarly, the extended warranty cost coefficient k has a different effect on each manufacturer. When warranty cost coefficient k increases, optimal price p^*_{m1} and demand q^*_{m1} of the manufacturer 1 increase, whereas those of the manufacturer 2 decrease. With a constant extended warranty length t, the cost of providing an extended warranty service will decrease when the cost coefficient k decreases. Suppose that the manufacturer 2 initially performs an unbundled extended warranty strategy to be more competitive and increase market demand. Thus, having a more significant market demand q_{m2}, the manufacturer 2 manages to increase the price p_{m2} of its product to gain higher profits. However, the manufacturer 1, which provides a shorter warranty length than the manufacturer 2, has a decrease in demand q_{m1} due to increased demand q_{m2}. Eventually, the manufacturer 1 charges a lower price p_{m1} to maintain its market share. ■

Proposition 4: In the mixed bundling scenario, when the manufacturer 2 provides extended warranty services t, we have $\frac{\partial t^*}{\partial c} < 0$, $\frac{\partial t^*}{\partial \beta} > 0$ and $\frac{\partial t^*}{\partial \upsilon_1} < 0$ if $3w - \upsilon_1 + c >0$, $6kw + \beta^2 > 0$, and $6kw - \beta^2 > 0$ are satisfied, respectively.

Proof: See the Appendix.

Proposition 4 demonstrates that an increase in marginal cost c and customer willingness to purchase from manufacturer υ_1 lead to a decrease in extended warranty length t from manufacturer 2. An increase in marginal cost c implies that customers incur additional costs when they purchase a product from manufacturer 2, decreasing customer willingness for an unbundled extended warranty service υ_2. At the same time, customers prefer to purchase products from manufacturer 1, which offers a bundled base warranty service. Thus, the warranty length t offered by manufacturer 2 decreases to reduce marginal cost c between the bundling and unbundling strategies. In contrast, since customer sensitivity to extended warranty service β increases, the manufacturer 2 can attract more customers and generate increased demand. When customer sensitivity β increases to a particular level, it will consistently yield a more significant increase in warranty length t. ■

V. COMPUTATIONAL RESULTS AND DISCUSSION

This section presents the analysis of optimal decision variables and profit using the model developed by comparing each scenario. We examine the optimal decision and the impact of key parameters on it. Afterwards, we present an analysis of strategic decisions in a comparative analysis of each scenario. Finally, we generate managerial insights for managers and decision-makers that can be implemented using similar practices.

A. NUMERICAL AND SENSITIVITY ANALYSIS

These analyzes are performed to verify the effectiveness of the developed model in the previous section. The case of Apple Inc. and Samsung is appropriate to represent this situation [7], [21]. Apple competes with Samsung by providing a bundled base warranty and an unbundled extended warranty to gain a competitive advantage in the global smartphone market. Referring to previous studies [6], [10], other relevant parameters have been selected to effectively manage results while ensuring analytical tractability. These parameter settings are as follows: $w = 0.6$, $\beta = 1.5$, $c = 0.1$, $k = 2$, $\upsilon_1 = 0.1$, and $\upsilon_2 = 0.4$. Accordingly, by substituting the given parameter settings into the optimal decision equations in Theorems 1 and 2, we obtain the optimal decision results in Table 3.

Table 3 summarizes the optimal decision results for price, demand, and warranty length for various manufacturers and market scenarios. We first investigate the impact of the optimal decisions on each manufacturer's profit. In this setting, the profit of the manufacturer 2 outperforms the manufacturer 1. Intuitively, a greater customer willingness υ_i induces increased demand q_i, so the manufacturer increases its price p_i and in turn increases profits π_i. Furthermore, the manufacturer 2 has an optimal extended warranty length t as

TABLE 3. **Optimal decision-making results.**

Market scenario (j)	Manufacturer i	p_i	q_i	t	π_i
Pure bundling (b)	1	0.50	0.42	-	0.21
	2	0.70	0.58	-	0.41
Mixed bundling (m)	1	0.42	0.35	-	0.18
	2	0.78	0.65	0.48	0.27

it provides an unbundled extended warranty service instead of a bundled base warranty in the mixed bundling scenario.

In the pure bundling scenario, the customer's willingness to purchase from a particular manufacturer affects its profit and the other profit. For example, Fig. 2(a) describes how an increase in υ_1 induces an increase in the optimal profit π_{b1}^* but decreases the optimal profit π_{b2}^*. This concurs with the work of Bian et al. [7] and Liu et al. [6], namely, that when a manufacturer offers better services to its customers, their willingness to purchase from that manufacturer increases. The service can be a lower bundle price, a longer base warranty period, or other benefits. Following Proposition 1, the quantity demanded and the profit acquired by that manufacturer also increases. On the other hand, this situation can also be detrimental by decreasing the competitor's profits. Considering the fairness of the manufacturer, providing bundling strategy options may increase a manufacturer's competitiveness to provide better service to their customers.

In the mixed bundling scenario, the manufacturer 2 should determine the optimal extended warranty length t for its unbundled extended warranty strategy. Consequently, this warranty length t represents a customer's willingness to purchase an extended warranty. Fig. 3 illustrates how the extended warranty length t varies with shifts in the marginal cost c, the customer willingness for manufacturer υ_1, and the extended warranty sensitivity β. In this setting, each parameter changes from 0 to 1. As observed, an increase in the marginal cost c and customer willingness υ_1 prompts a decrease in the extended warranty length t, while an increase in the sensitivity of extended warranty service β yields an increase in the extended warranty length t from manufacturer 2. This analysis coincides with that of Liu et al. [6] and Panda et al. [23] and confirms Proposition 4.

