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RESEARCH ARTICLE

Differential Game Model of Distributed Energy Sharing in Industrial Clusters Based on the Cap-and-Trade Mechanism

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ABSTRACT The sharing economy is a new economic model that can promote the optimal allocation of resources. Distributed energy sharing in the industrial cluster is of great significance for cluster enterprises to improve energy utilization efficiency and reduce carbon emissions. In this paper, we establish a differential game model for energy sharing in the industrial cluster under the cap-and-trade mechanism analyze the equilibrium strategies of core and supporting enterprises in the industrial cluster under three different decision scenarios. We then conduct a comparative analysis of the results, and the effect of the carbon cap and carbon trading prices on energy sharing in the industrial cluster is discussed in detail. Finally, the results of the theoretical analysis were verified through numerical simulations. The conclusions are as follows: 1) The energy sharing synergy, profit for both parties, and total system profit are the highest under the centralized decision, the Stackelberg game is better than the decentralized decision, and the cost-sharing contract can achieve the overall coordination of interests; 2) A higher carbon trading price can increase the low-carbon level of energy consumption, but interestingly, when the carbon cap is below a certain limit, the increase in carbon trading price within a certain time and interval will lead to a decrease in profits for both parties and total system profit; 3) In the Stackelberg game scenario where cost-sharing contracts are introduced, the cost-sharing ratio of core enterprises will increase as the proportion of benefits to core enterprises increases and government subsidies to supporting enterprises decrease.

INDEX TERMS Cap-and-trade mechanism, industrial cluster, energy sharing, cost-sharing contract, differential game.

I. INTRODUCTION

The problem of global warming brought about by excessive carbon emissions has attracted widespread international attention, and a low-carbon economy has become a mainstream trend in world economic development. China proposed at the UN Climate Ambition Summit that China's CO2 emissions per unit of GDP will drop by more than 65% in 2030 compared to 2005, that the share of non-fossil energy in primary energy consumption will reach approximately 25%, and that efforts will be made to achieve carbon neutrality by 2060. To complement the achievement of this goal, China completed the overall design of its carbon emissions trading system in 2017 and officially launched its operations. Under the government's carbon regulation requirements, enterprises are gradually shifting to low-carbon production and reducing their carbon emissions. The direct and indirect emissions from energy consumption account for a high proportion of corporate carbon emissions and distributed low-carbon energy sources such as photovoltaics offer new options for enterprises to improve their low-carbon energy levels. However, as the operation of distributed energy equipment is constrained by the external environment and the enterprises' energy use, the energy supply will be idle for a certain period of time and redundant in terms of energy, and the low carbon level of energy cannot be fully utilized [1]. There is still

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room for efficiency improvement in the energy system as a whole.

In recent years, the sharing economy model has undergone extensive commercial practice in areas such as transportation and accommodation, demonstrating a strong ability to optimize the allocation of resources and enable a mutually beneficial win-win situation for participants [2]. Seventy percent of industrial energy consumption is concentrated in industrial clusters, which have high energy consumption, diversified energy use patterns and large amounts of energy demand for cooling, heating, and electricity, providing favorable conditions for the development of distributed energy sharing based on complementarity and cooperative emission reduction [3]. Under the cap-and-trade mechanism, the implementation of energy sharing by enterprises not only aims to effectively reduce carbon emission costs but also to improve their profitability, which makes the management of energy sharing cooperation in industrial clusters even more important and complex.

On the basis of the above research background, this paper investigates the issue of energy sharing strategies in industrial clusters under the cap-and-trade mechanism. The objective is to explore the core elements affecting energy sharing in industrial clusters and analyze the mechanism of the carbon cap-and-trade mechanism and government energy low-carbon subsidies on energy sharing in industrial clusters. To grasp the key features of the dynamic process of the energy sharing synergy effect, this paper adopts the methodology of differential game theory and numerical simulation. We will discuss the following questions: (1) What is the optimal energy sharing strategy for cluster enterprises in the face of the cap-and-trade mechanism? (2) What is the optimal low-carbon level of energy and the profit of enterprises under different decision-making models? (3) How do energy sharing synergy and enterprises' profits change over time? (4) How do carbon caps, carbon trading prices, and government subsidies affect enterprises' energy sharing strategies?

To address these questions, we propose an improved model with consideration of the cap-and-trade mechanism and study the complex effects of corporate behavior, government subsidies, carbon cap and trading price on energy sharing decisions in industrial clusters in a dynamic scenario. The main contributions of this work are as follows: (1) We consider the impact of cap-and-trade mechanism on the energy sharing decision of industrial cluster enterprises and construct enterprise revenue models under centralized, decentralized and Stackelberg game scenarios with the introduction of cost-sharing contracts, to analyze the optimal energy low-carbon level and profit of enterprises. (2) It integrates energy sharing synergies and optimal decision-making into the proposed model from a dynamic perspective and obtains the optimal trajectory of energy sharing synergy. (3) The effects of carbon cap, carbon trading prices, and government subsidies on energy sharing synergy and total system profit are analyzed using mathematical derivations and numerical simulations to provide a reference for enterprise and government decision-making.

The paper is organized as follows. In Section II, we review the relevant literature. Section III presents the problem description and underlying assumptions. Section IV constructs and solves the Stackelberg game model with centralized decision-making, decentralized decision-making and the introduction of a cost-sharing contract. Section V presents a comparative analysis of the solution results of the models. Section VI performs numerical simulations to validate the analytical results. Section VII draws conclusions.

II. LITERATURE REVIEW

The relevant literature in this paper covers three main areas: (1) cap-and-trade mechanism, (2) energy sharing, (3) corporate cooperation strategies.

(1) Cap-and-trade mechanism. A number of studies have explored the effects of cap-and-trade mechanisms on economic transformation, green technology investment, and supply chain emissions.

In the research on economic transformation and green technology investment under the carbon cap and trade mechanism. Wang et al. discovered that under resource and environmental constraints, China's carbon trading mechanism is positively correlated with the transition to a low-carbon economy [4]. Yang et al. focused on the typical initial allowance allocation rules under a cap-and-trade mechanism and developed mathematical models to solve for optimal green technology investments and product pricing [5]. Li et al. studied the impact of two government subsidies based on a fixed green technology investment cost and the amount of emission reduction on the green decision of the supply chain under the cap-and-trade mechanism. Research was conducted on the supply chain emission reduction strategy under the carbon cap and trade mechanism [6]. Wang and Wu discovered that high initial carbon emissions can negatively affect carbon reduction and product recycling [7]. Shen et al. constructed a supply chain game model based on different dominant types under a hybrid carbon policy of carbon cap-and-trade and carbon tax [8]. Mondal and Giri investigated the competitive and cooperative strategies of retailers in closed-loop green supply chains under government intervention and cap-and-trade policies [9]. Chai et al. explored an appropriate carbon reduction strategy for firms regulated by cap-and-trade [10].

Other studies have explored the effects of cap-and-trade mechanisms on new energy, energy scheduling, and integrated energy systems. Fang et al. attempted to explore the impact of carbon trading mechanisms on new energy applications based on a novel nonlinear energy saving and emission reduction system [11]. Xie and Liu proposed a bilevel multi-objective model for cofiring biomass with coal under carbon cap-and-trade regulation [12]. Zhang et al. developed a computable general equilibrium model to analyze the impact of different ETS quota allocation schemes on the power sector to derive the optimal power sector quota allocation scheme [13]. Jin et al. constructed a stochastic dynamic economic dispatch model based on the uncertainty of wind power and carbon trading [14]. Qu et al. proposed a decentralized optimal multiple energy flow for a large integrated energy system in a carbon trading market [15]. Liu discussed the effects of a two-stage operation of a carbon trading mechanism and refined P2G on the results of the optimal allocation of integrated energy sources [16].

(2) Energy sharing. The development of the sharing economy is of great significance to achieving the goal of sustainable development [17]. The existing research on energy sharing is still in its infancy, but the preliminary research results mainly focus on the following aspects.

Regarding the research aspect of the energy sharing mechanism, Cui et al. proposed a two-stage energy sharing framework, including renewable energy generation, multiple storage units and load transfer, which overcame the impact of market price and the uncertainty of renewable energy and provided a stable energy sharing schedule for producers and retailers [18]. Petri et al. proposed an energy framework based on blockchain, which uses blockchain to support the formation and use of energy communities and support energy exchange in producer communities. Some scholars have focused on the energy sharing mechanism based on P2P [19]. Zhou et al. focus on P2P energy sharing. Based on the multiagent simulation framework, a systematic index system is developed to evaluate the performance of various P2P energy sharing mechanisms [20]. Long et al. proposed a two-stage aggregation control method to achieve P2P energy sharing in a community microgrid. The results show that compared with traditional P2P energy trading, P2P energy sharing can reduce the energy cost of communities by 30% [21]. Cui et al. studied the sustainable energy management of an energy building cluster with distributed transactions and proposed a two-stage energy sharing strategy [22]. Chen et al. proposed an interdisciplinary P2P energy sharing framework and a dynamic price profit model for energy sharing provider [23].

