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RESEARCH ARTICLE

Analyzing the Vulnerabilities in a Transshipment Network: A Bilevel Programming Approach

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ABSTRACT As one of the most common types of networks, transshipment networks face certain risks in their daily operations. This risk either stems from a deliberate intervention in the process or from an unpredictable natural effect. The owner of the network normally aims to minimize the total cost of the transportation, while a rival or an opponent might attempt to intervene in the process and cause an increase in the costs. Unpredictable natural setbacks, on the other hand, can also have a negative impact on the network necessitating decision-makers to be more vigilant, especially for the worst-case consequences of network disruptions. The problem can be designed as a bilevel mathematical model. In this study, we develop a bilevel max-min model considering that the arcs in a transshipment network are liable to a disruption imposed by an intervenor. We present an original model with a novel solution approach to identify vulnerabilities in the arcs of a given transshipment network. The results reveal the most vulnerable arcs in the network, which have the highest negative impact on transportation costs when a disruption occurs. The decision-makers can use such results to analyze the risk in distribution networks such as electrical grids and develop associated measures to decrease the cost and increase the flexibility of the distribution systems. The model also enables decision makers with a methodical understanding of worst cases if the distribution network is interrupted for some reason.

INDEX TERMS Network safety, bilevel mathematical modeling, linear programming, integer programming.

I. INTRODUCTION

In today‘s competitive and clash-likely environment, disagreements quite often emanate from conflicting goals or interests of at least two sides (players). Where there is a conflict, there is usually an intervention by an opponent with some certain means. It is likely that one of the two players having a conflicting goal with the other tries to maximize his/her own goal whereas the other attempts to minimize it with some limited resources. The problem has a vast area of applications, especially in network modeling and optimization.

With many different types, networks play an important role in our lives with increased intensity as never seen before. We see the network models in transportation, communication, projects, electricity distribution, manufacturing, etc. Ahuja et al. [1] give a comprehensive analysis of the networks in terms of applications, modeling, algorithms as well as the fundamental theoretical aspects. The more the networks get infiltrated into our lives, the more the scientists get involved in their vulnerabilities and protection of them with the interdiction of these models also gaining an impetus. The deliberate intervention of an opponent in the network process may have a huge impact on it and needs to be well analyzed by both the interfering side and the network owner.

In general, there are two players in a network interdiction problem, the network owner, and the opponent. The network owner tries to run the network optimally and is the defending side (defender). The opponent on the other hand tries to impede the network process and is the attacking one (interdictor/intervenor/attacker). With limited resources at hand, the interdictor inflicts some negative effects on the network arcs so that the objective of the network owner maximally deteriorates. Both players have enough information about the other concerning the courses of action to be selected. The attacker is the one who usually acts first to choose his/her strategy (decision). Then, the defender responds with his/her own decision to optimize the network process. Each player

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tries to optimize his/her objective, which is in complete conflict with the other's, in a sequential manner. The problem can be seen as a Stackelberg Game Von Stackelberg [34], Simaan and Cruz [28]. Depending on who moves first, the problem is also called an attacker-defender or defender-attacker model as a bilevel optimization programming model. Drawing the interest of scientists since the 1960s, the problem has been studied in many different fields and types of networks.

Wood [37] gives an overview of the chronological emergence of interdiction problems. Initial studies of the field including Wollmer [35], Corely and David [9], Malik et al. [22], Ball et al. [4], investigate the most vital arcs in a network problem. Interdiction models come up usually in Min-Max or Max-Min type bilevel models and are classified as NP-Hard from a computational complexity point of view Ball et al. [4]. Smith and Song give various interdiction models and algorithms including emerging methodologies and techniques for future studies. Kleinert et al. give a survey on mixed integer programming techniques in bilevel optimization. Solution procedures usually require using advanced linear programming approaches like duality Wood [36], Kleinert et al. [21], Xie and Aros-Vera [38] and mixed integer programming techniques like decomposition Enayaty-Ahangar et al. [10], Grimm [14], Ambrosius [3].

From a practical point of view, we see interdiction applications in various fields to handle the risk and figure out optimal strategies in case of deliberate intervention in a system. To mention a few examples, Salmerón et al. [27] analyzes the security of an electric grid under a terrorist attack, Brown et al. [6] provide a model for ballistic missile defense, and Brown et al. [7] propose a model to defend critical infrastructure. In another study, Brown et al. [8] study how to delay a nuclear weapons project. Roy et al. introduce some interdiction strategies to prevent the spread of infectious diseases. Using the idea, a more efficient project management approach is proposed by Kasimoglu and Akgün [18] in a competitive environment in which a deliberate intervention in the process is possible.

There are some studies quantitatively identifying the risk and suggesting mitigating strategies in the networks having potential life and environmental risks. Among those, Yin et al. [39], Utku and Soyöz [33], and Utku and Erol [32] propose a hazardous material (hazmat) transportation network design model to decrease the risk in transportation. Zhang et al. [40] suggest a model to determine the risk factors of hazmat crashes on a macro level and develop appropriate measures for improving hazmat transportation safety. Ke and Verma [19] propose a framework to identify terminal criticality and associated mitigation strategies in cases of random disruptions of rail intermodal terminals. Sun et al. [31] develop a model to minimize the total deprivation cost of casualties under the risk of disruptions in temporary medical centers. Zhou et al. [41] provide a method for investigating vulnerabilities in airport transportation networks facing disruptions such as Covid-19. Insuasty-Reina et al. [16] aim at the identification and prioritization of operational risks in a logistics network for the recovery of waste cooking oil. Alsokhiry et al. [2] apply a game theory-based method to deal with the strategy of offenders having malware attacks in a wireless sensor network-based transportation system.

While transportation safety is considered from a couple of different perspectives in the aforementioned studies, it is hardly ever possible to see in the literature the effect of a deliberate intervention on a transshipment network, which is especially critical in distribution systems like electric grids. Sperstad et al. [30] describe four different aspects of the security of an electricity distribution system, one of which is supply reliability. The reliability of supply is usually attributed to the failures in the system components as in the studies done by Guner and Ozdemir [12], Escalara et al. [11], Jimada and Teh [17], Ourahou et al. [24]. The existence of an external intervenor is not considered in the mentioned studies. However, in today's competitive environment, distribution systems may also be influenced by the intervention of rival companies and opponents. This will also enable decision-makers to analyze the effect of a deliberate intervention on the system as well as the worst cases when disruptions occur. Adding a new facet to the problem we can summarise the main contributions of our study as follows:

- For the first time, a generalized bilevel programming model integrating external intervention to discover the vulnerabilities of transshipment networks is presented.
- An exact solution procedure is developed with a novel solution approach combining the duality theory and the property of the coefficient matrix A.
- A generic model applicable in a broad area of study including electricity distribution networks is provided.
- Worst-case outcomes in a transshipment network due to disruptions in certain segments are uncovered.

In the remainder of the study, we first give a transshipment network model (TNM), show that the coefficient matrix A of the model is totally unimodular, and provide the dual of the model. Then, considering a deliberate intervention on our transshipment network we develop a bilevel (Max-Min) interdiction model (Bi-TNM). Next, using duality and total unimodularity concepts we transform our bilevel model into an ultimate standard mixed-integer programming (MIP) model. Finally, we apply our model in various scenarios in which different sizes of networks and various numbers of targeted disruptions by an intervener are contemplated.

II. A TRANSSHIPMENT NETWORK MODEL

Let G = (N, SA) be a directed graph denoting a transshipment network where N represents the node set (stations) and SA represents the set of existing arcs. A generic representation of the network is given in Figure 1 assuming that the distribution is done at two stages for the sake of simplicity. Here the arcs indicate existing distribution routes. The requirements at demand points must be met from existing supply points through the transshipment stations. We assume that the transportation is done from one or more supply points i through stations j and k in sequence to meet the demands at points l.

We first give a mathematical formulation for a balanced transshipment problem, in which total supply and total demand are equal to each other. The indices, sets, and parameters needed for modeling purposes are described and then the transshipment model, Model TNM, is presented as a mixed integer programming model below.

Sets and Indices

- i supply points, $(1, 2, \ldots, I)$.
- j Stage 1 stations, $(1, 2, \ldots, J)$.
- k Stage 2 stations, $(1, 2, \ldots, K)$.
- l demand points, $(1, 2, \ldots, L)$.
- SA Set of all existing arcs (lines) $\{(i, j)\text{'s}, (j, k)\text{'s}, (k, l)\text{'s}\}$.

Decision Variables

- x_{ij} quantity of transportation between i and j.
- x_{jk} quantity of transportation between j and k.
- x_{kl} quantity of transportation between k and l.

Parameters

- D_l Demand at arrival station l.
- S_i The capacity of supply station i.
- C_{ij} Cost coefficient between i and j.
- C_{jk} Cost coefficient between j and k.
- C_{kl} Cost coefficient between k and l.

