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RESEARCH ARTICLE

Output-Feedback Prescribed Performance Decentralized Controller for Interconnected Large-Scale Uncertain Nonlinear Systems With Unknown Input Gain Sign

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ABSTRACT A novel decentralized output-feedback controller is proposed in this paper for large-scale uncertain general nonautonomous nonlinear systems without prior knowledge of the input gain's sign. The subsystems are considered to be completely unknown and nonautonomous, except for their known full relative degree. The proposed controller uses a higher-order switching differentiator to estimate the time-derivatives of the output tracking error, resulting in a low-complexity output-feedback prescribed performance controller that compensates for uncertainties, including high-frequency gain sign and unstructured uncertainties. It is mathematically proven that the output tracking error and its time-derivatives are all maintained within the prescribed regions. To demonstrate the effectiveness of the proposed controller, numerical simulations of two interconnected inverted pendulums are conducted. To the best of the authors' knowledge, this paper presents the first results on this problem.

INDEX TERMS Large-scale nonlinear system, decentralized controller, prescribed performance control, unknown control sign.

I. INTRODUCTION

The study of Large-scale systems (LSSs) or interconnected systems has garnered significant attention due to their widespread prevalence in modern practical systems [1], such as multi-agent systems, multi-machine power systems, and contemporary mechanical systems, which all consist of interconnected subsystems. In order to control these LSSs, a decentralized control scheme is employed, utilizing only the locally available states without the need for communication between remote subsystems, as opposed to a distributed controller that necessitates the exchange of state information between each subsystem. The decentralized controller is typically more practical, as the controller often lacks sufficient knowledge of plant uncertainties and the interactions between subsystems.

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The extensive research conducted in the area of control for uncertain nonlinear systems [2], [3], [4], [5], [6], [7], [8] has had a significant impact on the typical approaches for controlling nonlinear large-scale systems with unstructured uncertainties, leading to the widespread adoption of universal approximators [9], [10], [11], [12], [13], [14], [15], [16], [17]. Universal approximators, such as fuzzy logic systems (FLSs) and neural networks (NNs), are capable of capturing and compensating for unknown functions in the controlled system dynamics, thus converting the unknown function problem into an unknown parameter problem that can be addressed through conventional adaptive control methods. However, control schemes that utilize universal approximators are encumbered by their computational complexity and high dynamic order of the controller, which is the result of a large number of adaptively tuning parameters. Additionally, in the context of strict-feedback or pure-feedback systems, combining the backstepping scheme with real-time tuning approximators significantly increases the complexity of the resultant control law [3], [5], [10] [11], [15].

Recently, prescribed performance control (PPC) [18] has been widely utilized to overcome the drawbacks of controlling uncertain nonlinear systems, and it has been applied to large-scale interconnected systems [19], [20], [21]. In the context of PPC, the complexity of the controller structure is significantly reduced as the use of universal approximators is not required. Nevertheless, the backstepping design steps are still a necessary component of PPC schemes. In [22], [23], state-feedback PPCs for uncertain strict-feedback nonlinear systems with unknown sign of the input gain have been proposed, and an output-feedback controller is also presented in [24]. The traditional method for addressing the issue of unknown control direction employs the Nussbaum gain technique [25]. While a substantial number of algorithms have been successfully developed using this technique (refer to [26], [27] and references therein), it is important to note that the Nussbaum gain technique consistently exhibits poor transient responses, as commented in [28]. The schemes in [22], [23], [24], [29] avoid this problem. In [29], an output-feedback PPC scheme for even more general nonautonomous nonlinear systems is proposed. However, all of these PPC algorithms consider only single system control problems.

A new approach to control uncertain nonlinear systems has emerged in the form of differentiator-based controllers (DBCs) [30], [31], [32], [33] more recently. These DBCs present several benefits when compared to conventional controllers for the control of uncertain nonlinear systems. Firstly, the absence of a requirement for the use of FLSs or NNs as estimators for unknown functions leads to a considerable simplification of the control law and stability proof. Additionally, the DBC does not rely on the backstepping design scheme, further reducing the complexity of the overall control scheme. Secondly, as reported in [33], the DBC can be applied to a wide range of nonlinear systems and the design of output-feedback controllers is made more feasible. The performance of the adopted differentiator (or time-derivative estimator) ensures either finite-time exact output tracking or asymptotic stability of the tracking error.

Most research in this area has focused on strict-feedback nonlinear systems, with limited research results available for the decentralized output-feedback control of LSSs with uncertain pure-feedback nonlinear subsystems [12], [13], [16], [17]. Despite this, some control schemes in this area adopt fuzzy logic systems (FLSs) or neural networks (NNs) as approximators of the unknown functions for adaptive observers or controllers. However, the use of these approximators in the closed-loop system increases the dynamic order of the control laws and makes stability analysis highly complex. Additionally, all previous research in this field [12], [13], [16], [17] has dealt with autonomous nonlinear subsystems.

