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RESEARCH ARTICLE

A New Dropout Compensation Scheme for Fuzzy Dynamic Output-Feedback Control of Discrete-Time System

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ABSTRACT This article under unreliable communication links considers the problem of dynamic compensation output-feedback control for non-linear discrete-time networked control systems (NCSs) with packet dropouts in the light of the Takagi-Sugeno (T-S) fuzzy method. Specifically, the packet dropouts are modeled by the Bernoulli random binary distribution in both the controller-to-actuator (C/A) and sensor-to-controller (S/C). In addition, to estimate the missing packets, attention is focused on the dynamic output-feedback controller, which is designed using a new improved compensation strategy. Here, sufficient conditions are obtained such that the closed-loop control system is stochastically stable with the prescribed H_∞ performance by constructing fuzzy Lyapunov function. To reduce conservativeness and deal with the non-convex problem, a cone-complementarity linearisation (CCL) program is introduced. Finally, the effectiveness of the method is demonstrated by the simulation results of the two numerical examples.

INDEX TERMS T-S fuzzy model, dropout compensation, NCSs, output feedback, CCL.

I. INTRODUCTION

In recent decades, nonlinearity has been a factor that must be considered when physical objects were modeled to obtain satisfactory or superior performance. However, nonlinear systems cannot be directly handled using the existing linear control theories. Especially, significant efforts have been devoted to many effective approximation methods. The T-S fuzzy model method has been extensively [1] proven to be a very effective tool for processing nonlinearity. Using the ''IF-THEN'' rules, a nonlinear system is transformed into a series of local linear systems blended with the corresponding membership functions to obtain a complete fuzzy system. So, in terms of application of conventional control theory, the T-S fuzzy model provides a basis for research on synthesis and stability analysis of nonlinear control systems. Recently, several results have been published in [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], and [12] based on this model. For T-S fuzzy systems, the criteria of stability were discussed based on the common quadratic Lyapunov function in the early research stage, which is conservative when coping with highly complicated nonlinear systems in [13]. Subsequently, based on the piecewise or fuzzy quadratic Lyapunov function, scholars have proposed some less conservative methods [14], [15]. Note that in different regions the common quadratic Lyapunov function is a particular situation of the piecewise quadratic Lyapunov function when the same Lyapunov function is used. For a controlled plant, the fuzzy quadratic Lyapunov function is composed of several local Lyapunov functions through the same membership function, which is another typical less conservative Lyapunov function in T-S fuzzy control.

The plant, sensor, controller, and actuator of a practical system are often scattered at different places, in which from one component to another signals are transmitted. The network is applied to compose the communication links to facilitate communication among them. In recent years, for nonlinear NCSs [16], [17], [18], [19], [20], the problems of control

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and modeling have attracted considerable attention owing to their obvious superiority (e.g., high reliability, simple maintenance and installation, and low cost). By constructing the scheme of state feedback control, for nonlinear NCSs amenable to network-induced constraints, the problems of stabilization and stability were researched in [21] and [22]. For imperfect communication channels, in [23], a novel model was proposed, in which only measurable states were suitable. The scheme of control is inaccessible if the states are unmeasurable. Furthermore, it was pointed out that: practical implementation, the serious drawback of state-feedback control is that an abundance of state vector messages are rarely available, which makes it difficult to adopt the control approaches proposed in [21] and [22]. In [24], based on the controlled system the authors proposed a method of dynamic output-feedback control that stabilizes the non-linearity network. However, it is noted that the incomplete communication links have not yet been adequately deliberated. Based on these studies, the design of a dynamic compensation output-feedback controller motivates our work.

If there are other network congestions, buffer overflows, or packet collisions as common phenomenon, the results mentioned above have considered packet dropouts, which is one of the critical troubles in controller programming. In general, for modeling packet dropout appearance, the methods can be classified into two categories which are called zero strategy and zero-hold strategy [25]. The so-called zero strategy [26], [27] applied the Bernoulli random variable to model unreliable communication links without any compensation, in which for analysis and synthesis of controlled systems, there exists a relatively simple mathematical expression. However, if packet dropouts occur continuously, the entire system will be an open loop for a certain period of time. Using neglecting the compensation scheme studied in [28], [29], and [30], packet dropouts are scheduled as a zero strategy, in which the performance of the system is abated. Clearly, a high packet-loss rate degrades the performance of the system or even makes it unstable. The other is called the zero-hold strategy [25] as an improved method, in which a "buffer" is presented to save the last successful signal transmitted so that the lost packets are compensated. However, for successive packet dropouts the compensation can not be updated on time because of the lack of regulating gains, which does not facilitate retaining the performance of the system. In light of the above policy, to tackle packet dropout it is urgent that we seek a better data transmission method. Therefore, under an unacceptable network environment in practice, it is very important for us to consider a new scheme that is needed to deal with the packet dropout phenomenon in order to obtain better performance of T-S fuzzy systems. To the best of the authors' knowledge, with dropout compensation strategy the issue of dynamic output feedback has not yet been researched. Consequently, the aforementioned gap will be filled, which motivates our research.

TABLE 1. Nomenclature.

Notation Definition
$\mathbf{T}$: the matrix transposition
R^n:n-dimensional Euclidean space (R stands for R^1)
$prob\{.\}$: the occurrence probability
$*$: the symmetry term in the complex symmetric block matrice
$0, I$: a zero matrix and the identity matrix respectively
$P > 0 (P \geq 0)$:real symmetric and positive definite (semi-definite)
$l_2[0, \infty]$: the space of square-summable vector among $[0, \infty]$
$\|M\| = \sqrt{tr(M^{\mathbf{T}}M)}$:$M$ matrix norm
$\lvert . \rvert$: the norm of Euclidean vector
$\| . \|_2$:the norm of common $l_2[0, \infty]$
$E\{\alpha\}$:the expectation of the event α
$E\{\alpha/\beta\}$: the conditional expectation of the event α on the event β
If the dimensions are not prescribed demonstrably it is assumed that matrices have compatible dimensions in this paper.

This paper proposes the key contributions as follows: (1) The fuzzy Lyapunov function, which is different from the existing common quadratic Lyapunov function, is used to achieve the main results, which can facilitate obtaining less conservative results. (2) A novel dynamic compensation output-feedback controller was proposed for the successive packet dropout and a compensation gain is applied to regulate the compensated data. (3) To reduce the conservatism without introducing other variables, we introduce the CCL algorithm to decouple the conditions.

The remainder of this paper is organized as follows. Section II describes the problem considered. Section III provides the main results, in which the output-feedback control scheme and stability analysis are presented. Two examples are provided to show the advantages of the proposed method in Section IV. Section V presents the conclusions.

II. ESSENTIAL CONDITION FORMULATION

Motivated by the aforementioned discussion, based on the T-S fuzzy model approach, the problem of dynamic compensation output-feedback was investigated for a class of discrete-time nonlinear NCSs with intermittent packet dropout. First, the T-S fuzzy model, which is a discrete-time non-linear system, is constructed. Then, a novel dynamic compensation output-feedback controller is proposed in order to effectively decrease the effect of unreliable communication.