Thus, the transshipment network model that minimizes the total distribution cost, **Model TNM,** is given as follows.

$$\textbf{\textit{Min}} \sum\nolimits_{(i,j)\in SA} C_{ij}x_{ij} + \sum\nolimits_{(j,k)\in SA} C_{jk}x_{jk} + \sum\nolimits_{(k,l)\in SA} C_{kl}x_{kl} \tag{1}$$

$$\sum\nolimits_{j|(i,j)\in SA} x_{ij} = S_i \quad \forall i \tag{2}$$

$$\sum\nolimits_{i|(i,j)\in SA} x_{ij} = \sum\nolimits_{k|(j,k)\in SA} x_{jk} \quad \forall j \tag{3}$$

$$\sum\nolimits_{j|(j,k)\in SA} x_{jk} = \sum\nolimits_{l|(k,l)\in SA} x_{kl} \quad \forall k \tag{4}$$

$$\sum\nolimits_{k|(k,l)\in SA} x_{kl} = D_l \quad \forall l \tag{5}$$

$$x_{ij}, x_{jk}, x_{kl} \geq 0 \textit{ and integers.} \tag{6}$$

In Model TNM, the objective function (1) minimizes the total cost of the distribution in our network. Constraint (2) defines supply amounts for each supply point i. Equations (3) and (4) are balance constraints guaranteeing that whatever quantity of material coming to a transition station j *or* k is transported to another station at the subsequent stage. Constraint (5) specify the required quantity of transportation needed at each demand point l. Constraint (6) is used to identify decision variables.

The Model TNM can be equivalently stated in a more standard way with some slight adjustments in constraints 3, 4, and 5 as given below.

$$\textbf{\textit{Min}} \sum\nolimits_{(i,j)\in SA} C_{ij}x_{ij} + \sum\nolimits_{(j,k)\in SA} C_{jk}x_{jk} + \sum\nolimits_{(k,l)\in SA} C_{kl}x_{kl} \tag{7}$$

$$\sum\nolimits_{j|(i,j)\in SA} x_{ij} = S_i \quad \forall i \tag{8}$$

$$\sum\nolimits_{k|(j,k)\in SA} x_{jk} - \sum\nolimits_{i|(i,j)\in SA} x_{ij} = 0 \quad \forall j \tag{9}$$

$$\sum\nolimits_{l|(k,l)\in SA} x_{kl} - \sum\nolimits_{j|(j,k)\in SA} x_{jk} = 0 \quad \forall k \tag{10}$$

$$-\sum\nolimits_{k|(k,l)\in SA} x_{kl} = -D_l \quad \forall l \tag{11}$$

$$x_{ij}, x_{jk}, x_{kl} \geq 0 \textit{ and integers.} \tag{12}$$

A. TOTAL UNIMODULARITY OF THE COEFFICIENT MATRIX A OF THE MODEL

Matrix A is said to be totally unimodular if every square submatrix has determinant 0, + 1, or −1. These matrices have been investigated in relation to transportation problems by Heller as well as to combinatorial mathematical problems see Pulleyblank et al. [25]. There are also other ways of characterizing a totally unimodular matrix. If a matrix $A = [a_{ij}]$ having entries 0, +1, or −1 has no more than two nonzero entries in each column, and if $\sum_i a_{ij} = 0$ whenever column j contains two nonzero entries, then A is totally unimodular Nemhauser and Wolsey [23].

It can be shown that the constraint matrix A corresponding to the constraint equations (8)-(12) is a totally unimodal matrix. We can write the system in (8)-(12) as a matrix-vector notation, $A\mathbf{x} = \mathbf{b}$,where A is the constraint matrix of the system consisting of $(I + J + K + L)$ rows and $(IxJ) + (JxK) + (KxL)$ columns; $\mathbf{x}$ is a column vector with $(IxJ)+(JxK)+(KxL)$ elements and $\mathbf{b}$ is a column vector with $(I + J + K + L)$ elements. One can partition the constraint matrix A as given in Figure 2.

In Figure 2, A_1 is a submatrix consisting of $(I + J)$ rows and (IxJ) columns; A_2 is a submatrix consisting of $(J + K)$ rows and (JxK) columns and A_3 is a submatrix consisting of $(K + L)$ rows and (KxL) columns as given in Figures 2.a, 2.b and 2.c respectively.

On the other hand, O_1 in Figure 2 is a zero matrix consisting of $(K + L)$ rows and $(I \times J)$ columns; O_2 is a zero matrix consisting of I rows and $(J \times K)$ columns, O_3 is a zero matrix consisting of L rows and $(J \times K)$ columns, and O_4 is a zero matrix consisting of $(I + J)$ rows and $(K \times L)$ columns.

Notice that the matrices A_1, A_2, and A_3 has exactly two nonzero entries +1 or −1 in each column, and the sum of the entries in each column is exactly zero. Let us show these submatrices are totally unimodular, indeed.

Proposition 1: The matrices $O_{\mathrm{k}}(k : 1, 2, 3, 4)$ and $A_i(i : 1, 2, 3)$ in the partitioned coefficient matrix A, are totally unimodular.

Proof: Since all entries of a zero matrix are zero, the determinant of any submatrix of it is zero. Hence, O_{k} is totally unimodular.

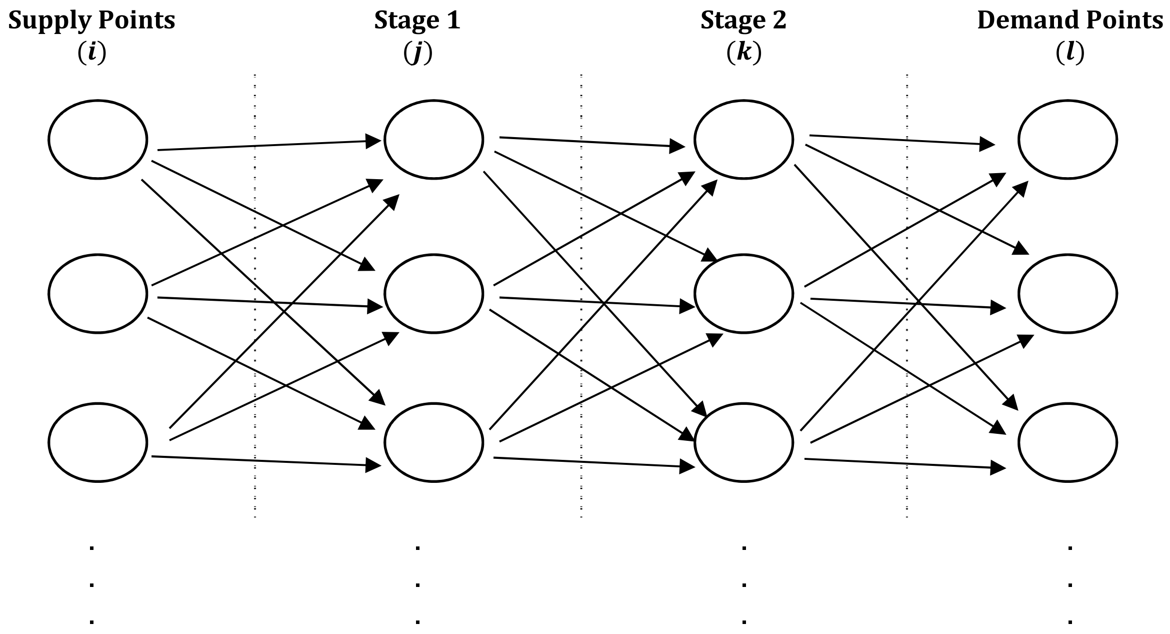

FIGURE 1. Network representation of transshipment process.

Let us now prove that A_i is totally unimodular. Let B be an $n \times n$ submatrix of A. We will show that $det\ (B) = -1, 0,$ or 1 using proof-by-induction on n.

Basis Step: If $n = 1$, then B has a single entry. Since all entries of A_i are $-1, 0,$ or 1, then $det(B) = -1, 0$ or 1.

Hypothesis Step: Assume that every $(n-1) \times (n-1)$ square submatrix B of A has determinant -1, 0, or 1 for any $n \geq 2$.

Inductive Step: Assume B is an $n \times n$ submatrix of A. Since all entries of A_i are $-1, 0,$ or 1, and every column of A_i has at most two non-zero entries of value -1 and 1, we have the following cases regarding determinants of B:

Case 1: If B is a square submatrix with at least one zero column, then $det(B) = 0$.

Case 2: Assume B has at least one column with exactly one non-zero entry equal to 1. Assume the j^{th} column of B has a single entry of value 1, say the entry is in i^{th} row. If we obtain the submatrix C from B by deleting the i^{th} row and j^{th} column of B, then by cofactor expansion we have,

$$det(B) = (-1)^{i+j} \det(C).$$

Since C is an $(n-1) \times (n-1)$ square submatrix of A, then $det(C)$ is $-1, 0,$ or 1 by the assumption in the Hypothesis Step. That is why, $det(B)$ is also $-1, 0,$ or 1.

Case 3: Assume B has at least one column with exactly one non-zero entry equal to -1. Assume the j^{th} column of B has a single entry of value -1, say the entry is in i^{th} row. If we obtain the submatrix C from B by deleting the i^{th} row and j^{th} column of B, then by cofactor expansion we have

$$det(B) = (-1)^{i+j+1} \det(C).$$

Since C is an $(n-1) \times (n-1)$ square submatrix of A, then $det(C)$ is $-1, 0,$ or 1 by the assumption in the Hypothesis Step. That is why, $det(B)$ is also $-1, 0,$ or 1.

