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RESEARCH ARTICLE

Topology Synthesis of Hybrid Space Control Mechanism Based on FIS Theory

LU ZHANG AND YANG QI
School of Mechanical Engineering, Tianjin University of Technology and Education, Tianjin 300222, China
Corresponding author: Yang Qi (qiyang@tju.edu.cn)

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ABSTRACT Driven by the significant demand for space control mechanism in the development of national space field, topology synthesis method for the hybrid mechanism based on finite and instantaneous screw (FIS theory) is studied in this paper. The number and types of degrees of freedom (DOF) required for capturing, detumbling, screwing and plugging control tasks are analyzed in depth, and the simplest mathematical expression of continuous motion corresponding to space control tasks are formed. According to the requirements of space control task, the synthesis principle of serial and parallel mechanism is proposed, and the arrangement principle of expected motion pattern is summarized. The standard limb is analytically characterized based on the expected motion pattern, and derivative limbs satisfying the expected motion pattern are derived based on joint equivalent transformation. By determining assembly conditions, the available topological structures satisfying the requirements of space control tasks are generated. Based on serial and parallel mechanisms that satisfy design principles, the simplest analytical expression of the topology of hybrid space control mechanism is obtained using finite screw principle. The general topology synthesis method of hybrid mechanisms is established, and various topological structures of hybrid space control mechanism are obtained, which lays a foundation for the research and development of novel hybrid space control mechanisms.

INDEX TERMS Space control mechanism, hybrid mechanism, FIS theory, topology synthesis.

I. INTRODUCTION

With the continuous in-depth research on space control mechanisms at home and abroad, serial mechanisms with simple structure, large workspace, and good flexibility are used in international and domestic space stations to complete operational tasks [1], [2], [3], [4], [5]. But the use of serial mechanism in different tasks, need to replace the corresponding special claw to complete the task. Although serial mechanisms are widely used in modern manufacturing, they are limited in special applications requiring high load, high precision, and large working space. In view of this phenomenon, combining the advantages of serial and parallel mechanisms, a hybrid mechanism with high load capacity, high positioning accuracy, high stiffness and large workspace is proposed, which leads to a new research direction for the innovative development of mechanism science [6].

According to the development history of mechanics, hybrid mechanisms are widely used as a novel type of mechanism in modern manufacturing industry. Because of its advantages of higher precision, stiffness, and load capacity, it is very popular in the fields of aerospace, high-speed machining, and medical instruments. Due to the complex topology of hybrid mechanisms, there are few innovative studies on hybrid mechanisms at home and abroad [7], [8], [9]. Currently, hybrid mechanisms include two structures: the first type is a combination of multiple parallel mechanisms,

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which are relatively limited in application due to their relatively complex structures [10], [11], [12]. The second type is a combination of serial and parallel mechanisms, which combines serial and parallel mechanisms to form a new mechanism. This hybrid mechanism is a high-quality mechanism that has the advantages of large workspace and good flexibility of serial mechanisms, as well as the characteristics of high stiffness, high load capacity, and high accuracy of parallel mechanisms.

The serial-parallel hybrid mechanism can be divided into two types according to actual application scenarios. One is the parallel mechanism as the base and the end as the serial mechanism. This type of hybrid mechanism is widely used in machine tool processing and multi legged robots due to its high load capacity, high stiffness, and large workspace. The other is the serial mechanism as the base and the end as parallel mechanism. Due to the characteristics of high vacuum, ultra-low temperature, and strong radiation in the space environment [13], as well as the increasingly complex and diverse space control tasks, the difficulty of astronauts performing extravehicular tasks has increased sharply. If it can be used to assist or replace astronauts to complete some complex and dangerous space control tasks, it can not only improve the efficiency of space control tasks, but also reduce the risk of astronauts' extravehicular activities. Therefore, the in-depth study of the space control mechanism, which is an important technical equipment for space exploration activities, has important scientific significance and research value.

Topology synthesis of mechanism is the first step in the innovative design of robot equipment, and it is one of the research hotspots in modern mechanism science [14], [15], [16]. Given the number and type of DOF of mechanisms, the goal of topology synthesis is to construct the number, type and position arrangement of limbs and joints, to obtain many mechanisms that satisfy the expected motion requirements of mechanisms. So far, scholars at home and abroad have conducted a large amount of research on the issue of mechanism topology synthesis and proposed various methods for topology synthesis [17], [18], [19], [20], [21], [22]. According to different description methods, topology synthesis methods can be divided into instantaneous motion and finite motion, including constrained screw synthesis, differential geometry synthesis, displacement subgroup synthesis, azimuth-characteristic synthesis. The description methods based on instantaneous motion all stay at the level of instantaneous constraints, which cannot fully reflect the limited motion characteristics of mechanisms. Moreover, some currently proposed measures to avoid instantaneous DOF of mechanisms still lack universality. The description method based on finite motion needs to analyze and distinguish specific structures, which is difficult to reflect the continuous motion characteristics of mechanisms accurately and comprehensively. Therefore, the FIS theory [23], [24], [25], [26], [27], [28] is used to topology synthesis of the mechanism in this paper, and the topology model of the mechanism is obtained through algebraic derivation, which lays a theoretical foundation for the subsequent performance modeling and optimization design, to ensure that the integrated analysis and optimization research of the mechanism is completed under the unified mathematical framework [29], [30], [31].

II. FINITE AND INSTANTANEOUS SCREW THEORY

According to Chasles' theorem, the axis of motion of a rigid body and the rotation and translation about/along the axis constitute the elements to describe the continuous motion of a rigid body. For finite motion of any rigid body, using finite screw, the continuous motion process of rigid body can be described as

$$\boldsymbol{S}_f = 2\tan\frac{\theta}{2}\begin{pmatrix}\boldsymbol{s}_f\\ \boldsymbol{r}_f \times \boldsymbol{s}_f\end{pmatrix} + t\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_f\end{pmatrix} \tag{1}$$

where $\boldsymbol{s}_f$ denotes the unit direction vector of the axis, $\boldsymbol{r}_f$ is the position vector of the axis, θ and t are the rotational angle and translational distance about/along the axis, respectively. $\boldsymbol{S}_f$ is the pose change process of rigid body rotating angular displacement θ about Chasles' axis and translation line displacement t along the axis. Therefore, $\boldsymbol{S}_f$ denotes finite screw.

Finite screw is a complex nonlinear operation based on screw triangle theorem, which is called screw triangle product for short. If the rigid body moves $\boldsymbol{S}_{f,1}$ and $\boldsymbol{S}_{f,2}$ continuously, they are denoted by finite screw respectively

$$\boldsymbol{S}_{f,1} = 2\tan\frac{\theta_1}{2}\begin{pmatrix}\boldsymbol{s}_{f,1}\\ \boldsymbol{r}_{f,1} \times \boldsymbol{s}_{f,1}\end{pmatrix} + t_1\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_{f,1}\end{pmatrix} \tag{2}$$

$$\boldsymbol{S}_{f,2} = 2\tan\frac{\theta_2}{2}\begin{pmatrix}\boldsymbol{s}_{f,2}\\ \boldsymbol{r}_{f,2} \times \boldsymbol{s}_{f,2}\end{pmatrix} + t_2\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_{f,2}\end{pmatrix} \tag{3}$$

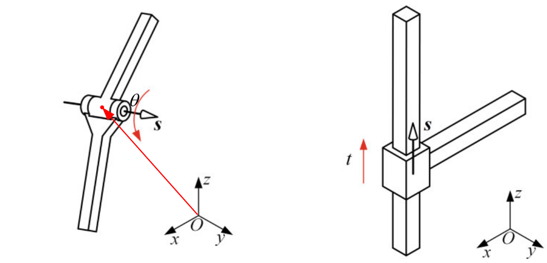
The final continuous motion $\boldsymbol{S}_{f,12}$ of the rigid body can be expressed as (4), as shown at the bottom of the next page, where "Δ" denotes the screw triangle product [24].

$$\boldsymbol{S}_{f,2} \times \boldsymbol{S}_{f,1} = \begin{pmatrix} 2\tan\frac{\theta_2}{2}\boldsymbol{s}_{f,2} \times 2\tan\frac{\theta_1}{2}\boldsymbol{s}_{f,1} \\ 2\tan\frac{\theta_2}{2}\boldsymbol{s}_{f,2} \times \left(2\tan\frac{\theta_1}{2}\left(\boldsymbol{r}_{f,1} \times \boldsymbol{s}_{f,1}\right) + t_1\boldsymbol{s}_{f,1}\right) + \\ \left(2\tan\frac{\theta_2}{2}\left(\boldsymbol{r}_{f,2} \times \boldsymbol{s}_{f,2}\right) + t_1\boldsymbol{s}_{f,2}\right) \times 2\tan\frac{\theta_1}{2}\boldsymbol{s}_{f,1} \end{pmatrix} \tag{5}$$

The finite motion of a rigid body can be described as the sum of several rotations and translations. As shown in Fig. 1, the finite motion of the jth joint of the ith limb can be respectively described as

$$\boldsymbol{S}_{f,i,j,r} = 2\tan\frac{\theta_{i,j}}{2}\begin{pmatrix}\boldsymbol{s}_{i,j}\\ \boldsymbol{r}_{i,j} \times \boldsymbol{s}_{i,j}\end{pmatrix} \tag{6}$$

$$\boldsymbol{S}_{f,i,j,t} = t_{i,j}\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_{i,j}\end{pmatrix} \tag{7}$$



$\boldsymbol{S}_{f,i,j,r} = 2\tan\dfrac{\theta_{i,j}}{2}\begin{pmatrix}\boldsymbol{s}_{i,j}\\ \boldsymbol{r}_{i,j}\times\boldsymbol{s}_{i,j}\end{pmatrix}$ $\boldsymbol{S}_{f,i,j,t} = t_{i,j}\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_{i,j}\end{pmatrix}$

(*a*) R joint (*b*) P joint

FIGURE 1. **Finite motion of the joints.**

III. CONTINUOUS MOTION DESCRIPTION OF SPACE CONTROL TASK

As shown in Fig. 2, space control tasks mainly include capture, detumbling, screw and plugging. The number and types of DOF required to realize the four control tasks are analyzed based on the FIS theory, and the simplest mathematical expression of the continuous motion corresponding to the space control tasks can be obtained.

The capture function requires the end of the mechanism to realize 3-DOF translation, it can be expressed as

$$\boldsymbol{S}_{f,1} = t_3\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_3\end{pmatrix}\Delta t_2\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_2\end{pmatrix}\Delta t_1\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_1\end{pmatrix} \tag{8}$$

The detumbling function requires the end of the mechanism to realize rotation about the axis of the controlled object, it can be expressed as

$$\boldsymbol{S}_{f,2} = 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix} \tag{9}$$

The screw function requires the end of the mechanism to realize rotation about and translation along the axis of the controlled object, it can be expressed as

$$\boldsymbol{S}_{f,3} = 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\Delta t_3\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_3\end{pmatrix} \tag{10}$$

The plugging function requires the end of the mechanism to align the plane and translation along the axis of the controlled object, it can be expressed as

$$\boldsymbol{S}_{f,4} = t_3\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix} \tag{11}$$

To complete all the tasks of space control, the hybrid mechanism needs to satisfy the above four finite continuous motions. Therefore, the expected motion $\boldsymbol{S}_{f,\mathrm{M}}$ of the hybrid space control mechanism can be expressed as

$$\begin{aligned}\boldsymbol{S}_{f,\mathrm{M}} = {} & 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\\ & \times\quad \Delta 2\tan\tfrac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta t_3\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_3\end{pmatrix}\Delta t_2\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_2\end{pmatrix}\\ & \times\quad \Delta t_1\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_1\end{pmatrix}\end{aligned} \tag{12}$$

IV. DESIGN PRINCIPLES OF HYBRID SPACE CONTROL MECHANISM

According to section III, the expected motion of the hybrid space control mechanism is three rotations and three translations (3R3T). To complete all tasks in space control, the hybrid mechanism topology designed in this paper takes serial mechanism as the base and parallel mechanism is installed at the end. The hybrid mechanism combines the advantages of serial and parallel mechanisms, and has the characteristics of large workspace, high accuracy, and high stability. How to assign the expected motion to serial and parallel mechanisms is the key content of this paper. According to the tasks to be completed by the hybrid mechanism, the corresponding arrangement principles are proposed as follows

(1) Considering the complex diversity of space control tasks, the serial mechanism is designed as a multi-DOF structure to ensure its good flexibility.

(2) Considering the influence of the stiffness deformation caused by the parallel mechanism at the end on the serial mechanism, the parallel mechanism is designed as a minor-DOF structure to reduce the load pressure of the serial mechanism.

(3) Due to the kinematic decoupling of between serial and parallel mechanisms, the kinematic relationship is not simply a linear superposition, but rather is obtained through a nonlinear operation - intersection algorithm (screw triangular product).

Based on the above three arrangement principles, the serial mechanism is defined as multi-DOF mechanism, and the parallel mechanism is defined as minor-DOF mechanism. The expected motion of the hybrid mechanism can satisfy the requirement of space control task.