A. T-S FUZZY MODEL

The i-th rule of the T-S is the following:

Model Rule i: if $z_1(t)$ is M_{i1} and $\cdots$ and $z_{\bar{s}}(t)$ is $M_{i\bar{s}}$,

$$\begin{cases} x(t+1) = A_i x(t) + B_i u(t) + E_i \varpi(t) \\ r(t) = C_{1i} x(t) + D_i u(t) + F_i \varpi(t) \\ y(t) = C_{2i} x(t) \end{cases} \tag{1}$$

where $Mi\bar{j}$ is the fuzzy set associated with the i-th model rule and $\bar{j}$-th premise variable component; $x(t) \in R^{n_x \times 1}$ is the state vector; $u(t) \in R^{n_u \times 1}$ is the control input vector; $y(t) \in R^{n_y \times 1}$ is the vector of measured output; $r(t) \in R^{n_r \times 1}$

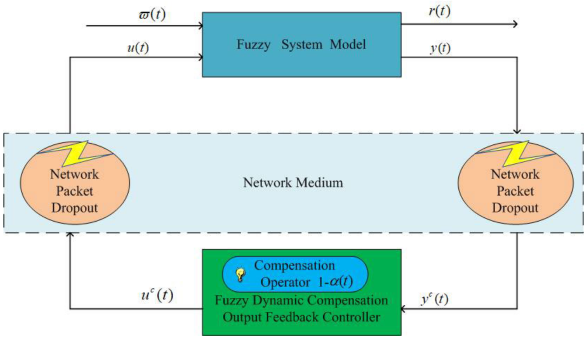

FIGURE 1. Plant flow chart.

is the vector of controlled output; $\varpi(t) \in l_2[0, \infty)$ is external disturbance and $\varpi(t) \in R^{n_\varpi \times 1}$. A_i, B_i, E_i, D_i, F_i, C_{1i}, C_{2i} are local system matrices with appropriate dimensions. $z(t) = [z_1(t) \cdots z_{\bar{s}}(t)]^{\mathrm{T}}$ are known premise variables. ($i = 1, 2, \cdots, r.\bar{j} = 1, 2, \cdots, \bar{s}.$), the scalar r is the number of rules. The final fuzzy system is the following:

$$\begin{cases} x(t+1) = \sum_{i=1}^{r} h_i[A_i x(t) + B_i u(t) + E_i \varpi(t)] \\ r(t) = \sum_{i=1}^{r} h_i[C_{1i} x(t) + D_i u(t) + F_i \varpi(t)] \\ \quad y(t) = \sum_{i=1}^{r} h_i C_{2i} x(t) \end{cases} \tag{2}$$

where for all t: $h_i \triangleq h_i(z(t))$ is the normalized membership function satisfying $h_i(z(t)) = \omega_i(z(t))/ \sum_{i=1}^{r} \omega_i(z(t)), \omega_i(z(t)) = \prod_{\bar{j}=1}^{\bar{s}} M_{i\bar{j}}(z_{\bar{j}}(t)), \omega_i(z(t)) \geq 0, \sum_{i=1}^{r} \omega_i(z(t)) > 0, \sum_{i=1}^{r} h_i(z(t)) = 1,$ $h_i(z(t)) \geq 0$.

B. FUZZY DYNAMIC COMPENSATION OUTPUT FEEDBACK CONTROLLER

In this section, we simultaneously consider both the C/A and the S/C communication channels. In order to decrease the effect of packet dropouts and compensate the lost data effectively for the system stability, the transmitted data is estimated by the constructing of the following subsystem in terms of the Bernoulli process. Based on the T-S fuzzy model (2), in this paper, the dynamic compensation in S/C channel has the following form:

C_i : If $z_1(t)$ is M_{i1} and $\cdots$ and $z_{\bar{s}}(t)$ is $M_{i\bar{s}}$

$$\begin{cases} \eta_c(t+1) = (1-\alpha(t))A_i^c \eta_c(t) + B_i^c y^c(t) \\ u^c(t) = C_i^c \eta_c(t) \end{cases} \tag{3}$$

where $\eta_c(k) \in R^{n_\eta \times 1}$ is the controller state vector ; $y^c(t) \in R^{n_y \times 1}$ is the controller input vector; $u^c(t) \in R^{n_u \times 1}$ is the controller output vector; A_i^c, B_i^c, C_i^c are matrices with appropriate dimensions. Then

$$\begin{cases} \eta_c(t+1) = \sum_{i=1}^{r} h_i \left[(1-\alpha(t))A_i^c \eta_c(t) + B_i^c y^c(t)\right] \\ u^c(t) = \sum_{i=1}^{r} h_i C_i^c \eta_c(t). \end{cases} \tag{4}$$

Remark 1: Based on the Bernoulli distribution, the new model (4) is a reformed model. For the system stability the effect of data packet dropouts can be decreased compared with the so called zero strategy and the zero-hold one. The model could compensate lost signals and update the compensated data on time.

C. IMPERFECT COMMUNICATION LINKS

From Fig. 1, it can be seen that the model contacts the communication network. It can be considered that a good few elements are introduced into the network in which data packet dropouts randomly occur in both the C/A and the S/C in this paper. Thus $y(t) \neq y^c(t)$ and $u^c(t) \neq u(t)$. Based on the stochastic method, the above phenomenon is described as following

$$\begin{cases} y^c(t) = \alpha(t) y(t) \\ u(t) = \beta(t) u^c(t) \end{cases} \tag{5}$$

in which $\alpha(t)$, $\beta(t)$ satisfy the process of Bernoulli random distribution. And $\alpha(t)$, $\beta(t)$ presents the S/C, C/A of imperfect communication, respectively. Suppose $\alpha(t)$, $\beta(t)$ as following

$$\begin{cases} \text{prob}\{\alpha(t) = 1\} = E\{\alpha(t)\} = \bar{\alpha} \\ \text{prob}\{\alpha(t) = 0\} = 1 - \bar{\alpha} \\ \text{prob}\{\beta(t) = 1\} = E\{\beta(t)\} = \bar{\beta} \\ \text{prob}\{\beta(t) = 0\} = 1 - \bar{\beta}. \end{cases} \tag{6}$$

According to the (5), we obtain

$$\begin{cases} \eta_c(t+1) = \sum_{i=1}^{r} h_i \left[(1-\alpha(t))A_i^c \eta_c(t) + \alpha(t) B_i^c y(t)\right] \\ u^c(t) = \sum_{i=1}^{r} h_i C_i^c \eta_c(t). \end{cases} \tag{7}$$

Combining (2) and (7), the augmented closed loop system is as

$$\begin{cases} \bar{\xi}(t+1) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [A_{ij} \bar{\xi}(t) + \Xi_i \varpi(t)] \\ r(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [C_{ij} \bar{\xi}(t) + F_i \varpi(t)] \end{cases} \tag{8}$$

where

$$A_{ij} = \begin{bmatrix} A_i & \beta(t) B_i C_j^c \\ \alpha(t) B_j^c C_{2i} & (1-\alpha(t)) A_j^c \end{bmatrix},$$

$$\Xi_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix},$$

$$C_{ij} = \begin{bmatrix} C_{1i} & \beta(t) D_i C_j^c \end{bmatrix}, \quad \bar{\xi}(t) = \begin{bmatrix} x(t) \\ \eta_c(t) \end{bmatrix}.$$