Case 4: Assume every column of B has at least one column with exactly two non-zero entries -1 and 1. Since all nonzero rows of B are distinct from each other and every column contains exactly one -1 and 1, then the sum of all rows of B is zero. And this means that the row space of B is linearly dependent. Hence, $det(B) = 0$.

Since A is a partitioned matrix of the totally unimodular matrices 0_k and A_i, then A is a totally unimodular matrix. Hence, we can give the following result.

Proposition 2: The coefficient matrix A of the model TNM is totally unimodular.

This property of matrix A enables us to relax the integrality constraint of the model without any loss of generality since any optimal solution to the problem is guaranteed to be an integer in the linearly relaxed version of the model (Bazaraa et al. [5] (2005)). That is, the variables of the model (x_{ij}, x_{jk}, x_{kl}) can be treated as linear ones instead of integers enabling us comfortably to exploit linear programming approaches.

B. THE DUAL OF THE MODEL

Since the transshipment model can be treated as a linear model, we can take the dual of the model and use the properties of duality. Having said that, we define the dual variables of the model as given follows.

Dual Variables

$$\sum_{j|(i,j)\in SA} x_{ij} = S_i \quad \forall i \qquad w_i \qquad (13)$$

$$\sum_{k|(j,k)\in SA} x_{jk} - \sum_{i|(i,j)\in SA} x_{ij} = 0 \quad \forall j \qquad w_j \qquad (14)$$

$$\sum_{l|(k,l)\in SA} x_{kl} - \sum_{j|(j,k)\in SA} x_{jk} = 0 \quad \forall k \qquad w_k \qquad (15)$$

$$-\sum_{k|(k,l)\in SA} x_{kl} = -D_l \quad \forall l \qquad w_l \qquad (16)$$

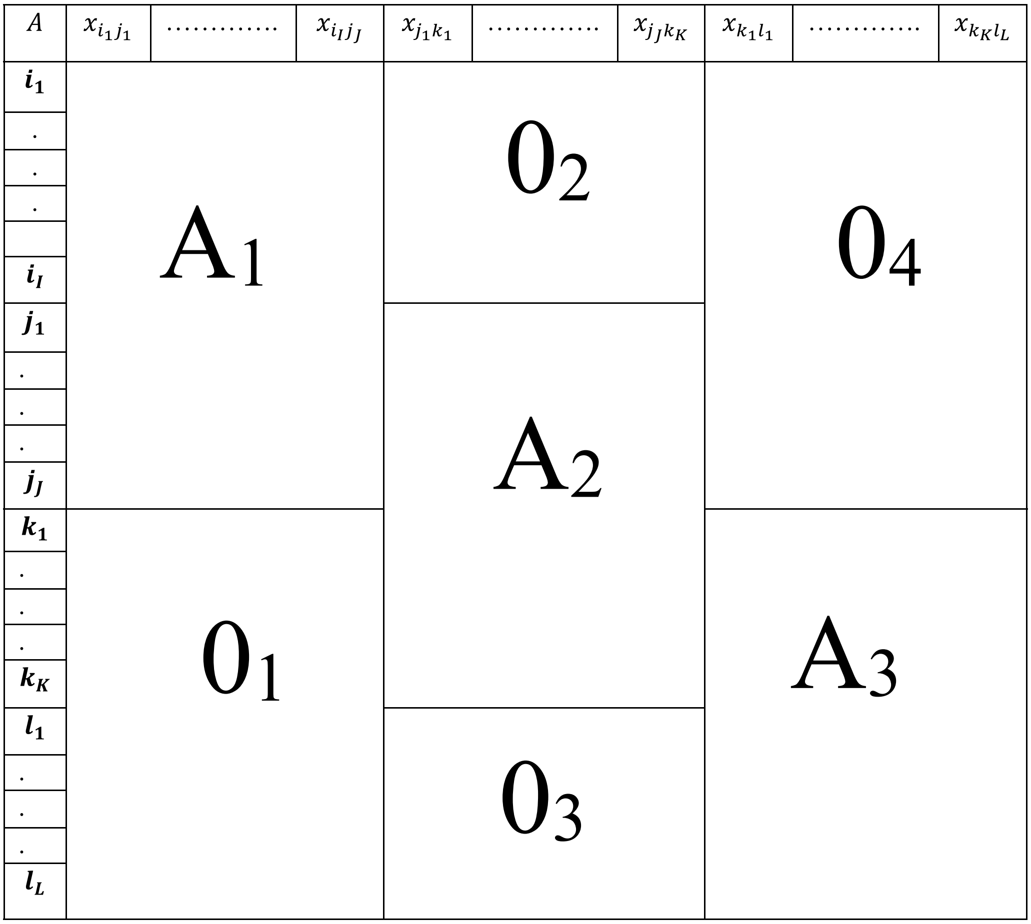

FIGURE 2. **Partitioning of the coefficient matrix A. (a) Submatrix A_1. (b) Submatrix A_2. (c) Submatrix A_3.**

The resulting dual model of Model TNM, DTNM, is given below.

$$\textbf{\textit{Max}} \sum\nolimits_i S_i w_i - \sum\nolimits_l D_l w_l \tag{17}$$

$$w_i - w_j \leq C_{ij} \quad \forall (i,j) \in SA \tag{18}$$

$$w_j - w_k \leq C_{jk} \quad \forall (j,k) \in SA \tag{19}$$

$$w_k - w_l \leq C_{kl} \quad \forall (k,l) \in SA \tag{20}$$

$$w_i, w_j, w_k, w_l \ unrestricted \quad \forall i,j,k,l \tag{21}$$

III. FORMULATING AN INTERDICTION MODEL FOR THE TRANSSHIPMENT NETWORK

Suppose that an opponent (interdictor/attacker) tries to impede the distribution in our network so that the cost of delivery is maximized. The opponent has the chance to disable transportation in some parts (arcs) of the network with his/her limited interdiction resources just before the distribution begins. The opponent is clever enough to choose the best interdiction plan (i.e., which arcs of the network to disable with available interdiction resources) in order to increase the distribution cost. Network users, on the other hand, need to figure out their best distribution plan taking into account that the network arcs are liable to intervention and thus some of them might not be used. The problem can be considered a classical Stackelberg Game Von Stackelberg [34], Simaan and Cruz [28]. The following assumptions apply to the problem.

- Interdiction is conducted just before the distribution starts.
- Both the interdictor and network owner know each other‘s most likely move.
- The interdictor is the one who moves first.

Note that here the opponent as the side acting first holds the initiative. Even though we consider a deliberate intervention in the network, the situation can also be thought of as the worst-case scenario for the network owner if the interruption stems from natural grounds. In this respect, we first propose a bilevel attack model and then develop this bilevel model into a standard one-level model that can be solved through regular optimization techniques.

A. A BILEVEL INTERDICTION MODEL FOR TRANSSHIPMENT NETWORK

Below we present a bilevel interdiction model for the transshipment network model, Bi-TNM, with the new parameters and decision variables introduced for interdiction.

	$x_{i_1j_1}$	$x_{i_1j_2}$	$x_{i_1j_3}$	…	$x_{i_1j_{J-1}}$	$x_{i_1j_J}$	$x_{i_2j_1}$	$x_{i_2j_2}$	$x_{i_2j_3}$	…	$x_{i_2j_{J-1}}$	$x_{i_2j_J}$	…	$x_{i_Ij_1}$	$x_{i_Ij_2}$	$x_{i_Ij_3}$	…	$x_{i_Ij_{J-1}}$	$x_{i_Ij_J}$
i_1	1	1	1	…	1	1	0	0	0	…	0	0	…	0	0	0	…	0	0
i_2	0	0	0	…	0	0	1	1	1	…	1	1	…	0	0	0	…	0	0
.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
i_I	0	0	0	…	0	0	0	0	0	…	0	0	…	1	1	1	…	1	1
j_1	-1	0	0	…	0	0	-1	0	0	…	0	0	…	-1	0	0	…	0	0
j_2	0	-1	0	…	0	0	0	-1	0	…	0	0	…	0	-1	0	…	0	0
.	.	.	-1	…	.	.	.	.	-1	…	.	.	…	.	.	-1	…	.	.
.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
.	.	.	.	…	-1	.	.	.	.	…	-1	.	…	.	.	.	…	-1	.
j_J	0	0	0	…	0	-1	0	0	0	…	0	-1	…	0	0	0	…	0	-1

(a)

	$x_{j_1k_1}$	$x_{j_1k_2}$	$x_{j_1k_3}$	…	$x_{j_1k_{K-}}$	$x_{j_1k_K}$	$x_{j_2k_1}$	$x_{j_2k_2}$	$x_{j_2k_3}$	…	$x_{j_2k_{K-}}$	$x_{j_2k_K}$	…	$x_{j_Jk_1}$	$x_{j_Jk_2}$	$x_{j_Jk_3}$	…	$x_{j_Jk_{K-}}$	$x_{j_Jk_K}$
j_1	1	1	1	…	1	1	0	0	0	…	0	0	…	0	0	0	…	0	0
j_2	0	0	0	…	0	0	1	1	1	…	1	1	…	0	0	0	…	0	0
.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
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.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
j_J	0	0	0	…	0	0	0	0	0	…	0	0	…	1	1	1	…	1	1
k_1	-1	0	0	…	0	0	-1	0	0	…	0	0	…	-1	0	0	…	0	0
k_2	0	-1	0	…	0	0	0	-1	0	…	0	0	…	0	-1	0	…	0	0
.	.	.	-1	…	.	.	.	.	-1	…	.	.	…	.	.	-1	…	.	.
.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
.	.	.	.	…	-1	.	.	.	.	…	-1	.	…	.	.	.	…	-1	.
k_K	0	0	0	…	0	-1	0	0	0	…	0	-1	…	0	0	0	…	0	-1