In addition, some scholars have also studied the problem of energy sharing and optimization of scheduling based on shared energy storage, integrated energy systems, and residential photovoltaics. Liu et al. proposed an energy sharing provider equipped with energy storage to facilitate energy sharing among multiple PV producers [24]. Liu et al. proposed a hybrid energy sharing framework with multiple micro-grids and established power and heat sharing models for cogeneration and photovoltaic systems [25]. Monsberger et al. found that within energy communities, both contractors and residents have high margins, the extent of which depends on accounting methods, assumed interest rates, and depreciation timing [26]. Quddus et al. optimized the power flow between commercial buildings, electric vehicle (EV) charging stations, and the grid under the condition of power demand uncertainty and established a two-stage stochastic programming model, which truly captured the different operational constraints between multiple commercial buildings and EV charging stations [27]. Xu et al. proposed a new two-stage game theory framework for residential photovoltaic panel planning and developed an efficient solution based on a descending search algorithm that could significantly improve computational efficiency [28].

(3) Corporate cooperation strategies. Many scholars use game theory to study corporate cooperative strategies such as cooperative innovation, cooperative emission reduction, and cooperation and sharing.

In research on collaborative innovation, Duan et al. proposed a kind of industry-university-research cooperative innovation evolutionary game method based on the GS algorithm for digital media enterprise clusters and proposed the evolutionary stability strategy of cooperative innovation between enterprises and research institutions [29]. Qin et al. discussed the decision-making of knowledge innovation and environmental social responsibility in a multiagent enterprise R&D innovation system composed of core enterprises and satellite enterprises [30]. For research on collaborative emission reduction, Zhou et al. proposed a difference game involving a manufacturer and a retailer in a two-channel supply chain in a low-carbon environment [31]. Li et al. investigated the optimal decision and performance of the CLSCS under four different play structures. The results show that the two-way cooperation structure of cooperation promotion and carbon emission reduction is the best in pricing decisions and carbon emission reduction levels [32]. In the study of collaborative sharing, Li et al. introduced Gaussian white noise into the stochastic evolutionary game model of PPP supply chain knowledge sharing. The results show that enterprise groups with strong knowledge strength are more sensitive to parameter changes than those with weak knowledge strength [33]. Yang et al. studied the selection strategy and information sharing strategy of an e-commerce sales model in a dual-channel supply chain. The results show that e-retailers are willing to share demand forecasting information only when the investment efficiency of the manufacturer's after-sales service is high [34].

In the above literature, although there has been much research on the service mechanism and technical solutions for energy sharing, the research has focused on the optimal scheduling and benefit distribution of energy sharing without considering the impact of energy sharing on enterprises' production decisions under the cap-and-trade mechanism, and few papers have used differential games and other methods to study the cooperative game strategy of energy sharing in a dynamic framework. In contrast, energy sharing in industrial clusters is a time-varying corporate decision-making behavior based on industrial chain cooperation and emission reduction, and there are more complex relationships between participating enterprises. It is difficult to provide theoretical support for the increasing number of energy sharing strategies in practice. The differential game, as a dynamic model for studying the competition between two parties in continuous time, can better portray the process of the synergistic effect

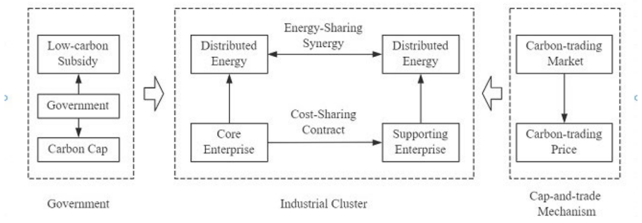

FIGURE 1. Distributed energy sharing in industrial clusters under the cap-and-trade mechanism.

of energy sharing over time through differential equations and solve the problem of energy sharing strategies in industrial clusters in a dynamic framework. Therefore, this paper attempts to introduce differential game theory into the field of energy sharing in industrial clusters and investigate the energy sharing strategies of core and supporting enterprises in industrial clusters based on a dynamic perspective under the carbon cap-and-trade mechanism.

III. PROBLEM DESCRIPTION AND ASSUMPTIONS

A. DESCRIPTION OF THE PROBLEM

During the development of an industrial cluster, an outsourced production network will gradually form with a core enterprise with resource and technology advantages as the core, accompanied by several supporting enterprises. Under the cap-and-trade policy, the core enterprises adopt distributed power generation and other means to enhance low-carbon energy production, and at the same time share low carbon energy with supporting enterprises through microgrids and other technical means to obtain energy-sharing synergy effects, improving energy use efficiency and reducing carbon emissions at the same time. When enterprises engage in energy sharing, the size of the energy sharing synergy depends on how much both parties invest in low carbon levels of energy. The government will subsidize the energy low-carbon costs of enterprises to achieve the carbon neutrality target. In this paper, a simple system consisting of core enterprise and supporting enterprise in an industrial cluster is selected as the research object, and the goal is to maximize the respective interests of the two parties involved. We study the energy sharing decision problem of the enterprises under the centralized, decentralized and Stackelberg game scenarios.as shown in Fig. 1.

B. MODEL ASSUMPTIONS

Assumption 1: The energy low carbon level of core enterprise in an industry cluster is $E_X(t)$; the energy low carbon level of supporting enterprise is $E_Y(t)$. Similar to many scholars, we assume that the energy low carbon cost of both parties is a quadratic function of their respective energy low carbon levels [35], [36]. The energy low carbon cost of both parties at time t is

$$C_X(t) = \frac{r_X}{2} E_X(t)^2 \tag{1}$$

$$C_Y(t) = \frac{r_Y}{2} E_Y(t)^2 \tag{2}$$

where r_X is the cost coefficient of low-carbon energy for core enterprise and r_Y is the cost coefficient of low-carbon energy for core enterprise.

Assumption 2: In the process of energy sharing in industrial clusters, the low-carbon level of energy of enterprises has a positive impact on energy sharing, and through the optimal dispatch of low-carbon energy, it is possible to generate energy sharing synergy effects and improve the efficiency of low-carbon energy use while improving the low-carbon level of energy of both parties. It is assumed that the energy sharing synergy effect decays continuously over time, i.e., there is a natural decay rate. Referring to the assumption of Jørgensen et al. and Wang et al. [37], [38], the differential equation for the energy sharing synergy effect is

$$\dot{K}(t) = \lambda_X E_X(t) + \lambda_Y E_Y(t) - \delta K(t) \tag{3}$$

where, in the initial state, $K(0) = K_0 \geq 0$; λ_X is the coefficient of influence of the low carbon level of energy of the core enterprises on the energy sharing synergy; λ_Y is the coefficient of influence of the low carbon level of energy of the supporting enterprises on the energy sharing synergy; and δ is the natural decay rate of the energy sharing synergy.

Assumption 3: This paper focuses on the low-carbon level of energy of the participating energy-sharing enterprises. To simplify the model, other factors affecting the benefits of energy sharing are not considered and both parties make decisions based on complete information. We assume that the total benefits of energy sharing are

$$S(K(t), t) = S_0 + \beta K(t) \tag{4}$$

where, $S_0\,(S_0 \geq 0)$ indicates the initial revenue of energy sharing.

Assumption 4: The total proceeds from energy sharing are distributed among the sharing enterprises, with the core enterprise receiving a share of the proceeds at $\alpha, \alpha \in (0, 1)$ and the supporting enterprise receiving a share of the proceeds at $1 - \alpha$. The share of proceeds is determined by mutual agreement.

Assumption 5: The initial carbon emissions of an enterprise are G_i, the government allocates a certain carbon emission cap based on the nature of the enterprise Q_i, and any excess emissions above the cap must be purchased from the carbon trading market. The carbon trading price p_e is used as an exogenous variable in the model and is influenced by climate, supply and demand, and the macro environment. The cost of carbon trading for enterprises on both sides is then

$$CT_X(K(t), t) = [G_X - \mu_X K(t) - Q_X]\, p_e \tag{5}$$

$$CT_Y(K(t), t) = [G_Y - \mu_Y K(t) - Q_Y]\, p_e \tag{6}$$

where μ_X is the coefficient of the impact of energy sharing synergy on the emission reductions of core enterprises, and

TABLE 1. Notation and definitions.