Assuming

$$\alpha(t) = \bar{\alpha} + \tilde{\alpha}(t),$$
$$\beta(t) = \bar{\beta} + \tilde{\beta}(t),$$
$$\text{then} \quad E\{\tilde{\alpha}(t)\tilde{\alpha}(t)\} = \bar{\alpha}(1-\bar{\alpha}),$$
$$E\{\tilde{\beta}(t)\tilde{\beta}(t)\} = \bar{\beta}(1-\bar{\beta}),$$
$$A_{ij} = A_{ij}^1 + \hat{A}_{ij}^2,$$
$$C_{ij} = C_{ij}^1 + \hat{C}_{ij}^2,$$
$$\hat{A}_{ij}^2 = \begin{bmatrix} 0 & \tilde{\beta}(t)B_iC_j^c \\ \tilde{\alpha}(t)B_j^cC_{2i} & -\tilde{\alpha}(t)A_j^c \end{bmatrix},$$
$$C_{ij}^1 = \begin{bmatrix} C_{1i} & \bar{\beta}D_iC_j^c \end{bmatrix},$$
$$A_{ij}^1 = \begin{bmatrix} A_i & \bar{\beta}B_iC_j^c \\ \bar{\alpha}B_j^cC_{2i} & (1-\bar{\alpha})A_j^c \end{bmatrix},$$
$$\hat{C}_{ij}^2 = \begin{bmatrix} 0 & \tilde{\beta}(t)D_iC_j^c \end{bmatrix}.$$

D. DEFINITION AND LEMMA

Definition 1 [22]: Any initial condition $\bar{\xi}(0)$ is considered and $\varpi(t) \equiv 0$, if there exists a matrix $W > 0$ such that

$$E\left\{\sum_{t=0}^{\infty} \left|\bar{\xi}(t)\right|^2 \Big|_{\bar{\xi}(0)}\right\} < \bar{\xi}^{\mathsf{T}}(0)W\bar{\xi}(0). \tag{9}$$

Then, the closed-loop system (8) is stochastically stable. Furthermore, the problems to be solved in this article are as follows: the augmented system (8) is stochastically stable with zero initial condition and satisfies

$$E\left\{\sum_{t=0}^{\infty} |r(t)|^2\right\} \le \gamma^2 \|\varpi\|_2^2 \tag{10}$$

where $\gamma > 0$, then the H_∞ performance γ of the output is obtained.

Lemma 1 [31]: If the following conditions are founded

$$M_{ii} < 0, \quad i = 1, 2, \cdots \lambda, \tag{11}$$
$$\frac{1}{\lambda - 1}M_{ii} + \frac{1}{2}(M_{il} + M_{li}) < 0, \quad i \neq l, i, l = 1, 2, \cdots \lambda, \tag{12}$$

then we have the following inequality

$$\sum_{i=1}^{\lambda}\sum_{l=1}^{\lambda} h_ih_lM_{il} < 0. \tag{13}$$

III. MAIN RESULTS

In this section, firstly, the H_∞ performance analysis condition is provided in the following theorem for the given controller gain matrices. The fuzzy Lyapunov function is adopted in the following results to introduce the less conservative condition.

Theorem 1: The system of closed-loop fuzzy model in (8) is stochastically stable and satisfies H_∞ performance with the supposed matrices A_j^c, B_j^c and C_j^c $(j = 1, \ldots, r)$ of the controller gain, if there exist matrices P_l, $l = 1, \ldots, r$ satisfying the following conditions

$$\bar{\Gamma}_{ii}^{\mathsf{T}}\hat{P}_l\bar{\Gamma}_{ii} + \Upsilon_{ii}^{\mathsf{T}}\Upsilon_{ii} - \Lambda_i < 0 \tag{14}$$
$$\begin{aligned}&(\frac{\bar{\Gamma}_{ij} + \bar{\Gamma}_{ji}}{2})^{\mathsf{T}}\hat{P}_l(\bar{\Gamma}_{ij} + \bar{\Gamma}_{ji})\\ &\quad + (\frac{\Upsilon_{ij} + \Upsilon_{ji}}{2})^{\mathsf{T}}(\Upsilon_{ij} + \Upsilon_{ji}) - \Lambda_i - \Lambda_j < 0\end{aligned} \tag{15}$$

where

$$\bar{\Gamma}_{ij} = \begin{bmatrix} A_{ij}^1 & \Xi_i \\ A_{ij}^2 & 0 \end{bmatrix}, \quad \Lambda_i = diag\left\{P_i \quad \gamma^2 I\right\},$$
$$\Upsilon_{ij} = \begin{bmatrix} C_{ij}^1 & F_i \\ C_{ij}^2 & 0 \end{bmatrix}, \quad \hat{P}_l = diag\left\{P_l \quad P_l\right\},$$
$$A_{ij}^2 = \begin{bmatrix} 0 & \sqrt{\bar{\beta}(1-\bar{\beta})}B_iC_j^c \\ \sqrt{\bar{\alpha}(1-\bar{\alpha})}B_j^cC_{2i} & -\sqrt{\bar{\alpha}(1-\bar{\alpha})}A_j^c \end{bmatrix},$$
$$C_{ij}^2 = \begin{bmatrix} 0 & \sqrt{\bar{\beta}(1-\bar{\beta})}D_iC_j^c \end{bmatrix}.$$

Proof: Defining $\varpi(t) \equiv 0$, the stochastically stability of the system (8) is proved. For system (8), the following Lyapunov function is chosen $V(t) = \bar{\xi}^{\mathsf{T}}(t)\left[\sum_{i=1}^{r} h_iP_i\right]\bar{\xi}(t)$ where $P_i > 0$, supposing $h_l^+ = h_l^+(z(t+1))$, and we have

$$\begin{aligned}
&E\{\Delta V(t)\}\\
&= E\left\{V(t+1)\left|\bar{\xi}(t)\right.\right\} - V(t)\\
&= E\left\{\bar{\xi}^{\mathsf{T}}(t)\sum_{l=1}^{r} h_l^+\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{\bar{t}=1}^{r} h_ih_jh_sh_{\bar{t}}(A_{ij}{}^{\mathsf{T}}P_lA_{s\bar{t}})\right]\right.\\
&\quad \left.\bar{\xi}(t)\right\} - \bar{\xi}^{\mathsf{T}}(t)\left[\sum_{i=1}^{r} h_iP_i\right]\bar{\xi}(t)\\
&= E\left\{\bar{\xi}^{\mathsf{T}}(t)\sum_{l=1}^{r} h_l^+\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{\bar{t}=1}^{r} h_ih_jh_sh_{\bar{t}}(\frac{A_{ij}+A_{ji}}{2})^{\mathsf{T}}\right.\right.\\
&\quad \left.\left.P_l(\frac{A_{s\bar{t}}+A_{\bar{t}s}}{2})\right]\bar{\xi}(t)\right\} - \bar{\xi}^{\mathsf{T}}(t)\left[\sum_{i=1}^{r} h_iP_i\right]\bar{\xi}(t)\\
&= \frac{1}{4}E\left\{\bar{\xi}^{\mathsf{T}}(t)\sum_{l=1}^{r} h_l^+\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{\bar{t}=1}^{r} h_ih_jh_sh_{\bar{t}}(A_{ij}+\right.\right.\\
&\quad \left.\left.A_{ji})^{\mathsf{T}}P_l(A_{s\bar{t}}+A_{\bar{t}s})\right]\bar{\xi}(t)\right\} - \bar{\xi}^{\mathsf{T}}(t)\left[\sum_{i=1}^{r} h_iP_i\right]\bar{\xi}(t)\\
&\le \bar{\xi}^{\mathsf{T}}(t)\sum_{l=1}^{r} h_l^+\left[\sum_{i=1}^{r} h_i{}^2(\Gamma_{ii}^{\mathsf{T}}\hat{P}_l\Gamma_{ii} - P_i)+\right.\\
&\quad \left.\sum_{i=1}^{r-1}\sum_{j=i+1}^{r} h_ih_j(\frac{\Gamma_{ij}+\Gamma_{ji}}{2})^{\mathsf{T}}\hat{P}_l(\Gamma_{ij}+\Gamma_{ji}) - P_i - P_j\right]\bar{\xi}(t)
\end{aligned} \tag{16}$$

where $\Gamma_{ij} = \begin{bmatrix} A_{ij}^1 \\ A_{ij}^2 \end{bmatrix}$, $\hat{P}_l = diag\left\{P_l \ P_l\right\}$. Via Schur complement, according to (14)-(15), one can obtain $\Gamma_{ii}^{\mathsf{T}}\hat{P}_l\Gamma_{ii} - P_i < 0$ and $(\frac{\Gamma_{ij}+\Gamma_{ij}}{2})^{\mathsf{T}}\hat{P}_l(\Gamma_{ij}+\Gamma_{ij}) - P_i - P_j < 0$.