(b)

	$x_{k_1l_1}$	$x_{k_1l_2}$	$x_{k_1l_3}$	…	$x_{k_1l_{L-}}$	$x_{k_1l_L}$	$x_{k_2l_1}$	$x_{k_2l_2}$	$x_{k_2l_3}$	…	$x_{k_2l_{L-}}$	$x_{k_2l_L}$	…	$x_{k_Kl_1}$	$x_{k_Kl_2}$	$x_{k_Kl_3}$	…	$x_{k_Kl_{L-}}$	$x_{k_Kl_L}$
k_1	1	1	1	…	1	1	0	0	0	…	0	0	…	0	0	0	…	0	0
k_2	0	0	0	…	0	0	1	1	1	…	1	1	…	0	0	0	…	0	0
.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
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.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
k_K	0	0	0	…	0	0	0	0	0	…	0	0	…	1	1	1	…	1	1
l_1	-1	0	0	…	0	0	-1	0	0	…	0	0	…	-1	0	0	…	0	0
l_2	0	-1	0	…	0	0	0	-1	0	…	0	0	…	0	-1	0	…	0	0
.	.	.	-1	…	.	.	.	.	-1	…	.	.	…	.	.	-1	…	.	.
.	.	.	.	…	.	.	.	.	.	…	.	.	…	.	.	.	…	.	.
.	.	.	.	…	-1	.	.	.	.	…	-1	.	…	.	.	.	…	-1	.
l_L	0	0	0	…	0	-1	0	0	0	…	0	-1	…	0	0	0	…	0	-1

(c)

FIGURE 2. ***(Continued.)*** **Partitioning of the coefficient matrix A. (a) Submatrix A_1. (b) Submatrix A_2. (c) Submatrix A_3.**

Parameters

- r_{ij} interdiction resources needed to disable arc $(i - j)$.
- r_{jk} interdiction resources needed to disable arc $(j - k)$.
- r_{kl} interdiction resources needed to disable arc $(k - l)$.
- TIR total interdiction resources available.
- M an arbitrarily large positive number.

Decision Variables

- y_{ij} binary decision variable for interdictor to indicate whether arc $(i-j)$ is interdicted or not (1 if interdicted, 0 otherwise).
- y_{jk} binary decision variable for interdictor to indicate whether arc $(j - k)$ is interdicted or not (1 if interdicted, 0 otherwise).

y_{kl} binary decision variable for interdictor to indicate whether arc $(k-l)$ is interdicted or not (1 if interdicted, 0 otherwise).

In light of these definitions, we can formulate the **Bi-TNM** model as follows:

$$\underset{y_{ij},y_{jk},y_{kl}}{\textbf{\textit{Max}}}\ \underset{\in\Omega}{\textbf{\textit{Min}}}\ \sum\nolimits_{(i,j)\in SA} C_{ij}x_{ij}+\sum\nolimits_{(j,k)\in SA} C_{jk}x_{jk} + \sum\nolimits_{(k,l)\in SA} C_{kl}x_{kl} \quad (22)$$

$$\sum\nolimits_{j|(i,j)\in SA} x_{ij} \le S_i \quad \forall i \quad (23)$$

$$\sum\nolimits_{k|(j,k)\in SA} x_{jk} - \sum\nolimits_{i|(i,j)\in SA} x_{ij} = 0 \quad \forall j \quad (24)$$

$$\sum\nolimits_{l|(k,l)\in SA} x_{kl} - \sum\nolimits_{j|(j,k)\in SA} x_{jk} = 0 \quad \forall k \quad (25)$$

$$-\sum\nolimits_{k|(k,l)\in SA} x_{kl} = -D_l \quad \forall l \quad (26)$$

$$x_{ij} \le M(1-y_{ij}) \quad \forall (i,j)\in SA \quad (27)$$

$$x_{jk} \le M(1-y_{jk}) \quad \forall (j,k)\in SA \quad (28)$$

$$x_{kl} \le M(1-y_{kl}) \quad \forall (k,l)\in SA \quad (29)$$

$$x_{ij}, x_{jk}, x_{kl} \ge 0 \quad \forall i,j,k,l \quad (30)$$

where Ω denotes the feasible set in the decision space of the opponent defined by the following constraints:

$$\sum\nolimits_{(i,j)\in SA} r_{ij}y_{ij} + \sum\nolimits_{(j,k)\in SA} r_{jk}y_{jk} + \sum\nolimits_{(k,l)\in SA} r_{kl}y_{kl} \le TIR \quad (31)$$

$$y_{ij}, y_{jk}, y_{kl} \in \{0,1\} \quad \forall i,j,k,l \quad (32)$$

In Model Bi-TNM the inner minimization model belongs to the network operator while the outer maximization problem belongs to an opponent with the intention of intervening in the distribution network with some limited resources. Normally, the objective function (22) representing the total distribution cost is minimized by the network runner. Here, an external player in conflict with the network runner tries to maximize the function. The courses of action for the opponent are defined by constraint (31) and constraint (32) in the outer model. Constraint (31) defines the intervenor‘s resource limitation, and constraint (32) describes binary decision variables y‘s, which indicate whether a specific segment is disabled by the opponent. The other constraints (23)-(30) in the inner model are similar to those in Model TNM.

From an efficient modeling perspective, one can set big M in the model to the sum of the supply as in (33) since the maximum value that x_{ij}, x_{jk}, x_{kl} can take, is guaranteed to be less than the total supply.

$$M = \sum\nolimits_i S_i \quad (33)$$

B. DEVELOPING A STANDARD MIXED INTEGER PROGRAMMING INTERDICTION MODEL FOR THE TRANSSHIPMENT NETWORK

Bi-TNM given above can be converted into a standard one-level model through duality. We have already given the dual of TNM in Section II. Here we will give the dual of the inner part of Bi-TNM. Notice that the inner model continues to be totally unimodular, even though there are some new constraints (27,28,29) added to the model associated with the possible interdicted arcs. The coefficient matrix of these additional constraints (27,28,29) forms an identity matrix. With these constraints, the resulting coefficient matrix can be given in the form of (34).

$$\begin{bmatrix} A \\ I \end{bmatrix} \quad (34)$$

Given any matrix A that is totally unimodular, the matrix given in (34) is totally unimodular too Nemhauser and Wolsey [23]. Thus, we can still treat the variables as linear ones, instead of integers without any loss of generality.

To take the dual of the inner part, we first introduce the following dual variables in addition to the dual variables already introduced in Section II for the associated constraints of the model.

	Dual Variables	
$x_{ij} \le M(1-y_{ij}) \quad \forall (i,j)\in SA$	v_{ij}	(35)
$x_{jk} \le M(1-y_{jk}) \quad \forall (j,k)\in SA$	v_{jk}	(36)
$x_{kl} \le M(1-y_{kl}) \quad \forall (k,l)\in SA$	v_{kl}	(37)

Fixing the variable y‘s in the model and taking the dual of the inner part we come up with a nonlinear programming (NLP) version of the interdiction model, **NLP TNM-I**, as given below.

$$\textbf{\textit{Max}} \sum\nolimits_i S_i w_i - \sum\nolimits_l D_l w_l + \sum\nolimits_{(i,j)\in SA} M(1-y_{ij})v_{ij} + \sum\nolimits_{(j,k)\in SA} M(1-y_{jk})v_{jk} + \sum\nolimits_{kl} M(1-y_{kl})v_{kl} \quad (38)$$

$$w_i - w_j + v_{ij} \le C_{ij} \quad \forall (i,j)\in SA \quad (39)$$

$$w_j - w_k + v_{jk} \le C_{jk} \quad \forall (j,k)\in SA \quad (40)$$

$$w_k - w_l + v_{kl} \le C_{kl} \quad \forall (k,l)\in SA \quad (41)$$

$$\sum\nolimits_{(i,j)\in SA} r_{ij}y_{ij} + \sum\nolimits_{(j,k)\in SA} r_{jk}y_{jk} + \sum\nolimits_{(k,l)\in SA} r_{kl}y_{kl} \le TIR \quad (42)$$

$$w_i \le 0; \quad w_j, w_k, w_l \ unrestricted \quad \forall i,j,k,l;$$
$$v_{ij}, v_{jk}, v_{kl} \le 0 \quad \forall\,(i,j),(j,k),(k,l);$$
$$y_{ij}, y_{jk}, y_{kl} \in \{0,1\} \quad \forall i,j,k,l \quad (43)$$

Notice that the objective function (38) of NLP TNM-I contains nonlinear terms (i.e., $(y_{ij}v_{ij})$, $(y_{jk}v_{jk})$, $(y_{kl}v_{kl})$). To solve the problem with linear programming techniques, we can linearize the objective function by introducing some new variables and constraints. Let the variables q_{ij}, q_{jk}, q_{kl} defined as below in an attempt to eliminate the nonlinearity in function (38).

$$q_{ij} = (1 - y_{ij})v_{ij} \quad (44)$$
$$q_{jk} = (1 - y_{jk})v_{jk} \quad (45)$$
$$q_{kl} = (1 - y_{kl})v_{kl} \quad (46)$$

Note that the variables q_{ij}, q_{jk}, q_{kl} cannot be positive since $v_{ij}, v_{jk}, v_{kl} \leq 0$ and y_{ij}, y_{jk}, y_{kl} are binary. Depending on the value of the variables y_{ij}, y_{jk}, y_{kl}, we have two cases for each of the equations (44), (45), and (46). Below we consider the cases for equation (44) as an example.