Notations	Definition
$E_i(t)$	low carbon levels of energy for enterprise i. $i\in\{X,Y\}$
r_i	The cost coefficient of low carbon energy for enterprise i. $i\in\{X,Y\}$, $r_i>0$
η_i	Government subsidy coefficient for low carbon costs of energy for enterprise i, $i\in\{X,Y\}$, $\eta_i>0$
λ_i	The impact coefficient of low carbon level of enterprise i energy on energy sharing synergy, $i\in\{X,Y\}$, $\lambda_i>0$
μ_i	Coefficient of impact of energy sharing synergy on corporate emission reductions, $i\in\{X,Y\}$, $\mu_i>0$
α	Proportion of core enterprise receiving energy-sharing benefits $\alpha\in(0,1)$
β	Coefficient of impact of energy sharing synergy on revenue. $\beta>0$
δ	Natural rate of decay of energy sharing synergy. $\delta>0$
φ	The cost-sharing ratio of core enterprise X to supporting enterprise Y. $\varphi\in(0,1)$
ρ	Discount rate, $\rho>0$
p_e	Carbon trading price. $p_e>0$
G_i	The initial carbon emissions of enterprise i. $i\in\{X,Y\}$
Q_i	Carbon emission cap granted by the government to enterprise i. $i\in\{X,Y\}$
$K(t)$	Energy sharing synergy at time t
$S(t)$	Total revenue from energy sharing at time t
J_i	The long-term profits of cluster enterprises X and Y. $i\in\{X,Y\}$
J_S	Total system profit

μ_Y is the coefficient of the impact of energy sharing synergy on the emission reductions of supporting enterprises.

Assumption 6: The government provides government subsidies for low-carbon energy sources such as photovoltaic power generation to achieve carbon neutrality and increase the incentive of enterprises to develop low-carbon energy sources. Currently, there are two types of government subsidies for new energy generation: investment subsidies and electricity subsidies, both of which can effectively reduce the low carbon cost of energy for enterprises. Using η_X and η_Y to denote the government subsidy rates for low carbon costs of energy for core and supporting enterprises respectively, the impact of government subsidies on the synergy effect of energy sharing can be examined and provide a basis for the government to formulate subsidy policies.

The parameters involved in the models and their meanings are shown in Table 1.

IV. MODEL CONSTRUCTION AND SOLUTION

A. CENTRALIZED DECISION-MAKING

The centralized decision is denoted by the upper corner marker U, emphasizing the profit maximization of the decision-makers as a whole, i.e., The cluster enterprises cooperate to decide on the low carbon level of energy to maximize the overall profit of both parties, thus enhancing the competitiveness of the chain. The objective function of decision-making at this point is

$$\begin{aligned}J_S^U(K^U) = \max_{E_X\geq 0,E_Y\geq 0}\int_0^\infty & e^{-\rho t}[(S_0+\beta K)-(1-\eta_X)\frac{r_X}{2}E_X^2\\ &-(1-\eta_Y)\frac{r_Y}{2}E_Y^2-(G_X-\mu_X K-Q_X)p_e\\ &-(G_Y-\mu_Y K-Q_Y)p_e]dt \end{aligned}\tag{7}$$

Proposition 1: In the centralized decision-making case, the optimal equilibrium strategy for core enterprise and supporting enterprise is

$$E_X^U = \frac{\lambda_X\left(\beta+p_e\mu_X+p_e\mu_Y\right)}{r_X\left(\rho+\delta\right)\left(1-\eta_X\right)}\tag{8}$$

$$E_Y^U = \frac{\lambda_Y\left(\beta+p_e\mu_X+p_e\mu_Y\right)}{r_Y\left(\rho+\delta\right)\left(1-\eta_Y\right)}\tag{9}$$

The optimal trajectory of energy sharing synergy is

$$K^U = e^{-\rho t}\left(K_0-H^U\right)+H^U\tag{10}$$

of which

$$H^U = \frac{\lambda_X^2\left(\beta+p_e\mu_X+p_e\mu_Y\right)}{\delta r_X\left(\rho+\delta\right)\left(1-\eta_X\right)}+\frac{\lambda_Y^2\left(\beta+p_e\mu_X+p_e\mu_Y\right)}{\delta r_Y\left(\rho+\delta\right)\left(1-\eta_Y\right)}$$

The optimal value of total system profit is.

$$J_S^U = e^{-\rho t}V_S^U\left(K\right)\tag{11}$$

of which

$$\begin{aligned}V_S^U&\left(K\right)\\ =&\ \frac{\beta+p_e\mu_X+p_e\mu_Y}{\rho+\delta}K\\ &+\frac{S_0-p_e(G_X-Q_X)-p_e(G_Y-Q_Y)}{\rho}\\ &+\frac{(\beta+p_e\mu_X+p_e\mu_Y)^2}{2\rho\left(\rho+\delta\right)^2}[\frac{\lambda_X^2}{r_X\left(1-\eta_X\right)}+\frac{\lambda_Y^2}{r_Y\left(1-\eta_Y\right)}]\end{aligned}$$

Proof: See the Appendix.

B. DECENTRALIZED DECISION-MAKING

Decentralized decision making is denoted by the superscript L and emphasises the maximization of the respective profits of the decision-makers, i.e., the cluster enterprises simultaneously decide independently on their respective low carbon levels of energy. The decision objective function at this point is

$$\begin{aligned}J_X^L\left(K^L\right) = \max_{e_X\geq 0}\int_0^\infty & e^{-\rho t}[\alpha(S_0+\beta K)-(1-\eta_X)\frac{r_X}{2}E_X^2\\ &-(G_X-\mu_X K-Q_X)p_e]dt\end{aligned}\tag{12}$$

$$\begin{aligned}J_Y^L\left(K^L\right) = \max_{e_Y\geq 0}\int_0^\infty & e^{-\rho t}[\alpha(S_0+\beta K)-(1-\eta_Y)\frac{r_Y}{2}E_Y^2\\ &-(G_Y-\mu_Y K-Q_Y)p_e]dt\end{aligned}\tag{13}$$

Proposition 2: In the decentralized decision-making case, the optimal equilibrium strategy for core enterprise and supporting enterprise is

$$E_X^L = \frac{\lambda_X\left(\alpha\beta+p_e\mu_X\right)}{r_X\left(\rho+\delta\right)\left(1-\eta_X\right)}\tag{14}$$

$$E_Y^L = \frac{\lambda_Y[\beta(1-\alpha)+p_e\mu_Y]}{r_Y(\rho+\delta)(1-\eta_Y)} \tag{15}$$

The optimal trajectory of energy sharing synergy is

$$K^L = e^{-\rho t}\left(K_0 - H^L\right) + H^L \tag{16}$$

of which

$$H^L = \frac{\lambda_X^2\,(\alpha\beta+p_e\mu_X)}{\delta r_X\,(\rho+\delta)\,(1-\eta_X)} + \frac{\lambda_Y^2[\beta\,(1-\alpha)+p_e\mu_Y]}{\delta r_Y\,(\rho+\delta)\,(1-\eta_Y)}$$

The optimal values for the profit of core enterprise, supporting enterprise and the total system profit are

$$J_X^L = e^{-\rho t}V_X^L(K) \tag{17}$$

$$J_Y^L = e^{-\rho t}V_Y^L(K) \tag{18}$$

$$J_S^L = e^{-\rho t}[V_X^L(K)+V_Y^L(K)] \tag{19}$$

of which

$$\begin{aligned} V_X^L\,(K) = {} & \frac{\alpha\beta+p_e\mu_X}{\rho+\delta}K + \frac{\alpha S_0 - p_e(G_X-Q_X)}{\rho} \\ & + \frac{\lambda_X^2(\alpha\beta+p_e\mu_X)^2}{2\rho r_X\,(1-\eta_X)\,(\rho+\delta)^2} \\ & + \frac{\lambda_Y^2(\alpha\beta+p_e\mu_X)[(1-\alpha)\beta+p_e\mu_Y]}{\rho r_Y\,(1-\eta_Y)\,(\rho+\delta)^2} \\ V_Y^L\,(K) = {} & \frac{(1-\alpha)\beta+p_e\mu_Y}{\rho+\delta}K + \frac{(1-\alpha)S_0 - p_e(G_X-Q_X)}{\rho} \\ & + \frac{\lambda_Y^2[(1-\alpha)\beta+p_e\mu_Y]^2}{2\rho r_Y\,(1-\eta_Y)\,(\rho+\delta)^2} \\ & + \frac{\lambda_X^2(\alpha\beta+p_e\mu_X)[(1-\alpha)\beta+p_e\mu_Y]}{\rho r_X\,(1-\eta_X)\,(\rho+\delta)^2} \end{aligned}$$

Proof: The proof is omitted and the procedure is similar to Proposition 1.

