IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 29 March 2023, accepted 8 May 2023, date of publication 22 May 2023, date of current version 1 June 2023.

*Digital Object Identifier 10.1109/ACCESS.2023.3278795*

RESEARCH ARTICLE

# Beamforming and Power Optimization for User Fairness in Cell-Free MIMO Systems

**KEUNYOUNG KIM, JUNGHO MYUNG, YOUNG-JO KO, AND WOO YONG LEE**
Electronics and Telecommunications Research Institute, Daejeon 34129, Republic of Korea

Corresponding author: Keunyoung Kim (kykim12@etri.re.kr)

This work was supported in part by the Institute of Information and Communications Technology Planning and Evaluation (IITP) Grant funded by the Korea Government, Ministry of Science and ICT (MSIT) (No. 2018-0-00218, Specialty Laboratory for Wireless Backhaul Communications Based on Very High Frequency); and in part by the Korea Institute of Marine Science and Technology Promotion (KIMST) funded by the Korea Government (MSIT) (No. 2021-0626, Development of Polar Region Communication Technology and Equipment for the Internet of Extreme Things (IoET)).

**ABSTRACT** Cell-free (CF) massive multiple-input multiple-output (MIMO) systems are expected to provide high spectral efficiency to all users, regardless of their locations, when equipped with a large number of evenly distributed access points (APs) in the area of coverage. In this paper, we investigate beamforming and power allocation schemes in CF MIMO systems to achieve user fairness, while maintaining high spectral efficiency without degrading the performance of users with good channel conditions, even as the number of users scales up. The system models and optimization problems for both downlink and uplink are described in a unified mathematical framework. For achieving user fairness, three different approaches are used, i.e., maximizing the minimum received signal power, minimizing the maximum interference power, and maximizing the minimum of signal to interference and noise ratio (SINR). By decoupling beamforming and power allocation problems, the beamforming problems can be formulated as the generalized eigenvalue problems, and the optimal solutions correspond to well-known schemes such as maximum ratio, zero-forcing, and minimum mean square error combining/transmission. In addition to beamforming, we provide closed-form solutions for optimal power allocation by incorporating additional objectives such as minimum total transmit power or evenly balanced SINR when needed. Our performance comparison suggests that to achieve evenly high data rates for all users irrespective of their locations or the number of users, active interference suppression is necessary instead of relying on maximum ratio combining/transmission even with many APs employed.

**INDEX TERMS** Cell-free massive MIMO, beamforming, eigenvalue problem, max-min optimization, power allocation.

## I. INTRODUCTION

Wireless communications have become an indispensable part of our lives, which demands enormous data exchanges. To accommodate the demands, wireless communication systems have evolved to transmit or receive a large amount of data through innovative technologies such as massive multiple-input multiple-output (MIMO), frequency use in millimeter-wave, and network ultra-densification [1]. However, users at cell edges are often limited to receiving only a small portion of data due to weak signal and strong interference power. Owing to the difficulties in controlling interference, most wireless systems emulate a point-to-point communication environment by attempting to avoid the interference generated from other users. Avoiding interference is generally realized by allocating frequency and time to each user orthogonally [2]. As a result, the data rate available for each user decreases as the number of users increases since the total system bandwidth is divided by multiple users.

We envision a wireless system that can provide high data rates for all users regardless of their locations or the number of users, even as the number of users scales up. This is

The associate editor coordinating the review of this manuscript and approving it for publication was Wei Feng.

possible if channel gains are uniformly high for all users, and the degrees of freedom (DoF) scale linearly with the number of users. Cell-free (CF) massive MIMO systems are a promising option for tackling these issues since they can provide uniform and improved quality of service for all users [3]. In CF massive MIMO systems, a very large number of access points (APs) are geographically distributed to serve a relatively small number of users in the same time-frequency resources. Channel station information (CSI) at each AP is estimated by receiving uplink (UL) pilot signals from users, and the information is used for processing both uplink and downlink (DL) data by assuming the channel reciprocity in time-division duplex (TDD) systems. A massive number of APs are expected to eliminate the effects of uncorrelated receiver noise, fast fading and interference by using only simple linear processing such as maximum ratio combining (MRC) or transmission (MRT) [4]. Each AP is connected to central processing units (CPUs) to combine or precode data for each user. If the connections between APs and CPUs are ideal to share all CSI and data, CF massive MIMO systems can be viewed as multi-user MIMO systems with distributed antennas. We can expect uniformly high channel gains when APs are evenly and densely deployed. We can also achieve linearly increasing DoF when signals are properly processed to suppress interference.

To fully realize the potential of CF massive MIMO, signals between users and APs should be coherently combined and precoded without loss and delay. After coherent processing, interference should be ideally suppressed. For this purpose, we need to acquire sufficiently accurate CSI and share enough data and the channel information between APs and CPUs. A massive number of APs are expected to contribute outstanding positive phenomena, such as channel hardening and favorable propagation, as a consequences of the law of large numbers [5], [6]. However, these desirable phenomena are less solid in CF massive MIMO compared to co-located massive MIMO [7], [8]. Even though taking the whole advantage of massive MIMO is not easy, CF MIMO systems, which are equipped with at least as many APs as the users in a geographically distributed fashion, provide as many DoF gains as the number of users and attain macro diversity gain by close-in distance between an AP and a user.

In this paper, we investigate the maximum achievable performance of CF MIMO systems while taking user fairness criteria into consideration. By adopting appropriate beamforming and power allocation, we demonstrate the possibility of realizing a vision that wireless systems, such as CF MIMO, can offer high data rates for all users, irrespective of their location or the number of users, even as the number of users increases. User fairness is modeled as optimization problems to provide uniformity of quantities such as received signal power, interference power, or signal to interference and noise ratio (SINR). Beamforming schemes are chosen to maximize or minimize these quantities, and power allocation schemes are optimized for the same level of them. We deal with various problems using a unified mathematical framework, for both the UL and the DL, to provide closed-form solutions for different schemes. Optimizations on beamforming schemes are mostly formulated as generalized eigenvalue problems, and optimizations on power allocation schemes are mostly max-min ones. The suggested solutions are used to compare the performances of the UL and the DL in the same channel environment. Additionally, we provide some relationship between the proposed schemes. For example, in terms of fairness for received signal power, the UL provides fairer performances than the DL. In the case of two users, the UL transmit power of each user is exactly the same under the schemes for fairness in the signal power and the interference power within our formulation, even though the optimal beamforming schemes may not be the same under both schemes. The achievable spectral efficiency shows the possibility of uniformly high performance regardless of locations and the number of users, especially when the number of APs is large enough to convert the DoF gain into the spectral efficiency.

Since MIMO technology has a long history, the schemes covered in this paper are partially presented in many papers [9]. Maximizing signal power is usually known as the matched filtering, sometimes called MRC in the UL or MRT in the DL. When the number of antennas is large enough to null out all interference, minimizing interference is named as zero-forcing (ZF). Maximizing SINR is related to the minimum mean-square error (MMSE) [10]. Focusing on CF massive MIMO, UL MR combining and MMSE schemes are presented in [5] and [11], DL MR transmission in [5], DL ZF in [12], and DL MMSE in [13]. For power allocation schemes, UL max-min SINR power control problems are shown in [5] and [14], DL max-min SINR power control in [12], [15], [16], and [17]. An extensive survey is given in [18] and [19].

The issue of user fairness is addressed in some recent papers. The authors in [20] presented the max-min optimization problem in uplink CF massive MIMO systems, in which the beamforming vector is given as MRC, and the optimal power is generated from meta-heuristic schemes such as simulated annealing, differential evolution, and particle swarm optimization. The proposed schemes are shown to provide near optimal solutions with reasonable computational effort. In [21], for large intelligent surfaces as an evolution of massive MIMO, the maximization of minimum SINR in the uplink was presented with a matched filter process at each panel and unitary transmitted power per terminal. The decision variables are related to the panel-terminal allocation and panel selection. The optimization problem is a mixed-integer linear programing and was solved using commercial optimization software. The authors in [22] considered an uplink power optimization problem in radio stripe communications, with the aim of maximizing two metrics, which are operationalized as objective functions to maximize the total sum spectral efficiency and the spectral efficiency of the worst user. In radio stripe networks, access points are sequentially connected in the same stripe, and thus, a sequential linear processing combining scheme is employed. Since the objective functions are non-convex, a meta-heuristic based on the differential evolution algorithm was adopted to find a near-optimal solution with low computation complexity. In our paper, most beamforming schemes are formulated as generalized eigenvalue problems, and power allocation schemes as max-min optimization problems. All solutions are given in closed forms by decoupling the beamforming and the power allocation problems.

*Notations*: We use uppercase boldface letters, lowercase boldface letters, standard lowercase letters for matrices, column vectors, and scalars, respectively. The notation $(\cdot)^H$ denotes the conjugate transpose (Hermitian) of a matrix or a vector. The $i$th column vector of matrix $\mathbf{X}$ is $\mathbf{x}_i$, The $i$th row vector of matrix $\mathbf{X}^H$ is $\mathbf{x}_i^H$, and the $i$th element of vector $\mathbf{x}$ is $x_i$. The $i$th row and $j$th column element of matrix $\mathbf{X}$ is represented as $x_{ij}$ or $[\mathbf{X}]_{i,j}$. The $L^2$ norm (length) of vector $\mathbf{x}$ is expressed as $\|\mathbf{x}\|$, and the absolute value of scalar $x$ as $|x|$. The notations $\text{diag}(\mathbf{x})$ and $\text{diag}(x_1)$ denote the diagonal matrices with the elements $\mathbf{x}$ and $x_i$, respectively. $\mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}\right)$ stands for a zero mean circularly symmetric complex Gaussian random vector with covariance $\sigma^2\mathbf{I}$. The field of complex numbers is denoted as $\mathbb{C}$. $\mathbf{X}_{-i}$ represents a matrix striping out the $i$th column vector in matrix $\mathbf{X}$. $\mathcal{N}(\mathbf{X}), \mathcal{C}(\mathbf{X})$, and $\dim(\cdot)$ denote the null space, the column space of matrix $\mathbf{X}$ and the dimension of a vector space, respectively. $\rho(\mathbf{X})$ is the spectral radius of matrix $\mathbf{X}$. In addition, we do not differentiate notations containing the same meaning but different values across sections unless they cause confusion.

## II. SYSTEM MODEL

We consider the UL and the DL transmissions of a CF MIMO system with $L$ single-antenna APs and $K$ single-antenna users. Each user is randomly located over specific areas such as smart factories, smart farms, and smart cities. We assume the number of APs is no less than that of users, i.e., $L \geq K$. All APs are connected to a CPU to collaboratively process the received or transmitted signals of each AP. We assume that the information between an AP and the CPU is transferred without loss and delay, enabling ideal sharing and processing of all data and CSI. This type of CF MIMO can be viewed as MU-MIMO with distributed antennas.

### A. CELL-FREE MIMO UPLINK

All $K$ users are equipped with a single antenna, thus, no transmit beamforming is applied. The transmitted signal of user $k$, $x_k$, is given by,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_K \end{bmatrix} = \sqrt{\mathbf{P}}\mathbf{s} = \begin{bmatrix} \sqrt{p_1}s_1 \\ \vdots \\ \sqrt{p_K}s_K \end{bmatrix}, \tag{1}$$

where $p_k$ is the allocated power to send information symbol of the $k$th user, $s_k$, $\sqrt{\mathbf{P}} = \text{diag}(\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_K})$, and $\mathbf{s} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. Power constraints are imposed on a per-user basis such that $0 \leq p_k \leq p^{\max}$ for all $k = 1, 2, \ldots, K$.

