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RESEARCH ARTICLE

Further Results on Stability of Sampled-Data Systems Considering Time-Delay and Its Application to Electric Power Markets

WEI-MIN WANG AND **WEI WANG**
School of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou 412007, China
Corresponding author: Wei Wang (wangwei9804@163.com)

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ABSTRACT This paper concentrates on analyzing the stability problems of aperiodic sampled-data systems with time delay. Based on Lyapunov theory, a new time-square-dependent two-side looped-functional (TTLF) is proposed, which can take full advantage of the second order terms with respect to time. And by using the intrinsic relationships of state vectors, a new zero equality is obtained. Then, a less conservative stability condition is gained. In addition, the method proposed is applied to an electric power market (EPM) to study the influence of market clearing time (MCT) and communication delay on system stability. Finally, the effectiveness of the proposed stability criterion is verified based on numerical experiments.

INDEX TERMS Sampled-data system, electric power market, looped-functional, stability.

I. INTRODUCTION

With the rapid development of computer network technology and modern communication technology, various digital control systems have been studied [1], [2], [3], [4], [5], [6], [7]. Furthermore, since sampled-data control reduce control cost and improve control accuracy, it has been diffusely used in various domains such as industrial production, scientific research and national defense construction. In some complex systems such as intelligent traffic, artificial intelligence and the smart power grid, it need to use wide area network to realize the collection of information, data exchange and resource sharing. Although these information are collected and sent in a fixed period, the sampling signal is actually an aperiodic signal received by the controller because of the communication delay of signal transmission in the network. For sampled-data system, sampling period is an important index to measure the performance of sampled-data system. The larger the sampling period, the lower the operating requirement for system communication rate, capacity and bandwidth, the requirement on system hardware will be relaxed. Therefore, various methods have been used to obtain a larger allowable sampling period under the premise of ensuring the stability of sampled-data system [8], [9], [10], [11], [12], [13].

At present, several methods have been proposed to investigate the stability of aperiodic sampled-data system. In [14] and [15], the stability condition is obtained by the discrete-time approach, which converts the sampled-data system into a discrete-time system. In [16], [17], [18], and [19], in terms of the input delay method, sampled-data system is treated as continuous-time system with a time-delayed input, which is designed to study the stability of these systems [20], [21], [22]. In [23], the stability condition is obtained by the impulsive system approach, which converts a sampled-data system to the modality of impulsive systems.

In fact, the conservativeness of the derived stability conditions based on Lyapunov functional approach depends on the construction of Lyapunov functionals [24], [25], [26], [27], [28] and the bounding approach for the functional derivative. In [29], a two-side looped-functional is proposed, which relaxes the restrictive condition of conventional Lyapunov functional and yields relaxed stability criterion. To get a tight bound for the integral terms in the derivative

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TABLE 1. Notations.

Notations	Explanations
$\mathcal{H}^T$	the transpose of matrix $\mathcal{H}$
$\mathcal{H}^{-1}$	the inverse of matrix $\mathcal{H}$
$\Re^n$	n-dimensional Euclidean space
$\Re^{n\times m}$	$n\times m$ real matrices
$M>0$	the matrix M is positive definite and symmetric
$diag\{\cdots\}$	a block-diagonal matrix
I	the identity matrix
0	a zero matrix
$*$	the symmetric terms of the matrix
He$\{\mathtt{C}\}$	$\mathtt{C}+\mathtt{C}^T$
$col\{l_0, l_1, \cdots, l_n\}$	$[l_0^T, l_1^T, \cdots, l_n^T]^T$
$\|\cdot\|$	the Euclidean vector norm

of Lyapunov functional, several integral inequalities are employed, such as Jensen inequality [30], Wirtinger-based inequality [31], [32], Bessel-Legendre inequality [33] and free-matrix-based inequalities [34], [35]. Lately, by using a method that guarantees that the sum of several matrices is positive definite instead of requiring each matrix to be positive definite, relaxed stability condition are obtained in [36]. Nevertheless, it only considers the first order terms with respect to t. Whether the conservativeness of the derived condition can be reduced by increasing the order of the terms with respect to t motivate current research.

This paper concentrates on the stability problem of aperiodic sampled-data systems with time delay. The main contributions of this paper are concluded as follows:

1) Based on the Lyapunov theory, a time-square-dependent two-side looped-functional (TTLF) is proposed, which introduces the second order terms with respect to t.

2) In order to reduce conservativeness, a new zero equality is established by using the intrinsic relationships of state vectors. Based on the proposed TTLF and zero equality, an improved stability condition is obtained.

3) To validate the practicability, the proposed method is applied to the model of electric power market (EPM), and the influence of market clearing time (MCT) and communication delay on the stability of the power market is discussed, which provides certain guiding significance to ensure the balance of energy supply and demand.

II. PRELIMINARIES

Consider the linear system of the form:

$$\dot{x}(\mathrm{t}) = \mathbb{A}x(\mathrm{t}) + \check{\mathbb{B}}u(\mathrm{t}) \tag{1}$$

where $u(\mathrm{t}) \in \Re^m$ and $x(\mathrm{t}) \in \Re^n$ are the control input and the state vector, $\mathbb{A} \in \Re^{n\times n}$ and $\check{\mathbb{B}} \in \Re^{n\times m}$ are system matrices, respectively. $s_k (k = 0, 1, 2\cdots)$ represent sampling instant time of sampler. When the measurement and control signals are transmitted through the network, communication delay can not be avoided. Therefore, the $u(\mathrm{t})$ is described as

$$u(\mathrm{t}) = Kx(s_k), \ \mathrm{t} \in [s_k + \tau, s_{k+1} + \tau) \tag{2}$$

Then, let $\mathrm{t}_k = s_k + \tau$ and $u(\mathrm{t})$ is described as

$$u(\mathrm{t}) = Kx(\mathrm{t}_k - \tau), \ \mathrm{t} \in [\mathrm{t}_k, \mathrm{t}_{k+1}) \tag{3}$$

where K is the state feedback matrix and t_k means the updating time instant of the system, which satisfy

$$h_k = \mathrm{t}_{k+1} - \mathrm{t}_k = s_{k+1} - s_k, h_k \in [h_1, h_2] \tag{4}$$

where h_k represents the sampling periods (update periods), h_1 and h_2 represent the minimum and the maximum of the sampling periods. The communication delay is denoted by τ and it is assumed that $\tau < h_2$. At this point, the closed-loop system is represented as

$$\dot{x}(\mathrm{t}) = \mathbb{A}x(\mathrm{t}) + \mathbb{B}x(\mathrm{t}_k - \tau), \ \mathrm{t} \in [\mathrm{t}_k, \ \mathrm{t}_{k+1}) \tag{5}$$

with $\mathbb{B} = \check{\mathbb{B}}K$. Let $\varphi_k = h_k + \tau$, and the maximum of φ_k is indicated by $\varphi_M = h_2 + \tau$. Let $d_k(\mathrm{t}) = \mathrm{t} - \mathrm{t}_k$ and (5) is indicated by

$$\dot{x}(\mathrm{t}) = \mathbb{A}x(\mathrm{t}) + \mathbb{B}x(\mathrm{t} - d_k(\mathrm{t}) - \tau) \tag{6}$$

where

$$d_k(\mathrm{t}) \in [0, \ h_k), \ \dot{d}_k(\mathrm{t}) = 1 \text{ for } \mathrm{t} \neq \mathrm{t}_k \tag{7}$$

III. MAIN RESULTS

In order to simplify the description, we define

$$\zeta_1(\beta,\alpha) = x(\beta) - x(\alpha), \ \zeta_2(\beta,\alpha) = \frac{1}{\beta-\alpha}\int_\alpha^\beta x(s)ds,$$

$$\zeta_3(\beta,\alpha) = \frac{2}{(\beta-\alpha)^2}\int_\alpha^\beta\int_\alpha^s x(u)duds,$$

$$\zeta_4(\beta,\alpha) = \frac{2}{(\beta-\alpha)^2}\int_\alpha^\beta\int_s^\beta x(u)duds,$$