Let

$$\begin{aligned}\Psi &= \sum_{l=1}^{r} h_l^+ \left[\sum_{i=1}^{r} h_i^2(\Gamma_{ii}^{\mathsf{T}}\hat{P}_l\Gamma_{ii} - P_i) \right.\\ &\left. + \sum_{i=1}^{r-1}\sum_{j=i+1}^{r} h_ih_j(\frac{\Gamma_{ij}+\Gamma_{ji}}{2})^{\mathsf{T}}\hat{P}_l(\Gamma_{ij}+\Gamma_{ji}) - P_i - P_j \right].\end{aligned} \tag{17}$$

From

$$\lambda_{\min}(-\Psi)\left|\bar{\xi}(t)\right|^2 \le \bar{\xi}^{\mathsf{T}}(t)(-\Psi)\bar{\xi}(t) \le \lambda_{\max}(-\Psi)\left|\bar{\xi}(t)\right|^2$$

we can obtain

$$\begin{aligned} E&\left\{\bar{\xi}^{\mathsf{T}}(t+1)\left[\sum_{l=1}^{r} h_l^+ P_l\right]\bar{\xi}(t+1)\right\}\\ &- \bar{\xi}^{\mathsf{T}}(t)\left[\sum_{i=1}^{r} h_iP_i\right]\bar{\xi}(t) \le -\lambda_{\min}(-\Psi)\bar{\xi}^{\mathsf{T}}(t)\bar{\xi}(t).\end{aligned} \tag{18}$$

From $t = 0, 1, \cdots k$ and $k \ge 1$ for the above inequality, to calculate and sum mathematical expectation, we can have

$$\begin{aligned} E&\left\{\bar{\xi}^{\mathsf{T}}(k+1)\left[\sum_{l=1}^{r} h_l^+ P_l\right]\bar{\xi}(k+1)\right\}\\ &- \bar{\xi}^{\mathsf{T}}(0)\left[\sum_{i=1}^{r} h_iP_i\right]\bar{\xi}(0) \le -\lambda_{\min}(-\Psi)E\left\{\sum_{t=0}^{k}\left|\bar{\xi}(t)\right|^2\right\}.\end{aligned} \tag{19}$$

Then, when $k = 1, \cdots \infty$ and given

$$E\left\{\bar{\xi}^{\mathsf{T}}(\infty)\left[\sum_{l=1}^{r} h_l^+ P_l\right]\bar{\xi}(\infty)\right\} \ge 0,$$

we obtain

$$\begin{aligned} E&\left\{\sum_{t=0}^{k}\left|\bar{\xi}(t)\right|^2\right\}\\ &\le (\lambda_{\min}(-\Psi))^{-1}\left\{\bar{\xi}^{\mathsf{T}}(0)\left[\sum_{i=1}^{r} h_iP_i\right]\bar{\xi}(0)\right.\\ &\left. - E\left\{\bar{\xi}^{\mathsf{T}}(k+1)\left[\sum_{l=1}^{r} h_l^+ P_l\right]\bar{\xi}(k+1)\right\}\right\}\\ &\le (\lambda_{\min}(-\Psi))^{-1}\bar{\xi}^{\mathsf{T}}(0)\left[\sum_{i=1}^{r} h_iP_i\right]\bar{\xi}(0)\\ &= \bar{\xi}^{\mathsf{T}}(0)[(\lambda_{\min}(-\Psi))^{-1}\sum_{i=1}^{r} h_iP_i]\bar{\xi}(0).\end{aligned} \tag{20}$$

Let $W = (\lambda_{\min}(-\Psi))^{-1}\sum_{i=1}^{r} h_iP_i$, we obtain $\Psi < 0$ and $W > 0$. Therefore, in terms of Definition 1 we obtain that the system (8) is stochastically stable. When the initial condition is zero the following section will present the H_∞ performance. The H_∞ index is following

$$\begin{aligned} J = E&\left\{r^{\mathsf{T}}(t)r(t)\left|_{\Theta(t)}\right.\right\} - \gamma^2\varpi^{\mathsf{T}}(t)\varpi(t)\\ &+ E\left\{V(t+1)\left|_{\Theta(t)}\right.\right\} - V(t).\end{aligned} \tag{21}$$

Let $\Theta(k) = \begin{bmatrix}\bar{\xi}(k) \\ \varpi(k)\end{bmatrix}$, then obtain

$$\begin{aligned} J &= E\left\{V(t+1)\left|_{\Theta(t)}\right.\right\} - V(t)\\ &+ E\left\{r^{\mathsf{T}}(t)r(t)\left|_{\Theta(t)}\right.\right\} - \gamma^2\varpi^{\mathsf{T}}(t)\varpi(t)\\ &= E\left\{\Theta^{\mathsf{T}}(t)\sum_{l=1}^{r} h_l^+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{\bar{t}=1}^{r} h_ih_jh_sh_{\bar{t}}\right.\\ &\left.\left(\begin{bmatrix}A_{ij} & \Xi_i\end{bmatrix}^{\mathsf{T}}P_l\begin{bmatrix}A_{s\bar{t}} & \Xi_s\end{bmatrix}\right)\Theta(t)\right\}\\ &- \gamma^2\varpi^{\mathsf{T}}(t)\varpi(t) - \bar{\xi}^{\mathsf{T}}(t)\left[\sum_{i=1}^{r} h_iP_i\right]\bar{\xi}(t)\\ &+ E\left\{\Theta^{\mathsf{T}}(t)\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{\bar{t}=1}^{r} h_ih_jh_sh_{\bar{t}}\right.\\ &\left.\left(\begin{bmatrix}C_{ij} & F_i\end{bmatrix}^{\mathsf{T}}\begin{bmatrix}C_{s\bar{t}} & F_s\end{bmatrix}\right)\Theta(t)\right.\\ &= \Theta^{\mathsf{T}}(t)\sum_{l=1}^{r} h_l^+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{\bar{t}=1}^{r} h_ih_jh_sh_{\bar{t}}\\ &\begin{bmatrix}(C_{ij}^1)^{\mathsf{T}}C_{s\bar{t}}^1 + (C_{ij}^2)^{\mathsf{T}}C_{s\bar{t}}^2 & (C_{ij}^1)^{\mathsf{T}}F_s \\ F_i^{\mathsf{T}}C_{s\bar{t}}^1 & F_i^{\mathsf{T}}F_s\end{bmatrix}\Theta(t)\\ &- \Theta^{\mathsf{T}}(t)\sum_{i=1}^{r} h_i\begin{bmatrix}P_i & 0 \\ 0 & \gamma^2 I\end{bmatrix}\Theta(t)\\ &+ \Theta^{\mathsf{T}}(t)\sum_{l=1}^{r} h_l^+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{\bar{t}=1}^{r} h_ih_jh_sh_{\bar{t}}\\ &\begin{bmatrix}(A_{ij}^1)^{\mathsf{T}}P_lA_{s\bar{t}}^1 + (A_{ij}^2)^{\mathsf{T}}P_lA_{s\bar{t}}^2 & (A_{ij}^1)^{\mathsf{T}}P_l\Xi_s \\ \Xi_i^{\mathsf{T}}P_lA_{s\bar{t}}^1 & \Xi_i^{\mathsf{T}}P_l\Xi_s\end{bmatrix}\Theta(t)\\ &\le \Theta^{\mathsf{T}}(t)\sum_{l=1}^{r} h_l^+\left[\sum_{i=1}^{r} h_i^2(\bar{\Gamma}_{ii}^{\mathsf{T}}\hat{P}_l\bar{\Gamma}_{ii} + \Upsilon_{ii}^{\mathsf{T}}\Upsilon_{ii} - \Lambda_i)\right.\\ &+ \sum_{i=1}^{r-1}\sum_{j=i+1}^{r} h_ih_j(\frac{\bar{\Gamma}_{ij}+\bar{\Gamma}_{ji}}{2})^{\mathsf{T}}\hat{P}_l(\bar{\Gamma}_{ij}+\bar{\Gamma}_{ji})\\ &\left. + (\frac{\Upsilon_{ij}+\Upsilon_{ji}}{2})^{\mathsf{T}}(\Upsilon_{ij}+\Upsilon_{ji}) - \Lambda_i - \Lambda_j\right]\Theta(t).\end{aligned} \tag{22}$$