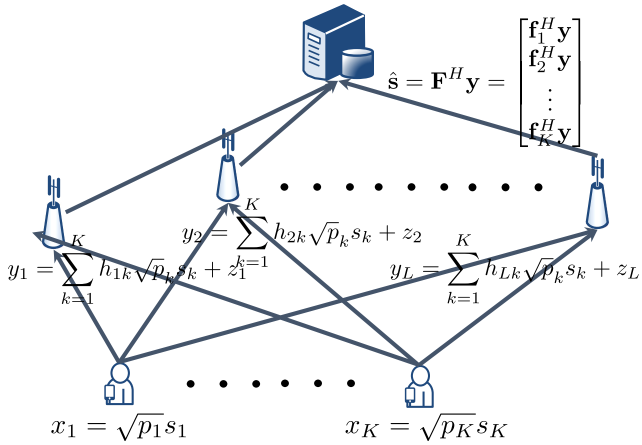


**FIGURE 1.** Cell-free MIMO uplink system model.

AP $l$ receives the signal, $y_l$, as

$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_L \end{bmatrix} = \sum_{k=1}^{K} \mathbf{h}_k x_k + \mathbf{z} = \mathbf{H}\sqrt{\mathbf{P}}\mathbf{s} + \mathbf{z}$$
$$= \begin{bmatrix} \sum_{k=1}^{K} h_{1k}\sqrt{p_k}s_k + z_1 \\ \sum_{k=1}^{K} h_{2k}\sqrt{p_k}s_k + z_2 \\ \vdots \\ \sum_{k=1}^{K} h_{Lk}\sqrt{p_k}s_k + z_L \end{bmatrix}. \tag{2}$$

Here, $\mathbf{H}$ is an $L \times K$ matrix whose element, $h_{lk} \in \mathbb{C}$, is the channel gain from user $k$ to AP $l$, and $\mathbf{z} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}\right)$ is the noise. We assume that $\mathbf{H}$ is full-rank.

The received signal is combined at the CPU to obtain

$$\hat{\mathbf{s}} = \mathbf{F}^H\mathbf{y} = \mathbf{F}^H\mathbf{H}\sqrt{\mathbf{P}}\mathbf{s} + \mathbf{F}^H\mathbf{z} = \begin{bmatrix} \mathbf{f}_1^H\mathbf{y} \\ \mathbf{f}_2^H\mathbf{y} \\ \vdots \\ \mathbf{f}_K^H\mathbf{y} \end{bmatrix}, \tag{3}$$

where $\mathbf{F}^H$ is a $K \times L$ combining matrix whose $k$th row vector $\mathbf{f}_k^H$ is used for detection of user $k$'s information symbol. The combining vector of user $k$, $\mathbf{f}_k^H$, can be considered as a receive beamforming vector in traditional MIMO systems. The common combining schemes include MRC, ZF, and MMSE. The cell-free MIMO uplink system model is summarized in Fig. 1.

The UL SINR of user $k$ is given by

$$\text{SINR}_k^{\text{UL}} = \frac{p_k\mathbf{f}_k^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{f}_k}{\mathbf{f}_k^H\left(\sum_{i=1,i\neq k}^{K} p_i\mathbf{h}_i\mathbf{h}_i^H + \sigma^2\mathbf{I}\right)\mathbf{f}_k} \tag{4}$$
$$= \frac{p_k|\mathbf{f}_k^H\mathbf{h}_k|^2}{\sum_{i=1,i\neq k}^{K} p_i|\mathbf{f}_k^H\mathbf{h}_i|^2 + \sigma^2}. \tag{5}$$

We impose constraints of $\|\mathbf{f}_k\|^2 = 1$ for all $k$, even though $\text{SINR}_k^{\text{UL}}$ is the same upto a scalar scaling factor for $\|\mathbf{f}_k\|^2$. As will be shown in Section III-A, the constraints decouple the optimal beamforming problem and the optimal power allocation problem without affecting the optimality.

An achievable spectral efficiency in the UL can be given as

$$\mathrm{SE}_k^{\mathrm{UL}} = \log\left(1 + \frac{\mathrm{SINR}_k^{\mathrm{UL}}}{\Gamma}\right), \tag{6}$$

where $\Gamma \geq 1$ is the signal to noise ratio (SNR) gap to capacity [19]. We assume $\Gamma = 1$ since $\mathbf{s} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ and all data are processed ideally.

### B. CELL-FREE MIMO DOWNLINK

At AP $l$, the transmitted signal, $x_l$ is precoded as,

$$\mathbf{x} = \begin{bmatrix} x_1 = \sqrt{q_{11}}g_{11}s_1 \cdots + \sqrt{q_{1K}}g_{1K}s_K \\ x_2 = \sqrt{q_{21}}g_{21}s_1 + \cdots + \sqrt{q_{2K}}g_{2K}s_K \\ \vdots \\ x_L = \sqrt{q_{L1}}g_{L1}s_1 \cdots + \sqrt{q_{LK}}g_{LK}s_K \end{bmatrix} = \mathbf{V}\mathbf{s}, \tag{7}$$

where $q_{lk}$ is the amount of power allocated to user $k$ at AP $l$, and $g_{lk}$ is the transmit beamforming coefficient (phase) for user $k$ and AP $l$. The precoding matrix containing the power and beamforming elements is given by

$$\mathbf{V} = \begin{bmatrix} \sqrt{q_{11}}g_{11} & \sqrt{q_{12}}g_{12} & \cdots & \sqrt{q_{1K}}g_{1K} \\ \sqrt{q_{21}}g_{21} & \sqrt{q_{22}}g_{22} & \cdots & \sqrt{q_{2K}}g_{2K} \\ \vdots & \vdots & \vdots & \vdots \\ \sqrt{q_{L1}}g_{L1} & \sqrt{q_{L2}}g_{L2} & \cdots & \sqrt{q_{LK}}g_{LK} \end{bmatrix}. \tag{8}$$

The beamforming component is set to be $|g_{lk}|^2 = 1$ for all $l$ and $k$, and power constraints are imposed on a per-AP basis such that $[\mathbf{V}\mathbf{V}^H]_{l,l} \leq q^{\max}$ for all $l = 1, 2, \ldots, L$. In general, the precoding matrix cannot be factorized into a power allocation matrix and a beamforming matrix as opposed to the uplink. However, in the case where the beamforming matrix is fixed, and each direction of the beamforming vectors should not change, the precoding matrix can be expressed as $\mathbf{G}\sqrt{\mathbf{P}}$, and the transmitted signal is given by

$$\mathbf{x} = \mathbf{V}\mathbf{s} = \mathbf{G}\sqrt{\mathbf{P}}\mathbf{s} = \sum_{k=1}^{K} \sqrt{p_k} s_k \mathbf{g}_k, \tag{9}$$

where $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_K]$ is a transmit beamforming matrix applied across APs, and $\sqrt{\mathbf{P}}$ is a diagonal matrix filled with the allocated power to each user in its main diagonal. The beamforming vector for each user should be $\|\mathbf{g}_k\|^2 = 1$ for all $k$, and the allocated power to each AP should be $[\mathbf{G}\mathbf{P}\mathbf{G}^H]_{l,l} \leq q^{\max}$ for all $l$.

The received signal at each user is

$$\mathbf{y} = \begin{bmatrix} y_1 = \mathbf{h}_1^H\mathbf{x} + z_1 \\ \vdots \\ y_K = \mathbf{h}_K^H\mathbf{x} + z_K \end{bmatrix} = \mathbf{H}^H\mathbf{V}\mathbf{s} + \mathbf{z}. \tag{10}$$

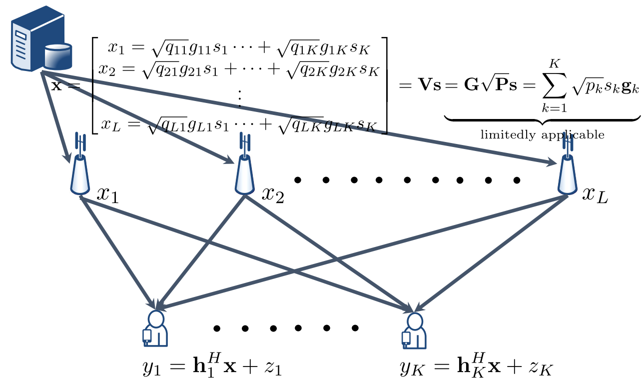


**FIGURE 2.** Cell-free MIMO downlink system model.

Since each user is equipped with a single antenna, no receive beamforming is applied. Thus, the detected information symbol is $\hat{\mathbf{s}} = \mathbf{y}$. The cell-free MIMO downlink system model is summarized in Fig. 2.

The DL SINR of user $k$ is given as

$$\mathrm{SINR}_k^{\mathrm{DL}} = \frac{\mathbf{v}_k^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{v}_k}{\sum_{i=1,i\neq k}^{K} \mathbf{v}_i^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{v}_i + \sigma^2} \tag{11}$$

$$= \frac{|\mathbf{h}_k^H\mathbf{v}_k|^2}{\sum_{i=1,i\neq k}^{K} |\mathbf{h}_k^H\mathbf{v}_i|^2 + \sigma^2}. \tag{12}$$

In case that the direction of a beamforming vector needs to be preserved, the DL SINR of user $k$ is

$$\mathrm{SINR}_k^{\mathrm{DL}} = \frac{p_k\mathbf{g}_k^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{g}_k}{\sum_{i=1,i\neq k}^{K} p_i\mathbf{g}_i^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{g}_i + \sigma^2} \tag{13}$$

$$= \frac{p_k|\mathbf{h}_k^H\mathbf{g}_k|^2}{\sum_{i=1,i\neq k}^{K} p_i|\mathbf{h}_k^H\mathbf{g}_i|^2 + \sigma^2}. \tag{14}$$

An achievable spectral efficiency in the DL can be calculated as the same as that in the UL (6).

In the DL, one of performance metrics can be the signal to leakage and noise ratio (SLNR), in which the leakage power generated from a user is presented in the denominator instead of the interference power received by a user [20]. The SLNR of user $k$ is given as,

$$\mathrm{SLNR}_k^{\mathrm{DL}} = \frac{\mathbf{v}_k^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{v}_k}{\sum_{i=1,i\neq k}^{K} \mathbf{v}_k^H\mathbf{h}_i\mathbf{h}_i^H\mathbf{v}_k + \sigma^2} \tag{15}$$

$$= \frac{|\mathbf{v}_k^H\mathbf{h}_k|^2}{\sum_{i=1,i\neq k}^{K} |\mathbf{v}_k^H\mathbf{h}_i|^2 + \sigma^2}, \tag{16}$$

and, for a fixed direction of a beamforming vector,

$$\mathrm{SLNR}_k^{\mathrm{DL}} = \frac{p_k\mathbf{g}_k^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{g}_k}{\mathbf{g}_k^H\left(p_k\sum_{i=1,i\neq k}^{K} \mathbf{h}_i\mathbf{h}_i^H + \sigma^2\mathbf{I}\right)\mathbf{g}_k} \tag{17}$$

$$= \frac{|\mathbf{g}_k^H\mathbf{h}_k|^2}{\sum_{i=1,i\neq k}^{K} |\mathbf{g}_k^H\mathbf{h}_i|^2 + \frac{\sigma^2}{p_k}}. \tag{18}$$

## III. OPTIMIZATION FOR USER FAIRNESS

We address user fairness in three ways: optimizing signal power, interference power, and the SINR/SLNR. To achieve fairness, we formulate signal power optimization as the maximization of the minimum received signal power, interference power optimization as the minimization of the maximum received interference power, SINR/SLNR optimization as the maximization of the minimum SINR/SLNR. Well-known beamforming schemes are adopted such as MRC/MRT, ZF combining/precoding, and MMSE combining/precoding. Specifically, MRC and MRT focus solely on maximizing the power of the received signal, while ZF is concerned with nullifying the power of received interference. On the other hand, MMSE considers both the received signal and interference powers in order to optimize the SINR in the UL or the SLNR the DL. In addition to beamforming schemes, we provide closed-form solutions for optimal power allocation under a unified mathematical framework. For optimization problems that do not have a unique solution, we impose additional objectives such as minimum total transmit power and evenly balanced SINR/SLNR.

### A. OPTIMIZATION ON RECEIVED SIGNAL POWER IN THE UPLINK

For fairness in the received signal power for all users, we consider a problem maximizing the minimum received signal power for each user. In the UL, the max-min problem for optimizing the received signal power can be formulated as

$$\begin{aligned} \max_{\mathbf{f}_k, p_k} \quad \min_{k=1,\ldots,K} \quad & p_k |\mathbf{f}_k^H \mathbf{h}_k|^2 = p_k \mathbf{f}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{f}_k \\ \text{subject to} \quad & \|\mathbf{f}_k\|^2 = 1, \quad \forall k, \\ & 0 \le p_k \le p^{\max}, \quad \forall k. \end{aligned} \tag{19}$$

We focus on only the received signal power of each user, which is not coupled with that of other users. This allows us to determine the optimal beamforming vector and power allocation for each user without considering the impact on other users. Moreover, a beamforming vector is not dependent on power allocation since multiplying a vector by a scalar does not change its direction. Consequently, we can compute the optimal beamforming vector and power allocation independently.