As shown in Fig. 3, the serial mechanism is composed of several revolute or prismatic joints with uni-DOF, and the output motion of each front joint is the input motion of the rear joint. The finite motion of the whole serial mechanism is the output motion of its end, and its motion is the triangular product of the continuous motion of n joints, which can be

$$\boldsymbol{S}_{f,12} = \boldsymbol{S}_{f,1}\Delta\boldsymbol{S}_{f,2} = \frac{\boldsymbol{S}_{f,1} + \boldsymbol{S}_{f,2} + \frac{\boldsymbol{S}_{f,2}\times\boldsymbol{S}_{f,1}}{2} - \tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\left(t_2\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_{f,1}\end{pmatrix} + t_1\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_{f,2}\end{pmatrix}\right)}{1 - \tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\boldsymbol{s}_{f,1}^{\mathrm{T}}\boldsymbol{s}_{f,2}} \tag{4}$$

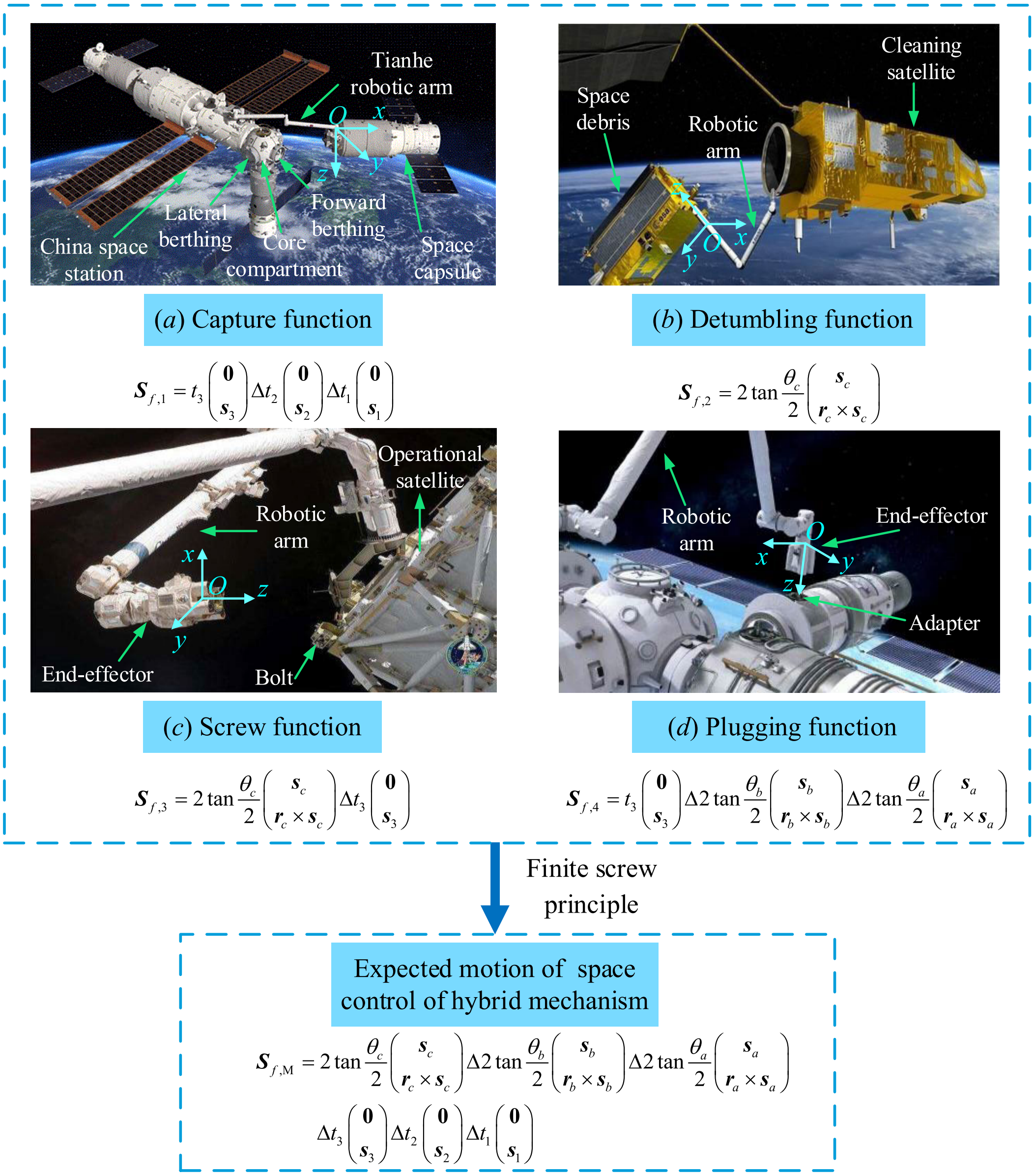


FIGURE 2. **Parameterized description of space control task.**

expressed as

$$S_{f,\text{serial}} = S_{f,n}\Delta S_{f,n-1}\Delta\cdots S_{f,1} \tag{13}$$

As shown in Fig. 4, the parallel mechanism is a closed-loop mechanism that consists of a moving platform and a fixed platform connected by at least two independent limbs. Therefore, the mechanism has at least two or more DOF and is driven in parallel. Each limb of the parallel mechanism can be regarded as an individual serial mechanism, and the continuous motion of each limb is the triangular product of the continuous motion of each joint of the limb. The motion of the moving platform in the parallel mechanism is the intersection of the finite motion of the ends of m limbs. It can be expressed as

$$S_{f,\text{parallel}} = S_{f,1}\cap S_{f,2}\cap\cdots S_{f,m} \tag{14}$$

As shown in Fig. 5, in the hybrid mechanism, the end of the serial mechanism is connected to the fixed platform of the parallel mechanism, and its continuous motion is equivalent to the synthesis of the finite motion of the serial mechanism and the finite motion of the parallel mechanism, which can be expressed as

$$S_f = S_{f,\text{serial}}\Delta S_{f,\text{parallel}} \tag{15}$$

V. TOPOLOGY SYNTHESIS OF HYBRID SPACE CONTROL MECHANISM

Based on the topology design principles of hybrid mechanisms, the finite instantaneous screw expression for space control tasks is analyzed, and the expected motion pattern 3R3T is divided into 2R1T and 1R2T, which makes it easier to implement the motion pattern. The hybrid mechanism

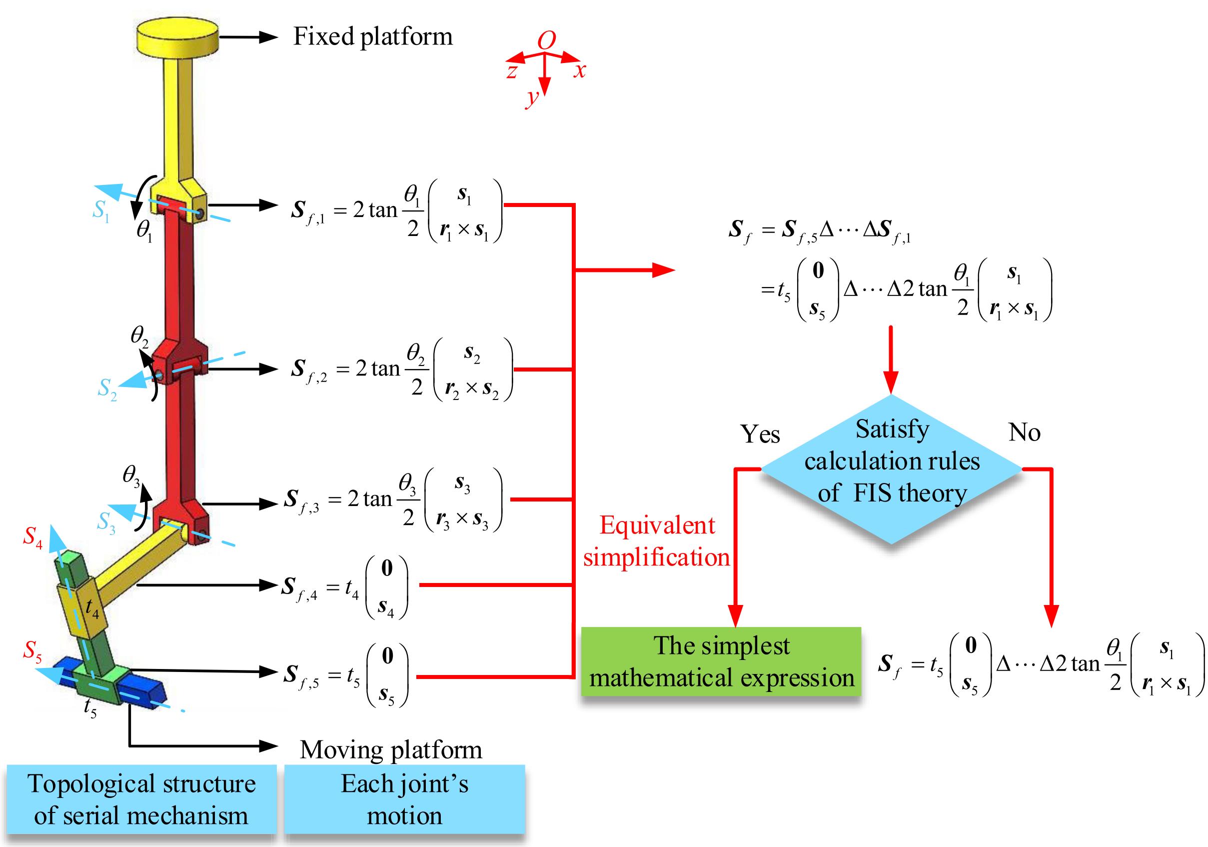


FIGURE 3. Topology of serial mechanism.

designed in this paper is based on the moving platform of the parallel mechanism with minor-DOF. Therefore, 2R1T is assigned as the expected motion pattern of the parallel mechanism, which can directly fulfill the task objective of plugging function. Then the expected motion pattern of the serial mechanism is 1R2T.

As shown in Fig. 6, the continuous motion parameterization of serial and parallel mechanism is performed based on the FIS theory. The derivative limb is obtained through position transformation and type replacement of joints, and the corresponding topology is obtained according to the assembly conditions. On this basis, the topology of serial and parallel are further assembled, and finally the topology of hybrid mechanism satisfying the space control task is obtained.

According to the finite and instantaneous screw theory, the expected motions of serial and parallel mechanisms can be parameterized as

$$S_{f,\text{parallel}} = t_3\begin{pmatrix} \mathbf{0} \\ s_3 \end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix} s_b \\ r_b \times s_b \end{pmatrix} \times \Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix} s_a \\ r_a \times s_a \end{pmatrix} \tag{16}$$

$$S_{f,\text{serial}} = t_2\begin{pmatrix} \mathbf{0} \\ s_2 \end{pmatrix}\Delta t_1\begin{pmatrix} \mathbf{0} \\ s_1 \end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix} s_c \\ r_c \times s_c \end{pmatrix} \tag{17}$$

As shown in Table 1, the following standard limb forms can be obtained by adding zero, one, or two parameter finite screws.

A. TOPOLOGY SYNTHESIS OF SERIAL MECHANISM

Adding one-parameter finite screws, based on the finite and instantaneous screw theory, the standard type of the limb of the serial mechanism is

$$\{S_{f,1\text{R3T}}\}_{\text{serial},1\text{T}} = t_2\begin{pmatrix} \mathbf{0} \\ s_2 \end{pmatrix}\Delta t_1\begin{pmatrix} \mathbf{0} \\ s_1 \end{pmatrix} \Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix} s_c \\ r_c \times s_c \end{pmatrix}\Delta t_3\begin{pmatrix} \mathbf{0} \\ s_3 \end{pmatrix} \tag{18}$$

$$\{S_{f,2\text{R2T}}\}_{\text{serial},1\text{R}} = t_2\begin{pmatrix} \mathbf{0} \\ s_2 \end{pmatrix}\Delta t_1\begin{pmatrix} \mathbf{0} \\ s_1 \end{pmatrix} \Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix} s_c \\ r_c \times s_c \end{pmatrix} \Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix} s_a \\ r_a \times s_a \end{pmatrix} \tag{19}$$

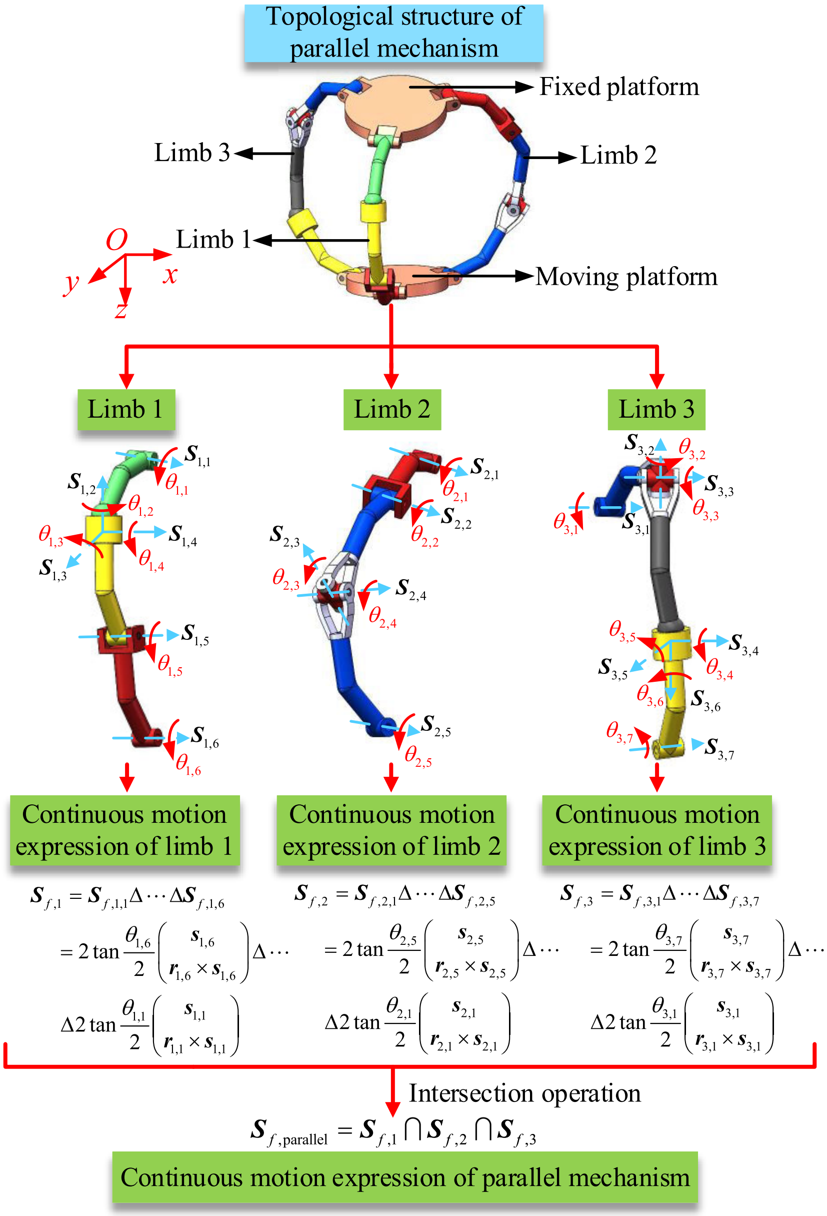


FIGURE 4. Topology of parallel mechanism.

Adding two-parameter finite screws, based on the finite and instantaneous screw theory, the standard type of the limb of the serial mechanism is

$$\begin{aligned}
&\{S_{f,2R3T}\}_{\text{serial},1T1R} \\
&= t_2 \begin{pmatrix} \mathbf{0} \\ \boldsymbol{s}_2 \end{pmatrix} \Delta t_1 \begin{pmatrix} \mathbf{0} \\ \boldsymbol{s}_1 \end{pmatrix} \Delta 2 \tan \frac{\theta_c}{2} \begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_c \times \boldsymbol{s}_c \end{pmatrix} \\
&\times \Delta 2 \tan \frac{\theta_a}{2} \begin{pmatrix} \boldsymbol{s}_a \\ \boldsymbol{r}_a \times \boldsymbol{s}_a \end{pmatrix} \Delta t_3 \begin{pmatrix} \mathbf{0} \\ \boldsymbol{s}_3 \end{pmatrix} \qquad (20)
\end{aligned}$$

In Eq. (18) to Eq. (20), t_i and $\boldsymbol{s}_i$ ($i = 1,2,3$) respectively denote unit vectors of different translation distances and axis directions; θ_a, θ_b and θ_c respectively denote three rotation angles; $\boldsymbol{r}_a$, $\boldsymbol{r}_b$ and $\boldsymbol{r}_c$ denote position vectors of three rotation axes.