This paper addresses the issue of decentralized output-feedback control for interconnected LSSs with uncertain nonautonomous general nonlinear subsystems with unknown sign of the input gain. Based on the author's previous work [29], [34], it is extended to the control of interconnected LSSs. The adoption of the higher-order switching differentiator (HOSD) [35] allows for estimation of time-varying signal derivatives, resulting in maintaining output tracking error within prescribed region. Unlike conventional control approaches, the proposed controller does not require the use of universal approximators, and avoids the occurrence of severe chattering or peaking in the control input. In this work, as in [24] and [29], the control strategy retains its simplicity and continuity by not incorporating techniques such as the Nussbaum-Gain method, the sliding-mode control method, or any tools for approximation, identification, estimation, or switching. In [22] and [24], the controller was designed by estimating the sign of the virtual control for all state variables. However, the controller presented in this paper has the advantage of being more concise since it only estimates the sign of the input gain that appears in the last transformed state equation. The advantages of the controller presented in this paper compared to existing research can be summarized as follows:

1) It considers a very general time-varying nonlinear subsystem in the broadest category.
2) The output tracking controller designed using HOSD guarantees prescribed performance.
3) The structure of the distributed controller and stability proof is relatively simple since the sign of the input gain is estimated only once in the last stage.

To the best of our knowledge, very few research results exist on designing output-feedback controllers for time-varying nonlinear subsystems as general as (1) without input gain sign information. Even the most recent research result [24] deals with a limited range of nonlinear systems with constant input gain, while the system (1) dealt with in this paper belongs to a much broader category.

II. PROBLEM FORMULATION

In the following sections, $|\mathbf{x}|$ denotes the 2-norm of vector $\mathbf{x}$, and $|v|$ denotes the absolute value of scalar v. The notation $a(t) \to 0$ is a shorthand for $\lim_{t\to\infty} a(t) = 0$.

This paper is concerned with the analysis and control of a general nonlinear system composed of N interconnected subsystems. The system's dynamics are uncertain, nonautonomous, and nonaffine in the control. The dynamics of each subsystem j can be expressed as follows:

$$\left.\begin{aligned} \dot{\mathbf{x}}_j &= \mathbf{f}_j(\mathbf{x}_j, u_j, t, \tilde{\mathbf{x}}_j) \\ y_j &= h_j(\mathbf{x}_j, t) \end{aligned}\right\}, \quad j = 1, 2, \cdots, N. \tag{1}$$

Here, $j \in 1, 2, \cdots, N$ is a unique index that identifies each subsystem. The functions $\mathbf{f}_j$ and h_j are unknown smooth functions, and $\mathbf{x}_j = [x_{j,1}, x_{j,2}, \cdots, x_{j,n_j}]^T$ is the state vector of the jth subsystem, where n_j is the subsystem's dynamic order. The variables y_j and u_j represent the output and input of the jth subsystem, respectively. The vector $\tilde{\mathbf{x}}_j = [\mathbf{x}_1^T, \cdots, \mathbf{x}_{j-1}^T, \mathbf{x}_{j+1}^T, \cdots, \mathbf{x}_N^T]^T$ denotes the total state vectors of remote subsystems, which is a collection of state vectors except for the jth one.

It should be noted that the considered subsystem belongs to a broader class of nonlinear systems, including both strict- and pure-feedback systems, that are also nonautonomous. The functions $\mathbf{f}_j$ and h_j are explicitly dependent on time. This class of systems may encompass systems with time-varying parameters and disturbances, either additive or multiplicative in nature. The assumption is made that only the output y_j of the jth subsystem is available.

In practical engineering systems, it is desirable to maintain the states within prescribed bounded operational regions and to restrict the control inputs within bounds imposed by physical constraints.

Assumption 1: The open set Ω_j, defined as

$$\Omega_j = \left\{ \mathbf{x}_j, u_j \middle| |\mathbf{x}_j| < \lambda_j^x, |u_j| < \lambda_j^u \right\} \tag{2}$$

encompasses the entire operational region of the jth subsystem (1), where λ_j^x and λ_j^u are positive constants.
The union of all Ω_j's constitutes the entirety of the set:

$$\Omega = \cup_{j=1}^N \Omega_j. \tag{3}$$

Assumption 2: All subsystems have full relative degree n_j, and the control input u_j appears first in the $y_j^{(n_j)}$ equation for all $j = 1, \cdots, N$.
The control objective is driving $y_j(t)$ to track the desired output $r_j(t)$ within predetermined region while maintaining all signals involved to be bounded. Let the tracking error be $z_j \triangleq y_j - r_j$ and error vector be $\mathbf{z}_j = [z_j, \dot{z}_j, \cdots, z_j^{(n_j-1)}]^T$. If the tracking error vector is regarded as a new state vector, then the original system can be redescribed as the following normal form

$$\begin{aligned}
\dot{z}_{j,1} &= g_{j,1}(\mathbf{z}_j, t) \triangleq z_{j,2} \\
\dot{z}_{j,2} &= g_{j,2}(\mathbf{z}_j, t) \triangleq z_{j,3} \\
&\vdots \\
\dot{z}_{j,n} &= g_{j,n}(\mathbf{z}_j, u_j, t)
\end{aligned} \tag{4}$$

where $z_{j,1} = z_j$, $\mathbf{r}_j \triangleq [r_j, \dot{r}_j, \cdots, r_j^{(n_j-1)}]^T$,

$$\begin{aligned}
g_{j,1}(\mathbf{z}_j, t) &= \frac{\partial h(\mathbf{z}_j + \mathbf{r}_j, t)}{\partial \mathbf{z}_j} \mathbf{f}_j(\mathbf{z}_j + \mathbf{r}_j, u_j, t, \tilde{\mathbf{x}}_j) \\
&\quad + \frac{\partial h_j(\mathbf{z}_j + \mathbf{r}_j, t)}{\partial t} - \dot{r}_j \\
g_{j,i}(\mathbf{z}_j, t) &= \frac{\partial g_{j,i-1}(\mathbf{z}_j, t)}{\partial \mathbf{z}_j} \mathbf{f}_j(\mathbf{z}_j + \mathbf{r}_j, u_j, t, \tilde{\mathbf{x}}_j) \\
&\quad + \frac{\partial g_{j,i-1}(\mathbf{z}_j, t)}{\partial t}, i = 2, \cdots, n_j - 1 \\
g_{j,n_j}(\mathbf{z}_j, u_j, t) &= \frac{\partial g_{j,n_j-1}(\mathbf{z}_j, t)}{\partial \mathbf{z}_j} \mathbf{f}_j(\mathbf{z}_j + \mathbf{r}_j, u_j, t, \tilde{\mathbf{x}}_j) \\
&\quad + \frac{\partial g_{j,n_j-1}(\mathbf{z}_j, t)}{\partial t}
\end{aligned} \tag{5}$$

