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RESEARCH ARTICLE

Toward Optimal DoF Maximization With Interference Categorization Using TIM and SIC

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ABSTRACT In this paper, a novel topological interference management (TIM) framework, combined with the successive interference cancellation (SIC) technique, is proposed for Device-to-Device (D2D) networks. While the TIM problem was originally studied in a partially connected network, the main contribution appears here in proposing a novel design that allows some interfering links to be very high so that SIC can be used to handle them efficiently. More specifically, this inter-twinning between the two techniques (TIM and SIC) is based on the design of the precoding and decoding vectors used by the transmitters and receivers, respectively. This scenario of having very strong interference may occur in crowded D2D networks, such as malls, concerts or stadiums, where D2D communications take place in very close proximity. The problem is formulated as a rank minimization framework and solved by building on the characteristic polynomial function (CPF) of the adjacency matrix of the network interference graph. Numerical results show the effectiveness of our approach by achieving promising gains in terms of degrees-of-freedom (DoF), as compared to existing solutions, with a polynomial complexity.

INDEX TERMS Successive interference cancellation (SIC), degrees-of-freedom (DoF), topological interference management (TIM), topology information.

I. INTRODUCTION

The Device-to-device (D2D) communication is considered as a promising candidate to handle the unprecedented growth of cellular traffic by allowing devices in proximity to communicate in a direct link and bypass the base station, allowing hence to offload the data from the network [1]. Despite of the D2D benefits, the severe interference, imposed on the D2D receivers by nearby D2D transmitters, is a serious limitation that faces the development of D2D. To deal with this interference, several management schemes have been proposed, such as mode selection [2], resource allocation [3], power control [4], and a combination of these methods [5]. Moreover, fifth generation (5G) wireless networks, with Internet of Things (IoT) applications, are expected to become highly dense, which makes the interference more complex, and hence the applicability of the aforementioned approaches no more sufficient [6]. Recently, interference alignment (IA) [7] has evolved in D2D communication, although it requires the knowledge of the channel state information (CSI) among the D2D devices. Acquiring CSI is a burdensome task for a UE with limited capabilities, and costs additional signaling overhead in the network.

To deal with the signaling problem, topological interference management (TIM) is recognized as a clever approach in the direction of IA, where instantaneous CSI is not required, and the interference can be managed by only knowing the network's topology, which is represented by an adjacency matrix $\boldsymbol{X}$. The entries of this matrix denote the interference strength (which is an indicator variable denoting if the interference is weaker or stronger than the "noise floor") [8]. It has been established that TIM is equivalent, in terms of degrees-of-freedom (DoF), to index coding under linear schemes [8]. The TIM problem was recast as a low-rank matrix completion problem (LRMC), knowing that the achievable symmetric DoF is inversely proportional to the

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rank value of the output matrix of LRMC [9], [10]. In the topic of interference management, TIM has proved its performance efficiency in terms of increasing the DoF of the system.

Another technique that is usually used to deal with interference (and hence the resulting corrupted signal) is to separate the superimposed information, so that each receiver can retrieve its signal and decode its own data. This can be achieved by non-linear receivers that employ the Successive Interference Cancellation (SIC) technique [11]. SIC is usually employed to decode the stronger signal, subtract it from the combined signal, and then extract the weaker one from the residue. However, two of the main problems in using SIC are 1) its demanding computational requirements [12], and 2) its practicality when different power levels are used by the transmitters.

To this end, we propose a novel design of interference management by combining the two aforementioned techniques, namely TIM and SIC, intelligently, while only relying on the topology information at the transmitters. Note here that applying TIM-only in D2D networks may not be sufficient, since D2D communications typically occur in close proximity, along with heterogeneous path losses among the users, and the use of different levels of transmit powers [13]. Hence, there may be scenarios in which the interference may be stronger than the desired signal. Consequently, the precoding and decoding vectors adopted in the traditional TIM are then changed in this framework in order to take into account the use of SIC. Here appears the importance of our proposed approach that incorporates SIC with TIM while building the adjacency matrix. In this case, the dot product between the precoding and decoding vectors (this product represents an entry in the adjacency matrix) is represented by a very large value, allowing the very strong interference to be eliminated by SIC. It is worth mentioning that, contrary to standard SIC that is applied to all links, we emphasize that SIC is applied in our case to only *some* links (i.e. where the interference is very strong), which allows implementing SIC at a reduced complexity. Also, nowadays and in the near future, the computational capacity of both handsets and access points is expected to be high enough to run such algorithms [14]. Considering then the potentials of TIM, and SIC, our interest here is then to boost the system D2D network's DoF w.r.t to TIM only, while maintaining a complexity that is comparable to TIM [15].

To the best of our knowledge, there is no existing work that investigates the joint TIM-SIC approach and applies it in D2D communications scenarios. The main contributions of this work are:

- We propose a new design for Topological Interference Management (TIM), while taking into account the successive interference cancellation (SIC) technique. To that end, we integrate the notion of interference classification into Topological Interference Management (TIM), by differentiating between two levels: strong and very strong interference, which are likely to occur in D2D networks. The combination of these two techniques results in a new design of the signal precoding and decoding vectors.
- By formulating the problem as a low-rank matrix completion (LRMC) problem, we develop a novel rank minimization method by building on the characteristic polynomial function (CPF) of the adjacency matrix of the network interference graph. The resulting method is a two-stage successive rank minimization, which reduces the rank further as compared to existing work (and hence improve the DoF of the system).
- We provide numerical results that show the superiority of our approach with respect to existing work. We also provide discussions, along with numerical analyses of some network topologies, that explain why by combining SIC and TIM a performance improvement is achieved.

Additionally, we propose a topology exchange framework among the communicating D2D devices in order to explain how the D2D devices can become aware of the network topology.

In the remainder of this paper, Section II discusses the related work, Section III describes the system model of combining TIM with SIC. Section IV details the mathematical model, while Section V discusses the complexity of the proposed framework. Section VI presents the experimental results, and Section VII provides a discussion along with numerical results to show the importance of combining TIM and SIC. Finally, Section VII concludes the paper.

II. LITERATURE REVIEW

Due to the detrimental effects of interference in D2D networks, extensive research has been conducted on the topic of interference management to offer reliable communications. Most proposed schemes can be classified into three categories. First, there is interference avoidance that builds on orthogonal time-frequency multiple access schemes, such as TDMA, FDMA, and CDMA techniques, which are regarded as special cases of a more general scheme, namely TIM [16]. The second scheme is interference cancellation which applies advanced signal processing techniques on the D2D links. Third, the interference coordination method that employs intelligent power control and link scheduling schemes. In this paper, we focus on the combination of the first two types of techniques.

Distributed interference avoidance solutions using Machine Learning methods have been studied in [17]. Joint interference avoidance and resource allocation has been investigated as well in [18] and [19]. These works [17], [18], and [19] require channel state information knowledge at the transmitter, which results in high signaling overhead in the network. In our work, we focus on topological interference management (TIM) by assuming the knowledge of the interference graph topology only, which hence allows overcoming the high signaling overhead issue. Interference avoidance, and TIM in specific, attracted a lot of follow-up research with various assumptions, such as with transmitter/receiver cooperation and message passing [20], that have proved their efficiency in enhancing the achieved DoF. TIM has also been extended in various directions, e.g., with multiple antennas [21], with reconfigurable antennas [22], [23], and with confidential messages [24]. A TIM problem with adversarial topology perturbations is considered in [25] and [26] where a dynamic graph coloring is proposed. TIM has also been studied in cellular networks [27], where multiple layers of interference are included and its effect on DoF is analyzed. Additionally, TIM has been explored for multilevel connectivity scenarios [16], [28], where the network is decomposed into two components, one that includes the interfering links on which TIM is applied and another that consists of the weak links which could be handled via power control. An analytical approach is proposed to allocate power and TIM precoding and decoding. A distributed method is also proposed in [28] and its convergence is proved. Other work based on combining TIM with clustering [29], [30] or low-rank tensor completion in time-varying topology networks [31] have also been developed in the literature. Reference [32] investigates the problem of interference graph estimation to determine if the interference link is weak or strong. However, the work in [32] does not account for the channel fluctuations. Furthermore, TIM has been studied in partial connectivity situations, like alternating connectivity [33] and fast fading scenarios [34]. In these works, a matrix rank-loss approach is proposed to find the network topology conditions, leading to a certain DoF. For slow fading conditions, the design of TIM precoding and decoding has been done by establishing a direct relation between TIM and index coding in [8]. In this respect, the work in [10] proposes a matrix-completion-based scheme for TIM design with only some known matrix entries [35], while the missing values are filled so that the rank is minimized. In this work, we also use the strong relation to linear index coding, and adopt the LRMC approach to solve the TIM part of the problem, but we formulate this part differently, i.e., by using the characteristic polynomial function of the adjacency matrix. While the aforementioned papers deal with TIM only, our work in this paper is different as it combines both SIC and TIM, which leads to system performance improvements.

Many existing works study the NP-hardness and the non-convexity of the rank minimization problem, and propose sub-optimal solutions by finding relaxation methods such as the nuclear norm [35], the alternating projections [36], the directional alternating projections [37], the alternating minimization [38], and the least greedy algorithm [39]. However, the adjacency matrix considered in TIM is characterized by a special structure (having all ones on the diagonal) that prohibits the application of the aforementioned methods. For example, the nuclear [35] and the trace [40] norms fill the missing entries of the TIM matrix with 0 values, and thus always return the diagonal matrix as the optimal solution. Our proposed method succeeds in overcoming this diagonal constraint, while using the coefficients of the characteristic polynomial function. The other methods listed above need the optimal rank of the matrix as input, in contrast to our work, which does not require it, boosting by this its practicality. Moreover, these methods suffer from convergence issues, which were solved by using the Riemannian optimization for example in [41]. However, this technique has a very high computational complexity, opposite to the polynomial one of our algorithm. It helps to also note that these algorithms were not directly applied on D2D networks. In our previous works [42], [43], we improved the existing solutions and overcame the aforementioned constraints by developing novel approximation solutions based on SDP to solve TIM in D2D environments. Those solutions outperform other existing methods in terms of the minimum rank attained (while offering a lower complexity), and thus correspond to a lessening in signal interference. However, relying on SDP still suffers from a relatively high complexity, and thus motivated a part of this work, where we propose to build on the characteristic polynomial function to solve the LRMC-based TIM, which reduces the rank further.

In addition to the promising benefits offered by TIM, other interference management techniques proved their efficiency in dealing with interference, such as the successive interference cancellation (SIC) technique. SIC is one of several interference cancellation techniques that include parallel as well as iterative interference cancellation. However, SIC remains the mostly used technique, since its architecture (in terms of hardware complexity and cost) is similar to the traditional non-SIC receivers, and its decoder is the same for the different decoding stages of the composite signal, without the need for complicated decoders or multiple antennas [44]. Recently, there have been increasing interests in SIC-based physical layer communication schemes [45], such as full duplex technology and non-orthogonal multiple access (NOMA) [46]. In this regard, NOMA has gained attention due to the network level performance gains that SIC brought. More specifically, NOMA is applied in networks that schedule the same frequency resources to multiple users.

Several approaches also investigated the coexistence of NOMA and D2D communications with SIC-enabled receivers [44], [47], [48]. The authors in [47] mainly addressed the optimization issues associated with power control and data rate. As for [44], the authors use stochastic geometry tools to analyze the interference management and resource allocation problems in D2D-enabled multi-cell cellular networks, and to study the effect of the SIC technique. These tools allow deriving the successful transmission probabilities for both the cellular uplinks and D2D links with SIC, which reveals the gain of SIC in large-scale wireless networks. In [48], the authors introduce the concept of group D2D communications, where a D2D transmitter can simultaneously communicate with multiple D2D receivers with the aid of the NOMA protocol. They model the optimal resource allocation strategy of the NOMA-based D2D groups as a many-to-one matching problem, which leads to managing the interference from the underlying uplink cellular communication, and hence yield a better D2D sum-rate performance. Results have shown that the proposed NOMA-based D2D scheme is capable of delivering higher throughput than conventional D2D communication. In [49], the authors propose a new mechanism that jointly coordinates beamforming-based multiuser multiple-input multiple-output (MIMO), NOMA, and D2D communications in a downlink cellular network, to maximize the total system throughput. In [50], the authors specify two types of D2D-NOMA integrations: forward-D2D NOMA in which one D2D transmitter sends signals to a group of D2D receivers, and reverse-D2D NOMA, where D2D transmitters as part of a cluster transmit to a single D2D receiver.