From Schur complement, according to (15), we can obtain $J \le 0$ and $E\left\{\sum_{t=0}^{\infty}\left|r(t)\right|^2\right\} \le \gamma^2\left\|\varpi\right\|_2^2$.

The proof is finished.

Theorem 1 can not be directly applied to controller design because the conditions are not in the form of linear matrix inequalities in Theorem 1. Next, the following theorem is derived to design the output-feedback controller and stabilize the fuzzy system (8) for the compensation gains.

Theorem 2: The system of closed-loop fuzzy model in (8) is stochastically stable and satisfies H_∞ performance γ with the supposed matrices A_j^c, B_j^c and C_j^c $(j = 1, ..., r)$ of the controller gain, if there exist matrices P_l, $(l = 1, ..., r)$ satisfying the following conditions

$$\begin{bmatrix} -I & 0 & \Upsilon_{ij} \\ * & -\hat{P}_l^{-1} & \bar{\Gamma}_{ij} \\ * & * & -\Lambda_i \end{bmatrix} < 0. \tag{23}$$

Proof: According to (14)-(15), one can have

$$\begin{bmatrix} (C_{ij}^1)^{\mathsf{T}} C_{ij}^1 + (C_{ij}^2)^{\mathsf{T}} C_{ij}^2 & (C_{ij}^1)^{\mathsf{T}} F_i \\ F_i^{\mathsf{T}} C_{ij}^1 & F_i^{\mathsf{T}} F_i \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix}$$
$$+ \begin{bmatrix} (A_{ij}^1)^{\mathsf{T}} P_l A_{ij}^1 + (A_{ij}^2)^{\mathsf{T}} P_l A_{ij}^2 & (A_{ij}^1)^{\mathsf{T}} P_l \Xi_i \\ \Xi_i^{\mathsf{T}} P_l A_{ij}^1 & \Xi_i^{\mathsf{T}} P_l \Xi_i \end{bmatrix} < 0,$$

and one can have

$$\begin{bmatrix} C_{ij}^1 & F_i \\ C_{ij}^2 & 0 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} C_{ij}^1 & F_i \\ C_{ij}^2 & 0 \end{bmatrix} + \begin{bmatrix} A_{ij}^1 & \Xi_i \\ A_{ij}^2 & 0 \end{bmatrix}^{\mathsf{T}}$$
$$\times \begin{bmatrix} P_l & 0 \\ 0 & P_l \end{bmatrix} \begin{bmatrix} A_{ij}^1 & \Xi_i \\ A_{ij}^2 & 0 \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix}$$
$$= \Upsilon_{ij}^{\mathsf{T}} \Upsilon_{ij} + \bar{\Gamma}_{ij}^{\mathsf{T}} \hat{P}_l \bar{\Gamma}_{ij} - \Lambda_i < 0. \tag{24}$$

By using the Schur complement, one can have (23). It was searched that the aforementioned condition in Theorem 2 is more conservative. In terms of Lemma 1, in the following theorem we propose the less conservative condition to achieve a large scope of feasible solutions.

Remark 2: We note that when getting LMI conditions for the design of the controller the basis-dependent Lyapounov function is introduced in this paper, in which a nonconvex condition is produced. It is difficult to solve the matrices gains of the controller. Therefore, the CCL algorithm is applied to show the controller design procedure.

Theorem 3: The system of closed-loop fuzzy model in (8) is stochastically stable and satisfies H_∞ performance γ with the supposed matrices A_j^c, B_j^c and C_j^c $(j = 1, ..., r)$ of the controller gain, if there exist matrices P_l, $(l = 1, ..., r)$ satisfying the following conditions

$$\Phi_{iil} < 0, \quad (i, l = 1, 2, \ldots, r), \tag{25}$$
$$\frac{1}{\lambda - 1}\Phi_{iil} + \frac{1}{2}(\Phi_{ijl} + \Phi_{jil}) < 0, \quad (i \neq j, i, j, l = 1, 2, \ldots, r), \tag{26}$$
$$P_l L_l = I, \tag{27}$$

where $\Phi_{ijl} = \begin{bmatrix} -I & 0 & \Upsilon_{ij} \\ * & \hat{L}_l & \bar{\Gamma}_{ij} \\ * & * & -\Lambda_i \end{bmatrix}, \hat{L}_l = diag\left\{-L_l \ -L_l\right\}.$

Proof: Let $P_l^{-1} = L_l$, one can have

$$\begin{bmatrix} -I & 0 & \Upsilon_{ij} \\ * & \hat{L}_l & \bar{\Gamma}_{ij} \\ * & * & -\Lambda_i \end{bmatrix} < 0. \tag{28}$$

That is

$$\sum_{l=1}^{r} h_l^+ \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \Phi_{ijl} < 0. \tag{29}$$

Based on Lemma 1, therefore, we can obtain (28) from (27). The matrix inequalities (25)-(26) are satisfied, then the inequality (29) holds.

The proof is completed.

In this theorem, we can achieve the parameters of the dynamic output-feedback controller in the form of (3) if $(P_i, L_i, A_i^c, B_i^c, C_i^c)$ is a feasible solution of (25)-(27).

We introduce the basic notion of the CCL algorithm. If $P_l(n \times n) > 0$, $L_l(n \times n) > 0$, $(l = 1, ..., r)$ are solution for the condition of LMI:

$$\begin{bmatrix} P_l & I \\ I & L_l \end{bmatrix} \geq 0, \tag{30}$$

then, $tr(P_l L_l) \geq n$; furthermore, if and only if $P_l L_l = I$,

$$tr(P_l L_l) = n. \tag{31}$$

In this paper, the dynamic compensation output feedback controller design problem is as follows: $tr(\sum_l P_l L_l)$ subject to (25)-(26) and (30). Then the conclusions in Theorem 3 are resoluble if there have solutions that min $tr(\sum_l P_l L_l)$ is subject to (25)-(26) and (30). The algorithm in Table 2 is proposed to solve the above problem by us.