C. THE STACKELBERG GAME WITH THE INTRODUCTION OF COST-SHARING CONTRACT

Suppose that in the Stackelberg primary-secondary game scenario, a cost-sharing contract is introduced to achieve coordination between the core and supporting enterprises of the industry cluster as a whole. Core enterprise is the leader in energy sharing and supporting enterprise is the follower. To increase the motivation of both enterprises to share energy, core enterprise provides incentives to supporting enterprise by offering to bear a proportion of the low carbon energy costs of $\varphi(0 \le \varphi \le 1)$ for the support enterprise. In this hypothesis, core enterprise first decides on its low-carbon energy level and cost-sharing ratio φ, while supporting enterprise decides on its low-carbon energy level after observing the core enterprise's decision. The Stackelberg game is denoted by the superscript R. The objective function of both parties' decisions at this point is

$$\begin{aligned} J_X^R\left(K^R\right) = {} & \max_{e_X\ge 0}\int_0^\infty e^{-\rho t}[\alpha(S_0+\beta K)-(1-\eta_X)\frac{r_X}{2}E_X^2 \\ & - \varphi\frac{r_Y}{2}E_Y^2 - (G_X-\mu_X K - Q_X)p_e]dt \end{aligned} \tag{20}$$

$$\begin{aligned} J_Y^R\left(K^R\right) = {} & \max_{e_Y\ge 0}\int_0^\infty e^{-\rho t}[(1-\alpha)(S_0+\beta K) \\ & - (1-\varphi-\eta_Y)\frac{r_Y}{2}E_Y - (G_Y-\mu_Y K - Q_Y)p_e]dt \end{aligned} \tag{21}$$

Proposition 3: In the Stackelberg game scenario with the introduction of a cost-sharing contract, the core enterprise cost-sharing ratio and the optimal equilibrium strategy for both parties are

$$E_X^R = \frac{\lambda_X\,(\alpha\beta+p_e\mu_X)}{r_X\,(\rho+\delta)\,(1-\eta_X)} \tag{22}$$

$$E_Y^R = \frac{\lambda_Y[\beta(1+\alpha)+2p_e\mu_X+p_e\mu_Y]}{2r_Y(\rho+\delta)(1-\eta_Y)} \tag{23}$$

$$\varphi = \begin{cases} \dfrac{[(3\alpha-1)\beta+2p_e\mu_X-p_e\mu_Y](1-\eta_Y)}{(1+\alpha)\beta+2p_e\mu_X+p_e\mu_Y}, \\ \qquad 1>\alpha>\dfrac{\beta+p_e\mu_Y-2p_e\mu_X}{3\beta} \\ 0, \quad \dfrac{\beta+p_e\mu_Y-2p_e\mu_X}{3\beta}>\alpha>0 \end{cases} \tag{24}$$

The optimal trajectory of energy sharing synergy is

$$K^R = e^{-\rho t}\left(K_0 - H^R\right) + H^R \tag{25}$$

of which

$$\begin{aligned} H^R = {} & \frac{\lambda_X^2\,(\alpha\beta+p_e\mu_X)}{\delta r_X\,(\rho+\delta)\,(1-\eta_X)} \\ & + \frac{\lambda_Y^2[\beta(1+\alpha)+2p_e\mu_X+p_e\mu_Y]}{2\delta r_Y(\rho+\delta)(1-\eta_Y)} \end{aligned}$$

The optimal values of the profits of core enterprise, supporting enterprise and the total profit of the chain system are

$$J_X^R = e^{-\rho t}V_X^R(K) \tag{26}$$

$$J_Y^R = e^{-\rho t}V_Y^R(K) \tag{27}$$

$$J_S^R = e^{-\rho t}[V_X^R(K)+V_Y^R(K)] \tag{28}$$

of which

$$\begin{aligned} V_X^R\,(K) = {} & \frac{\alpha\beta+p_e\mu_X}{\rho+\delta}K + \frac{\alpha S_0 - p_e(G_X-Q_X)}{\rho} \\ & + \frac{\lambda_X^2(\alpha\beta+p_e\mu_X)^2}{2\rho r_X\,(1-\eta_X)\,(\rho+\delta)^2} \\ & + \frac{\lambda_Y^2[(1+\alpha)\beta+2p_e\mu_X+p_e\mu_Y]^2}{8\rho r_Y\,(1-\eta_Y)\,(\rho+\delta)^2} \\ V_Y^R\,(K) = {} & \frac{(1-\alpha)\beta+p_e\mu_Y}{\rho+\delta}K + \frac{(1-\alpha)S_0 - p_e(G_X-Q_X)}{\rho} \\ & + \frac{\lambda_X^2(\alpha\beta+p_e\mu_X)[(1-\alpha)\beta+p_e\mu_Y]}{\rho r_X\,(1-\eta_X)\,(\rho+\delta)^2} \\ & + \frac{\lambda_Y^2[(1-\alpha)\beta+p_e\mu_Y][(1+\alpha)\beta+2p_e\mu_X+p_e\mu_Y]}{4\rho r_Y\,(1-\eta_Y)\,(\rho+\delta)^2} \end{aligned}$$

Proof: See the Appendix.

V. COMPARATIVE ANALYSIS OF RESULTS

Comparing the optimal energy low carbon levels, energy sharing synergy and total profits of the core and supporting enterprises in the above three scenarios respectively, the following inferences can be drawn.

Corollary 1: Comparing the results of the decisions in the three cases shows that

(1) $E_X^U > E_X^L = E_X^R$.

(2) When $1 > \alpha > \frac{\beta + p_e\mu_Y - 2p_e\mu_X}{3\beta}$, $E_Y^U > E_Y^R > E_Y^L, K^U > K^R > K^L, J_X^R > J_X^L, J_Y^R > J_Y^L, J_S^U > J_S^R > J_S^L$.

(3) When $\frac{\beta + p_e\mu_Y - 2p_e\mu_X}{3\beta} > \alpha > 0$, $E_Y^U > E_Y^R = E_Y^L, K^U > K^R = K^L, J_X^R = J_X^L, J_Y^R = J_Y^L, J_S^U > J_S^R = J_S^L$.

Proof: The proof is omitted; it is easy to prove by observing Eqs. (8) - (11), (14) - (19) and (22) - (28).

Corollary 1 shows that under centralized decision-making, the optimal low carbon level of energy, the synergy effect of energy sharing and the total system profit of both parties are the highest, which indicates that centralized decision-making can enhance the motivation of both parties to share energy and the overall efficiency of energy sharing, reduce carbon transaction costs and thus increase the total system profit. However, it is worth noting that although centralized decision-making can maximize total system profit, certain constraints need to be met for the core and supporting enterprise to voluntarily implement centralized decision-making, i.e., the profit shared by both parties under centralized decision-making must be higher than the other two models, namely

$$J_X^U - J_X^R > 0, J_X^U - J_X^L > 0$$
$$J_Y^U - J_Y^R > 0, J_Y^U - J_Y^L > 0$$

In this case, under centralized decision-making, the proportion of each party's incremental profit to the total incremental profit of the system depends on, for example, the negotiating power and access to the information of both parties. In the Stackelberg game scenario of cost-sharing, the cost-sharing ratio of the core enterprise to the supporting enterprise is influenced by the proportion of revenue obtained by the core enterprise α. When α is higher than a certain proportion, the core enterprise bears part of the low-carbon energy costs for the supporting enterprise, and the supporting enterprise gains incentives to improve the low-carbon energy level significantly. That is, compared to decentralized decision-making, cost-sharing contracts have an incentive effect and can increase energy sharing synergy and the respective profits of both parties as well as the total system profit. When α is below a certain percentage, the core enterprise does not share the cost with the supporting enterprise, there is no incentive for the supporting enterprise, and the level of low carbon energy and energy sharing synergy and total system profit for both parties are the same as in the decentralized decision. This indicates that whether the cost-sharing contract can be reached is related to the proportion of revenue obtained by both parties, and when the proportion of revenue of the core enterprise is low, no cost-sharing triggers incentives for the supporting enterprise, resulting in lower energy-sharing synergy as well as system profit.

Corollary 2: In all three decision scenarios, the optimal energy low carbon level of both parties is positively proportional to the coefficient of the impact of energy sharing synergy on benefits (β), the coefficient of the impact of respective energy low carbon levels on energy sharing synergy (λ_i), the coefficient of the impact of energy sharing synergy on respective emission reductions (μ_i), and the coefficient of respective government energy low carbon cost subsidies (η_i) and inversely proportional to the coefficient of respective energy low carbon costs (r_i), the discount rate (ρ), and the natural rate of decay of energy sharing synergy (δ). In the decentralized decision, the optimal energy low carbon level for both parties are proportional to their respective benefits (α and $1 - \alpha$). In the Stackelberg game scenario, the optimal low-carbon energy level for both parties is proportional to the revenue share of the core enterprise (α).

In the three decision scenarios, the total system profit is positively proportional to the coefficient of the impact of energy sharing synergy on benefits (β), the coefficient of the impact of both sides' low carbon energy levels on energy sharing synergy (λ_i), the coefficient of the impact of both sides' energy sharing synergy on their respective emission reductions (μ_i), and the coefficient of government subsidies for low carbon energy costs on both sides (η_i) and inversely proportional to the coefficient of low carbon energy costs on both sides (r_i), the discount rate (ρ), and the natural rate of decay of the synergy (δ).

Proof: The proof is omitted; it is easy to prove by observing Eqs. (8) - (11), (14) - (19) and (22) - (28).