If the variable $y_{ij} = 1$ in (44), then the associated new variable introduced for linearization, q_{ij}, will take the value of 0 in the objective function (38) as shown below.

$$M\left(1 - y_{ij}\right) v_{ij} = Mq_{ij} = 0 \rightarrow q_{ij} = 0 \quad (47)$$

Considering that M is an arbitrarily large positive number and $q_{ij} \leq 0$, (47) can be expressed with the following constraint to be used in the model.

$$q_{ij} \geq -M(1 - y_{ij}) \quad (48)$$

Notice that when $y_{ij} = 0$, constraint (48) turns out to be a redundant one.

On the other hand, if the variable $y_{ij} = 0$ in (44), then the associated new variable, q_{ij}, will take the value of v_{ij} in the objective function (38) as given in (49).

$$M\left(1 - y_{ij}\right) v_{ij} = Mq_{ij} = Mv_{ij} \rightarrow q_{ij} = v_{ij} \quad (49)$$

Statement (49) can be equivalently given by the following constraint in the model.

$$q_{ij} \leq v_{ij} + My_{ij} \quad (50)$$

Note that if y_{ij} is 0, q_{ij} will have an upper value, v_{ij}, set by constraint (50). Since the model is a maximization problem and there is no other constraint for q_{ij}, it is guaranteed that q_{ij} will take a value at its upper bound, v_{ij}. Notice also that when $y_{ij} = 1$, constraint (50) turns out to be a redundant one.

We have developed the constraints (48) and (50) to satisfy equation (44) to linearize the nonlinear term associated with q_{ij} in the objective function (38). Similarly, we can also develop the constraints to satisfy equations (45) and (46) so that the nonlinear terms associated with q_{jk} and q_{kl} are also linearized (i.e. constraints 58,59,60,61).

With the new variables and constraints used for linearization, we end up with the following standard MIP interdiction model for the transshipment network, **Model TNM-I**.

$$\textbf{\textit{Max}} \sum_i S_i w_i - \sum_l D_l w_l + \sum_{(i,j)\in SA} Mq_{ij} + \sum_{(j,k)\in SA} Mq_{jk} + \sum_{kl} Mq_{kl} \quad (51)$$

$$w_i - w_j + \mathbf{v_{ij}} \leq C_{ij} \quad \forall (i,j) \in SA \quad (52)$$

$$w_j - w_k + \mathbf{v_{jk}} \leq C_{jk} \quad \forall (j,k) \in SA \quad (53)$$

$$w_k - w_l + v_{kl} \leq C_{kl} \quad \forall (k,l) \in SA \quad (54)$$

$$\sum_{(i,j)\in SA} r_{ij}y_{ij} + \sum_{(j,k)\in SA} r_{jk}y_{jk} + \sum_{(k,l)\in SA} r_{kl}y_{kl} \leq TIR \quad (55)$$

$$q_{ij} \geq -M(1 - y_{ij}) \quad \forall (i,j) \quad (56)$$

$$q_{ij} \leq v_{ij} + My_{ij} \quad \forall (i,j) \quad (57)$$

$$q_{jk} \geq -M(1 - y_{jk}) \quad \forall (j,k) \quad (58)$$

$$q_{jk} \leq v_{jk} + My_{jk} \quad \forall (j,k) \quad (59)$$

$$q_{kl} \geq -M(1 - y_{kl}) \quad \forall (k,1) \quad (60)$$

$$q_{kl} \leq v_{kl} + My_{kl} \quad \forall (k,1) \quad (61)$$

$$w_i \leq 0; \quad w_j, w_k, w_l \; unrestricted \quad \forall i,j,k,l;$$
$$v_{ij}, v_{jk}, v_{kl} \leq 0 \quad \forall (i,j), (j,k), (k,l);$$
$$q_{ij}, q_{jk}, q_{kl} \leq 0 \quad \forall (i,j), (j,k), (k,l);$$
$$y_{ij}, y_{jk}, y_{kl} \in \{0,1\} \quad \forall i,j,k,l \quad (62)$$

To summarize, in our eventual model TNM-I, the objective function (51) is now linear and indicates the total distribution cost, which is maximized by an opponent's move. Constraints (52), (53), and (54) result from the dual of the inner part of Bi-TNM. Constraint (55) ensures not to exceed the total interdiction resource available. Constraints (56), (57), (58), (59), (60), and (61) are used for the linearization of the nonlinear terms in the objective function. (62) defines the variables in the model. In this regard, w_i, w_j, w_k, w_l and v_{ij}, v_{jk}, v_{kl} are the associated dual variables. The variables q_{ij}, q_{jk}, q_{kl} are the ones used for linearization purposes. y_{ij}, y_{jk}, y_{kl} indicate whether an interdiction exists on the associated arc (line) of the transshipment network. These segments with y_{ij}, y_{jk}, y_{kl} values of 1 will naturally be the most vulnerable parts of the distribution network. As an MIP our eventual model can easily be solved through standard optimization packages.

IV. EXPERIMENTAL STUDY

In this part of the study, we use GAMS (2010) with CPLEX 12.2 solver on a computer with a 3.2 GHz processor to solve our models for the experimental instances we created. We set a limit of 1.000-second (16,67-minute) time interval for the solver to find a solution, which is defined by the user. Depending on the available time and the goal of the study one can change the mentioned time limit. However, since a lot of runs need to be done for experimental purposes, we allow the solver 1.000 seconds so that the analysis is conducted in a timewise efficient manner.

In our problem setting, we create generic networks in which transportation from every i to every j, from every j to every k, and from every k to every l is possible. So, our node-arc incidence matrix is dense in terms of possible transportation routes. However, it is possible to use some arbitrarily big cost values to model non-existing routes between nodes in real life situations. In our computational part, we generate the parameters of our problem randomly

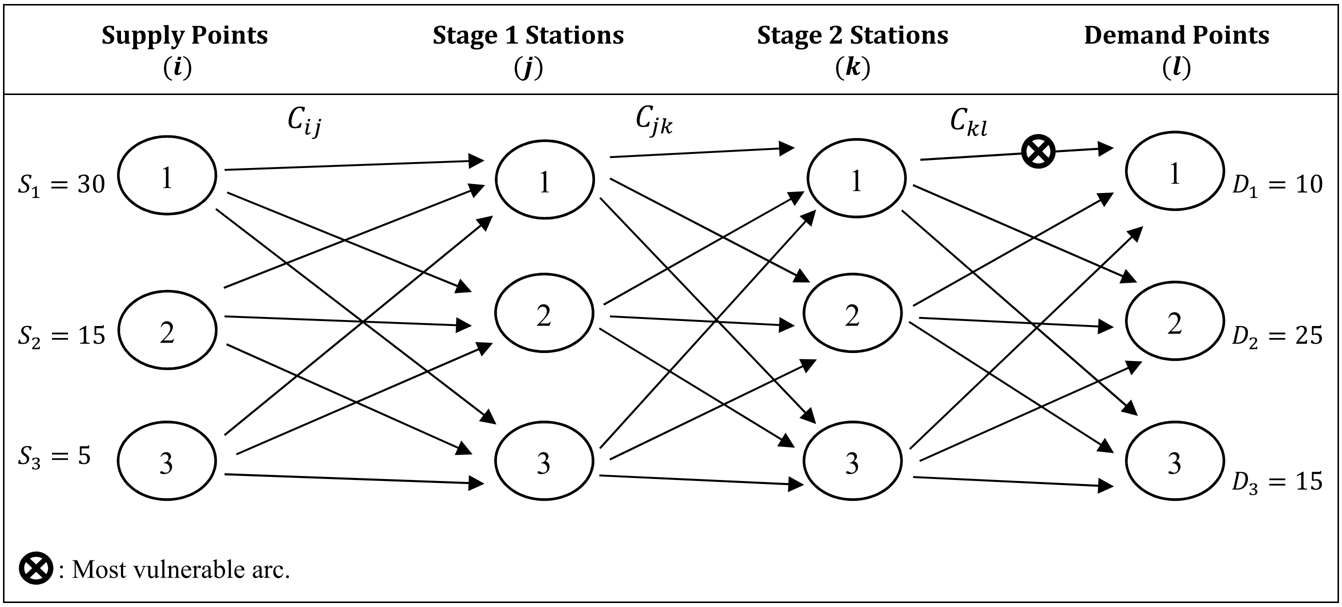

FIGURE 3. Network representation of distribution process.

from a uniform distribution. We use integer-valued parameters $S_i, D_l, C_{ij}, C_{jk}, C_{kl}, r_{ij}, r_{jk}, r_{kl}$ assign them values from the uniform intervals [10, 15], [5, 10], [50, 400], [20, 80], [150, 700] , [1, 2] , [2, 4], [1, 2] respectively.