The $g_{j,i}(\cdot)$s for $i = 1, \cdots, n_j$ are also unknown smooth functions, and the transformed system (4) is also nonautonomous. Note that, although $\mathbf{f}_j$ is a function of u_j, the $g_{j,n_j}(\cdot)$ is the only function of control input u_j since all the original subsystems are assumed to have full relative degrees.

Assumption 3: For the following control gain function

$$b_j(\mathbf{z}_j, u_j, t) \triangleq \frac{\partial g_{j,n_j}(\mathbf{z}_j, u_j, t)}{\partial u_j} \tag{6}$$

there is a constant λ_j^b that is not required to be known such that $|b_j(\mathbf{x}, u, t)| > \lambda_j^b$ over Ψ for $t \geq 0$, and the sign of the control direction function $b_j(\cdot)$ is assumed to be unknown.

Assumption 4: The time-derivatives of the desired output $r_j^{(i)}(t)$ for $i = 1, 2, \cdots, n_j + 2$ are all bounded.
The proposed output-feedback controller adopts HOSD to estimate time-derivatives of the tracking error $z_j(t)$.

A. TIME-DERIVATIVES ESTIMATOR

To facilitate the discussion of HOSD dynamics, we introduce the following definitions. Let Φ be a set of all strictly increasing infinite time sequences such that

$$\Phi \triangleq \{(t_i)_{i=0}^{\infty} | t_0 = 0, t_i < t_{i+1} \forall i \in \mathbb{N}_0\} \tag{7}$$

where $\mathbb{N}_0 = \{0, 1, 2, \cdots\}$. For a sequence $T = (t_i) \in \Phi$, Ω_T denotes a set of functions that discontinuous at some or all t_i.

Definition 1 [35]: For $T = (t_i) \in \Phi$, define the set of functions as follows:

$$\overline{\Omega}_T^L \triangleq \left\{ f(t) \middle| f(t) \in \Omega_T, \sup_{\substack{t_i \leq t < t_{i+1} \\ \forall i \in \mathbb{N}_0}} |f(t)| \leq L < \infty \right\} \tag{8}$$

where $L > 0$ is a constant. The functions in $\overline{\Omega}_T^L$ are bounded in the piecewise sense (BPWS) below L.

Park [35] proposed the original HOSD and its dynamics is modified such that it has only one design constant in [33] as the following Lemma.

Lemma 1 [33]: Suppose the time-derivatives of the tracking error of the jth subsystem $z_j(t)$ are BPWS such that $z_j^{(i+1)} \in \overline{\Omega}_T^{L_{j,i}^*}$ for $i = 1, 2, \cdots, n_j$ where $L_{j,i}^*$s are positive constants and $T \in \Phi$. $z_j^{(n+2)}$ is also assumed to be BPWS. We define the HOSD dynamics as follows

$$\left.\begin{aligned}
\dot{\alpha}_{j,i} &= \beta_i L_j e_{\alpha_{j,i}} + \sigma_{j,i} \\
\dot{\sigma}_{j,i} &= L_j \, sgn(e_{\alpha_{j,i}})
\end{aligned}\right\}, \quad i = 1, 2, \cdots, n_j \tag{9}$$

where $e_{\alpha_{j,i}} = \sigma_{j,i-1} - \alpha_{j,i}$ with $\sigma_{j,0} = z_j$. Choosing the design constants $\beta_i > 0$ for all i and $L_j > \max\{L_{j,1}^*, \cdots, L_{j,n_j}^*\}$ ensures that

$$\sigma_{j,i}(t) \to z_j^{(i)}, \quad i = 1, 2, \cdots, n_j. \tag{10}$$

The detailed proof of Lemma 1 is shown in [35]. In [33], the constants β_is up to $i = 6$ have been suggested as

$$\beta_1 = 10, \quad \beta_2 = 7, \; \beta_3 = 5.5, \; \beta_4 = 4.8, \; \beta_5 = 4.4, \; \beta_6 = 4.2. \tag{11}$$

The only design constant L_j in HOSD (9) must be increased to improve the estimating performance of the HOSD. In Fact 3 in [35], it is proven that the following holds

$$\dot{\sigma}_{j,i} \to \sigma_{j,i+1} \quad (12)$$

for $i = 1, \cdots, n_j - 1$. Together the fact $\sigma_{j,n_j} \to z_j^{(n_j)}$ with (12) for $i = n_j - 1$, the following is trivially holds.

$$\dot{\sigma}_{j,n_j-1} \to z_j^{(n_j)} \quad (13)$$

Compared to conventional time-derivative estimators, such as HGO [36] or HOSMD [37], HOSD has the advantages of asymptotic tracking performance and no peaking or chattering in the generated estimations.