Corollary 2 shows that when the carbon trading price is higher, both parties' energy low carbon levels and energy sharing synergy increase, and an appropriate increase in carbon trading price has a facilitating effect on low carbon energy promotion and energy sharing cooperation; when energy sharing synergy has a greater impact on total revenue (i.e., the coefficient of energy sharing synergy on total revenue β is larger), energy low carbon is more likely to generate energy sharing synergy (i.e., the coefficient of energy sharing synergy on their respective energy low carbon levels is larger) and energy sharing synergy is more likely to reduce carbon emissions. (i.e., the coefficient of the impact of respective energy low carbon levels on energy sharing synergy is larger λ_i), the energy sharing synergy is more likely to reduce carbon emissions (i.e., the coefficient of the impact of energy sharing synergy on respective emission reductions μ_i) and the government subsidy rate is larger η_i, the respective energy low carbon levels, profit of both parties and total system profit will be increased. Cost reduction and revenue enhancement can be achieved by increasing government subsidies and other means to promote energy sharing; under decentralized decision-making, both parties focus on the proportion of revenue each obtains, and the greater they benefit themselves, the more motivated they are to share energy; in the Stackelberg game scenario, the core enterprise bears part of the low carbon cost of energy for the supporting enterprise, and when the proportion of revenue obtained by the core enterprise is above a certain limit, the higher the proportion of the core enterprise's revenue, the greater the proportion of cost sharing for supporting enterprise, and the higher the energy low-carbon level of both parties. Therefore, appropriately increasing the proportion of revenues obtained by the core enterprise will promote the improvement of the low-carbon energy level and the enthusiasm for energy sharing on both sides; when the energy low-carbon cost coefficient (r_i) is larger or the natural decay rate of the synergy effect of energy sharing (δ) is larger, both will have a negative impact on the low-carbon energy level and profits and reduce the enthusiasm of energy sharing.

Corollary 3: In all three decision scenarios, the level of low-carbon energy and energy sharing synergy between the two parties increases with the increase in the carbon trading price (p_e). When the carbon cap is small, the respective profit of both parties and the total system profit decrease and then increase as the carbon trading price (p_e) increases; when the carbon cap is large, the respective profit of both parties and the total system profit increase as the carbon trading price (p_e) increases.

Proof: See the Appendix.

Corollary 3 illustrates that when the carbon trading price p_e is higher, the energy low-carbon level of both parties as well as the synergy effect of energy sharing increases, and an appropriate increase in the carbon trading price has a catalytic effect on improving the energy low-carbon level and reducing the carbon emissions of enterprises; when the carbon cap is small, an increase in the carbon trading price (p_e) within a certain range will reduce the profits of enterprises and inhibit the incentive of energy sharing, which can be adjusted by increasing the carbon cap.

Corollary 4: In the Stackelberg game scenario,

When $1 > \alpha > \frac{\beta + p_e\mu_Y - 2p_e\mu_X}{3\beta}$,

The core enterprise bears part of the low carbon cost of energy for the supporting enterprise, and the core enterprise's share of the cost is proportional to its share of the benefits received and inversely proportional to the rate of government subsidies to the supporting enterprise.

Proof: See the Appendix

Corollary 4 illustrates that the cost-sharing ratio of core enterprises is related to the proportion of revenue distribution and the rate of government subsidies. When the proportion of revenue obtained by core enterprises is within a certain range, the greater the revenue of core enterprises, the higher the cost-sharing ratio. When government subsidies to supporting enterprises are high, core enterprises will reduce their cost-sharing ratio.

VI. NUMERICAL SIMULATION ANALYSIS

To further analyze the results of the three decision situations and to compare the decision results more intuitively, this paper uses MATLAB software to conduct numerical simulations to discuss the long-term scenarios of energy sharing synergy, the trend of total system profit over time, and the effect of parameter changes on the decision variables to verify the validity of the model. Based on the above analysis and assumptions for each parameter, and with reference to Liu et al. and Ji et al. [39], [40], the parameters are assigned as follows: $r_X = 20, r_Y = 15, \lambda_X = 0.6, \lambda_Y = 0.3, \mu_X = 0.8, \mu_Y = 0.6, G_X = 20, G_Y = 10, Q_X = 12, Q_Y = 6, S_0 = 20, p_e = 1, \eta_X = 0.2, \eta_Y = 0.1, \beta = 20, \delta = 0.1, \rho = 0.8, t = 1$.

TABLE 2. Equilibrium results of the game in the three models.

Projects	Centralized decision-making	Decentralized decision-making	Stackelberg game
Energy low carbon level of X	0.8917	0.5333	0.5333
Energy low carbon level of Y	0.5284	0.2123	0.4222
Energy sharing synergy	3.8190	2.1129	2.4597
Cost-sharing ratio	-	-	0.3977
Profit of core enterprise X	-	39.0277	44.3305
Profit of supporting enterprise Y	-	24.3930	28.0822
Total system profit	111.1139	63.4208	72.4127

A. COMPARATIVE ANALYSIS OF THE EQUILIBRIUM RESULTS OF THE GAME

Bringing the above parameters into the relevant propositions, the equilibrium results of the game in the three modes of centralized decision-making, decentralized decision-making, and Stackelberg game are obtained, as shown in Table 2.

From the calculation results in Table 2, it can be seen that, compared with the centralized decision, the energy low-carbon level of the core enterprise decreases by 40.19%, the low-carbon level of the supporting enterprise decreases by 59.81%, the energy sharing synergy decreases by 44.67%, and the total profit of the system decreases by 42.92%, indicating that the decentralized decision limits the enthusiasm of both parties to share energy, resulting in a decrease in the total profit of the system The introduction of a cost-sharing contract is therefore needed for coordination; after the introduction of a cost-sharing contract, compared with decentralized decision-making, the energy-sharing synergy increase by 16.41%, the profits of core enterprise and supporting enterprise increase by 13.59% and 15.12% respectively, and the total system profit increase by 14.18%, which is closer to the level of centralized decision-making, further illustrating the effectiveness of the contracts.

Figure. 2 and Figure. 3 show the curves of the energy sharing synergy and the total system profit over time for the three game scenarios, respectively.

As shown in Figure. 2 and Figure. 3, the energy sharing synergy and total system profit of the industry cluster under the centralized decision are always higher than those under

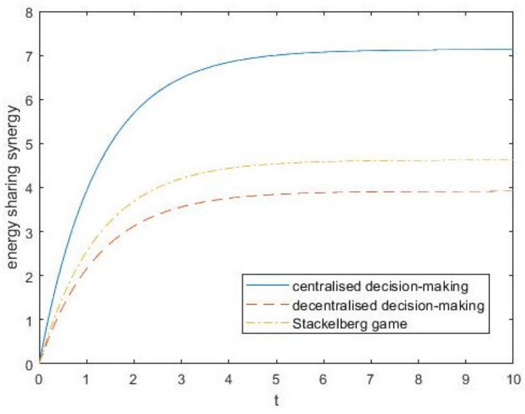

FIGURE 2. Energy sharing synergy for three game scenarios.

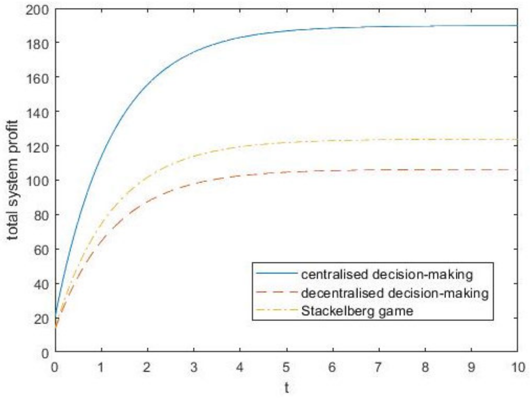

FIGURE 3. Total system profit for three game scenarios.

the decentralized decision over time, while the Stackelberg game with a cost-sharing contract can effectively enhance the equilibrium results of the decentralized decision, which is consistent with the findings of Corollary 1.

B. SENSITIVITY ANALYSIS OF RELEVANT PARAMETERS

To investigate the impact of changes in the low-carbon energy-related parameters on the energy sharing synergy under the three game scenarios, two parameters, the low-carbon cost of energy coefficient r and the low-carbon level of energy on the energy sharing synergy coefficient λ, were selected for sensitivity analysis. Fig. 4 and Fig. 5 examine r_X (the cost coefficient of low-carbon energy for the core enterprise) and λ_X(the impact coefficient of the low-carbon level of energy on the energy sharing synergy for the core enterprise) on the energy sharing synergy respectively.

As seen from Fig. 4, the energy sharing synergy effect increases at the same moment with the parameter λ_X for all three game scenarios (and with λ_Y, simulation omitted), indicating that the easier low carbon levels of energy improve energy sharing efficiency, the more pronounced the energy sharing synergy effect is, which is consistent with the findings of Corollary 2.

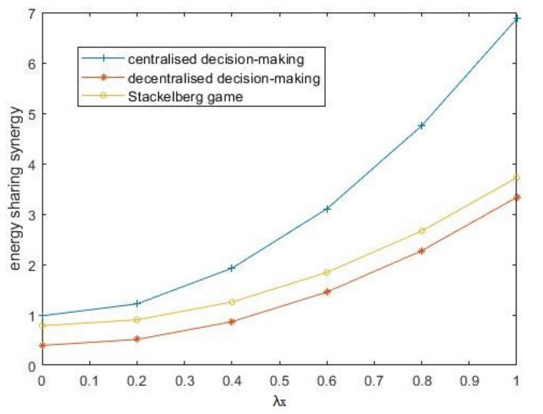

FIGURE 4. Sensitivity analysis of the impact coefficient of the low carbon level of energy on energy sharing synergy.