B. OPTIMAL STRATEGIC DECISION ANALYSIS

In the following analysis, we compare the optimal decision variables in each scenario, including price p_i^*, demand q_i^*, and profit π_i^* based on customer willingness to purchase υ_i. However, since only manufacturer 2 performs an unbundled extended warranty strategy, we omit to compare the length of warranty t here. Furthermore, we simultaneously compare the optimal profit π_i^* under different scenarios for each manufacturer to indicate the best strategy for decision-making.

Corollary 1: In the pure bundling scenario, we have $p_{b2}^* < p_{b1}^*$, $q_{b2}^* < q_{b1}^*$, and $\pi_{b2}^* < \pi_{b1}^*$ when $-3w \leq \upsilon_2 - \upsilon_1 < 0$. Meanwhile, we have $p_{b2}^* > p_{b1}^*$, $q_{b2}^* > q_{b1}^*$, and $\pi_{b2}^* > \pi_{b1}^*$ when $0 < \upsilon_2 - \upsilon_1 \leq 3w$.

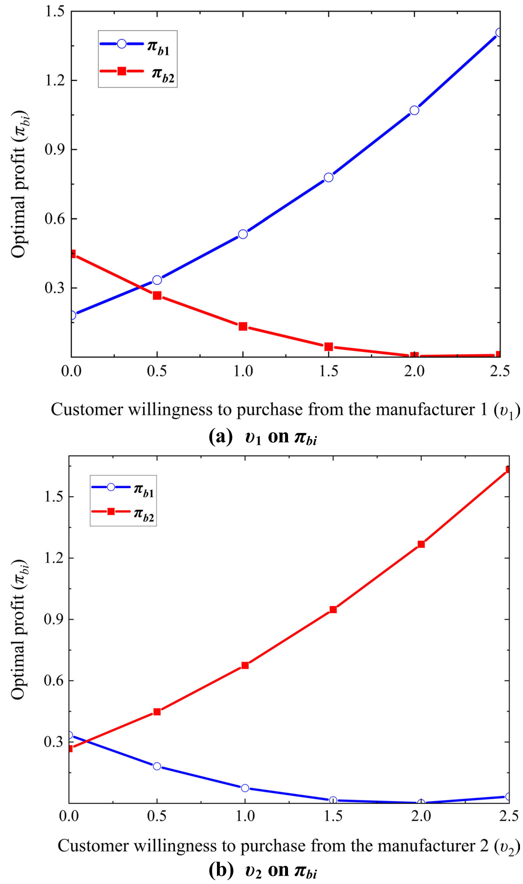


FIGURE 2. **Effect of customer willingness to purchase υ_i on the optimal profit of two manufacturers in the pure bundling scenario.**

Proof: See the Appendix.

Corollary 1 indicates the effect of the difference in customers' willingness to purchase the product between two manufacturers, $\upsilon_2 - \upsilon_1$, on their optimal decision variables and profits in the pure bundling scenario. As illustrated in Fig. 4, differences in customer willingness yield changes in the optimal price p_{bi}^*, the demand q_{bi}^*, and the profit π_{bi}^* of each manufacturer. This is consistent with the theory of interdependence and customer behaviour.

We can observe that the equilibrium of the price, the demand, and the profit for manufacturers 1 and 2 is perpetually changed at $\upsilon_2 - \upsilon_1 = 0$. Accordingly, we identify two situations that signify different decision-making effects, $-3w \leq \upsilon_2 - \upsilon_1 < 0$ and $0 < \upsilon_2 - \upsilon_1 \leq 3w$. When $-3w \leq \upsilon_2 - \upsilon_1 < 0$ exists, the customer's willingness to purchase the product from manufacturer 1 is more significant than that from manufacturer 2. The difference in customer willingness between the two manufacturers is more than $-3w$. This explains that the manufacturer 1 provides customers with better warranty and bundling strategies than the manufacturer 2. Therefore, the manufacturer 1 can obtain more customers demand q_{b1} through its bundled base warranty strategy. In such a situation, the manufacturer 1 increases the selling price p_{b1}, thus obtaining higher profits π_{b1}.

The reverse situation occurs when $0 < \upsilon_2 - \upsilon_1 \leq 3w$ exists. In this situation, the customer's willingness to purchase the product from manufacturer 2 is more significant than that from manufacturer 1. By providing the unbundled extended warranty strategy, the manufacturer 2 outperforms the manufacturer 1. Therefore, the manufacturer 2 can acquire more customers' demand q_{b2} and increase the product price p_{b2} to earn higher profits π_{b2}. ■

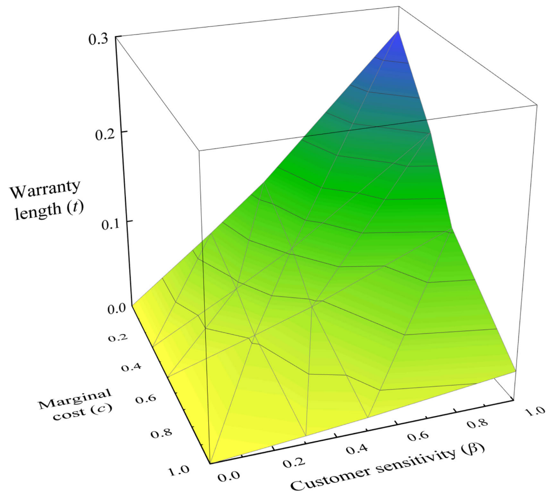
Corollary 2: In the mixed bundling scenario, we have $p^*_{m1} \geq p^*_{m2}$ and $q^*_{m1} \geq q^*_{m2}$ if $\beta \in \left(0, \sqrt{2k(\upsilon_1 + c)}\right)$, otherwise we have $p^*_{m1} < p^*_{m2}$ and $q^*_{m1} < q^*_{m2}$ if $\beta > \sqrt{2k(\upsilon_1 + c)}$. Moreover, we have $\pi^*_{m2} < \pi^*_{m1}$ when $0 < \beta^2 < \underline{M}$, and we have $\pi^*_{m2} > \pi^*_{m1}$ when $\underline{M} < \beta^2 < \overline{M}$.