$$\begin{aligned}
\psi_1(\mathrm{t}) = col\{&x(\mathrm{t}), \ x(\mathrm{t}-\tau), \ \tau\zeta_2(\mathrm{t},\mathrm{t}-\tau), \\
&\frac{\tau^2}{2}\zeta_3(\mathrm{t},\mathrm{t}-\tau), \ \frac{\tau^2}{2}\zeta_4(\mathrm{t},\mathrm{t}-\tau)\},\\
\psi_2(\mathrm{t},s) = col\{&x(s), \ \dot{x}(s), \ x(\mathrm{t}), \ x(\mathrm{t}-\tau), \ \tau\zeta_2(s,\mathrm{t}-\tau), \\
&\tau\zeta_2(\mathrm{t},s)\},\\
\psi_3(\mathrm{t}) = col\{&x(\mathrm{t}_k), \ x(\mathrm{t}_k-\tau), \ x(\mathrm{t}_{k+1}), \ x(\mathrm{t}_{k+1}-\tau),\\
&\tau\zeta_2(\mathrm{t}_k,\mathrm{t}_k-\tau), \ x(\mathrm{t}), \ x(\mathrm{t}-\tau), \ \zeta_2(\mathrm{t}_{k+1},\mathrm{t}),\\
&\zeta_4(\mathrm{t}_{k+1},\mathrm{t}), \ \zeta_2(\mathrm{t}_{k+1}-\tau,\mathrm{t}-\tau),\\
&\zeta_4(\mathrm{t}_{k+1}-\tau,\mathrm{t}-\tau), \ \tau\zeta_2(\mathrm{t},\mathrm{t}-\tau),\\
&\frac{\tau^2}{2}\zeta_3(\mathrm{t},\mathrm{t}-\tau), \ \frac{\tau^2}{2}\zeta_4(\mathrm{t},\mathrm{t}-\tau)\},\\
\psi_4(\mathrm{t}) = col\{&x(\mathrm{t}_k), \ x(\mathrm{t}_k-\tau), \ x(\mathrm{t}_{k+1}), \ x(\mathrm{t}_{k+1}-\tau),\\
&\tau\zeta_2(\mathrm{t}_k,\mathrm{t}_k-\tau), \ x(\mathrm{t}), \ x(\mathrm{t}-\tau), \ \zeta_2(\mathrm{t},\mathrm{t}_k),\\
&\zeta_3(\mathrm{t},\mathrm{t}_k), \ \zeta_2(\mathrm{t}-\tau,\mathrm{t}_k-\tau), \ \zeta_3(\mathrm{t}-\tau,\mathrm{t}_k-\tau),\\
&\tau\zeta_2(\mathrm{t},\mathrm{t}-\tau), \ \frac{\tau^2}{2}\zeta_3(\mathrm{t},\mathrm{t}-\tau), \ \frac{\tau^2}{2}\zeta_4(\mathrm{t},\mathrm{t}-\tau)\},\\
\psi_5(\mathrm{t}) = col\{&x(\mathrm{t}_k), \ x(\mathrm{t}_k-\tau), \ x(\mathrm{t}_{k+1}), \ x(\mathrm{t}_{k+1}-\tau),\\
&\tau\zeta_2(\mathrm{t}_k,\mathrm{t}_k-\tau), \ x(\mathrm{t}), \ x(\mathrm{t}-\tau), \ \zeta_2(\mathrm{t},\mathrm{t}_k),\\
&\zeta_3(\mathrm{t},\mathrm{t}_k), \ \zeta_2(\mathrm{t}-\tau,\mathrm{t}_k-\tau), \ \zeta_3(\mathrm{t}-\tau,\mathrm{t}_k-\tau),
\end{aligned}$$

$$\zeta_2(t_{k+1}, t),\ \zeta_4(t_{k+1}, t),\ \zeta_2(t_{k+1}-\tau, t-\tau),$$
$$\zeta_4(t_{k+1}-\tau, t-\tau),\ \tau\zeta_2(t, t-\tau),$$
$$\frac{\tau^2}{2}\zeta_3(t, t-\tau),\ \frac{\tau^2}{2}\zeta_4(t, t-\tau)\},$$
$$\psi_6(t) = col\{\zeta_1(t, t_k),\ \zeta_1(t-\tau, t_k-\tau)\},$$
$$\psi_7(t) = col\{\zeta_1(t, t_{k+1}),\ \zeta_1(t-\tau, t_{k+1}-\tau)\},$$
$$\psi_8(t) = col\{\zeta_2(t, t-\tau),\ \zeta_3(t, t-\tau),\ \zeta_2(t, t_k),\ \zeta_3(t, t_k),$$
$$\zeta_2(t_{k+1}, t),\ \zeta_4(t_{k+1}, t),\ \zeta_2(t-\tau, t_k-\tau),$$
$$\zeta_3(t-\tau, t_k-\tau),\ \zeta_2(t_{k+1}-\tau, t-\tau),$$
$$\zeta_4(t_{k+1}-\tau, t-\tau),\ \zeta_2(t_k, t_k-\tau)\},$$
$$\breve{\psi}_1(t) = col\{x(t),\ x(t_k),\ x(t_{k+1}),\ \zeta_2(t_{k+1}, t),\ \zeta_4(t_{k+1}, t)\},$$
$$\breve{\psi}_2(t) = col\{x(t),\ x(t_k),\ x(t_{k+1}),\ \zeta_2(t, t_k),\ \zeta_3(t, t_k)\},$$

$$\breve{\psi}_3(t) = col\{x(t),\ x(t_k),\ x(t_{k+1}),\ \zeta_2(t, t_k),\ \zeta_3(t, t_k),$$
$$\zeta_2(t_{k+1}, t),\ \zeta_4(t_{k+1}, t)\},$$
$$\psi(t) = col\{x^T(t),\ x(t-\tau),\ x(t_k),\ x(t_k-\tau),\ x(t_{k+1}),$$
$$x(t_{k+1}-\tau),\ x(t-\varphi_k),\ \dot{x}(t-\tau),\ \psi_8(t)\},$$
$$\breve{\psi}(t) = col\{x(t),\ x(t_k),\ \zeta_2(t, t_k),\ \zeta_3(t, t_k),\ x(t_{k+1}),$$
$$\zeta_2(t_{k+1}, t),\ \zeta_4(t_{k+1}, t)\},$$
$$e_j = \left[0_{n\times(j-1)n}\ I_n\ 0_{n\times(19-j)n}\right],\ j = 1, 2, \cdots, 19,$$
$$\breve{e}_j = \left[0_{n\times(j-1)n}\ I_n\ 0_{n\times(7-j)n}\right],\ j = 1, 2, \cdots, 7.$$

Now, the following theorem is presented.

Theorem 1: For given $\tau > 0$ and $h_2 \geq h_1 \geq 0$, system (5) with the control input (3) satisfying (4) is stable if there exist matrices $P > 0$, $S > 0$, $D_1 > 0$, $D_2 = D_2^T$, $D_3 > 0$, Q_1, Q_2, X, $G = G^T$, $\mathcal{W}_1 > 0$, $\mathcal{W}_2 > 0$, $\mathcal{W}_3 > 0$, $\mathcal{W}_4 > 0$, $\mathcal{N}_i$, $\mathcal{M}_i$, $\mathcal{L}_i$, $i = 1, 2, 3$, $\mathcal{Y}_j$, $j = 1, 2, \cdots, 13$, such that LMIs (8)-(14) are satisfied.

$$D_2 + D_3 > 0 \tag{8}$$
$$\mathcal{W}_1 + D_3 > 0 \tag{9}$$
$$\mathcal{W}_3 + D_2 > 0 \tag{10}$$
$$\begin{bmatrix} \Lambda_1 & \Psi_1 & \sqrt{\tau}\Psi_2 \\ * & -\Pi_1 & 0 \\ * & * & -\Pi_2 \end{bmatrix}_{h_k\in[h_1,\ h_2]} < 0 \tag{11}$$
$$\begin{bmatrix} \Lambda_1 + h_k\Lambda_2 + h_k^2\Lambda_3 & \Psi_{32} \\ * & -\Pi_{32} \end{bmatrix}_{h_k\in[h_1,\ h_2]} < 0 \tag{12}$$
$$\begin{bmatrix} \Lambda_1 + \frac{h_k}{4}\Lambda_2 & \Psi_{132} \\ * & -\Pi_{132} \end{bmatrix}_{h_k\in[h_1,\ h_2]} < 0 \tag{13}$$
$$\begin{bmatrix} \Lambda_1 + \frac{3}{4}h_k\Lambda_2 + \frac{1}{2}h_k^2\Lambda_3 & \Psi_{312} \\ * & -\Pi_{312} \end{bmatrix}_{h_k\in[h_1,\ h_2]} < 0 \tag{14}$$

where

$$\begin{aligned}\Lambda_1 &= He\{\rho_1^T P\rho_2 + \rho_{32}^T X\rho_{31} + \rho_{30}^T X\rho_{32} + \mathcal{M}_1\rho_{11} + \mathcal{M}_2\rho_{12} \\
&+ \mathcal{M}_3\rho_{13} + \mathcal{N}_1\rho_{14} + \mathcal{N}_2\rho_{15} + \mathcal{N}_3\rho_{16} + \mathcal{Y}_1\rho_{17} + \mathcal{Y}_2\rho_{36} \\
&+ \mathcal{Y}_3\rho_{37} + \mathcal{Y}_4\rho_{18} + \mathcal{Y}_5\rho_{38} + \mathcal{Y}_6\rho_{39} + \mathcal{L}_1\rho_{19} + \mathcal{L}_2\rho_{20} \\
&+ \mathcal{L}_3\rho_{21} + \mathcal{Y}_7\rho_{22} + \mathcal{Y}_8\rho_{42} + \mathcal{Y}_9\rho_{23} - \mathcal{Y}_{10}\rho_{11} - \mathcal{Y}_{11}\rho_{14} \\
&- \mathcal{Y}_{12}\rho_{28} - \mathcal{Y}_{13}\rho_{29} + \rho_{40}^T S\rho_{41} \\
&+ h_k(-2\rho_{30}^T Q_1\rho_5 + \rho_{30}^T Q_1\rho_8 + \mathcal{Y}_{11}\rho_{25} + \mathcal{Y}_{13}\rho_{27} \\
&+ \rho_{34}^T G\rho_6) + h_k^2(\rho_{32}^T Q_1\rho_5 + \rho_{30}^T Q_1\rho_7)\} + \rho_3^T S\rho_3 \\
&- \rho_4^T S\rho_4 + \tau\eta^T D_1\eta + h_2\eta^T D_2\eta + \varphi_M\eta^T D_3\eta \\
&+ h_k(\rho_6^T G\rho_6 + \eta^T\mathcal{W}_1\eta + e_8^T\mathcal{W}_3 e_8)\end{aligned}$$