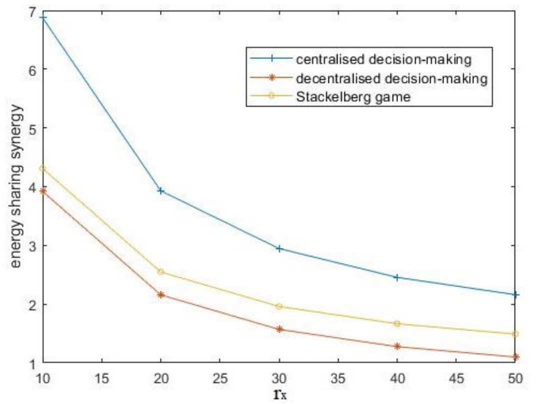

FIGURE 5. Sensitivity analysis of the cost coefficient of low carbon energy.

As seen from Fig. 5, the energy sharing synergy effect decreases at the same moment in all three game scenarios with the increase of the energy low carbon cost coefficient r_X(also decreases with the increase of r_Y, simulation omitted), indicating that the higher the energy low carbon cost, the worse the enthusiasm of industrial cluster energy sharing, which has an opposite effect on the energy sharing synergy effect, which is consistent with the conclusion of Corollary 2.

C. CARBON TRADING PRICE ANALYSIS

Figure. 6 and Figure. 7 examine the effect of the carbon trading price p_e over time on energy sharing synergy and total system profit under centralized decision-making, respectively.

As shown in Fig. 6, at any given moment, the higher the price of carbon trading is, the higher the energy sharing synergy effect of enterprises. This indicates that the higher the price of carbon trading, the more motivated enterprises are to improve their low carbon level of energy, the more willing they are to engage in energy sharing, and the more likely they are to generate energy sharing synergy, which is consistent with the findings of Corollary 3.

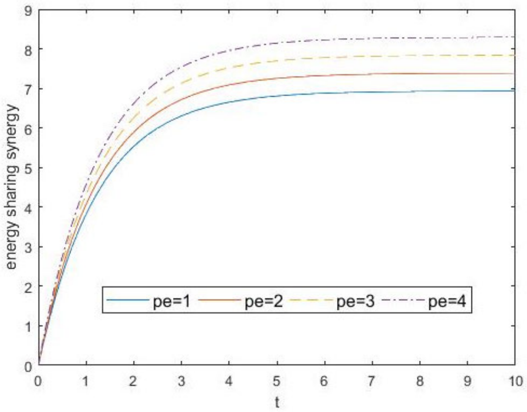

FIGURE 6. Effect of the carbon trading price on energy sharing synergy.

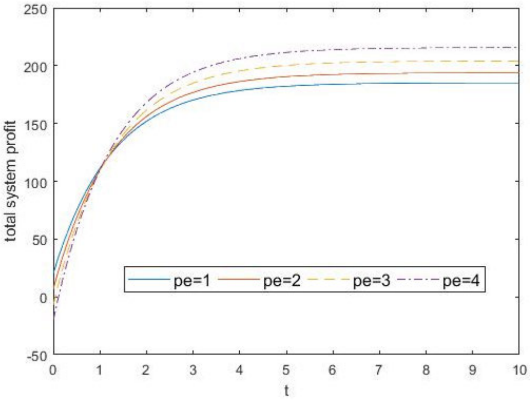

FIGURE 7. Effect of the carbon trading price on total system profit.

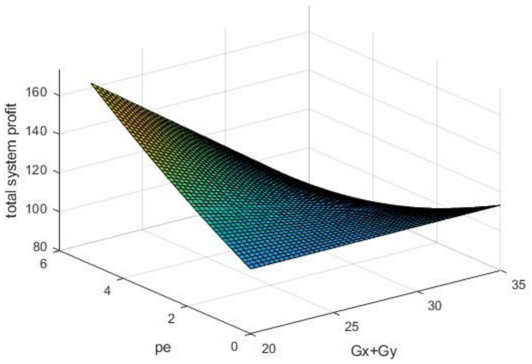

FIGURE 8. Effect of carbon trading price and initial carbon emissions on total system profit.

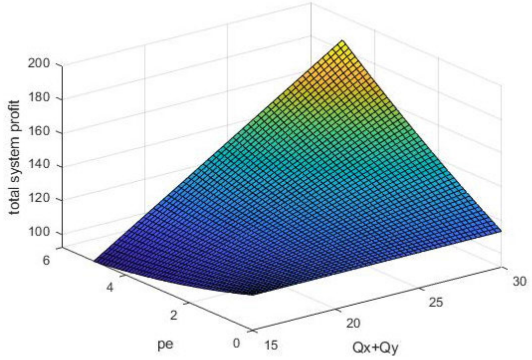

FIGURE 9. Effect of carbon trading price and carbon cap on total system profit.

As shown in Fig. 7, total system profit decreases with an increase in the carbon trading price within a certain time frame after the start and increases with an increase in the carbon trading price beyond a certain time point. This indicates that although an increase in the carbon trading price has a negative impact on the profit of the enterprise in the short term, in the long term, a higher carbon trading price can enhance the profit of the enterprise.

Fig. 8 and Fig. 9 examine the effect of corporate initial carbon emissions, carbon cap and carbon trading price p_e on total system profit under centralized decision-making.

As shown in Fig. 8, when corporate carbon emissions are high, total system profit decreases and then increases with a higher carbon trading price, and when corporate carbon emissions are low, total system profit increases with a higher carbon trading price. This suggests that when corporate carbon emissions are high, a lower carbon trading price may reduce the profits of enterprises and discourage energy sharing. When corporate carbon emissions are low, an increase in the carbon trading price increases the profits of enterprises.

As shown in Fig. 9, total system profit decreases and then increases with a higher carbon trading price when the carbon cap is low and increases with a higher carbon trading price when the carbon cap is high. This suggests that when the carbon cap is low, a lower carbon trading price may reduce corporate profits and discourage energy sharing. When the carbon cap is higher, an increase in the carbon trading price increases the enterprise's profit, which is consistent with the conclusion of Corollary 4.

D. ANALYSIS OF GOVERNMENT SUBSIDY POLICIES

Figure. 10 and Figure. 11 show the effect of the government's low carbon energy subsidy factor for cluster enterprises η_i on energy sharing synergy and total system profit under centralized decision making.

As shown in Fig. 10, the energy sharing synergy effect increases with the increase in government subsidies, indicating that government subsidies can effectively incentivize energy sharing among cluster enterprises. However, the rate of subsidies to core and supporting enterprises will have different degrees of impact on the energy sharing synergy effect. This is a result of the different other relevant parameters of different enterprises.

As shown in Fig. 11, as government subsidies increase, so do total system profit. The core enterprise, as the dominant player in the chain, can profit not only from their government subsidies but also indirectly through government subsidies to supporting enterprises. It can thus be seen that government

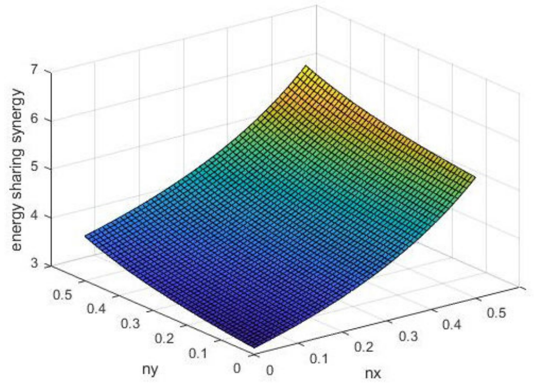

FIGURE 10. Effect of government subsidy rates on energy sharing synergy.

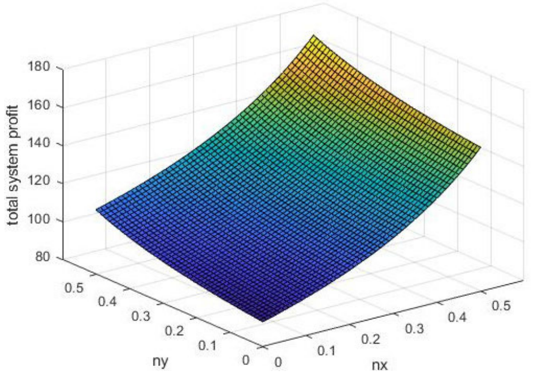

FIGURE 11. Effect of government subsidy rates on total system profit.

subsidies can help to coordinate energy sharing in industrial clusters.

