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RESEARCH ARTICLE

Generalized Defensive Modeling of Malware Propagation in WSNs Using Atangana–Baleanu–Caputo (ABC) Fractional Derivative

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ABSTRACT The malware spreading in Wireless Sensor Network (WSN) has lately attracted the attention of many researchers as a hot problem in nonlinear systems. WSN is a collection of sensor nodes that communicate with each other wirelessly. These nodes are linked in a decentralised and distributed structure, allowing for efficient data collection and communication. Due to their decentralised architecture and limited resources, WSN is vulnerable to security risks, including malware attacks. Malware can attack sensor nodes, causing them to malfunction and consume more energy. These attacks can spread from one infected node to others in the network, making it essential to protect WSN against malware attacks. In this paper, we focus on the analysis of a novel fractional epidemiology model, specifically the fractional order SEIVR epidemic model in the sense of Caputo's fractional derivative of order $0< \alpha \leq 1$ with the goal of examining the efficacy of vaccination strategies and the heterogeneity of a scale-free network on epidemic spreading. First, using the next-generation technique and obtain the basic reproduction number of the proposed epidemic model, which is essential for determining both the locally asymptotically stable equilibrium point of the worm-free system and the unique existence of the endemic equilibrium point. To numerically solve the model, the Adam-Bashforth-Moulton predictor-corrector (ABM) method is applied. The fractional calculus enables us to deal directly with the "memory effect" of numerous phenomena, taking into account the system's dependence on previous stages. This method provides the results of a complex system. Additionally, research demonstrates that vaccine treatments are quite effective at preventing the spread of malware. The outcome of the study reveals that the applied ABM predictor-corrector method is computationally strong and effective to analyse fractional order dynamical systems in the SEIVR epidemic model for malware propagation in WSN. The results show that the order of the fractional derivative has a significant effect on the dynamic process. Also, from the result, it is obvious that the memory effect is zero for $\alpha = 1$. When the fractional order α is decreased from 1, the memory effect appears, and its dynamics vary according to the time. This memory effect points out the difference between derivatives of fractional and integer orders. The theorems and their proofs are presented to validate the validity of the proposed model. To validate the proposed model, extensive theoretical study and computational analysis have also been applied.

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INDEX TERMS Basic reproduction number, stability analysis, fractional order derivative.

I. INTRODUCTION

Wireless Sensor Network (WSN) is a type of network where a large number of small sensor nodes are deployed in a specific area to collect environmental data and transmit it to a central node or sink. The applications of WSN are very wide including defense, agriculture, healthcare, defense, mobility, commerce, industry, personal use and more [1], [2]. One of the major challenges faced by WSN is the limited resources available to sensor nodes, including processing power, energy source, memory, and coverage area. As a result, WSN needs to be designed in a way that optimizes these resources and maximizes the network's lifetime.

Another challenge faced by WSNs is security. The sensor nodes are vulnerable to malware attacks, and the data transmitted by the nodes can be intercepted or manipulated. This can lead to a range of problems, including privacy breaches, data tampering, and disruption of network operations.

To address these challenges, researchers have developed various techniques and protocols for optimizing resource usage, improving network security, and extending the lifetime of WSN. These include energy-efficient routing algorithms, data aggregation techniques, and secure communication protocols.

WSNs also have a role to play in the emerging fields of Internet of Things (IoT) and the Internet of Vehicles (IoV) [3], where sensor nodes can be used to collect data from a wide range of sources [4], [5], including vehicles, wearable devices, and smart home appliances. However, the challenges faced by WSNs in these contexts are even greater, as the scale of the networks is much larger, and the data collected is more diverse and complex. Overall, WSNs have enormous potential in a range of applications, but their success depends on addressing the operational challenges faced by these networks such as limited energy, security especially malware attack, and the placement of nodes [6], [7] etc.

One of the most important issues is the malware attacks on WSN. Malware that is created purposely and deliberately installed does not normally enter into a network without human interaction, malware may spread across WSNs since it can be autonomously activated and do so by self-transmission [8]. Wireless device-targeting malicious malware has already been discovered. Cabir is a such type of virus that can spread via the air interface, and is seriously damage the wireless devices.

Researchers have developed various security mechanisms to protect WSNs against malware attacks [9], [10], [11], [12], [13], [14].These include secure routing protocols, intrusion detection systems, and data encryption techniques. However, the nature of WSNs makes it challenging to implement these security mechanisms effectively, given the limited resources available to the sensor nodes.