B. REDEFINING OUTPUT-FEEDBACK CONTROL PROBLEM

From (10) for $i = 1$, $\dot{z}_j \to \sigma_{j,1}$ holds and it can be rewritten as

$$\dot{z}_{j,1} = \sigma_{j,1} + d_{j,1}(t) \quad (14)$$

where $d_{j,1}(t)$ is a time-varying function that decays asymptotically. From (12), it can be redescribed as

$$\dot{\sigma}_{j,i-1} = \sigma_{j,i} + d_{j,i}(t) \quad (15)$$

for $i = 2, \cdots, n_j - 1$ where $d_{j,i}(t)$s are estimation errors that vanish asymptotically. Finally, combining (13) with (4), we obtain

$$\begin{aligned} \dot{\sigma}_{j,n_j-1} &= z_j^{(n_j)} + d_{j,n}(t) \\ &= g_{j,n}(\mathbf{z}_j, u_j, t) + d_{j,n_j}(t) \quad (16) \end{aligned}$$

where $d_{j,n_j}(t)$ is also an observation error that goes to zero asymptotically. Equations (14), (15), and (16) together form a new system with state vector

$$\hat{\mathbf{z}}_j = [z_j, \sigma_{j,1}, \cdots, \sigma_{j,n_j-1}]^T. \quad (17)$$

Equations (14), (15), and (16) constitute a pure-feedback nonlinear system with respect to $\hat{\mathbf{z}}_j$ whose disturbances are unmatched. Thus, low-complexity prescribed performance control scheme in [22] can be applied.

C. CONTROLLER DESIGN

The predetermined tracking performance is

$$\begin{aligned} -c_j k_{j,1}(t) < z_j(t) < k_{j,1}(t), \quad & \textit{if } z_j(0) \geq 0 \\ -k_{j,1}(t) < z_j(t) < c_j k_{j,1}(t), \quad & \textit{if } z_j(0) < 0 \quad (18) \end{aligned}$$

where $0 < c_j < 1$ is a design constant, and

$$k_{j,1}(t) = (k_{j,1}^0 - k_{j,1}^\infty)e^{-\mu_{j,1}t} + k_{j,1}^\infty \quad (19)$$

with $k_{j,1}^0 > k_{j,1}^\infty > 0$ and $\mu_{j,1} > 0$ are design constants. The convergence rate is specified by $\mu_{j,1}$ and the ultimate bound of $z_j(t)$ is depicted by $k_{j,1}^\infty$. The overshoot of the tracking error is specified to be less than $c_j k_{j,1}(t)$.

To transform asymmetric constraint (18) into a symmetric form, let us define

$$\underline{k}_{j,1}(t) = c_j k_{j,1}(t), \quad \overline{k}_{j,1}(t) = k_{j,1}(t), \textit{ if } z_j(0) \geq 0$$

$$\underline{k}_{j,1}(t) = k_{j,1}(t), \overline{k}_{j,1}(t) = c_j k_{j,1}(t), \textit{ if } z_j(0) < 0 \quad (20)$$

and

$$\begin{aligned} p_{j,1}(t) &= \frac{1}{2}(\underline{k}_{j,1}(t) + \overline{k}_{j,1}(t)) \\ \delta_j(t) &= \frac{1}{2}(\underline{k}_{j,1}(t) - \overline{k}_{j,1}(t)) \\ e_{j,1}(t) &= z_j(t) + \delta_j(t) \quad (21) \end{aligned}$$

Then, it can be easily deduced that the following inequality

$$-p_{j,1}(t) < e_{j,1}(t) < p_{j,1}(t) \quad (22)$$

is equivalent to (18).

To proceed, the state errors are defined as

$$e_{j,i+1} = \hat{z}_{j,i+1} - \zeta_{j,i}, \; i = 1, \cdots, n_j - 1 \quad (23)$$

where $\zeta_{j,i}$ is the intermediate control signal that will be designed in what follows. The performance bounds of this state errors are selected as

$$p_{j,i}(t) = (p_{j,i}^0 - p_{j,i}^\infty)e^{-\mu_{j,i}t} + p_{j,i}^\infty, \; i = 2, \cdots, n_j \quad (24)$$

where the positive design paramters $p_{j,i}^0$ and $p_{j,i}^\infty$ are chosen such that $|e_{j,i}(0)| < p_{j,i}^0$ and $p_{j,i}^\infty < p_{j,i}^0$ hold.

To drive $|e_{j,i}(t)| < p_{j,i}(t)$, $\forall t \geq 0$, the control law is determined as

$$\zeta_{j,i} = -\gamma_{j,i}\eta_{j,i}, \; i = 1, \cdots, n_j - 1 \quad (25)$$

$$u_j = \gamma_{j,n_j} v(\eta_{j,n_j})\eta_{j,n_j} \quad (26)$$

where $\gamma_{j,i}$ s are positive control gains,

$$\eta_{j,i} = \tan\left(\frac{\pi e_{j,i}}{2p_{j,i}}\right), \; i = 1, \cdots, n_j, \quad (27)$$

and the function $v(\eta_{j,n_j})$ that estimates the control direction is smooth with respect to η_{j,n_j} and satisfies that there exist positive constants $a_{j,1}$ and $a_{j,2}$ such that

$$\limsup_{\eta_{j,n_j} \to \infty} v(\eta_{j,n_j}) = a_{j,1} \quad (28)$$

$$\liminf_{\eta_{j,n_j} \to \infty} v(\eta_{j,n_j}) = -a_{j,2}. \quad (29)$$

for example $v(\cdot) = \sin(\cdot)$ or $v(\cdot) = \cos(\cdot)$.