A. AN ILLUSTRATIVE EXAMPLE

Consider a distribution network having 3 supply and 3 demand points in which the distribution is conducted through two stages (Stage 1 and Stage 2) with 3 transshipment stations at each stage. Each supply point has an associated supply capacity (S_i) and each demand point has an associated demand value (D_l). Unit transportation costs between nodes are known and denoted by C_{ij}, C_{jk}, C_{kl} respectively. The network representation of the distribution process is given in Figure 3.

The associated supply (S_i) and demand (D_l) values can be seen in Figure 3 and are also given in Table 1.a. Unit distribution costs between supply points and Stage 1 stations, C_{ij}'s, between stage 1 and stage 2 stations, C_{jk}'s, and between stage 2 stations and demand points, C_{kl}'s are given in Table 1.b, Table 1.c and Table 1.d respectively.

For simplicity, suppose that an opponent needs one unit of resource to disable an arc connecting any two nodes such that r_{ij}, r_{jk}, r_{kl} values are all 1.

Normally when there is no intervention in the network, the minimum cost of distribution in our example can be found using our classical model TNM. The solution of TNM through GAMS/CPLEX gives a minimum delivery cost of 3800.

However, if an opponent had the chance to disable one arc in the distribution network with the given data above, which arc would this be and how much would the distribution cost increase?

TABLE 1. (a) Supply and demand values. (b) Cost values-C_{ij}. (c) Cost values-C_{jk}. (d) Cost values-C_{kl}.

(a)

Supply Point (i)	Supply Amount	Demand Point (l)	Demand Amount
1	30	1	10
2	15	2	25
3	5	3	15

(b)

Supply Points (i)	Stage 1 Stations (j)		
	1	2	3
1	40	80	20
2	50	30	90
3	60	40	20

(c)

Stage 1 Stations (j)	Stage 2 Stations (k)		
	1	2	3
1	20	40	80
2	30	80	60
3	80	70	20

(d)

Stage 2 Stations (k)	Demand Points (l)		
	1	2	3
1	20	10	40
2	70	30	60
3	80	20	70

The answer to the question can easily be figured out by setting total interdiction resource (TIR) to 1 and solving model TNMI. The solution of TNMI through GAMS/CPLEX gives

TABLE 2. **(a) Computational results (*N* = 24, 40, 60, 80, 100, 120, 160, 200, 240, 280; *TIR* = 0, 1, 2).**

	TIR=0		TIR=1		TIR=2	
Number of Stations	Total Cost	Arc(s) to Interdict	Total Cost	Arc(s) to Interdict	Total Cost	Arc(s) to Interdict
$I,J,K,L=6$ $N=24$	15017	-	16005	k_1-l_2	16458	i_6-j_2 k_1-l_2
$I,J,K,L=10$ $N=40$	23177	-	23511	k_8-l_2	24361	$k_{10}-l_9$
$I,J,K,L=15$ $N=60$	28345	-	28714	$k_{11}-l_2$	29492	k_4-l_{14}
$I,J,K,L=20$ $N=80$	37381	-	37848	$k_{13}-l_6$	38557	k_8-l_9 $k_{19}-l_9$
$I,J,K,L=25$ $N=100$	44817	-	45157	$k_{12}-l_1$	45418	$k_{12}-l_1$ $k_{21}-l_{17}$
$I,J,K,L=30$ $N=120$	53219	-	53632	$k_{15}-l_{10}$	53780	$k_{13}-l_{26}$ $k_{15}-l_{10}$
$I,J,K,L=40$ $N=160$	67964	-	68304	$k_{16}-l_{13}$	68608	$k_{16}-l_{13}$ $k_{32}-l_{27}$
$I,J,K,L=50$ $N=200$	84805	-	85129	$k_{37}-l_{26}$	85334	$k_{30}-l_{40}$ $k_{37}-l_{26}$
$I,J,K,L=60$ $N=240$	98688	-	98928	$k_{52}-l_1$	99017	$k_{24}-l_{59}$ $k_{52}-l_1$
$I,J,K,L=70$ $N=280$	114711	-	114897	$k_{59}-l_{57}$	115145	$k_{25}-l_{26}$ $k_{57}-l_{26}$

a delivery cost of 4200 with the interdicted arc (k_1-l_1) shown with a cross mark in Figure 3. Note that originally the cost was 3800; now that arc (k_1-l_1) is out of use, the total distribution cost has increased by 400 units. This is because when the relatively less costly arc (k_1-l_1) is not in use, the other arcs have to be used for transshipment from Stage 2 to demand points, which makes the transportation more costly. Under the given circumstances, arc (k_1-l_1) is the most vulnerable arc for intervention since it has the largest detrimental impact on the total distribution cost when disrupted. The disruption in the network can either stem from a deliberate intervention or from some natural setbacks, which are mostly out of the control of the network owner.

Of course, an opponent can also disable more than one arc in the network depending on his/her available resources and more than one unit of resource might be needed to disable an arc. In this case, one needs to update r_{ij}, r_{jk}, r_{kl} and *TIR* values properly and solve model TNMI to find out vulnerable arcs and the corresponding increase in the total delivery cost.

B. COMPUTATIONAL RESULTS

We use various quantities of total interdiction resources, *TIR*, and different-size networks constituting different cases for our analysis. We test up to 70 nodes (stations) in each station type i, j, k, l; which means 280 total stations in our distribution network. In our network with 280 nodes, there are $(ixj) + (jxk) + (kxl) = (70x70) + (70x70) + (70x70) =$ 14.700 arcs. This means that there are 14.700 cost coefficients to be handled giving us an idea about the complexity of our experimentation. In order to handle the big data, we get use of GDX (GAMS Data eXchange) facilities available in GAMS; which make it possible to read data from Excel files.

TABLE 2. ***(Continued.)*** **(b) Computational results (N = 24, 40, 60, 80, 100, 120, 160, 200, 240, 280; TIR = 3, 4, 5).**

	TIR=3		*TIR=4*		*TIR=5*	
Number of Stations	*Total Cost*	*Arc(s) to Interdict*	*Total Cost*	*Arc(s) to Interdict*	*Total Cost*	*Arc(s) to Interdict*
$I,J,K,L=6$ $N=24$	*17385*	k_1-l_2 k_3-l_5 k_6-l_5	*17915*	k_1-l_2 k_2-l_2 k_4-l_2	*18809*	k_1-l_2 k_2-l_2 k_3-l_2 k_4-l_2
$I,J,K,L=10$ $N=40$	*24647*	k_8-l_2 $k_{10}-l_9$	*25321*	k_3-l_5 $k_{10}-l_9$	*25607*	k_3-l_5 k_8-l_2 $k_{10}-l_9$
$I,J,K,L=15$ $N=60$	*29860*	k_4-l_{14} $k_{11}-l_2$	*30221*	k_4-l_{14} k_7-l_7	*30554*	k_4-l_{14} k_7-l_7 $k_{11}-l_2$ $k_{12}-l_2$
$I,J,K,L=20$ $N=80$	*39027*	k_2-l_9 $k_{13}-l_6$ $k_{19}-l_9$	*39443*	k_2-l_9 k_3-l_3 $k_{13}-l_6$ $k_{19}-l_9$	*39773*	k_2-l_9 k_3-l_3 $k_{13}-l_6$ $k_{19}-l_9$ $k_{20}-l_{18}$
$I,J,K,L=25$ $N=100$	*45726*	$k_{12}-l_1$ $k_{18}-l_{24}$	*46053*	k_2-l_7 $k_{12}-l_1$ $k_{19}-l_7$	*46311*	k_2-l_7 $k_{12}-l_1$ $k_{19}-l_7$ $k_{21}-l_{17}$
$I,J,K,L=30$ $N=120$	*54139*	k_1-l_{19} $k_{15}-l_{10}$ $k_{29}-l_{19}$	*54291*	k_1-l_{19} $k_{13}-l_{26}$ $k_{15}-l_{10}$ $k_{29}-l_{19}$	*54553*	k_1-l_{19} $k_{15}-l_{10}$ $k_{22}-l_7$ $k_{29}-l_{19}$ $k_{30}-l_7$
$I,J,K,L=40$ $N=160$	*68842*	$k_{16}-l_{13}$ $k_{24}-l_{16}$ $k_{32}-l_{27}$	*69071*	$k_{16}-l_{13}$ $k_{24}-l_{16}$ $k_{32}-l_{27}$ $k_{36}-l_5$	*69278*	$k_{12}-l_1$ $k_{16}-l_{13}$ $k_{24}-l_{16}$ $k_{32}-l_{27}$
$I,J,K,L=50$ $N=200$	*85524* (Gap: 0,0601)*	$k_{15}-l_{14}$ $k_{30}-l_{40}$ $k_{37}-l_{26}$	*85708* (Gap: 0,1559)*	k_5-l_{27} $k_{15}-l_{14}$ $k_{30}-l_{40}$ $k_{37}-l_{26}$	*85732* (Gap: 0,1996)*	$k_{12}-l_{39}$ $k_{15}-l_{14}$ $k_{30}-l_{40}$ $k_{36}-l_{46}$ $k_{37}-l_{26}$
$I,J,K,L=60$ $N=240$	*99250* (Gap: 0,0477)*	$k_{38}-l_{60}$ $k_{52}-l_1$	*99374* (Gap: 0,1611)*	k_6-l_{14} k_8-l_{14} $k_{57}-l_{14}$	*99160* (Gap: 0,1886)*	i_2-j_4 $i_{25}-j_{20}$ $i_{46}-j_{60}$ $k_{38}-l_{60}$
$I,J,K,L=70$ $N=280$	*115327* (Gap: 0,0583)*	$k_{25}-l_{26}$ $k_{57}-l_{26}$ $k_{59}-l_{57}$	*115180* (Gap: 0,1631)*	$k_{18}-l_{62}$ $k_{57}-l_{26}$ $k_{58}-l_{62}$	*115581* (Gap: 0,1861)*	k_1-l_{56} $i_{18}-l_9$ $k_{25}-l_{26}$ $k_{57}-l_{26}$ $k_{59}-l_{57}$