Eq. (18) to Eq. (20) correspond to standard limbs $P_3R_cP_1P_2$, $R_aR_cP_1P_2$ and $P_3R_aR_cP_1P_2$, respectively. Next, by changing the type and position of the joints in Eq. (18) to Eq. (20), the derivative limb with 4-DOF and 5-DOF of the serial mechanism will be synthesized.

As shown in Fig. 7, 4-DOF limb synthesis is taken as an example.

Type 1: 1R3T

(1) Type replacement of joints

The derivative limb of $P_3R_cP_1P_2$ can be constructed by type replacement of joint in the following two methods.

TABLE 1. Standard limb structure.

	Adding parameter	Parallel mechanism	Serial mechanism
Arrangement mode 1	zero	2R1T	1R2T
Arrangement mode 2	one-DOF translation	2R2T	1R3T
Arrangement mode 3	one-DOF rotation	3R1T	2R2T
Arrangement mode 4	two single DOF	3R2T	2R3T

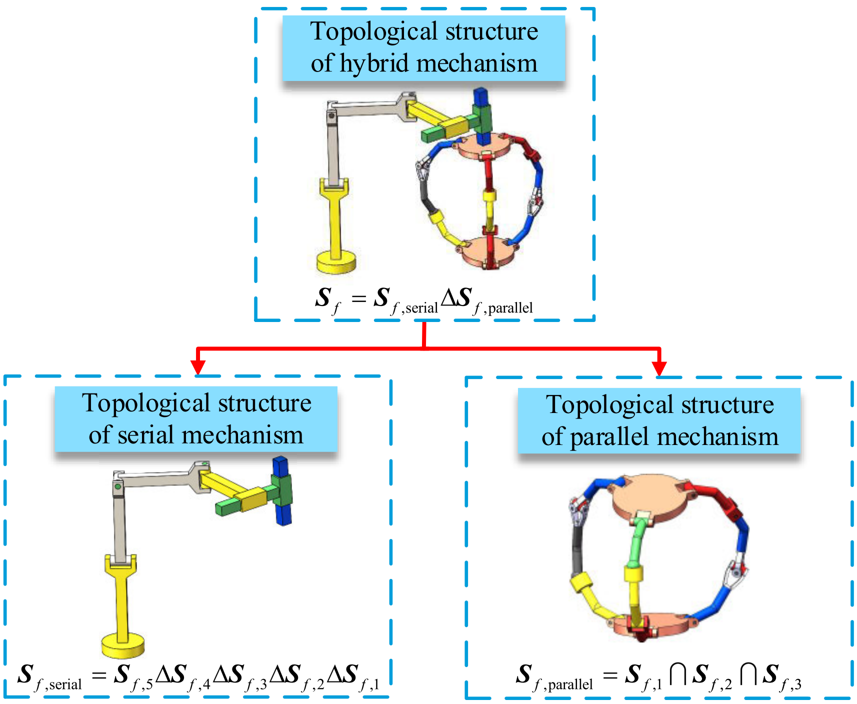


FIGURE 5. Topology of hybrid mechanism.

Method 1: Replace P_3R_c in $P_3R_cP_1P_2$ limb with R_cR_c to obtain $R_cR_cP_1P_2$ limb, which can be expressed as

$$\{\boldsymbol{S}'_{f,\mathrm{1R3T}}\}_{\mathrm{serial,1T}} = t_2\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_2\end{pmatrix}\Delta t_1\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_1\end{pmatrix} \Delta 2\tan\frac{\theta_{c_2}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_2}\times\boldsymbol{s}_c\end{pmatrix} \Delta 2\tan\frac{\theta_{c_1}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_1}\times\boldsymbol{s}_c\end{pmatrix} \tag{21}$$

According to the screw triangle product algorithm, it can be rewritten as

$$\{\boldsymbol{S}'_{f,\mathrm{1R3T}}\}_{\mathrm{serial,1T}} = t_2\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_2\end{pmatrix}\Delta t_1\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_1\end{pmatrix} \times \Delta\left(\begin{array}{c}\boldsymbol{0}\\ \left(\exp\left(-\theta_{c_2}\tilde{\boldsymbol{s}}_c\right)-\exp\left(-\sum_{i=1}^{2}\theta_{c_i}\tilde{\boldsymbol{s}}_c\right)\right)\left(\boldsymbol{r}_{c_2}-\boldsymbol{r}_{c_1}\right)\end{array}\right) \times \Delta 2\tan\frac{\sum_{i=1}^{2}\theta_{c_i}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix} \tag{22}$$

Without changing the value range of the above equation, the following substitutions of symbols can be made

$$\sum_{i=1}^{2}\theta_{c_i}\mapsto\theta_c,\ \boldsymbol{r}_{c_2}\mapsto\boldsymbol{r}_c$$

$$\{\boldsymbol{S}'_{f,\mathrm{1R3T}}\}_{\mathrm{serial,1T}} = t_2\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_2\end{pmatrix}\Delta t_1\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_1\end{pmatrix} \Delta\left(\begin{array}{c}\boldsymbol{0}\\ \left(\exp\left(-\theta_{c_2}\tilde{\boldsymbol{s}}_c\right)-\exp\left(-\theta_c\tilde{\boldsymbol{s}}_c\right)\right)\left(\boldsymbol{r}_c-\boldsymbol{r}_{c_1}\right)\end{array}\right) \Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix} \tag{23}$$

where $\tilde{\boldsymbol{s}}_c$ denotes the skew-symmetric matrix of $\boldsymbol{s}_c$, and the third finite screw denotes a move along a ring of radius $|\boldsymbol{r}_c-\boldsymbol{r}_{c_1}|$ that is perpendicular to $\boldsymbol{s}_c$. When $|\boldsymbol{r}_c-\boldsymbol{r}_{c_1}|\to\infty$ and $\boldsymbol{s}_c\times(\boldsymbol{s}_1\times\boldsymbol{s}_2)\neq 0$, the third finite screw can denote the third translation that is linearly independent of $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$. It is proved that Eq. (21) is equivalent to Eq. (18), that is, the derivative limb $R_cR_cP_1P_2$ is equivalent to the standard limb $P_3R_cP_1P_2$.

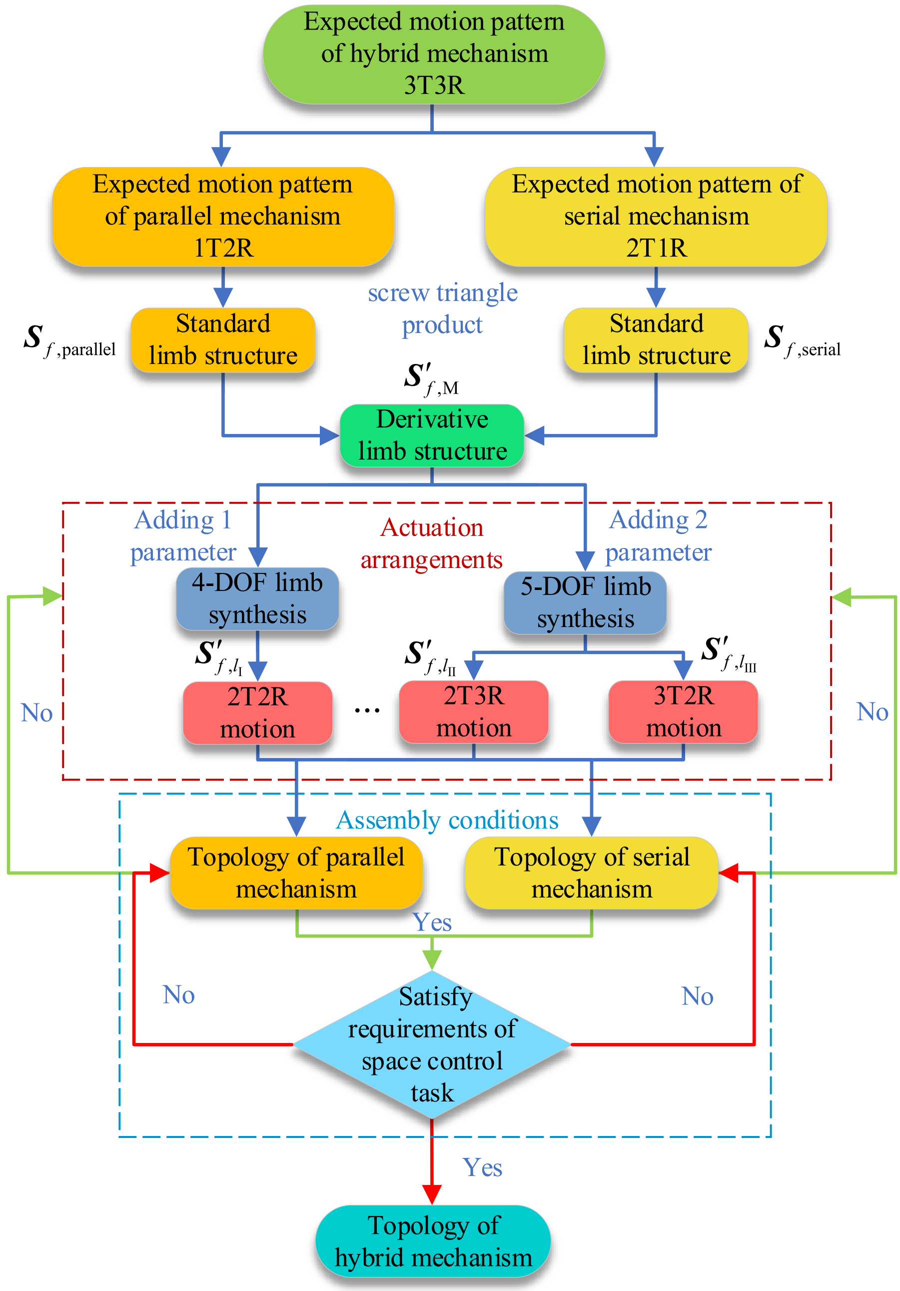


FIGURE 6. Topology synthesis flow of hybrid mechanism.

Method 2: Replace $R_cP_1P_2$ in $P_3R_cP_1P_2$ limb with $P_3R_cR_cR_c$ to obtain $P_3R_cR_cR_c$ limb, which can be expressed as

$$\{S'_{f,1R3T}\}_{\mathrm{serial},1T} = 2\tan\frac{\theta_{c_3}}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_{c_3}\times\boldsymbol{s}_c \end{pmatrix} \Delta 2\tan\frac{\theta_{c_2}}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_{c_2}\times\boldsymbol{s}_c \end{pmatrix} \Delta 2\tan\frac{\theta_{c_1}}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_{c_1}\times\boldsymbol{s}_c \end{pmatrix} \Delta t_3 \begin{pmatrix} \mathbf{0} \\ \boldsymbol{s}_3 \end{pmatrix} \quad (24)$$

According to the properties of screw triangular product, Eq. (24) is equivalent to Eq. (18), that is, the derivative limb $P_3R_cR_cR_c$ is equivalent to the standard limb $P_3R_cP_1P_2$.

By replacing prismatic joints with one or two revolute joints, the corresponding number of independent ring translations are replaced by directional translations, but the one rotation and three translations generated by limbs remained the same. By using this rule, two derivative limbs are synthesized.

(2) Position transformation of joints

Two derivative limbs are obtained based on the type replacement of joints, and other types of limb structures are

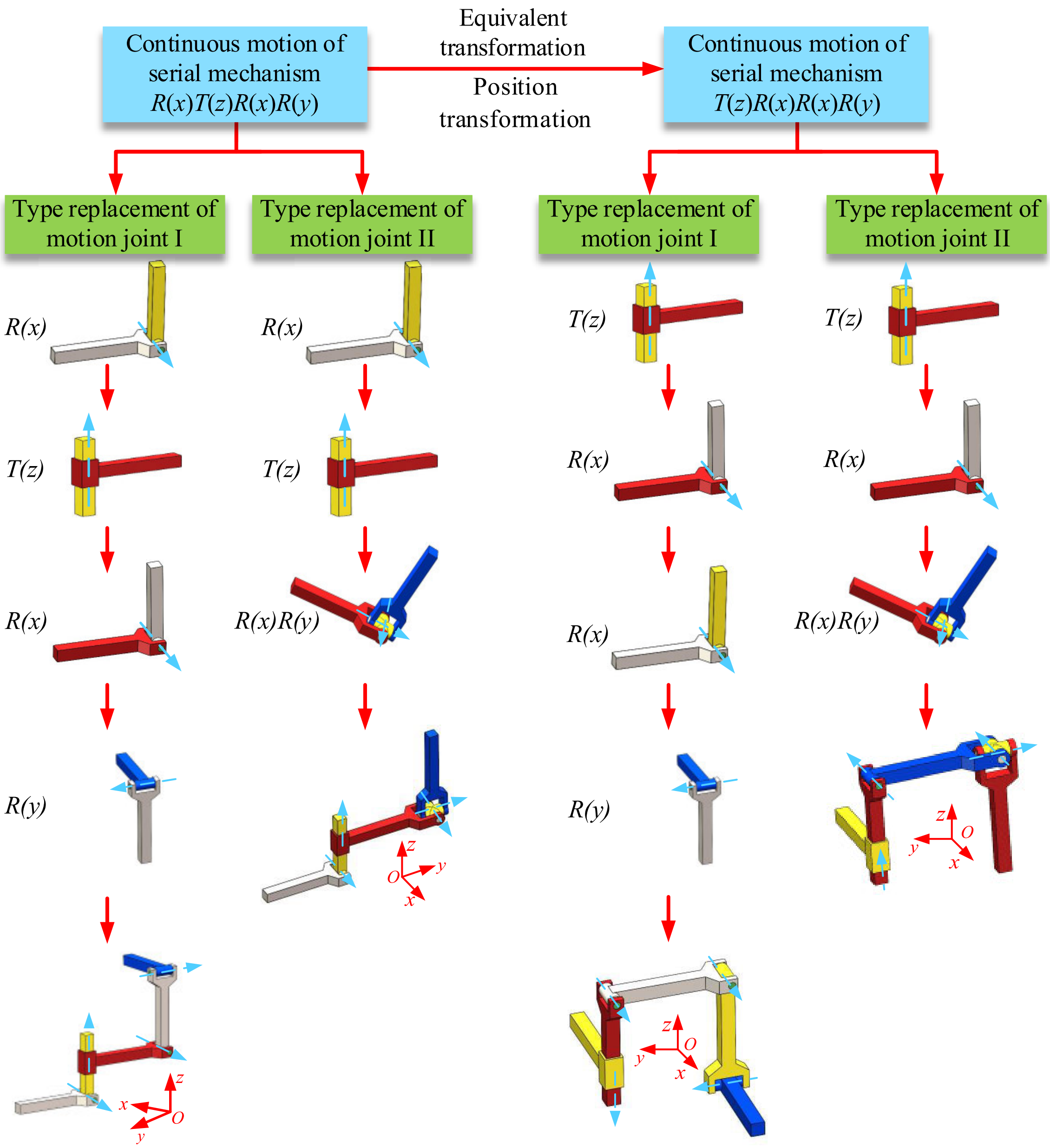


FIGURE 7. Topology synthesis of serial mechanism.

obtained through the position transformation of P joint in the limb.