While both SIC and D2D continue to mature in terms of their theoretical and practical aspects, TIM is also maturing. However, the TIM framework for NOMA users (with SIC capabilities) has not been well investigated in the context of D2D. Recently, few papers have appeared on the subject of combining TIM with NOMA. So far, Kalokidou et al. proposed to combine TIM principles with power-domain NOMA in single-input single-output (SISO) [51] and MIMO [52] systems. They introduced a two-stage process. In the first stage, users with different channel gains are clustered into groups, and TIM manages the "inter-cluster" interference among the clusters. In the second stage, the SIC technique is applied to cancel the "intra-cluster" interference. The employment of this scheme significantly improved the sum-rate performance of the system. However, the combination of these two techniques in [51] and [52] is hierarchical and is not seen as one entity, in contrast to our work, where SIC and TIM are both represented by the same LRMC model, allowing hence a more powerful precoding and decoding design.

Inspired by the potential benefits of the aforementioned interference management techniques, it becomes desirable to invoke intelligent joint interference management approaches that amalgamate TIM with the existing SIC technique, and to apply this approach in a D2D environment for further performance improvement in terms of system DoF.

III. SYSTEM MODEL

In this work, we consider a network of d D2D pairs, communicating in a multiple unicast setting. Without loss of generality, these devices can be divided into k clusters, knowing that in a D2D network, the D2D devices, that exist in close proximity, have tendency to interact more, and hence can be grouped in the same cluster. Grouping the devices into clusters can be done using various existing methods (e.g. [43]). The objective of this work is to develop an efficient TIM scheme in order to manage the interference within each cluster of devices, considering the fact that the clusters are either using orthogonal resources or the interference between clusters is small (since devices in different clusters are not in close proximity). For illustration, we show, in Fig. 1, a network of $d = 18$ D2D pairs that are divided into $k = 3$ groups: $G1$, $G2$ and $G3$, and each cluster operates at a different frequency (illustrated by a circle with a different color). Within each cluster, each pair of D2D devices (denoted by DTx and DRx in Figure 1) communicates between each other and exerts interference on the other devices. The interference can be classified into weak, strong and very strong as it will be explained in subsection III-A.

Note that in this paper, similar to previous works in TIM, [42], [43], [53], we assume that the transmit devices in each cluster are only aware of the network topology, and not of the instantaneous CSI. It is good to mention here that our framework holds of course when the network is composed of a single cluster, i.e. when all network devices are in the same cluster. However, we have chosen multiple clusters intentionally, knowing that in practice the network will be composed of multiple clusters since the devices will not interact between each other if they are not in close proximity. The network will be then naturally composed of multiple clusters.

In the following, we start by describing our proposed joint TIM-SIC framework to cancel the interference before discussing how these devices can get the information about the network topology that is necessary for the interference cancellation framework.

A. INTERFERENCE CANCELLATION USING JOINT TIM-SIC

Fig. 1 illustrates a scenario of D2D communications in which some of the communicating devices are in very close proximity, leading to very strong interferences that can be even stronger than the desired signal. Consequently, as shown in the figure, a D2D-receiver could receive the sum of the desired signal (yellow arrow), a very strong interference component (red arrow), a strong interference component (green arrow), and some weak interferences from far transmitters that could be neglected (and hence deleted from the figure for simplicity). *Without loss of generality*, we assume one very strong interference component in the received signal. However, the method we integrate in our approach, can manage (i.e., cancel) multiple very strong interference components. Managing all these kinds of interferences (specially the very strong one that overwhelms the desired signal) through TIM is not doable, since TIM may be inefficient when these interferences strongly differ in strength. We consequently propose to combine TIM with the Successive Interference Cancellation (SIC) method. We should note that our work is inline with the one in [54] in that our clustering part of our framework keeps the number of D2D devices within the cluster relatively small, given that they will be assigned the same subchannel (frequency range). This in turn keeps the receiver complexity low as was decribed in [48], [55], [56], and [57]. This is attributed to the fact that the hardware complexity and processing delay increase with the number of receivers that are multiplexed on the same subchannel [56]. In our framework, a finer classification of interference takes place, where a strong interference can be managed by TIM while the very strong one is handled by SIC. In pure SIC, the

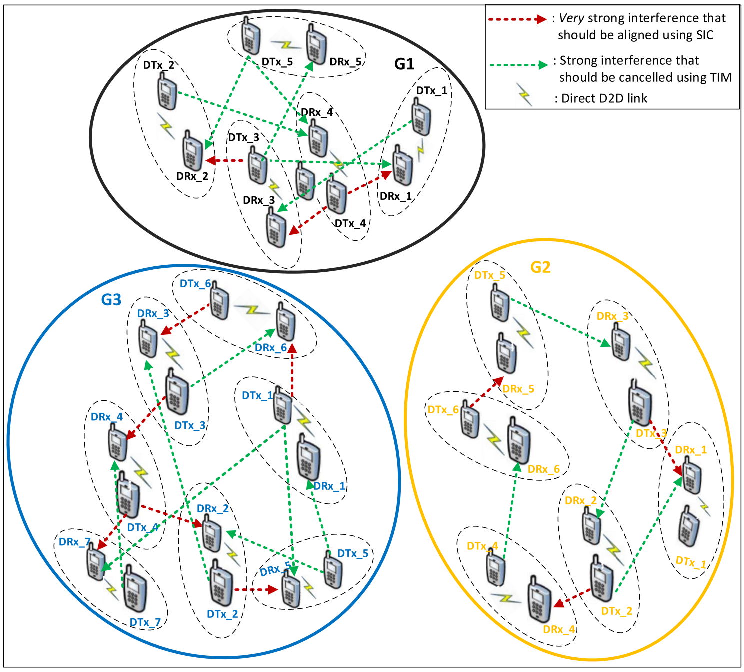


FIGURE 1. A network of 18 × 18 D2D-enabled devices with SIC capabilities.

signal separation is typically performed sequentially: each receiver decodes the strongest signal first, and then subtracts the decoded signal from the received signal to find its own signal. In this paper, we assume that all receivers are equipped with SIC which operates ideally, in that it makes perfect cancellation of the signals in the iteration steps. We remind here that the previously mentioned assumptions related to the absence of the instantaneous channel state information at the devices remain valid in the SIC scheme [58]. This results in a very smooth and successful combination of TIM and SIC.

Practically, each transmitter T_i in a cluster sends its intended signal s_i to its corresponding receiver R_i via the sidelink channel of the PC5 interface (as defined in the 3GPP standards [59]) after multiplying it by $\boldsymbol{v}_i$, the corresponding precoding vector. To recover its desired message, R_i decodes s_i by projecting the received vector $\boldsymbol{y}_i$ (that includes other undesired signals like s_j and s_k) into the space $\boldsymbol{u}_i$, which represents its decoding vector. Due to space constraint, the reader can refer to our previous works in [42], [43], and [53]) for more details about the notations usually adopted in the initial TIM framework and its background. Recall here that in this work the clusters are using orthogonal frequencies and hence the inter-cluster interference is absent. The input-output relationship within each cluster can be then written as:

$$\boldsymbol{u}_i\boldsymbol{y}_i = \boldsymbol{u}_i(\boldsymbol{v}_i h_{i,i} s_i + \sum_{k,(i,k)\in\mathcal{S}'} \boldsymbol{v}_k h_{i,k} s_k + \sum_{j,(i,j)\in\mathcal{S}} \boldsymbol{v}_j h_{i,j} s_j + z_i) \tag{1}$$

where $h_{i,i}$ and $h_{i,j}$ are the channel coefficients between $T_i - R_i$ and $T_j - R_i$ respectively, $z_i \sim \mathcal{N}(0, 1)$ is the noise, and $\mathcal{S}$ is the set of all pairs (i, j) such that R_i has strong interference from T_j, e.g., in group $G2$ in Fig. 1, $\mathcal{S}$ includes (1,2) since R_1 desires a signal from T_1, but it is suffering from the interference resulting from T_2. We define here a new set $\mathcal{S}'$ containing the indices of the nodes (T_k, R_i) with a *very strong* interference element (even stronger than the desired signal) that should be cancelled using SIC, e.g., $\mathcal{S}'$ includes (1,3) in group $G2$ in Fig. 1 since R_1 is also suffering from a *very strong* interference coming from T_3.

Thus, to preserve the wanted signal, we start by following the original TIM framework conditions on the precoding and decoding matrices, $\boldsymbol{V} = [\boldsymbol{v}_i]$ and $\boldsymbol{U} = [\boldsymbol{u}_j]$, respectively, as in [10]:

$$\begin{aligned} &\boldsymbol{u}_i\boldsymbol{v}_j = 0, \quad \text{for } \forall i \neq j,\ (i, j) \in \mathcal{S}, \text{ and} \\ &\boldsymbol{u}_i\boldsymbol{v}_i = 1, \quad \text{otherwise} \end{aligned} \tag{2}$$

These TIM strategies allow a part of the undesired signals, i.e., the strong interference to be removed ($\boldsymbol{u}_i \sum_{(i,j)\in\mathcal{S}, i\neq j} \boldsymbol{v}_j h_{i,j}$), as in (3). We therefore employ SIC to remove the other *very strong* interference, and recover the intended signal.

$$\begin{aligned} \boldsymbol{u}_i\boldsymbol{y}_i &= \boldsymbol{u}_i\boldsymbol{v}_i h_{i,i} s_i + \sum_{k,(i,k)\in\mathcal{S}'} \boldsymbol{u}_i\boldsymbol{v}_k h_{i,k} s_k \\ &\quad + \sum_{j,(i,j)\in\mathcal{S}} \underbrace{\boldsymbol{u}_i\boldsymbol{v}_j}_{0} h_{i,j} s_j + \boldsymbol{u}_i z_i \\ &= \boldsymbol{u}_i\boldsymbol{v}_i h_{i,i} s_i + \sum_{k,(i,k)\in\mathcal{S}'} \boldsymbol{u}_i\boldsymbol{v}_k h_{i,k} s_k + \boldsymbol{u}_i z_i \end{aligned} \tag{3}$$

According to [10], the conditions in (2) can be rewritten in a matrix form such as $\boldsymbol{X}_{G\ell} = [X_{i,j/k}] = \boldsymbol{u}_i\boldsymbol{v}_{j/k}$, which is an $n \times n$ real matrix ($n < d$ since it corresponds to the sub-matrix that results after clustering) and ℓ refers to the ℓ^{th} cluster. Note here that this matrix can be created by a base station (BS), i.e., in network-assisted mode, after receiving a sequence of bits from each D2D-receiver [60], where every bit corresponds to a transmitter heard by this receiver, based on the pilot signals.