$$\begin{aligned}\Lambda_2 &= He\{\mathcal{Y}_{10}\rho_{24} + \mathcal{Y}_{12}\rho_{26} - (-2\rho_{30}^T Q_1\rho_5 + \rho_{30}^T Q_1\rho_8 \\
&+ \mathcal{Y}_{11}\rho_{25} + \mathcal{Y}_{13}\rho_{27}) + 2\rho_{31}^T Q_2\rho_9 \\
&+ \rho_{31}^T Q_2\rho_{10} + h_k\rho_{33}^T G\rho_6 + \rho_{35}^T G\rho_6 - \rho_{34}^T G\rho_6 \\
&- 2h_k(\rho_{32}^T Q_1\rho_5 + \rho_{30}^T Q_1\rho_7)\} - \rho_6^T G\rho_6 + \eta^T\mathcal{W}_2\eta \\
&+ e_8^T\mathcal{W}_4 e_8 - (\rho_6^T G\rho_6 + \eta^T\mathcal{W}_1\eta + e_8^T\mathcal{W}_3 e_8)\end{aligned}$$

$$\begin{aligned}\Lambda_3 &= He\{-\rho_{33}^T G\rho_6 + \rho_{32}^T Q_1\rho_5 + \rho_{30}^T Q_1\rho_7 + \rho_{32}^T Q_2\rho_9 \\
&+ \rho_{31}^T Q_2\rho_7\}\end{aligned}$$

with

$$\Psi_1 = [h_k\mathcal{N}_1\ \ h_k\mathcal{N}_2\ \ h_k\mathcal{N}_3\ \ h_k\mathcal{Y}_4\ \ h_k\mathcal{Y}_5\ \ h_k\mathcal{Y}_6\ \ h_k\mathcal{Y}_9]$$
$$\Psi_2 = [\mathcal{L}_1\ \ \mathcal{L}_2\ \ \mathcal{L}_3\ \ \mathcal{Y}_7\ \ \mathcal{Y}_8]$$
$$\Psi_3 = [h_k\mathcal{M}_1\ \ h_k\mathcal{M}_2\ \ h_k\mathcal{M}_3\ \ h_k\mathcal{Y}_1\ \ h_k\mathcal{Y}_2\ \ h_k\mathcal{Y}_3]$$
$$\Pi_1 = \text{diag}\{h_k\mathcal{W}_2, 3h_k\mathcal{W}_2, 5h_k\mathcal{W}_2, h_k\mathcal{W}_4, 3h_k\mathcal{W}_4, 5h_k\mathcal{W}_4,\ h_k(D_2 + D_3)\}$$
$$\Pi_2 = \text{diag}\{D_1, 3D_1, 5D_1, D_3, 3D_3\}$$
$$\Pi_3 = \text{diag}\{h_k(\mathcal{W}_1 + D_3), 3h_k(\mathcal{W}_1 + D_3), 5h_k(\mathcal{W}_1 + D_3), h_k(\mathcal{W}_3 + D_2), 3h_k(\mathcal{W}_3 + D_2), 5h_k(\mathcal{W}_3 + D_2)\}$$
$$\Psi_{32} = \left[\Psi_3 \quad \sqrt{\tau}\Psi_2\right],\ \Pi_{32} = \text{diag}\{\Pi_3, \Pi_2\}$$
$$\Psi_{132} = \left[\Psi_1\ \Psi_3\ \sqrt{\tau}\Psi_2\right]$$
$$\Pi_{132} = \text{diag}\{\frac{4}{3}\Pi_1, 4\Pi_3, \Pi_2\}$$
$$\Psi_{312} = \left[\Psi_3\ \Psi_1\ \sqrt{\tau}\Psi_2\right]$$
$$\Pi_{312} = \text{diag}\{\frac{4}{3}\Pi_3, 4\Pi_1, \Pi_2\}$$
$$\eta = \mathbb{A}e_1 + \mathbb{B}e_4$$
$$\rho_1 = [e_1^T\ e_2^T\ \tau e_9^T\ \frac{\tau^2}{2}e_{10}^T\ \tau^2 e_9^T - \frac{\tau^2}{2}e_{10}^T]^T$$
$$\rho_2 = [\eta^T\ e_8^T\ e_1^T - e_2^T\ \tau(e_9^T - e_2^T)\ \tau(e_1^T - e_9^T)]^T$$
$$\rho_3 = [e_1^T\ \eta^T\ e_1^T\ e_2^T\ \tau e_9^T\ 0]^T$$
$$\rho_4 = [e_2^T\ e_8^T\ e_1^T\ e_2^T\ 0\ \tau e_9^T]^T$$
$$\rho_5 = [e_3^T\ e_4^T\ e_5^T\ e_6^T\ \tau e_{19}^T\ e_1^T\ e_2^T\ e_{13}^T\ e_{14}^T\ e_{17}^T\ e_{18}^T\ \tau e_9^T\ \frac{\tau^2}{2}e_{10}^T\ \tau^2 e_9^T - \frac{\tau^2}{2}e_{10}^T]^T$$
$$\rho_6 = [e_3^T\ e_4^T\ e_5^T\ e_6^T\ \tau e_{19}^T\ e_1^T\ e_2^T\ e_{11}^T\ e_{12}^T\ e_{15}^T\ e_{16}^T\ e_{13}^T\ e_{14}^T\ e_{17}^T\ e_{18}^T\ \tau e_9^T\ \frac{\tau^2}{2}e_{10}^T\ \tau^2 e_9^T - \frac{\tau^2}{2}e_{10}^T]^T$$

$$\rho_7 = [0\,0\,0\,0\,0\,\eta^T\; e_8^T\; 0\,0\,0\,0\; e_1^T - e_2^T\; \tau(e_9^T - e_2^T)\; \tau(e_1^T - e_9^T)]^T$$
$$\rho_8 = [0\,0\,0\,0\,0\,0\,0\; e_{13}^T - e_1^T\; 2(e_{14}^T - e_{13}^T)\; e_{17}^T - e_2^T\; 2(e_{18}^T - e_{17}^T)\; 0\,0\,0]^T$$
$$\rho_9 = [e_3^T\; e_4^T\; e_5^T\; e_6^T\; \tau e_{19}^T\; e_1^T\; e_2^T\; e_{11}^T\; e_{12}^T\; e_{15}^T\; e_{16}^T\; \tau e_9^T\; \frac{\tau^2}{2}e_{10}^T\; \tau^2 e_9^T - \frac{\tau^2}{2}e_{10}^T]^T$$
$$\rho_{10} = [0\,0\,0\,0\,0\,0\,0\; e_1^T - e_{11}^T\; 2(e_{11}^T - e_{12}^T)\; e_2^T - e_{15}^T\; 2(e_{15}^T - e_{16}^T)\; 0\,0\,0]^T$$
$$\rho_{11} = e_1 - e_3,\ \rho_{12} = e_1 + e_3 - 2e_{11}$$
$$\rho_{13} = e_1 - e_3 - 6e_{11} + 6e_{12},\ \rho_{14} = e_5 - e_1$$
$$\rho_{15} = e_5 + e_1 - 2e_{13},\ \rho_{16} = e_5 - e_1 + 6e_{13} - 6e_{14}$$
$$\rho_{17} = e_2 - e_4,\ \rho_{18} = e_6 - e_2,\ \rho_{19} = e_1 - e_2$$
$$\rho_{20} = e_1 + e_2 - 2e_9,\ \rho_{21} = e_1 - e_2 - 6e_9 + 6e_{10}$$
$$\rho_{22} = e_3 - e_4,\ \rho_{23} = e_4 - e_7,\ \rho_{24} = \mathbb{A}e_{11} + \mathbb{B}e_4$$
$$\rho_{25} = \mathbb{A}e_{13} + \mathbb{B}e_4,\ \rho_{26} = \mathbb{A}e_{12} + \mathbb{B}e_4,\ \rho_{27} = \mathbb{A}e_{14} + \mathbb{B}e_4$$
$$\rho_{28} = 2e_{11} - 2e_3,\ \rho_{29} = 2e_5 - 2e_{13},\ \rho_{32} = [\eta^T\; e_8^T]^T$$
$$\rho_{30} = [e_1^T - e_3^T\; e_2^T - e_4^T]^T,\ \rho_{31} = [e_1^T - e_5^T\; e_2^T - e_6^T]^T$$
$$\rho_{33} = [0\,0\,0\,0\,0\,\eta^T\; e_8^T\; 0\,0\,0\,0\,0\,0\,0\,0\; e_1^T - e_2^T\; \tau(e_9^T - e_2^T)\; \tau(e_1^T - e_9^T)]^T$$
$$\rho_{34} = [0\,0\,0\,0\,0\,0\,0\; e_1^T - e_{11}^T\; 2(e_{11}^T - e_{12}^T)\; e_2^T - e_{15}^T\; 2(e_{15}^T - e_{16}^T)\; 0\,0\,0\,0\,0\,0\,0]^T$$
$$\rho_{35} = [0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\; e_{13}^T - e_1^T\; 2(e_{14}^T - e_{13}^T)\; e_{17}^T - e_2^T\; 2(e_{18}^T - e_{17}^T)\; 0\,0\,0]^T$$
$$\rho_{36} = e_2 + e_4 - 2e_{15},\ \rho_{37} = e_2 - e_4 - 6e_{15} + 6e_{16}$$
$$\rho_{38} = e_6 + e_2 - 2e_{17},\ \rho_{39} = e_6 - e_2 + 6e_{17} - 6e_{18}$$
$$\rho_{40} = [\tau e_9^T\; e_1^T - e_2^T\; \tau e_1^T\; \tau e_2^T\; \frac{\tau^2}{2}e_{10}^T\; \tau^2 e_9^T - \frac{\tau^2}{2}e_{10}^T]^T$$
$$\rho_{41} = [0\,0\,\eta^T\; e_8^T\; -e_2^T\; e_1^T]^T,\ \rho_{42} = e_3 + e_4 - 2e_{19}.$$

