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RESEARCH ARTICLE

A Partial Order OWA Operator for Solving the OWA Weighing Dilemma

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ABSTRACT Prior weights are necessary for the application of ordered weighted averaging (OWA) operators, but obtaining them is expensive and contentious, which restricts the application of operators. To address the weighting issue, the weight space is used to "replace" the conventional weight vector, and the operator comparison is then extended to a partial order comparison on the weight space. The results show that the partial order OWA operator can be used as long as the weight order is clear, that is, there is no need to take accurate values. The evaluation result is represented by a Hasse diagram. The partial order OWA operator retains the properties of the conventional operator, and the running cost is low. It can be seen from the example that the partial order OWA operator solves the time weight problem. It can compare, sort, and optimize data using the Hasse graph, and it can also implement hierarchical clustering. The comparison results have strong robustness.

INDEX TERMS OWA operator, partially ordered set, weight, robustness.

I. INTRODUCTION

In 1988, the American scholar Yager proposed the ordered weighted averaging operator (OWA) operator [1]. This operator is gradually gaining worldwide attention and has been widely used in neural networks [2], [3], Fuzzy control and fuzzy modeling [4], [5], information fusion [6], expert system [7], decision-making [8], [9], [10], communication network [11], and many other fields. Inspired by the OWA operator, scholars have continuously proposed HOWA [12], IOWA [13], and LOWA [14] operators, among others. There are nearly 100 kinds of derivative operators. Although the OWA operator can flexibly gather information according to the change of the situation, both the OWA and its derivative inevitably encounter the difficult problem of weighting, which restricts the application of this method.

At present, weighting methods are mainly classified in two categories: subjective and objective weighting methods. The main achievements of subjective weighting method so far include expert survey [15], analytic hierarchy process [16], preference ratio [17], chain scale scoring [18], binomial coefficient [19], comparison matrix [24], and importance ordering [20] methods. Objective weighting methods include principal component analysis [21], "open grade" [22], entropy technique [17], deviation maximization [23], mean square error [24], and multi-objective programming [25] methods. Despite the wide variety of subjective and objective weighting techniques, none of them is fully recognized.

Although objective weighting methods make use of the perfect mathematical theoretical knowledge, they ignore the decision maker's subjective preferences. On the other hand, subjective weighting methods are easily affected by human factors due to the influence of the decision makers' or expert's knowledge, experience, and preferences. There is currently a more accurate and effective way to deal with multi-criteria decision making: the Stochastic Multi-criteria Acceptability Analysis (SMAA) [26]. SMAA can effectively solve the challenging weighting problem, and has a similar effect to the partially ordered set method. Both use a weight space to replace the conventional single weight, but the latter has more obvious robustness advantages.

In the SMAA model, it is necessary to clarify the weight variation boundary. The boundary obtained by various decision makers may be different, so the simulation

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results are not entirely consistent. The partially ordered set decision-making method [27] resolves the issue that the boundary of the SMAA weight space is unknown by obtaining the weight order and producing the weight space through preference information or expert judgment. Partially ordered sets can be widely combined with multi-criteria decision models, including the partially ordered TOPSIS model [28], the partially ordered PROMETHEE model [29], and partial order comprehensive evaluation model [30]. The multi-criterion model expressed by partial order not only overcomes the weighting problem, but also significantly enhances the robustness of the model. The partial order decision method is used to embed the weight space into the OWA operator because the weight space determination method is simple and the boundary is clear. The result is a robust partial order-OWA operator that can express the decision makers' preferences.

II. PARTIAL ORDER-OWA OPERATOR

A. OWA OPERATORS ON WEIGHT SPAC

The OWA operator is assembled through specific weight rules, which have a wide range of applications and are more adaptable. Compared to the conventional assembly rules, some unreasonable situations can be avoided.

Definition 1 [3]: Setup $f:R^n \to R$, if $f(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} \omega_j a_j$, where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weighted vector ($\omega_j \in [0, 1], \sum_{j=1}^{n} \omega_j = 1$) associated with f, and a_j is the largest element in a set of data $(a_1, a_2, \cdots, a_n)$, the function f is said to be an n-dimensional ordered weighted average (OWA) operator.