Back to matrix $\boldsymbol{X}_{G\ell}$, composition, we show in (4) the different entry values that reflect the product between the precoding and decoding vectors and hence their designs. The first three values in (4) (i.e., 1, 0,*) can be deduced from the original TIM framework in (2). Our twist here is that we divide the interferers into two sets $\mathcal{S}$ and $\mathcal{S}'$ depending if the induced intereference is strong or *very* strong. Therefore, the interference resulting from the interferers present in set $\mathcal{S}$ should be cancelled by TIM: this will appear as a 0 in $\boldsymbol{X}_{G\ell}$. Now to handle the incorporation of the very strong interferer, we add a new symbol † denoting that the interference coming from T_k should be cancelled using SIC. This † value is well-defined, and is computed in Section IV.

$$\boldsymbol{X}_{G\ell} = [X_{i,j/k}] = \boldsymbol{u}_i \boldsymbol{v}_{j/k} = \begin{cases} 1 & \text{if } i = j = k \text{ the desired signal} \\ 0 & \text{if } (i,j) \in \mathcal{S} \ \& \ i \neq j \text{ interference cancelled} \\ & \text{by TIM} \\ * & \text{otherwise, i.e., not affecting interference} \\ \dagger & \text{if } (i,k) \in \mathcal{S}' \ \& \ i \neq k \text{ interference} \\ & \text{cancelled by SIC} \end{cases} \tag{4}$$

It is worth mentioning that, as usually considered in the TIM literature, $X_{i,j/k} = *$ in (4) means that there is no constraints on the values of $v_{j/k}$ and $\boldsymbol{u}_i$, since the link between devices j and i is weak. Regarding †, its value is kept general here since it can vary from one topology to another, but in all cases, † is greater than 2, to show that it is stronger than the desired signal ($X_{i,i} = \boldsymbol{u}_i \boldsymbol{v}_i = 1$). This can be hence considered as an LRMC approach for the joint TIM-SIC problem. The rank of the resulting matrix $\boldsymbol{X}$ is related to DoF, knowing the equivalence between TIM and index coding with linear schemes. More specifically, we consider, in this work, the symmetric DoF (DoF_{sym}) of the network (the largest DoF that can be achieved by all users simultaneously [10]) as our main figure of merit, in line with many existing works [16]. We assume for convenience a single data stream transmission per D2D pair, and so, DoF_{sym} that is attainable at high signal-to-noise ratio (SNR) (when the interference dominates the noise) will be $\text{DoF}_{\text{sym}} = \frac{1}{\text{rank}(\boldsymbol{X}^*)} = \frac{1}{r}$ per user. Thus, our objective is to minimize the matrix rank, and hence maximize the DoF, which is the total number of spatial streams that the network can support simultaneously without interference. Back to (3), replacing the values of $X_{i,j}$ in this equation gives:

$$\tilde{\boldsymbol{y}}_i = h_{i,i} s_i + \dagger h_{i,k} s_k + \boldsymbol{u}_i z_i \tag{5}$$

Thus, to cancel the *very strong* interference, R_i can use SIC. However, prior to SIC, users should be ordered according to

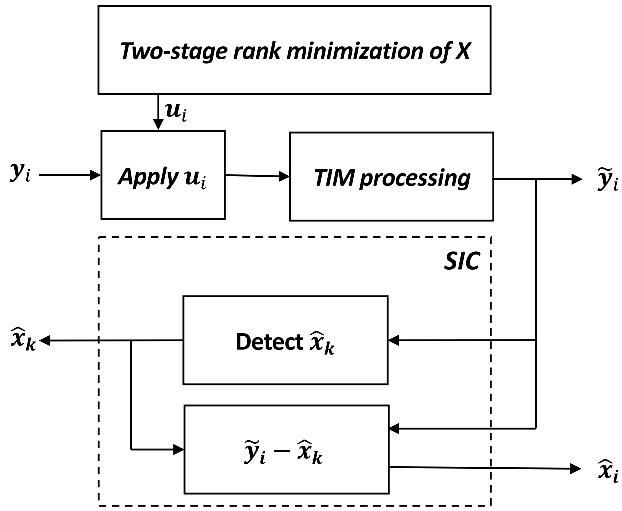


FIGURE 2. Interference cancellation at receiver using TIM and SIC.

their signal strengths, so that the SIC-enabled receivers can work properly [61]. Reordering (5) accordingly leads to:

$$\tilde{\boldsymbol{y}}_i = \dagger h_{i,k} s_k + h_{i,i} s_i + \boldsymbol{u}_i z_i \tag{6}$$

By this, SIC can first decode the signal x_k by extracting s_k (where s_k was defined in (3) to belong to $\mathcal{S}'$), considering its own signal as noise, and substracting the estimate $\hat{x}_k$ from the output $\tilde{y}_i$ of the TIM block (which cancels the strong interference signal component), so that it can decode its own signal $\hat{x}_i$:

$$\begin{aligned} \text{First stage: } \hat{x}_k &= \dagger h_{i,k} s_k + \boldsymbol{u}_i z_i \\ \text{Second stage: } \hat{x}_i &= \tilde{\boldsymbol{y}}_i - \dagger h_{i,k} s_k = h_{i,i} s_i + \boldsymbol{u}_i z_i \end{aligned} \tag{7}$$

The block diagram in Fig. 2 illustrates the process of cancelling the interference using the joint TIM-SIC approach. The top block shown in the figure is detailed in the next section, but as shown, its purpose is to produce the decoding vector $\boldsymbol{u}_i$.

It is worthwhile to note here that this framework assumes that the devices are aware of the network topology. For this, we propose, in the following subsection, possible schemes to learn about this topology information.

B. LEARNING ABOUT TOPOLOGY INFORMATION

We describe here two possible schemes for learning about the D2D network topology: one that only involves the D2D devices themselves, whereas the other is based on the network-assisted mode, where the base station (BS) collects information about the topology and builds the adjacency matrix.

The first scheme builds on the works in [62], [63], [64], and [65], according to which, devices can learn about the topology information of neighboring devices through the device discovery process. This discovery is made possible by each UE broadcasting its own proximity beacon signals at given times, thus enabling nearby UEs to capture such beacons [66] that can contain the identity of each potential

D2D user and also serve as reference signals for measuring the interference level. Moreover, we propose for each device to propagate the list of interfering and communicating neighbors that it learns about in its beacons, which is doable, as inferred from [67] and [68]. This information that each device sends, is a list of tuples, where each one includes the Id of the device and one of two associated values: interference ($v0$) or an ongoing or pending D2D session ($v1$). For example, when device R_2 transmits a beacon that contains (T_1, $v0$) and (T_2, $v1$), it says that the strength of the beacon it heard from T_1 constitutes interference ($v0$), while $v1$ indicates that it (i.e., R_2) has an ongoing or pending D2D communication session with T_2. In this respect, we note that a $v1$ value is only sent by D2D receivers, not transmitters. Hence, a device T_3 which hears the beacon from R_2, adds a row in its neighborhood list that associates the values $v0$ and $v1$ with T_1 and T_2, respectively. With this, if T_3 now wishes to establish a D2D session with another device using our scheme, it has to first construct the adjacency matrix, which it can do from the information it has collected in its neighborhood list, in particular by only considering devices that have sent $v1$ values (i.e., D2D receivers).

The above proposed scheme ensures that all neighboring (i.e., connected directly, or indirectly) transmitters (and receivers) have identical neighborhood lists, and thus can construct the same adjacency matrix ($\boldsymbol{X}$). Any of those transmitters, e.g., transmitter j (T_j) can then perform rank minimization on the adjacency matrix it has built, and extract the relevant precoding and decoding vectors $\boldsymbol{v}_j$ and $\boldsymbol{u}_j$ to cancel the interference (i.e., using TIM). Related to $\boldsymbol{v}_j$ and $\boldsymbol{u}_j$, two options are available: T_j can produce both vectors, and send $\boldsymbol{u}_j$ to R_j (i.e., its corresponding receiver), or rely on R_j to extract $\boldsymbol{u}_j$ itself (since it has the same $\boldsymbol{X}$).

Since broadcasting beacons, if uncontrolled, can lead to flooding the wireless network, we propose the following four measures to make the process of sending beacons more energy efficient and less loading on the network:

- The values $v0$ and $v1$ can be specified using one bit in the beacons (0 for $v0$, and 1 for $v1$).
- The discovery process of transmitting beacons between the devices, using Orthogonal Frequency Division Multiple Access (OFDMA) as in [69] and [70], allows for the possibility of the devices to transmit beacon signals in parallel slots. As for the beacon structure, it can use the one proposed in [69] (which is built on the existing beacon design of the 3GPP Long Term Evolution (LTE)), to fit the list of neighbors in their "future use" field.
- The problem of synchronization when beacons are multiplexed together in the same OFDMA symbols can be resolved by dividing the devices into groups that use different patterns to transmit in different beaconing opportunities, as was also proposed in [69].
- The frequency of transmitting beacons can be tied to mobility: A device sends the next beacon only when its location significantly changes (using its GPS sensor), or when it receives beacons from devices it did not hear from recently.

We note that according to [67], the minimum unit of beacon is a Resource Block (RB) which carries 72 OFDM symbols [71], whereas the maximum can be two RBs [72]. This limits the amount of information carried in a beacon, but with the application of the measures above, we enable a device to send more tuples in each beacon. Nevertheless, in case a device was not able to include all the information in one beacon, it can distribute them among consecutive ones.

On the other hand, the second scheme for learning about the D2D network topology, and consequently building the adjacency matrix is based on the D2D network-assisted mode. Here, each receiver can send a sequence of bits to a base station (BS), where every bit corresponds to a transmitter (that is heard by this receiver) based on the pilot signals. This 1-bit feedback, which is in line with [8], refers to the average interference channel strength after comparing the average power of the received links against a prechosen threshold value (i.e., the acceptable noise floor): it is equal to "0" for weak interference links, and "1" for significant ones. Having this information, the BS can then deduce the interfering pairs and build the connectivity pattern (i.e., construct the adjacency matrix), apply the rank minimization method on it, and then build the precoding and decoding vectors (by applying QR factorization). These vectors will be sent to the transmitters and receivers, respectively.

IV. TWO-STAGE RANK MINIMIZATION

To solve the joint TIM-SIC problem, we rely on the low-rank matrix completion approach. Based on (4), we can write, in the following, the adjacency matrices $\boldsymbol{X}_{G1}$, $\boldsymbol{X}_{G2}$ and $\boldsymbol{X}_{G3}$ that correspond to each of the groups present in Fig.1, such that the column and row numbers of these matrices correspond to the transmitter and receiver indices, respectively:

$$\boldsymbol{X}_{G1} = \begin{pmatrix} 1 & a_1 & 0 & \dagger_{1,1} & b_1 \\ c_1 & 1 & \dagger_{2,1} & d_1 & 0 \\ 0 & e_1 & 1 & \dagger_{3,1} & f_1 \\ g_1 & 0 & h_1 & 1 & 0 \\ i_1 & j_1 & 0 & k_1 & 1 \end{pmatrix},$$

$$\boldsymbol{X}_{G2} = \begin{pmatrix} 1 & 0 & \dagger_{1,2} & a_2 & b_2 & c_2 \\ d_2 & 1 & 0 & e_2 & f_2 & g_2 \\ h_2 & i_2 & 1 & j_2 & 0 & k_2 \\ l_2 & \dagger_{2,2} & m_2 & 1 & n_2 & o_2 \\ p_2 & q_2 & r_2 & s_2 & 1 & \dagger_{3,2} \\ t_2 & u_2 & v_2 & 0 & w_2 & 1 \end{pmatrix},$$

$$\boldsymbol{X}_{G3} = \begin{pmatrix} 1 & a_3 & b_3 & c_3 & 0 & d_3 & e_3 \\ f_3 & 1 & g_3 & \dagger_{1,3} & 0 & h & i \\ j_3 & 0 & 1 & k_3 & l_3 & \dagger_{2,3} & m_3 \\ n_3 & o & \dagger_{3,3} & 1 & p_3 & q_3 & 0 \\ 0 & \dagger_{4,3} & r_3 & s_3 & 1 & t_3 & u_3 \\ \dagger_{5,3} & v_3 & 0 & w_3 & x_3 & 1 & y_3 \\ 0 & z_3 & aa_3 & \dagger_{6,3} & bb_3 & cc_3 & 1 \end{pmatrix} \quad (8)$$

where $\boldsymbol{X}_{G1}$, $\boldsymbol{X}_{G2}$ and $\boldsymbol{X}_{G3}$ have all ones on their diagonals to represent the desired communication between the D2D pairs, and predefined values as 0s to denote the strong interferences within each cluster. As for $\{a_\ell, \ldots, w_\ell\}$ where $\ell = \{1, 2, 3\}$ and $\{aa_3, bb_3, cc_3\} \in \mathcal{T}$, they represent the missing values "*" (don't care values) of (4). As for $\dagger_{\bullet,\ell}$, its place in the matrix indicates the source of the *very strong* interference. For a better explanation, we consider cluster $G1$ in Fig. 1 and its corresponding adjacency matrix $\boldsymbol{X}_{G1}$ in (8) as an example: the desired communication between T_1 and R_1 appears as a value of 1 in the 1st row, 1st column in $\boldsymbol{X}_{G1}$, and the strong interference (green link) coming from T_3 towards R_1 is denoted as 0 in the 3rd column, 1st row of $\boldsymbol{X}_{G1}$ to indicate that it should be cancelled using TIM. As for the *very* strong interference going from T_4 to R_1 (red link), it is represented by $\dagger_{1,1}$ in the 1st row, 4th column of $\boldsymbol{X}_{G1}$. Concerning the remaining values a_1 and b_1, they indicate that T_2 and T_5 have no effect on R_1.