Proof: First we constract a new Lyapunov functional for the system (5)

$$V(\mathrm{t}) = V_c(\mathrm{t}) + V_d(\mathrm{t}) \tag{15}$$

where $V_c(\mathrm{t}) = \sum_{j=1}^{5} V_{cj}(\mathrm{t})$ and $V_d(\mathrm{t}) = \sum_{j=1}^{8} V_{dj}(\mathrm{t})$ are as follows

$$V_{c1}(\mathrm{t}) = \psi_1^T(\mathrm{t})P\psi_1(\mathrm{t}),$$
$$V_{c2}(\mathrm{t}) = \int_{\mathrm{t}-\tau}^{\mathrm{t}} \psi_2^T(\mathrm{t}, s)S\psi_2(\mathrm{t}, s)ds,$$
$$V_{c3}(\mathrm{t}) = \int_{-\tau}^{0}\int_{\mathrm{t}+\theta}^{\mathrm{t}} \dot{x}^T(u)D_1\dot{x}(u)dud\theta,$$
$$V_{c4}(\mathrm{t}) = \int_{-\varphi_M}^{-\tau}\int_{\mathrm{t}+\theta}^{\mathrm{t}} \dot{x}^T(u)D_2\dot{x}(u)dud\theta,$$
$$V_{c5}(\mathrm{t}) = \int_{-\varphi_M}^{0}\int_{\mathrm{t}+\theta}^{\mathrm{t}} \dot{x}^T(u)D_3\dot{x}(u)dud\theta,$$
$$V_{d1}(\mathrm{t}) = 2(h_k - d_k(\mathrm{t}))^2\psi_6^T(\mathrm{t})Q_1\psi_3(\mathrm{t}),$$
$$V_{d2}(\mathrm{t}) = 2d_k^2(\mathrm{t})\psi_7^T(\mathrm{t})Q_2\psi_4(\mathrm{t}),$$
$$V_{d3}(\mathrm{t}) = (h_k - d_k(\mathrm{t}))d_k(\mathrm{t})\psi_5^T(\mathrm{t})G\psi_5(\mathrm{t}),$$
$$V_{d4}(\mathrm{t}) = 2\psi_6^T(\mathrm{t})X\psi_7(\mathrm{t}),$$
$$V_{d5}(\mathrm{t}) = (h_k - d_k(\mathrm{t}))\int_{\mathrm{t}_k}^{\mathrm{t}} \dot{x}^T(s)\mathcal{W}_1\dot{x}(s)ds,$$
$$V_{d6}(\mathrm{t}) = -d_k(\mathrm{t})\int_{\mathrm{t}}^{\mathrm{t}_{k+1}} \dot{x}^T(s)\mathcal{W}_2\dot{x}(s)ds,$$
$$V_{d7}(\mathrm{t}) = (h_k - d_k(\mathrm{t}))\int_{\mathrm{t}_k-\tau}^{\mathrm{t}-\tau} \dot{x}^T(s)\mathcal{W}_3\dot{x}(s)ds,$$
$$V_{d8}(\mathrm{t}) = -d_k(\mathrm{t})\int_{\mathrm{t}-\tau}^{\mathrm{t}_{k+1}-\tau} \dot{x}^T(s)\mathcal{W}_4\dot{x}(s)ds.$$

Taking the derivative of $V(\mathrm{t})$ along the trajectories of system (5) yields

$$\dot{V}_{c1}(\mathrm{t}) = 2\psi_1^T(\mathrm{t})P\dot{\psi}_1(\mathrm{t}),$$
$$\dot{V}_{c2}(\mathrm{t}) = \psi_2^T(\mathrm{t}, \mathrm{t})S\psi_2(\mathrm{t}, \mathrm{t}) - \psi_2^T(\mathrm{t}, \mathrm{t}-\tau)S\psi_2(\mathrm{t}, \mathrm{t}-\tau) + 2\int_{\mathrm{t}-\tau}^{\mathrm{t}} \psi_2^T(\mathrm{t}, s)S\frac{\partial\psi_2(\mathrm{t}, s)}{\partial \mathrm{t}}ds,$$
$$\dot{V}_{c3}(\mathrm{t}) = \tau\dot{x}^T(\mathrm{t})D_1\dot{x}(\mathrm{t}) + \wp_1,$$
$$\dot{V}_{c4}(\mathrm{t}) = h_2\dot{x}^T(\mathrm{t})D_2\dot{x}(\mathrm{t}) + \wp_2,$$
$$\dot{V}_{c5}(\mathrm{t}) = \varphi_M\dot{x}^T(\mathrm{t})D_3\dot{x}(\mathrm{t}) + \wp_3,$$
$$\dot{V}_{d1}(\mathrm{t}) = -4(h_k - d_k(\mathrm{t}))\psi_6^T(\mathrm{t})Q_1\psi_3(\mathrm{t}) + 2(h_k - d_k(\mathrm{t}))^2 \times \dot{\psi}_6^T(\mathrm{t})Q_1\psi_3(\mathrm{t}) + 2(h_k - d_k(\mathrm{t}))^2\psi_6^T(\mathrm{t})Q_1\dot{\psi}_3(\mathrm{t}),$$
$$\dot{V}_{d2}(\mathrm{t}) = 4d_k(\mathrm{t})\psi_7^T(\mathrm{t})Q_2\psi_4(\mathrm{t}) + 2d_k^2(\mathrm{t})\dot{\psi}_7^T(\mathrm{t})Q_2\psi_4(\mathrm{t}) + 2d_k^2(\mathrm{t})\psi_7^T(\mathrm{t})Q_2\dot{\psi}_4(\mathrm{t}),$$
$$\dot{V}_{d3}(\mathrm{t}) = -d_k(\mathrm{t})\psi_5^T(\mathrm{t})G\psi_5(\mathrm{t}) + (h_k - d_k(\mathrm{t}))\psi_5^T(\mathrm{t})G\psi_5(\mathrm{t}) + 2(h_k - d_k(\mathrm{t}))d_k(\mathrm{t})\dot{\psi}_5^T(\mathrm{t})G\psi_5(\mathrm{t}),$$
$$\dot{V}_{d4}(\mathrm{t}) = 2\dot{\psi}_6^T(\mathrm{t})X\psi_7(\mathrm{t}) + 2\psi_6^T(\mathrm{t})X\dot{\psi}_7(\mathrm{t}),$$
$$\dot{V}_{d5}(\mathrm{t}) = (h_k - d_k(\mathrm{t}))\dot{x}^T(\mathrm{t})\mathcal{W}_1\dot{x}(\mathrm{t}) + \wp_4,$$
$$\dot{V}_{d6}(\mathrm{t}) = d_k(\mathrm{t})\dot{x}^T(\mathrm{t})\mathcal{W}_2\dot{x}(\mathrm{t}) + \wp_5,$$
$$\dot{V}_{d7}(\mathrm{t}) = (h_k - d_k(\mathrm{t}))\dot{x}^T(\mathrm{t}-\tau)\mathcal{W}_3\dot{x}(\mathrm{t}-\tau) + \wp_6,$$
$$\dot{V}_{d8}(\mathrm{t}) = d_k(\mathrm{t})\dot{x}^T(\mathrm{t}-\tau)\mathcal{W}_4\dot{x}(\mathrm{t}-\tau) + \wp_7.$$