IV. ILLUSTRATIVE EXAMPLE

In the section, the effectiveness of the proposed result is demonstrated by some simulation results of two numerical examples. A Pulse-Width-Modulation (PWM)-driven boost converter [31], [32], [33] and the mass-spring-damping system [22] are applied to illustrate the validity of the proposed design, respectively. We use the compensation method and the zero-hold method respectively in Example 1, and compare the trajectories which are produced by these two methods. Similarly, the results which are produced by the compensation method and the zero strategy method in Example 2 are compared.

A. EXAMPLE 1

Considering the PWM-driven boost converter as described in [32], the $s(t)$ is controlled by the PWM device and it switches once at most in each switched period T. R is the load resistance, $e_s(t)$ is the source voltage, L is the inductance, and C is the capacitance. This kind of power converter can be modeled as switched system in recent years. By introducing variables $\tilde{\tau} = {}^t/_T$, $L_1 = {}^L/_T$, $C_1 = {}^C/_T$, and $e_c(t) \leq \widehat{E}$. The

TABLE 2. Design algorithm.

Programme Algorithm
Step 1: Set $k = 0$. Seek a feasible set $(P_l^0, L_l^0, A_i^{c0}, B_i^{c0}, L_i^{c0})$ to satisfy (25)-(26) and (30).
Step 2: Solve the following issue $\min tr(\sum_l (P_l L_l^k + P_l^k L_l))$ s.t. (25)-(26) and (30).
Step 3: The achieved variables $(P_l, L_l, A_i^c, B_i^c, L_i^c)$ are substituted into the inequality (23). If the inequality (23) are hold with $\left\|tr(\sum_l P_l L_l) - rn\right\| < \bar{\delta}$ for any sufficiently small scalar $\bar{\delta} > 0$, then obtain the feasible solutions $(P_l, L_l, A_i^c, B_i^c, L_i^c)$. EXIT.
Step 4: If $k > \bar{N}$, where $\bar{N}$ is the allowed maximum number of iterations , EXIT.
Step 5: Set $k = k+1$, $({P_l}^k, {L_l}^k, A_i^{ck}, B_i^{ck}, L_i^{ck}) = (P_l, L_l, A_i^c, B_i^c, L_i^c)$ and go to Step 2.

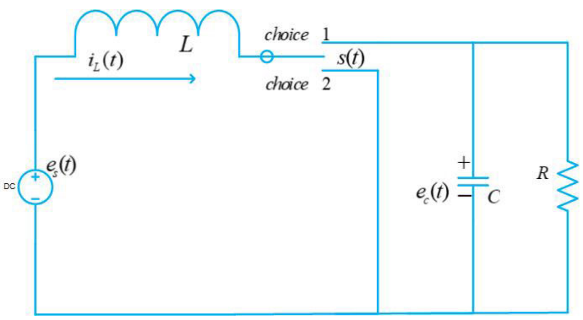

FIGURE 2. PWM-driven boost converter.

differential equations of the model are as follows:

$$\begin{cases} \dot{i}_L(\tilde{\tau}) = -(1 - s(\tilde{\tau}))\dfrac{1}{L_1}e_c(\tilde{\tau}) + s(\tilde{\tau})\dfrac{1}{L_1}e_s(\tilde{\tau}) \\ \dot{e}_c(\tilde{\tau}) = -\dfrac{1}{RC_1}e_c(\tilde{\tau}) + (1 - s(\tilde{\tau}))\dfrac{1}{C_1}i_L(\tilde{\tau}) \end{cases} \tag{32}$$

Then, from (32) and Fig. 2 we can obtain that choice 1 is in the continuous-time system,

$$\dot{x}(\tilde{\tau}) = A(\tilde{\tau})x(\tilde{\tau}) + f(\tilde{\tau}, x(\tilde{\tau})) \tag{33}$$

where, $x(\tilde{\tau}) = [i_L, e_c]^{\mathsf{T}}, f(\tilde{\tau}, x(\tilde{\tau})) = 0$,

$$A = \begin{bmatrix} 0 & {}^{T}\!/_{L} \\ -{}^{T}\!/_{C} & -{}^{T}\!/_{RC} \end{bmatrix}.$$

And as in [32] and [33], we assume the control matrices to be $B_1 = B_2 = \begin{bmatrix} 0.5 & 0 \end{bmatrix}^{\mathsf{T}}$, and set the sampling time to be $T_s = 0.5$, $C = 1$, $L = 1$, $R = 1$, $\widehat{E} = 1$. Respectively, T taking values 1 and 0.5, by the normalization and discretization technique the matrixes of parameters for the system (1) are listed as follows:

Plant Rule 1 : if $x_1(t)$ is $h_1(x_1(t))$ then

$$\begin{cases} x(t+1) = A_1x(t) + B_1u(t) + E_1\varpi(t) \\ r(t) = C_{11}x(t) + D_1u(t) + F_1\varpi(t) \\ y(t) = C_{21}x(t). \end{cases} \tag{34}$$

Plant Rule 2 : if $x_1(t)$ is $h_2(x_1(t))$ then

$$\begin{cases} x(t+1) = A_2x(t) + B_2u(t) + E_2\varpi(t) \\ r(t) = C_{12}x(t) + D_2u(t) + F_2\varpi(t) \\ y(t) = C_{22}x(t). \end{cases}$$

$$A_1 = \begin{bmatrix} 0.8956 & 0.3773 \\ -0.3773 & 0.5182 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2409 \\ -0.0522 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.9713 & 0.2189 \\ -0.2189 & 0.7524 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2476 \\ -0.0287 \end{bmatrix}. \tag{35}$$

Else

$$E_1 = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.3 \\ 0.4 \end{bmatrix}, \quad C_{11} = \begin{bmatrix} -1.5 & 0.5 \end{bmatrix},$$

$$C_{12} = \begin{bmatrix} 0.2 \ 0.5 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} 1.5 & 0.5 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 0.7 & 0.5 \end{bmatrix},$$

$$D_1 = -0.5, D_2 = 0.5, F_1 = 0.3, F_2 = 0.4.$$

Membership functions for rule 1, 2 are given as follows:

$$h_1(x_1(t)) = \begin{cases} 1 + (1/3)x_1(t) \ q - 3 \le x_1(t) \le 0 \\ 1 - (1/3)x_1(t) \quad 0 \le x_1(t) \le 3 \\ 0 \quad \text{else}, \end{cases}$$

$$h_2(x_1(t)) = 1\text{-}h_1(x_1(t)).$$

In order to obtain the stochastically stable closed-loop system (8) and H_∞ performance attenuation level, our goal is the design of the gain in (3). We apply the LMIs of theorem 3 and the CCL algorithm when $\bar{\alpha} = 0.6$, $\bar{\alpha} = 0.7$. The gains are listed below

$$A_1^c = \begin{bmatrix} 0.1171 & 0.0193 \\ 0.1846 & -0.0297 \end{bmatrix},$$

$$A_2^c = \begin{bmatrix} 0.0603 & -0.0501 \\ -0.0966 & 0.6995 \end{bmatrix},$$

$$B_1^c = \begin{bmatrix} 0.0097 \\ 0.0074 \end{bmatrix}, \quad B_2^c = \begin{bmatrix} -0.1859 \\ 0.0672 \end{bmatrix},$$

$$C_1^c = \begin{bmatrix} 0.4954 & -0.4945 \end{bmatrix}, \quad C_2^c = \begin{bmatrix} -1.2667 & 1.3544 \end{bmatrix}.$$