Recall that the objective here is to minimize the rank of each of these matrices in order to increase the system DoF. In this framework, we start in Stage 1 by computing the adequate value of $\dagger_{\bullet,\ell}$ for each of the groups, so that it can be used in Stage 2, where we apply *eRM-TIM* (the rank minimization method that we developed in [42]).

A. STAGE 1

To determine the appropriate value of $\dagger_\ell$ (recognizing that the *very strong* interference should be cancelled using SIC), we rely on the characteristic polynomial of a square $n \times n$ matrix $\boldsymbol{X}_{G\ell}$ (having the eigenvalues λ_i as roots of this polynomial) [73], given by:

$$\begin{aligned} p(\lambda) &= \det(\boldsymbol{X}_{G\ell} - \lambda \boldsymbol{I}) \\ &= (-1)^n(\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_n) \\ &= (-1)^n[\lambda^n + c_1\lambda^{n-1} + c_2\lambda^{n-2} + \ldots + c_{n-1}\lambda + c_n] \\ &= (-1)^n[\lambda^n - \lambda^{n-1}\text{Trace}(\boldsymbol{X}_{G\ell}) + c_2\lambda^{n-2} + \ldots \\ &\quad + (-1)^{n-1}c_{n-1}\lambda + (-1)^n\det(\boldsymbol{X}_{G\ell})] \end{aligned} \tag{9}$$

where $\boldsymbol{I}$ is the identity matrix, and the coefficients $c_2, \ldots, c_{n-1}$ can be expressed in terms of traces of powers of $\boldsymbol{X}_{G\ell}$ (as c_m in (10)):

$$\begin{aligned} c_m &= -\frac{t_m}{m} + \frac{1}{2!}\underbrace{\sum_{i=1}^{m-1}\sum_{j=1}^{m-1}}_{i+j=m}\frac{t_it_j}{ij} - \frac{1}{3!}\underbrace{\sum_{i=1}^{m-2}\sum_{j=1}^{m-2}\sum_{k=1}^{m-2}}_{i+j+k=m}\frac{t_it_jt_k}{ijk} \\ &\quad + \ldots + \frac{(-1)^m t_1^m}{m!}, \\ &\text{where } m = 1, 2, \ldots, n, \text{ and } t_k = \text{Trace}((\boldsymbol{X}_{G\ell})^k) \end{aligned} \tag{10}$$

or in terms of its eigenvalues (as c_k in (11)):

$$c_k = (-1)^k \sum_{\substack{i_1=1 \\ i_1<i_2<\ldots<i_k}}^{n} \sum_{i_2=1}^{n} \ldots \sum_{i_k=1}^{n} \underbrace{\lambda_{i_1}\lambda_{i_2}\ldots\lambda_{i_k}}_{\text{k factors}}, \quad \text{for } k = 1, 2, \ldots, n \tag{11}$$

Recall that our objective is to minimize the rank of each sub-matrix that corresponds to a cluster, or, equivalently, to minimize the number of non-zero eigenvalues of $\boldsymbol{X}_{G\ell}$, knowing that the number of non-zero eigenvalues of $\boldsymbol{X}_{G\ell}$ is at most the rank of $\boldsymbol{X}_{G\ell}$. Building on the relation between the c_k coefficients and the eigenvalues of $\boldsymbol{X}_{G\ell}$ in (11), we recognize that the matrix rank (rank$(\boldsymbol{X}_{G\ell}) = n$ when it is full rank) gets reduced by z, if and only if:

$$\begin{aligned} &c_n = 0,\ c_{n-1} = 0, \ldots, \text{ and } c_{n-(z-1)} = 0 \\ &\rightarrow c_n^2 + c_{n-1}^2 \ldots + c_{n-(z-1)}^2 = 0 \end{aligned} \tag{12}$$

To this end, for each sub-matrix, we define a vector $\boldsymbol{f}_\ell$ that contains all the c_k coefficients, such that $\boldsymbol{f}_\ell = [c_n, \ldots, c_2]$ (where $k = (n-1), \ldots, 2$). Note here that the c_k coefficients, which are included in $\boldsymbol{f}_\ell$, are the ones that contain variables, excluding by this the c_k factors that represent constant values (e.g., $c_1 = -\text{Trace}(\boldsymbol{X}_{G\ell}) = \text{cst}$, since the TIM adjacency matrix always has 1 on its diagonal entries). We have also chosen to work with the square of these values since the c_k factors belong to $\mathbb{R}$, i.e., c_k can take positive or negative values. The objective then becomes to minimize the square of the ℓ_2-norm of $\boldsymbol{f}_\ell$ that corresponds to each sub-matrix, i.e., $||\boldsymbol{f}_\ell||_2^2 = c_n^2 + c_{n-1}^2 + \ldots + c_2^2$ (the dimension of $\boldsymbol{f}_\ell$ is then $1 \times (n-1)$). As for the constraints, on one hand, $\dagger$ should be greater than 2, so that $X_{i,k} = \boldsymbol{u}_i\boldsymbol{v}_k = \dagger$, representing a *very strong* interference, should be greater than the desired signal $X_{i,i} = \boldsymbol{u}_i\boldsymbol{v}_i = 1$ (as previously mentioned) to be placed first in (6). On the other hand, the variables $\{a_\ell, \ldots, w_\ell\}$ and $\{aa_3, bb_3, cc_3\} \in \mathcal{T}$ that are assigned "*" (don't care values) in (4) are not subjected to any constraint, since their effect do not appear in the received signal in (1). The optimization problem (that we call *cRM-TIM*) can hence be written as:

$$\begin{aligned} &\min\ ||\boldsymbol{f}_\ell||_2^2 \\ &\text{s.t. } \dagger_{\bullet,\ell} \geq 2 \end{aligned} \tag{13}$$

This problem is a strictly convex one, since the vector 2-norm squared is a strictly convex function. Note also that (13) can be linearized and written in trace format, as follows:

$$\begin{aligned} &\min\ \text{Tr}\left(\boldsymbol{f}_\ell\boldsymbol{f}_\ell^T\right) \\ &\text{s.t. } \dagger_{\bullet,\ell} \geq 2 \end{aligned} \tag{14}$$

since $\text{Tr}\left(\boldsymbol{f}_\ell\boldsymbol{f}_\ell^T\right) = ||\boldsymbol{f}_\ell||_2^2$. Knowing this equivalence, we solve the optimization problem in (13) using the interior point method in order to compute the appropriate value of $\dagger$ that should be adopted. This is because $\dagger_{\bullet,\ell}$ is dependent on the topology (i.e., each topology is represented by a different matrix, with different entries). Therefore, each of the adjacency sub-matrices $\boldsymbol{X}_{G1}, \boldsymbol{X}_{G2}$ and $\boldsymbol{X}_{G1}$ will have its own $\dagger_{\bullet,\ell}$ value.

B. STAGE 2

Once the value of $\dagger_{\bullet,\ell}$ is known, the rank optimization method *eRM-TIM* that we proposed in [43] can be applied to get the appropriate * entries (or represented as variables in (8)) that minimize the rank of matrix $\boldsymbol{X}_{G\ell}$. We refer here to $\boldsymbol{X}_{G\ell}$ as $\boldsymbol{X}$ for simplicity. The solution $\boldsymbol{X}^*$ can be then factorized

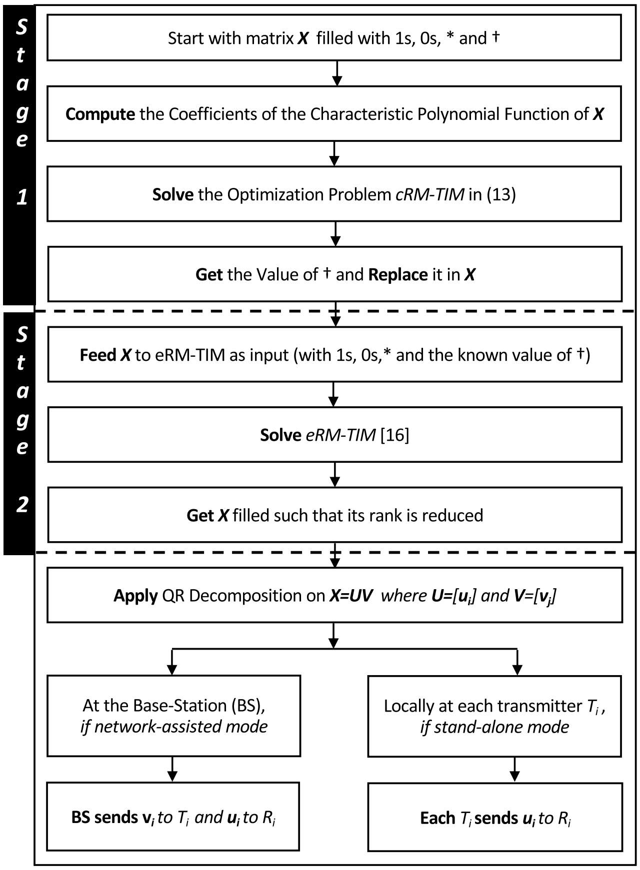


FIGURE 3. Two-stage rank minimization of X.

using QR decomposition to get the precoding and decoding matrices, $\boldsymbol{V}$ and $\boldsymbol{U}$, respectively. The steps of the algorithm are summarized in Fig. 3. Note here that to decide which singular values can be neglected while doing the rank minimization of $\boldsymbol{X}$ (knowing that the number of non-zero singular values indicates the rank of $\boldsymbol{X}$), we rely on the Eckart and Young Theorem [74], which states that $||\boldsymbol{X} - \hat{\boldsymbol{X}}_k||_F$ can be approximated by $\sqrt{\sigma_{k+1}^2 + \cdots + \sigma_n^2}$. To quantify the error resulting from this approximation, we define the following metric:

$$error = \frac{||\boldsymbol{X} - \hat{\boldsymbol{X}}_k||_F}{||\boldsymbol{X}||_F} = \frac{\sqrt{\sigma_{k+1}^2 + \cdots + \sigma_n^2}}{||\boldsymbol{X}||_F} \tag{15}$$

As a side note, to assess the performance of this minimization, the resulting matrix rank can be compared to the optimal rank value, which can be deduced from the characteristic polynomial function: the optimal rank that can be achieved is when all c_k factors (with variables) are equal to 0, or in other words, the optimal rank value is equal to:

$$\text{Optimal rank} = (n+1) - (\text{nb of } c_k \text{ factors that are variables}) \tag{16}$$

Based on our experimental results (as will be shown later) and for the scenarios that we have tested, we made some observations: 1) solving (8) by using *eRM-TIM* only (without feedback) gives a higher rank than when we combine both methods ((8) and the method in [43]), and 2) when † is replaced by a 0 value, it reverts back to the previous minimization problem, while giving a higher rank. This implies that there is a benefit behind doing a classification (by using a threshold) of the experienced interferences and distinguishing between two levels of interference: strong and *very strong*.

V. COMPLEXITY

The overall complexity of our proposed framework results from solving TIM, and from implementing SIC, after computing the coefficients of the characteristic polynomial function.