Power allocation through max-min optimization on the received signal power does not necessarily result in the same received signal power value for all users since the value larger than the minimum received signal power does not affect the optimal value. However, by introducing an additional constraint to minimize the total transmit power, the optimal value can be obtained with the same received signal power for all users, and the optimal power allocation is uniquely determined. In this case, the equalized maximum value of the minimum received signal power is obtained by setting the maximum allowable power, $p^{\max}$, to the user in the worst channel condition and by setting the power inversely proportional to the channel condition for other users.

The optimal $\mathbf{f}_k$ maximizing $|\mathbf{f}_k^H \mathbf{h}_k|^2$ with the constraints of $\|\mathbf{f}_k\|^2 = 1$ is known as MRC or the matched filtering, which has the form of

$$\mathbf{f}_k^* = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}. \tag{20}$$

The optimal beamforming can be derived by using Caushy-Schwarz inequality or the Rayleigh quotient. Since $\mathbf{h}_k \mathbf{h}_k^H$ is a positive semidefinite matrix of rank one, it has a single eigenvalue greater than 0. The corresponding eigenvector produces the optimal beamforming vector.

With the optimal beamforming vector, the received power is $p_k \|\mathbf{h}_k\|^2$, and the optimal power allocated to user $k$ is inversely proportional to its channel gain such that

$$p_k^* = \begin{cases} p^{\max} & \text{if } k = \arg\min_i \|\mathbf{h}_i\|, \\ \dfrac{s_{\max}^{\mathrm{UL}}}{\|\mathbf{h}_k\|^2} & \text{otherwise.} \end{cases} \tag{21}$$

The evenly balanced maximum received power in the UL, $s_{\max}^{\mathrm{UL}}$, is given as

$$s_{\max}^{\mathrm{UL}} = p^{\max} \min_i \|\mathbf{h}_i\|^2. \tag{22}$$

With the optimal beamforming vector and power allocation, the interference of user $k$ is given as

$$\begin{aligned} r_k^{\mathrm{UL}} &= \mathbf{f}_k^H \left( \sum_{i=1, i\neq k}^{K} p_i \mathbf{h}_i \mathbf{h}_i^H \right) \mathbf{f}_k \\ &= \frac{\mathbf{h}_k^H}{\|\mathbf{h}_k\|} \left( \sum_{i=1, i\neq k}^{K} \frac{s_{\max}^{\mathrm{UL}}}{\|\mathbf{h}_i\|^2} \mathbf{h}_i \mathbf{h}_i^H \right) \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|} \\ &= \sum_{i=1, i\neq k}^{K} \frac{s_{\max}^{\mathrm{UL}}}{\|\mathbf{h}_k\|^2} \frac{|\mathbf{h}_k^H \mathbf{h}_i|^2}{\|\mathbf{h}_i\|^2}. \end{aligned} \tag{23}$$

The interference is balanced to attain the same level if $K = 2$, but generally not if $K \ge 2$. Then, the SINR of user $k$ is

$$\mathrm{SINR}_k^{\mathrm{UL}} = \frac{s_{\max}^{\mathrm{UL}}}{\frac{s_{\min}^{\mathrm{UL}}}{\|\mathbf{h}_k\|^2} \sum_{i=1, i\neq k}^{K} \frac{|\mathbf{h}_k^H \mathbf{h}_i|^2}{\|\mathbf{h}_i\|^2} + \sigma^2}. \tag{24}$$

### B. OPTIMIZATION ON RECEIVED SIGNAL POWER IN THE DOWNLINK

In the DL, the max-min problem for optimizing the received power of each user can be formulated as

$$\begin{aligned} \max_{\mathbf{v}_k} \quad \min_{k=1,\ldots,K} \quad & |\mathbf{h}_k^H \mathbf{v}_k|^2 = \mathbf{h}_k^H \mathbf{v}_k \mathbf{v}_k^H \mathbf{h}_k \\ \text{subject to} \quad & |g_{lk}{}^2 = 1, \quad \forall k \text{ and } \forall l, \\ & \sum_{k=1}^{K} q_{lk} = q_l \le q^{\max}, \quad \forall l, \end{aligned} \tag{25}$$

where $v_{lk} = \sqrt{q_{lk}} g_{lk}$ is the precoding weight, $g_{lk}$ is the beamforming coefficient, and $q_{lk}$ is the amount of allocated power for user $k$ at AP $l$.

If AP $l$ or the CPU knows $h_{kl}$ through an appropriate channel estimation algorithm, we can adopt the conjugate beamforming so that the intended signal is coherently combined at each receiver to maximize the received signal power of each user. This beamforming coefficient is not unique since we receive the same power by multiplying the same constant phase to the conjugate beamforming vector, i.e., $|\mathbf{h}_k^H \mathbf{v}_k e^{j\phi_k}|^2 = |\mathbf{h}_k^H \mathbf{v}_k|^2$. Without loss of generality, we may choose $g_{lk}$ so that $\mathbf{h}_k^H \mathbf{v}_k$ is real. Then, the beamforming coefficient is

$$g_{lk} = \frac{h_{lk}}{|h_{lk}|} = e^{j\theta_{lk}}, \tag{26}$$

where $\theta_{lk}$ is the phase component of $h_{lk}$. With the conjugate beamforming, the received power of user $k$ is given as

$$s_k^{\mathrm{DL}} = |\mathbf{h}_k^H \mathbf{v}_k|^2 = (|h_{1k}^H|\sqrt{q_{1k}} + \cdots + |h_{Lk}^H|\sqrt{q_{Lk}})^2. \tag{27}$$

The max-min problem, which optimizes the received signal power in the DL, results in equal power distribution among users since, at each AP $l$, the power can be shared by all users. If a user receives less power than other users, the power can be redistributed to balance it. Thus, the max-min problem can be converted into a problem that maximizes the balanced received power. The optimal power allocation $q_{lk}$ can be obtained by solving the following quadratically constrained linear programming.

$$\begin{aligned} \max_{t, \sqrt{q_{lk}}} \quad & t = \sqrt{s_{\max}^{\mathrm{DL}}} \\ \text{subject to } & |h_{1k}^H|\sqrt{q_{1k}} + \cdots + |h_{Lk}^H|\sqrt{q_{Lk}} = t, \quad \forall k, \\ & \sum_{k=1}^{K} \sqrt{q_{lk}}^2 \le q^{\max}, \quad \forall l, \\ & \sqrt{q_{lk}} \ge 0, \quad \forall k \text{ and } \forall l. \end{aligned} \tag{28}$$

In the DL with the convex problem (28), the balanced received signal power of each user reaches its maximum, and all power of each AP is fully utilized at the boundary of the constraints.

Even though the convex problem can be solved efficiently using optimization software packages, the computational burden may not be negligible for the case that the number of APs or users grows large. An alternative is to fix the direction of the beamforming vector and adapt it in an optimal way. One example is applying matched filtering to the channel, which is traditionally known as MRT [21]. The MRT is the scheme of lowest complexity, maximizes the received power when power is constrained by a per-user basis, and uses the local CSI at each AP. By separating the precoding matrix, $\mathbf{V}$, into the beamforming matrix, $\mathbf{G}$, and the power allocation matrix, $\mathbf{P}$, the optimization problem maximizing the minimum received power is formulated as

$$\begin{aligned} \max_{\mathbf{g}_k, p_k} \quad \min_{k=1,\ldots,K} \quad & p_k |\mathbf{g}_k^H \mathbf{h}_k|^2 = p_k \mathbf{g}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{g}_k \\ \text{subject to} \quad & \|\mathbf{g}_k\|^2 = 1, \quad \forall k, \\ & 0 \le [\mathbf{G}\mathbf{P}\mathbf{G}^H]_{l,l} \le q^{\max}, \quad \forall l. \end{aligned} \tag{29}$$

As the same case as the UL, the optimal $\mathbf{g}_k$ is given as

$$\mathbf{g}_k^* = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}. \tag{30}$$

Again, after adopting the beamforming vector, the optimally allocated power to each user $k$ should be inversely proportionally to its channel gain, that is,

$$p_k^* = \frac{s_{\max}^{\mathrm{DL}}}{\|\mathbf{h}_k\|^2}. \tag{31}$$

The balanced maximum received power in the downlink can be calculated by inserting the above beamforming vector and the power allocation into the per-AP power constraints. Thus, the balanced maximum received power in the downlink is given as

$$s_{\max}^{\mathrm{DL}} = \frac{q^{\max}}{\max_l \left[ \sum_{k=1}^{K} \frac{\mathbf{h}_k \mathbf{h}_k^H}{\|\mathbf{h}_k\|^4} \right]_{l,l}}, \tag{32}$$

and the interference power of user $k$ is

$$\begin{aligned} r_k^{\mathrm{DL}} &= \sum_{i=1, i\neq k}^{K} p_i \mathbf{h}_k^H \mathbf{g}_i \mathbf{g}_i^H \mathbf{h}_k \\ &= \mathbf{h}_k^H \left( \sum_{i=1, i\neq k}^{K} \frac{s_{\max}^{\mathrm{DL}}}{\|\mathbf{h}_i\|^2} \frac{\mathbf{h}_i}{\|\mathbf{h}_i\|} \frac{\mathbf{h}_i^H}{\|\mathbf{h}_i\|} \right) \mathbf{h}_k \\ &= \sum_{i=1, i\neq k}^{K} s_{\max}^{\mathrm{DL}} \frac{|\mathbf{h}_k^H \mathbf{h}_i|^2}{\|\mathbf{h}_i\|^4}. \end{aligned} \tag{33}$$

Then, the SINR of each user $k$ is given as

$$\mathrm{SINR}_k^{\mathrm{DL}} = \frac{s_{\max}^{\mathrm{DL}}}{s_{\max}^{\mathrm{DL}} \sum_{i=1, i\neq k}^{K} \frac{|\mathbf{h}_k^H \mathbf{h}_i|^2}{\|\mathbf{h}_i\|^4} + \sigma^2}. \tag{34}$$

For the max-min fairness on the received signal power and the minimization of the total transmit power, $p_k$ should be inversely proportional to $\|\mathbf{h}_k\|^2$ in both in the uplink and the downlink. The allocation schemes look like exactly the same, but the optimal power allocation in the UL is entirely determined by the column of the channel ($\|\mathbf{h}_k\|^2$), whereas the optimal power allocation in the DL is determined by both the column and row of the channel ($\mathbf{h}_k$ and $\mathbf{h}_k^H$). Thus, even if $p^{\max} = q^{\max}$ and $\mathbf{H}$ is square, i.e., $K = L$, the optimal power allocation schemes in the UL and in the DL are not the same, neither are $s_{\max}^{\mathrm{UL}}$ and $s_{\max}^{\mathrm{DL}}$.

In the UL, each pair of two users shares a common component in their interference. For example, user $i$ and $j$ have the same interference component consisting of $\frac{|\mathbf{h}_i^H \mathbf{h}_j|^2}{\|\mathbf{h}_i\|^2 \|\mathbf{h}_j\|^2}$. When $K = 2$, the interference is the same for the both users. On the other hand, in the DL, interference is inversely proportional to the other users' channel gain. Thus, in the DL, the difference of interference among users seems to be generally greater than that in the UL, resulting in a more even distribution of SINR in the UL. The simulation results support this conjecture.

### C. OPTIMIZATION ON INTERFERENCE AND SIGNAL POWER IN THE UPLINK

Through beamforming and power allocation, we can control the interference power of each user. For the fairness on interference in the UL, we formulate the min-max problem of interference as follows:

$$\begin{aligned}\min_{\mathbf{f}_k,p_k} \quad \max_{k=1,\ldots,K} \quad & \sum_{i=1,i\neq k}^{K} p_i|\mathbf{f}_k^H\mathbf{h}_i|^2 = \mathbf{f}_k^H\mathbf{H}_{-k}\mathbf{P}_{-k}\mathbf{H}_{-k}^H\mathbf{f}_k \\ \text{subject to} \quad & \|\mathbf{f}_k\|^2 = 1, \quad \forall k, \\ & 0 \leq p_k \leq p^{\max}, \quad \forall k. \end{aligned} \tag{35}$$

Again, in the UL, $\|\mathbf{f}_k\|^2$ does not need to be normalized to be 1, but can be set to an arbitrary scalar. Nevertheless, we introduce the constrains to separate power allocation from beamforming.