VII. CONCLUSION

The government's low-carbon energy subsidy policy and the carbon trading mechanism have had an important impact on the energy sharing problem in industrial clusters. In this paper, we use differential game theory to study the core and supporting enterprises in an industrial cluster and construct a differential game model with the synergy effect of energy sharing as the state variable. Based on a dynamic perspective, the energy sharing strategies of the core and supporting enterprises in the industry cluster are investigated. The effects of carbon trading price, government subsidies and unilateral incentives of the core enterprises as leaders on energy sharing strategies are explored. The optimal equilibrium strategies of both enterprises, the optimal trajectory of energy sharing synergy over time and the total system profit are discussed in three scenarios: centralized decision-making, decentralized decision-making and the Stackelberg game with the introduction of a cost-sharing contract, and the results are compared and analyzed. Finally, the theoretical derivations are verified by numerical simulation analysis, and the main conclusions are as follows.

(1) The optimal energy low carbon level, energy sharing synergy, and the respective profit and total system profit of both parties are the highest under the centralized decision, which achieves the Pareto optimum. This indicates that the centralized decision can improve the motivation of energy sharing between enterprises and is the optimal decision for energy sharing in industrial clusters. In the Stackelberg game scenario, by introducing a cost-sharing contract, the core enterprise bears part of the low-carbon cost of energy for the supporting enterprise, resulting in an increase in both parties' motivation to share energy and total system profit compared to the decentralized decision, indicating that the incentive mechanism effectively regulates the energy sharing strategy under the decentralized decision. The optimal equilibrium strategies for both parties in all three cases are independent of time, and the obtained energy sharing strategies for industrial clusters have some practical implications for management.

(2) As the impact of the respective low carbon level of energy on the energy sharing synergy effect and the impact of the energy sharing synergy effect on the emission reduction of both parties increased, the respective low carbon level of energy, profit and total system profit of both parties increased. This indicates that the greater the sensitivity of the energy sharing synergy to the low carbon level of energy of the enterprise and the greater the effect of the energy sharing synergy on emission reductions, the more beneficial it is to the energy sharing participating enterprises. As the cost factor and the natural decay rate of the energy sharing synergy increase, the energy sharing synergy, the respective profits of both parties and the total system profit decrease. This suggests that the higher the low carbon cost of energy, the more it discourages energy sharing and reduces the synergy and profits of both parties, resulting in poorer energy sharing, which is consistent with the actual situation. Government subsidies and cost-sharing contracts can be introduced to increase the incentive for energy sharing.

(3) As the carbon trading price increases, the energy low-carbon level and energy sharing synergy between the two parties increases, indicating that a higher carbon trading price can improve the energy low-carbon level of enterprises. However, when the carbon cap is below a certain limit, the increase in the carbon trading price within a certain time and interval will lead to a decrease in the profit of both parties and the total profit of the system, reducing the incentive of the enterprises to share energy. This suggests that the government should set a carbon cap based on factors such as initial carbon emissions and government subsidies. Enterprises should also choose an appropriate energy sharing strategy based on the carbon cap and the market price of carbon trading, which provides a reference for enterprises' energy sharing decisions under the carbon trading mechanism.

(4) In the Stackelberg game scenario where cost-sharing contracts are introduced, the cost-sharing ratio of core enterprises to supporting enterprises will increase as the proportion of benefits to core enterprises increases and government subsidies to supporting enterprises decrease, indicating that the larger the proportion of benefits to core enterprises, the more obvious the improvement to cost-sharing contracts, and

the higher cost subsidies supporting enterprises can obtain from core enterprises, thus making it easier to achieve energy sharing in industrial clusters. Thus, it is easier to achieve energy sharing in the industry cluster. In addition, government subsidies can guide the coordination of energy sharing in industrial clusters, which provides a theoretical reference for the government to develop an energy sharing subsidy strategy.

Further research in this paper could be carried out in the following areas. First, this paper has studied the energy sharing strategy of industry clusters under the consideration of carbon trading mechanisms, but it has not considered the influence of government actions on the energy sharing decision of industry clusters, and government participation can be considered as the next research direction. Second, this paper does not explore in depth the issue of benefit distribution between the two parties under the cost-sharing contract, which does not enable the industry chain to achieve complete coordination and could be achieved by designing a contractual mechanism for centralized decision-making for profit. Finally, this paper only considers energy sharing between a core enterprise and a supporting enterprise in an industrial cluster, and an equilibrium model of energy sharing in an industrial cluster consisting of multiple core and supporting enterprises can be considered in future research.

APPENDIX

Proof of Proposition 1: Referring to the dynamic stochastic control method of solution, note that after moment t, the objective function of the optimal value of the total system profit of both core and supporting enterprises of the industrial cluster is: $J_S^U = e^{-\rho t} V_S^U(K)$, $V_S^U(K)$ For all K $\geq$ 0, the following HJB equation is satisfied.

$$\begin{aligned}\rho V_S^U(K) &= \max_{E_X\geq 0, E_Y \geq 0}[(S_0+\beta K)-(G_X-\mu_X K-Q_X)p_e \\ &\quad -(G_Y-\mu_Y K-Q_Y)p_e-(1-\eta_X)\frac{r_X}{2}E_X^2 \\ &\quad -(1-\eta_Y)\frac{r_Y}{2}E_Y^2+V_S^{U'}(K)(\lambda_X E_X+\lambda_Y E_Y-\delta K)]\end{aligned} \tag{A.1}$$

Solving the first-order condition yields the optimal strategy for both parties.

$$E_X^U = \frac{\lambda_X V_S^{U'}(K)}{r_X(1-\eta_X)} \tag{A.2}$$

$$E_Y^U = \frac{\lambda_Y V_S^{U'}(K)}{r_Y(1-\eta_Y)} \tag{A.3}$$

Substituting Eq. (A.2) and Eq. (A.3) into Eq. (A.1) gives

$$\begin{aligned}V_S^U(K) &= \frac{\beta+p_e\mu_X+p_e\mu_Y-\delta V_S^{U'}}{\rho}K \\ &\quad +\frac{S_0-p_e(G_X-Q_X)-p_e(G_Y-Q_Y)}{\rho} \\ &\quad +V_S^{U'}[\frac{\lambda_x^2}{r_X(1-\eta_X)}+\frac{\lambda_Y^2}{r_Y(1-\eta_Y)}]\end{aligned} \tag{A.4}$$

The analysis of Eq. (A.4) shows that the linear optimal value function with respect to G is a solution to the HJB equation. Let $V_S^U(K) = s_1 K + s_2$, where both s_1 and s_2 are constants, give

$$s_1 = \frac{\beta+p_e\mu_X+p_e\mu_Y}{\rho+\delta} \tag{A.5}$$

$$\begin{aligned}s_2 &= \frac{S_0-p_e(G_X-Q_X)-p_e(G_Y-Q_Y)}{\rho} \\ &\quad +\frac{(\beta+p_e\mu_X+p_e\mu_Y)^2}{2\rho(\rho+\delta)^2}[\frac{\lambda_x^2}{r_X(1-\eta_X)}+\frac{\lambda_Y^2}{r_Y(1-\eta_Y)}]\end{aligned} \tag{A.6}$$

Substituting Eq. (A.5) into Eq. (A.2) and Eq. (A.3) to find the optimal equilibrium strategy for the low carbon level of energy of the core and supporting enterprises under centralized decision-making, as in Eq. (8) and Eq. (9); then, substitute Eq. (8) and Eq. (9) into Eq. (3) to obtain the optimal trajectory of the energy sharing synergy effect, as in Eq. (10); finally, substitute Eq. (A.5) and Eq. (A.6) into $V_S^U(K) = s_1 K + s_2$ and then substitute the result into $J_S^U = e^{-\rho t} V_S^U(K)$ to further find the total system profit, as in Eq. (11). we can obtain Proposition 1.

Proof of Proposition 3: Using the inverse induction method for analytical solution, in the Stackelberg game, the supporting enterprise Y takes the observed low carbon energy level E_X and cost sharing ratio φ of the core enterprise X as the given parameters to decide its own low carbon energy level E_Y transforming the decision problem into a unilateral optimal control problem for the supporting enterprise Y. At this point, assume that there exists a continuous bounded differential function $V_Y^R(K)$ such that the optimal value of the profit of the supporting enterprise Y, $J_Y^R = e^{-\rho t} V_Y^R(K)$, for all K $\geq$ 0, conforms to the following HJB equation.

$$\begin{aligned}\rho V_Y^R(K) &= \max_{E_Y\geq 0}[(1-\alpha)(S_0+\beta K) \\ &\quad -(G_Y-\mu_Y K-Q_Y)p_e-(1-\varphi-\eta_Y)\frac{r_Y}{2}E_Y^2 \\ &\quad +V_Y^{R'}(K)\left(\lambda_X E_X+\lambda_Y E_Y-\delta K\right)\Big]\end{aligned} \tag{A.7}$$