* Solutions given with a relative gap value after a run time of 1.000 seconds.

We summarize the results obtained for the networks in different sizes and with several *TIR* values in Tables 2.a through Table 2.f As seen in Table 2.a, when there are 6 available stations for each type of nodes i, j, k, l, ($I, J, K, L = 6$ *and* $N = 24$) the total cost of transportation is normally 15017, where there is, in fact, no interdiction resource (*TIR* = *0*). With *TIR* values 1 and 2 the total cost turns out to be 16005 and 16458 respectively. The disabled arcs that are

TABLE 2. ***(Continued.)*** **(c) Computational results (N = 24, 40; TIR = 10, 20, 30).**

	TIR=10		*TIR=20*		*TIR=30*	
Number of Stations	*Total Cost*	*Arc(s) to Interdict*	*Total Cost*	*Arc(s) to Interdict*	*Total Cost*	*Arc(s) to Interdict*
$I,J,K,L = 6$ $N = 24$	*26458*	$i_6 - j_2$ $k_1 - l_2$ $k_1 - l_5$ $k_2 - l_5$ $k_3 - l_5$ $k_4 - l_5$ $k_5 - l_5$ $k_6 - l_5$	*35915*	$k_1 - l_2$ $k_1 - l_4$ $k_1 - l_5$ $k_2 - l_2$ $k_2 - l_4$ $k_2 - l_5$ $k_3 - l_4$ $k_3 - l_5$ $k_4 - l_2$ $k_4 - l_4$ $k_4 - l_5$ $k_5 - l_4$ $k_5 - l_5$ $k_6 - l_4$ $k_6 - l_5$	*44307*	$i_6 - j_2$ $k_1 - l_2$ $k_1 - l_4$ $k_1 - l_5$ $k_2 - l_2$ $k_2 - l_4$ $k_2 - l_5$ $k_3 - l_2$ $k_3 - l_3$ $k_3 - l_4$ $k_3 - l_5$ $k_4 - l_2$ $k_4 - l_4$ $k_4 - l_5$ $k_5 - l_2$ $k_5 - l_4$ $k_5 - l_5$ $k_6 - l_2$ $k_6 - l_3$ $k_6 - l_4$ $k_6 - l_5$
$I,J,K,L = 10$ $N = 40$	*2731*	$k_1 - l_1$ $k_3 - l_5$ $k_5 - l_1$ $k_6 - l_1$ $k_{10} - l_9$	*35607*	$k_1 - l_1$ $k_2 - l_1$ $k_3 - l_1$ $k_3 - l_5$ $k_4 - l_1$ $k_5 - l_1$ $k_6 - l_1$ $k_7 - l_1$ $k_8 - l_1$ $k_8 - l_2$ $k_9 - l_1$ $k_{10} - l_1$ $k_{10} - l_9$	*41511*	$k_1 - l_2$ $k_1 - l_9$ $k_2 - l_1$ $k_2 - l_9$ $k_3 - l_1$ $k_3 - l_9$ $k_4 - l_1$ $k_4 - l_9$ $k_5 - l_1$ $k_5 - l_9$ $k_6 - l_1$ $k_6 - l_9$ $k_7 - l_1$ $k_7 - l_9$ $k_8 - l_1$ $k_8 - l_2$ $k_8 - l_9$ $k_9 - l_1$ $k_9 - l_9$ $k_{10} - l_1$ $k_{10} - l_9$

chosen by the interdictor for each of the mentioned cases correspondingly are $(k_1 - l_2)$ *and* $(i_6 - j_2, k_1 - l_2)$. We test our model with different sets of nodes in the network increasing each time the number of nodes. The results for the instances

TABLE 2. ***(Continued.)*** **(d) Computational results (N = 80, 160; TIR = 10, 20, 30).**

	TIR=10		*TIR=20*		*TIR=30*	
Number of Stations	*Total Cost*	*Arc(s) to Interdict*	*Total Cost*	*Arc(s) to Interdict*	*Total Cost*	*Arc(s) to Interdict*
$I,J,K,L=20$ $N=80$	*40950** *(Gap: 0,0520)*	$i_{11}-j_{13}$ k_2-l_9 k_3-l_3 k_9-l_8 $k_{13}-l_6$ $k_{19}-l_9$ $k_{20}-l_8$ $k_{20}-l_{18}$	*42368** *(Gap: 1,3397)*	k_2-l_9 k_3-l_3 k_3-l_9 k_4-l_9 k_5-l_9 k_7-l_9 k_9-l_9 $k_{10}-l_9$ $k_{12}-l_9$ $k_{13}-l_6$ $k_{14}-l_9$ $k_{15}-l_9$ $k_{17}-l_9$ $k_{18}-l_9$ $k_{19}-l_9$ $k_{20}-l_9$	*128826** *(Gap: 0,0472)*	k_1-l_9 k_2-l_9 k_3-l_3 k_3-l_9 k_4-l_9 k_5-l_9 k_6-l_9 k_7-l_9 k_8-l_9 k_9-l_9 $k_{10}-l_9$ $k_{11}-l_9$ $k_{11}-l_{10}$ $k_{12}-l_9$ $k_{13}-l_6$ $k_{13}-l_9$ $k_{14}-l_9$ $k_{15}-l_9$ $k_{16}-l_9$ $k_{17}-l_9$ $k_{18}-l_9$ $k_{19}-l_9$ $k_{20}-l_9$ $k_{20}-l_{18}$
$I,J,K,L=40$ $N=160$	*69700** *(Gap: 0,3753)*	$k_{10}-l_{28}$ $k_{16}-l_{13}$ $k_{21}-l_{28}$ $k_{27}-l_9$ $k_{31}-l_9$ $k_{32}-l_{27}$ $k_{38}-l_{28}$ $k_{40}-l_{28}$	*70783** *(Gap: 0,5854)*	k_1-l_{28} k_3-l_{28} k_7-l_{28} $k_{10}-l_{28}$ $k_{12}-l_1$ $k_{14}-l_{28}$ $k_{17}-l_{28}$ $k_{18}-l_{28}$ $k_{18}-l_{36}$ $k_{21}-l_{28}$ $k_{23}-l_{28}$ $k_{24}-l_{36}$ $k_{25}-l_{28}$ $k_{32}-l_{27}$ $k_{38}-l_{28}$ $k_{39}-l_{28}$ $k_{40}-l_{28}$ $k_{40}-l_{36}$	*71929** *(Gap: 0,7757)*	k_1-l_{28} k_1-l_{36} k_3-l_{28} k_7-l_{28} $k_{10}-l_{28}$ $k_{12}-l_{36}$ $k_{13}-l_7$ $k_{14}-l_{28}$ $k_{15}-l_{36}$ $k_{16}-l_{13}$ $k_{17}-l_{28}$ $k_{18}-l_{28}$ $k_{18}-l_{36}$ $k_{21}-l_{28}$ $k_{23}-l_{28}$ $k_{24}-l_{36}$ $k_{25}-l_{28}$ $k_{26}-l_{23}$ $k_{30}-l_{36}$ $k_{36}-l_5$ $k_{38}-l_{28}$ $k_{39}-l_{28}$ $k_{39}-l_{36}$ $k_{40}-l_{28}$ $k_{40}-l_{36}$

* Solutions given with a relative gap value after a run time of 1.000 seconds.

where $I,J,K,L = 10$ *and* $N = 40$, $I,J,K,L = 15$ *and* $N = 60$, $I,J,K,L = 20$ *and* $N = 80$, $I,J,K,L = 25$ *and* $N = 100$, $I,J,K,L = 40$ *and* $N = 160$, $I,J,K,L = 30$ *and* $N = 120$, $I,J,K,L = 50$ *and* $N = 200$, $I,J,K,L = 60$