Firstly, the position of the P joint in $P_3R_cP_1P_2$ limb is transformed to obtain 3 types of limbs $R_cP_3P_1P_2$, $P_3P_1R_cP_2$ and $P_3P_1P_2R_c$. Where, $R_cP_3P_1P_2$ limb can be expressed as

$$\{S'_{f,1R3T}\}_{serial,1T} = t_2 \begin{pmatrix} \mathbf{0} \\ \boldsymbol{s}_2 \end{pmatrix} \Delta t_1 \begin{pmatrix} \mathbf{0} \\ \boldsymbol{s}_1 \end{pmatrix} \Delta t_3 \begin{pmatrix} \mathbf{0} \\ \boldsymbol{s}_3 \end{pmatrix} \Delta 2\tan\frac{\theta_c}{2} \begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_c \times \boldsymbol{s}_c \end{pmatrix} \quad (25)$$

According to the properties of screw triangular product, Eq. (25) is equivalent to Eq. (18), that is, the derivative limb $R_cP_3P_1P_2$ is equivalent to the standard limb $P_3R_cP_1P_2$. Similarly, other derivative limbs $P_3P_1R_cP_2$ and $P_3P_1P_2R_c$ are equivalent to the standard limb $R_cR_cP_1P_2$.

Secondly, the position of the P joint in $R_cR_cP_1P_2$ limb is transformed to obtain 5 types of limbs $P_1R_cR_cP_2$, $P_1P_2R_cR_c$, $R_cP_1R_cP_2$, $R_cP_1P_2R_c$ and $P_1R_cP_2R_c$. Where, $P_1R_cR_cP_2$ limb can be expressed as

$$\{S'_{f,1R3T}\}_{serial,1T} = t_2 \begin{pmatrix} \mathbf{0} \\ \boldsymbol{s}_2 \end{pmatrix} \Delta 2\tan\frac{\theta_{c_2}}{2} \begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_{c_2} \times \boldsymbol{s}_c \end{pmatrix} \Delta 2\tan\frac{\theta_{c_1}}{2} \begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_{c_1} \times \boldsymbol{s}_c \end{pmatrix} \Delta t_1 \begin{pmatrix} \mathbf{0} \\ \boldsymbol{s}_1 \end{pmatrix} \quad (26)$$

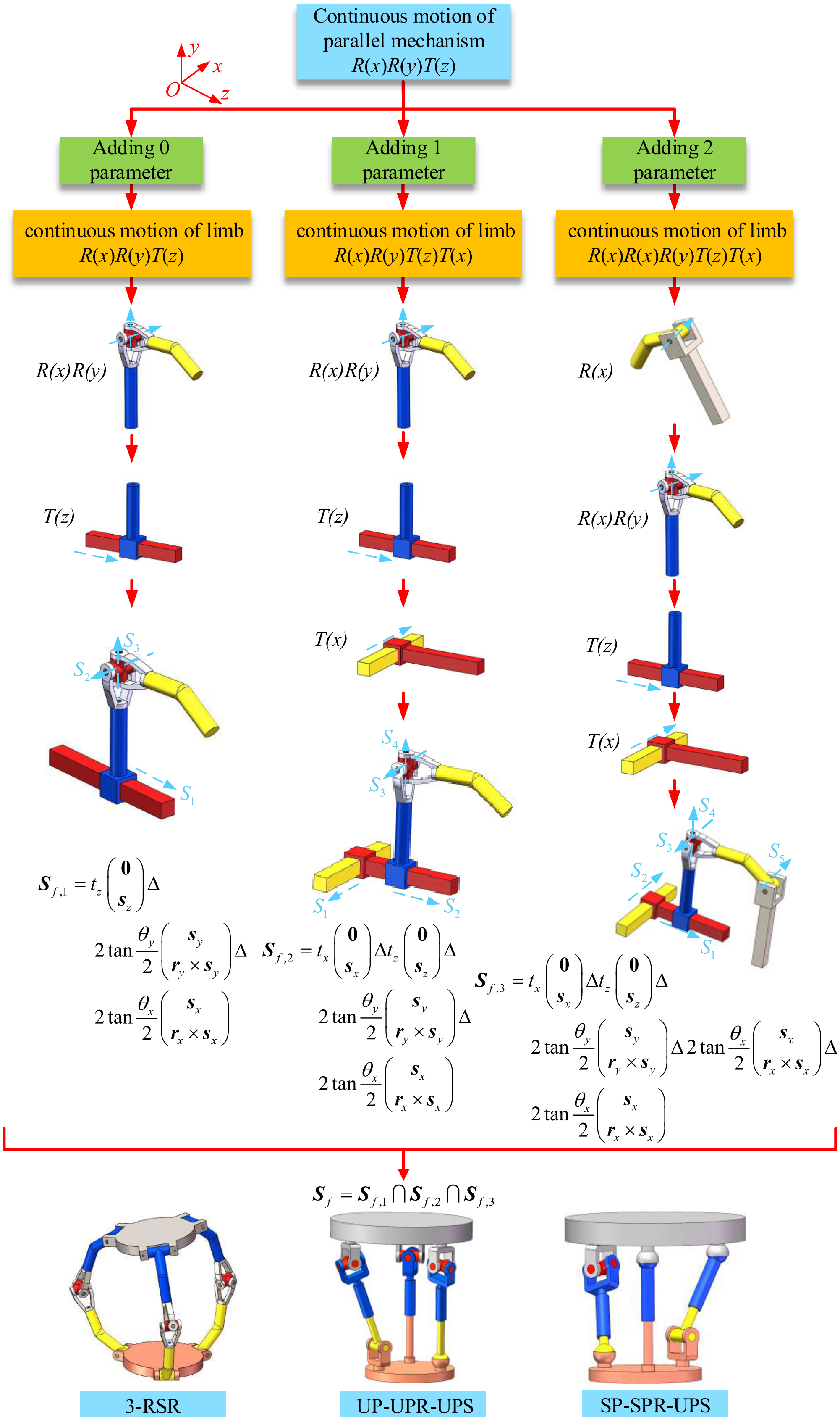


FIGURE 8. Topology synthesis of parallel mechanism.

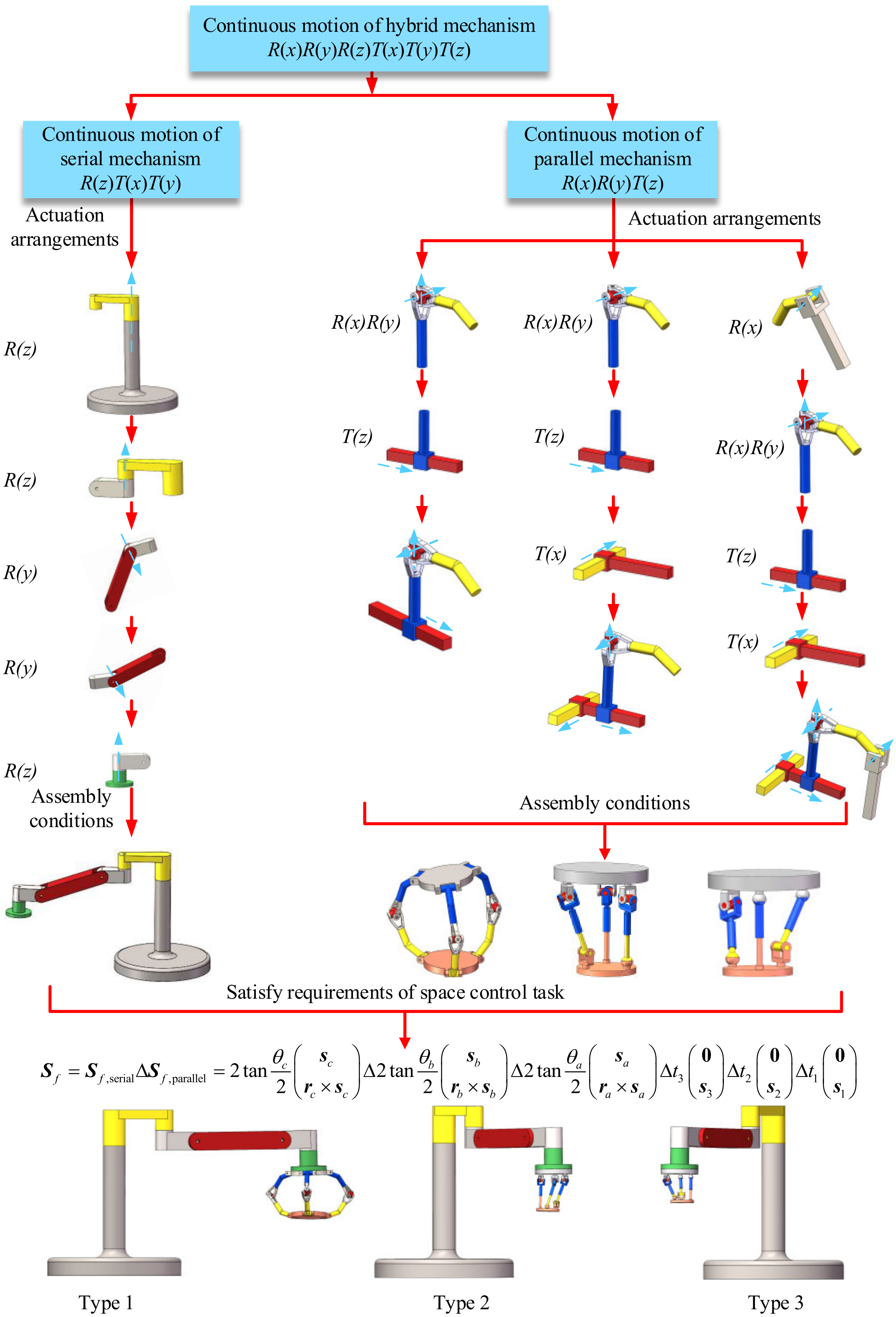


FIGURE 9. **Topology synthesis of hybrid mechanism.**

According to the properties of screw triangular product, Eq. (26) is equivalent to Eq. (21), that is, the derivative limb $P_1R_cR_cP_2$ is equivalent to the standard limb $R_cR_cP_1P_2$. Similarly, other derivative limbs $P_1P_2R_cR_c$, $R_cP_1R_cP_2$,

$R_cP_1P_2R_c$ and $P_1R_cP_2R_c$ are equivalent to the standard limb $R_cR_cP_1P_2$.

Finally, the position of the P joint in $P_3R_cR_cR_c$ limb is transformed to obtain 3 types of limbs $R_cR_cR_cP_3$, $R_cP_3R_cR_c$ and $R_cR_cP_3R_c$. Where, $R_cR_cR_cP_3$ limb can be expressed as

$$\begin{aligned}&\{S'_{f,1R3T}\}_{serial,1T}\\&=t_3\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_{c_3}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_3}\times\boldsymbol{s}_c\end{pmatrix}\\&\quad\Delta 2\tan\frac{\theta_{c_2}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_2}\times\boldsymbol{s}_c\end{pmatrix}\Delta 2\tan\frac{\theta_{c_1}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_1}\times\boldsymbol{s}_c\end{pmatrix}\end{aligned}\tag{27}$$

According to the properties of screw triangular product, Eq. (27) is equivalent to Eq. (24), that is, the derivative limb $R_cR_cR_cP_3$ is equivalent to the standard limb $P_3R_cR_cR_c$. Similarly, other derivative limbs $R_cR_cR_cP_3$, $R_cP_3R_cR_c$ and $R_cR_cP_3R_c$ are equivalent to the standard limb $R_cR_cP_1P_2$.

Type 2: 2R2T

(1) Type replacement of joints

The derivative limb of $R_aR_cP_1P_2$ can be constructed by type replacement of joint in the following two methods.

Method 1: Replace R_cP_1 in $R_aR_cP_1P_2$ limb with R_cR_c to obtain $R_aR_cR_cP_2$ limb, which can be expressed as

$$\begin{aligned}&\{S'_{f,2R2T}\}_{serial,1R}\\&=t_2\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_2\end{pmatrix}\Delta 2\tan\frac{\theta_{c_2}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_2}\times\boldsymbol{s}_c\end{pmatrix}\\&\quad\Delta 2\tan\frac{\theta_{c_1}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_1}\times\boldsymbol{s}_c\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\end{aligned}\tag{28}$$

According to the properties of screw triangular product, Eq. (28) is equivalent to Eq. (19), that is, the derivative limb $R_aR_cR_cP_2$ is equivalent to the standard limb $R_aR_cP_1P_2$.

Method 2: Replace $R_cP_1P_2$ in $R_aR_cP_1P_2$ limb with $R_cR_cR_c$ to obtain $R_aR_cR_cR_c$ limb, which can be expressed as

$$\begin{aligned}&\{S'_{f,2R2T}\}_{serial,1R}\\&=2\tan\frac{\theta_{c_3}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_3}\times\boldsymbol{s}_c\end{pmatrix}\Delta 2\tan\frac{\theta_{c_2}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_2}\times\boldsymbol{s}_c\end{pmatrix}\\&\quad\Delta 2\tan\frac{\theta_{c_1}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_1}\times\boldsymbol{s}_c\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\end{aligned}\tag{29}$$

According to the properties of screw triangular product, Eq. (29) is equivalent to Eq. (19), that is, the derivative limb $R_aR_cR_cR_c$ is equivalent to the standard limb $R_aR_cP_1P_2$.

By replacing prismatic joints with one or two revolute joints, the corresponding number of independent ring translations are replaced by directional translations, but the one rotation and three translations generated by limbs remained the same. By using this rule, two derivative limbs are synthesized.

(2) Position transformation of joints

Based on the type replacement of joints, two derivative limbs are obtained. As the $R_aR_cR_cR_c$ limb did not contain P joints, it only considered the position transformation of P joints by other limbs to obtain other types of limb structure.