From (26)-(29), the following attributes of the control input are evident

$$\begin{aligned} \limsup_{(e_{j,n_j} - p_{j,n_j}) \to 0^-} u_j &= +\infty \\ \liminf_{(e_{j,n_j} - p_{j,n_j}) \to 0^-} u_j &= -\infty \quad (30) \end{aligned}$$

and

$$\begin{aligned} \limsup_{(e_{j,n_j} + p_{j,n_j}) \to 0^+} u_j &= +\infty \\ \liminf_{(e_{j,n_j} + p_{j,n_j}) \to 0^+} u_j &= -\infty. \quad (31) \end{aligned}$$

Lemma 2: For $i = 1, \cdots, n_j - 1$, $\dot{\zeta}_{j,i}$ remains bounded if $e_{j,i}$, $\dot{e}_{j,i}$, and $\eta_{j,i}$ are all bounded.

Proof: From (25), the following is easily induced

$$\begin{aligned}\dot{\zeta}_{j,i} &= \gamma_{j,i}\dot{\eta}_{j,i}\\ &= \gamma_{j,i}\frac{\pi}{2}\frac{\dot{e}_{j,i}p_{j,i}-e_{j,i}\dot{p}_{j,i}}{p_{j,i}^2}\frac{1}{\cos^2(\frac{\pi e_{j,i}}{2p_{j,i}})} \end{aligned} \tag{32}$$

From (19) and (21)

$$\begin{aligned}p_{j,1} &= \frac{1}{2}(c_j+1)k_{j,1}\\ &= \frac{1}{2}(c_j+1)\{(k_{j,1}^0-k_{j,1}^\infty)e^{-\mu_{j,1}t}+k_{j,1}^\infty\},\end{aligned} \tag{33}$$

the followings are easily induced.

$$\begin{aligned}&\frac{2}{(1+c_j)k_{j,1}^0}\le\frac{1}{p_{j,1}}\le\frac{2}{(1+c_j)k_{j,1}^\infty}\\ &\frac{\mu_{j,1}}{2}(k_{j,1}^\infty-k_{j,1}^0)\le\dot{p}_{j,1}<0.\end{aligned} \tag{34}$$

From (24), the followings are also easily derived.

$$\begin{aligned}&\frac{1}{p_{j,i}^0}\le\frac{1}{p_{j,i}}\le\frac{1}{p_{j,i}^\infty},\quad i=2,\cdots,n_j\\ &\mu_{j,i}(p_{j,i}^\infty-p_{j,i}^0)\le\dot{p}_{j,i}<0,\quad i=2,\cdots,n_j\end{aligned} \tag{35}$$

And by defining $\theta_{j,i}=\frac{\pi e_{j,i}}{2p_{j,i}}$, from the boundedness of $\eta_{j,i}$ and the definition of (27), since it can be deduced that $|\theta_{j,i}|\neq\frac{m\pi}{2}$ for $m=1,3,5,\cdots$, which guarantees that $1/cos^2\theta_{j,i}$ is also bounded. Together all these facts with the assumption that $e_{j,i}$ and $\dot{e}_{j,i}$ are bounded, $\dot{\zeta}_{j,i}$s are concluded to be bounded. □

The following theorem summarizes the main result of the proposed control scheme.

Theorem 1: Consider system (1) under Assumptions 1 through 3. The control input (26) using the HOSD (9) makes the tracking error e_j remain within a prescribed region.

Proof: By (21) and (14), we have

$$\begin{aligned}\dot{e}_{j,1} &= \dot{\hat{z}}_{j,1}+\dot{\delta}_j\\ &= \hat{z}_{j,2}+d_{j,1}+\dot{\delta}\end{aligned} \tag{36}$$

where $\hat{z}_{j,1}=z_j$. Using (23) and (15), we can write

$$\begin{aligned}\dot{e}_{j,i} &= \dot{\hat{z}}_{j,i}-\dot{\zeta}_{j,i-1}\\ &= \hat{z}_{j,i+1}+d_{j,i}-\dot{\zeta}_{j,i-1},\quad i=2,\cdots,n_j-1\end{aligned} \tag{37}$$

From (23) and (16), we obtain

$$\begin{aligned}\dot{e}_{j,n_j} &= \dot{\hat{z}}_{j,n_j}-\dot{\zeta}_{j,n_j-1}\\ &= g_{j,n_j}(\mathbf{z}_j,u_j,t)+d_{j,n_j}-\dot{\zeta}_{j,n_j-1}\\ &= g_{j,n_j}(\mathbf{z}_j,0,t)+d_{j,n_j}-\dot{\zeta}_{j,n_j-1}\\ &\quad+g_{j,n_j}(\mathbf{z}_j,u_j,t)-g_{j,n_j}(\mathbf{z}_j,0,t).\end{aligned} \tag{38}$$

By the mean-value theorem and (6), there exists a ξ_j between u_j and 0 such that

$$b_j(\mathbf{z}_j,\xi_j,t)=\frac{g_{j,n_j}(\mathbf{z}_j,u_j,t)-g_{j,n_j}(\mathbf{z}_j,0,t)}{u_j} \tag{39}$$

Using this, (38) becomes

$$\dot{e}_{j,n_j}=g_{j,n_j}(\mathbf{z}_j,0,t)+d_{j,n_j}-\dot{\zeta}_{j,n_j-1}+b_j(\mathbf{z}_j,\xi_j,t)u_j \tag{40}$$