Fig. 3 describes that the dropout of random data packet is shown. $\varpi(t) = 1/(2+t)$ is the external disturbance. Moreover, we assume that $x(0) = \tilde{x}(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^{\mathsf{T}}$, $\eta_c(0) = \tilde{x}_c(0) = \begin{bmatrix} -2 & 2 \end{bmatrix}^{\mathsf{T}}$, in which $x(0)$, $\eta_c(0)$ are the initial value of the state and the initial value of the controller state in the compensation method, respectively. And $\tilde{x}(t), \tilde{x}_c(t), \tilde{r}(t)$ under the zero-hold strategy are the state, the controller state and the output in Fig. 4 - Fig. 6. It can be seen from Fig. 4, Fig. 5 that the states $x(t)$ and the controller states $\eta_c(t)$ may be compared by the CCL, in which the merits of the proposed control approach are further indicated. Under traditional schemes without the compensation, in a short period of time the trajectories of the controller states shown in Fig. 4, obviously have converged to zero as quickly as the compensated state, but which represents the zero-hold method is not as effective

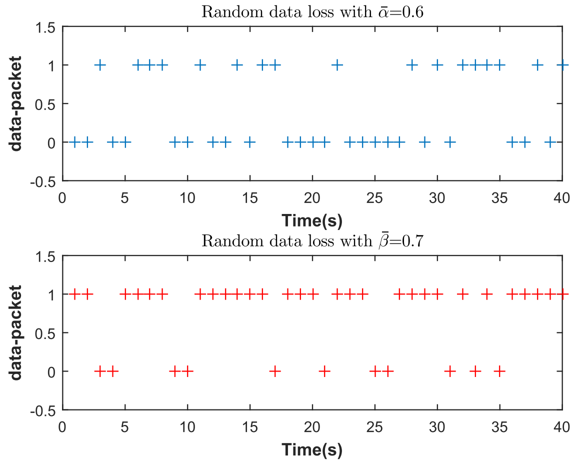

FIGURE 3. The dropout of random data packet.

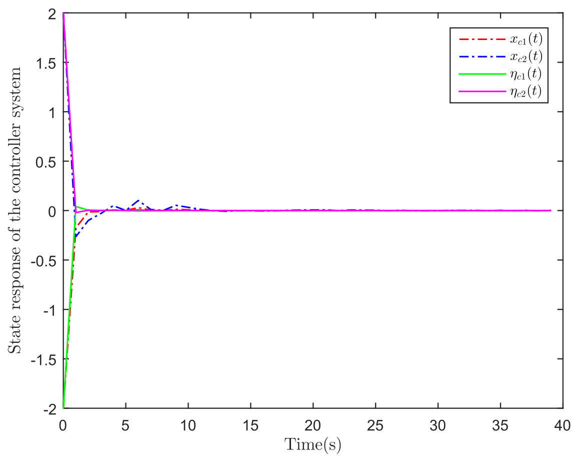

FIGURE 4. The compared controller states.

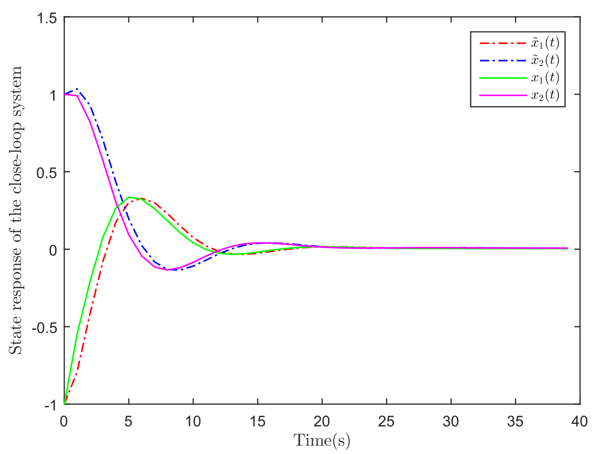

FIGURE 5. The compared system states.

as compensation methods. Finally, the output of the plant and the one of the traditional schemes are presented with H_∞ performance in Fig. 6. From Fig. 4 - Fig. 6, it can be found that the amplitudes of our compensation schemes are denser

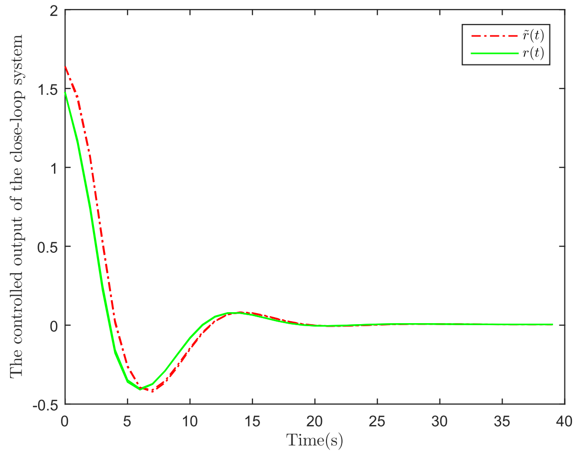

FIGURE 6. The compared output signals of the systems.

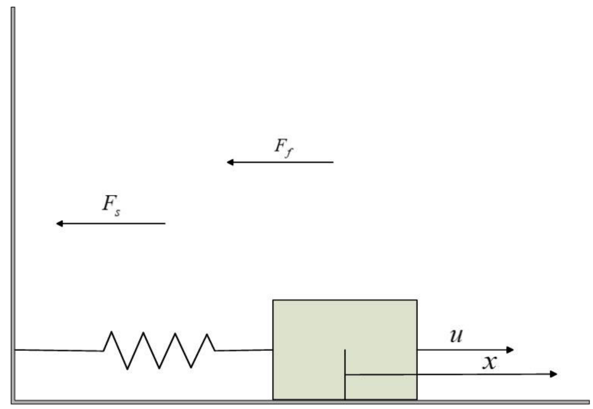

FIGURE 7. The mass-spring-damping system.

and smaller, in which that a better performance in the system is further shown.

B. EXAMPLE 2

The mass-spring-damping system [22] is considered in Fig. 7 and according to Newton's law, one can obtain

$$m\ddot{x} + F_f + F_s = u(t) \tag{36}$$

where F_f is the friction force; m is the mass; F_s and u are the restoring force of the spring and the external control input, respectively. The hardening spring force $F_s = k(1 + a^2x^2)x$ with constants k and a and the friction force $F_f = c\dot{x}$ with $c > 0$. Thus, the dynamic equation is as follows:

$$m\ddot{x} + c\dot{x} + kx + ka^2x^3 = u(t) \tag{37}$$

in which x is the displacement from a reference point. Define $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$. Let $x_1(t) \in [-1\ 1]^{\mathsf{T}}$, $\eta_c(0) = [-2\ 2]^{\mathsf{T}}$, $m = 1kg$, $c = 2Nm/s$, $k = 8N/m$, and $a = 0.3\ m^{-1}$. Then, one can obtains follows:

$$\dot{x}(t) = \sum_{i=1}^{2} H_i(x_1(t))(A_i x(t) + B_i u(t)) \tag{38}$$

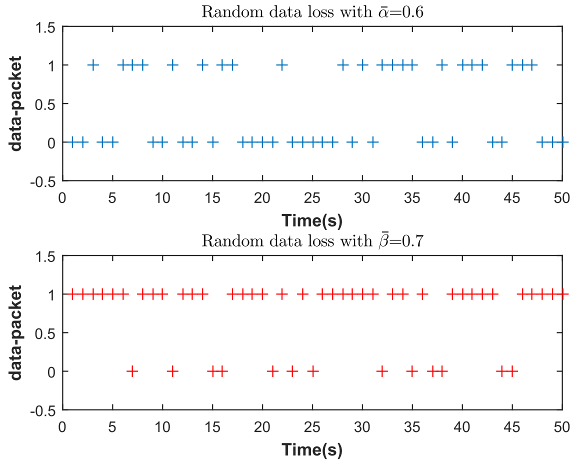

FIGURE 8. The dropout of random data packet.