Proof: See the Appendix.

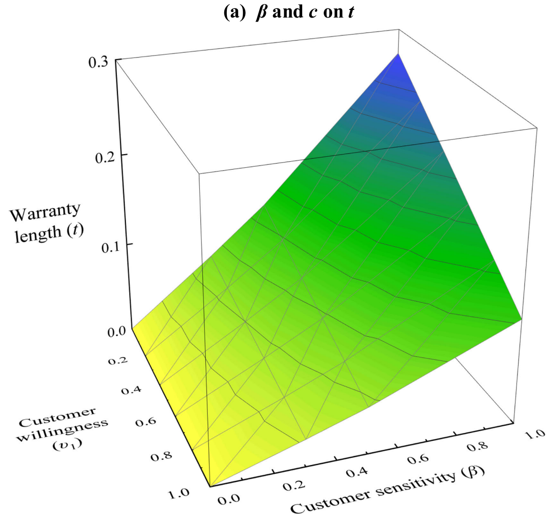
Corollary 2 demonstrates that the price and demand of the manufacturer 2 are smaller than the manufacturer 1 when β is in a smaller interval. At the same time, both the price and demand of the manufacturer 2 are larger than the manufacturer 1 when β is in a larger interval. Figs. 5 (a) and 5(b) demonstrate how the price and demand of the two manufacturers vary with the shift of extended warranty sensitivity β. When the extended warranty sensitivity β increases, the price p_{m1} and demand q_{m1} decrease, while the price p_{m2} and demand q_{m2} increase. Accordingly, for manufacturer 2, a longer extended warranty length can increase customers' demand q_{m2}, thus increasing the price p_{m2} and obtaining higher profits π_{m2}.

In addition, Corollary 4 indicates that in the mixed bundling market scenario, the profit of the manufacturer 2 is less than the manufacturer 1 when the extended warranty sensitivity β is small, and vice versa. The more significant the extended warranty sensitivity β, the more profitable it is for manufacturer 2, which can perform different warranty and bundling strategies. Thus, providing this service can increase customers' demand and profit. Fig. 5(c) illustrates how the profit of manufacturers 1 and 2 varies with the extended warranty sensitivity β.

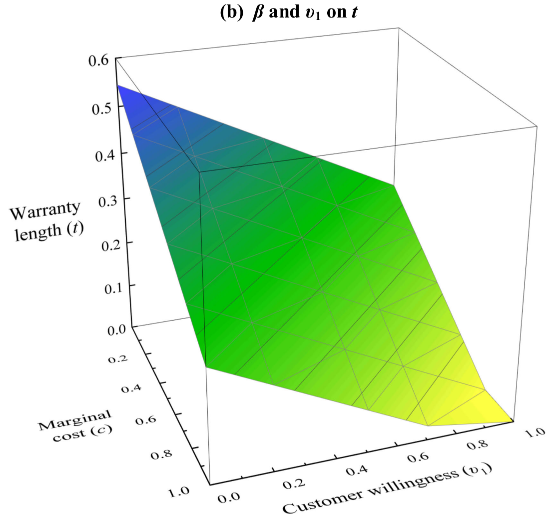
The profit of the manufacturer 1 decreases with the increase in β. Meanwhile, the profit of the manufacturer 2 first slightly increases and then decreases with the increase in β. In such a case, when the extended warranty sensitivity β is exorbitant, the unbundled extended warranty strategy provided by manufacturer 2 does not satisfy customers well. It confirms Lin et al. [35] that it instead drives customer dissatisfaction, leading to a decrease in profits. Furthermore, both manufacturers compete in the same market, and customer sensitivity generates a relationship between the equilibriums of the two manufacturer profits to change. In detail, when $0 < \beta < \sqrt{\underline{M}}$, the profit of the manufacturer 1 is larger than that of the manufacturer 2. When $\sqrt{\underline{M}} < \beta < 2$, the profit of the manufacturer 2 is larger than that of the manufacturer 1. It means that the manufacturer 2 does not always gain higher profits by adopting an unbundled extended warranty strategy in this duopoly. Therefore, these analyzes support Propositions 2 and 3. ■



(a) β and c on t



(b) β and υ_1 on t



(c) υ_1 and c on t

FIGURE 3. Effect of unbundled extended warranty strategy parameters on the extended warranty length t of the manufacturer 2 in the mixed bundling scenario.

Corollary 3: For manufacturer 1, we have $\pi^*_{b1} > \pi^*_{m1}$ if $\upsilon_1 < \omega_1$. Meanwhile, we have $\pi^*_{b1} \leq \pi^*_{m1}$ if $\upsilon_1 \geq \omega_1$, where $\omega_1 = 3w + \upsilon_2 - \frac{6kw(\upsilon_2+c)}{\beta^2}$.

Proof: See the Appendix.