To address this challenge, researchers have turned to mathematical modelling to better understand the dynamics of malware propagation in WSNs. Epidemiological models, which are commonly used to study the spread of diseases in populations, can be adapted to study the spread of malware in WSNs. By developing mathematical models of propagation, researchers can better understand and monitor the patterns of malware transfer from one node to the next. The analysis of disease dynamics continues to be a hot topic among scientists [15], [16], [17]. The transmission of malware in WSN is similar to the spread of disease in populations. In both cases, the spread of the contagion depends on various factors such as the network topology, connectivity, and transmission rates. The various epidemiological models have been proposed by researchers to investigate that stability and performance of WSN [18], [19], [20], [21], [22], [23].

The ordinary differential equation (ODE) is used to describe the various physical phenomena of the model. The ODE helps in the formulation of the models and analyses the dynamics of malware spreading in the WSN. But classical calculus is not able to describe the exact complex phenomena of the system. To describe the exact complex phenomena of the system, use fractional calculus (FC). Fractional calculus extends the concepts of differentiation and integration to non-integer orders. Fractional differential equations (FDEs) have been shown to model many physical phenomena more accurately than ODEs, particularly in systems that exhibit memory effects, or long-range dependence. One of the key advantages of FDEs is that they provide a more accurate description of the behaviour of complex systems than ODEs, which assume instantaneous and proportional responses. FDEs can capture the dynamics of systems that have a complex history or memory, where past events can influence the system's behaviour [24].

To replicate practical problems, fractional derivatives like Caputo, Grünwald Letnikov, Riemann-Liouville, Jumarie etc. have gained popularity among scholars. These derivatives' theoretical underpinnings have advanced greatly over time [25], [26]. Some applications of the FC have been explained by the researchers [27], [28], [29].

In the case of dynamical systems, the incorporation of the fractional derivative is significant since the definition includes integration, and as a result, the function stores information regarding the past memory. The study by Bolton et al. [30] shown the superiority of fractional-order models versus integer-order models in terms of their applicability to a given data set for analysis. Many researchers [31], [32], [33], [34] use applications of these derivatives in their work to comprehend many phenomena in mathematical biology and their multidisciplinary disciplines. In this context, different researchers have also begun investigating the classical order epidemiological epidemic models by integrating a wide variety of fractional derivatives [35], [36], [37].

In spite of all above said applications, both the derivative in the Riemann-Liouville sense and the derivative in the Caputo sense have some drawbacks, as has been noted. (1) There was no nonlocality in the kernel. (2) The integral associate is the average of the function and its integral, not a fractional operator. Some academics draw the conclusion that the operator was a fractional parameter filter and not a derivative with fractional order. Consequently, the fractional parameter may be seen as a filter regulator. The fact that their kernels are solitary, although being nonlocal, poses a significant challenge for the well-known Caputo and Riemann-Liouville orders. When simulating situations in the actual world, this flaw has an impact

Recently, new forms of nonlocal fractional derivatives have been proposed in the literature to handle the power-law reduction of derivative operators. The Caputo-Fabrizio derivative with fractional order was the new operator presented by Caputo and Fabrizio [38]. Numerous researchers successfully used their derivative in a small number of real-world issues as a result of the originality of their findings [39], [40]. Their operator is new in that the derivative, which has applicability in several groundwater and thermal science issues, lacks a single kernel. In addition to the innovative idea's real-world implementations, several theoretical papers were also presented.

Atangana and Baleanu proposed a novel operator based on the Mittag-Leffler function with fractional order to address the aforementioned issues [41]. In addition to using a nonlocal kernel, its operators have all the advantages of Caputo and Fabrizio. All the advantages of the Riemann-Liouville and Caputo operators are present in the operators, and the kernel is non-singular. It's interesting to note that their fractional integral is the fractional average of the supplied function's Riemann-Liouville fractional integral and the function itself. Along with the aforementioned advantages, the derivative was discovered to be quite helpful in thermal science and material sciences [41], [42]. These novel derivatives with fractional orders are both fractional derivatives and filters. The Atangana–Baleanu–Caputo (ABC) fractional derivative provides an accurate description of the memory [43]. The important applications of the ABC operator can be found in [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], and [58]. Recently, it has been observed that fractional differential equations can be employed to modeling phenomena of worldwide more accurately. The global problem of the spread of the disease attracted the attention of researchers from various fields, which led to the emergence of a number of proposals to analyze and anticipate the development of the epidemic [53].