It is clear from the above definition that weights are needed to run an ordered weighting operator. The weighting problem is always a realistic constraint that limits how the model can be applied. To this end, SMAA suggests an effective processing method to replace the conventional weight vector with a weight space. With this approach, the weights' uncertainty is fully reflected and the difficulty of "representativeness" of weight vectors is addressed properly. According to the idea of SMAA dealing with weights, an OWA operator with weight space is given. Let $\omega \in \Lambda$, where Λ is the set of weights satisfying certain constraints. Thus, we have:

$$f_\Lambda(a_1, a_2, \cdots a_n) = \{\sum_{j=1}^{n} \omega_j a_j \,|\omega \in \Lambda\} \tag{1}$$

For an arbitrary sum $b = (b_1, b_2, \cdots, b_n)$, the comparison of the two in the weight space Λ can be organized as:

$$\Delta(a, b) = \{f(a_1, a_2, \cdots, a_n) - f(b_1, b_2, \cdots, b_n) \,|\omega \in \Lambda\} \tag{2}$$

According to (2), if $\min \Delta(a, b) \geq 0$, then it holds for $\forall\omega \in \Lambda, f(a_1, a_2, \cdots, a_n) - f(b_1, b_2, \cdots, b_n) \geq 0$. The OWA operator extended according to (1) has the following properties:

Property 1: For any $a = (a_1, a_2, \cdots, a_n)$, $b = (b_1, b_2, \cdots, b_n)$, formula (1) has the following properties:

1) Boundedness. The range of values of the $f_\Lambda(a_1, a_2, \cdots, a_n)$ belongs to the interval consisting of the minimum and maximum values of the vector, that is, $f_\Lambda(a_1, a_2, \cdots, a_n) \subseteq [\min\{a_1, a_2, \cdots a_n\}, \max\{a_1, a_2, \cdots, a_n\}]$.

2) Consistency. At that time $a_j = \theta, j = 1, \ldots, n$, then for $\forall\omega \in \Lambda$, there is $f(a_1, a_2, \cdots, a_n) = \theta$.

3) Exchangeability. Let $(a_{\pi(1)}, a_{\pi(2)}, \cdots, a_{\pi(n)})$ be any permutation of $(a_1, a_2, \cdots, a_n)$, then there is always: $f_\Lambda(a_1, a_2, \cdots, a_n) = f_\Lambda(a_{\pi(1)}, a_{\pi(2)}, \cdots, a_{\pi(n)})$.

4) Monotonicity. When $a_j \geq b_j, j = 1, \ldots, n$, then we have $\min \Delta(a, b) \geq 0$.

Proof: The procedure for proving the above property is similar. (Thus only equation 4 is proved).

For any $\omega \in \Lambda$, at that time $a_j \geq b_j, j = 1, \ldots, n$, it is known that $f(a_1, a_2, \cdots, a_n) \geq f(b_1, b_2, \cdots, b_n)$ according to the monotonicity property of OWA, so any element of the set $\Delta(a, b) = \{f(a_1, a_2, \cdots, a_n) - f(b_1, b_2, \cdots, b_n) \,|\omega \in \Lambda\}$ is greater than or equal to zero, and therefore, $\min \Delta(a, b) \geq 0$.

B. PARTIAL ORDER RELATIONS FOR CONSTRUCTING OWA ON WEIGHT SPACE

The OWA operator on weight space Λ has good robustness in addition to retaining the beneficial properties of conventional OWA. When the OWA operator is compared on the weight space, it actually expands the comparison of two real-valued functions to the comparison of two sets. It is not difficult to verify that the comparison relation satisfies the partial order relation.

Definition 2: Let R be a binary relation on a non-empty set A, R is said to be a partial order relation on A if it satisfies self-reflexivity, anti-symmetry, and transiti-vity [27].

(1) Self-reflexivity: for any $x \in A$, with xRx.

(2) Anti-symmetry: for any $x, y \in A$, when xRy and yRx, with $x = y$.

(3) Transitivity: for any $x, y, z \in A$, when xRy and yRz, there is xRz, then R is said to be a partial order relation on A (denoted as $\succeq$).

Property 2: When $\min \Delta(a, b) \geq 0$, there is $a \succeq b$, then the binary relation $\succeq$ is a partial order relation.

Proof: If the binary relation ($\succeq$) satisfies the three properties in Definition 2, then it is a partial order relation. For any a, it is obvious that $\min \Delta(a, a) \geq 0$, and it follows that $\succeq$ satisfies self-reflexivity, that is, $a \succeq a$.