TABLE 2. **4-DOF limb of serial mechanism.**

Type	Limb structure			
1R3T	$P_3R_cP_1P_2$	$R_cP_3P_1P_2$	$P_3P_1R_cP_2$	$P_3P_1P_2R_c$
	$R_cR_cP_1P_2$	$P_1R_cR_cP_2$	$P_1P_2R_cR_c$	$R_cP_1R_cP_2$
	$R_cP_1P_2R_c$	$P_1R_cP_2R_c$	$P_3R_cR_cR_c$	$R_cR_cR_cP_3$
	$R_cP_3R_cR_c$	$R_cR_cP_3R_c$		
2R2T	$R_aR_cP_1P_2$	$R_aP_1R_cP_2$	$R_aP_1P_2R_c$	$R_aR_cR_cP_2$
	$R_aR_cP_2R_c$	$R_aP_2R_cR_c$	$R_aR_cR_cR_c$	

Firstly, the position of the P joint in $R_aR_cP_1P_2$ limb is transformed to obtain 2 types of limbs $R_aP_1R_cP_2$ and $R_aP_1P_2R_c$. Where, $R_aP_1R_cP_2$ limb can be expressed as

$$\begin{aligned}&\{S'_{f,2R2T}\}_{serial,1R}\\&=t_2\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_2\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\\&\quad\Delta t_1\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_1\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\end{aligned}\tag{30}$$

According to the properties of screw triangular product, Eq. (30) is equivalent to Eq. (19), that is, the derivative limb $R_aP_1R_cP_2$ is equivalent to the standard limb $R_aR_cP_1P_2$. Similarly, other derivative limb $R_aP_1P_2R_c$ is equivalent to the standard limb.

Secondly, the position of the P joint in $R_aR_cR_cP_2$ limb is transformed to obtain 2 types of limbs $R_aR_cP_2R_c$ and $R_aP_2R_cR_c$. Where, $R_aP_2R_cR_c$ limb can be expressed as

$$\begin{aligned}&\{S'_{f,2R2T}\}_{serial,1R}\\&=2\tan\frac{\theta_{c_2}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_2}\times\boldsymbol{s}_c\end{pmatrix}\Delta 2\tan\frac{\theta_{c_1}}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_{c_1}\times\boldsymbol{s}_c\end{pmatrix}\\&\quad\Delta t_2\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_2\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\end{aligned}\tag{31}$$

According to the properties of screw triangular product, Eq. (31) is equivalent to Eq. (28), that is, the derivative limb $R_aP_2R_cR_c$ is equivalent to the standard limb $R_aR_cR_cP_2$. Similarly, other derivative limb $R_aR_cP_2R_c$ is equivalent to the standard limb.

In summary, the 4-DOF limb synthesis of the serial mechanism can be obtained, as shown in Table 2.

The following is the 5-DOF limb synthesis with type of 2R3T.

(1) Type replacement of joints

Like the synthesis process of 4-DOF limb, for 5-DOF limb standard limb $P_3R_aR_cP_1P_2$, the derivative limb of $P_3R_aR_cP_1P_2$ can be constructed in the following four methods.

Method 1: Replace $P_3R_aR_c$ in $P_3R_aR_cP_1P_2$ limb with $R_aR_aR_c$ and $R_aR_cR_c$ to obtain $R_aR_aR_cP_1P_2$ and $R_aR_cR_cP_1P_2$ limb. Where, $R_aR_aR_cP_1P_2$ limb can be expressed as

$$\begin{aligned}&\{S'_{f,2R3T}\}_{serial,1T1R}\\&=t_2\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_2\end{pmatrix}\Delta t_1\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_1\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\\&\quad\Delta 2\tan\frac{\theta_{a_2}}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_{a_2}\times\boldsymbol{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_{a_1}}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_{a_1}\times\boldsymbol{s}_a\end{pmatrix}\end{aligned}\tag{32}$$

According to the properties of screw triangular product, Eq. (32) is equivalent to Eq. (20), that is, the derivative limb $R_aR_aR_cP_1P_2$ is equivalent to the standard limb $P_3R_aR_cP_1P_2$. Similarly, other derivative limb $R_aR_cR_cP_1P_2$ is equivalent to the standard limb $P_3R_aR_cP_1P_2$.

Method 2: Replace $R_aR_cP_1P_2$ in $P_3R_aR_cP_1P_2$ limb with $R_aR_aR_cR_c$ to obtain $P_3R_aR_aR_cR_c$ limb, which can be expressed as

$$\begin{aligned}&\{S'_{f,2R3T}\}_{serial,1T1R}\\&=2\tan\frac{\theta_{c_2}}{2}\begin{pmatrix}s_c\\r_{c_2}\times s_c\end{pmatrix}\Delta 2\tan\frac{\theta_{c_1}}{2}\begin{pmatrix}s_c\\r_{c_1}\times s_c\end{pmatrix}\\&\Delta 2\tan\frac{\theta_{a_2}}{2}\begin{pmatrix}s_a\\r_{a_2}\times s_a\end{pmatrix}\Delta 2\tan\frac{\theta_{a_1}}{2}\begin{pmatrix}s_a\\r_{a_1}\times s_a\end{pmatrix}\\&\Delta t_3\begin{pmatrix}\mathbf{0}\\s_3\end{pmatrix}\end{aligned}\tag{33}$$

According to the properties of screw triangular product, Eq. (33) is equivalent to Eq. (20), that is, the derivative limb $P_3R_aR_aR_cR_c$ is equivalent to the standard limb $P_3R_aR_cP_1P_2$.

Method 3: Replace $R_aR_cP_1P_2$ in $P_3R_aR_cP_1P_2$ limb with $R_aR_aR_aR_c$ and $R_aR_cR_cR_c$ to obtain $P_3R_aR_aR_aR_c$ and $P_3R_aR_cR_cR_c$ limb. Where, $P_3R_aR_aR_aR_c$ limb can be expressed as

$$\begin{aligned}&\{S'_{f,2R3T}\}_{serial,1T1R}\\&=2\tan\frac{\theta_c}{2}\begin{pmatrix}s_c\\r_c\times s_c\end{pmatrix}\Delta 2\tan\frac{\theta_{a_3}}{2}\begin{pmatrix}s_a\\r_{a_3}\times s_a\end{pmatrix}\\&\Delta 2\tan\frac{\theta_{a_2}}{2}\begin{pmatrix}s_a\\r_{a_2}\times s_a\end{pmatrix}\Delta 2\tan\frac{\theta_{a_1}}{2}\begin{pmatrix}s_a\\r_{a_1}\times s_a\end{pmatrix}\\&\Delta t_3\begin{pmatrix}\mathbf{0}\\s_3\end{pmatrix}\end{aligned}\tag{34}$$

According to the properties of screw triangular product, Eq. (34) is equivalent to Eq. (20), that is, the derivative limb $P_3R_aR_aR_aR_c$ is equivalent to the standard limb $P_3R_aR_cP_1P_2$. Similarly, other derivative limb $P_3R_aR_cR_cR_c$ is equivalent to the standard limb $P_3R_aR_cP_1P_2$.

Method 4: Replace all the P joints in the $P_3R_aR_cP_1P_2$ limb with R joints to obtain $R_aR_aR_aR_cR_c$ and $R_aR_aR_cR_cR_c$ limb. Where, $R_aR_aR_aR_cR_c$ can be expressed as

$$\begin{aligned}&\{S'_{f,2R3T}\}_{serial,1T1R}\\&=2\tan\frac{\theta_{c_2}}{2}\begin{pmatrix}s_c\\r_{c_2}\times s_c\end{pmatrix}\Delta 2\tan\frac{\theta_{c_1}}{2}\begin{pmatrix}s_c\\r_{c_1}\times s_c\end{pmatrix}\\&\Delta 2\tan\frac{\theta_{a_3}}{2}\begin{pmatrix}s_a\\r_{a_3}\times s_a\end{pmatrix}\Delta 2\tan\frac{\theta_{a_2}}{2}\begin{pmatrix}s_a\\r_{a_2}\times s_a\end{pmatrix}\\&\Delta 2\tan\frac{\theta_{a_1}}{2}\begin{pmatrix}s_a\\r_{a_1}\times s_a\end{pmatrix}\end{aligned}\tag{35}$$

According to the properties of screw triangular product, Eq. (35) is equivalent to Eq. (20), that is, the derivative limb $R_aR_aR_aR_cR_c$ is equivalent to the standard limb $P_3R_aR_cP_1P_2$. Similarly, other derivative limb $R_aR_aR_cR_cR_c$ is equivalent to the standard limb $P_3R_aR_cP_1P_2$.

(2) Position transformation of joints

Based on the type replacement of joints, seven derivative limbs are obtained. As the $R_aR_aR_aR_cR_c$ and $R_aR_aR_cR_cR_c$ limb did not contain P joints, it only considered the position transformation of P joints by other limbs to obtain other types of limb structure.

Firstly, the position of the P joint in $P_3R_aR_cP_1P_2$ limb is transformed to obtain 9 types of limbs $R_aR_cP_3P_1P_2$, $P_3P_1R_aR_cP_2$, $P_3P_1P_2R_aR_c$, $R_aP_3R_cP_1P_2$, $R_aP_3P_1R_cP_2$, $R_aP_3P_1P_2R_c$, $P_3R_aP_1R_cP_2$, $P_3R_aP_1P_2R_c$ and $P_3P_1R_aP_2R_c$. Where, $R_aR_cP_3P_1P_2$ limb can be expressed as

$$\begin{aligned}&\{S'_{f,2R3T}\}_{serial,1T1R}\\&=t_2\begin{pmatrix}\mathbf{0}\\s_2\end{pmatrix}\Delta t_1\begin{pmatrix}\mathbf{0}\\s_1\end{pmatrix}\Delta t_3\begin{pmatrix}\mathbf{0}\\s_3\end{pmatrix}\\&\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}s_c\\r_c\times s_c\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}s_a\\r_a\times s_a\end{pmatrix}\end{aligned}\tag{36}$$

According to the properties of screw triangular product, Eq. (36) is equivalent to Eq. (20), that is, the derivative limb $R_aR_cP_3P_1P_2$ is equivalent to the standard limb $P_3R_aR_cP_1P_2$. Similarly, other 8 derivative limbs are equivalent to the standard limb $P_3R_aR_cP_1P_2$.

Secondly, the position of the P joint in $R_aR_aR_cP_1P_2$ limb is transformed to obtain 9 types of limbs $R_aR_aP_1P_2R_c$, $R_aP_1P_2R_aR_c$, $P_1P_2R_aR_aR_c$, $R_aR_aP_1R_cP_2$, $R_aP_1R_aR_cP_2$, $P_1R_aR_aR_cP_2$, $R_aP_1R_aP_2R_c$, $P_1R_aR_aP_2R_c$ and $P_1R_aP_2R_aR_c$. Where, $R_aR_aP_1P_2R_c$ limb can be expressed as

$$\begin{aligned}&\{S'_{f,2R3T}\}_{serial,1T1R}\\&=2\tan\frac{\theta_c}{2}\begin{pmatrix}s_c\\r_c\times s_c\end{pmatrix}\Delta t_2\begin{pmatrix}\mathbf{0}\\s_2\end{pmatrix}\Delta t_1\begin{pmatrix}\mathbf{0}\\s_1\end{pmatrix}\\&\Delta 2\tan\frac{\theta_{a_2}}{2}\begin{pmatrix}s_a\\r_{a_2}\times s_a\end{pmatrix}\Delta 2\tan\frac{\theta_{a_1}}{2}\begin{pmatrix}s_a\\r_{a_1}\times s_a\end{pmatrix}\end{aligned}\tag{37}$$

According to the properties of screw triangular product, Eq. (37) is equivalent to Eq. (32), that is, the derivative limb $R_aR_cP_3P_1P_2$ is equivalent to the standard limb $P_3R_aR_cP_1P_2$. Similarly, other 8 derivative limbs are equivalent to the standard limb $P_3R_aR_cP_1P_2$. Using the same method, 9 derived derivative limbs equivalent to $R_aR_cR_cP_1P_2$ are obtained. $R_aR_cP_1P_2R_c$, $R_aP_1P_2R_cR_c$, $P_1P_2R_aR_cR_c$, $R_aR_cP_1R_cP_2$, $R_aP_1R_cR_cP_2$, $P_1R_aR_cR_cP_2$, $R_aP_1R_cP_2R_c$, $P_1R_aR_cP_2R_c$ and $P_1R_aP_2R_cR_c$.

Thirdly, the position of the P joint in $P_3R_aR_aR_cR_c$ limb is transformed to obtain 4 types of limbs $R_aR_aP_3R_cR_c$, $R_aR_aR_cR_cP_3$, $R_aR_aR_cP_3R_c$ and $R_aP_3R_aR_cR_c$. Where, $R_aR_aP_3R_cR_c$ limb can be expressed as

$$\begin{aligned}&\{S'_{f,2R3T}\}_{serial,1T1R}\\&=2\tan\frac{\theta_{c_2}}{2}\begin{pmatrix}s_c\\r_{c_2}\times s_c\end{pmatrix}\Delta 2\tan\frac{\theta_{c_1}}{2}\begin{pmatrix}s_c\\r_{c_1}\times s_c\end{pmatrix}\\&\Delta t_3\begin{pmatrix}\mathbf{0}\\s_3\end{pmatrix}\Delta 2\tan\frac{\theta_{a_2}}{2}\begin{pmatrix}s_a\\r_{a_2}\times s_a\end{pmatrix}\\&\Delta 2\tan\frac{\theta_{a_1}}{2}\begin{pmatrix}s_a\\r_{a_1}\times s_a\end{pmatrix}\end{aligned}\tag{38}$$

According to the properties of screw triangular product, Eq. (38) is equivalent to Eq. (33), that is, the derivative limb $R_aR_aP_3R_cR_c$ is equivalent to the standard limb $P_3R_aR_aR_cR_c$. Similarly, other 3 derivative limbs are equivalent to the standard limb $P_3R_aR_cP_1P_2$.

Finally, the position of the P joint in $P_3R_aR_aR_aR_c$ limb is transformed to obtain 4 types of limbs $R_aR_aR_aP_3R_c$, $R_aR_aP_3R_aR_c$, $R_aP_3R_aR_aR_c$ and $R_aR_aR_aR_cP_3$. Where, $R_aR_aR_aR_cP_3$ limb can be expressed as

$$\begin{aligned}\{\boldsymbol{S'}_{f,2R3T}\}_{serial,1T1R} &= t_3\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\\ &\Delta 2\tan\frac{\theta_{a_3}}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_{a_3}\times\boldsymbol{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_{a_2}}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_{a_2}\times\boldsymbol{s}_a\end{pmatrix}\\ &\Delta 2\tan\frac{\theta_{a_1}}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_{a_1}\times\boldsymbol{s}_a\end{pmatrix}\end{aligned} \quad (39)$$

According to the properties of screw triangular product, Eq. (39) is equivalent to Eq. (34), that is, the derivative limb $R_aR_aR_aR_cP_3$ is equivalent to the standard limb $P_3R_aR_aR_aR_c$. Similarly, other 3 derivative limbs are equivalent to the standard limb $P_3R_aR_aR_aR_c$. Using the same method, 4 derived derivative limbs equivalent to $P_3R_aR_cR_cR_c$ are obtained. $R_aR_cR_cP_3R_c$, $R_aR_cP_3R_cR_c$, $R_aP_3R_cR_cR_c$ and $R_aR_cR_cR_cP_3$.

In summary, the 5-DOF limb synthesis of the serial mechanism can be obtained, as shown in Table 3.