TABLE 2. ***(Continued.)*** **(e) Computational results (N = 200, 240; TIR = 10, 20, 30).**

	TIR=10		*TIR=20*		*TIR=30*	
Number of Stations	*Total Cost*	*Arc(s) to Interdict*	*Total Cost*	*Arc(s) to Interdict*	*Total Cost*	*Arc(s) to Interdict*
$I,J,K,L = 50$ $N = 200$	*86992** *(Gap: 0,3093)*	$i_{12}-j_{36}$ $i_{48}-j_{47}$ k_5-l_{27} $k_{15}-l_{14}$ $k_{30}-l_{40}$ $k_{36}-l_{40}$ $k_{37}-l_{26}$ $k_{40}-l_{40}$ $k_{46}-l_{40}$ $k_{48}-l_{40}$	*87492** *(Gap: 0,4514)*	k_3-l_{40} k_5-l_{27} $k_{11}-l_{17}$ $k_{15}-l_{14}$ $k_{19}-l_{40}$ $k_{23}-l_{40}$ $k_{26}-l_{40}$ $k_{30}-l_{40}$ $k_{31}-l_{41}$ $k_{32}-l_{40}$ $k_{36}-l_{40}$ $k_{39}-l_9$ $k_{40}-l_{40}$ $k_{42}-l_{40}$ $k_{46}-l_{40}$ $k_{47}-l_{40}$ $k_{48}-l_{40}$	*87042** *(Gap: 0,6302)*	k_1-l_{10} k_2-l_{44} k_8-l_{38} $k_{12}-l_{44}$ $k_{13}-l_{29}$ $k_{14}-l_{29}$ $k_{15}-l_{14}$ $k_{17}-l_{29}$ $k_{18}-l_{29}$ $k_{20}-l_{29}$ $k_{21}-l_{29}$ $k_{22}-l_{44}$ $k_{23}-l_{29}$ $k_{27}-l_{29}$ $k_{30}-l_{40}$ $k_{31}-l_{29}$ $k_{36}-l_{29}$ $k_{36}-l_{44}$ $k_{38}-l_{44}$ $k_{42}-l_{29}$ $k_{43}-l_{29}$ $k_{46}-l_{29}$ $k_{48}-l_{40}$
$I,J,K,L = 60$ $N = 240$	*99646** *(Gap: 0,2815)*	k_5-l_{50} k_6-l_{14} k_8-l_{14} $k_{37}-l_{38}$ $k_{45}-l_{27}$ $k_{48}-l_{50}$ $k_{49}-l_{50}$ $k_{57}-l_{14}$	*99831** *(Gap: 0,4032)*	k_8-l_{19} $k_{12}-l_{19}$ $k_{14}-l_9$ $k_{16}-l_{25}$ $k_{24}-l_9$ $k_{25}-l_{19}$ $k_{26}-l_{19}$ $k_{31}-l_7$ $k_{38}-l_{20}$ $k_{52}-l_1$ $k_{58}-l_{19}$ $k_{12}-l_{30}$ $k_{21}-l_{51}$ $k_{38}-l_{60}$ $k_{41}-l_{37}$ $k_{42}-l_{47}$ $k_{57}-l_{33}$	*100309** *(Gap: 0,5024)*	k_3-l_{38} k_6-l_{14} $k_{14}-l_3$ $k_{25}-l_{19}$ $k_{32}-l_3$ $k_{41}-l_4$ $k_{48}-l_{17}$ $k_{52}-l_1$ $k_{52}-l_4$ $k_{57}-l_{14}$ $k_{58}-l_{19}$ $k_{15}-l_{50}$ $k_{16}-l_{38}$ $k_{27}-l_{31}$ $k_{28}-l_{31}$ $k_{37}-l_{38}$ $k_{48}-l_{50}$ $k_{49}-l_{50}$

* Solutions given with a relative gap value after a run time of 1.000 seconds.

and $N = 240$, $I,J,K,L = 70$ *and* $N = 280$, are given in Table 2.a as well.

Similarly associated results for *TIR values 3,4 and 5* are presented in Table 2.b for each of our instances. So, the results clearly show the most vulnerable arcs in a certain scenario and make it available for the network owner to figure out the arcs in the transshipment network requiring special concern when a possible intervention is possible. It should also be noted that the results in Table 2.a. are given as exact solutions for all existing instances whereas the solutions for large networks ($N = 200, 240, 280$) in Table 2.b are given with some solution gaps within the defined solution run time for GAMS/CPLEX solver, which is 1000 seconds. This clearly shows that the performance of the model depends

TABLE 2. ***(Continued.)*** **(f) Computational results (N = 280; TIR = 10, 20, 30).**

	TIR=10		*TIR=20*		*TIR=30*	
Number of Stations	*Total Cost*	*Arc(s) to Interdict*	*Total 3ost*	*Arc(s) to Interdict*	*Total Cost*	*Arc(s) to Interdict*
$I, J, K, L = 70$ $N = 280$	*115633** *(Gap: 0,2495)*	$i_3 - j_{51}$ $k_{18} - l_{60}$ $k_{41} - l_{60}$ $k_{57} - l_{26}$ $k_{57} - l_{32}$ $k_{59} - l_{57}$ $k_{62} - l_6$ $k_{65} - l_6$	*115578** *(Gap: 0,3638)*	$i_{68} - j_{46}$ $j_{27} - k_{61}$ $k_9 - l_{24}$ $k_{11} - l_{24}$ $k_{12} - l_4$ $k_{18} - l_{62}$ $k_{21} - l_{20}$ $k_{40} - l_{20}$ $k_{58} - l_4$ $k_{58} - l_{62}$ $k_{59} - l_{57}$ $k_{60} - l_{40}$	*115865** *(Gap: 0,4609)*	$k_2 - l_{21}$ $k_7 - l_{21}$ $k_{12} - l_{21}$ $k_{17} - l_{21}$ $k_{18} - l_{60}$ $k_{19} - l_{21}$ $k_{20} - l_{21}$ $k_{28} - l_{21}$ $k_{32} - l_{21}$ $k_{34} - l_{21}$ $k_{39} - l_{21}$ $k_{40} - l_{21}$ $k_{45} - l_{21}$ $k_{47} - l_{21}$ $k_{58} - l_{21}$ $k_{64} - l_{21}$ $k_{66} - l_{21}$ $k_{68} - l_{21}$ $k_{69} - l_{21}$

* Solutions given with a relative gap value after a run time of 1.000 seconds.

on both the network size, N, and the available interdiction resource the opponent has, *TIR*.

In an attempt to better elaborate the performance of the model and identify its limitations, we expand our experimentation with larger *TIR* (10, 20, 30) values as well. As seen in Table 2.c, exact solution values are provided for relatively smaller networks with $N = 24, 40$. However, as given in Tables 2.d, 2.e and 2.f, the solutions are provided with some certain gaps for larger N values, namely 80, 160, 200, 240, 280.

One can say that solvers using exact solution algorithms can easily yield solutions for small *TIR* values (i.e. 1, 2) even when N has larger values (i.e. 240, 280), which can be observed in Table2.a. The same is also true for the cases in which N is small (i.e. 24, 40) and TIR is large (i.e. 10, 20, 30), which is observable in Table 2.c. So, when the standard MIP solvers is used for solution, it will be time-consuming to get an exact solution with large values of N and *TIR*, which is a limitation of the proposed method.

V. CONCLUSION AND FUTURE WORK

In this study, we develop an interdiction model for a transshipment network (TNM-I) to analyze the vulnerabilities in the distribution process. We show that the coefficient matrix A of the transshipment network is totally unimodular, which enables us to use linear programming approaches regardless of the discreteness of variables. Using duality and linearizing certain discrete variables, we further show that the initial bilevel interdiction model (Bi-TNM) we develop, can be converted into a standard MIP model solvable through standard MIP solvers (i.e. CPLEX).

The resulting Model TNM-I helps network analysts reveal the courses of action of an opponent trying to increase the cost of transportation by interdicting certain arcs of the transportation network with certain interdiction resources. In our application, we observe the critical arcs as well as the resulting increase in the distribution cost in case the network is interdicted by an opponent using limited interdiction resources. The total cost substantially increases depending on the available interdiction resources on the opponent side. The decision-makers can figure out the segments of a transportation network that are potentially more costly in case some are disabled. The decision-makers can also increase their awareness with respect to the effect of any such intervention on their objective function by observing the change in the total cost depending on an opponent's available interdiction resources. Such information can help decision-makers develop appropriate courses of action to reduce transportation costs as well as get a deep understanding of worst cases in cases of disruptions. A more sustainable distribution process can be designed using the findings in this study. The generic models developed in this study can easily be used in typical transshipment networks like electric grids.

One limitation to the proposed method we identify in our experimental study is that it will require much time to get an exact solution through the standard MIP solvers when large N and TIR values exist. Even though the model can be solved via off-the-shelf optimization software for networks having N less than 200 and TIR values less than 5 within short time limits, further study would be beneficial to develop some decomposition or heuristic techniques to solve problems with larger N and TIR values so that the timewise efficiency of the solution is improved. It would also be interesting to apply the idea to the capacitated network problems since in this study capacity of the arcs is not taken into account.

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