To understand the attacking and spreading dynamics of malware in WSN, Author [59] proposed a compartmental epidemic model to examined the projected non-linear structure of the model by using generalized Adams-Bashforth-Moultan method.

The Adams-Bashforth has been recognized as a great and powerful numerical method able to provide a numerical solution closer to the exact solution [60]. The correct version of the fractional Adams-Bashforth [61] methods which take into account the nonlinearity of the kernels including Mittag-Leffler law for the Atangana-Baleanu Differential operator, which is used here to model WSN.

The vaccination technique is not entirely effective because vaccinations may only provide partial protection from infection after a particular period of time. To address this problem, we propose a fractional epidemic (SEIRV) model with a factor $\rho \in [0, 1]$ in the contact rate of vaccinated nodes, where $\rho = 0$ means perfect efficacy of vaccination and $\rho = 1$ means complete inefficacy of vaccination. The FDEs will help in the exact analysis of vaccination effect on WSN.

The key goal of proposed models is to develop method to control the malware transmission as well as to enhance WSN lifetime. The model describes the transmission of malware dynamics in WSN, the contributions of proposed model are:

1. The proposed model studies the transmission dynamics of worms in WSN. The model suggests the mechanism against malware attack.

2. For deterrence of malware transmission and consumption of sensor node's energy in WSN, utilize the concept of Exposed state.

3. To include the idea of a vaccination strategy in the epidemic model. This idea is used for the maintenance of network operations. The vaccination method is used to increase network lifetime, suppress malicious activity, and improve WSN security.

4. To analyses the system's response to a malware assault and look into quick WSN recovery techniques under steady state conditions.

5. To investigate the stability of the system under various conditions and to validate the analytical analysis through the results of simulations.

The following order determines how the remaining portion of the paper is structured. Numerical analysis and graphical outcome of the study is presented in the section of results and discussion. In Section II presented related work. Section III presents basic definitions and results. Section IV provides details on the description of the proposed model and the analysis's assumptions. The presence of non-negative solutions and their uniqueness are covered in Section V. Section VI focuses on equilibrium points and their stability analysis. The numerical solution and its analysis are employed in Section VII. Section VIII contains the findings and a discussion of the simulation results for the proposed models. The conclusion and future research are discussed in section IX.

II. RELATED WORK

This section presents a comprehensive review of wireless sensor networks. The number of researchers who have studies spreading of malware in WSN apply the concept of epidemic modeling.

Wu et al. [62] considered a STSIR model to analyse the virus spread among the devices with consideration of variable number of devices. They applied the idea of game theory to curb the virus spread among the devices. Zhang et al. [63] proposed a epidemic theory based model to supress the spreading of malware in WSNs. They consider heterogeneity in the sensor network as sensor nodes as well as sink nodes. They analysed the effect of malware attack on different types of topologies. They suggest the mechanism of the design, distribution, and maintenance of the sensor network. Further, Ye et al. [64] proposed a SIR1R2 model that discussed about the mechanism of secondary immunity. To analyse the spreading behaviour of virus in WSN computed the value of basic reproduction number (R_0) and found that if $R_0 < 1$ the malware will no longer survive in WSN, whereas if $R_0 > 1$ malware will spread in the entire WSN. They studied the impact of secondary immunity on propagation of malware in WSN.

Shen et al. [65] proposed VCQPS model that uses to defend the WSN against the attack of malware. They considered both heterogeneous types of sensor nodes and mobile nodes in the investigation of malware propagation in WSN. To study the dynamics of malware propagation in WSN the differential equations is derived. The changes among the transition states presented by the differential equations. The existence of stationary points of the model is discussed. They compare the proposed model with the traditional models based on epidemic theory and suggest the method to defend WSN against malware attack. Zhou et al. [66] used game theory to analyse the propagation dynamics of malware in WSN. The Nash equilibrium strategy is applied to investigate the propagation behaviour of malware in WSN. The relationship established between ratio of infection steady-state with parameters of the game and discussed the existence of malwares in WSN. They also used cellular automaton and simulate the process of malwares' propagation in WSN and verify the theoretical model correctness with simulation results.