Taking the derivative of E_Y, which can be solved by the first order condition equal to 0, gives

$$E_Y^R = \frac{\lambda_Y V_Y^{R'}(K)}{r_Y(1-\varphi-\eta_Y)} \tag{A.8}$$

The core enterprise will rationally predict the optimal decision E_Y of the supporting enterprise. Therefore, the core enterprise will decide its own optimal low carbon level of energy E_X and cost sharing ratio φ according to the rational reflection of the supporting enterprise to satisfy its own interest maximization. Assume that there exists a continuous bounded differential function $V_X^R(K)$ such that the optimal value of the profit of core enterprise $J_X^R = e^{-\rho t} V_X^R(K)$, for all K ≥ 0, conforms to the following HJB equation.

$$\begin{aligned}\rho V_X^R(K) = \max_{E_X \geq 0}[&\alpha(S_0+\beta K)-(G_X-\mu_X K-Q_X)p_e\\&-(1-\eta_X)\frac{r_X}{2}E_X^2-\varphi\frac{r_Y}{2}E_Y^2\\&+V_X^{R'}(K)(\lambda_X E_X+\lambda_Y E_Y-\delta K)].\end{aligned}\tag{A.9}$$

Substituting Eq. (A.8) into Eq. (A.9) and deriving E_X and φ, the first-order condition equals zero to solve for

$$E_X^R = \frac{\lambda_X V_X^{R'}(K)}{r_X(1-\eta_X)}\tag{A.10}$$

$$\varphi = \frac{(1-\eta_Y)\,[2V_X^{R'}(K)-V_Y^{R'}(K)]}{2V_X^{R'}(K)+V_Y^{R'}(K)}\tag{A.11}$$

Substituting Eq. (A.8), Eq. (A.10) and Eq. (A.11) into Eq. (A.7) and Eq. (A.9), and assuming that the linear optimal value function with respect to K is a solution of the HJB equation based on order characteristics, such that $V_X^R(K) = p_1K + p_2$, $V_Y^R(K) = q_1K + q_2$, where p_1, p_2, q_1, q_2 are constants, leads to

$$p_1 = \frac{\alpha\beta + p_e\mu_X}{\rho+\delta}\tag{A.12}$$

$$\begin{aligned}p_2 = {}&\frac{\alpha S_0 - p_e(G_X-Q_X)}{\rho} + \frac{\lambda_X^2(\alpha\beta+p_e\mu_X)^2}{2\rho r_X(1-\eta_X)(\rho+\delta)^2}\\&+\frac{\lambda_Y^2[(1+\alpha)\beta+2p_e\mu_X+p_e\mu_Y]^2}{8\rho r_Y(1-\eta_Y)(\rho+\delta)^2}\end{aligned}\tag{A.13}$$

$$q_1 = \frac{(1-\alpha)\beta + p_e\mu_Y}{\rho+\delta}\tag{A.14}$$

$$\begin{aligned}q_2 = {}&\frac{(1-\alpha)S_0 - p_e(G_X-Q_X)}{\rho}\\&+\frac{\lambda_X^2(\alpha\beta+p_e\mu_X)[(1-\alpha)\beta+p_e\mu_Y]}{\rho r_X(1-\eta_X)(\rho+\delta)^2}\\&+\frac{\lambda_Y^2[(1-\alpha)\beta+p_e\mu_Y][(1+\alpha)\beta+2p_e\mu_X+p_e\mu_Y]}{4\rho r_Y(1-\eta_Y)(\rho+\delta)^2}\end{aligned}\tag{A.15}$$

Substituting Eq. (A.12) and Eq.(A.14) into Eq. (A.8), Eq. (A.10) and Eq. (A.11), we can find the equilibrium strategies of core enterprise and supporting enterprise under the Stackelberg game (E_X, E_Y) and the proportion of costs shared by the core enterprise for supporting enterprise φ, as in Eq. (22), Eq. (23), Eq. (24); then the results are substituted into Eq. (3) to obtain the optimal trajectory of energy sharing synergy, as in Eq. (25); finally, Eq. (A.12), Eq. (A.13), Eq. (A.14), Eq. (A.15) are taken into $V_X^R(K) = p_1K + p_2$ and $V_Y^R(K) = q_1K + q_2$, and the resulting $V_X^R(K)$ and $V_Y^R(K)$ are taken into $J_X^R = e^{-\rho t}V_X^R(K)$ and $J_Y^R = e^{-\rho t}V_Y^R(K)$ to obtain the respective profits of the two parties and the total profit of the system, as in Eq. (26), Eq. (27) and Eq. (28). We can obtain Proposition 3.

Proof of Corollary 3: Taking centralized decision-making as an example, according to Eq. (8), (9) and (10) on the first-order conditions of p_e

$$\frac{\partial E_X^U}{\partial p_e} > 0,\quad \frac{\partial E_Y^U}{\partial p_e} > 0,\quad \frac{\partial K^U}{\partial p_e} > 0\tag{A.16}$$

The low carbon level of energy and the energy sharing synergy of both parties increase with the price of carbon trading (p_e).

Based on the total system profit function in Eq. (10), the first- and second-order conditions on the carbon trading price p_e are obtained as

$$\begin{aligned}\frac{\partial J_S^U}{\partial p_e} = {}&2(p_e+\beta)[\frac{(1-e^{-\rho t})}{\delta}+\frac{1}{2\rho}]\\&\times[\frac{\lambda_x^2}{r_X(1-\eta_X)}+\frac{\lambda_Y^2}{r_Y(1-\eta_Y)}]\frac{(\mu_X+\mu_Y)^2}{(\rho+\delta)^2}\\&-\frac{(G_X+G_Y-Q_X-Q_Y)}{\rho}\end{aligned}\tag{A.17}$$

$$\begin{aligned}\frac{\partial^2 J_S^U}{\partial p_e^2} = {}&2[\frac{(1-e^{-\rho x})}{\delta}+\frac{1}{2\rho}][\frac{\lambda_x^2}{r_X(1-\eta_X)}+\frac{\lambda_Y^2}{r_Y(1-\eta_Y)}]\\&\times\frac{(\mu_X+\mu_Y)^2}{(\rho+\delta)^2}\end{aligned}\tag{A.18}$$

From $\frac{\partial^2 J_S^U}{\partial p_e^2} > 0$, the total system profit is a convex function with respect to the carbon trading price p_e, and making its first-order condition equal to zero yields the most unfavorable carbon trading price. (A.19), as shown at the top of the next page.

Since the carbon trading price $p_e \geq 0$, when $p_e^* > 0$, i.e.

$$\begin{aligned}Q_X+Q_Y < {}&G_X+G_Y\\&-2\rho\beta[\frac{(1-e^{-\rho t})}{\delta}+\frac{1}{2\rho}][\frac{\lambda_x^2}{r_X(1-\eta_X)}\\&+\frac{\lambda_Y^2}{r_Y(1-\eta_Y)}]\frac{(\mu_X+\mu_Y)^2}{(\rho+\delta)^2},\end{aligned}$$

Total system profit decreases and then increases as the carbon trading price p_e increases.

When $p_e^* \leq 0$, i.e.

$$\begin{aligned}Q_X+Q_Y \geq {}&G_X+G_Y\\&-2\rho\beta[\frac{(1-e^{-\rho t})}{\delta}+\frac{1}{2\rho}][\frac{\lambda_x^2}{r_X(1-\eta_X)}\\&+\frac{\lambda_Y^2}{r_Y(1-\eta_Y)}]\frac{(\mu_X+\mu_Y)^2}{(\rho+\delta)^2},\end{aligned}$$

Total system profit rises with higher negotiated trading prices p_e. We can obtain Corollary 3.

Proof of Corollary 4: According to Eq. (24), finding the first order derivatives of φ with respect to α and η_Y, we can find

$$\frac{\partial\varphi}{\partial\alpha} = \frac{2[(2+3\alpha)\beta+2p_e\mu_X+2p_e\mu_Y](1-\eta_Y)}{[(1+\alpha)\beta+2p_e\mu_X+p_e\mu_Y]^2} > 0\tag{A.20}$$

$$\frac{\partial\varphi}{\partial\eta_Y} = -[(3\alpha-1)\beta+2p_e\mu_X-p_e\mu_Y] < 0\tag{A.21}$$

$$p_e^* = \frac{(G_X + G_Y - Q_X - Q_Y) - 2\rho\beta\left[\frac{(1-e^{-\rho t})}{\delta} + \frac{1}{2\rho}\right]\left[\frac{\lambda_X^2}{r_X(1-\eta_X)} + \frac{\lambda_Y^2}{r_Y(1-\eta_Y)}\right]\frac{(\mu_X+\mu_Y)^2}{(\rho+\delta)^2}}{2\rho\left[\frac{(1-e^{-\rho t})}{\delta} + \frac{1}{2\rho}\right]\left[\frac{\lambda_X^2}{r_X(1-\eta_X)} + \frac{\lambda_Y^2}{r_Y(1-\eta_Y)}\right]\frac{(\mu_X+\mu_Y)^2}{(\rho+\delta)^2}} \tag{A.19}$$

The cost-sharing ratio of core enterprises φ is positively proportional to their share of revenues α and inversely proportional to the rate of government subsidies to supporting enterprises η_Y. We can obtain Corollary 4.

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