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{-k-4ka^2}{m} & \dfrac{-c}{m} \end{bmatrix}, \ A_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{-k}{m} & \dfrac{-c}{m} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix}.$$

The membership function is the same as the example 1. Under sampling time $T_s = 0.5s$, the parameters of the matrices can be obtained

$$A_1 = \begin{bmatrix} 0.1925 & 0.1930 \\ -2.0994 & -0.1935 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0.0742 \\ 0.1930 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.3711 & 0.2222 \\ -1.7779 & -0.0734 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.0786 \\ 0.2222 \end{bmatrix}.$$

Else

$$E_1 = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix}, \ E_2 = \begin{bmatrix} -0.3 \\ 0.4 \end{bmatrix}, \ C_{11} = \begin{bmatrix} -1.0789 & 0.5 \end{bmatrix},$$
$$C_{12} = \begin{bmatrix} 0.0585 & 0.5 \end{bmatrix}, \ C_{21} = \begin{bmatrix} 0.9962 & 0.5 \end{bmatrix}, D_1 = -0.5,$$
$$C_{22} = \begin{bmatrix} 0.4795 & 0.5 \end{bmatrix}, \ D_2 = 0.5, F_1 = 0.3, F_2 = 0.4.$$

The gains are listed below

$$A_1^c = \begin{bmatrix} 0.2986 & 0.0220 \\ 0.1592 & -0.1091 \end{bmatrix}, \ A_2^c = \begin{bmatrix} 0.0037 & -0.0709 \\ 0.1555 & 0.0570 \end{bmatrix},$$
$$B_1^c = \begin{bmatrix} -0.1608 \\ -0.0213 \end{bmatrix}, \ B_2^c = \begin{bmatrix} 0.0175 \\ 0.1334 \end{bmatrix},$$
$$C_1^c = \begin{bmatrix} 0.1630 & -0.0461 \end{bmatrix}, \ C_2^c = \begin{bmatrix} 1.8295 & -0.6331 \end{bmatrix}.$$

All initial conditions are the same as in Example 1. Fig. 8 describes the dropout of random data packet. And $\tilde{x}(t)$, $\tilde{x}_c(t)$, $\tilde{r}(t)$ under the zero strategy are the state, the controller state and the output in Fig. 9 - Fig. 11. Fig. 9 plots the compared trajectories of the controller. Fig. 10 indicates the compared state trajectories of the system. From Fig. 9 - Fig. 11, it can be found that the amplitudes of our compensation schemes are denser and smaller. In terms of the simulation results, the effectiveness of the proposed approach is better than the zero strategy.

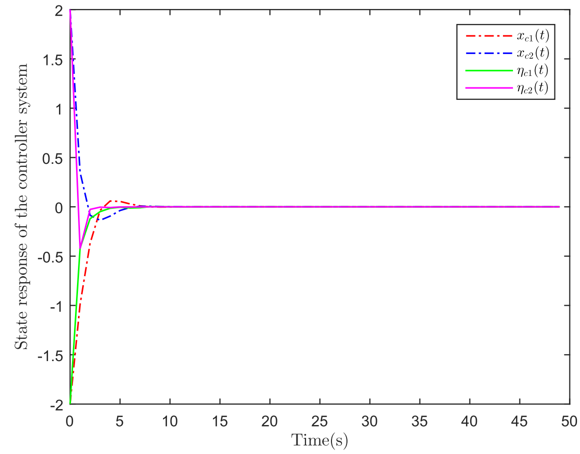

FIGURE 9. The compared controller states.

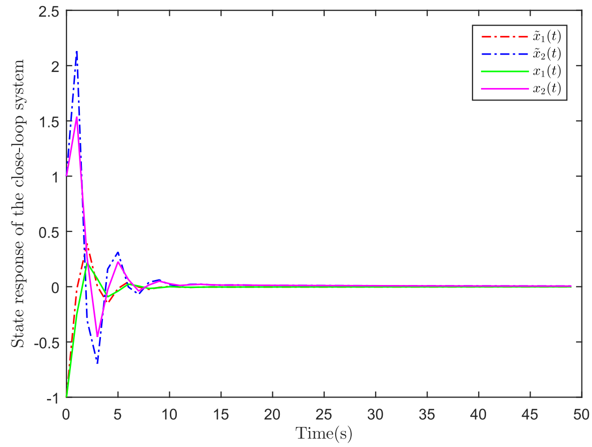

FIGURE 10. The compared system states.

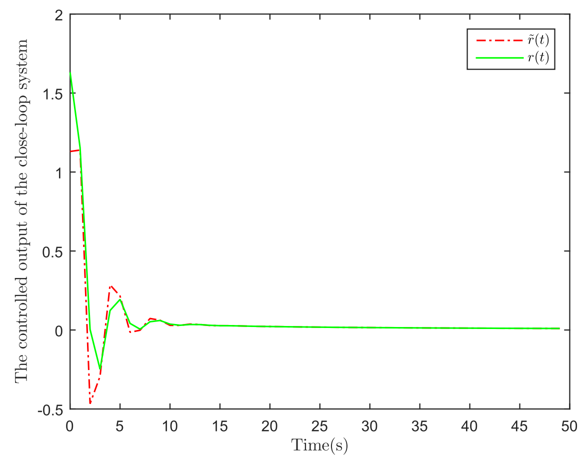

FIGURE 11. The compared output signals of the systems.

Remark 3: From both of the previous examples, it is noted that the traditional strategies don't get the best performance of system. From Fig. 4 - Fig. 6 and Fig. 9 - Fig. 11, in comparison with other strategies it can be observed that system response and better H_∞ performance can be obtained by the presented compensation method. Between the compensation strategy proposed and the existing strategies the more detailed comparison is illustrated in both of the previous examples in this paper, which clearly demonstrates that by the proposed strategy much better performance may be obtained.

V. CONCLUSION

Under an imperfect communication network, for nonlinear NCSs we investigated the problem of data dropout compensation via a fuzzy approach in this paper. The S/C and C/A channels in which data loss occurs are considered so that the effect of measurement loss is decreased for the system stability by constructing a new data compensation scheme. In addition, attention is focused on the dynamic output-feedback controller which is designed via a newly adopted compensation strategy for estimations of the missing packets. Additionally, to effectively decrease conservativeness and deal with the non-convex problem, the CCL method was applied to obtain the main results. Finally, the effectiveness of the proposed results is demonstrated using two illustrative examples.

DECLARATION

The authors declare that there is no conflict of interests on the research, authorship, and publication of this article.

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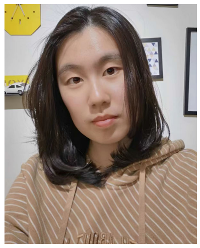

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