Zhang et al. [67] proposed a MDBCA model which describes the diffusion of malware through use of cellular automata. The set of differential equation deduced which explains the dynamics of various states of the model. The points of equilibria of the model are obtained which determine the threshold value whether malware will diffuse or die out in WSN. They also compute the value of basic reproduction number and investigate the equilibrium points' stability through use of next-generation matrix. The simulation has been performed to validate the effectiveness of the model. A SNIRD model is suggested by Shen et al. [68] to supress spreading of malware in WSN. The suggested model considered the connectivity with heterogeneity of sensor nodes. The model also discussed the hiding characteristics of malware and dysfunctional types of sensor nodes. The existence of equilibrium for the suggested model has been verified, and the threshold value is computed which determines the status of malware in the network, the computed value determine that whether malware will fade out or spread. The SNIRD model compared with the conventional SIS and SIR and demonstrates the effectiveness over SIS and SIR models. Further, for Heterogeneous wireless sensor networks (HWSNs) a HSIRD model is proposed by Shen et al. [69] to analyse the malware propagation dynamics. The model categorise the sensor nodes are of different types. They considered that the sensor nodes may be damage due to attack of malware or physical attack and in both cases sensor nodes will loss the functionality. The connectivity and heterogeneity are considered in the analysis of the malware spread in the system. The points of existence of equilibria of the HSIRD model is discussed. The value of the basic reproduction number is computed which govern the equilibrium points stability. The condition of malware or die out or diffuse is obtained. The model is validated through simulation results. The HSIRD model compared with the traditional SIR and SIS models and showed efficiency over them.

However, there are still some WSN malware spreading issues need to be addressed. The SEIVR model and fractional calculus approach can be used to address the challenges of malware propagation in WSNs.

III. BASIC DEFINITION AND RESULTS

First of all, we recall some basic definitions and results of the fractional calculus.

Definition 1: Let f be a function defined on [a, b] and $\alpha > 0$. The Riemann – Liouville fractional integral of order $\alpha > 0$ for the function f is defined by

$$ {}^{RL}_{a}D_x^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-s)^{\alpha-1} f(s)ds, \quad x \in [a,b]. $$

Provided the right – hand side is point wise defined on [a, b]. where $\Gamma(.)$ is the gamma function. For $\alpha = 0$, we get ${}_aD_x^0 = I$, the identity operator.

Definition 2: Let $\alpha \geq 0$, $n \in \mathbb{N}$ and $f \in C^n[a, b]$. The Caputo fractional derivatives of order α for the function f is defined by

$$ {}^{C}_{a}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_a^t \frac{f'(s)}{(t-s)^{\alpha}} ds, \quad t \in [a,b], \ 0 < \alpha \leq 1 $$

Definition 3: Let $\alpha > 0$, the function E_α defined by

$$ E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+1)}, \quad \alpha > 0, \ z \in \mathbb{C} $$

is called the Mittage – Leffler function of order α. The function $E_\alpha(z)$ is entire function. For the special cases when $\alpha = 1$, $E_1(z) = e^z$ and when $\alpha = 2$, $E_2(z) = \cosh(\sqrt{z})$. It's general form

$$ E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+\beta)}, \quad \alpha, \beta \in C, \quad z \in \mathbb{C} \ and \ R(\alpha) > 0, \ R(\beta) > 0 $$

Definition 4: Caputo-Fabrizio fractional order derivative

$$ {}^{CF}_{a}D_t^{\alpha} f(t) = \frac{B(\alpha)}{1-\alpha}\int_a^t f'(x)\exp\left(-\frac{\alpha(t-x)}{1-\alpha}\right)dx, $$
$$ t \in [a,b],\ 0 < \alpha \leq 1 $$

where $B(\alpha)$ is the normalization function such that $B(1) = B(0) = 1$.

Definition 5: Atangana–Baleanu fractional derivative in Caputo sense:

$$ {}^{ABC}_{a}D_t^{\alpha} f(t) = \frac{B(\alpha)}{1-\alpha}\int_a^t f'(x)\,\mathrm{E}_{\alpha}\left(-\frac{\alpha(t-x)^{\alpha}}{1-\alpha}\right)dx, $$
$$ t \in [a,b],\ 0 < \alpha \leq 1 $$

Lemma 1: . If f is continuous and $\alpha \geq 0$ then ${}^{c}_{a}D_t^{\alpha}\, {}_{a}D_t^{-\alpha} f(t) = f(t)$.