If $a \succeq b$, there is $\min \Delta(a, b) \geq 0$; if $b \succeq a$, there is $\min \Delta(b, a) \geq 0$. Both hold simultaneously, there must be $\Delta(b, a) = \{0\}$. That is $f(a_1, a_2, \cdots, a_n) - f(b_1, b_2, \cdots, b_n) = 0$, for any $\omega \in \Lambda$. Then, using the converse method, it can be proved that for any component there must be $b_j = a_j$. If $b_j \neq a_j$, then let $\omega_j = 1$, so that there are non-zero elements in $\Delta(b, a)$, which contradicts the premise. It follows that the relation $a = b$, $\succeq$ satisfies anti-symmetry.

Let $a \succeq b$, $b \succeq c$, correspond to that $\min \Delta(a, b) \geq 0$, $\min \Delta(b, c) \geq 0$ and according to (2), it is known that for any

$\omega \in \Lambda$, we have:

$$f(a_1, a_2, \cdots, a_n) - f(b_1, b_2, \cdots, b_n) \geq 0$$
$$f(b_1, b_2, \cdots, b_n) - f(c_1, c_2, \cdots, c_n) \geq 0$$

Therefore, it follows that there is $\min \Delta(a, c) \geq 0$ and thus the relation satisfies transitivity.

It follows that $f(a_1, a_2, \cdots, a_n) - f(c_1, c_2, \cdots, c_n) \geq 0$ and therefore, $\min \Delta(a, c) \geq 0$, so that we have $a \succeq c$ and hence, the relation $\succeq$ satisfies transitivity.

C. PRINCIPLE OF PARTIAL ORDER REPRESENTATION OF OWA OPERATOR

The partial order relation can be constructed according to Property 2, but it is not actually possible to traverse the weights and complete the comparison of the OWA operator with $a = (a_1, a_2, \cdots, a_n)$, $b = (b_1, b_2, \cdots, b_n)$. The theorem that follows provides a method for comparing under the weight space.

Theorem 1 [28]: Given the evaluation set $M = (A, \mathrm{IC}, D)$, where $A = \{a_1 \cdots a_n\}$ is the set of evaluation options, $\mathrm{IC} = \{c_1, c_2, \cdots c_n\}$ is the set of criteria, and $D = (d_1, d_2, \cdots, d_n)^T$ is the initial evaluation matrix, $d_i = (d_{i1}, d_{i2}, \cdots, d_{in}) \in R^n$, where d_{ij} denotes the evaluation value of option a_j under criterion c_j. Assumption the criterion weights $\omega_1 \geq \omega_2 \geq \cdots \omega_n \geq 0$; if $\sum_{j=1}^{t} a_j \geq \sum_{j=1}^{t} b_j$ $(t = 1, 2, \cdots, n)$, then $g(a) \geq g(b)$, relevant proofs can be found in the literature.

In this theorem, the function g is a simple linear function, i.e., $g(a) = \omega_1 a_1 + \omega_2 a_2 + \cdots + \omega_n a_n$. The theorem characterizes a weight space $\Lambda = \big\{(\omega_1, \omega_2, \cdots, \omega_n) \,|\omega_1 \geq \omega_2 \geq \cdots \omega_n \geq 0\big\}$. In this space, if the relationship between the two vectors satisfies $\sum_{j=1}^{t} a_j \geq \sum_{j=1}^{t} b_j$ $(t = 1, 2, \cdots, n)$, namely,

$$\begin{cases} a_1 \leq b_1 \\ a_1 + a_2 \leq b_1 + b_2 \\ \cdots \\ a_1 + a_2 + \cdots + a_n \leq b_1 + b_2 + \cdots + b_n \end{cases} \tag{3}$$

then there is $g(a) \leq g(b)$. This theorem demonstrates that only the ordering information of the weights must be known to compare two simple linear weighting functions, knowledge of the exact weights is not necessary. Further research shows that the comparison of (3) can be converted into a matrix problem [28], that is, given the upper triangular matrix $\boldsymbol{E}$,

$$\boldsymbol{E} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{4}$$

when $\omega_1 \geq \omega_2 \geq \cdots \omega_n \geq 0$, the upper triangular matrices $\boldsymbol{E}$ and $\boldsymbol{D}$ perform the following operations to obtain the matrix $\boldsymbol{Y}$:

$$\begin{aligned} \boldsymbol{Y} = \boldsymbol{D} \cdot \boldsymbol{E} &= \begin{bmatrix} a_1 & a_2 & \cdots & a_n \\ b_1 & b_2 & \cdots & b_n \\ \vdots & \vdots & \ddots & \vdots \\ z_1 & z_n & \cdots & z_n \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \\ &= \begin{bmatrix} a_1 & a_1 + a_2 & \cdots & a_1 + a_2 + \cdots + a_n \\ b_1 & b_1 + b_2 & \cdots & b_1 + b_2 + \cdots + b_n \\ \vdots & \vdots & \ddots & \vdots \\ z_1 & z_1 + z_n & \cdots & z_1 + z_2 + \cdots + z_n \end{bmatrix} \end{aligned} \tag{5}$$

By adding the right end of (5) to (3), it can be determined that the first row is less than or equal to the second row. Then, it can be known that $f(a) \leq f(b)$, and we obtain $\min \Delta(a, b) \geq 0 \Leftrightarrow a \succeq b$. Thus, the partial order relation can be established based on the comparison between rows in matrix $\boldsymbol{Y}$.

According to Theorem 1, the indicator determines the weight size, which is fixed based on the indicator, and different indicators correspond to different weights. However, the OWA operator's weight assignment has diverse characteristics. The weight can depend on the indicator, on its assignment size, or even on the design of a given situation. For instance, if the indicator is monthly data, the longer the month, the greater the indicator weight.

Any scheme's values can be fully arranged in accordance with the exchange invariance of the OWA operator. For the convenience of research and application, the first component corresponds to the first important index, the second component corresponds to the second important index, and so on, until the n-th component and index. Let the adjusted decision matrix be $\boldsymbol{T}(\boldsymbol{D})$, and then apply Theorem 1 on this basis, that is, according to (5), it is obtained that: $\boldsymbol{T}(\boldsymbol{D}) \cdot \boldsymbol{E}$.

The schemes are compared in pairs according to the partial order relationship. The following partial order relationship matrix $\boldsymbol{R} = (r_{ab})_{m \times n}$ is established, that is $\forall a, b \in A$, where:

$$r_{ab} = \begin{cases} 1, & a_i \succeq a_j; \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

However, the partial order relation matrix must be simplified to the Hasse matrix $\boldsymbol{H_R}$ because it contains redundant information. Fan [31] gives the transformation formula:

$$\boldsymbol{H_R} = (\boldsymbol{R} - \boldsymbol{I})\text{-}(\boldsymbol{R} - \boldsymbol{I}) * (\boldsymbol{R} - \boldsymbol{I}) \tag{7}$$

where $\boldsymbol{I}$ is the identity matrix and the operator $*$ represents Boolean multiplication, operation by element when applied. The Hasse graph can be drawn according to $\boldsymbol{H_R}$.

Under the condition that preprocessing of the data has been completed, the operation steps represented by the partially ordered set are as follows:

Step 1: Sort the evaluation data from left to right in decreasing weight order to obtain $\boldsymbol{T}(\boldsymbol{D})$.

Step 2: Obtain the matrix $\boldsymbol{Y} = \boldsymbol{T}(\boldsymbol{D}) \cdot \boldsymbol{E}$ from (5).

Step 3: Perform a row-by-row comparison of matrix $\boldsymbol{Y}$, that is, obtain the partial order relationship matrix R from (6) and the Hasse matrix $\boldsymbol{H_R}$ according to (7).

Step 4: Draw a Hasse diagram according to the matrix $\boldsymbol{H_R}$. On this basis, analyze the schemes by sorting, comparing, etc.

III. EXAMPLE ANALYSIS

In Liaoning, Baijiu products sold by large-scale supermarket channels include national brands like Moutai and Wuliangye, but provincial regional brands like FX, Daoguang 25, Lingta, Peking University Cang, Lao Village Chief, and Yushu Qian are more prevalent. As a well-known brand of Liaoning Baijiu, FX Baijiu first entered the large-scale supermarket channel ten years ago. According to sales data for Baijiu in Liaoning, the sales performance of RT-Mart supermarket chain FX Baijiu (41 varieties in total) is relatively high. Compared with other provincial regional brands, FX Baijiu has a certain representativeness. Therefore, taking the 2020 annual sales of FX Baijiu's 18 stores in RT-Mart Liaoning as an example, the sales status of FX Baijiu and the sales target for the next year are analyzed, and decision-making reference is provided for store marketing strategy adjustment and sales performance improvement.