By redescribing (23) as

$$\hat{z}_{j,i+1}=e_{j,i+1}+\zeta_{j,i},\quad i=1,\cdots,n_j-1 \tag{41}$$

equations (36),(37), and (40) can be rewritten as

$$\dot{e}_{j,i}=\phi_{j,i}+\zeta_{j,i},\quad i=1,\cdots,n_j-1 \tag{42}$$

$$\dot{e}_{j,n_j}=\phi_{j,n_j}+b_j(\mathbf{z}_j,\xi_j,t)u_j \tag{43}$$

where

$$\begin{aligned}\phi_{j,1} &= e_{j,2}+d_{j,1}+\dot{\delta}_j\\ \phi_{j,i} &= e_{j,i+1}+d_{j,i}-\dot{\zeta}_{j,i-1},\quad i=2,\cdots,n_j-1\\ \phi_{j,n_j} &= g_{j,n_j}(\mathbf{z}_j,0,t)+d_{j,n_j}-\dot{\zeta}_{j,n_j-1}.\end{aligned} \tag{44}$$

Suppose there exists a time instance $t_1>0$ in which an error variable $e_{j,i}$ reaches the boundary $p_{j,i}$ first while other error variables $e_{j,m}(m\neq i)$ are maintained within their boundaries such that

$$|e_{j,i}(t_1)|=p_{j,i}(t_1) \tag{45}$$

$$|e_{j,m}(t)|<p_{j,m}(t),\quad t\in[0,t_1),\ \forall m\neq i \tag{46}$$

The necessary condition for (45) is that the followings hold

$$\lim_{(e_{j,i}-p_{j,i})\to0^-}\dot{e}_{j,i}\ge\dot{p}_{j,i},\quad \lim_{(e_{j,i}+p_{j,i})\to0^+}\dot{e}_{j,i}\le\dot{p}_{j,i} \tag{47}$$

Recalling (34) and (35), (47) further follows that

$$\lim_{(e_{j,i}-p_{j,i})\to0^-}\dot{e}_{j,i}\ge h_{j,i},\quad \lim_{(e_{j,i}+p_{j,i})\to0^+}\dot{e}_{j,i}\le -h_{j,i} \tag{48}$$

where $h_{j,i}$s are negative constants defined as

$$\begin{aligned}h_{j,1} &\triangleq \frac{\mu_{j,1}}{2}(k_{j,1}^\infty-k_{j,1}^0)\\ h_{j,i} &\triangleq \mu_{j,i}(p_{j,i}^\infty-p_{j,i}^0),\ i=2,\cdots,n_j\end{aligned} \tag{49}$$

Case 1 ($i=1,\cdots,n_j-1$): From the fact $\phi_{j,i}\in L^\infty$ and

$$\lim_{(e_{j,i}-p_{j,i})\to0^-}\zeta_{j,i}=-\infty,\quad \lim_{(e_{j,i}+p_{j,i})\to0^+}\zeta_{j,i}=+\infty \tag{50}$$

the following are trivially deduced

$$\lim_{(e_{j,i}-p_{j,i})\to0^-}\dot{e}_{j,i}=-\infty,\quad \lim_{(e_{j,i}+p_{j,i})\to0^+}\dot{e}_{j,i}=+\infty. \tag{51}$$

which evidently violate the necessary condition (48).

Case 2($i=n_j$): From the boundedness of ϕ_{j,n_j}, the equalities (30), and Assumption 2, it holds that

$$\begin{aligned}&\lim_{(e_{j,n_j}-p_{j,n_j})\to0^-}g_{j,n_j}(\mathbf{z}_j,\xi_j,t)u_j=-\infty\\ &\lim_{(e_{j,n_j}+p_{j,n_j})\to0^+}g_{j,n_j}(\mathbf{z}_j,\xi_j,t)u_j=+\infty\end{aligned} \tag{52}$$

regardless of the sign of $g_{j,n_j}(\mathbf{z}_j,\xi_j,t)$. Thus, the following are deduced

$$\begin{aligned}&\liminf_{(e_{j,n_j}-p_{j,n_j})\to0^-}\dot{e}_{j,n_j}=-\infty,\\ &\limsup_{(e_{j,n_j}+p_{j,n_j})\to0^+}\dot{e}_{j,n_j}=+\infty.\end{aligned} \tag{53}$$

which also violate the necessary condition (48).

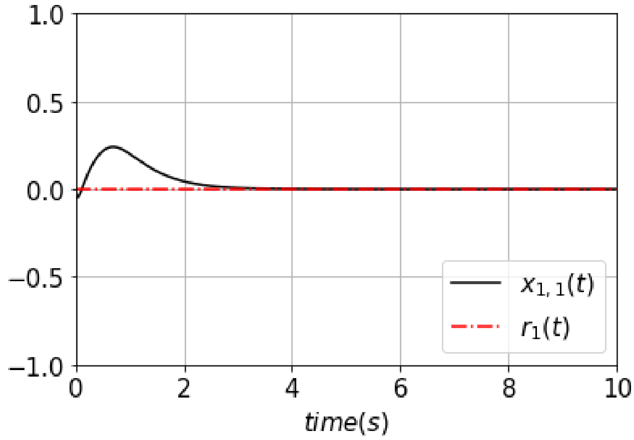

FIGURE 1. Trajectories of $x_{1,1}(t)$ and $r_1(t)$.