Lemma 2: If $\alpha \geq 0,\ r > 0,\ \theta \in [-\pi, \pi] \geq 0,\ r > 0,\ \theta \in [-\pi, \pi]$ and

$$ \lim_{t\to\infty} E_{\alpha}\left(-\lambda t^{\alpha}\right) = 0 \text{ for } |\theta| < \frac{\alpha\pi}{2}\lambda = re^{-i\theta} $$

Now, some result related to the stability of the fractional order systems. Let D be an open subset of $\mathbb{R}^n$. For $\alpha \in (0, 1]$, we consider the initial value problem (IVP) consisting of an autonomous fractional – order system.

$$ {}^{c}_{a}D_t^{\alpha} u(t) = h(u),\quad u(t_0) = u_0 \tag{2.1} $$

where $h : D \to \mathbb{R}^n$ is locally Lipschitz continuous in t.

Definition 6: . A point u^* is called an equilibrium of the system (2.1) if $h(u^*) = 0$.

Lemma 3: [36], [49] Let $J(u^*)$ denote the cobian matrix of the system evaluated at an equilibrium point u^*, and let $\lambda_j,\ j = 1,\ 2,\ 3,\ \ldots\ldots,\ n$ be the eigen values of $J(u^*)$. Then u^* is locally asymptotically stable if and only if $\left|arg\lambda_j\right| > \frac{\alpha\pi}{2},\ j = 1,\ 2,\ 3,\ \ldots\ldots,\ n.$

To prove stability result, we have following lemma:

Lemma 4: Assume that $\alpha \in (0, 1]$ and $g \in C([a, \infty), \mathbb{R}^+)$. For any $\geq a$, we have

${}^{c}_{a}D_t^{\alpha} g^2(t) \leq g(t)\, {}^{c}_{a}D_t^{\alpha} g(t)$ [[50], Lemma 1]

$$ {}^{c}_{a}D_t^{\alpha}[g(t) - g^* - g^*\ln\frac{g(t)}{g^*}] \leq (1 - \frac{g^*}{g(t)})\, {}^{c}_{a}D_t^{\alpha} g(t), $$

where $g^* \in \mathbb{R}^+$.

Let $H \in C^1(D, \mathbb{R})$ and $\alpha \in (0, 1]$, the α^{th} order Caputo derivatives of H(u) along the solution u(t) of the system ${}^{c}_{a}D_t^{\alpha} u(t) = h(u),\ t \in [a, \infty)$ is given by [65]

$$ {}^{c}_{a}D_t^{\alpha} H(u(t)) = {}_{a}D_t^{-(1-\alpha)}(\frac{dH}{du}\frac{du}{dt}) $$

Now, next is, fractional version of the well-known LaSalle's invariance Principle:

Lemma 5: [37] Assume that A is a bounded closed set in $\mathbb{R}^n$ and that every solution of ${}^{c}_{a}D_t^{\alpha} u(t) = h(u),\ t \in [a, \infty)$, starting from a point in A, remains in A for all time t. Assume, further, that $H \in C'(A, \mathbb{R})$ such that ${}^{c}_{a}D_t^{\alpha} H(u(t)) \leq 0$, where $u(t)$ is any solution of the system ${}^{c}_{a}D_t^{\alpha} u(t) = h(u)$. Let $E = \left\{u \in A : {}^{c}_{a}D_t^{\alpha} H = 0\right\}$, and let S be the largest invariant subset of E. Then every solution $u(t)$ of ${}^{c}_{a}D_t^{\alpha} u(t) = h(u), t \in [a, \infty)$; originating in A tends to S as $t \to \infty$. In particular, if $S = \{0\}$, $u(t) \to 0\ as\ t \to \infty$.

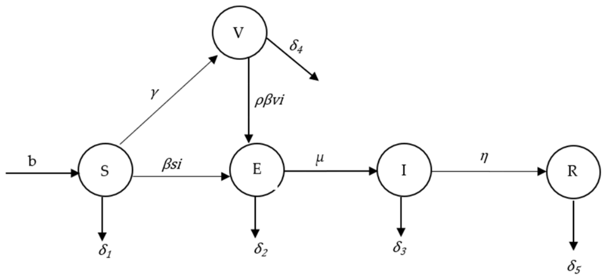

FIGURE 1. Transition state diagram of the model.