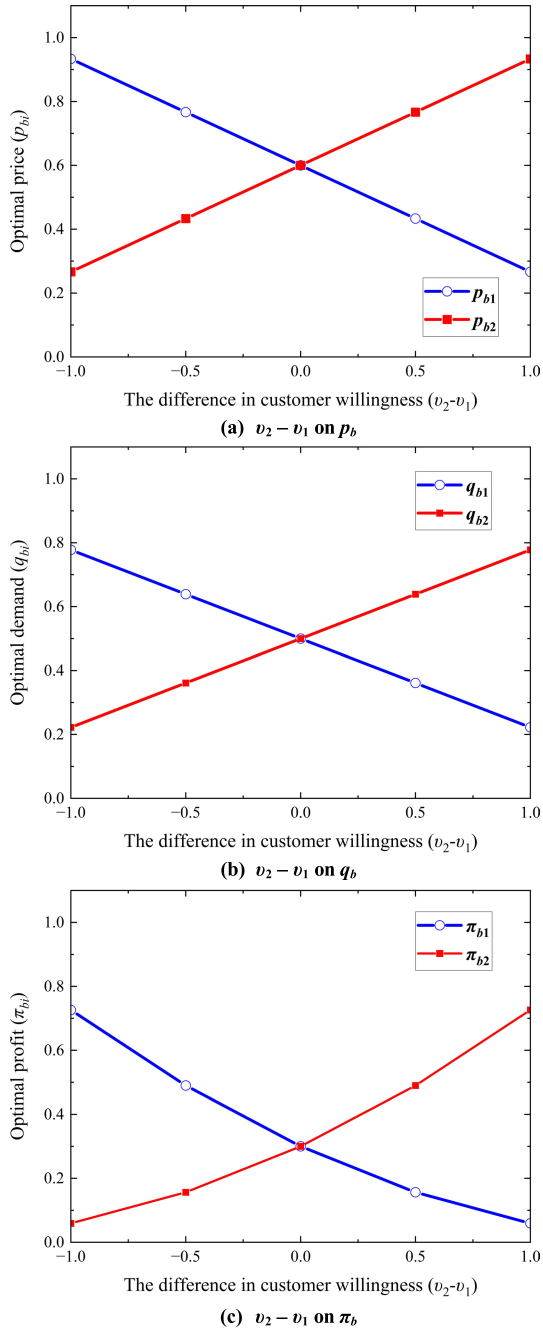


FIGURE 4. **Effect of the difference in customer willingness to purchase, $\upsilon_2 - \upsilon_1$, on optimal decisions for two manufacturers in pure bundling scenario.**

In this study, the manufacturer 1 provides a base warranty and performs a bundling strategy to sell its products in pure or mixed bundling scenarios. Corollary 3 presents the change in profit decision of the manufacturer 1 for each developed scenario considering customers' willingness to purchase a product from this manufacturer, υ_1. The manufacturer 1 earns a higher profit in the pure bundling scenario π_{b1} when the customer willingness υ_1 is less than ω_1. Thus, the manufacturer 1 earns higher profit in the mixed bundling scenario π_{m1} when the customer willingness υ_1 is greater than or equal to ω_1. It can occur since the extended warranty length t of the manufacturer 2 is specified, but customer willingness υ_1 is larger. As a result, the manufacturer 1 would profit more from performing the bundled base warranty strategy in pure bundling scenario when customer willingness υ_1 is lower. Otherwise, the manufacturer 1 would profit more in the mixed bundling scenario. ■

Corollary 4: For manufacturer 2, we have $\pi^*_{m2} \geq \pi^*_{b2}$ if $\upsilon_1 \geq \omega_2$. Meanwhile, we have $\pi^*_{m2} < \pi^*_{b2}$ if $\upsilon_1 < \omega_2$, where $\omega_2 = \frac{(3w+\upsilon_2)(6kw-\beta^2)-3(3w-c)\sqrt{kw(4kw-\beta^2)}}{6kw-\beta^2-3\sqrt{kw(4kw-\beta^2)}}$.

Proof: See the Appendix.

Similarly, Corollary 4 presents the change in profit decision of the manufacturer 2 for each developed scenario considering customer willingness, υ_1. Despite only providing a bundled base warranty in pure bundling scenario, the manufacturer 2 enhances this by providing an extended warranty in the unbundling strategy to sell its products in mixed bundling scenario. In Corollary 4, the manufacturer 2 earns a higher profit in mixed bundling scenario π_{m2} when customer willingness υ_1 is greater than or equal to ω_2. In contrast, the manufacturer 2 earns a higher profit in the pure bundling scenario π_{b2} when customer willingness υ_1 is less than ω_2. In such cases where customer willingness υ_1 is greater and customers are sensitive to the extended warranty service, the manufacturer 2 generates more profit by differentiating its decision to offer an unbundled extended warranty strategy. Accordingly, the manufacturer 2 would profit more from performing an unbundled extended warranty strategy in the mixed bundling scenario when customer willingness υ_1 is greater. Otherwise, the manufacturer 2 would profit more from performing only the bundled base warranty strategy in pure bundling scenario. Fig. 6 describes the profit comparison of manufacturers 1 and 2 in two different scenarios considering customer willingness to purchase, υ_1.

In pure bundling scenario, manufacturers 1 and 2 provide a bundled base warranty strategy. Suppose customer willingness υ_2 is constant and υ_1 is large. The profit of manufacturer π_{b1} is greater. In other words, customers will purchase more and more from any manufacturer according to the more significant customer willingness for a corresponding manufacturer. This coincides with the conclusion of Fang [15]. In the mixed bundling scenario, manufacturers 1 and 2 perform different warranty and bundling strategies, which indicates price bundling discrimination. With a particular extended warranty sensitivity β, marginal cost c, and customer willingness υ_1, a greater value of υ_1 will have a more significant impact on the profit of the manufacturer 2 in a pure bundling scenario and vice versa. Therefore, the manufacturer 2 should adapt strategic decisions regarding the equilibrium of customer willingness υ_1. ■

C. MANAGERIAL INSIGHTS

This analysis generates several managerial insights that may be valuable in making pricing decisions under warranty