So far, 21 types of 4-DOF limbs and 75 types of 5-DOF limbs have been synthesized. According to the design principles defined in Section IV, the serial mechanism is a 3-DOF mechanism (one rotation and two translations). Therefore, the assembly conditions can be defined by the cooperative relationship between joint motions, and the serial mechanism needs to satisfy the following conditions

$$\boldsymbol{S}_{f,serial} = t_2\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_2\end{pmatrix}\Delta t_1\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_1\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix} \quad (40)$$

Considering that the mechanism is applied in a space environment, $R_aR_cR_cR_aR_a$ limb with revolute structure is selected, and its adjacent axes are arranged in parallel, satisfying Eq. (40). Therefore, it can be determined that the limb $R_aR_cR_cR_aR_a$ is a serial mechanism that satisfies practical engineering requirements, and its topology can be expressed as follows

$$\begin{aligned}\boldsymbol{S}_{f,serial} &= 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\\ &\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\\ &\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\end{aligned} \quad (41)$$

B. TOPOLOGY SYNTHESIS OF PARALLEL MECHANISM

Adding one-parameter finite screws, based on the finite and instantaneous screw theory, the standard type of the limb of the parallel mechanism is

$$\begin{aligned}\{\boldsymbol{S}_{f,3R1T}\}_{parallel,1R} &= t_3\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\\ &\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\end{aligned} \quad (42)$$

$$\begin{aligned}\{\boldsymbol{S}_{f,2R2T}\}_{parallel,1T} &= t_3\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\\ &\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta t_1\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_1\end{pmatrix}\end{aligned} \quad (43)$$

Adding two-parameter finite screws, based on the finite and instantaneous screw theory, the standard type of the limb of the parallel mechanism is

$$\begin{aligned}\{\boldsymbol{S}_{f,3R2T}\}_{parallel,1T1R} &= t_3\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\\ &\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\\ &\Delta t_1\begin{pmatrix}\mathbf{0}\\ \boldsymbol{s}_1\end{pmatrix}\end{aligned} \quad (44)$$

In Eq. (42) to Eq. (44), t_i and $\boldsymbol{s}_i$ (i = 1,2,3) respectively denote unit vectors of different translation distances and axis directions; θ_a, θ_b and θ_c respectively denote three rotation angles; $\boldsymbol{r}_a$, $\boldsymbol{r}_b$ and $\boldsymbol{r}_c$ denote position vectors of three rotation axes.

Eq. (42) to Eq. (44) correspond to standard limbs $R_cR_aR_bP_3$, $P_1R_aR_bP_3$ and $P_1R_cR_aR_bP_3$, respectively. As shown in Fig. 8, by changing the type and position of the joints in Eq. (42) to Eq. (44), the derivative limb with 4-DOF and 5-DOF of the parallel mechanism will be synthesized. According to the assembly conditions, the typical topology of 3-DOF parallel mechanism is obtained.

4-DOF limb synthesis is carried out below, including two standard limb types.

Type 1: 3R1T

(1) Type replacement of joints

Replace all the P joints in the $R_cR_aR_bP_3$ limb with R joints to obtain $R_cR_aR_b\underline{\underline{R}}_1$ limb (denoting the R joint without the origin of the original R joint as $\underline{\underline{R}}$), which can be expressed as

$$\begin{aligned}\{\boldsymbol{S'}_{f,3R1T}\}_{parallel,1R} &= 2\tan\frac{\theta_1}{2}\begin{pmatrix}\boldsymbol{s}_1\\ \boldsymbol{r}_1\times\boldsymbol{s}_1\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\\ &\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\end{aligned} \quad (45)$$

(2) Position transformation of joints

Firstly, the position of the P joint in $R_cR_aR_bP_3$ limb is transformed to obtain 3 types of limbs $P_3R_cR_aR_b$, $R_cP_3R_aR_b$ and $R_cR_aP_3R_b$. Where, $R_cR_aP_3R_b$ limb can be

TABLE 3. 5-DOF limb of serial mechanism.

Type	Limb structure				
2R3T	$P_3R_aR_cP_1P_2$	$R_aR_cP_3P_1P_2$	$P_3P_1R_aR_cP_2$	$P_3P_1P_2R_aR_c$	$R_aP_3R_cP_1P_2$
	$R_aP_3P_1R_cP_2$	$R_aP_3P_1P_2R_c$	$P_3R_aP_1R_cP_2$	$P_3R_aP_1P_2R_c$	$P_3P_1R_aP_2R_c$
	$R_aR_aR_cP_1P_2$	$R_aR_aP_1P_2R_c$	$R_aP_1P_2R_aR_c$	$P_1P_2R_aR_aR_c$	$R_aR_aP_1R_cP_2$
	$R_aP_1R_aR_cP_2$	$P_1R_aR_aR_cP_2$	$R_aP_1R_aP_2R_c$	$P_1R_aR_aP_2R_c$	$P_1R_aP_2R_aR_c$
	$R_aR_cR_aP_1P_2$	$R_aR_cP_1P_2R_a$	$R_aP_1P_2R_cR_a$	$P_1P_2R_aR_cR_a$	$R_aR_cP_1R_aP_2$
	$R_aP_1R_cR_aP_2$	$P_1R_aR_cR_aP_2$	$R_aP_1R_cP_2R_a$	$P_1R_aR_cP_2R_a$	$P_1R_aP_2R_cR_a$
	$R_aR_cR_cP_1P_2$	$R_aR_cP_1P_2R_c$	$R_aP_1P_2R_cR_c$	$P_1P_2R_aR_cR_c$	$R_aR_cP_1R_cP_2$
	$R_aP_1R_cR_cP_2$	$P_1R_aR_cR_cP_2$	$R_aP_1R_cP_2R_c$	$P_1R_aR_cP_2R_c$	$P_1R_aP_2R_cR_c$
	$P_3R_aR_aR_cR_c$	$R_aR_aP_3R_cR_c$	$R_aR_aR_cR_cP_3$	$R_aR_aR_cP_3R_c$	$R_aP_3R_aR_cR_c$
	$R_aP_3R_cR_cR_a$	$R_aR_cR_cP_3R_a$	$R_aR_cP_3R_cR_a$	$P_3R_aR_cR_cR_a$	$R_aR_cR_cR_aP_3$
	$P_3R_aR_aR_aR_c$	$R_aR_aR_aP_3R_c$	$R_aR_aP_3R_aR_c$	$R_aP_3R_aR_aR_c$	$R_aR_aR_aR_cP_3$
	$R_aR_aR_cR_aP_3$	$R_aR_aR_cP_3R_a$	$R_aR_aP_3R_cR_a$	$R_aP_3R_aR_cR_a$	$P_3R_aR_aR_cR_a$
	$R_aR_cR_aR_aP_3$	$R_aR_cR_aP_3R_a$	$R_aR_cP_3R_aR_a$	$R_aP_3R_cR_aR_a$	$P_3R_aR_cR_aR_a$
	$P_3R_aR_cR_cR_c$	$R_aR_cR_cP_3R_c$	$R_aR_cP_3R_cR_c$	$R_aP_3R_cR_cR_c$	$R_aR_cR_cR_cP_3$
	$R_aR_aR_aR_cR_c$	$R_aR_aR_cR_cR_a$	$R_aR_cR_cR_aR_a$	$R_aR_aR_cR_cR_c$	$R_aR_cR_cR_cR_a$

expressed as

$$\begin{aligned}&\{\boldsymbol{S}'_{f,3R1T}\}_{\text{parallel},1R}\\&=2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\\boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\Delta t_3\begin{pmatrix}\boldsymbol{0}\\\boldsymbol{s}_3\end{pmatrix}\\&\quad\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\\boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\\boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\end{aligned}\tag{46}$$

According to the properties of screw triangular product,

$$\begin{aligned}&\{\boldsymbol{S}'_{f,3R1T}\}_{\text{parallel},1R}\\&=t_3\begin{pmatrix}\boldsymbol{0}\\\exp(-\theta_b\tilde{\boldsymbol{s}}_b)\boldsymbol{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\\boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\\&\quad\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\\boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\\boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\end{aligned}\tag{47}$$

As can be seen from the above, $\{\boldsymbol{S}_{f,3R1T}\}_{\text{parallel},1R}\subseteq\{\boldsymbol{S}'_{f,3R1T}\}_{\text{parallel},1R}$ is valid, but it is not equivalent to Eq. (42). The $R_cR_aP_3R_b$ limb is a feasible limb, but it is not equivalent to $R_cR_aR_bP_3$. Similarly, the other 2 limbs are feasible limbs that are not equivalent to $R_cR_aR_bP_3$.

Secondly, the position of the $\underline{\underline{R}}_1$ joint in $R_cR_aR_b\underline{\underline{R}}_1$ limb is transformed to obtain 3 types of limbs $R_cR_a\underline{\underline{R}}_1R_b$, $R_c\underline{\underline{R}}_1R_aR_b$ and $\underline{\underline{R}}_1R_cR_aR_b$. Where, $R_cR_a\underline{\underline{R}}_1R_b$ limb can be expressed as

$$\begin{aligned}&\{\boldsymbol{S}'_{f,3R1T}\}_{\text{parallel},1R}\\&=2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\\boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_1}{2}\begin{pmatrix}\boldsymbol{s}_1\\\boldsymbol{r}_1\times\boldsymbol{s}_1\end{pmatrix}\\&\quad\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\\boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\\boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix}\end{aligned}\tag{48}$$

According to the properties of screw triangular product, $\{\boldsymbol{S}_{f,3R1T}\}_{\text{parallel},1R}\subseteq\{\boldsymbol{S}'_{f,3R1T}\}_{\text{parallel},1R}$ is valid, but it is not equivalent to Eq. (45). The $R_cR_a\underline{\underline{R}}_1R_b$ limb is a feasible limb, but it is not equivalent to $R_cR_aR_b\underline{\underline{R}}_1$. Similarly, the other 2 limbs are feasible limbs that are not equivalent to $R_cR_aR_b\underline{\underline{R}}_1$.

Type 2: 2R2T

(1) Type replacement of joints

The derivative limb of $P_1R_aR_bP_3$ can be constructed by type replacement of joint in the following two methods.

Method 1: Replace R_bP_3 in $P_1R_aR_bP_3$ limb with R_bR_b to obtain $P_1R_aR_bR_b$ limb, which can be expressed as

$$\begin{aligned}&\{\boldsymbol{S}'_{f,2R2T}\}_{\text{parallel},1T}\\&=2\tan\frac{\theta_{b_2}}{2}\begin{pmatrix}\boldsymbol{s}_b\\\boldsymbol{r}_{b_2}\times\boldsymbol{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_{b_1}}{2}\begin{pmatrix}\boldsymbol{s}_b\\\boldsymbol{r}_{b_1}\times\boldsymbol{s}_b\end{pmatrix}\\&\quad\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\\boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\Delta t_1\begin{pmatrix}\boldsymbol{0}\\\boldsymbol{s}_1\end{pmatrix}\end{aligned}\tag{49}$$

According to the properties of screw triangular product, Eq. (49) is equivalent to Eq. (43), that is, the derivative limb $P_1R_aR_bR_b$ is equivalent to the standard limb $P_1R_aR_bP_3$.

Method 2: Replace all the P joints in the $P_1R_aR_bP_3$ limb with R joints to obtain $R_aR_bR_bR_b$ limb, which can be expressed as

$$\begin{aligned}&\{\boldsymbol{S}'_{f,2R2T}\}_{\text{parallel},1T}\\&=2\tan\frac{\theta_{b_3}}{2}\begin{pmatrix}\boldsymbol{s}_b\\\boldsymbol{r}_{b_3}\times\boldsymbol{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_{b_2}}{2}\begin{pmatrix}\boldsymbol{s}_b\\\boldsymbol{r}_{b_2}\times\boldsymbol{s}_b\end{pmatrix}\\&\quad\Delta 2\tan\frac{\theta_{b_1}}{2}\begin{pmatrix}\boldsymbol{s}_b\\\boldsymbol{r}_{b_1}\times\boldsymbol{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\\boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}\end{aligned}\tag{50}$$

According to the properties of screw triangular product, Eq. (50) is equivalent to Eq. (43), that is, the derivative limb $R_aR_bR_bR_b$ is equivalent to the standard limb $P_1R_aR_bP_3$.

(2) Position transformation of joints

Based on the type replacement of joints, two derivative limbs are obtained. As the $R_aR_bR_bR_b$ limb did not contain P joints, it only considered the position transformation of P joints by other limbs to obtain other types of limb structure.

TABLE 4. 4-DOF limb of parallel mechanism.

Type	Limb structure			
3R1T	$R_cR_aR_bP_3$	$P_3R_cR_aR_b$	$R_cP_3R_aR_b$	$R_cR_aP_3R_b$
	$R_cR_aR_b\underline{\underline{R}}_1$	$R_cR_a\underline{\underline{R}}_1R_b$	$R_c\underline{\underline{R}}_1R_aR_b$	$\underline{\underline{R}}_1R_cR_aR_b$
2R2T	$P_1R_aR_bP_3$	$R_aP_1R_bP_3$	$R_aP_1P_3R_b$	$P_1R_aR_bR_b$
	$R_aP_1R_bR_b$	$R_aR_bP_1R_b$	$R_aR_bR_bR_b$	

Firstly, the position of the P joint in $P_1R_aR_bP_3$ limb is transformed to obtain 2 types of limbs $R_aP_1R_bP_3$ and $R_aP_1P_3R_b$. Where, $R_aP_1R_bP_3$ limb can be expressed as

$$\begin{aligned}\{S'_{f,\mathrm{2R2T}}\}_{\mathrm{parallel,1T}} &= t_3\begin{pmatrix}\mathbf{0}\\ \mathbf{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\mathbf{s}_b\\ \mathbf{r}_b\times\mathbf{s}_b\end{pmatrix}\\ &\quad\Delta t_1\begin{pmatrix}\mathbf{0}\\ \mathbf{s}_1\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\mathbf{s}_a\\ \mathbf{r}_a\times\mathbf{s}_a\end{pmatrix}\end{aligned}\tag{51}$$

According to the properties of screw triangular product, Eq. (51) is equivalent to Eq. (43), that is, the derivative limb $R_aP_1R_bP_3$ is equivalent to the standard limb $P_1R_aR_bP_3$. Similarly, other derivative limb $R_aP_1P_3R_b$ is equivalent to the standard limb.