Theorem 1 [60], [61]: Let f be a continuous function on [a, b]. then, the following inequality is satisfied on a closed interval [a, b]:

$$ \left\|{}^{ABC}_{0}D_t^{\alpha}[f(t)]\right\| < \frac{M(\alpha)}{1-\alpha}B,\quad \|g(t)\| = \max_{a\leq t\leq b}|g(t)| $$

Theorem 2 [61]: The Atangana-Baleanu fractional derivative in Caputo sense satisfy the Lipschitz condition, for the given functions f and g, the following inequalities can be obtained:

$$ \left\|{}^{ABC}_{0}D_t^{\alpha}[f(t)] - {}^{ABC}_{0}D_t^{\alpha}[g(t)]\right\| < H\ \|f(t) - g(t)\| $$

Theorem 3 ([61] Condition for Stability): Let u(t) be a solution of ${}^{ABC}_{0}D_t^{\alpha} u(t) = f(t, u(t))$ with f being continuous and bounded;

if f satisfies a Lipschit condition, then the required condition for Adams – Bashforth method when applied to approximate the Atangana – Baleanu derivative of fractional order in Caputo sense is achieved if

$$ \|f(t_n,\ u_n) - f(t_{n-1},\ u_{n-1})\|_{\infty} \to 0\ as\ n \to \infty. $$

IV. MODEL DESCRIPTION AND ASSUMPTIONS MADE IN ITS ANALYSIS

The defense mechanism is required for protection of WSN against worm attack. For this purpose, fractional epidemic model SEIVR (Susceptible-Exposed-Infectious-Vaccinated-Recovered) is proposed. The model helps in the study of worm propagation dynamics in WSN, which formulation is described in Fig. 1. There are two types of infectious state of the sensor nodes are considered in the model. These states are: Exposed state sensor node and Infectious state sensor node. Worms transmit in WSN with useful data through the neighboring active mode sensor nodes.

The different states of the proposed models:

Susceptible State(S): The nodes that are not infected but are vulnerable to worms are referred to as susceptible states.

Exposed State(E): The exposed sate contains the infected but non-infectious nodes. This type of nodes does not spread infection to other nodes.

Infectious State(I): Infected nodes are those that have the potential to infect other nodes; these nodes belong to the infected state.

Recovered State(R): When a node that was previously infected is now free of infection, it is considered to be in a state of recovery.

Vaccinated State(V): Those nodes that are vaccinated are considered to be part of the vaccinated state.

For formulation of model the following assumptions has been made:

• Initially sensor nodes are of Susceptible state (S) and they are vulnerable against malware attack. When malware installs in susceptible sensor node then susceptible sensor node becomes exposed node. Malware executed partially in exposed node. The rate of conversion of susceptible node into exposed node is β. Due to normal operation some of susceptible sensor nodes become dead with rate δ_1.

• The abnormal behavior exhibitsby the exposed state (E) of sensor node in WSN. In exposed node malware has installed successfully but not executed completely. Therefore, to stop the further transmission of malware in WSN apply a corrective measure on exposed state of nodes in time. Otherwise, these nodes become infectious with rate μ. Some of the exposed class of sensor nodes become dead with rate δ_2 because of malware attack.

• Sensor node of WSN gets compromise with malware and begin to malicious activity in the network. Infected nodes start to spread malware with surrounding nodes and increase their own energy dissipation. At the same time, malicious programs may decide not to continue attacking the infected node, and these infected nodes are only infectious and not destructive. The corrective measure applies in time and install anti-malware successfully manner, the node will be safely converted to the recovered state at the rate η. As the degree of damage from malicious programs increases, the node will die faster. Some of the infectious sensor nodes become dead with rate δ_3 because of attack of worms.

• Recovered state of nodes in WSN not only immune to malware attack but they have also level of high-energy. Sensor nodes of the network turn into recovered state infectious state. Some of the recovered class of sensor nodes become dead with rate δ_5 .

• Susceptible sensor nodes are adding in the in the system is at the rate of b.

Total number of sensor nodes in WSN at any time t is divided into five states. The N(t) number of nodes in the system these names are as Susceptible State S(t); Vaccinated V (t); Exposed State E(t)); Infectious State I(t); Recovered State R(t).

Therefore, mathematically satisfy the following equation $N(t) = S(t) + E(t) + I(t) + V(t) + R(t)$, for any time $t \geqslant 0$.

TABLE 1. Description of used parameter.