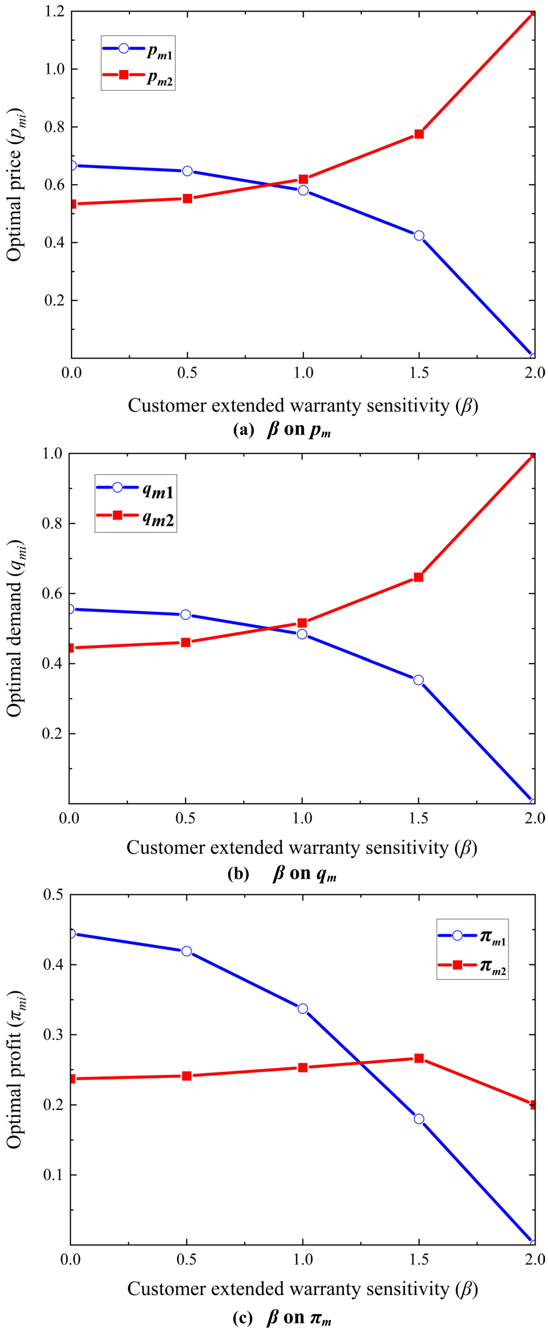


FIGURE 5. Effect of extended warranty sensitivity β on optimal decisions for two manufacturers in the mixed bundling scenario.

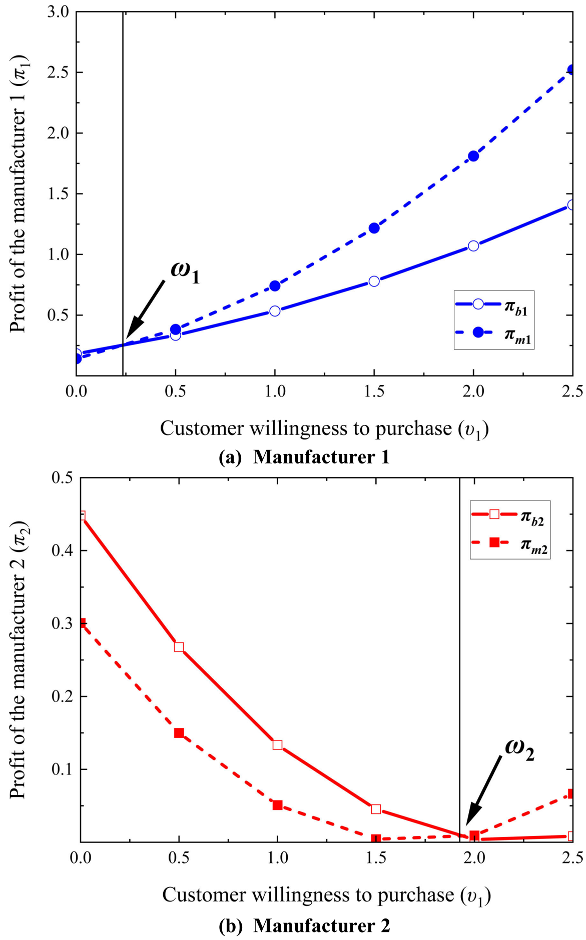


FIGURE 6. Profit comparison of two different scenarios for each manufacturer.

and bundling strategies in a competitive market. Initially, the warranty strategy design has a significant impact on pricing decisions. Generally speaking, incorporating a base warranty into product sales is an effective way to attract customers, while offering extended warranties may increase customer loyalty and satisfaction. This study demonstrates that providing a base warranty may benefit manufacturers in capturing market share by decreasing product price and cost. In addition, offering an extended warranty may increase a manufacturer's profitability by enabling them to charge a higher price. Managers and decision-makers might apply this insight to design a warranty strategy that balances customer demands with the manufacturer's profitability.

We also consider the effect of price bundling on offering different warranty services, a topic that has yet to be considered in the literature thus far. Bundling is an effective pricing strategy for manufacturers in a competitive supply chain. Bundling products and warranties together may increase both customer demand and manufacturer profit. Meanwhile, unbundling is a worthwhile strategy for reducing costs and increasing profit margins. Offering unbundled products and warranties also provides price- and service-sensitive customers more flexibility. However, managers and decision-makers should realize that each of these strategies is not always profitable in every situation. Therefore, they should carefully evaluate customer demand and market conditions to determine a warranty and bundling strategy that is aligned with their overall business strategy.

Devising an optimal pricing strategy depends on the level of competition and a customer's willingness to purchase a product from a particular manufacturer. When there is intense competition, duopolists must decrease their prices and offer a base warranty by engaging a bundled strategy to gain a competitive advantage. In contrast, when competition is moderate, duopolists may charge a higher price and offer an extended warranty by exploiting an unbundled strategy to maximize profits.

This study also emphasizes the trade-off between profitability and market demand. Offering a lower price with a bundled base warranty may help a manager or a decision-maker capture market demand but could simultaneously reduce profitability. While offering a higher price with an unbundled extended warranty may increase profitability, it may limit market demand. Thus, managers and decision-makers should consider competition when developing a pricing strategy, as the actions of competitors may significantly impact the effectiveness of any pricing strategy. Given the influential nature of a manufacturer's warranty and bundling strategies, exploiting them may enable decision-makers to align their pricing strategy and other needs with their broader business goals. Furthermore, they can apply these insights to a wide range of industries and products and thereby enhance their competitive position in the market.