To calculate the polynomial function in Stage 1, many algorithms were proposed. One of those is Berkowitz's algorithm [75], which is widely used in simulation software like Matlab. It computes the coefficients by calculating iterated matrix products and performing $\mathcal{O}(n^4)$ multiplications in $\mathbb{R}$ [76], where n is the dimension of the matrix which in our case corresponds to the average cluster size. Both rank minimization methods adopted in Stage 1 and Stage 2 use the interior point method of complexity around $\mathcal{O}(n^6)$ [43], [77], which constitutes the major factor of the overall complexity. In this regard, Newton's method is often used to approximate the central path for non-linear programming, as it requires at most $\mathcal{O}(\sqrt{n}\ \log\epsilon^{-1})$ iterations, where ϵ is the desired accuracy [78]. The most computationally expensive step in each iteration here is the LDL factorization of the $p \times p$ system matrix, where p is the number of constraints [79] present in the optimization problem (13), which we had called the *cRM-TIM method*. In the worst case, this LDL factorization could take $\mathcal{O}(p^3)$ time [80], [81], [82]. The total complexity of this step will become $\mathcal{O}(n^{\frac{1}{2}}p^3)$. As for the implementation complexity of SIC at the receiver, it increases with the number of interfering users, m, i.e., $\mathcal{O}(m^3)$ [55].

Thus, the overall complexity resulting from the combination of TIM and SIC becomes equal to $\mathcal{O}(n^6) + \mathcal{O}(n^4) + \mathcal{O}(m^3) + \mathcal{O}(n^{\frac{1}{2}}p^3) = \mathcal{O}(n^6)$, where, again, n is the total number of D2D pairs *within* a cluster, m represents the number of interfering D2D users ($m < n$), and p denotes the number of constraints present in *cRM-TIM*.

All of this shows the importance of using a clustering scheme in the proposed framework. This is because, even if the network is of large dimension, the complexity will be reduced by the virtue of dividing the network into groups. By this, the joint TIM-SIC framework will be executed in parallel across all sub-matrices that correspond to the different clusters, and hence this complexity is computed per cluster. This certainly leads to a gain in the computation because if the network was not divided into ℓ clusters, then the value of n above would have been substituted by d, where d is the total number of D2D pairs in the whole network, i.e., $d = n \times \ell$.

Finally, it is worth commenting on the overall complexity of the proposed two-stage interference mitigation framework of this paper. First, it is not worse than the complexity of *eRM-TIM* proposed in [43], and second, by being a function of the

Algorithm 1 Rank Approximation Decision

Input: $\boldsymbol{X}$
Output: $\hat{\boldsymbol{X}}_k$, $rank(\boldsymbol{X})$, $\boldsymbol{U}$, $\boldsymbol{V}$
Solve the problem in [31] to get X;
Calculate $\|\boldsymbol{X}\|_F$;
Find the SVD decomposition $\boldsymbol{X} = \boldsymbol{Y}\boldsymbol{S}\boldsymbol{Z}^T$ with $\boldsymbol{S} = \text{diag}\,(\sigma_1, \ldots, \sigma_n)\,, \sigma_1 \geq \ldots \geq \sigma_n$
Define errot $= \frac{\|X - X_k\|_F}{\|X\|_F}$
while *error* $\leq 10\%$ **do**
  Initialize $i = n + 1$;
  repeat
    /*zeroing the smallest singular values */
    $i = i - 1$;
    Set $[\sigma_i, \ldots, \sigma_n] = 0$ in diag(S);
    Get new $\hat{S}$;
    Compute $\hat{\boldsymbol{X}} = \boldsymbol{Y}\hat{S}Z^T$;
    Calculate *error*;
  until *error* $>10\%$;
  return $i + 1, \hat{\boldsymbol{X}}_{i+1}$;
end
$rank(\boldsymbol{X}) = n - (i + 1)$;
$\hat{\boldsymbol{X}}_k = \hat{\boldsymbol{X}}$
/* Get the decoding and precoding matrices, $\boldsymbol{U}$ and $\boldsymbol{V}$ respectively */
Apply QR factorization on $\hat{\boldsymbol{X}}_k$ to get $\boldsymbol{U}$ and $\boldsymbol{V}$, since $\hat{\boldsymbol{X}}_k = \boldsymbol{U}\boldsymbol{V}$

number of D2D pairs (n) within each cluster, the runtime is kept under check. Moreover, we note that even if we factor in our clustering method which we proposed in [43], the overall complexity will not increase. In other words, no matter how the network size increases, the complexity will always depend on the number of D2D pairs *within* the cluster and not on the whole network.

VI. EXPERIMENTAL RESULTS

We implement the combination of TIM and SIC using the built-in Optimization Toolbox [83], as well as the convex optimization toolbox CVX [84] in Matlab R2016b on a desktop of 64GB RAM with Intel Xeon CPU E5-2620 v4 working at 2.10 GHz. To demonstrate the relative performance gains of the proposed algorithm, we first consider, as an example, the three different clusters $G1$, $G2$ and $G3$ (with 5×5, 6×6, and 7×7 topologies, respectively) of the 18×18 D2D network represented in Fig. 1 in three different scenarios (will be described later on in this section). We study the gain of implementing our two-stage proposed algorithm as compared to the cases where only stage 1 or stage 2 of our algorithm are used. This allows assessing the interest of differentiating between strong and very strong interference, as done in our framework, and thus the interest of combining SIC and TIM as proposed in our work. We then consider various network topologies with 5×5, 6×6, 7×7, 10×10, 11×11, 12×12, 13×13, and 18×18 devices (i.e. 36 devices) and compare between our proposed algorithm and existing TIM methods in the literature [41], [42]. We show by simulation the gain in terms of percentage of DoF gain that our method brings as compared to the DoF achieved by the TIM solutions in [41] and [42].

A. SCENARIO 1: NO DIFFERENTIATION AMONG THE INTERFERENCES

In this scenario, we consider that all interferences within the same cluster are recognized in the same fashion, i.e., no difference between strong and *very strong* interferences, and hence $\dagger_{\bullet,\ell} = 0$. Consequently, the SIC role vanishes. We consider this scenario as a performance baseline. In this context, by solving (13) using the built-in function "fmincon" of Matlab with the matrices $\boldsymbol{X}_{G1}$, $\boldsymbol{X}_{G2}$ and $\boldsymbol{X}_{G3}$ in (8) as inputs (each one at a time), then the ranks of the 5×5, 6×6, and 7×7 topologies get minimized only to 3, 5, and 4, respectively, as shown in Table 1. On the other hand, if the same matrices are provided as inputs for our previous rank minimization *eRM-TIM* in [43], then the ranks get reduced to 4, 4, and 4, respectively. However, for these specific topologies, the obtained values have not reached the optimal values yet (which is equal to 2 for all). Recall that this optimal rank can be deduced from (16), with $n = 5, 6, 7$ and the numbers of c_k factors which are variables in the corresponding characteristic polynomial functions are equal to 4, 5 and 6 respectively. These results show that an additional step can be added to decrease the rank further, namely by distinguishing between the levels of interference: strong and *very strong*. This is covered by the following two scenarios.

B. ONE STAGE AT A TIME RANK MINIMIZATION

In this scenario, we adopt the interference classification approach, which we denote mathematically by putting $\dagger_{\bullet,\ell}$ back in the adjacency matrix, and setting it to a value that is greater than 2 (to differentiate it from the desired signal, as previously discussed). We test here the importance of integrating the two rank minimization methods, by showing the effect of its non-existence. Thus, we measure the achieved rank while using each method independently. Results have shown that applying *cRM-TIM* reduces the ranks only to 2, 3 and 5, respectively, for the considered topologies of Fig. 1. When *eRM-TIM* in [43] is applied alone (without integrating the $\dagger$ obtained from *cRM-TIM* into *eRM-TIM*) and after defining the input $\dagger_{\bullet,\ell} \geq 2$, the ranks are minimized to 3, 3 and 4, respectively. This demonstrates the effectiveness of working with interference classification. From an LRMC perspective, differentiating between the levels of interference gives more flexibility to the matrix by removing the 0s, and hence reduces the rank further (in both rank minimization methods). From a communication perspective, it allows for making use of the SIC function, which, by the way, is a capability that is supposed to be implemented in hardware in 5G devices [14]. Again, the use of SIC is limited here to *some* interfering links and not all of them (i.e., where the dot product between the precoding and decoding vector is very large) which reduces the complexity resulting from the use of SIC. As with the previous scenario, the optimal rank is still not reached while using each method separately. For this, we try to combine both rank minimization methods, while implementing a feedback from *cRM-TIM* to *eRM-TIM* to pass the value of $\dagger_{\bullet,\ell}$, which leads us to the third scenario.

C. SCENARIO 3: RANK MINIMIZATION METHOD WITH INTEGRATION OF STAGE 1 INTO STAGE 2

Finally, we test here the whole framework represented in Fig. 2. For ease of discussion, we refer to *cRM-TIM* as Stage 1 or $S1$, and to our *eRM-TIM* method as Stage 2 or $S2$. We start by solving the problem in (13) in order to get the appropriate value of $\dagger_{\bullet,\ell}$ for each sub-matrix of (8). Once obtained, the value is inserted in the corresponding entries of $\boldsymbol{X}_{G\ell}$. Next, the matrix $\boldsymbol{X}_{G\ell}$ is fed to *eRM-TIM*. As a result, the rank $\boldsymbol{X}_{G\ell}$ attained the optimal value of 2 in the tested networks of sizes $n = 5$ and $n = 6$. This proves that establishing a feedback step between the two rank minimization methods is beneficial. We remind here that both scenarios 1 and 2, apply either $S1$ or $S2$ alone. In this regard, applying $S1$ alone does not always achieve the minimum rank as in the case of the 5×5 and 6×6 topologies, and the same goes for $S2$. Moreover, it is not only about the rank achieved, but also about the value of $\dagger$ it is achieving. More specifically, $S1$ forces $\dagger_{\bullet,\ell}$ to be much greater than 2, whereas the *eRM-TIM* formulation has a tendency to make $\dagger_{\bullet,\ell}$ close to 2. In this regard, having a $\dagger_{\bullet,\ell}$ value much greater than 2 makes more sense since while sorting the factors in the input-output relation, some gap needs to exist between the desired signal and the *very strong* interference, so that the latter can be detected and managed by SIC. Table 1 summarizes all the aforementioned results, showing a remarkable improvement in terms of the minimum rank achieved (and hence in terms of DoF), as compared to the conventional TIM. As for the order of the rank minimization methods that correspond to SIC and TIM in the LRMC framework, one can ask if the order of the stages can be swapped: Stage 1 can become the one corresponding to *eRM-TIM* (i.e., TIM) followed by *cRM-TIM* (i.e, SIC). In this case, the initial guess for $\dagger_{\bullet,\ell}$ will always be very close to 2, which makes this step unnecessary. On the other hand, if we start by *cRM-TIM*, then it can give the value of $\dagger_{\bullet,\ell}$ that helps in the rank minimization while applying *eRM-TIM*. In this case, *eRM-TIM* will have the value of $\dagger_{\bullet,\ell}$ adequately computed, and not assigned a value very close to 2. This $\dagger_{\bullet,\ell}$ value will lead then to a better ordering for SIC application.

D. PERFORMANCE GAIN

We now compare our proposed framework to the TIM optimization methods in [41] and [42]. For that we consider various network topologies with 5×5, 6×6, 7×7, 10×10, 11×11, 12×12, 13×13, 14×14, and 18×18 devices (in Figure 1) and compare between our proposed algorithm and the TIM methods in [41] and [42]. We plot the percentage DoF (Degree of Freedom) gain achieved by our method with respect to existing ones in [41] and [42], given by: $Gain = 100 * (DoF_{proposed} - DoF_{existing})/DoF_{existing}$. The obtained results, provided in Figure 4, shows the superiority of our method as compared to the ones in [41] and [42]. We can see that the DoF gain varies between 50% and 100% as compared to [41]. It is worth noticing that a gain of 100% means, according to the definition above that the achieved DoF by our method is twice the one in [41]. One can notice also that the gain between our proposed framework here and the work in [42] is between 50 and 70%, which means that the achieved DoF is 1.5 to 1.7 times the one in [42].

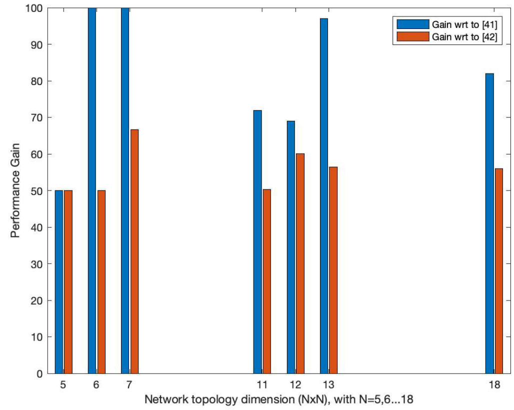


FIGURE 4. Gain of our Framework as compared to [41] and [42] for various network size.