S. No.	Used Parameters	Description
1	b	Susceptible recruitment rate
2	ϒ	Vaccination rate
3	ρ	Rate of effectiveness of vaccination
4	µ	Rate of exposed node become infectious
5	η	Recovery rate of infectious node
6	δ_1	Mortality rate of susceptible nodes
7	δ_2	Mortality rate of exposed nodes
8	δ_3	Mortality rate of infectious nodes
9	δ_4	Mortality rate of vaccinated nodes
10	δ_5	Mortality rate of recovered nodes

To analyses the transmission dynamics of malware and state of change of sensor node presented by the set of differential equation. The state transition equations are written as:

$$\begin{aligned} \frac{dS}{dt} &= b - \beta SI - (\delta_1 + \gamma)\, S \\ \frac{dE}{dt} &= \beta SI + \rho\beta VI - (\delta_2 + \mu)\, E \\ \frac{dI}{dt} &= \mu E - (\delta_3 + \eta)\, I \\ \frac{dV}{dt} &= \gamma S - \rho\beta VI - \delta_4 V \\ \frac{dR}{dt} &= \eta I - \delta_5 R \end{aligned} \tag{3.1}$$

To model a dimensionally consistent model, we assume that the initial value of the five compartments satisfy the initial conditions (ICs):

$$\begin{aligned} &\mathrm{S}(0) = S_0, \quad E(0) = E_0, \quad I(0) = I_0, \\ &V(0) = V_0, \quad R(0) = R_0, \end{aligned} \tag{3.2}$$

where S_0, E_0, I_0, V_0 and R_0 are nonnegative real numbers.

In corporate of memory effect, we wish to convert the above model (3.1) into fractional order problem. To achieve this, equivalent integral form of (3.1) are given by

$$\begin{aligned} S(t) &= S_0 + \int_0^t [b - \beta S(p)I(p) - (\delta_1 + \gamma)\, S(p)]\, dp \\ E(t) &= E_0 + \int_0^t [\beta S(p)I(p) + \rho\beta V(p)I(p) \\ &\quad - (\delta_2 + \mu)\, E(p)\ ]dp \\ I(t) &= I_0 + \int_0^t [\mu E(p) - (\delta_3 + \eta)\, I(p)\ ]dp \end{aligned}$$

$$V(t) = V_0 + \int_0^t [\gamma S(p) - \rho\beta V(p)I(p) - \delta_4 V(p)]\, dp$$
$$R(t) = R_0 + \int_0^t [\eta I(p) - \delta_5 R(p)]\, dp \tag{3.3a}$$

In order to include the influence of memory effect [66], we re-write (3.3a) in terms of time dependent integrals:

$$S(t) = S_0 + \int_0^t \varphi(t,p)\,[b - \beta S(p)I(p) - (\delta_1 + \gamma) S(p)]\, dp$$
$$\mathrm{E}(t) = E_0 + \int_0^t \varphi(t,p)\,[\beta S(p)I(p) + \rho\beta V(p)I(p) - (\delta_2 + \mu) E(p)]dp$$
$$I(t) = I_0 + \int_0^t \varphi(t,p)\,[\mu E(p) - (\delta_3 + \eta) I(p)]dp$$
$$V(t) = V_0 + \int_0^t \varphi(t,p)\,[\gamma S(p) - \rho\beta V(p)I(p) - \delta_4 V(p)]\, dp$$
$$R(t) = R_0 + \int_0^t \varphi(t,p)\,[\eta I(p) - \delta_5 R(p)]\, dp \tag{3.3b}$$

where $\varphi(t,p)$ plays the role of a time – dependent memory kernel as a delta function $\delta(t,p)$ in the classical Markov process. The proper choice of $\varphi(t,p)$ can be a power – law correlation function, to incorporate long – term memory effect, which contributes to the evolution of the system. Thus, we can assume the non-local and nonsingular kernel as:

$$\varphi(t,p) = \frac{B(\alpha)}{1-\alpha} \mathrm{E}_\alpha\left(-\frac{\alpha(t-x)^{\alpha-1}}{1-\alpha}\right), \qquad \alpha \in (0, 1].$$

Putting this choice of $\varphi(t,p)$ into equation (3.3b) and using definition 2.1, we get:

$$\begin{aligned}
S(t) - S_0 &= {}^{ABC}_{\ 0}D_t^{-\alpha}[b - \beta SI - (\delta_1 + \gamma) S] \\
E(t) - E_0 &= {}^{ABC}_{\ 0}D_t^{-\alpha}[\beta SI + \rho\beta VI - (\delta_2 + \mu) E] \\
I(t) - I_0 &= {}^{ABC}_{\ 0}D_t^{-\alpha}[\mu E - (\delta_3 + \eta) I] \\
V(t) - V_0 &= {}^{ABC}_{\ 0}D_t^{-\alpha}[\gamma S - \rho\beta VI - \delta_4 V] \\
R(t) - R_0 &= {}^{ABC}_{\ 0}D_t^{-\alpha}[\eta I - \delta_5 R]
\end{aligned} \tag{3.4}$$