with

$$\wp_1 = -\int_{\mathrm{t}-\tau}^{\mathrm{t}} \dot{x}^T(s)D_1\dot{x}(s)ds,$$
$$\wp_2 = -\int_{\mathrm{t}-\varphi_M}^{\mathrm{t}-\tau} \dot{x}^T(s)D_2\dot{x}(s)ds,$$
$$\wp_3 = -\int_{\mathrm{t}-\varphi_M}^{\mathrm{t}} \dot{x}^T(s)D_3\dot{x}(s)ds,$$
$$\wp_4 = -\int_{\mathrm{t}_k}^{\mathrm{t}} \dot{x}^T(s)\mathcal{W}_1\dot{x}(s)\mathrm{d}s,$$
$$\wp_5 = -\int_{\mathrm{t}}^{\mathrm{t}_{k+1}} \dot{x}^T(s)\mathcal{W}_2\dot{x}(s)\mathrm{d}s,$$
$$\wp_6 = -\int_{\mathrm{t}_k-\tau}^{\mathrm{t}-\tau} \dot{x}^T(s)\mathcal{W}_3\dot{x}(s)\mathrm{d}s,$$
$$\wp_7 = -\int_{\mathrm{t}-\tau}^{\mathrm{t}_{k+1}-\tau} \dot{x}^T(s)\mathcal{W}_4\dot{x}(s)\mathrm{d}s.$$

For $\varphi_M \geq \varphi_k$ and $D_2 + D_3 > 0$, it follows that

$$\begin{aligned}\wp_2 + \wp_3 &= -\int_{t-\varphi_M}^{t-\tau} \dot{x}^T(s)D_2\dot{x}(s)ds - \int_{t-\varphi_M}^{t} \dot{x}^T(s)D_3\dot{x}(s)ds \\ &\leq -\int_{t_k-\tau}^{t-\tau} \dot{x}^T(s)D_2\dot{x}(s)ds - \int_{t_k}^{t} \dot{x}^T(s)D_3\dot{x}(s)ds \\ &\quad - \int_{t_k-\tau}^{t_k} \dot{x}^T(s)D_3\dot{x}(s)ds \\ &\quad - \int_{t-\varphi_k}^{t_k-\tau} \dot{x}^T(s)(D_2+D_3)\dot{x}(s)ds\end{aligned}$$

Then,we obtain

$$\wp_2 + \wp_3 + \wp_4 + \wp_6 \leq \bar{\wp}_2 + \bar{\wp}_3 + \bar{\wp}_4 + \bar{\wp}_6 \tag{16}$$

where

$$\begin{aligned}\bar{\wp}_2 &= -\int_{t_k}^{t} \dot{x}^T(s)(\mathcal{W}_1 + D_3)\dot{x}(s)ds, \\ \bar{\wp}_3 &= -\int_{t_k-\tau}^{t-\tau} \dot{x}^T(s)(\mathcal{W}_3 + D_2)\dot{x}(s)ds, \\ \bar{\wp}_4 &= -\int_{t-\varphi_k}^{t_k-\tau} \dot{x}^T(s)(D_2 + D_3)\dot{x}(s)ds, \\ \bar{\wp}_6 &= -\int_{t_k-\tau}^{t_k} \dot{x}^T(s)D_3\dot{x}(s)ds.\end{aligned}$$

Applying free-matrix-based integral inequality in [35], it follows from (9) and (10) that

$$\begin{aligned}\wp_1 \leq \psi^T(t)[\tau \sum_{i=1}^{3} \frac{1}{2i-1}\mathcal{L}_i D_1^{-1}\mathcal{L}_i^T \\ + He\{\mathcal{L}_1\rho_{19} + \mathcal{L}_2\rho_{20} + \mathcal{L}_3\rho_{21}\}]\psi(t),\end{aligned} \tag{17}$$

$$\begin{aligned}\bar{\wp}_2 \leq \psi^T(t)[d_k(t) \sum_{i=1}^{3} \frac{1}{2i-1}\mathcal{M}_i(\mathcal{W}_1 + D_3)^{-1}\mathcal{M}_i^T \\ + He\{\mathcal{M}_1\rho_{11} + \mathcal{M}_2\rho_{12} + \mathcal{M}_3\rho_{13}\}]\psi(t),\end{aligned} \tag{18}$$

$$\begin{aligned}\bar{\wp}_3 \leq \psi^T(t)[d_k(t) \sum_{i=1}^{3} \frac{1}{2i-1}\mathcal{Y}_i(\mathcal{W}_3 + D_2)^{-1}\mathcal{Y}_i^T \\ + He\{\mathcal{Y}_1\rho_{17} + \mathcal{Y}_2\rho_{36} + \mathcal{Y}_3\rho_{37}\}]\psi(t),\end{aligned} \tag{19}$$

$$\begin{aligned}\bar{\wp}_4 \leq \psi^T(t)[(h_k - d_k(t))\mathcal{Y}_9 D_{23}^{-1}\mathcal{Y}_9^T \\ + He\{\mathcal{Y}_9\rho_{23}\}]\psi(t),\end{aligned} \tag{20}$$

$$\begin{aligned}\wp_5 \leq \psi^T(t)[(h_k - d_k(t)) \sum_{i=1}^{3} \frac{1}{2i-1}\mathcal{N}_i\mathcal{W}_2^{-1}\mathcal{N}_i^T \\ + He\{\mathcal{N}_1\rho_{14} + \mathcal{N}_2\rho_{15} + \mathcal{N}_3\rho_{16}\}]\psi(t),\end{aligned} \tag{21}$$

$$\begin{aligned}\bar{\wp}_6 \leq \psi^T(t)[\tau \sum_{i=1}^{2} \frac{1}{2i-1}\mathcal{Y}_{i+6}D_3^{-1}\mathcal{Y}_{i+6}^T \\ + He\{\mathcal{Y}_7\rho_{22} + \mathcal{Y}_8\rho_{42}\}]\psi(t),\end{aligned} \tag{22}$$

$$\begin{aligned}\wp_7 \leq \psi^T(t)[(h_k - d_k(t)) \sum_{i=1}^{3} \frac{1}{2i-1}\mathcal{Y}_{i+3}\mathcal{W}_4^{-1}\mathcal{Y}_{i+3}^T \\ + He\{\mathcal{Y}_4\rho_{18} + \mathcal{Y}_5\rho_{38} + \mathcal{Y}_6\rho_{39}\}]\psi(t).\end{aligned} \tag{23}$$

Similar to arguments used in [29], we can gain

$$0 = 2\psi^T(t)\mathcal{Y}_{10}[d_k(t)\rho_{24} - \rho_{11}]\psi(t) \tag{24}$$

$$0 = 2\psi^T(t)\mathcal{Y}_{11}[(h_k - d_k(t))\rho_{25} - \rho_{14}]\psi(t) \tag{25}$$

$$0 = 2\psi^T(t)\mathcal{Y}_{12}[d_k(t)\rho_{26} - \rho_{28}]\psi(t) \tag{26}$$

$$0 = 2\psi^T(t)\mathcal{Y}_{13}[(h_k - d_k(t))\rho_{27} - \rho_{29}]\psi(t) \tag{27}$$

Then, we have

$$\dot{V}(t) \leq \psi^T(t)\Xi(d_k(t))\psi(t) \tag{28}$$

where

$$\begin{aligned}&\Xi(d_k(t)) \\ &= d_k^2(t)\Lambda_3 + d_k(t)\Lambda_2 + \Lambda_1 + \Upsilon, \\ &\Upsilon \\ &= \sum_{i=1}^{3} \frac{1}{2i-1}\{\tau\mathcal{L}_i D_1^{-1}\mathcal{L}_i^T + d_k(t)[\mathcal{M}_i(\mathcal{W}_1 + D_3)^{-1}\mathcal{M}_i^T \\ &\quad + \mathcal{Y}_i(\mathcal{W}_3 + D_2)^{-1}\mathcal{Y}_i^T] + (h_k - d_k(t))[\mathcal{N}_i\mathcal{W}_2^{-1}\mathcal{N}_i^T \\ &\quad + \mathcal{Y}_{i+3}\mathcal{W}_4^{-1}\mathcal{Y}_{i+3}^T]\} + (h_k - d_k(t))\mathcal{Y}_9(D_2 + D_3)^{-1}\mathcal{Y}_9^T \\ &\quad + \tau \sum_{i=1}^{2} \frac{1}{2i-1}\mathcal{Y}_{i+6}D_3^{-1}\mathcal{Y}_{i+6}^T,\end{aligned}$$

with Λ_1, Λ_2 and Λ_3 being defined in Theorem 1.