VI. CONCLUSION

By developing a Nash game theoretic decision-making model, this study determines optimal pricing decisions under different warranty and bundling strategies to maximize profits and gain a competitive advantage in a duopolistic market. With two manufacturers competing spatially, the main results of this study are as follows. First, deciding to perform an unbundled extended warranty strategy can make a manufacturer more competitive. On the one hand, the unbundled extended warranty strategy may increase customer demand and maximize profit amongst manufacturers. On the other hand, the provision of this strategy also imposes an additional cost on the manufacturer, which also impacts its pricing decisions. Second, it is noteworthy that the unbundled extended warranty strategy is not consistently profitable. When customers are highly sensitive to the unbundled extended warranty strategy within a specific range, an increase in the length of the warranty offered by the manufacturer may improve the customer's satisfaction and the likelihood of an eventual purchase, thus generating increased demand and greater profit. Third, when two manufacturers compete to maintain a bundled base and an unbundled extended warranty, warranty length and customer sensitivity to an extended warranty impact the manufacturers' strategy decisions. Interestingly, when a customer's willingness to select the bundled base warranty is more significant than a particular value, the other manufacturer may gain higher profits by implementing an unbundled extended warranty strategy. Otherwise, the manufacturer should stick to the bundled base warranty strategy.

Our study yields several theoretical and practical implications. This analysis provides a theoretical framework for analysing optimal pricing decisions considering warranty and bundling strategies in a duopolistic, competitive supply chain. Thus, it contributes to the literature on pricing and supply chain management since the joint effects of warranty and bundling strategies influence optimal pricing decisions. These effects vary depending on the particular market situation. The results also contribute to developing game theory models for pricing models in a duopolistic market. For practical implications, managers and decision-makers may implement optimal pricing decisions that maximize profits and satisfy customer needs by better understanding customer preferences for various warranty and bundling strategies. These findings help supply chain managers better understand their competitors' strategies and make more informed decisions to differentiate their products to stay ahead in the market.

We should acknowledge a few limitations of this study. First, it focuses on a duopolistic and does not involve multiple firms or a more complex market structure. Second, we assume that perfect information about market conditions and competitors' strategies is available. Third, our pricing decision-making analysis does not consider other factors such as advertising, environmental sustainability, customer behaviour, and other risks. Therefore, we suggest these potential directions for future studies to address these limitations. First, future studies could expand the analysis to include more complex supply chain structures and more than only two firms. Second, exploring the impact of imperfect information on pricing decisions may provide a more comprehensive representation of a real-world scenario. Lastly, investigating other factors, such as advertising, environmental sustainability, customer behaviour, and other risks, could extend pricing decision-making analysis in a supply chain.

APPENDIX

A. THE PROOF OF THEOREM 1

In the pure bundling market scenario, taking the second derivative of π_{b1} and π_{b2} subject to p_{b1} and p_{b2}, respectively, we have:

$$\frac{\partial^2 \pi_{b1}}{\partial p_{b1}^2} = \frac{\partial^2 \pi_{b2}}{\partial p_{b2}^2} = -w < 0.$$

Thus, π_{b1} and π_{b2} are concave in p_{b1} and p_{b2}, respectively. Then, solving $\partial \pi_{b1} / \partial p_{b1} = 0$ and $\partial \pi_{b2} / \partial p_{b2} = 0$, we have the optimal prices, p_{b1}^* and p_{b2}^*, as in Theorem 1.

Accordingly, substituting the p_{b1}^* and p_{b2}^* into (1) and (2), we have the optimal demands, q_{b1}^* and q_{b2}^*. Since $q_{b1}^* \geq 0$ and $q_{b2}^* \leq 1$, the sufficient and necessary condition is to be satisfied: $-3w \leq \upsilon_2 - \upsilon_1 \leq 3w$. Hence, substituting p_{b1}^* and p_{b2}^* in (3) and (4), we have the optimal profits, π_{b1}^* and π_{b2}^*, as in Theorem 1.

B. THE PROOF OF THEOREM 2

In the mixed bundling market scenario, taking the first derivatives of π_{m2} subject to t, we have:

$$\frac{\partial \pi_{m2}}{\partial t} = \frac{\beta p_{m2}}{2w} - kt.$$

Solving $\partial \pi_{m2}/\partial t = 0$ and let $t' = \beta p_{m2}/2kw$, we substitute t' into (8) and (9), then we have:

$$\pi_{m1} = \frac{p_{m1}}{2w}\left(w - p_{m1} + p_{m2} + \upsilon_1 + c - \frac{\beta^2 p_{m2}}{2kw}\right),$$
$$\pi_{m2} = \frac{p_{m2}}{2w}\left(w + p_{m1} - p_{m2} - \upsilon_1 - c + \frac{\beta^2 p_{m2}}{2kw}\right) - \frac{1}{2}k\left(\frac{\beta p_{m2}}{2kw}\right)^2.$$

Afterward, taking the second derivative of π_{m1} and π_{m2} subject to p_{m1} and p_{m2}, respectively, we have:

$$\frac{\partial^2 \pi_{m1}}{\partial p_{m1}^2} = -\frac{1}{w} < 0 \text{ and } \frac{\partial^2 \pi_{m2}}{\partial p_{m2}^2} = -\frac{1}{w} + \frac{\beta^2}{4kw^2} < 0.$$

Hence, π_{m1} is concave in p_{m1} and π_{m2} is concave in p_{m2} when $\beta^2 < 4kw$.