VII. DISCUSSION

In this section we discuss two important points. The first is about proving that the source of the achieved rank minimization is the entire framework of performing TIM and then SIC, and not due to applying TIM alone. The second addresses a likely question, which is why to perform TIM and then SIC rather than performing SIC before TIM.

A. SOURCE OF RANK MINIMIZATION

In this section, we prove the effectiveness of the interference classification concept, which we integrated into an LRMC framework. From a matrix perspective, we show here that the rank reduction results from the concept adopted (in terms of changing some 0s to $\dagger$s such that their values are greater than 2) and not from the method used (*whether cRM-TIM or eRM-TIM*). To prove this claim, we illustrate it by considering two cases:

1) CASE 1

We consider a network of n D2D pairs, where there is no differentiation between interference levels, and hence, there is no need for applying SIC. This can be written in a matrix form as an $n \times n$ matrix $\boldsymbol{X}$, having only 1s in the diagonal entries and 0s in some predefined positions. We consider an

TABLE 1. Interest of our two-stage rank minimization (TIM+SIC): Rank minimization performance with different values for †.

Scenario		Rank Achieved for the network of size		
		5 × 5	**6 × 6**	**7 × 7**
No interference classification ($\dagger = 0$)	Stage 1-*only*	3	5	4
	Stage 2-*only*	4	4	4
One stage Rank minimization (either TIM or SIC)	Stage 1-*only*	2	3	5
	Stage 2-*only*	3	3	4
Two-stage Rank minimization (TIM+SIC)	Stage 1 + Stage 2	2(optimal)	2(optimal)	3

example for $n = 3$, as follows

$$\boldsymbol{X} = \begin{pmatrix} 1 & 0 & a \\ b & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{17}$$

The characteristic polynomial function of this matrix can be then written as:

$$\begin{aligned} p(\lambda) = det &\begin{pmatrix} \lambda - 1 & 0 & -a \\ -b & \lambda - 1 & 0 \\ 0 & 0 & \lambda - 1 \end{pmatrix} \\ &= (\lambda - 1)[(\lambda - 1)^2 - 0] - 0(-b(\lambda - 1) - 0) \\ &\quad - a(-b \times 0 - 0 \times (\lambda - 1)) \\ &= (\lambda - 1)^3 = \lambda^3 - 3\lambda^2 + 3\lambda - 1 \end{aligned} \tag{18}$$

where λ are $\boldsymbol{X}$'s eigenvalues, and $\boldsymbol{I}$ is the identity matrix. Clearly, the resulting coefficients of the characteristic polynomial function do not include any variables, and so the corresponding matrix $\boldsymbol{X}$ is of full rank (all the roots of $p(\lambda)$, i.e., the eigenvalues are equal to 1). This can be explained by the fact that computing $p(\lambda) = det(\lambda\boldsymbol{I} - \boldsymbol{X})$ will always give $(\lambda - 1)$ in the diagonal entries, because $\boldsymbol{X}$ has all ones in the diagonal. Thus, having a 0 in a certain position will affect the determinant computation by canceling out some variable entries and keeping the $(\lambda - 1)$ factor. This will then produce $(\lambda - 1)$ as a common factor, and so a root of 1.

On the other hand, if some 0s are replaced by †s (or in other words, some interferences are now considered as very strong ones and should be cancelled using SIC), then the corresponding matrix can be written as:

$$\boldsymbol{X} = \begin{pmatrix} 1 & 0 & a \\ b & 1 & \dagger_1 \\ \dagger_2 & 0 & 1 \end{pmatrix} \tag{19}$$

and its adjacency matrix as:

$$\begin{aligned} p(\lambda) = det &\begin{pmatrix} \lambda - 1 & 0 & -a \\ -b & \lambda - 1 & -\dagger_1 \\ -\dagger_2 & 0 & \lambda - 1 \end{pmatrix} \\ &= (\lambda - 1)[(\lambda - 1)^2 - 0] + 0(-b(\lambda - 1) - \dagger_1\dagger_2) \\ &\quad - a(-b \times 0 - (-\dagger_2)(\lambda - 1)) \\ &= \lambda^3 - 3\lambda^2 + \lambda(3 - a\dagger_2) + a\dagger_2 - 1 \end{aligned} \tag{20}$$

Zeroing out the coefficients of $p(\lambda)$ leads to the following conditions: $c_3 = a\dagger_2 - 1 = 0$ and $c_2 = 3 - a\dagger_2 = 0$. By solving these two equations, the values of a and $\dagger_2$ can be obtained as 0.5 and 2, respectively. Note that these values satisfy one of the conditions (i.e., the first one only), which means that only the coefficient c_3 can be zeroed out. The resulting eigenvalues will be then equal to 1, 2, 0.0000, and hence the rank will be equal to 2. All of this proves that replacing the predefined 0s in $\boldsymbol{X}$ by † values (such that $\dagger > 2$) affects the rank and its minimization. This shows clearly the gain of interference classifications used in this paper and hence the interest of the TIM+SIC combination.

2) CASE 2

We consider here a special case when the matrix is upper or lower diagonal. This could be an example of a D2D network topology, where each receiver is suffering from interferences resulting from transmitters with preceding indices. Knowing that in a triangular matrix the eigenvalues are equal to the entries on the main diagonal, then the eigenvalues of $\boldsymbol{X}$ will be equal to 1 (since in TIM, the diagonal entries are all equal to 1). This implies that the TIM adjacency matrix with this special structure (as triangular matrix) will always be a full rank matrix, regardless of the remaining "*" values. To illustrate this, we consider an example of a lower diagonal matrix $\boldsymbol{X}$ as in (21), which is of full rank (equal to 3):

$$\boldsymbol{X} = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \tag{21}$$

To show the importance of replacing some zeros by daggers, we show in (22) a modified version of (21), where the 0 at $X_{3,1}$ is replaced by a †. From a communication perspective, this means that the interference coming from transmitter T_1 to receiver R_3 is now a very strong one instead of strong, and hence should be handled using SIC:

$$\boldsymbol{X} = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ \dagger & 0 & 1 \end{pmatrix} \tag{22}$$

This replacement makes the eigenvalues more flexible, since the coefficients of the characteristic polynomial function will include more variables, as shown below:

$$p(\lambda) = det\begin{pmatrix} \lambda-1 & -a & -b \\ 0 & \lambda-1 & -c \\ -\dagger & 0 & \lambda-1 \end{pmatrix}$$
$$= \lambda^3 - 3\lambda^2 + (3-\dagger b)\lambda + (\dagger b - \dagger ac - 1) \quad (23)$$

To zero out the coefficients, the following conditions should be considered: $3 - \dagger b = 0$ and $\dagger b - \dagger ac - 1 = 0$. Then, $\dagger = 6$, $a = 0.3$, $b = 0.5$, $c = \frac{1}{0.9}$. Replacing these values in $\boldsymbol{X}$ reduces the rank from 3 to 2. This example then shows the importance of integrating the † concept.

B. REORDERING OF INTERFERENCE MANAGEMENT SCHEMES

In this subsection, we discuss the alternative of starting with SIC before TIM, in contrast to the approach taken in our proposed framework.

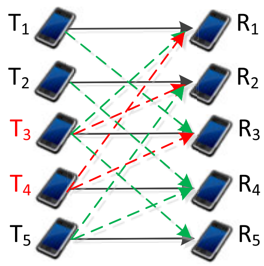
1) EXAMPLE 1

In this example, we consider the network topology of group $G1$ in Fig. 1 and its corresponding adjacency matrix $\boldsymbol{X}_{G1}$ presented in (8). We reproduce the connectivity pattern and the matrix in Fig. 5 for better illustration. In this D2D network and based on the new scheme, the very strong interference resulting from the transmitters T_3 and T_4 towards the receivers R_1, R_2 and R_3 will be cancelled first using SIC. Then, the strong interferences (e.g., those coming from T_1 to R_3, T_2 to R_4, T_3 to R_1 and R_5, T_5 to R_2 and R_4) will be managed by TIM. From an LRMC perspective, the adjacency matrix will not include the daggers "†" (denoting very strong interference) in its entries anymore. More specifically, the columns and rows that comprise these daggers will be deleted. Applying this elimination to the adjacency matrix in Fig. 5(b) will produce the following sub-matrix:

$$\boldsymbol{X}_{G1_{sub}} = \begin{pmatrix} 1 & a_1 & b_1 \\ c_1 & 1 & 0 \\ i_1 & j_1 & 1 \end{pmatrix} \quad (24)$$

In other words, the columns and rows that correspond to T_3, T_4 and R_3, R_4, respectively, are eliminated. The remaining columns and rows correspond to T_1, T_2, T_5 and R_1, R_2, R_5. By applying *eRM-TIM* on $\boldsymbol{X}_{G1_{sub}}$, the matrix gets filled in such a way that its rank is minimized from 3 to 2. Thus, the dimensions of the resulting precoding and decoding matrices, $\boldsymbol{V}_{G1_{sub}}$ and $\boldsymbol{U}_{G1_{sub}}$, respectively, (after factorizing $\boldsymbol{X}_{G1_{sub}} = \boldsymbol{U}_{G1_{sub}}\boldsymbol{V}_{G1_{sub}}$) get also reduced.

We note here that although the rows and columns are eliminated from the matrix, the effect of the excluded nodes, namely T_3 and T_4, cannot be neglected. This is because in addition to the fact that T_3 is affecting R_3 with a very strong interference (as shown in Fig. 5), it is also affecting R_1 with a strong interference. This explains why these nodes should be also taken into account. In Fig. 6(a), we illustrate the effect of the excluded nodes on the residual ones, whereas in Fig. 6(b), we show how the residual nodes affect the excluded ones. Note here that the hidden icons and lines refer to the omitted



$$\boldsymbol{X}_{G1} = \begin{pmatrix} 1 & a_1 & 0 & \dagger_{1,1} & b_1 \\ c_1 & 1 & \dagger_{2,1} & d_1 & 0 \\ 0 & e_1 & 1 & \dagger_{3,1} & f_1 \\ g_1 & 0 & h_1 & 1 & 0 \\ i_1 & j_1 & 0 & k_1 & 1 \end{pmatrix}$$

(a) (b)

FIGURE 5. (a) 5 × 5 D2D network topology, and (b) its corresponding adjacency matrix.