We consider the case that the number of APs is no less than that of users (i.e., $L \geq K$) and the channel is full-rank. Thus, the interference of each user can be always cancelled out by choosing $\mathbf{f}_k$ in the left null space of $\mathbf{H}_{-k}$ such that $\mathbf{f}_k^H\mathbf{H}_{-k}\mathbf{P}_{-k}\mathbf{H}_{-k}^H\mathbf{f}_k = 0$ for all $k$ since $\dim\left(\mathcal{N}(\mathbf{H}_{-k}^H)\right) = L - (K-1) \geq 1$. If $L = K$, i.e., $\dim\left(\mathcal{N}(\mathbf{H}_{-k}^H)\right) = 1$, the optimal $\mathbf{f}_k$ is uniquely determined by the single basis in the left null space of $\mathbf{H}_{-k}$ for all $k$. Otherwise, there are multiple candidates of $\mathbf{f}_k$ nulling out interference. Among them, we select the one which maximizes the received signal power (i.e., SINR with zero interference) using the matched filtering. It is known as the decorrelator, or ZF combining [22], [23]. The projection matrix on the left null space of $\mathbf{H}_{-k}$ is given as

$$\mathbf{Q}_k = \mathbf{I} - \mathbf{H}_{-k}(\mathbf{H}_{-k}^H\mathbf{H}_{-k})^{-1}\mathbf{H}_{-k}^H. \tag{36}$$

The ZF beamforming which is the matched filtering after the projection can be written as

$$\mathbf{f}_k^* = \frac{\mathbf{Q}_k^H(\mathbf{Q}_k\mathbf{h}_k)}{\left\|\mathbf{Q}_k^H(\mathbf{Q}_k\mathbf{h}_k)\right\|} = \frac{\mathbf{Q}_k\mathbf{h}_k}{\|\mathbf{Q}_k\mathbf{h}_k\|}, \tag{37}$$

or, equivalently, in the well known form of the left inverse, the optimal beamforming without norm normalization on each column vector is

$$\mathbf{F}_{\text{unnormalized}}^* = \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}. \tag{38}$$

The normalized optimal vector of each user is given by each column of $\mathbf{F}_{\text{unnormalized}}^*$ divided by its norm. For example, when the number of users is 2, the norms of first and second column of (38) are $\frac{\|\mathbf{h}_2\|}{\sqrt{D}}$ and $\frac{\|\mathbf{h}_1\|}{\sqrt{D}}$ respectively, where $D$ is the determinant of $\mathbf{H}^H\mathbf{H}$, that is, $\|\mathbf{h}_1\|^2\|\mathbf{h}_2\|^2 - |\mathbf{h}_1^H\mathbf{h}_2|^2$. Thus, the normalized optimal vectors for the case of $K = 2$ is given as,

$$(\mathbf{F}_{\text{2-user}}^*)^H = \frac{1}{\sqrt{D}}\begin{bmatrix} \frac{1}{\|\mathbf{h}_2\|}\left(\|\mathbf{h}_2\|^2\mathbf{h}_1^H - \mathbf{h}_1^H\mathbf{h}_2\mathbf{h}_2^H\right) \\ \frac{1}{\|\mathbf{h}_1\|}\left(\|\mathbf{h}_1\|^2\mathbf{h}_2^H - \mathbf{h}_2^H\mathbf{h}_1\mathbf{h}_1^H\right)\end{bmatrix}. \tag{39}$$

With the beamforming vectors residing in the left null space of $\mathbf{H}_{-k}$, the power allocation does not change the direction of each beamforming vector because of interference cancelled out to be zero and signal multiplied by a scalar. This can also be identified by the following equation, which represents the received power of each user with the optimal beamforming matrix given as,

$$\left((\mathbf{H}\sqrt{\mathbf{P}})^H\mathbf{H}\sqrt{\mathbf{P}}\right)^{-1}(\mathbf{H}\sqrt{\mathbf{P}})^H = (\sqrt{\mathbf{P}})^{-1}(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H. \tag{40}$$

Noting that multiplying any diagonal matrix on the left side of a matrix does not change the direction of each row of the matrix, we can also separately optimize beamforming and power allocation without affecting the optimality. To optimize power allocation maximizing the SINR of each user, or equivalently, the minimum received power, we formulate an additional power allocation problem as

$$\begin{aligned}\max_{p_k} \quad \min_{k=1,\ldots,K} \quad & \frac{p_k|\mathbf{h}_k^H\mathbf{f}_k^*|^2}{\sigma^2} \\ \text{subject to} \quad & 0 \leq p_k \leq p^{\max}, \quad \forall k.\end{aligned} \tag{41}$$

Once again, the optimal solution is not unique, as in the case of optimizing the received signal power in the UL. Following the same philosophy to choose the solution that minimizes the total transmit power of all users, the optimal allocated power is given as

$$p_k^* = \begin{cases} p^{\max} & \text{if } k = \arg\min_i |\mathbf{h}_i^H\mathbf{f}_i^*|^2, \\ \frac{s_{\max}^{\text{UL}}}{|\mathbf{h}_k^H\mathbf{f}_k^*|^2} & \text{otherwise,}\end{cases} \tag{42}$$

where the evenly balanced maximum received power is given as

$$s_{\max}^{\text{UL}} = p^{\max}\min_i |\mathbf{h}_i^H\mathbf{f}_i^*|^2. \tag{43}$$

Note that the optimal allocated power in (42) is the same as the one in (21) when $K = 2$. That is, even with the different beamforming scheme given as ZF or MRC, the optimally allocated power is regardlessly the same when the number of users is 2. This can be verified directly by inserting (39) into (42) and comparing the resulting equation with (21).

With the optimal beamforming vectors and power allocation, the interference of each user is zero, and the SINR of user $k$ is balanced to the same value as

$$\text{SINR}_k^{\text{UL}} = \frac{s_{\max}^{\text{UL}}}{\sigma^2}. \tag{44}$$

### D. OPTIMIZATION ON INTERFERENCE AND SIGNAL POWER IN THE DOWNLINK

In the DL, the min-max problem for optimizing the interference of each user is formulated as

$$\begin{aligned}\min_{\mathbf{v}_k} \quad \max_{k=1,\ldots,K} \quad & \sum_{i=1,i\neq k} |\mathbf{v}_i^H\mathbf{h}_k|^2 = \mathbf{h}_k^H\mathbf{V}_{-k}\mathbf{V}_{-k}^H\mathbf{h}_k \\ \text{subject to} \quad & |g_{lk}|^2 = 1, \quad \forall k \text{ and } \forall l, \\ & \sum_{k=1}^{K} q_{lk} = q_l \leq q^{\max}, \quad \forall l,\end{aligned} \tag{45}$$

where $v_{lk} = \sqrt{q_{lk}}g_{lk}$ is the precoding weight.

Without loss of generality, we may choose $\mathbf{v}_k$ so that $\mathbf{h}_k^H\mathbf{v}_k$ is real. As the same case of the uplink, we can completely null out all the interference such that $\max_k \mathbf{h}_k^H\mathbf{V}_{-k}\mathbf{V}_{-k}^H\mathbf{h}_k = 0$ since $\dim\left(\mathcal{N}(\mathbf{h}_i^H)\right) = L-1 \geq K-1 = \dim\left(\mathcal{C}(\mathbf{V}_{-k})\right)$. Under per-AP constraints, ZF based on the pseudo-inverse is not generally optimal, but a numerical optimization programing based on the generalized inverse is needed to find the optimal beamforming and power allocation [24]. This programing for maximizing the minimum SINR is a convex second order cone programing (SOCP), and the SOCP with $KL$ unknowns can be solved via the interior-point algorithm, which has computational complexity of $\mathcal{O}(K^3L^3)$ [25], [26]. Even though the algorithm is efficient, the complexity may not be ignored for the case that a large number of $L$ or $K$ are involved. Thus, instead of adopting the precoding scheme from the generalized inverse, we choose the one using the pseudo-inverse. The pseudo-inverse is optimal when the constraint is given as the total transmit power, which is not quite adequate for per-AP constraints. However, the pseudo-inverse returns a closed-form solution and shows a considerable performance improvement comparing to schemes based on MRT.

The ZF precoding using the pseudo-inverse with the per-AP constraints is given by [27],

$$\mathbf{V} = \sqrt{s_{\max}^{\mathrm{DL}}}\mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}, \tag{46}$$

where $s_{\max}^{\mathrm{DL}}$ is a scale factor for satisfying power constraint and also corresponds to the balanced received signal power.

To meet the per-AP constraints, $s_{\max}^{\mathrm{DL}}$ should be given as

$$s_{\max}^{\mathrm{DL}} = \frac{q^{\max}}{\max_l\left[\mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\right]_{l,l}}. \tag{47}$$

For the AP of $l \neq \arg\max_i\left[\mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\right]_{i,i}$, the available power of it is not fully utilized.

With the ZF precoding, the SINR of each user is balanced to have the same value for all $k$, which is given as

$$\mathrm{SINR}_k^{\mathrm{DL}} = \frac{s_{\max}^{\mathrm{DL}}}{\sigma^2}. \tag{48}$$

### E. OPTIMIZATION ON SIGNAL TO INTERFERENCE AND NOISE RATIO IN THE UPLINK

In the UL, we will optimize the SINR of each user to reach an evenly balanced value. For fairness on the SINR, we formulate the optimization problem as the following:

$$\begin{aligned}\max_{\mathbf{f}_k,p_k}\ \min_{k=1,\ldots,K}\ & \frac{p_k\mathbf{f}_k^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{f}_k}{\mathbf{f}_k^H\left(\sum_{i=1,i\neq k}^K p_i\mathbf{h}_i\mathbf{h}_i^H + \sigma^2\mathbf{I}\right)\mathbf{f}_k}\\ &= \frac{p_k|\mathbf{f}_k^H\mathbf{h}_k|^2}{\sum_{i=1,i\neq k}^K p_i|\mathbf{f}_k^H\mathbf{h}_i|^2+\sigma^2}\\ \text{subject to}\ \ & \|\mathbf{f}_k\|^2 = 1, \quad \forall k,\\ & 0 \leq p_k \leq p^{\max}, \quad \forall k.\end{aligned} \tag{49}$$

The same problem was covered in [14], which includes channel estimation and formulates the power allocation problem into a standard geometric programming. However, in this paper, all solutions related to the beamforming and power allocation are given in closed-forms.

As in the case of the optimization of received signal power in the UL, the max-min fairness of SINR does not imply the same value of SINR for all users. Among optimal solutions, we choose the beamforming and power allocation scheme which minimizes total transmit power and maximizes an evenly balanced SINR. The optimal beamforming cannot be independently obtained without considering the power allocation. However, given the power allocation $\mathbf{P}$, maximizing SINR through beamforming corresponds to the generalized Rayleigh quotient. Thus, we can obtain the optimal beamforming vector by solving the generalized eigenvalue problem. This beamforming vector is known as MMSE and given as

$$\mathbf{f}_k^* = \frac{\left(\mathbf{HPH}^H - p_k\mathbf{h}_k\mathbf{h}_k^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_k}{\left\|\left(\mathbf{HPH}^H - p_k\mathbf{h}_k\mathbf{h}_k^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_k\right\|} \tag{50}$$

$$= \frac{\left(\mathbf{HPH}^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_k}{\left\|\left(\mathbf{HPH}^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_k\right\|}. \tag{51}$$

Equation (51) is derived from (50) by using Sherman-Morrison formula. On the contrary to most beamforming and power allocation schemes in the previous sections, $\mathbf{f}_k^*$ depends on $\mathbf{P}$. Thus, the beamforming cannot be independently optimized without considering the power allocation.

With the optimal beamforming and a given power allocation, the SINR of user $k$ is

$$\begin{aligned}\mathrm{SINR}_k^{\mathrm{UL}} &= p_k\mathbf{h}_k^H\left(\mathbf{HPH}^H - p_k\mathbf{h}_k\mathbf{h}_k^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_k\\ &= \frac{1}{1 - p_k\mathbf{h}_k^H\left(\mathbf{HPH}^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_k} - 1.\end{aligned} \tag{52}$$

Since it is not easy to find the optimal $\mathbf{P}$ balancing the SINR of each user directly from (52), we may use the approach in [28], [29], and [30].