Then, taking the first derivative of π_{m1} and π_{m2} subject to p_{m1} and p_{m2}, respectively, we obtain:

$$\frac{\partial \pi_{m1}}{\partial p_{m1}} = \frac{1}{2w}\left(w - 2p_{m1} + p_{m2} + \upsilon_1 + c - \frac{\beta^2 p_{m2}}{2kw}\right),$$
$$\frac{\partial \pi_{m2}}{\partial p_{m2}} = \frac{1}{2w}\left(w + p_{m1} - 2p_{m2} - \upsilon_1 - c + \frac{\beta^2 p_{m2}}{kw}\right) - \frac{\beta^2 p_{m2}}{4kw^2}.$$

Solving $\partial \pi_{m1}/\partial p_{m1} = 0$ and $\partial \pi_{m2}/\partial p_{m2} = 0$, we have the optimal prices, p_{m1}^* and p_{m2}^*, as in Theorem 2. Furthermore, we have the optimal warranty length t^*.

Afterwards, substituting p_{m1}^*, p_{m2}^*, and t^* into (5) and (6), we have the optimal demands, q_{m1}^* and q_{m2}^*. As $q_{m1}^* \geq 0$ and $q_{m2}^* \leq 1$, the sufficient and necessary condition is to be satisfied, $\upsilon_1 + c \leq 3w$ and $\beta^2 \leq k(3w+ \upsilon_1 + c)$. Thus, substituting p_{m1}^*, p_{m2}^*, and t^* into (7) and (8), we have the optimal the optimal profits, π_{m1}^* and π_{m2}^*, as in Theorem 2.

C. THE PROOF OF PROPOSITION 1

In the pure bundling market scenario, taking the first derivative of the optimal profit $\pi_{bi}*$ subject to υ_i, we have:

$$\frac{\partial \pi_{b1}*}{\partial \upsilon_1} = \frac{wp_1}{2} > 0,$$
$$\frac{\partial \pi_{b1}*}{\partial \upsilon_2} = -\frac{wp_1}{2} < 0,$$
$$\frac{\partial \pi_{b2}*}{\partial \upsilon_1} = -\frac{wp_2}{2} < 0, \text{and} \frac{\partial \pi_{b2}*}{\partial \upsilon_2} = \frac{wp_2}{2} > 0.$$

D. THE PROOF OF PROPOSITION 2

In the mixed bundling market scenario, taking the first derivative of optimal profit π_{m2}^* subject to υ_1, c, and β, respectively, and we have:

$$\frac{\partial \pi_{m2}^*}{\partial \upsilon_1} = -\frac{k(3w - \upsilon_1 - c)(4kw - \beta^2)}{(6kw - \beta^2)^2} < 0,$$
$$\frac{\partial \pi_{m2}^*}{\partial c} = -\frac{k(3w - \upsilon_1 - c)(4kw - \beta^2)}{(6kw - \beta^2)^2} < 0,$$
$$\frac{\partial \pi_{m2}^*}{\partial \beta} = \frac{\beta k(3w - \upsilon_1 - c)^2(2kw - \beta^2)}{(6kw - \beta^2)^3} > 0.$$

E. THE PROOF OF PROPOSITION 3

In the mixed bundling market scenario, taking the first derivative of the optimal price p_{mi}^* and demand q_{mi}^* are subject to β and k, respectively, and we have:

$$\frac{\partial p_{m1}^*}{\partial \beta} = -\frac{4\beta kw(3w - \upsilon_1 - c)}{(6kw - \beta^2)^2} < 0,$$
$$\frac{\partial q_{m1}^*}{\partial \beta} = -\frac{2\beta k(3w - \upsilon_1 - c)}{(6kw - \beta^2)^2} < 0,$$
$$\frac{\partial p_{m2}^*}{\partial \beta} = \frac{4\beta kw(3w - \upsilon_1 - c)}{(6kw - \beta^2)^2} > 0,$$
$$\frac{\partial q_{m2}^*}{\partial \beta} = \frac{2\beta k(3w + \upsilon_1 + c)}{(6kw - \beta^2)^2} > 0,$$
$$\frac{\partial p_{m1}^*}{\partial k} = \frac{2w\beta^2(3w - \upsilon_1 - c)}{(6kw - \beta^2)^2} > 0,$$
$$\frac{\partial q_{m1}^*}{\partial k} = \frac{\beta^2(3w - \upsilon_1 - c)}{(6kw - \beta^2)^2} > 0,$$
$$\frac{\partial p_{m2}*}{\partial k} = -\frac{2w\beta^2(3w - \upsilon_1 - c)}{(6kw - \beta^2)^2} < 0,$$
$$\frac{\partial q_{m2}^*}{\partial k} = -\frac{\beta^2(3w + \upsilon_1 + c)}{(6kw - \beta^2)^2} < 0.$$

F. THE PROOF OF PROPOSITION 4

In the mixed bundling market scenario, taking the first derivative of the optimal extended warranty length t^* subject to c, υ_1, and β, respectively, and we have:

$$\frac{\partial t^*}{\partial c} = -\frac{\beta}{6kw - \beta^2} < 0,$$
$$\frac{\partial t^*}{\partial \upsilon_1} = -\frac{\beta}{6kw - \beta^2} < 0,$$
$$\frac{\partial t^*}{\partial \beta} = \frac{(3w - \upsilon_1 - c)(6kw + \beta^2)}{(6kw - \beta^2)^2} > 0.$$

G. THE PROOF OF COROLLARY 1

Comparing the optimal prices and demands of manufacturers 1 and 2 in the pure bundling market scenario, we have price and demand discrimination:

$p_{b2} * -p_{b1}^* = \frac{2(\upsilon_2 - \upsilon_1)}{3}$ and $q_{b2}^* - q_{b1} * = \frac{w(\upsilon_2 - \upsilon_1)}{3}$.

Furthermore, we have:

$$\pi_{b2}^* - \pi_{b1}^* = \frac{2w^2(\upsilon_2 - \upsilon_1)}{3}.$$

Therefore, we acquire $p_{b2}^* > p_{b1}^*$, $q_{b2}^* > q_{b1}^*$, and $\pi_{b2}^* > \pi_{b1}^*$ when $0 < \upsilon_2 - \upsilon_1 \le 3w$. Meanwhile, we obtain $p_{b2}^* < p_{b1}^*$, $q_{b2}^* < q_{b1}^*$, and $\pi_{b2}^* < \pi_{b1}^*$ when $-3w \le \upsilon_2 - \upsilon_1 < 0$.