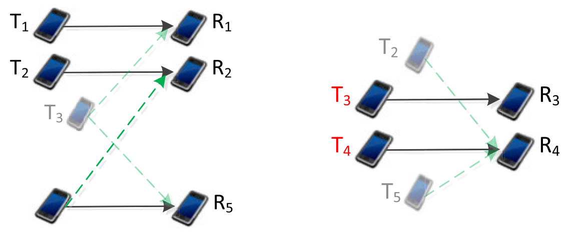


FIGURE 6. After removing the very strong interferences, the remaining effect (a) of the excluded nodes on the D2D pairs, and (b) vice versa.

nodes. For this, the corresponding precoding and decoding vectors, $\boldsymbol{v}$-s and $\boldsymbol{u}$-s, should be designed such that

$$\boldsymbol{v}_3 \cdot \boldsymbol{u}_1 = 0 \text{ and } \boldsymbol{v}_3 \cdot \boldsymbol{u}_5 = 0, \quad (25)$$

so that the strong interferences (shown in Fig. 6(a)) between the $(T_3 - R_3)$ and $(T_1 - R_1)$ pairs on one hand, and between (T_3-R_3) and (T_5-R_5) on the other hand get cancelled. In this case, $\boldsymbol{u}_1$ and $\boldsymbol{u}_5$ can be known after applying the QR decomposition on $\boldsymbol{X}_{G1_{sub}}$ in (24), and hence $\boldsymbol{v}_3$ can be deduced after solving (25). Moreover, the dot product $\boldsymbol{u}_3 \cdot \boldsymbol{v}_3 = 1$, denoting the desired signal between the $(T_3 - R_3)$ pair, can lead to obtaining $\boldsymbol{u}_3$. Moreover, to remove the strong interferences between $(T_2 - R_2)$ and $(T_4 - R_4)$ on one hand, and between $(T_5 - R_5)$ and $(T_4 - R_4)$ on the other hand, the TIM strategies should be designed such that

$$\boldsymbol{v}_2 \cdot \boldsymbol{u}_4 = 0 \text{ and } \boldsymbol{v}_5 \cdot \boldsymbol{u}_4 = 0. \quad (26)$$

The vectors $\boldsymbol{v}_2$ and $\boldsymbol{v}_5$ can also be known from the QR decomposition of $\boldsymbol{X}_{G1_{sub}}$ in (24), thus allowing us to obtain $\boldsymbol{u}_4$ by computing the cross product between $\boldsymbol{v}_2$ and $\boldsymbol{v}_5$. For this, the dimensions of $\boldsymbol{v}_2$ and $\boldsymbol{v}_5$ (and hence those of $\boldsymbol{U}_{G1_{sub}}$ and $\boldsymbol{V}_{G1_{sub}}$) cannot be reduced to 2, although the rank of $\boldsymbol{X}$ is minimized to 2. This is because the vector must be of length 3 in the dimension in which the cross product is taken. Then, the new $\boldsymbol{U}_{G1_{sub}}$ and $\boldsymbol{V}_{G1_{sub}}$ (of dimensions 5×3 and 3×5, respectively) can be constructed as $\boldsymbol{U}_{\text{new}} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4, \boldsymbol{u}_5]$, $\boldsymbol{V}_{\text{new}} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4, \boldsymbol{v}_5]$, and hence $\boldsymbol{X}_{\text{new}} = \boldsymbol{U}_{\text{new}}\boldsymbol{V}_{\text{new}}$. Hence, the rank of $\boldsymbol{X}_{\text{new}}$ gets reduced from 5 to 3, but the dimensions of $\boldsymbol{U}_{\text{new}}$ and $\boldsymbol{V}_{\text{new}}$ are still the same. Not to mention that alternatively if *eRM-TIM* was to be applied to $\boldsymbol{X}_{G1_{sub}}$ from the beginning, the rank of the 5×5 matrix would go down from 5 to 3. This shows that even if we keep the dimensions of $\boldsymbol{u}$-s and $\boldsymbol{v}$-s to overcome the dimensionality problem, there is no additionnal gain when applying SIC before TIM.

2) EXAMPLE 2

For further illustration, we consider another example of an $n = 6$ topology:

$$\boldsymbol{X}'_{G2} = \begin{pmatrix} 1 & 0 & 0 & a_2 & b_2 & c_2 \\ d_2 & 1 & 0 & e_2 & f_2 & g_2 \\ h_2 & i_2 & 1 & j_2 & 0 & k_2 \\ l_2 & \dagger_{2,2} & m_2 & 1 & n_2 & o_2 \\ p_2 & q_2 & r_2 & s_2 & 1 & \dagger_{3,2} \\ t_2 & u_2 & v_2 & 0 & w_2 & 1 \end{pmatrix} \tag{27}$$

The importance of this example is to show that the alternative of applying SIC before TIM does not only suffer from a dimensionality issue, but also has a feasibility problem. The elimination of the columns and rows, where the daggers exist, leads to the following sub-matrix:

$$\boldsymbol{X}'_{G2_{sub}} = \begin{pmatrix} 1 & 0 & a_2 & b_2 \\ h_2 & 1 & j_2 & 0 \\ l_2 & m_2 & 1 & n_2 \\ p_2 & r_2 & s_2 & 1 \end{pmatrix} \tag{28}$$

After applying rank minimization to the above sub-matrix, the rank of $\boldsymbol{X}'_{G2_{sub}}$ gets minimized to 2. Applying the QR decomposition to it produces the corresponding precoding and decoding matrices $\boldsymbol{V}'_{G2_{sub}}$ and $\boldsymbol{U}'_{G2_{sub}}$ respectively. If these matrices get truncated according to the new dimension ($rank = 2$), then some of the vectors $\boldsymbol{u}$-s and $\boldsymbol{v}$-s (i.e., $\boldsymbol{u}_1, \boldsymbol{u}_3, \boldsymbol{u}_4, \boldsymbol{u}_5, \boldsymbol{v}_1, \boldsymbol{v}_3, \boldsymbol{v}_4$ and $\boldsymbol{v}_5$) become known. However, to get the remaining ones (i.e., $\boldsymbol{u}_6, \boldsymbol{v}_6, \boldsymbol{u}_2, \boldsymbol{v}_2$), the following conditions on some $\boldsymbol{u}$-s and $\boldsymbol{v}$-s should be simultaneously satisfied:

$$\begin{aligned} &\boldsymbol{v}_2 \cdot \boldsymbol{u}_1 = 0 \cap \boldsymbol{v}_2 \cdot \boldsymbol{u}_1 = 1, \text{ and} \\ &\boldsymbol{v}_3 \cdot \boldsymbol{u}_2 = 0 \cap \boldsymbol{v}_4 \cdot \boldsymbol{u}_6 = 0 \cap \boldsymbol{u}_6 \cdot \boldsymbol{v}_6 = 1 \end{aligned} \tag{29}$$

This leads then to a problem composed of 5 equations with 4 unknowns (i.e., $\boldsymbol{u}_6, \boldsymbol{v}_6, \boldsymbol{u}_2, \boldsymbol{v}_2$). However, solving this overdetermined problem will produce vectors that cannot satisfy all the conditions in (29) at the same time. This example therefore shows a feasiblity problem that could arise when we apply SIC before TIM.

VIII. CONCLUSION

In this paper, the application of topological interference management (TIM) along with successive interference cancellation (SIC) to device-to-device (D2D) communications has been studied. By building on the characteristic polynomial function of the adjacency matrix of the interference graph, a novel algorithm involving the combination of these two techniques has been proposed. Numerical results showed that incorporating SIC in the TIM scheme provides a good performance gain in terms of DoF as compared to existing TIM solutions in the literature.

REFERENCES

[1] C. Xu, L. Song, and Z. Han, *Resource Management for Device-to-Device Underlay Communication*. Cham, Switzerland: Springer, 2014.
[2] K. Mahmood, G. K. Kurt, and I. Ali, "Mode selection rules for device-to-device communication: Design criteria and performance metrics," in *Proc. IEEE Int. Symp. Signal Process. Inf. Technol.*, Dec. 2013, pp. 315–320.
[3] Y. Li, D. Jin, J. Yuan, and Z. Han, "Coalitional games for resource allocation in the device-to-device uplink underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 7, pp. 3965–3977, Jul. 2014.
[4] C. Yu, K. Doppler, C. B. Ribeiro, and O. Tirkkonen, "Resource sharing optimization for device-to-device communication underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 2752–2763, Aug. 2011.
[5] X. Li, J. Li, W. Liu, Y. Zhang, and H. Shan, "Group-sparse-based joint power and resource block allocation design of hybrid device-to-device and LTE-advanced networks," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 1, pp. 41–57, Jan. 2016.
[6] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang, "What will 5G be?" *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1065–1082, Jun. 2014.
[7] S. Chiu, K. C. Lin, G. Lin, and H. Wei, "Empowering device-to-device networks with cross-link interference management," *IEEE Trans. Mobile Comput.*, vol. 16, no. 4, pp. 950–963, Apr. 2017.
[8] S. A. Jafar, "Topological interference management through index coding," *IEEE Trans. Inf. Theory*, vol. 60, no. 1, pp. 529–568, Jan. 2014.
[9] H. Esfahanizadeh, F. Lahouti, and B. Hassibi, "A matrix completion approach to linear index coding problem," in *Proc. IEEE Inf. Theory Workshop (ITW)*, Nov. 2014, pp. 531–535.
[10] B. Hassibi, "Topological interference alignment in wireless networks," in *Proc. Smart Antennas Workshop*, 2014, pp. 1–12.
[11] T. Cover, "Broadcast channels," *IEEE Trans. Inf. Theory*, vol. IT-18, no. 1, pp. 2–14, Jan. 1972.
[12] L. Dai, B. Wang, Y. Yuan, S. Han, I. Chih-lin, and Z. Wang, "Non-orthogonal multiple access for 5G: Solutions, challenges, opportunities, and future research trends," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 74–81, Sep. 2015.
[13] X. Li, C. He, H. Shan, J. Li, and Z. Jane Wang, "Feasibility-aware partial interference alignment for hybrid D2D and cellular communication networks," *IEEE Access*, vol. 6, pp. 71069–71083, 2018.
[14] R. C. Kizilirmak, "Non-orthogonal multiple access (NOMA) for 5G networks," in *Towards 5G Wireless Networks-A Physical Layer Perspective*. Rijeka, Croatia: InTech, 2016.
[15] S. Doumiati, "Interference management for device-to-device communications in 5G networks," Ph.D. dissertation, Dept. Elect. Comput. Eng., Amer. Univ. Beirut, Beirut, Lebanon, 2019. [Online]. Available: http://hdl.handle.net/10938/23083
[16] C. Geng, H. Sun, and S. A. Jafar, "Multilevel topological interference management," in *Proc. IEEE Inf. Theory Workshop (ITW)*, Sep. 2013, pp. 1–5.
[17] R. K. Mishra, K. Chahine, H. Kim, S. Jafar, and S. Vishwanath, "Distributed interference alignment for K-user interference channels via deep learning," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jul. 2021, pp. 2614–2619.
[18] C. Wang, D. Deng, L. Xu, W. Wang, and F. Gao, "Joint interference alignment and power control for dense networks via deep reinforcement learning," *IEEE Wireless Commun. Lett.*, vol. 10, no. 5, pp. 966–970, May 2021.
[19] W. Liu, K. Liu, L. Tian, C. Zhang, and Y. Yang, "Joint interference alignment and subchannel allocation in ultra-dense networks," *IEEE Trans. Veh. Technol.*, vol. 71, no. 7, pp. 7287–7296, Jul. 2022.
[20] X. Yi and G. Caire, "Topological interference management with decoded message passing," *IEEE Trans. Inf. Theory*, vol. 64, no. 5, pp. 3842–3864, May 2018.
[21] W. Liu, D. Chen, J. Li, and Y. Ma, "Topological interference management for MIMO interference broadcast channels," *IEEE Access*, vol. 7, pp. 38055–38067, 2019.
[22] H. Yang, N. Naderializadeh, A. S. Avestimehr, and J. Lee, "Topological interference management with reconfigurable antennas," *IEEE Trans. Commun.*, vol. 65, no. 11, pp. 4926–4939, Nov. 2017.
[23] X. Yi and H. Sun, "Opportunistic topological interference management," *IEEE Trans. Commun.*, vol. 68, no. 1, pp. 521–535, Jan. 2020.
[24] J. de Dieu Mutangana and R. Tandon, "Topological interference management with confidential messages," 2020, *arXiv:2010.14503*.

[25] Y. Liang, C. Liao, and X. Yi, "Topological interference management with adversarial perturbation," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jul. 2021, pp. 2608–2613.

[26] Y. Liang, C. Liao, and X. Yi, "Topological interference management with adversarial topology perturbation: An algorithmic perspective," *IEEE Trans. Commun.*, vol. 70, no. 12, pp. 8153–8166, Dec. 2022.

[27] Y. Gao, G. Wang, and S. A. Jafar, "Topological interference management for hexagonal cellular networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 5, pp. 2368–2376, May 2015.

[28] C. Geng, H. Sun, and S. A. Jafar, "Multilevel topological interference management: A TIM-TIN perspective," *IEEE Trans. Commun.*, vol. 69, no. 11, pp. 7350–7362, Nov. 2021.

[29] X. Jiang, B. Zheng, L. Wang, and X. Hou, "Clustering for topological interference management," *Chin. J. Electron.*, vol. 31, no. 5, pp. 844–850, Sep. 2022.

[30] X. Jiang, B. Zheng, W. Zhu, L. Wang, and X. Hou, "Topological interference management with inaccurate topology of network," *IEEE Commun. Lett.*, vol. 25, no. 11, pp. 3724–3728, Nov. 2021.