The evenly balanced SINR should be given as

$$\gamma^{\mathrm{UL}} = \frac{p_k|\mathbf{f}_k^H\mathbf{h}_k|^2}{\sum_{i=1,i\neq k}^K p_i|\mathbf{f}_k^H\mathbf{h}_i|^2+\sigma^2}, \quad \forall k. \tag{53}$$

By rearranging the above equation, we can draw the same equation as

$$p_k - \sum_{i\neq k}^K p_i\frac{\gamma^{\mathrm{UL}}|\mathbf{f}_k^H\mathbf{h}_i|^2}{|\mathbf{f}_k^H\mathbf{h}_k|^2} = \frac{\gamma^{\mathrm{UL}}\sigma^2}{|\mathbf{f}_k^H\mathbf{h}_k{}^2}, \quad \forall k. \tag{54}$$

In a matrix form, the equation is equivalently written as

$$(\mathbf{I} - \gamma^{\mathrm{UL}}\mathbf{A})\mathbf{p} = \gamma^{\mathrm{UL}}\mathbf{b}, \tag{55}$$

where $\mathbf{p} = [p_1, p_2, \ldots, p_K]^H$, and $\mathbf{A}$ is the $K \times K$ matrix with strictly positive off-diagonal elements

$$a_{ij} = \begin{cases} 0 & \text{if } i = j,\\ \dfrac{|\mathbf{f}_i^H\mathbf{h}_j|^2}{|\mathbf{f}_i^H\mathbf{h}_i|^2} & \text{if } i \neq j,\end{cases} \tag{56}$$

and,

$$\mathbf{b} = \left[ \frac{\sigma^2}{|\mathbf{f}_1^H \mathbf{h}_1|^2}, \frac{\sigma^2}{|\mathbf{f}_2^H \mathbf{h}_2|^2}, \ldots, \frac{\sigma^2}{|\mathbf{f}_K^H \mathbf{h}_K|^2} \right]^H. \quad (57)$$

$\mathbf{A}$ is a primitive matrix since $\mathbf{A}^2$ is a matrix with strictly positive elements. Thus, $\mathbf{A}$ is irreducible non-negative matrix. It is shown that $(\mathbf{I} - \gamma^{\mathrm{UL}}\mathbf{A})^{-1} \geq \mathbf{0}$ iff $\rho(\mathbf{A}) < 1/\gamma^{\mathrm{UL}}$ [31]. By Perron-Frobenius theorem, if $\gamma^{\mathrm{UL}} < 1/\rho(\mathbf{A}) = \gamma^{\max}$, the unique power vector is given as,

$$\tilde{\mathbf{p}}^* = \gamma^{\mathrm{UL}}(\mathbf{I} - \gamma^{\mathrm{UL}}\mathbf{A})^{-1}\mathbf{b}. \quad (58)$$

If the optimally balanced SINR $\gamma^* \leq \gamma^{\max}$, then, $\gamma^*\rho(\mathbf{A}) < 1$. Thus, $\tilde{\mathbf{p}}^*$ is guaranteed to be a positive vector.

With given $\mathbf{A}$ and $\mathbf{b}$, if $\gamma^{\mathrm{UL}} < \gamma^{\max}$, $\tilde{\mathbf{p}}^*$ is a (element-wise) monotonically increasing function of $\gamma^{\mathrm{UL}}$. It follows from

$$\tilde{\mathbf{p}}^* = \gamma^{\mathrm{UL}}(\mathbf{I} - \gamma^{\mathrm{UL}}\mathbf{A})^{-1}\mathbf{b} = \gamma^{\mathrm{UL}} \sum_{k=0}^{\infty} (\gamma^{\mathrm{UL}}\mathbf{A})^k \mathbf{b}, \quad (59)$$

and, from the fact that if $\gamma_1 > \gamma_2$, then,

$$(\gamma_1 \mathbf{A})^k > (\gamma_2 \mathbf{A})^k. \quad (60)$$

Since $\tilde{\mathbf{p}}^*$ is a monotonically increasing function of $\gamma^{\mathrm{UL}}$, $\gamma^{\mathrm{UL}}$ is bounded by $1/\rho(\mathbf{A})$, and $\tilde{\mathbf{p}}^*$ is also bounded above, we can obtain a converging value of $\tilde{\mathbf{p}}^*$ by increasing $\gamma^{\mathrm{UL}}$ a small amount at each step to find a suitable $\gamma^{\mathrm{UL}}$ which returns a positive power allocation vector and meets the power constraints at the boundary. However, at each iteration, it is required to calculate the matrix inverse of $\mathbf{I} - \gamma^{\mathrm{UL}}\mathbf{A}$, which might not be computationally affordable. One of alternatives is to apply some reference values of $\gamma^{\mathrm{UL}}$ and increase $\gamma^{\mathrm{UL}}$ appropriately. A maximum value for such reference values can be a maximum achievable SINR for given $\mathbf{P}$ and $\mathbf{F}$, or the value given as the following,

$$\tilde{\gamma}^{\max} = \frac{p^{\max}}{\max_i(b_i) + p^{\max}\rho(\mathbf{A})} < 1/\rho(\mathbf{A}). \quad (61)$$

For a nonnegative matrix $\mathbf{X}$ and a positive vector $\mathbf{b}$, if $\alpha \geq 0$ is such that $\mathbf{Xb} \leq \alpha\mathbf{b}$, then $\rho(\mathbf{X}) \leq \alpha$ [32]. To meet the power constraints, we need $\tilde{\mathbf{p}}^* = \gamma^{\mathrm{UL}}(\mathbf{I} - \gamma^{\mathrm{UL}}\mathbf{A})^{-1}\mathbf{b} \leq \frac{p^{\max}}{\max_k(b_k)}\mathbf{b}$. This implies that $\frac{\gamma^{\mathrm{UL}}}{1-\gamma^{\mathrm{UL}}\rho(\mathbf{A})} \leq \frac{p^{\max}}{\max_k(b_k)}$. Thus, we get (61). In general, $\mathbf{b}$ is not a eigenvector of $\gamma^{\mathrm{UL}}(\mathbf{I} - \gamma^{\mathrm{UL}}\mathbf{A})^{-1}$, thus, the equality does not hold. To meet the power constraints at the boundary, we need to scale up or down $\tilde{\mathbf{p}}^*$ as

$$\tilde{\mathbf{p}} = \frac{p^{\max}}{\max_k(\tilde{p}_k^*)}\tilde{\mathbf{p}}^*. \quad (62)$$

Note that $\tilde{\mathbf{p}}$ does not returns a balance value of SINR since scaling $\tilde{\mathbf{p}}^*$ with a scalar does not imply scaling SINR.

Using the above results, we propose the joint beamforming and power allocation algorithm 1 to achieve the maximum balanced SINR in the uplink. As the iteration goes on, $\tilde{\mathbf{p}}^*$ converges to have the same value of $\tilde{\mathbf{p}}$.

**Algorithm 1**: Joint Beamforming and Power Allocation for Optimizing the Uplink SINR

$\tilde{\mathbf{p}} \leftarrow [p^{\max}, p^{\max}, \ldots, p^{\max}]^H$
**while** 1 **do**
  $\tilde{\mathbf{f}}_k^* \leftarrow \frac{\left(\mathbf{H}\tilde{\mathbf{P}}\mathbf{H}^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_k}{\left\|\left(\mathbf{H}\tilde{\mathbf{P}}\mathbf{H}^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_k\right\|}$
  $\tilde{\gamma}^{\mathrm{UL}} \leftarrow \min_i(\mathrm{SINR}_i^{\mathrm{UL}})$
  $\tilde{a}_{ij} \leftarrow \begin{cases} 0 & \text{if } i = j, \\ \frac{|\tilde{\mathbf{f}}_i^H \mathbf{h}_j|^2}{|\tilde{\mathbf{f}}_i^H \mathbf{h}_i|^2} & \text{if } i \neq j, \end{cases}$
  $\tilde{\mathbf{b}} \leftarrow \left[ \frac{\sigma^2}{|\tilde{\mathbf{f}}_1^H \mathbf{h}_1|^2}, \frac{\sigma^2}{|\tilde{\mathbf{f}}_2^H \mathbf{h}_2|^2}, \ldots, \frac{\sigma^2}{|\tilde{\mathbf{f}}_K^H \mathbf{h}_K|^2} \right]^H$
  $\tilde{\gamma}^{\max} \leftarrow \min\left[1/\rho(\mathbf{A}), \frac{p^{\max}}{\max_i(b_i) + p^{\max}\rho(\mathbf{A})}, \max_i(\mathrm{SINR}_i^{\mathrm{UL}})\right]$
  $\tilde{\gamma}^* \leftarrow$ set a proper value of $\tilde{\gamma}^{\mathrm{UL}} \leq \tilde{\gamma}^{\max}$
  $\tilde{\mathbf{p}}^* \leftarrow \tilde{\gamma}^*(\mathbf{I} - \tilde{\gamma}^*\mathbf{A})^{-1}\mathbf{b}$
  $\tilde{\mathbf{p}} = \frac{p^{\max}}{\max_k(\tilde{p}_k^*)}\tilde{\mathbf{p}}^*$
  **if** $\max_i(\tilde{p}_i^*) = p^{\max}$ **then**
    break
  **end if**
**end while**

### F. OPTIMIZATION ON SIGNAL TO LEAKAGE AND NOISE RATIO IN THE DOWNLINK

One of the approaches dealing with the downlink problem for optimizing SINR is using uplink - downlink duality in which some downlink problems can be converted into an uplink problem with a sum power constraint [19]. Iterative algorithms are usually adopted to achieve a local optimum solution [33]. Another approach is to optimize SLNR instead of SINR since it has a simple closed solution and shows a considerable performance [20], [34].

We consider the following max-min problem for optimizing SLNR of each user.

$$\begin{aligned} \max_{\mathbf{g}_k, p_k} \min_{k=1,\ldots,K} \quad & \frac{\mathbf{g}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{g}_k}{\mathbf{g}_k^H \left((\mathbf{H}\mathbf{H}^H - \mathbf{h}_k\mathbf{h}_k^H) + \frac{\sigma^2}{p_k}\mathbf{I}\right)\mathbf{g}_k} \\ & = \frac{|\mathbf{g}_k^H \mathbf{h}_k|^2}{\sum_{i=1, i\neq k}^K |\mathbf{g}_k^H \mathbf{h}_i|^2 + \frac{\sigma^2}{p_k}} \\ \text{subject to} \quad & \|\mathbf{g}_i\|^2 = 1, \quad \forall i \\ & 0 \leq [\mathbf{G}\mathbf{P}\mathbf{G}^H]_{l,l} \leq q^{\max}, \quad \forall l. \end{aligned} \quad (63)$$

As the same case in the UL, optimizing SLNR problems can also be viewed as the generalized Rayleigh quotient. Thus, the optimal beamforming vector for a given allocated power of user $k$ is given as

$$\mathbf{g}_k^* = \frac{\left(\mathbf{H}\mathbf{H}^H + \frac{\sigma^2}{p_k}\mathbf{I}\right)^{-1}\mathbf{h}_k}{\left\|\left(\mathbf{H}\mathbf{H}^H + \frac{\sigma^2}{p_k}\mathbf{I}\right)^{-1}\mathbf{h}_k\right\|}, \tag{64}$$

$$= \frac{\mathbf{U}\left(\Sigma\Sigma^H + \frac{\sigma^2}{p_k}\mathbf{I}\right)^{-1}\mathbf{U}^H\mathbf{h}_k}{\left\|\mathbf{U}\left(\Sigma\Sigma^H + \frac{\sigma^2}{p_k}\mathbf{I}\right)^{-1}\mathbf{U}^H\mathbf{h}_k\right\|}, \tag{65}$$

where $\mathbf{U}$ and $\Sigma$ are the matrices in singular value decomposition of $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^H$. Equation (64) looks similar to (51), but, on contrary to the UL, we may need inverse operations of matrices as many as the number of users. In that case, (65) is helpful since $\Sigma\Sigma^H + \frac{\sigma^2}{p_k}\mathbf{I}$ is a diagonal matrix, thus only element-wise inversion is needed for matrix inversion.