Step 1: The closer the month is to the forecast time, the larger the impact factor and the greater the weight given to the month with the closer time distance, given the recessionary effect of its time series. The ranking size between the adjusted original data source evaluation criteria is December > November > … > January, i.e., $\omega_{12} \geq \omega_{11} \geq \cdots \geq \omega_1 \geq 0$, as shown in Table 1.

Step 2: Carry out the cumulative transformation through (5) (c.f. Table 2) and obtain the comparison relationship matrix according to (6) (c.f. Table 3).

Step 3: The matrix transformation method of the Hasse matrix can be determined from the comparison relationship matrix R, and (6) can be used to determine the Hasse matrix (omitted) of the sales performance level of 18 stores.

Step 4: Draw the Hasse diagram according to the matrix $\boldsymbol{H_R}$ as shown in Figure.

To demonstrate the advantages of the methods in this paper, a comparative analysis is carried out using a simple linear weighting model (SAW), which is the most commonly used of the multi-attribute decision models. Although other decision methods are emerging, the SAW model has been the preferred choice for practical applications [32]. The SAW model, based on the actual decision context, first needs to specify the weights of each evaluation attribute, in this case there is a comparative relationship between the weights of $\omega_{12} \geq \omega_{11} \geq \cdots \geq \omega_1 \geq 0$, so it is assumed that "$\omega_{12} = 0.18, \omega_{11} = 0.15, \omega_{10} = 0.13, \omega_9 = 0.12, \omega_8 = 0.1, \omega_7 = 0.08, \omega_6 = 0.07, \omega_5 = 0.06, \omega_4 = 0.05, \omega_3 = 0.03, \omega_2 = 0.02, \omega_1 = 0.01$," the initial evaluation matrix is shown in the data in Table 1, and then according to the simple linear function $g(a) = \omega_1 a_1 + \omega_2 a_2 + \cdots + \omega_n a_n$ in Theorem 1 to find out the weighting value of each scheme, as a basis for the ranking and comparison of schemes. The final composite score for each shop was obtained as follows:

Comparing the ranking positions of the shops in Table 4 with the Hasse diagram in Figure 1, the

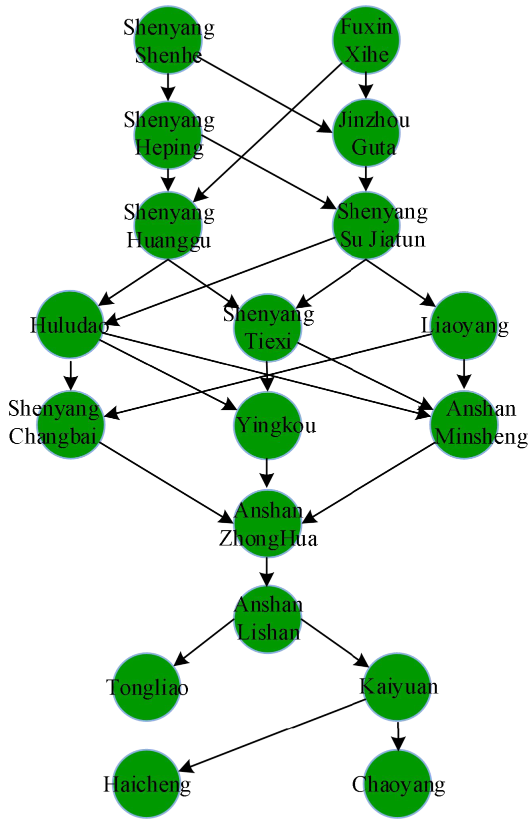

FIGURE 1. Hasse chart of FX Baijiu sales by store in 2020.

advantages of the methodology in this paper can be clearly identified:

(1) Hasse chart stratified to show the sales performance level of the 18 shops

Through the Hasse diagram, one can intuitively understand the comparison of the annual sales revenue of the 18 stores. The higher the income level is, the higher it is. The 18 stores are divided into 9 layers; the first layer is set {Shenyang Shenhe Store, Fuxin Xihe Store}, the second layer is set {Shenyang Heping Store, Jinzhou Ancient Pagoda Store}, and so on. The ninth layer includes {Haicheng Store, Chaoyang Store}. The more layers are set, the greater the difference between the schemes is. The first layer set which includes Shenyang Shenhe Store and Fuxin Xihe Store, is the best. The more stores at the top are comparatively better, whereas the stores at the bottom, Haicheng Store and Chaoyang Store, are the worst. In the SAW method, on the other hand, the supplier ranking of each shop is fixed, and if the same graphical representation is used, the graph is in the form of a single chain, which does not show a multi-level comparison of the sales performance of the 18 shops.