[31] X. Jiang, B. Zheng, W.-P. Zhu, L. Wang, and X. Hou, "Topological interference management via low-rank tensor completion for time-varying topology networks," *Signal Process.*, vol. 208, Jul. 2023, Art. no. 108972.

[32] H. Kallam, L. S. Cardoso, and J. Marie Gorce, "On the impact of normalized interference threshold for topological interference management," in *Proc. Eur. Conf. Netw. Commun. (EuCNC)*, Jun. 2020, pp. 105–110.

[33] H. Sun, C. Geng, and S. A. Jafar, "Topological interference management with alternating connectivity," in *Proc. IEEE Int. Symp. Inf. Theory*, Jul. 2013, pp. 399–403.

[34] N. Naderializadeh and A. S. Avestimehr, "Interference networks with no CSIT: Impact of topology," *IEEE Trans. Inf. Theory*, vol. 61, no. 2, pp. 917–938, Feb. 2015.

[35] E. J. Candès and B. Recht, "Exact matrix completion via convex optimization," *Found. Comput. Math.*, vol. 9, no. 6, pp. 717–772, Dec. 2009.

[36] B. Recht, M. Fazel, and P. A. Parrilo, "Guaranteed minimum-rank solutions of linear matrix equations via nuclear norm minimization," *SIAM Rev.*, vol. 52, no. 3, pp. 471–501, Jan. 2010.

[37] L. El Ghaoui and S.-L. Niculescu, *Advances in Linear Matrix Inequality Methods in Control*. Philadelphia, PA, USA: SIAM, 2000.

[38] M. Hardt, "Understanding alternating minimization for matrix completion," in *Proc. IEEE 55th Annu. Symp. Found. Comput. Sci.*, Oct. 2014, pp. 651–660.

[39] Y. Birk and T. Kol, "Coding on demand by an informed source (ISCOD) for efficient broadcast of different supplemental data to caching clients," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2825–2830, Jun. 2006.

[40] E. J. Candes and T. Tao, "The power of convex relaxation: Near-optimal matrix completion," *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2053–2080, May 2010.

[41] Y. Shi, J. Zhang, and K. B. Letaief, "Low-rank matrix completion for topological interference management by Riemannian pursuit," *IEEE Trans. Wireless Commun.*, vol. 15, no. 7, pp. 4703–4717, Jul. 2016.

[42] S. Doumiati, M. Assaad, and H. A. Artail, "Topological interference management framework for device-to-device communication," *IEEE Wireless Commun. Lett.*, vol. 7, no. 4, pp. 602–605, Aug. 2018.

[43] S. Doumiati, M. Assaad, and H. A. Artail, "A framework of topological interference management and clustering for D2D networks," *IEEE Trans. Commun.*, vol. 67, no. 11, pp. 7856–7871, Nov. 2019, doi: 10.1109/TCOMM.2019.2931319.

[44] C. Ma, W. Wu, Y. Cui, and X. Wang, "On the performance of successive interference cancellation in D2D-enabled cellular networks," in *Proc. IEEE Conf. Comput. Commun. (INFOCOM)*, Apr. 2015, pp. 37–45.

[45] N. I. Miridakis and D. D. Vergados, "A survey on the successive interference cancellation performance for single-antenna and multiple-antenna OFDM systems," *IEEE Commun. Surveys Tuts.*, vol. 15, no. 1, pp. 312–335, 1st Quart., 2013.

[46] S. Han, Z. Xu, S. Wang, Q. Sun, and Y. Chen, "New paradigm of 5G wireless internet," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 3, pp. 474–482, Mar. 2016.

[47] H. Song, J. Y. Ryu, W. Choi, and R. Schober, "Joint power and rate control for device-to-device communications in cellular systems," *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5750–5762, Oct. 2015.

[48] J. Zhao, Y. Liu, K. K. Chai, Y. Chen, M. Elkashlan, and J. Alonso-Zarate, "NOMA-based D2D communications: Towards 5G," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2016, pp. 1–6.

[49] H. Sun, Y. Xu, and R. Q. Hu, "A NOMA and MU-MIMO supported cellular network with underlaid D2D communications," in *Proc. IEEE 83rd Veh. Technol. Conf. (VTC Spring)*, May 2016, pp. 1–5.

[50] K. S. Ali, H. Elsawy, A. Chaaban, and M. Alouini, "Non-orthogonal multiple access for large-scale 5G networks: Interference aware design," *IEEE Access*, vol. 5, pp. 21204–21216, 2017.

[51] V. Kalokidou, O. Johnson, and R. Piechocki, "A hybrid TIM-NOMA scheme for the SISO broadcast channel," in *Proc. IEEE Int. Conf. Commun. Workshop (ICCW)*, Jun. 2015, pp. 387–392.

[52] V. Kalokidou, O. Johnson, and R. Piechocki, "A hybrid TIM-NOMA scheme for the broadcast channel," 2015, *arXiv:1508.03658*.

[53] S. Doumiati, H. A. Artail, and M. Assaad, "Managing interference in D2D networks via clustering and topological awareness," in *Proc. IEEE 23rd Int. Workshop Comput. Aided Model. Design Commun. Links Netw. (CAMAD)*, Sep. 2018, pp. 1–7.

[54] S. Sen, N. Santhapuri, R. R. Choudhury, and S. Nelakuditi, "Successive interference cancellation: Carving out MAC layer opportunities," *IEEE Trans. Mobile Comput.*, vol. 12, no. 2, pp. 346–357, Feb. 2013.

[55] S. M. A. Kazmi, N. H. Tran, T. M. Ho, A. Manzoor, D. Niyato, and C. S. Hong, "Coordinated device-to-device communication with non-orthogonal multiple access in future wireless cellular networks," *IEEE Access*, vol. 6, pp. 39860–39875, 2018.

[56] J. Zhao, Y. Liu, K. K. Chai, Y. Chen, and M. Elkashlan, "Joint subchannel and power allocation for NOMA enhanced D2D communications," *IEEE Trans. Commun.*, vol. 65, no. 11, pp. 5081–5094, Nov. 2017.

[57] F. Fang, H. Zhang, J. Cheng, and V. C. M. Leung, "Energy-efficient resource allocation for downlink non-orthogonal multiple access network," *IEEE Trans. Commun.*, vol. 64, no. 9, pp. 3722–3732, Sep. 2016.

[58] A. Benjebbour, Y. Saito, Y. Kishiyama, A. Li, A. Harada, and T. Nakamura, "Concept and practical considerations of non-orthogonal multiple access (NOMA) for future radio access," in *Proc. Int. Symp. Intell. Signal Process. Commun. Syst.*, Nov. 2013, pp. 770–774.

[59] *Technical Specification Group Services and System Aspects; Study on Architecture Enhancements to Support Proximity-Based Services (ProSe) (Release 12)*, 3GPP, document TR 23.703, 2014.

[60] *Technical Specification Group Services and System Aspects; Study on Architecture Enhancements to Support Proximity-Based Services (ProSe) (Release 12)*, 3GPP, document TR 23.703, 2014. [Online]. Available: https://www.3gpp.org/ftp/Specs/archive/23_series/23.703/

[61] S. M. R. Islam, N. Avazov, O. A. Dobre, and K.-S. Kwak, "Power-domain non-orthogonal multiple access (NOMA) in 5G systems: Potentials and challenges," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 2, pp. 721–742, 2nd Quart., 2017.

[62] G. A. Safdar, M. Ur-Rehman, M. Muhammad, M. A. Imran, and R. Tafazolli, "Interference mitigation in D2D communication underlaying LTE-A network," *IEEE Access*, vol. 4, pp. 7967–7987, 2016.

[63] F. Baccelli, N. Khude, R. Laroia, J. Li, T. Richardson, S. Shakkottai, S. Tavildar, and X. Wu, "On the design of device-to-device autonomous discovery," in *Proc. 4th Int. Conf. Commun. Syst. Netw. (COMSNETS)*, Jan. 2012, pp. 1–9.

[64] A. Vigato, L. Vangelista, C. Measson, and X. Wu, "Joint discovery in synchronous wireless networks," *IEEE Trans. Commun.*, vol. 59, no. 8, pp. 2296–2305, Aug. 2011.

[65] D. Tsolkas, N. Passas, and L. Merakos, "Device discovery in LTE networks: A radio access perspective," *Comput. Netw.*, vol. 106, pp. 245–259, Sep. 2016.

[66] *Discussion of D2D Discovery Methods*, 3GPP, document R1-132068, Alcatel-Lucent, May 2013. [Online]. Available: https://www.3gpp.org/ftp/tsg_ran/WG1_RL1/TSGR1_73/Report/

[67] J. Hong, S. Park, H. Kim, S. Choi, and K. B. Lee, "Analysis of device-to-device discovery and link setup in LTE networks," in *Proc. IEEE 24th Annu. Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC)*, Sep. 2013, pp. 2856–2860.

[68] Qualcomm-Whitepaper. (Aug. 2014). *LTE Direct Always-on Device-to-Device Proximal Discovery*. [Online]. Available: https://www.qualcomm.com/media/documents/files/lte-direct-always-on-device-to-device-proximal-discovery.pdf

[69] K. Doppler, C. B. Ribeiro, and J. Kneckt, "Advances in D2D communications: Energy efficient service and device discovery radio," in *Proc. 2nd Int. Conf. Wireless Commun., Veh. Technol., Inf. Theory Aerosp. Electron. Syst. Technol.*, Feb. 2011, pp. 1–6.

[70] F. Jameel, Z. Hamid, F. Jabeen, S. Zeadally, and M. A. Javed, "A survey of device-to-device communications: Research issues and challenges," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 3, pp. 2133–2168, 3rd Quart., 2018.

[71] *Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation (Release 15)*, 3GPP, document TS 36.211, 2019. [Online]. Available: https://www.3gpp.org/ftp/Specs/archive/36_series/36.211/

[72] *Technical Specification Group Radio Access Network; LTE Device to Device (D2D) Proximity Services (ProSe); User Equipment (UE) Radio Transmission and Reception (Release 12)*, 3GPP, document TR 36.877, 2015. [Online]. Available: https://www.3gpp.org/ftp/Specs/archive/36_series/36.877/

[73] B. P. Brooks, "The coefficients of the characteristic polynomial in terms of the eigenvalues and the elements of an n×n matrix," *Appl. Math. Lett.*, vol. 19, no. 6, pp. 511–515, Jun. 2006.

[74] C. Eckart and G. Young, "The approximation of one matrix by another of lower rank," *Psychometrika*, vol. 1, no. 3, pp. 211–218, Sep. 1936.

[75] M. Soltys, "Berkowitz's algorithm and clow sequences," *Electron. J. Linear Algebra*, vol. 9, no. 1, p. 5, Jan. 2002.

[76] S. Lo, M. Monagan, and A. Wittkopf, "A modular algorithm for computing the characteristic polynomial of an integer matrix in Maple," in *Proc. Maple Summer Workshop*, 2005, pp. 1–12.

[77] *Minimum of Constrained Nonlinear Multi-Variable Function*. [Online]. Available: https://www.mathworks.com/help/optim/ug/fmincon-interior-point-algorithm-with-analytic-hessian.html

[78] W.-K. Ma, C.-C. Su, J. Jaldén, and C.-Y. Chi, "Some results on 16-QAM MIMO detection using semidefinite relaxation," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process.*, Mar. 2008, pp. 2673–2676.

[79] *MATLAB Documentation*. [Online]. Available: https://www.mathworks.com/help/optim/ug/constrained-nonlinear-optimization-algorithms.html#brnpd5f

[80] G. H. Golub and C. F. Van Loan, *Matrix Computations*. London, U.K.: John Hopkins University Press, 1996.

[81] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 2012.

[82] J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis*, vol. 12. Cham, Switzerland: Springer, 2013.

[83] *MATLAB Optimization Toolbox*, The MathWorks, Natick, MA, USA.

[84] M. Grant, S. Boyd, and Y. Ye, "CVX: MATLAB software for disciplined convex programming," Tech. Rep., 2008.

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