With the optimal beamforming and a given power allocation, the SLNR of user $k$ is given as

$$\mathrm{SLNR}_k^{\mathrm{DL}} = \mathbf{h}_k^H\left(\mathbf{H}\mathbf{H}^H - \mathbf{h}_k\mathbf{h}_k^H + \frac{\sigma^2}{p_k}\mathbf{I}\right)^{-1}\mathbf{h}_k$$

$$= \frac{1}{1 - \mathbf{h}_k^H\left(\mathbf{H}\mathbf{H}^H + \frac{\sigma^2}{p_k}\mathbf{I}\right)^{-1}\mathbf{h}_k} - 1. \tag{66}$$

We can follow similar steps used in the UL to derive the power allocation for an evenly balanced SLNR. For a balanced SLNR, the allocated power to each user should meet the equation given as

$$\gamma^{\mathrm{DL}} = \frac{|\mathbf{g}_k^H\mathbf{h}_k|^2}{\sum_{i=1,i\neq k}^K |\mathbf{g}_k^H\mathbf{h}_i|^2 + \frac{\sigma^2}{p_k}}, \quad \forall k. \tag{67}$$

By rearranging (67), we get

$$\left(1 - \sum_{i\neq k}^K \frac{\gamma^{\mathrm{DL}}|\mathbf{g}_k^H\mathbf{h}_i{}^2}{|\mathbf{g}_k^H\mathbf{h}_k{}^2}\right)p_k = \frac{\gamma^{\mathrm{DL}}\sigma^2}{|\mathbf{g}_k^H\mathbf{h}_k|^2} \quad \forall k, \tag{68}$$

or, in a matrix form,

$$\left[(\mathbf{I} - \gamma^{\mathrm{DL}}\mathbf{A})\mathbf{1}_K\right] \odot \mathbf{p} = \gamma^{\mathrm{DL}}\mathbf{b}, \tag{69}$$

where $\odot$ is an element-wise product, $\mathbf{1}_K$ is a size $K$ vector with elements of all 1's, $\mathbf{A}$ is the $K \times K$ matrix with strictly positive off-diagonal elements of

$$a_{ij} = \begin{cases} 0 & \text{if } i = j, \\ \frac{|\mathbf{g}_i^H\mathbf{h}_j|^2}{|\mathbf{g}_i^H\mathbf{h}_i|^2} & \text{if } i \neq j, \end{cases} \tag{70}$$

and,

$$\mathbf{b} = \left[\frac{\sigma^2}{|\mathbf{g}_1^H\mathbf{h}_1|^2}, \frac{\sigma^2}{|\mathbf{g}_2^H\mathbf{h}_2|^2}, \ldots, \frac{\sigma^2}{|\mathbf{g}_K^H\mathbf{h}_K|^2}\right]^H. \tag{71}$$

With given $\mathbf{G}$ and $\mathbf{H}$, $p_k$ is a monotonically increasing function of $\gamma^{\mathrm{DL}}$. It follows from the fact that

$$p_k = \frac{\sigma^2}{\frac{|\mathbf{g}_k^H\mathbf{h}_k|^2}{\gamma^{\mathrm{DL}}} - \sum_{i\neq k}^K |\mathbf{g}_k^H\mathbf{h}_i|^2}. \tag{72}$$

The transmit power of each AP is also a monotonically increasing function of $\gamma^{\mathrm{DL}}$ since $p_k$ is so. If and only if $0 < \gamma^{\mathrm{DL}} < \min_k \sum_{i\neq k}^K \frac{|\mathbf{g}_k^H\mathbf{h}_k|^2}{|\mathbf{g}_k^H\mathbf{h}_i|^2}$, the allocated power to each user is positive, and the power vector is given as

$$\tilde{\mathbf{p}}^* = \gamma^{\mathrm{DL}}\mathbf{b} \oslash \left[(\mathbf{I} - \gamma^{\mathrm{DL}}\mathbf{A})\mathbf{1}_K\right] \tag{73}$$

where $\oslash$ is an element-wise division, and $\tilde{\mathbf{p}}^*$ is the power allocation vector achieving the balanced SLNR value, $\gamma^{\mathrm{DL}}$. Since $p_k$ is a monotonically increasing function of $\gamma^{\mathrm{DL}}$ for all $k$, we adopt the bisection method to set a proper value of $\gamma^{\mathrm{DL}}$. In the method, we iteratively set the value of $\gamma^{\mathrm{DL}}$ until we achieve a value as large as possible, that is, the largest value meeting the power constraints, $\max_l[\mathbf{G}\mathbf{P}\mathbf{G}^H]_{l,l} \leq q^{\max}$.

To meet the per-AP power constraints at the boundary, the transmit power of each user should be updated as

$$\tilde{\mathbf{p}} = \frac{q^{\max}}{\max_l\left[\mathbf{G}\tilde{\mathbf{P}}^*\mathbf{G}^H\right]_{l,l}}\tilde{\mathbf{p}}^*. \tag{74}$$

Note that $\tilde{\mathbf{p}}$ does not return a balanced value of SLNR, but one of the transmit power of each AP is set to be $q^{\max}$.

The optimal power allocation can be calculated given the optimal beamforming matrix, and vice versa. Thus, we propose the joint beamforming and power allocation algorithm for optimizing the downlink SLNR in which we iteratively update $\gamma^{\mathrm{DL}}$, $\mathbf{p}$, and $\mathbf{G}$ until $\gamma^{\mathrm{DL}}$ converges enough.

**Algorithm 2** : Joint Beamforming and Power Allocation for Optimizing the Downlink SLNR

$\tilde{\mathbf{p}} \leftarrow \left[\frac{q^{\max}}{K}, \frac{q^{\max}}{K}, \ldots, \frac{q^{\max}}{K}\right]^H$

**while** 1 **do**

$\quad \tilde{\mathbf{g}}_k^* \leftarrow \frac{\mathbf{U}\left(\Sigma\Sigma^H + \frac{\sigma^2}{\tilde{p}_k}\mathbf{I}\right)^{-1}\mathbf{U}^H\mathbf{h}_k}{\left\|\mathbf{U}\left(\Sigma\Sigma^H + \frac{\sigma^2}{\tilde{p}_k}\mathbf{I}\right)^{-1}\mathbf{U}^H\mathbf{h}_k\right\|}$

$\quad \tilde{\gamma}^{\mathrm{DL}} \leftarrow \min_i(\mathrm{SLNR}_i^{\mathrm{DL}})$

$\quad \tilde{a}_{ij} \leftarrow \begin{cases} 0 & \text{if } i = j, \\ \frac{|\mathbf{g}_i^H\mathbf{h}_j|^2}{|\mathbf{g}_i^H\mathbf{h}_i|^2} & \text{if } i \neq j, \end{cases}$

$\quad \tilde{\mathbf{b}} \leftarrow \left[\frac{\sigma^2}{|\tilde{\mathbf{g}}_1^H\mathbf{h}_1|^2}, \frac{\sigma^2}{|\tilde{\mathbf{g}}_2^H\mathbf{h}_2|^2}, \ldots, \frac{\sigma^2}{|\tilde{\mathbf{g}}_K^H\mathbf{h}_K|^2}\right]^H$

$\quad \tilde{\gamma}^{\max} \leftarrow \min_i \sum_{k\neq i}^K \frac{|\mathbf{g}_i^H\mathbf{h}_i|^2}{|\mathbf{g}_i^H\mathbf{h}_k|^2}$

$\quad \tilde{\gamma}^* \leftarrow$ set a proper value of $\tilde{\gamma}^{\mathrm{DL}} \leq \tilde{\gamma}^{\max}$

$\quad \tilde{\mathbf{p}}^* \leftarrow \tilde{\gamma}^*\mathbf{b} \oslash \left[(\mathbf{I} - \tilde{\gamma}^*\mathbf{A})\mathbf{1}_K\right]$

$\quad \tilde{\mathbf{p}} = \frac{\tilde{\mathbf{p}}^*}{\max_l\left[\mathbf{G}\tilde{\mathbf{P}}^*\mathbf{G}^H\right]_{l,l}}q^{\max}$

$\quad$ **if** $\tilde{\gamma}^{\mathrm{DL}} = \tilde{\gamma}^*$ **then**

$\quad\quad$ break

$\quad$ **end if**

**end while**

## IV. NUMERICAL RESULTS AND DISCUSSIONS

We study the performance of the beamforming and power allocation schemes and discuss the numerical results.

**TABLE 1. Simulation parameters.**

| Parameter | Value |
|---|---|
| Carrier frequency | 30 GHz |
| Bandwidth | 80 MHz |
| AP antenna height | 3 m |
| User antenna height | 1.5 m |
| Maximum transmit power per AP | 23 dBm |
| Maximum transmit power per user | 23 dBm |
| AP noise figure | 7 dB |
| User noise figure | 10 dB |
| Antenna element gain | 0 dBi |
| Thermal noise level | -174 dBm/Hz |
| Number of APs | 20 or 200 |
| Number of users | 2 or 20 |

We assume that a cell-free MIMO system is deployed at certain geographical areas such as smart factories, stadiums, shopping malls and offices.

### A. SIMULATION PARAMETERS AND SETUP

The simulation parameters and setup are mostly taken from the indoor hotspot model in the ITU-R guideline document for evaluation of 5G technologies [35]. In the indoor hotspot model, the test site consists of one floor of a building. The height of the floor is 3m, and the surface of the floor is 120 m × 50 m. The number of APs in our simulation setup is 20 or 200, and APs are evenly placed in length and width on the ceiling. The number of users is 2 or 20, and users are randomly and uniformly dropped throughout the floor. These distinct figures represent two scenarios: One where the number of APs is as small as the number of users, and another where that of APs is large enough to provide high SNR to each user. The simulation parameters are summarized in Table 1.

The channel model is also drawn from [35] and specifically defined as follows. Internal walls are not explicitly shown but are modeled via the stochastic line of sight (LoS) probability model given as

$$P_{\text{LoS}} = \begin{cases} 1, & d_{2\text{D}} \leq 5m, \\ \exp\left(-\dfrac{d_{2\text{D}}-5}{70.8}\right), & 5m \leq d_{2\text{D}} \leq 49m, \\ \exp\left(-\dfrac{d_{2\text{D}}-49}{211.7}\right)\cdot 0.54, & 49m \leq d_{2\text{D}}, \end{cases} \tag{75}$$

where $d_{2\text{D}}$ is the distance considering only length and width between an AP and a user. The pathloss model in the LoS case is given by

$$PL_{\text{LoS}} = 32.4 + 17.3\log_{10}(d_{3\text{D}}) + 20\log_{10}(f_c), \tag{76}$$

where $d_{3\text{D}}$ is the 3-dimensional distance, and $f_c$ is the carrier frequency. The pathloss model in non-line of sight (NLoS) case is given by $\max(PL_{\text{LoS}}, PL_{\text{NLoS}})$, and $PL_{\text{NLoS}}$ is given as

$$PL_{\text{NLoS}} = 17.3 + 38.3\log_{10}(d_{3\text{D}}) + 24.9\log_{10}(f_c). \tag{77}$$

The shadow fading is modeled as log-normal distribution, and its standard deviations in the Los and the NLoS are 3 dB and 8.03 dB, respectively. The distribution of the fast fading is assumed to be the Rayleigh distribution. In the LoS case, the channel is given by the sum of the LoS channel and the NLoS channel coefficients scaled by the desired Ricean K-factor as

$$H_{\text{LoS}} = \sqrt{\frac{1}{K+1}}H_{\text{NLoS}}^{\text{comp}} + \sqrt{\frac{K}{K+1}}H_{\text{LoS}}^{\text{comp}}, \tag{78}$$

where $K$ is the K-factor given by the normal distribution with the mean of 7 and the standard deviation of 4, $H_{\text{NLoS}}^{\text{comp}}$ and $H_{\text{NLoS}}^{\text{comp}}$ are the channel gain containing pathloss, shadow fading, and fast fading for the LoS and the NLoS channels, respectively.

### B. RESULTS AND DISCUSSIONS

The performance of the beamforming and power allocation schemes is compared in terms of spectral efficiency and total transmit power with varying numbers of users and APs. To prevent misinterpretation, particularly for the cases involving a different number of APs, all figures are drawn to the same scale. In the figures related to the UL, UL MR, UL ZF, and UL MSINR represent different optimization schemes for the received signal power, interference and signal power, and signal-to-interference and noise ratio in the UL, respectively. Similarly, in the figures related to the DL, DL MR, DL CB, DL ZF, and DL MSLNR represent different optimization schemes for the received signal power, received signal power with conjugate beamforming, interference and signal power, and signal-to-leakage and noise ratio in the DL, respectively. The thick lines indicate the cases in which 20 users are served simultaneously in a time and frequency resource, while the thin lines correspond to the cases of 2 users. The green lines represent the optimization for the received signal power, the red lines for the interference and signal power, and the blue lines for the signal-to-interference/leakage and noise ratio.