(2) Both deterministic and non-deterministic relation-ships can be reflected in the Hasse diagram

Direct and indirect connection schemes are comparable schemes, reflecting the comparative relationship of high robustness, such as Shenyang Shenhe Store and Shenyang

TABLE 1. Adjusted original data source (10,000 yuan).

Store Name	December	November	October	September	...	April	March	February	January
Shenyang Heping Store	4.396	2.727	3.875	5.726	...	3.784	2.418	9.556	8.396
Jinzhou Guta Store	3.693	2.119	3.657	5.151	...	3.779	2.224	12.087	10.827
Yingkou Store	2.398	2.055	2.036	2.715	...	2.669	1.557	6.261	6.35
Anshan Minsheng Store	2.237	1.776	2.324	4.150	...	2.117	1.707	9.426	5.327
...	...	...	...	...	...	...	...	...	...
Fuxin Hoshe Shop	4.577	1.768	3.330	9.035	...	2.947	2.416	51.321	41.741
Kaiyuan Store	1.013	5.959	1.346	2.107	...	1.039	0.545	4.436	3.500
Shenyang Tiexi Store	2.818	2.048	3.623	2.903	...	2.805	2.066	6.737	6.384
Liaoyang Store	2.568	1.764	3.175	5.155	...	2.842	2.540	8.872	7.855
Shenyang Huanggu Store	3.693	2.320	3.221	4.231	...	3.811	2.427	8.393	6.012
Anshan Lisan Store	1.491	0.877	1.040	1.944	...	1.091	0.998	4.098	3.769

TABLE 2. Accumulate the transformed data.

Store Name	December	November	...	January
Shenyang Heping Store	4.396	2.727+4.396	...	8.396+... +2.727+4.396=51.367
Jinzhou Guta Store	3.693	5.812	...	53.221
Yingkou Store	2.398	4.445	...	32.961
Anshan Minsheng Store	2.237	4.013	...	34.825
...	...	...	...	...
Anshan Lisan Store	1.491	2.368	...	17.926

TABLE 3. Comparison matrix after adjusting the weight order.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Shenyang Shenhe Store	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Fuxin Hoshe Shop	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Shenyang Heping Store	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Jinzhou Guta Store	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
Shenyang Huanggu Store	0	0	0	0	1	0	1	1	0	2	1	1	1	1	1	1	1	1
Shenyang Sujiatun Store	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
Huludao Store	0	0	0	0	0	0	1	0	0	1	1	1	1	1	1	1	1	1
Shenyang Tiexi Store	0	0	0	0	0	0	0	1	0	0	1	1	1	1	1	1	1	1
Liaoyang Store	0	0	0	0	0	0	0	0	1	1	0	1	1	1	1	1	1	1
Shenyang Changbai Store	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	1	1	1
Yingkou Store	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
Anshan Minsheng Store	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
Anshan Zhonghua Store	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
Anshan Lisan Store	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
Tongliao Store	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
Kaiyuan Store	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
Haicheng Store	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Chaoyang Store	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Heping Store, as well as Fuxin Xihe Store and Yingkou Store. The former's annual sales must be better than the latter's under the assumption that the time weight sequence doesn't change. A scheme without path connectivity is an incomparable scheme, which reflects an uncertain comparison relationship. For example, Fuxin Xihe Store and Shenyang Heping Store are incomparable, indicating that the former may be better or worse than the latter, and the ranking relationship between the two may be "flipped."

Only the order of the weights is necessary for the improved OWA operator decision-making method based on partial order set. It does not require specific weight values, it is rather assembled according to the weight space generated by the weight order. This method not only makes assignment easier,

TABLE 4. Calculation results using the SAW method.