H. THE PROOF OF COROLLARY 2

Comparing the optimal prices and demands of manufacturers 1 and 2 in the mixed bundling market scenario, we have price and demand discrimination as follows:

$$p_{m2} * -p_{m1}^* = \frac{2w(\beta^2 - 2kc - 2k\upsilon_1)}{6kw - \beta^2},$$

$$q_{m2}^* - q_{m1}^* = \frac{\beta^2 - 2kc - 2k\upsilon_1}{6kw - \beta^2}.$$

Therefore, we have $p_{m2}^* \le p_{m1}^*$ and $q_{m2}^* \le q_{m1}^*$ when $0 < \beta \le \sqrt{2k\left(\upsilon_1 + c\right)}$. Meanwhile, we have: $p_{m2}^* > p_{m1}^*$ and $q_{m2}^* > q_{m1}^*$ when $\sqrt{2k\left(\upsilon_1 + c\right)} < \beta \le \sqrt{k\left(3w + \upsilon_1 + c\right)}$. In addition, we have profit discrimination, $\pi_{m2}^* - \pi_{m1}^*$, as shown at the bottom of the page.

$$M = \frac{k\left(15w - \upsilon_1 - c\right)\left(w + \upsilon_1 + c\right)}{8w}.$$

Let $M = \beta^2$ and $M = 0$, we have $f(M)$ and $f'(0)$, respectively. Meanwhile, taking the derivative of $f(M)$ subject to M, we have $f'(M)$. All proofs are shown at the bottom of the page. Suppose $f'(M) = 0$, we have:

Therefore, when $f(M) = 0$, we have $0 < \underline{M} < \overline{M}$, where $\underline{M}$ and $\overline{M}$ are defined as shown at the bottom of the page.

I. THE PROOF OF COROLLARY 3

Comparing the optimal profits of the manufacturer 1 under different bundling market scenarios, $\pi_{m1}^* - \pi_{b1}^* > 0$, as shown at the bottom of the page. Accordingly, we have:

$$\upsilon_1 > 3w + \upsilon_2 - \frac{6kw(\upsilon_2 + c)}{\beta^2}.$$

Let $\omega_1 = 3w + \upsilon_2 - \frac{6kw(\upsilon_2+c)}{\beta^2}$, we acquire $\upsilon_1 \ge \omega_1$, thus $\pi_{m1}^* \ge \pi_{b1}^*$, and vice versa.

J. THE PROOF OF COROLLARY 4

Comparing the optimal profits of the manufacturer 2 under different bundling market scenarios, $\pi_{m2}^* - \pi_{b2}^* > 0$, as shown at the bottom of the page. Thus, we have:

$$\upsilon_1 > \frac{\left(3w + \upsilon_2\right)\left(6kw - \beta^2\right) - 3\left(3w - c\right)\sqrt{kw\left(4kw - \beta^2\right)}}{6kw - \beta^2 - 3\sqrt{kw\left(4kw - \beta^2\right)}}.$$

Let $\omega_2 = \frac{(3w+\upsilon_2)(6kw-\beta^2)-3(3w-c)\sqrt{kw(4kw-\beta^2)}}{6kw-\beta^2-3\sqrt{kw(4kw-\beta^2)}}$, we acquire $\upsilon_1 \ge \omega_2$, thus $\pi_{m2}^* \ge \pi_{b2}^*$, and vice versa.

$$\pi_{m2}^* - \pi_{m1}^* = \frac{-4w\beta^4 + \left(15w - \upsilon_1 - c\right)\left(w + \upsilon_1 + c\right)k\beta^2 - 48k^2w^2\left(\upsilon_1 + c\right)}{2\left(6kw - \beta^2\right)^2}$$

$$f\left(M\right) = -4wM^2 + \left(15w - \upsilon_1 - c\right)\left(w + \upsilon_1 + c\right)kM - 48k^2w^2\left(\upsilon_1 + c\right)$$
$$f\left(0\right) = -48k^2w^2\left(\upsilon_1 + c\right) < 0$$
$$f'\left(M\right) = -8wM + k\left(15w - \upsilon_1 - c\right)\left(w + \upsilon_1 + c\right)$$

$$\underline{M} = -\frac{k\left(\upsilon_1 + c\right)^2 - 15kw^2 - 14kw\left(\upsilon_1 + c\right) - k\left(\upsilon_1 + c - 3w\right)\sqrt{25w^2 + \left(\upsilon_1 + c\right)^2 - 22w\left(\upsilon_1 + c\right)}}{8w}$$
$$\overline{M} = -\frac{k\left(\upsilon_1 + c\right)^2 - 15kw^2 - 14kw\left(\upsilon_1 + c\right) + k\left(\upsilon_1 + c - 3w\right)\sqrt{25w^2 + \left(\upsilon_1 + c\right)^2 - 22w\left(\upsilon_1 + c\right)}}{8w}$$

$$\pi_{m1}^* - \pi_{b1}^* = \frac{36w^2\left(\left(3w + \upsilon_1 + c\right)k - \beta^2\right)^2 - \left(6kw - \beta^2\right)^2\left(3w + \upsilon_1 - \upsilon_2\right)^2}{18w\left(6kw - \beta^2\right)^2} > 0$$

$$\pi_{m2}^* - \pi_{b2}^* = \frac{9kw\left(4kw - \beta^2\right)\left(3w - \upsilon_1 - c\right)^2 - \left(6kw - \beta^2\right)^2\left(3w - \upsilon_1 + \upsilon_2\right)^2}{18w\left(6kw - \beta^2\right)^2} > 0$$

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