Fig. 3 illustrates the cumulative distribution of spectral efficiency of each user in the UL when the number of APs is 20. The UL MR exhibits the least performance, while UL MSINR shows the highest. When APs significantly outnumber users, the difference in performance is minimal. However, when the number of APs is similar to or on par with that of users, a noticeable difference in performance can be observed. This means that if the dimension of the left null space is large enough to achieve a significantly high value of SINR, a simple scheme such as UL ZF approaches the best performance without requiring the computationally burdensome iterative algorithm. As the number of users increases, the spectral efficiency of each user decreases for all applied schemes. In the case of UL MR, this decrease is due to the increase in interference power as the number of users increases. On the other hand, in UL ZF, interference from other users is completely eliminated. However, spectral efficiency still

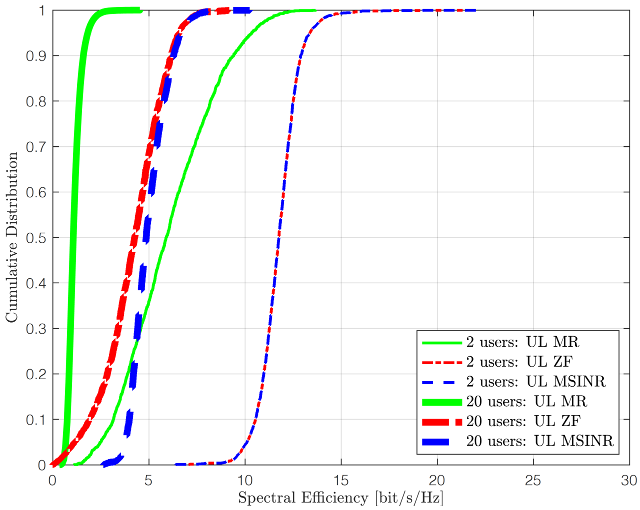


**FIGURE 3.** Spectral efficiency of each user in the UL when the number of APs is 20.

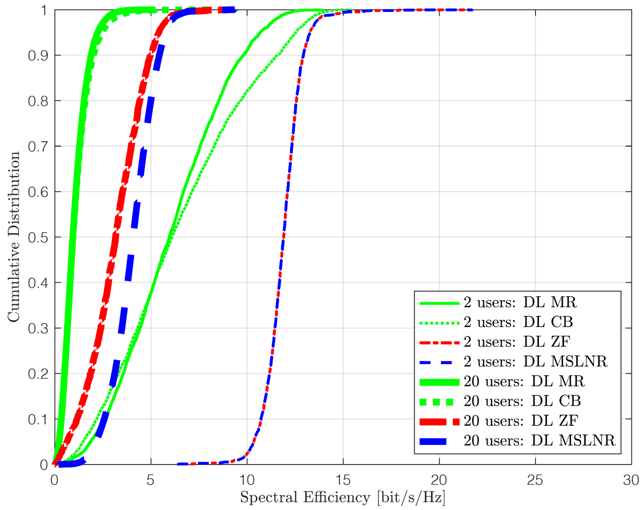


**FIGURE 5.** Spectral efficiency of each user in the DL when the number of APs is 20.

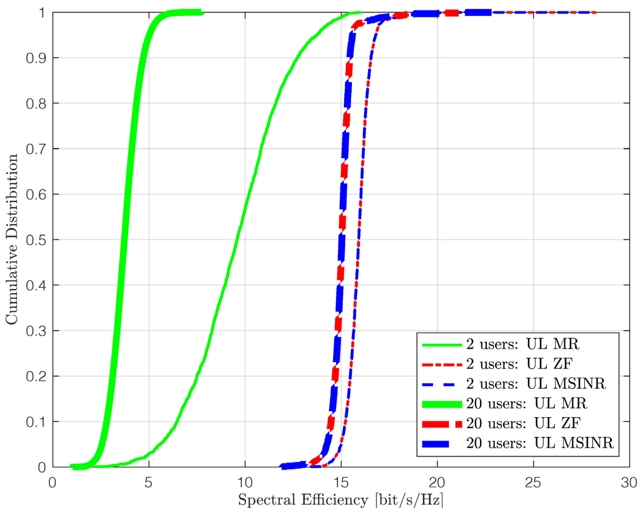


**FIGURE 4.** Spectral efficiency of each user in the UL when the number of APs is 200.

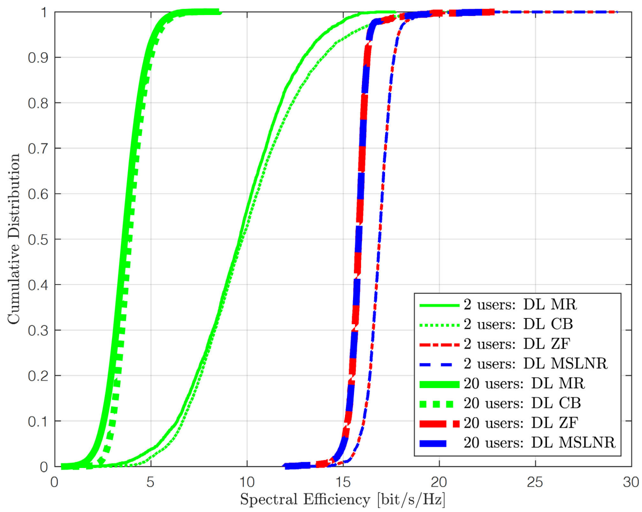


**FIGURE 6.** Spectral efficiency of each user in the DL when the number of APs is 200.

decreases as the matched filtering within the left null space does not significantly increase received signal power when the null space dimension is insufficient.

Fig. 4 shows the cumulative distribution of spectral efficiency of each user in the DL when the number of APs is 200. When comparing UL ZF and UL MSINR, their spectral efficiency is almost identical, just like the case of 20 APs. In contrast to the case of 20 APs, the degradation in performance is relatively minor as the number of users increases, particularly for UL ZF and UL MSINR. Taking into account the fact that the DoF of each user remains constant irrespective of the number of users, provided that there are more APs than users and all APs are cooperating fully to transmit and receive data, it can be concluded that every user's DoF is utilized to its full potential, especially in the case of 200 APs compared to 20 APs. The DoF can be converted into spectral efficiency under the condition that the signal power received from each user is substantial and the interference power from other users is effectively mitigated. By deploying APs densely and evenly across a coverage area and implementing an effective interference control scheme, we can provide every user with high SINR to uniformly high performance. Another aspect we need to consider is the slope of each line. A steep slope indicates that there is minimal variation between users. Steep slopes are observed in UL ZF and UL MSINR, which suggests that there is little variation in performance between users, regardless of their location.

Figs. 5 and 6 depict the cumulative distribution of spectral efficiency for each user in the DL when the number of APs is 20 and 200, respectively. Similar to the UL, we see that by increasing the number of APs and implementing effective interference mitigation schemes, high data rates can be delivered to all users regardless of their locations or the number of users, even as the number of users increases. When

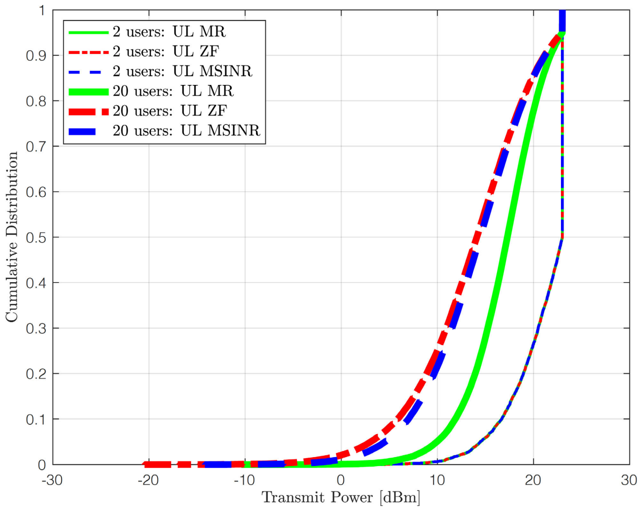


FIGURE 7. Transmit power of each user when the number of APs is 20.

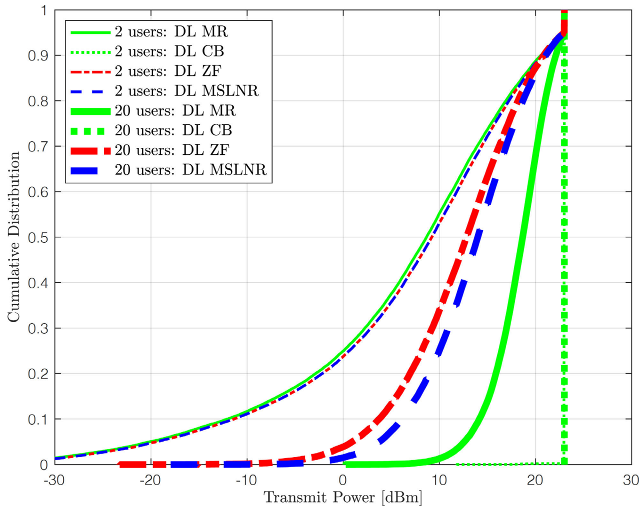


FIGURE 9. Transmit power of each AP when the number of APs is 20.

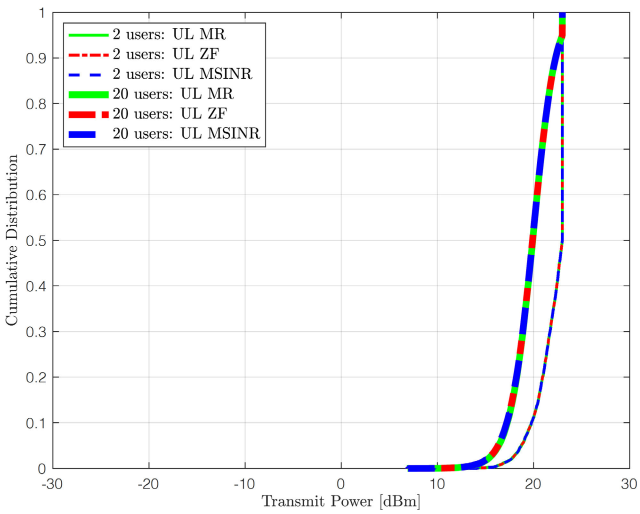


FIGURE 8. Transmit power of each user when the number of APs is 200.

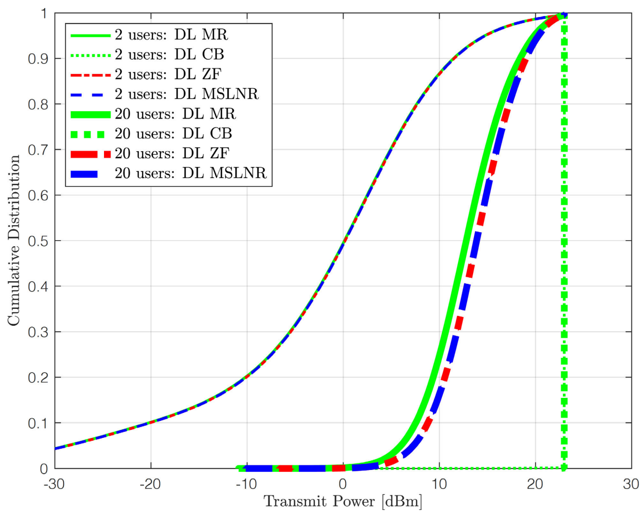


FIGURE 10. Transmit power of each AP when the number of APs is 200.

comparing DL MR and DL CB, we can observe that DL CB offers a slightly higher gain, and this gain is more prominent when the number of APs and users is small. In the case of DL MR and DL CB, especially for the 2-user scenario, the slopes of lines are not very steep, which indicates low fairness between users. This relatively low fairness for the 2-user case in DL MR and DL CB is due to high variations in SINR values when the number of users is small.

Fig. 7 depicts the cumulative distribution of transmit power for each user in the UL when the number of APs is 20. Note that when the number of users is 2, the transmit power for each user is identical for both UL MR and UL ZF, as verified in Section III-C. Additionally, UL ZF shows almost the same performance as UL MSINR, resulting in all three lines overlapping into a single line. When the number of users is 20, UL MR utilizes more power than UL ZF or UL MSINR. This implies that despite using more power, UL MR achieves even lower spectral efficiency. Thus, effective interference mitigation schemes play an important role when the number of APs is small. When comparing the cases of 2 users and 20 users, higher power is transmitted when there are 2 users, leading to greater spectral efficiency.

Fig. 8 illustrates the cumulative distribution of transmit power for each user in the UL when the number of APs is 200, which represents a scenario with a significantly larger number of APs than users. As the number of APs increases, they become more densely and evenly located in the area, leading to users experiencing similar and generally larger channel gains compared to scenarios with fewer APs. As a result, there is minimal variation in the transmit power of each user. Additionally, with users experiencing similar channel conditions, we we can expect each user to transmit with higher power. When the number of APs gets large and evenly deployed over the coverage area, all schemes exhibit nearly identical distribution.