By using the Lemma 4 in [37] for $N = 1$, $\dot{V}(t) < 0$ for $d_k(t) \in [0, h_k)$ is ensured by the following inequalities:

$$\Xi_{d_k(t)\in[0,h_k)} < 0 \tag{29}$$

$$\Xi_{d_k(t)=\frac{i-1}{2}h_k} + \frac{1}{4}h_k\ddot{\Xi}_{d_k(t)=\frac{i-1}{2}h_k} < 0, \quad i = 1, 2 \tag{30}$$

Obviously, if (29) and (30) are satisfied, then $\dot{V}(t) \leq -\Im\|x(t)\|^2$ for a sufficiently small $\Im > 0$. Based on Schur complement, it follows that (29) and (30) are equal to (11)-(12) and (13)-(14), respectively. Therefore, if (11)-(14) are satisfied, then system (5) with the control input (3) satisfying (4) is asymptotically stable. This completes the proof.

Remark 1: Inspired by the work [37], the TTLF (15) is constructed, which take full advantage of the second order terms with respect to t. It is made up of two parts, $V_c(t)$ and $V_d(t)$, where $V_d(t)$ satisfies the boundary conditions of the two-side looped-functional obtained in [29], $V_{dj}(t_k) = V_{dj}(t_{k+1}) = 0, j = 1, 2, \cdots, 8$.

Remark 2: Based on the condition given in Theorem 1, the allowable upper bound of time delay and sampling periods that the system keep to be stable can be obtained by using the dichotomy method presented in [24].

Remark 3: Similar to arguments used in [29], four zero equalities, (24)-(27) are introduced, which is helpful to reduce the conservativeness of the derived stability condition.

In addition, by using the intrinsic relationships of state vectors, $\int_{t_k}^{t} x(s)ds + \int_{t_k-\tau}^{t_k} x(s)ds = \int_{t-\tau}^{t} x(s)ds + \int_{t_k-\tau}^{t-\tau} x(s)ds$, a new zero equality (31) is obtained for any matrix $\mathcal{Y}_{14}$ with appropriate dimensions.

$$0 = 2\psi^T(t)\mathcal{Y}_{14}[d_k(t)\rho_{44} + \rho_{43}]\psi(t) \tag{31}$$

where, $\rho_{43} = \tau(e_9 - e_{19})$, $\rho_{44} = e_{15} - e_{11}$.

By adding the zero equality (31) to (28), the following theorem is obtained.

Theorem 2: For given $\tau > 0$ and $h_2 \geq h_1 \geq 0$, system (5) with the control input (3) satisfying (4) is stable, if there exist matrices $P > 0$, $S > 0$, $D_1 > 0$, $D_2 = D_2^T$, $D_3 > 0$, Q_1, Q_2, X, $G = G^T$, $\mathcal{W}_1 > 0$, $\mathcal{W}_2 > 0$, $\mathcal{W}_3 > 0$, $\mathcal{W}_4 > 0$, $\mathcal{N}_i$, $\mathcal{M}_i$, $\mathcal{L}_i$, $i = 1, 2, 3$, $\mathcal{Y}_j$, $j = 1, 2, \cdots, 14$, such that LMIs (8)-(10), (32)-(35) are satisfied.

$$\begin{bmatrix} \bar{\Lambda}_1 & \Psi_1 & \sqrt{\tau}\Psi_2 \\ * & -\Pi_1 & 0 \\ * & * & -\Pi_2 \end{bmatrix}_{h_k\in[h_1,\, h_2]} < 0 \tag{32}$$

$$\begin{bmatrix} \bar{\Lambda}_1 + h_k\bar{\Lambda}_2 + h_k^2\Lambda_3 & \Psi_{32} \\ * & -\Pi_{32} \end{bmatrix}_{h_k\in[h_1,\, h_2]} < 0 \tag{33}$$

$$\begin{bmatrix} \bar{\Lambda}_1 + \frac{h_k}{4}\bar{\Lambda}_2 & \Psi_{132} \\ * & -\Pi_{132} \end{bmatrix}_{h_k\in[h_1,\, h_2]} < 0 \tag{34}$$

$$\begin{bmatrix} \bar{\Lambda}_1 + \frac{3}{4}h_k\bar{\Lambda}_2 + \frac{1}{2}h_k^2\Lambda_3 & \Psi_{312} \\ * & -\Pi_{312} \end{bmatrix}_{h_k\in[h_1,\, h_2]} < 0 \tag{35}$$

where

$$\bar{\Lambda}_1 = \Lambda_1 + He\{\mathcal{Y}_{14}\rho_{43}\}$$
$$\bar{\Lambda}_2 = \Lambda_2 + He\{\mathcal{Y}_{14}\rho_{44}\}$$

with

$$\rho_{43} = \tau(e_9 - e_{19}), \quad \rho_{44} = e_{15} - e_{11}.$$

and other notations are defined as the same in Theorem 1.

Based on the Theorem 1, the following corollary can be easily obtained for sampled-data system without considering communication delay.

Corollary 1: For given $h_2 \geq h_1 \geq 0$, system (5) with the sampling periods satisfying (4) and $\tau = 0$ is stable if there exist matrices $\breve{P} > 0$, $\breve{Q}_1$, $\breve{Q}_2$, $\breve{X}$, $\breve{Z}$, $\breve{W}_1 > 0$, $\breve{W}_2 > 0$, $\breve{N}_i$, $\breve{M}_i$, $i = 1, 2, 3$, $\breve{Y}_j$, $j = 1, 2, 3, 4$, such that LMIs (36)-(39) are satisfied.

$$\begin{bmatrix} \breve{\Lambda}_1 & \breve{\Psi}_2 \\ * & -\breve{\Pi}_2 \end{bmatrix}_{h_k\in[h_1,\, h_2]} < 0 \tag{36}$$

$$\begin{bmatrix} \breve{\Lambda}_1 + h_k\breve{\Lambda}_2 + h_k^2\breve{\Lambda}_3 & \breve{\Psi}_1 \\ * & -\breve{\Pi}_1 \end{bmatrix}_{h_k\in[h_1,\, h_2]} < 0 \tag{37}$$

$$\begin{bmatrix} \breve{\Lambda}_1 + \frac{h_k}{4}\breve{\Lambda}_2 & \breve{\Psi}_2 & \breve{\Psi}_1 \\ * & -\frac{4}{3}\breve{\Pi}_2 & 0 \\ * & * & -4\breve{\Pi}_1 \end{bmatrix}_{h_k\in[h_1,\, h_2]} < 0 \tag{38}$$

$$\begin{bmatrix} \breve{\Lambda}_1 + \frac{3}{4}h_k\breve{\Lambda}_2 + \frac{1}{2}h_k^2\breve{\Lambda}_3 & \breve{\Psi}_1 & \breve{\Psi}_2 \\ * & -\frac{4}{3}\breve{\Pi}_1 & 0 \\ * & * & -4\breve{\Pi}_2 \end{bmatrix}_{h_k\in[h_1,\, h_2]} < 0 \tag{39}$$