Finally, the position of the P joint in $P_1R_aR_bR_b$ limb is transformed to obtain 2 types of limbs $R_aP_1R_bR_b$ and $R_aR_bP_1R_b$. Where, $R_aP_1R_bR_b$ limb can be expressed as

$$\begin{aligned}\{S'_{f,\mathrm{2R2T}}\}_{\mathrm{parallel,1T}} &= 2\tan\frac{\theta_{b_2}}{2}\begin{pmatrix}\mathbf{s}_b\\ \mathbf{r}_{b_2}\times\mathbf{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_{b_1}}{2}\begin{pmatrix}\mathbf{s}_b\\ \mathbf{r}_{b_1}\times\mathbf{s}_b\end{pmatrix}\\ &\quad\Delta t_1\begin{pmatrix}\mathbf{0}\\ \mathbf{s}_1\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\mathbf{s}_a\\ \mathbf{r}_a\times\mathbf{s}_a\end{pmatrix}\end{aligned}\tag{52}$$

According to the properties of screw triangular product, Eq. (52) is equivalent to Eq. (49), that is, the derivative limb $R_aP_1R_cP_2$ is equivalent to the standard limb $R_aR_cP_1P_2$. Similarly, other derivative limb $R_aP_1P_2R_c$ is equivalent to the standard limb.

In summary, the 4-DOF limb synthesis of the parallel mechanism can be obtained, as shown in Table 4.

The following is the 5-DOF limb synthesis with type of 3R2T.

(1) Type replacement of joints

Like the synthesis process of 4-DOF limb, for 5-DOF limb standard limb $P_1R_cR_aR_bP_3$, the derivative limb of $P_1R_cR_aR_bP_3$ can be constructed in the following two methods.

Method 1: Replace $R_aR_bP_3$ in $P_1R_cR_aR_bP_3$ limb with $R_aR_b\underline{\underline{R}}_3$ to obtain $P_1R_cR_aR_b\underline{\underline{R}}_3$ limb, which can be expressed as

$$\begin{aligned}\{S'_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}} &= 2\tan\frac{\theta_3}{2}\begin{pmatrix}\mathbf{s}_3\\ \mathbf{r}_3\times\mathbf{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\mathbf{s}_b\\ \mathbf{r}_b\times\mathbf{s}_b\end{pmatrix}\\ &\quad\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\mathbf{s}_a\\ \mathbf{r}_a\times\mathbf{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\mathbf{s}_c\\ \mathbf{r}_c\times\mathbf{s}_c\end{pmatrix}\\ &\quad\Delta t_1\begin{pmatrix}\mathbf{0}\\ \mathbf{s}_1\end{pmatrix}\end{aligned}\tag{53}$$

According to the properties of screw triangular product, Eq. (53) is equivalent to Eq. (44), that is, the derivative limb $P_1R_cR_aR_b\underline{\underline{R}}_3$ is equivalent to the standard limb $P_1R_cR_aR_bP_3$.

Method 2: Replace all the P joints in the $P_1R_cR_aR_bP_3$ limb with R joints to obtain $\underline{\underline{R}}_1R_cR_aR_b\underline{\underline{R}}_3$ limb, which can be expressed as

$$\begin{aligned}\{S'_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}} &= 2\tan\frac{\theta_3}{2}\begin{pmatrix}\mathbf{s}_3\\ \mathbf{r}_3\times\mathbf{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\mathbf{s}_b\\ \mathbf{r}_b\times\mathbf{s}_b\end{pmatrix}\\ &\quad\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\mathbf{s}_a\\ \mathbf{r}_a\times\mathbf{s}_a\end{pmatrix}\\ &\quad\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\mathbf{s}_c\\ \mathbf{r}_c\times\mathbf{s}_c\end{pmatrix}\Delta 2\tan\frac{\theta_1}{2}\begin{pmatrix}\mathbf{s}_1\\ \mathbf{r}_1\times\mathbf{s}_1\end{pmatrix}\end{aligned}\tag{54}$$

According to the properties of screw triangular product, Eq. (54) is equivalent to Eq. (44), that is, the derivative limb $\underline{\underline{R}}_1R_cR_aR_b\underline{\underline{R}}_3$ is equivalent to the standard limb $P_1R_cR_aR_bP_3$.

(2) Position transformation of joints

Firstly, the position of the P joint in $P_1R_cR_aR_bP_3$ limb is transformed to obtain 9 types of limbs $P_1P_3R_cR_aR_b$, $R_cP_1P_3R_aR_b$, $R_cR_aP_1P_3R_b$, $R_cR_aR_bP_1P_3$, $P_1R_cP_3R_aR_b$, $P_1R_cR_aP_3R_b$, $R_cP_1R_aP_3R_b$, $R_cP_1R_aR_bP_3$ and $R_cR_aP_1R_bP_3$. Where, $P_1P_3R_cR_aR_b$ limb can be expressed as

$$\begin{aligned}\{S'_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}} &= 2\tan\frac{\theta_b}{2}\begin{pmatrix}\mathbf{s}_b\\ \mathbf{r}_b\times\mathbf{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\mathbf{s}_a\\ \mathbf{r}_a\times\mathbf{s}_a\end{pmatrix}\\ &\quad\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\mathbf{s}_c\\ \mathbf{r}_c\times\mathbf{s}_c\end{pmatrix}\Delta t_3\begin{pmatrix}\mathbf{0}\\ \mathbf{s}_3\end{pmatrix}\Delta t_1\begin{pmatrix}\mathbf{0}\\ \mathbf{s}_1\end{pmatrix}\end{aligned}\tag{55}$$

According to the properties of screw triangular product, $\{S_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}} \subseteq \{S'_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}}$ is valid, but it is not equivalent to Eq. (44). The $P_1P_3R_cR_aR_b$ limb is a feasible limb, but it is not equivalent to $P_1R_cR_aR_bP_3$. Similarly, the other 8 limbs are feasible limbs that are not equivalent to $R_cR_aR_b\underline{\underline{R}}_1$.

Secondly, the position of the P_1 and $\underline{\underline{R}}_3$ joint in $P_1R_cR_aR_b\underline{\underline{R}}_3$ limb is transformed to obtain 19 types of limbs $P_1\underline{\underline{R}}_3R_cR_aR_b$, $R_cP_1\underline{\underline{R}}_3R_aR_b$, $R_cR_aP_1\underline{\underline{R}}_3R_b$, $R_cR_aR_bP_1\underline{\underline{R}}_3$, $P_1R_c\underline{\underline{R}}_3R_aR_b$, $P_1R_cR_a\underline{\underline{R}}_3R_b$, $R_cP_1R_a\underline{\underline{R}}_3R_b$, $R_cP_1R_aR_b\underline{\underline{R}}_3$, $R_cR_aP_1R_b\underline{\underline{R}}_3$, $\underline{\underline{R}}_3P_1R_cR_aR_b$, $R_c\underline{\underline{R}}_3P_1R_aR_b$, $R_cR_a\underline{\underline{R}}_3P_1R_b$, $R_cR_aR_b\underline{\underline{R}}_3P_1$, $\underline{\underline{R}}_3R_cP_1R_aR_b$, $\underline{\underline{R}}_3R_cR_aP_1R_b$, $\underline{\underline{R}}_3R_cR_aR_bP_1$, $R_c\underline{\underline{R}}_3R_aP_1R_b$, $R_c\underline{\underline{R}}_3R_aR_bP_1$ and $R_cR_a\underline{\underline{R}}_3R_bP_1$. Where, $P_1R_cR_a\underline{\underline{R}}_3R_b$ limb can be expressed as

$$\begin{aligned}\{S'_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}} &= 2\tan\frac{\theta_b}{2}\begin{pmatrix}\mathbf{s}_b\\ \mathbf{r}_b\times\mathbf{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_3}{2}\begin{pmatrix}\mathbf{s}_3\\ \mathbf{r}_3\times\mathbf{s}_3\end{pmatrix}\\ &\quad\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\mathbf{s}_a\\ \mathbf{r}_a\times\mathbf{s}_a\end{pmatrix}\Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\mathbf{s}_c\\ \mathbf{r}_c\times\mathbf{s}_c\end{pmatrix}\\ &\quad\Delta t_1\begin{pmatrix}\mathbf{0}\\ \mathbf{s}_1\end{pmatrix}\end{aligned}\tag{56}$$

According to the properties of screw triangular product, $\{S_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}} \subseteq \{S'_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}}$ is valid, but it is

TABLE 5. 5-DOF limb of parallel mechanism.

Type	Limb structure				
	$P_1R_cR_aR_bP_3$	$P_1P_3R_cR_aR_b$	$R_cP_1P_3R_aR_b$	$R_cR_aP_1P_3R_b$	$R_cR_aR_bP_1P_3$
	$P_1R_cP_3R_aR_b$	$P_1R_cR_aP_3R_b$	$R_cP_1R_aP_3R_b$	$R_cP_1R_aR_bP_3$	$R_cR_aP_1R_bP_3$
	$P_1P_3\underline{\underline{R}}_cR_aR_b$	$P_1\underline{\underline{R}}_cP_3R_aR_b$	$\underline{\underline{R}}_cP_1P_3R_aR_b$	$R_cP_1P_3\underline{\underline{R}}_aR_b$	$R_cP_1\underline{\underline{R}}_aP_3R_b$
	$R_c\underline{\underline{R}}_aP_1P_3R_b$	$R_cR_aP_1P_3\underline{\underline{R}}_b$	$R_cR_aP_1\underline{\underline{R}}_bP_3$	$R_cR_a\underline{\underline{R}}_bP_1P_3$	
	$P_1R_cR_aR_b\underline{\underline{R}}_3$	$P_1\underline{\underline{R}}_3R_cR_aR_b$	$R_cP_1\underline{\underline{R}}_3R_aR_b$	$R_cR_aP_1\underline{\underline{R}}_3R_b$	$R_cR_aR_bP_1\underline{\underline{R}}_3$
	$P_1R_c\underline{\underline{R}}_3R_aR_b$	$P_1R_cR_a\underline{\underline{R}}_3R_b$	$R_cP_1R_a\underline{\underline{R}}_3R_b$	$R_cP_1R_aR_b\underline{\underline{R}}_3$	$R_cR_aP_1R_b\underline{\underline{R}}_3$
3R2T	$\underline{\underline{R}}_3P_1R_cR_aR_b$	$R_c\underline{\underline{R}}_3P_1R_aR_b$	$R_cR_a\underline{\underline{R}}_3P_1R_b$	$R_cR_aR_b\underline{\underline{R}}_3P_1$	$\underline{\underline{R}}_3R_cP_1R_aR_b$
	$\underline{\underline{R}}_3R_cR_aP_1R_b$	$\underline{\underline{R}}_3R_cR_aR_bP_1$	$R_c\underline{\underline{R}}_3R_aP_1R_b$	$R_c\underline{\underline{R}}_3R_aR_bP_1$	$R_cR_a\underline{\underline{R}}_3R_bP_1$
	$P_1\underline{\underline{R}}_c\underline{\underline{R}}_cR_aR_b$	$\underline{\underline{R}}_cP_1\underline{\underline{R}}_cR_aR_b$	$\underline{\underline{R}}_c\underline{\underline{R}}_cP_1R_aR_b$	$R_cP_1\underline{\underline{R}}_a\underline{\underline{R}}_aR_b$	$R_c\underline{\underline{R}}_aP_1\underline{\underline{R}}_aR_b$
	$R_c\underline{\underline{R}}_a\underline{\underline{R}}_aP_1R_b$	$R_cR_aP_1\underline{\underline{R}}_b\underline{\underline{R}}_b$	$R_cR_a\underline{\underline{R}}_bP_1\underline{\underline{R}}_b$	$R_cR_a\underline{\underline{R}}_b\underline{\underline{R}}_bP_1$	
	$\underline{\underline{R}}_1R_cR_aR_b\underline{\underline{R}}_3$	$\underline{\underline{R}}_1\underline{\underline{R}}_3R_cR_aR_b$	$R_c\underline{\underline{R}}_1\underline{\underline{R}}_3R_aR_b$	$R_cR_a\underline{\underline{R}}_1\underline{\underline{R}}_3R_b$	$R_cR_aR_b\underline{\underline{R}}_1\underline{\underline{R}}_3$
	$\underline{\underline{R}}_1R_c\underline{\underline{R}}_3R_aR_b$	$\underline{\underline{R}}_1R_cR_a\underline{\underline{R}}_3R_b$	$R_c\underline{\underline{R}}_1R_a\underline{\underline{R}}_3R_b$	$R_c\underline{\underline{R}}_1R_aR_b\underline{\underline{R}}_3$	$R_cR_a\underline{\underline{R}}_1R_b\underline{\underline{R}}_3$
	$\underline{\underline{R}}_c\underline{\underline{R}}_c\underline{\underline{R}}_cR_aR_b$	$R_c\underline{\underline{R}}_a\underline{\underline{R}}_a\underline{\underline{R}}_aR_b$	$R_cR_a\underline{\underline{R}}_b\underline{\underline{R}}_b\underline{\underline{R}}_b$		

TABLE 6. Typical topologies of hybrid mechanism.

Type	Serial mechanism	Parallel mechanism	Hybrid mechanism
Type 1		3-RSR	$R_aR_cR_cR_aR_a$+3-RSR
Type 2	$R_aR_cR_cR_aR_a$	UP-UPR-UPS	$R_aR_cR_cR_aR_a$+UP-UPR-UPS
Type 3		SP-SPR-UPS	$R_aR_cR_cR_aR_a$+SP-SPR-UPS

not equivalent to Eq. (53). The $P_1R_cR_a\underline{\underline{R}}_3R_b$ limb is a feasible limb, but it is not equivalent to $P_1R_cR_aR_b\underline{\underline{R}}_3$. Similarly, the other 18 limbs are feasible limbs that are not equivalent to $P_1R_cR_aR_b\underline{\underline{R}}_3$.

Finally, the position of the $\underline{\underline{R}}$ joint in $\underline{\underline{R}}_1R_cR_aR_b\underline{\underline{R}}_3$ limb is transformed to obtain 9 types of limbs $\underline{\underline{R}}_1\underline{\underline{R}}_3R_cR_aR_b$, $R_c\underline{\underline{R}}_1\underline{\underline{R}}_3R_aR_b$, $R_cR_a\underline{\underline{R}}_1\underline{\underline{R}}_3R_b$, $R_cR_aR_b\underline{\underline{R}}_1\underline{\underline{R}}_3$, $\underline{\underline{R}}_1R_c\underline{\underline{R}}_3R_aR_b$, $\underline{\underline{R}}_1R_cR_a\underline{\underline{R}}_3R_b$, $R_c\underline{\underline{R}}_1R_a\underline{\underline{R}}_3R_b$, $R_c\underline{\underline{R}}_1R_aR_b\underline{\underline{R}}_3$ and $R_cR_a\underline{\underline{R}}_1R_b\underline{\underline{R}}_3$. Where, $\underline{\underline{R}}_1R_cR_a\underline{\underline{R}}_3R_b$ limb can be expressed as

$$\begin{aligned}
&\{\boldsymbol{S}'_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}} \\
&= 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_3}{2}\begin{pmatrix}\boldsymbol{s}_3\\ \boldsymbol{r}_3\times\boldsymbol{s}_3\end{pmatrix} \\
&\quad \Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix} \\
&\quad \Delta 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_c\times\boldsymbol{s}_c\end{pmatrix} \\
&\quad \Delta 2\tan\frac{\theta_1}{2}\begin{pmatrix}\boldsymbol{s}_1\\ \boldsymbol{r}_1\times\boldsymbol{s}_1\end{pmatrix}
\end{aligned} \tag{57}$$

According to the properties of screw triangular product, $\{\boldsymbol{S}_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}} \subseteq \{\boldsymbol{S}'_{f,\mathrm{3R2T}}\}_{\mathrm{parallel,1T1R}}$ is valid, but it is not equivalent to Eq. (54). The $\underline{\underline{R}}_1R_cR_a\underline{\underline{R}}_3R_b$ limb is a feasible limb, but it is not equivalent to $\underline{\underline{R}}_1R_cR_aR_b\underline{\underline{R}}_3$. Similarly, the other 18 limbs are feasible limbs that are not equivalent to $\underline{\underline{R}}_1R_cR_aR_b\underline{\underline{R}}_3$.