Fig. 9 shows the cumulative distribution of transmit power for each AP in the DL when the number of APs is 20. The transmit power of each AP in DL CB is at its maximum allowed value since the optimal power allocation solution for each AP is achieved at the boundary of constraints. For the two-user case, each AP in DL MR transmits less power compared to other schemes, but all schemes except DL CB show almost the same distributions of transmit power. When the number of users is comparable to that of APs, each AP in DL MR transmits the highest power, while those in DL ZF transmit the least amount of power.

Fig. 10 shows the cumulative distribution of transmit power for each AP in the DL when the number of APs is 200. As the number of APs increases, the difference in transmit power between the different schemes decreases, except for DL CB in which each AP transmits at its maximum power.

In terms of computational complexity, UL MR and DL MR require the least amount of computation since they only involve a Hermitian operation. UL ZF and DL ZF, on the other hand, require the inversion of a $K \times K$ matrix, which has a complexity of $\mathcal{O}(K^3)$ when using Gauss-Jordan elimination. At each iteration, UL MSINR requires inversion of an $L \times L$ matrix regardless of the number of users, while DL MSLNR requires $L \times L$ matrix inversion for each user, resulting in $L \times L \times K$ matrix inversion operations. To reduce the complexity, we proposed an element-wise inversion as shown in (65) that leverages the diagonal structure in the matrix instead of using matrix inversion.

## V. CONCLUSION

By deploying CF MIMO systems, we can expect very high and almost evenly balanced spectral efficiency for all users regardless of their locations or the number of users when a number of APs are evenly deployed over some area and interference is efficiently suppressed. This can be observed from the very steep slope of the cumulative distribution lines and the small differences between those lines in the spectral efficiency. These results are due to the effectively same and high channel gain between users in cell-free MIMO systems.

A relatively large gap is observed between the signal optimization schemes and the interference suppression schemes even with a large number of APs deployed. This seems to indicate that the favorable channel effect meaning the orthogonal channel between users is not profound in CF massive MIMO systems. Thus, some cooperative operation between APs may be effective for significant performance improvement, instead of a simple operation such as MRT, which requires only local channel information at each AP.

The beamforming schemes presented in this paper are reduced to existing schemes, such as MRC, MRT, ZF, and MMSE. These schemes are shown to be solutions of (generalized) eigenvalue problems through a unified mathematical framework for both the UL and the DL. Using the framework, some simulation results and relationships are compared across the different schemes. For instance, by decoupling beamforming and power allocation in the downlink MRT scheme, we obtain a spectral efficiency that is comparable to that of the downlink CB scheme. When the number of users is 2, the SINR of the uplink MRC is the same for both users, which is not the case in the downlink MRT, and the optimal power allocation in the uplink ZF scheme is the same as that in the uplink MRC.

The full potential of CF MIMO systems is investigated assuming perfect channel estimation without delay between a CPU and APs. Further work may include exploring performance degradation without relying on ideal assumptions, investigating multiple antenna extension for both APs and users, and developing a new performance metric such as the outage probability, using different mathematical formulations.

## REFERENCES


[1] A. Yadav and O. A. Dobre, "All technologies work together for good: A glance at future mobile networks," *IEEE Wireless Commun.*, vol. 25, no. 4, pp. 10–16, Aug. 2018.

[2] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[3] J. Zhang, S. Chen, Y. Lin, J. Zheng, B. Ai, and L. Hanzo, "Cell-free massive MIMO: A new next-generation paradigm," *IEEE Access*, vol. 7, pp. 99878–99888, 2019.

[4] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[5] H. Q. Ngo, A. Ashikhmin, H. Yang, E. G. Larsson, and T. L. Marzetta, "Cell-free massive MIMO versus small cells," *IEEE Trans. Wireless Commun.*, vol. 16, no. 3, pp. 1834–1850, Mar. 2017.

[6] G. Interdonato, E. Björnson, H. Q. Ngo, P. Frenger, and E. G. Larsson, "Ubiquitous cell-free massive MIMO communications," *EURASIP J. Wireless Commun. Netw.*, vol. 2019, no. 1, pp. 1–13, Aug. 2019.

[7] Z. Chen and E. Björnson, "Channel hardening and favorable propagation in cell-free massive MIMO with stochastic geometry," *IEEE Trans. Commun.*, vol. 66, no. 11, pp. 5205–5219, Nov. 2018.

[8] G. Interdonato, H. Q. Ngo, P. Frenger, and E. G. Larsson, "Downlink training in cell-free massive MIMO: A blessing in disguise," *IEEE Trans. Wireless Commun.*, vol. 18, no. 11, pp. 5153–5169, Nov. 2019.

[9] S. Yang and L. Hanzo, "Fifty years of MIMO detection: The road to large-scale MIMOs," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 4, pp. 1941–1988, 4th Quart., 2015.

[10] E. Bjornson, M. Bengtsson, and B. Ottersten, "Optimal multiuser transmit beamforming: A difficult problem with a simple solution structure [lecture notes]," *IEEE Signal Process. Mag.*, vol. 31, no. 4, pp. 142–148, Jul. 2014.

[11] E. Björnson and L. Sanguinetti, "Making cell-free massive MIMO competitive with MMSE processing and centralized implementation," *IEEE Trans. Wireless Commun.*, vol. 19, no. 1, pp. 77–90, Jan. 2020.

[12] E. Nayebi, A. Ashikhmin, T. L. Marzetta, H. Yang, and B. D. Rao, "Precoding and power optimization in cell-free massive MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 16, no. 7, pp. 4445–4459, Jul. 2017.

[13] E. Björnson and L. Sanguinetti, "Scalable cell-free massive MIMO systems," *IEEE Trans. Commun.*, vol. 68, no. 7, pp. 4247–4261, Jul. 2020.

[14] M. Bashar, K. Cumanan, A. G. Burr, M. Debbah, and H. Q. Ngo, "On the uplink max–min SINR of cell-free massive MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 18, no. 4, pp. 2021–2036, Apr. 2019.

[15] A. Zhou, J. Wu, E. G. Larsson, and P. Fan, "Max-min optimal beamforming for cell-free massive MIMO," *IEEE Commun. Lett.*, vol. 24, no. 10, pp. 2344–2348, Oct. 2020.

[16] J. Myung, K. Kim, and Y. Ko, "Transmit power control game with a sum power constraint in cell-free massive MIMO downlink," *IEEE Wireless Commun. Lett.*, vol. 11, no. 3, pp. 632–635, Mar. 2022.

[17] S. Elhoushy, M. Ibrahim, and W. Hamouda, "Cell-free massive MIMO: A survey," *IEEE Commun. Surveys Tuts.*, vol. 24, no. 1, pp. 492–523, 1st Quart., 2022.

[18] O. T. Demir, E. Björnson, and L. Sanguinetti, "Foundations of user-centric cell-free massive MIMO," *Found. Trends Signal Process.*, vol. 14, nos. 3–4, pp. 162–472, 2021.

[19] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2646–2660, Jun. 2007.
[20] M. Sadek, A. Tarighat, and A. Sayed, "A leakage-based precoding scheme for downlink multi-user MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1711–1721, May 2007.
[21] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, pp. 1458–1461, Oct. 1999.
[22] Y. Jiang, M. K. Varanasi, and J. Li, "Performance analysis of ZF and MMSE equalizers for MIMO systems: An in-depth study of the high SNR regime," *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 2008–2026, Apr. 2011.
[23] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. New York, NY, USA: Cambridge Univ. Press, 2005.
[24] A. Wiesel, Y. C. Eldar, and S. Shamai (Shitz), "Zero-forcing precoding and generalized inverses," *IEEE Trans. Signal Process.*, vol. 56, no. 9, pp. 4409–4418, Sep. 2008.
[25] S. P. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge Univ. Press, 2004.
[26] Q. Shi, M. Razaviyayn, M. Hong, and Z. Luo, "SINR constrained beamforming for a MIMO multi-user downlink system: Algorithms and convergence analysis," *IEEE Trans. Signal Process.*, vol. 64, no. 11, pp. 2920–2933, Jun. 2016.
[27] S.-R. Lee, J.-S. Kim, S.-H. Moon, H.-B. Kong, and I. Lee, "Zero-forcing beamforming in multiuser MISO downlink systems under per-antenna power constraint and equal-rate metric," *IEEE Trans. Wireless Commun.*, vol. 12, no. 1, pp. 228–236, Jan. 2013.
[28] S. V. Hanly, "An algorithm for combined cell-site selection and power control to maximize cellular spread spectrum capacity," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1332–1340, 1995.
[29] F. Rashid-Farrokhi, L. Tassiulas, and K. J. R. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Trans. Commun.*, vol. 46, no. 10, pp. 1313–1324, Oct. 1998.
[30] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, no. 1, pp. 57–62, Feb. 1992.
[31] S. U. Pillai, T. Suel, and S. Cha, "The perron-frobenius theorem: Some of its applications," *IEEE Signal Process. Mag.*, vol. 22, no. 2, pp. 62–75, Mar. 2005.
[32] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 2012. [Online]. Available: https://books.google.co.kr/books?id=O7sgAwAAQBAJ
[33] S. S. Christensen, R. Agarwal, E. De Carvalho, and J. M. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4792–4799, Dec. 2008.
[34] P. Cheng, M. Tao, and W. Zhang, "A new SLNR-based linear precoding for downlink multi-user multi-stream MIMO systems," *IEEE Commun. Lett.*, vol. 14, no. 11, pp. 1008–1010, Nov. 2010.
[35] *Guidelines for Evaluation of Radio Interface Technologies for IMT-2020*, document ITU-R M.2412-0, Oct. 2017.


**KEUNYOUNG KIM** received the B.S. degree in electronics from Korea University, in 1998, and the M.S. and Ph.D. degrees from the Korea Advanced Institute of Science and Technology (KAIST), South Korea, in 1999 and 2005, respectively. He is currently a Principal Researcher with the Mobile Communication Research Division, Electronics and Telecommunications Research Institute (ETRI), where he is with the 6G Wireless Technology Research Section. His research interests include the optimization of wireless communication systems, machine learning and deep learning applications, and information theory.

**JUNGHO MYUNG** received the B.S. degree in electrical engineering from Chungnam National University, Daejeon, Republic of Korea, in 2008, and the M.S. and Ph.D. degrees in engineering from the Korea Advanced Institute of Science and Technology, Daejeon, in 2010 and 2014, respectively. Since 2014, he has been with ETRI, where he is currently a Senior Engineer. His research interests include signal processing and optimization for wireless communications, with a focus on MIMO, cooperative systems, and machine learning.

**YOUNG-JO KO** received the B.S., M.S., and Ph.D. degrees in physics from KAIST, in 1992, 1994, and 1998, respectively. He joined the Electronics and Telecommunications Research Institute (ETRI) in March 1998, and has been working on mobile communications. He has actively participated in the 3GPP Standardization of LTE/LTE-Advanced. He is currently the Director of the 6G Wireless Technology Research Section, Mobile Communication Research Division, ETRI. He serves as the Chair for the 6G Technology Committee of 5G Forum, South Korea. His current research interests include 5G new radio and B5G/6G technologies, focusing on the physical layer aspects of radio access.

**WOO YONG LEE** received the B.S. degree in electronics engineering from Korea University, Seoul, South Korea, in 1989, and the M.S. and Ph.D. degrees in electrical and electronics from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 1991 and 1997, respectively. Upon completion of his graduate studies, he joined the Electronics and Telecommunications Research Institute (ETRI), Deajeon, where he worked on the research and development of the high speed wireless local and personal area networks systems. From August 2013 to July 2014, he was with the Department of Electrical Engineering, San Jose State University (SJSU), San Jose, CA, USA, as a Visiting Scholar. He was the Executive Director of the Korean Institute of Communications and Information Sciences (KICS), from 2017 to 2019, where he has been the Vice President, since 2020. Currently, he is a Principle Engineer of the 6G Wireless Technology Research Section, Telecommunication and Media Research Division, ETRI. His research and development activities are focused on transmission technologies for the next generation mobile communication systems. He made the standardization activities of the ECMA TC47, IEEE 802.15.3c, IEEE 802.11ad, over 15 years. He developed PHY/MAC algorithms and the architecture of wireless LAN/PAN IEEE 802.11ad/802.15.3c chipsets.