Store	Score	Ranking	Store	Score	Ranking
Shenyang Heping Store	3.7463	3	Huludao Store	2.6328	8
Jinzhou Guta Store	3.4148	4	Haicheng Store	0.8336	16
Yingkou Store	2.2374	12	Shenyang Changbai Store	2.2696	11
Anshan Minsheng Store	2.3019	10	Fuxin Hoshe Shop	5.1415	1
Shenyang Shenhe Store	5.1412	2	Kaiyuan Store	1.0892	15
Anshan Zhonghua Store	1.6093	13	Shenyang Tiexi Store	2.5361	9
Chaoyang Store	0.6846	17	Liaoyang Store	2.8649	7
Shenyang Sujiatun Store	3.1248	5	Shenyang Huanggu Store	2.9855	6
Tongliao Store	0.4696	18	Anshan Lisan Store	1.1685	14

but also gives full play to expert experience and resolves weight disputes, making this application strongly convenient and robust.

IV. CONCLUSION

(1) The OWA operator is expanded and upgraded based on the relevant theorems of partially ordered sets. While the original OWA operator requires exact weights, the partially ordered OWA operator only needs the weight order to carry out operations, which can make the operations easier for the original operator and reduce the operation cost. At the same time, the evaluation results can be more robust. The original comparable relationship remains unchanged as long as the weight order structure remains the same no matter how the weights change, so the result will not change much.

(2) The Hasse diagram intuitively illustrates the structural relationship between the schemes. The vertical relationship of the schemes reflects the hierarchical information between them, whereas the horizontal relationship, that is, the scheme within the same layer, reflects the clustering information. From the example analysis, it is clear that the structured information of each store in large commercial supermarkets, which identifies the uncertainty of store comparison, lays the foundation for the subsequent quantitative analysis and identification of key influencing factors. There are numerous types of OWA derivative operators, whether weighted or multiplied, and they can all be combined with partially ordered sets, demonstrating good universality and generalization.

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CONFLICT OF INTEREST STATEMENT

The topic selection, conception, design, writing, data, and statistical analysis of the thesis are not related to the fund support, and the fund support is limited to the article page fee. The data of Baijiu presented in the article is the data after deep processing, which only retains the time variation characteristics of the original data and the data has no practical reference value. The analysis and interpretation of Baijiu data are limited to the use of this method, and its views and conclusions cannot be used in any business management activities, otherwise, risks may arise. The enterprise name in the example is not a real name.

AUTHOR CONTRIBUTIONS

Mingyu Li conceived the idea of this study and focused on the related literature of OWA operators; Mingyu Li and Ruize Xu provided the theory of partially ordered sets and completed all the proofs; Mingyu Li desensitized the data; Qinghua Chen completed the application, analysis, and interpretation of data; Mingyu Li and Ruize Xu wrote the article; and all the authors discussed the results and revised the manuscript.

REFERENCES

[1] R. R. Yager, "On ordered weighted averaging aggregation operators in multi-criteria decision making," *IEEE Trans., Man, Cybern.*, vol. 18, no. 1, pp. 183–190, Jan./Feb. 1988.

[2] P. Eklund and F. Klawonn, "Neural fuzzy logic programming," *IEEE Trans. Neural Netw.*, vol. 3, no. 5, pp. 815–818, Sep. 1992.

[3] R. R. Yager, "OWA neurons: A new class of fuzzy neurons," in *Proc. Int. Joint Conf. Neural Netw. (IJCNN)*, Jun. 1992, pp. 226–231.

[4] R. R. Yager, *Including Importances in OWA Aggregations Using Fuzzy Systems Modeling*. Piscataway, NJ, USA: IEEE Press, 1998.

[5] R. R. Yager and D. P. Filew, "Fuzzy logic controllers with flexible structures," in *Proc. Int. Conf. Fuzzy Sets Neural Netw. Izuka*, 1992, pp. 317–320.

[6] R. R. Yager, "New modes of OWA information fusion," *Int. J. Intell. Syst.*, vol. 13, no. 7, pp. 661–681, Jul. 1998.

[7] M. O'Hagan, "Aggregating template or rule antecedents in real-time expert systems with fuzzy set logic," in *Proc. 22nd Asilomar Conf. Signals, Syst. Comput.*, Oct. 1988, pp. 681–689.

[8] P. A. Schaefer and H. B. Mitchell, "A generalized OWA operator," *Int. J. Intell. Syst.*, vol. 14, no. 2, pp. 123–143, Feb. 1999.

[9] Z. Xu, "A method based on linguistic aggregation operators for group decision making with linguistic preference relations," *Inf. Sci.*, vol. 166, nos. 1–4, pp. 19–30, Oct. 2004.