where

$$\begin{aligned}
\breve{\Lambda}_1 &= He\{\breve{e}_1^T\breve{P}\breve{\eta} + \breve{\eta}^T\breve{X}\breve{\rho}_5 - \breve{\rho}_1^T\breve{X}\breve{\eta} + \breve{M}_1\breve{\rho}_1 + \breve{M}_2\breve{\rho}_{12} \\
&\quad + \breve{M}_3\breve{\rho}_{13} + \breve{N}_1\breve{\rho}_5 + \breve{N}_2\breve{\rho}_{14} + \breve{N}_3\breve{\rho}_{15} - \breve{Y}_1\breve{\rho}_1 - \breve{Y}_2\breve{\rho}_5 \\
&\quad - \breve{Y}_3\breve{\rho}_{20} - \breve{Y}_4\breve{\rho}_{21} + h_k(-2\breve{\rho}_1^T\breve{Q}_1\breve{\rho}_2 + \breve{\rho}_1^T\breve{Q}_1\breve{\rho}_4 \\
&\quad + \breve{\rho}_{10}^T\breve{Z}\breve{\rho}_8 + \breve{Y}_2\breve{\rho}_{17} + \breve{Y}_4\breve{\rho}_{19}) + h_k^2(\breve{\eta}^T\breve{Q}_1\breve{\rho}_2 \\
&\quad + \breve{\rho}_1^T\breve{Q}_1\breve{\rho}_3)\} + h_k(\breve{\rho}_8^T\breve{Z}\breve{\rho}_8 + \breve{\eta}^T\breve{W}_1\breve{\eta}), \\
\breve{\Lambda}_2 &= He\{2\breve{\rho}_5^T\breve{Q}_2\breve{\rho}_6 + \breve{\rho}_5^T\breve{Q}_2\breve{\rho}_7 + \breve{\rho}_{11}^T\breve{Z}\breve{\rho}_8 + \breve{Y}_1\breve{\rho}_{16} + \breve{Y}_3\breve{\rho}_{18} \\
&\quad - (-2\breve{\rho}_1^T\breve{Q}_1\breve{\rho}_2 + \breve{\rho}_1^T\breve{Q}_1\breve{\rho}_4 + \breve{\rho}_{10}^T\breve{Z}\breve{\rho}_8 + \breve{Y}_2\breve{\rho}_{17} + \breve{Y}_4\breve{\rho}_{19}) \\
&\quad + h_k(\breve{\rho}_9^T\breve{Z}\breve{\rho}_8) - 2h_k(\breve{\eta}^T\breve{Q}_1\breve{\rho}_2 + \breve{\rho}_1^T\breve{Q}_1\breve{\rho}_3)\} - \breve{\rho}_8^T\breve{Z}\breve{\rho}_8 \\
&\quad + \breve{\eta}^T\breve{W}_2\breve{\eta} - (\breve{\rho}_8^T\breve{Z}\breve{\rho}_8 + \breve{\eta}^T\breve{W}_1\breve{\eta}), \\
\breve{\Lambda}_3 &= He\{-\breve{\eta}^T\breve{Q}_2\breve{\rho}_6 + \breve{\rho}_5^T\breve{Q}_2\breve{\rho}_3 + \breve{\eta}^T\breve{Q}_1\breve{\rho}_2 + \breve{\rho}_1^T\breve{Q}_1\breve{\rho}_3 \\
&\quad - \breve{\rho}_9^T\breve{Z}\breve{\rho}_8\}, \\
\breve{\Psi}_1 &= \left[h_k\breve{M}_1 \;\; h_k\breve{M}_2 \;\; h_k\breve{M}_3\right], \\
\breve{\Psi}_2 &= \left[h_k\breve{N}_1 \;\; h_k\breve{N}_2 \;\; h_k\breve{N}_3\right], \\
\breve{\Pi}_1 &= \mathrm{diag}\{h_k\breve{W}_1, 3h_k\breve{W}_1, 5h_k\breve{W}_1\}, \\
\breve{\Pi}_2 &= \mathrm{diag}\{h_k\breve{W}_2, 3h_k\breve{W}_2, 5h_k\breve{W}_2\}, \\
\breve{\eta} &= A\breve{e}_1 + B\breve{e}_2, \;\; \breve{\rho}_1 = [\breve{e}_1^T - \breve{e}_2^T]^T, \\
\breve{\rho}_2 &= [\breve{e}_1^T \; \breve{e}_2^T \; \breve{e}_5^T \; \breve{e}_6^T \; \breve{e}_7^T]^T, \;\; \breve{\rho}_3 = [\breve{\eta}^T \; 0\; 0\; 0\; 0]^T, \\
\breve{\rho}_4 &= [0\; 0\; 0\; \breve{e}_6^T - \breve{e}_1^T \; 2(\breve{e}_7^T - \breve{e}_6^T)]^T, \\
\breve{\rho}_5 &= [\breve{e}_5^T - \breve{e}_1^T]^T, \;\; \breve{\rho}_6 = [\breve{e}_1^T \; \breve{e}_2^T \; \breve{e}_5^T \; \breve{e}_3^T \; \breve{e}_4^T]^T, \\
\breve{\rho}_7 &= [0\; 0\; 0\; \breve{e}_1^T - \breve{e}_3^T \; 2(\breve{e}_3^T - \breve{e}_4^T)]^T, \\
\breve{\rho}_8 &= [\breve{e}_1^T \; \breve{e}_2^T \; \breve{e}_5^T \; \breve{e}_3^T \; \breve{e}_4^T \; \breve{e}_6^T \; \breve{e}_7^T]^T, \\
\breve{\rho}_9 &= [\breve{\eta}^T \; 0\; 0\; 0\; 0\; 0\; 0]^T, \\
\breve{\rho}_{10} &= [0\; 0\; 0\; \breve{e}_1^T - \breve{e}_3^T \; 2(\breve{e}_3^T - \breve{e}_4^T)\; 0\; 0]^T, \\
\breve{\rho}_{11} &= [0\; 0\; 0\; 0\; 0\; \breve{e}_6^T - \breve{e}_1^T \; 2(\breve{e}_7^T - \breve{e}_6^T)]^T, \\
\breve{\rho}_{12} &= \breve{e}_1 + \breve{e}_2 - 2\breve{e}_3, \;\; \breve{\rho}_{13} = \breve{e}_1 - \breve{e}_2 - 6\breve{e}_3 + 6\breve{e}_4, \\
\breve{\rho}_{14} &= \breve{e}_5 + \breve{e}_1 - 2\breve{e}_6, \;\; \breve{\rho}_{15} = \breve{e}_5 - \breve{e}_1 + 6\breve{e}_6 - 6\breve{e}_7, \\
\breve{\rho}_{16} &= A\breve{e}_3 + B\breve{e}_2, \;\; \breve{\rho}_{17} = A\breve{e}_6 + B\breve{e}_2, \\
\breve{\rho}_{18} &= A\breve{e}_4 + B\breve{e}_2, \;\; \breve{\rho}_{19} = A\breve{e}_7 + B\breve{e}_2, \\
\breve{\rho}_{20} &= 2\breve{e}_3 - 2\breve{e}_2, \;\; \breve{\rho}_{21} = 2\breve{e}_5 - 2\breve{e}_6.
\end{aligned}$$

IV. NUMERICAL EXAMPLES

In this section, a typical numerical example is used for experimental simulation to verify the superiority of the method.

Example 1: Consider system (5) with

$$\mathbb{A} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad \mathbb{B} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

In terms of different communication delay, the maximal allowable sampling periods calculated by Theorem 1 and 2

TABLE 2. Maximal value of h_2 for $h_1 = 10^{-5}$ and different τ.

Methods	$\tau = 0.1$	$\tau = 0.2$	$\tau = 0.4$	$\tau = 0.6$
[36]	2.7999	2.5538	2.2545	2.0660
Theorem 1	2.8516	2.5914	2.2831	2.1110
Theorem 2	2.9186	2.6772	2.3328	2.1183

TABLE 3. Maximal value of h_2 for $h_1 = h_2$ and different τ.

Methods	$\tau = 0.1$	$\tau = 0.2$	$\tau = 0.4$	$\tau = 0.6$
[36]	2.8671	2.6359	2.4080	2.3338
Theorem 1	3.0641	2.9176	2.7673	2.7626
Theorem 2	3.2553	3.2145	3.1191	3.0867
Analytic bound	3.37	3.47	3.68	3.89

TABLE 4. Maximal value of h_2 for $h_1 = 10^{-5}$ and $\tau = 0$.

Methods	h_2
[19]	2.8554
[29]	3.2632
Corollary 1	3.2713
Corollary 1($h_1 = h_2$)	3.2715

are listed in Table 2 and Table 3, respectively, for the cases of aperiodic sampling and periodic sampling, which are along with the results given in [36]. It is observed in Table 2 and Table 3 that the results obtained by Theorem 1 and Theorem 2 are superior over that in [36]. In comparison with the results obtained by Theorem 1 and Theorem 2, it is observed that the results computed by Theorem 2 are much better than that by Theorem 1, which indicate that the new zero equality (31) in this paper play important role in the reduction of conservatism.

Next, the proposed method is applied to system (5) without time delay and the results obtained by Corollary 1 and other methods are summarized in Table 4. Then, as can be seen from the data in the table, the new TTLF and the zero equality proposed in this paper can produce less conservative results than [19] and [29] and even the sampling period reaches 3.2715 when $h_1 = h_2$. Thus, the superiority of the method proposed is verified again.

Moreover, the state response of the system (5) in the cases of $\tau = 4, h_1 = h_2 = 3.1191$ and $\tau = 0, h_1 = h_2 = 3.2715$ are, respectively, provided in Figure 1 and Figure 2 under the initial state $x(0) = [2, -1.8]^T$. It is shown in Figure 1 that the provided maximal allowable bound can ensure the stability of the system with $h_1 = h_2 = 3.1191$ and $\tau = 0.4$. In addition, as shown in Figure 2, the results obtained by our method make the system reach the critical stable state in the case of $h_1 = h_2 = 3.2715$ and $\tau = 0$. Therefore, the effectiveness and superiority of the proposed method are confirmed.

V. APPLICATION OF THE PROPOSED METHOD IN EPM

In this section, an electric power market (EPM) is studied by the proposed method and a dynamic model of the EPM models is constructed to analyze the influence of market clearing time (MCT) and communication delay on the stability of the system.

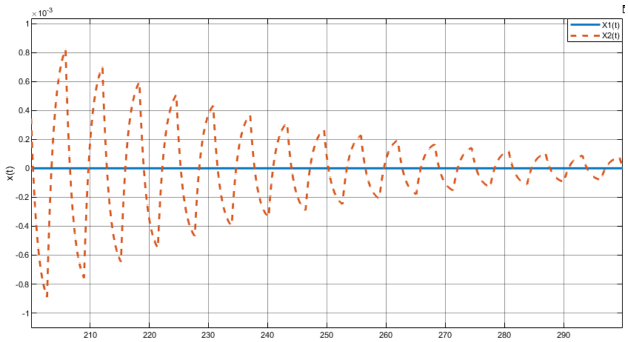

FIGURE 1. State response of the system (5) in the case of $\tau = 0.4$ and $h_1 = h_2 = 3.1191$.