In summary, the 5-DOF limb synthesis of the parallel mechanism can be obtained, as shown in Table 5.

So far, 21 types of 4-DOF limbs and 75 types of 5-DOF limbs have been synthesized. When using these limbs to assemble parallel mechanism, it is necessary to consider the assembly relationship between the limb motions. According to the design principles defined in Section IV, the parallel mechanism is a 3-DOF mechanism (two rotations and one translation), that is, it should contain three limbs to satisfy the requirements of DOF. Therefore, assembly conditions can be defined based on the cooperative relationship between limb motions. Three limbs in the parallel mechanism need to satisfy the following conditions

$$\begin{aligned}
&\boldsymbol{S}_{f,\mathrm{parallel}} \\
&= \boldsymbol{S}_{f,1}\cap\boldsymbol{S}_{f,2}\cap\boldsymbol{S}_{f,3} \\
&= t_3\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_b\times\boldsymbol{s}_b\end{pmatrix}\Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_a\times\boldsymbol{s}_a\end{pmatrix}
\end{aligned} \tag{58}$$

$$\begin{cases}\boldsymbol{s}_{1,a}=\boldsymbol{s}_{2,a}=\boldsymbol{s}_{3,a}\\ \boldsymbol{s}_{1,b}=\boldsymbol{s}_{2,b}=\boldsymbol{s}_{3,b}\end{cases} \tag{59}$$

Considering that the mechanism is applied in a space environment, select three 3R2T limbs with identical limbs according to assembly conditions, and arrange them symmetrically, satisfying eq. (58) to eq. (59). Assemble a 3-RSR parallel mechanism with fixed and moving platform that satisfies practicalengineering requirements, and its topology can be expressed as follows

$$\begin{aligned} S_{f,i} = & 2\tan\frac{\theta_{i,b}}{2}\begin{pmatrix} s_{i,b} \\ r_{i,b}\times s_{i,b}\end{pmatrix}\Delta 2\tan\frac{\theta_{i,a}}{2}\begin{pmatrix} s_{i,a} \\ r_{i,a}\times s_{i,a}\end{pmatrix} \\ & \Delta 2\tan\frac{\theta_{i,c}}{2}\begin{pmatrix} s_{i,c} \\ r_{i,c}\times s_{i,c}\end{pmatrix}\Delta 2\tan\frac{\theta_{i,b}}{2}\begin{pmatrix} s_{i,b} \\ r_{i,b}\times s_{i,b}\end{pmatrix} \\ & \Delta 2\tan\frac{\theta_{i,b}}{2}\begin{pmatrix} s_{i,b} \\ r_{i,b}\times s_{i,b}\end{pmatrix} i = 1, 2, 3 \end{aligned} \quad (60)$$

$$\begin{aligned} S_{f,\text{parallel}} = & S_{f,1}\cap S_{f,2}\cap S_{f,3} \\ = & t_3\begin{pmatrix} 0 \\ s_3\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix} s_b \\ r_b\times s_b\end{pmatrix} \\ & \Delta 2\tan\frac{\theta_a}{2}\begin{pmatrix} s_a \\ r_a\times s_a\end{pmatrix} \end{aligned} \quad (61)$$

As shown in Fig. 9, according to the design principles defined in Section IV, the hybrid mechanism topology designed in this paper takes serial mechanism as the base and parallel mechanism installed at the end. Finally, the following three typical topologies of hybrid mechanism are obtained, as shown in Table 6.

The hybrid mechanism can complete all the tasks in space control, combine the advantages of serial and parallel mechanisms, and has the characteristics of large workspace, high precision, and high stability. According to the calculation rule of finite and instantaneous screw theory in Section II, the topology of $R_aR_cR_cR_aR_a$+3-RSR mechanism can be obtained

$$\begin{aligned} S_f = & S_{f,\text{serial}}\Delta S_{f,\text{parallel}} \\ = & 2\tan\frac{\theta_c}{2}\begin{pmatrix} s_c \\ r_c\times s_c\end{pmatrix}\Delta 2\tan\frac{\theta_b}{2}\begin{pmatrix} s_b \\ r_b\times s_b\end{pmatrix} \\ & \Delta 2\tan\tfrac{\theta_a}{2}\begin{pmatrix} s_a \\ r_a\times s_a\end{pmatrix}\Delta t_3\begin{pmatrix} 0 \\ s_3\end{pmatrix}\Delta t_2\begin{pmatrix} 0 \\ s_2\end{pmatrix}\Delta t_1\begin{pmatrix} 0 \\ s_1\end{pmatrix} \end{aligned} \quad (62)$$

VI. CONCLUSION

To improve the on-orbit service technology in the field of aerospace in China, the parametric research system for the topology synthesis of hybrid space control mechanism is proposed in this paper, which is of great significance for promoting the basic theoretical research of mechanism science and accelerating the independent innovation and engineering application of space control mechanism.

(1) The continuous motion of capture, detumbling, screw and plugging is described parametrically, and the simplest mathematical expression of the continuous motion of space control task is formed through the intersection algorithm.

(2) Based on the requirements of space control tasks, the topology design principles of serial and parallel mechanism are proposed, and the joint arrangements of serial and parallel mechanism are determined.

(3) The standard limb of the mechanism is described parametrically based on the expected motion pattern, and various derivative limbs are generated by joint equivalent transformation. Through the limb cooperative relationship, novel available topologies satisfying the conditions are synthesized.

(4) According to the analytical expressions of the serial and parallel mechanism satisfying the design principles, the analytical expression of the hybrid space control mechanism is deduced by using screw triangle product.

REFERENCES

[1] Y. Y. Wang, C. W. Hu, Z. X. Tang, and S. Gao, "Key technologies development of the space station manipulator system," *Spacecraft Eng.*, vol. 31, no. 6, pp. 147–155, 2022.
[2] J. T. Li, H. B. Xu, and X. Y. Zhai, "Research on the architecture of in-orbit robot service system based on space station platform," *Robot Technique Appl.*, vol. 200, no. 2, pp. 45–48, 2021.
[3] Q.-X. Jia, P. Ye, H.-X. Sun, and J.-Z. Song, "Kinematics of a trinal-branch space robotic manipulator with redundancy," *Chin. J. Aeronaut.*, vol. 18, no. 4, pp. 378–384, Nov. 2005.
[4] H. Liu, D. Y. Liu, and Z. N. Jiang, "Space manipulator technology: Review and prospect," *Acta Aeronauticaet Astronautica Sinica*, vol. 42, no. 1, pp. 33–46, 2021.
[5] J. Peng, H. Wu, T. Liu, and Y. Han, "Workspace, stiffness analysis and design optimization of coupled active-passive multilink cable-driven space robots for on-orbit services," *Chin. J. Aeronaut.*, vol. 36, no. 2, pp. 402–416, Feb. 2023.
[6] C. D. Zeng, H. P. Ai, and L. Chen, "Force/pose impedance control for space manipulator orbit insertion and extraction," *CJME*, vol. 58, no. 3, pp. 84–94, 2022.
[7] M. H. Wang and M. X. Wang, "Dynamic modeling and performance evaluation of a new five-degree-of-freedom hybrid robot," *CJME*, vol. 59, no. 59, pp. 1–13, 2023.
[8] S. L. Li, Y. W. Wang, and Z. Wu, "Design of a four-degree-of-freedom concrete spraying device with tandem and hybrid," *Sci. Technol. Innov.*, vol. 209, no. 17, pp. 63–65, 2022.
[9] Y. B. Li, K. Chen, and P. Sun, "Kinematic analysis of a hybrid humanoid mechanical leg based on screw theory," *Chin. High Technol. Lett.*, vol. 32, no. 5, pp. 511–520, 2022.
[10] W. B. Chen, Y. Yue, and G. Meng, "Analytical kinematics modeling and workspace simulation of a 5-DOF hybrid robot," *Mach. Des. Res.*, vol. 38, no. 2, pp. 82–87, 2022.
[11] H. P. Quan, G. C. Lin, and J. Y. Huang, "Kinematic analysis of 3-PRS & 3P hybrid mechanism," *Mach. Tool Hydraul.*, vol. 50, no. 4, pp. 36–40, 2022.
[12] C. S. Ma, J. Y. Zhang, and X. Q. Yin, "Kinematic analysis of new series-parallel mechanism," *Packag. Eng.*, vol. 42, no. 13, pp. 241–245, 2021.
[13] L. D. Zhou, Y. W. Sun, and Z. C. Meng, "The influence of complex space environment on spacecraft," *AST*, no. 7, pp. 65–68 and 76, 2017.
[14] Q. Li and J. M. Herve, "Type synthesis of 3-DOF RPR-equivalent parallel mechanisms," *IEEE Trans. Robot.*, vol. 30, no. 6, pp. 1333–1343, Dec. 2014.
[15] Y. Qi, T. Sun, Y. Song, and Y. Jin, "Topology synthesis of three-legged spherical parallel manipulators employing lie group theory," *Proc. Inst. Mech. Eng., C, J. Mech. Eng. Sci.*, vol. 229, no. 10, pp. 1873–1886, Jul. 2015.
[16] T.-L. Yang, A.-X. Liu, Q. Jin, Y.-F. Luo, H.-P. Shen, and L.-B. Hang, "Position and orientation characteristic equation for topological design of robot mechanisms," *J. Mech. Des.*, vol. 131, no. 2, pp. 021001-1–021001-17, Feb. 2009.
[17] T.-L. Yang, A.-X. Liu, H.-P. Shen, Y.-F. Luo, L.-B. Hang, and Z.-X. Shi, "On the correctness and strictness of the position and orientation characteristic equation for topological structure design of robot mechanisms," *J. Mech. Robot.*, vol. 5, no. 2, pp. 021009-1–021009-18, May 2013.
[18] Y. Q. Wu, H. Wang, and Z. X. Li, "Quotient kinematics machines: Concept, analysis and synthesis," *J. Mech. Robot.*, vol. 3, no. 4, pp. 041004-1–041004-11, 2011.
[19] Z. B. Li, Y. J. Lou, and Z. X. Li, "Type synthesis and kinematic analysis of a new class Schonflies motion parallel manipulator," in *Proc. IEEE Int. Conf. Inf. Automat.*, Shenzhen, China, Jun. 2011, pp. 267–272.
[20] Y. M. Song, Z. Guo, and P. F. Wang, "Type synthesis of hybrid compliant mechanisms for peg-in-hole assembly," *J. Tianjin Univ., Sci. Technol.*, vol. 53, no. 6, pp. 582–592, 2020.

[21] H. L. Gui, B. K. Zhu, and Y. Cao, "Type synthesis of three-rotational hybrid mechanisms based on fractal theory," *Mach. Des. Res.*, vol. 34, no. 1, pp. 60–64, 2018.

[22] S. Ge, Y. Cao, R. Zhou, J. Zhu, and Z. Ding, "Type synthesis of three-translational hybrid mechanisms," *China Mech. Eng.*, vol. 28, no. 3, pp. 258–266, 2017.

[23] T. Sun, S. F. Yang, and B. B. Lian, *Finite and Instantaneous Screw Theory in Robotic Mechanism*. Singapore: Springer, 2020.

[24] T. Sun, S. Yang, T. Huang, and J. S. Dai, "A way of relating instantaneous and finite screws based on the screw triangle product," *Mechanism Mach. Theory*, vol. 108, pp. 75–82, Feb. 2017.

[25] S. Yang, T. Sun, T. Huang, Q. Li, and D. Gu, "A finite screw approach to type synthesis of three-DOF translational parallel mechanisms," *Mechanism Mach. Theory*, vol. 104, pp. 405–419, Oct. 2016.

[26] T. Sun, S.-F. Yang, T. Huang, and J. S. Dai, "A finite and instantaneous screw based approach for topology design and kinematic analysis of 5-axis parallel kinematic machines," *Chin. J. Mech. Eng.*, vol. 31, no. 1, pp. 31–44, Dec. 2018.

[27] B. Lian, T. Sun, Y. Song, Y. Jin, and M. Price, "Stiffness analysis and experiment of a novel 5-DoF parallel kinematic machine considering gravitational effects," *Int. J. Mach. Tools Manuf.*, vol. 95, pp. 82–96, Aug. 2015.

[28] D. Liang, Y. Song, T. Sun, and G. Dong, "Optimum design of a novel redundantly actuated parallel manipulator with multiple actuation modes for high kinematic and dynamic performance," *Nonlinear Dyn.*, vol. 83, nos. 1–2, pp. 631–658, Jan. 2016.

[29] T. Sun, B. Lian, Y. Song, and L. Feng, "Elastodynamic optimization of a 5-DoF parallel kinematic machine considering parameter uncertainty," *IEEE/ASME Trans. Mechatronics*, vol. 24, no. 1, pp. 315–325, Feb. 2019.

[30] T. Sun and B. Lian, "Stiffness and mass optimization of parallel kinematic machine," *Mechanism Mach. Theory*, vol. 120, pp. 73–88, Feb. 2018.

[31] Y. Song, Y. Qi, G. Dong, and T. Sun, "Type synthesis of 2-DoF rotational parallel mechanisms actuating the inter-satellite link antenna," *Chin. J. Aeronaut.*, vol. 29, no. 6, pp. 1795–1805, Dec. 2016.

LU ZHANG was born in Shaanxi, China, in 1999. She received the bachelor's degree in mechanical engineering from the Tianjin University of Technology and Education, in 2021, where she is currently pursuing the master's degree in mechanical engineering. She is the author of one SCI paper.

YANG QI was born in Tianjin, China, in 1988. He received the bachelor's degree in mechanical design manufacturing and automation and the M.S. and Ph.D. degrees in mechanical engineering from Tianjin University, in 2011 and 2018, respectively.

From 2018 to 2022, he was a Lecturer with the College of Mechanical Engineering, Tianjin University of Technology and Education. He is the author of 12 patents of innovation and five SCI papers. His research interests include configuration synthesis of parallel mechanisms with complex motion types, integrated design of scale parameters and section parameters for multi performance requirements, flight dynamics in microgravity environment, and the prototype construction of parallel mechanisms.