[10] Z. Xu, "Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment," *Inf. Sci.*, vol. 168, nos. 1–4, pp. 171–184, Dec. 2004.

[11] T. Rubinson, *Communication Networks*. New York, NY, USA: Polytechnic Institute, 1992.

[12] X. F. Wang, "Hybrid weighted integration operator and its application in decision making," *J. Hunan Univ. Technol.*, vol. ED-24, no. 2, pp. 34–38, 2010.

[13] H. Zhang, Q. Chen, and C. Xiong, "Optimal gray combination model based on IOWA operator and its application," *Statist. Decis. Making*, vol. 19, pp. 73–77, Jan. 2017.
[14] L. F. Cai, "Research on scheme preference problem based on LOWA operator," *Experim. Sci. Technol.*, to be published.
[15] C. Zhen, "A method for determining weight investigation in multi-objective decision making," *Syst. Eng. Theory Pract.*, vol. 2, pp. 16–24, Jan. 1987.
[16] X. Youmin, Y. Wang, and Q. Tao, "A discussion of valuation methods and rights of decision indicators," *Syst. Eng. Theory Pract.*, vol. 3, pp. 12–16, Jan. 1986.
[17] W. Chen and J. H. Xia, "Optimal combination of weighting methods for integrating subjective and objective weight information," *Pract. Understand. Math.*, vol. 1, pp. 17–22, Jan. 2007.
[18] M. S. Lu, "Weight coefficients in multi-objective decision making," *Syst. Eng. Theory Pract.*, vol. 4, pp. 77–78, 1986.
[19] Z. Wang, "Multi-objective power coefficient assignment methods and their selection strategies," *Syst. Eng. Electron. Technol.*, vol. 6, pp. 35–41, 1993.
[20] J. Chen, "Comparison matrix construction method and its application in paste hierarchy analysis," *J. Chengdu Univ. Sci. Technol.*, vol. 3, pp. 81–86, 1989.
[21] Y. Han, Y. Xiong, and H. Liao, "A comparative study of multi-objective decision making methods," *J. Univ. Electron. Sci. Technol.*, vol. S2, pp. 283–287, 1996.
[22] Y. Guo, *Comprehensive Evaluation Theory and Methods*. Beijing, China: Science Press, 2002.
[23] Y.-M. Wang, "Multi-indicator decision making and ranking using the disparity maximization method," *Syst. Eng. Electron. Technol.*, vol. 7, pp. 26–28, Jan. 1998.
[24] M. T. Wang, "Discrepancy and mean squared deviation decision method for determining weights in comprehensive evaluation of multiple indicators," *China Soft Sci.*, vol. 8, pp. 100–101, Jan. 1999.
[25] Y. Wang and M. Fu, "Finite scheme multi-objective decision making using infinite scheme multi-objective decision making method," *Control Decis. Making*, vol. 1, pp. 25–29, Jan. 1993.
[26] R. Lahdelma, J. Hokkanen, and P. Salminen, "SMAA-stochastic multi-objective acceptability analysis," *Eur. J. Oper. Res.*, vol. ED-106, no. 1, pp. 137–143, 1998.
[27] L. Z. Yue, Z. J. Zhang, and Y. Yan, "A multi-criteria decision making method for partial order sets with implied weights," *Oper. Res. Manage.*, vol. ED-27, no. 2, pp. 26–31, 2018.
[28] L. Z. Yue and L. Li, "Application of partial order sets to represent TOPSIS models with hard-to-get weights," *Fuzzy Syst. Math.*, vol. ED-31, no. 4, pp. 167–174, 2017.
[29] F. Zhang, L. Z. Yue, and G. Wang, "Research on optimization of PROMETHEE method based on partial order set," *Oper. Res. Manage.*, vol. ED-29, no. 1, pp. 10–16, 2020.
[30] L. Z. Yue, C. Lu, and Z. J. Zhang, "Partial order set representation for comprehensive evaluation models," *Oper. Res. Manage.*, vol. ED-31, no. 5, pp. 101–105, 2022.
[31] Y. Fan, "An analytical approach to Hashtubs," *J. Shanghai Second Inst. Technol.*, vol. 1, pp. 17–22, Jan. 2003.
[32] I. Kaliszewski and D. Podkopaev, "Simple additive weighting—A metamodel for multiple criteria decision analysis methods," *Expert Syst. Appl.*, vol. 54, pp. 155–161, 2016.

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