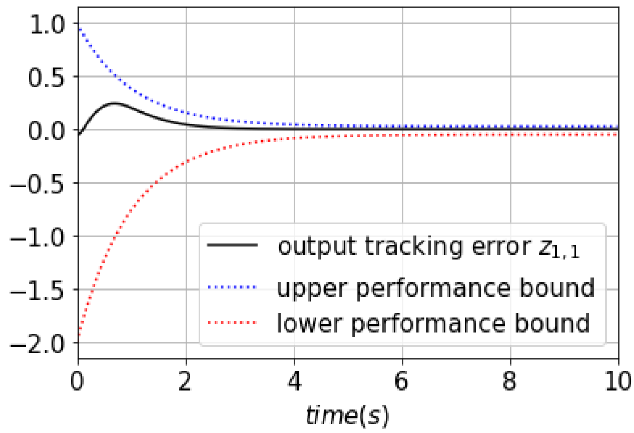

FIGURE 2. Trajectories of $z_{1,1}(t)$, $\bar{k}_{1,1}(t)$, and $\underline{k}_{1,1}(t)$.

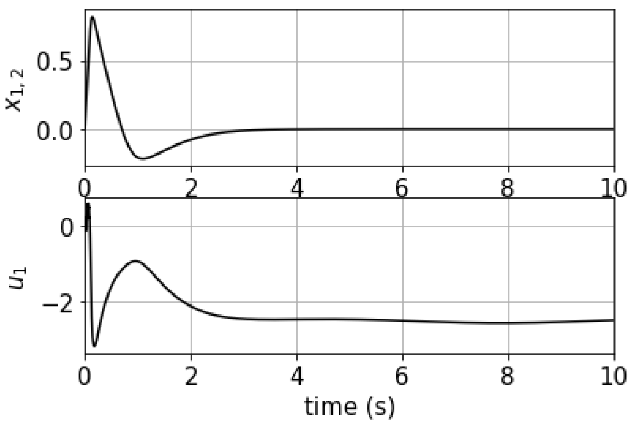

FIGURE 3. Trajectories of $x_{1,2}(t)$ and control input $u_1(t)$.

Therefore, considering case 1 and case 2 together, it can be concluded that the time t_1 at which an error variable reaches its boundary does not exist. This means that all the errors remain within their prescribed boundaries. □

Previous works [22], [24] required the determination of the sign of the virtual control term for all state variables, which necessitated a stability analysis at each iteration. In contrast, the technique introduced in this paper, as proven in Theorem 1, facilitates the estimation of a single gain sign in

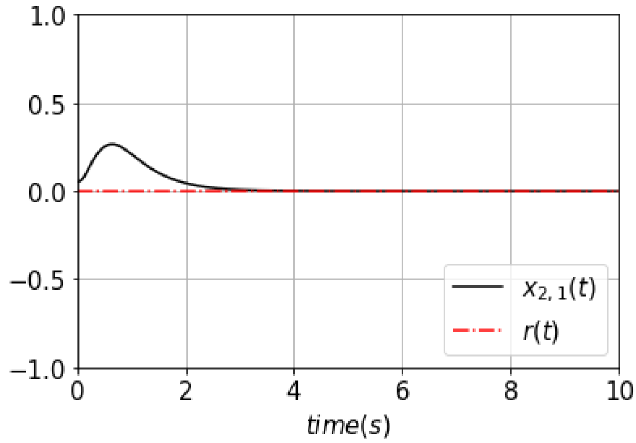

FIGURE 4. Trajectories of $x_{2,1}(t)$ and $r_2(t)$.

the final state equation of order n_j, thereby streamlining both the control formulation and stability verification process. Furthermore, the scope of the system (1) examined in this study surpasses that of the dynamic characteristic equations scrutinized in [22] and [24], as it encompasses a broader range of nonlinear time-varying systems.

III. SIMULATIONS

This section presents a numerical simulation of a system comprising two interconnected inverted pendulums to demonstrate the efficacy of the proposed controller design methodology and its corresponding performance. The system's state-space equations are articulated as follows:

$$\Sigma_1 : \begin{cases} \dot{x}_{1,1} &= x_{1,2} \\ \dot{x}_{1,2} &= \left(\frac{m_1 GH}{J_1} - \frac{KH}{2J_1}\right)\sin(x_{1,1}) + \frac{KH}{2J_1}(l-D) \\ & \quad + \frac{sat(u_1)}{J_1} + \frac{KH^2}{4J_1}\sin(x_{2,1}) + \Delta_1(t) \\ y_1 &= x_{1,1} \end{cases} \tag{54}$$

$$\Sigma_2 : \begin{cases} \dot{x}_{2,1} &= x_{2,2} \\ \dot{x}_{2,2} &= \left(\frac{m_2 GH}{J_2} - \frac{KH}{2J_2}\right)\sin(x_{2,1}) + \frac{KH}{2J_2}(l-D) \\ & \quad + \frac{sat(u_2)}{J_2} + \frac{KH^2}{4J_2}\sin(x_{1,1}) + \Delta_2(t) \\ y_2 &= x_{2,1} \end{cases} \tag{55}$$

where the system outputs $x_{j,1}$ $(j = 1, 2)$ denote the vertical angular displacements that can be directly measured. The angular velocities $x_{j,2}$ $(j = 1, 2)$ are treated as unknown states. The inputs u_j $(j = 1, 2)$ correspond to the torques generated by the servomotors, while Δ_j $(j = 1, 2)$ indicate external disturbances that are not known a priori. The specific disturbances in this simulation are assumed to be $\Delta_1(t) = 0.1\sin(t)$ and $\Delta_2(t) = 0.2 + 0.1\cos(2t)$. The gravitational acceleration is $G = 9.8m/s^2$, the spring constant is $K = 100N/m$, the height of the pendulum is $H = 0.5m$, the length of the spring