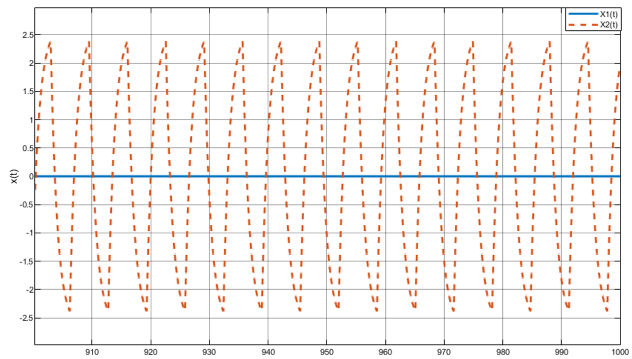

FIGURE 2. State response of the system (5) in the case of $\tau = 0$ and $h_1 = h_2 = 3.2715$.

A. DYNAMICS OF THE EPM

The dynamic balance between power generation and load at all time, which is an important index to measure the reliable and stable operation of power grid. In order to realize the balance of energy supply and demand, a method that measure the balance of energy supply and demand by using prices is proposed in recent years. This method can accurately reflect the actual situation of regional energy supply and demand, so as to provide effective means for formulating corresponding countermeasures.

A simplified EPM is composed of electricity suppliers, electricity consumers and real-time market.

1) SUPPLIER MODEL

The marginal production costs can be expressed as follows

$$\chi_g = b_g + c_g P_g \tag{40}$$

where χ_g, b_g, c_g, and P_g are the marginal cost, the fixed cost, the fixed coefficient, and the amount of the generated power, respectively.

If power suppliers see that the market price of electricity is higher than its production costs, the suppliers will expand production. The expansion rate is proportional to the difference between the observed price and the actual production cost. In addition, if there is a glut in supply and demand, the power suppliers must pay additional cost.

Therefore, the dynamic of the supplier is described as

$$\tau_g \dot{P}_g = \chi - b_g - c_g P_g - kE \tag{41}$$

where τ_g, χ, k, and E are the time constant that is described as the rate of change of supply, the observed power price, a constant gain, and the time integral of the difference between supply and demand, respectively.

2) CONSUMER MODEL

The marginal benefit functions can be expressed as follows

$$\chi_d = b_d + c_d P_d(\mathrm{t}) \tag{42}$$

where χ_d, b_d, c_d, and P_d are the marginal benefit, the fixed benefit, the fixed coefficient, and the amount of the consumed power, respectively.

When the marginal benefit function in consumer demand is greater than the marginal price, it will stimulate consumers to expand consumption, and the speed of expansion depends on consumers. The behavior of the consumer is defined by

$$\tau_d \dot{P}_d = b_d + c_d P_d - \chi \tag{43}$$

where τ_d is the time constant that denote the rate of change of demand.

3) ENERGY IMBALANCE AND PRICE RESPONSE

The imbalance between supply and demand is defined as the integral of the difference between them over time,

$$\dot{E} = P_g - P_d \tag{44}$$

At the same time, the price of electricity is changed by the observed electric grid supply and market demand,

$$\tau_\chi \dot{\chi} = -E \tag{45}$$

where τ_χ is the time constant representing the rate of change of prices in response to market perturbations.

In the real market, the discrete price signal are received by the participants that is actually equal to the MCT, namely the updating period of electricity price. In addition, when electricity price signals are sent through various communication networks and equipment, it will introduce communication delay. Combining those two aspects, the following linear model that take into account sampling periods and communication delay is obtained,

$$\begin{cases} \tau_g \dot{P}_g(\mathrm{t}) = \chi(\mathrm{t}_k - \tau) - b_g - c_g P_g(\mathrm{t}) - kE(\mathrm{t}), \\ \tau_d \dot{P}_d(\mathrm{t}) = b_d + c_d P_d(\mathrm{t}) - \chi(\mathrm{t}_k - \tau), \\ \dot{E}(\mathrm{t}) = P_g(\mathrm{t}) - P_d(\mathrm{t}), \\ \tau_\chi \dot{\chi}(\mathrm{t}) = -E(\mathrm{t}) \end{cases} \tag{46}$$

where t_k is the updating instants of the price satisfying

$$0 < \mathrm{t}_{k+1} - \mathrm{t}_k = T_{mc\mathrm{t}_k} \leq T_{mc\mathrm{t}}$$

with $T_{mc\mathrm{t}_k}$ and $T_{mc\mathrm{t}}$ being the MCT for k and its maximal value, respectively, and τ being the communication delay.

B. STABILITY ANALYSIS OF THE EPM

The stability region of an EPM is analyzed in [38] and the parameters of the system are given as $\tau_g = 0.2$, $c_g = 0.1, b_g = 2, \tau_d = 0.1, c_d = -0.2, b_d = 10$, $\tau_\chi = 100$ and $k = 0.1$.

1) STATE-SPACE EQUATIONS

Model (46) can be expressed as the following continuous-time state equation form:

$$\dot{z}(\mathrm{t}) = Az(\mathrm{t}) + Bz(\mathrm{t} - d_k(\mathrm{t}) - \tau) + C \tag{47}$$

where $0 < \mathrm{t}_{k+1} - \mathrm{t}_k = T_{mc\mathrm{t}_k} \leq T_{mc\mathrm{t}}$ and

$$z(\mathrm{t}) = \begin{bmatrix} P_g(\mathrm{t}) \\ P_d(\mathrm{t}) \\ E(\mathrm{t}) \\ \chi(\mathrm{t}) \end{bmatrix},$$

$$A = \begin{bmatrix} -\frac{c_g}{\tau_g} & 0 & -\frac{k}{\tau_g} & 0 \\ 0 & \frac{c_d}{\tau_d} & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_\chi} & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{\tau_g} \\ 0 & 0 & 0 & -\frac{1}{\tau_d} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -\frac{b_g}{\tau_g} \\ \frac{b_d}{\tau_d} \\ 0 \\ 0 \end{bmatrix}$$

The constant term C can be dealt with in the above system by using the method in [39] and [40]. Suppose z^* is the balance point of system (47). By defining a new state $x = z - z^*$, the stability of system (47) at its equilibrium point can be mathematically equivalent to that of the following system at zero-point:

$$\dot{x}(\mathrm{t}) = Ax(\mathrm{t}) + Bx(\mathrm{t} - d_k(\mathrm{t}) - \tau) \tag{48}$$

Notice that the above system is the closed-loop system (6) described in the paper. And, the relevant parameters such as $A, B, d_k(\mathrm{t})$, and τ in the system have the same physical meaning as those in closed-loop system (6). Therefore, the new stability criterion proposed in this paper can be used to discuss the influence of communication delay τ and MCT $d_k(\mathrm{t})$ on power market system.

2) ALGORITHM OF CALCULATION, CALCULATION RESULTS AND DISCUSSION

Define the following functions

$$tan(\theta) = \frac{T_{mct}}{\tau}, \quad \theta \in [0^\circ, 90^\circ], \; h = \sqrt{T_{mct}^2 + \tau^2} \tag{49}$$

where θ is the polar angle, which represents all value ranges corresponding to the possible proportion of MCT maximal value T_{mct} to the communication delay τ; h is the polar axis, and when given different θ, its maximum value is defined as

TABLE 5. $h_{max}(\theta)$ for $h_1 = h_2$ and different θ.

Methods	$\theta = 20°$	$\theta = 40°$	$\theta = 45°$	$\theta = 50°$
[7]	7.14	6.75	6.66	6.36
Theorem 2	8.46	8.58	8.64	8.72

$h_{max}(\theta)$. In terms of different θ, the values of h_{max} calculated by Theorem 2 are listed in Table 5, which are along with the results given in [7]. In Table 5, the method proposed in this paper is obviously superior to the result in [7]. Thus, this demonstrate that the method proposed in this paper is effective in the application of EPM.

VI. CONCLUSION

This paper proposes a new method for stability analysis of aperiodic sampled-data systems with communication delay. A TTLF is proposed, which take full advantage of the second order terms with respect to t. Improved stability criteria are derived by employing the TTLF and introducing some zero equalities with free matrices. The effectiveness of the proposed method has been validated by a given numerical example. In addition, the proposed method is applied to the dynamic model of the EPM, and the influence of MCT and communication delay on the stability of the power market is discussed, which provide certain guiding significance to ensure the balance of energy supply and demand.

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WEI-MIN WANG received the B.Sc. degree in electrical engineering and automation from Linyi University, Shandong, China, in 2020. He is currently pursuing the M.S. degree in electrical engineering with the Hunan University of Technology, Zhuzhou, China. His current research interests include neural networks, sampled-data control, and time-delay systems.

WEI WANG received the B.Sc. degree in electrical engineering from the Tianjin University of Technology and Education, Tianjin, China, in 2003, and the M.Sc. degree in electrical engineering from Hunan University, Changsha, China, in 2019. From April 2016 to February 2017, she was a Research Associate with the Department of Mathematics and Statistics, Curtin University, Perth, WA, Australia. She is currently an Associate Professor with the Department of Electrical and Information Engineering, Hunan University of Technology, China. Her current research interests include neural networks, sampled-data control, and time-delay systems.