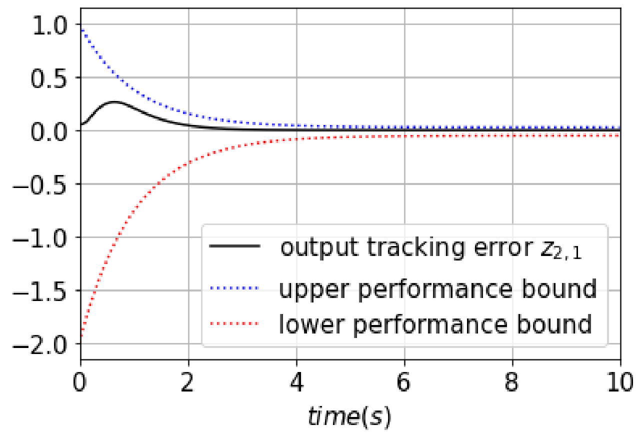

FIGURE 5. Trajectories of $z_{2,1}(t)$, $\overline{k}_{2,1}(t)$, and $\underline{k}_{2,1}(t)$.

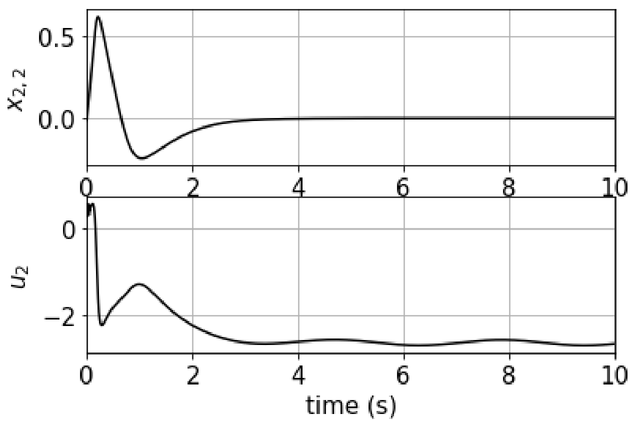

FIGURE 6. Trajectories of $x_{2,2}(t)$ and control input $u_2(t)$.

is $l = 0.5m$, the moments of inertia of the pendulums are $J_1 = 0.5kg \cdot m^2$ and $J_2 = 0.625kg \cdot m^2$, and the distance between the hinges is $D = 0.4m < l$. The pendulum masses are $m_1 = 2$ kg and $m_2 = 2.5$ kg respectively. The control inputs are subject to saturation constraints of $sat(u_j) = sgn(u_j) \min(|u_j|, \lambda_j^u)$, where $\lambda_1^u = \lambda_2^u = 25$ represent the maximum torques that can be generated by the servomotors.

The following design parameters have been chosen for the system: the constant L for HOSDs is set to 20. For the controller in the first subsystem, the values of c_1, $k_{1,1}^0$, $k_{1,2}^0$, $k_{1,1}^\infty$, $k_{1,2}^\infty$, $\mu_{1,1}$, $\mu_{1,2}$, $\gamma_{1,1}$, and $\gamma_{1,1}$ are 0.4, 2, 2, 0.05, 0.4, 1, 1, 0.4, and 0.1, respectively. Similarly, for the second subsystem, the values of c_2, $k_{2,1}^0$, $k_{2,2}^0$, $k_{2,1}^\infty$, $k_{2,2}^\infty$, $\mu_{2,1}$, $\mu_{2,2}$, $\gamma_{2,1}$, and $\gamma_{2,2}$ are 0.4, 2, 2, 0.05, 0.4, 1, 1, 0.4, and 1, respectively. It should be emphasized that the dynamic equations and disturbances affecting the system are unknown to the controller. Therefore, to facilitate the illustration, the controllers have been designed to regulate the outputs to their respective origins, with the desired outputs $r_1(t)$ and $r_2(t)$ set to zero for all $t \geq 0$. The simulations have been carried out using Python with the Scipy library.

In Figure 1, it can be observed that the output $x_{1,1}(t)$ closely follows the desired output $r_1(t) = 0$ after a short transient period. In Figure 2, it is evident that the output error $z_{1,1}(t)$ moves within the upper bound $\overline{k}_{1,1}(t)$ and lower bound $\underline{k}_{1,1}(t)$, and asymptotically converges within the range of (0.025, -0.05). Furthermore, Figure 3 displays that $x_{1,2}(t)$ and the control input $u_1(t)$ are also well-bounded. For the 2nd subsystem, the simulation results are illustrated in Figures 4 through 6.

IV. CONCLUSION

This paper addresses the problem of decentralized output-feedback control for interconnected LSSs with uncertain nonautonomous general nonlinear subsystems and unknown input gain sign. This general class of interconnected systems with subsystems that has no information on high-frequency gain sign has not been previously examined in the literature. The proposed controller employs HOSD to estimate the time-derivatives of the output tracking error, thereby maintaining the error within a prescribed region. In comparison to existing research, the proposed controller is more concise and avoids severe chattering or peaking in the control input. Furthermore, the controller is applicable to a very general time-varying nonlinear subsystem within the broadest category and only estimates the sign of the input gain that appears in the last transformed state equation. This results in a relatively simple structure for the distributed controller and stability proof.

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