Now, applying the Atangana–Baleanu fractional derivative of order α in Caputo sense on both sides of each equation in (3.4) and Lemma1, our fractional order epidemic model is given by

$$\begin{aligned}
{}^{ABC}_{\ 0}D_t^{\alpha}\mathrm{S}(t) &= b - \beta SI - (\delta_1 + \gamma) S \\
{}^{ABC}_{\ 0}D_t^{\alpha}E(t) &= \beta SI + \rho\beta VI - (\delta_2 + \mu) E \\
{}^{ABC}_{\ 0}D_t^{\alpha}I(t) &= \mu E - (\delta_3 + \eta) I \\
{}^{ABC}_{\ 0}D_t^{\alpha}V(t) &= \gamma S - \rho\beta VI - \delta_4 V \\
{}^{ABC}_{\ 0}D_t^{\alpha}R(t) &= \eta I - \delta_5 R
\end{aligned} \tag{3.5}$$

Both sides of the equations in (3.5) are assuming to be same in dimensions. Now, from here our full attention will be on this model (3.5) only. Fractional derivatives are very effective to describe memory effect in the modelling of dynamics of malware propagation in WSNs. The decay rate of the memory kernel $\varphi(t,p)$ depends on the value of α. A smaller value of α corresponds to more slowly decaying time – correlation functions. The fractional order α can be denoted as an index of memory of the given system, for detail concern [73].

V. EXISTENCE OF NON-NEGATIVE SOLUTION AND ITS UNIQUENESS

To discuss the existence and uniqueness of our model (3.5), we need to proof the following Theorem:

Theorem 4: The model (3.5) with the initial conditions (ICs) (3.2), has a unique positive solution for every $(S_0, E_0, I_0, V_0, R_0) \in \mathbb{R}_+^5$. Moreover, the compact set $\mathrm{Y} = \left\{(S, \mathrm{E}, \mathrm{I}, \mathrm{V}, \mathrm{R}) \in \mathbb{R}_+^5 : 0 \le S + E + I + V + R \le \frac{b}{\delta}\right\} \ldots$ (4.1) is a positively invariant set and having all solutions of the model (3.5) initiating in $\mathbb{R}_+^5$. Where $\delta = Max\{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}$

Proof:

Let

$$X(t) = \begin{bmatrix} S(t) \\ E(t) \\ I(t) \\ V(t) \\ R(t) \end{bmatrix};\ X(0) = X_0 = \begin{bmatrix} S(0) \\ E(0) \\ I(0) \\ V(0) \\ R(0) \end{bmatrix} = \begin{bmatrix} S_0 \\ E_0 \\ I_0 \\ V_0 \\ R_0 \end{bmatrix} W(X(t))$$
$$= \begin{bmatrix} W_1(X(t)) \\ W_2(X(t)) \\ W_3(X(t)) \\ W_4(X(t)) \\ W_5(X(t)) \end{bmatrix} = \begin{bmatrix} b - \beta SI - (\delta_1 + \gamma) S \\ \beta SI + \rho\beta VI - (\delta_2 + \mu) E \\ \mu E - (\delta_3 + \eta) I \\ \gamma S - \rho\beta VI - \delta_4 V \\ \eta I - \delta_5 R \end{bmatrix}$$

Therefore, the model (3.5) with the ICs (3.2), can be written in matrix form as:

$$ {}^{ABC}_{\ 0}D_t^{\alpha}X(t) = W(\mathrm{X})\text{with}\ X(0) = X_0.$$

The Jacobian matrix $J = \frac{\partial W}{\partial X} = \frac{\partial(W_1, W_2, W_3, W_4, W_5)}{\partial(\mathrm{S,E,I,V,R})}$ of W is continuous on $\mathbb{R}_+^5$ by [70], Remark 1.2.1] W is locally Lipschitz on $\mathbb{R}_+^5$ by [71], Remark 3.8], the model (3.5) with I.C. (3.2) has a unique solution for every $(S_0, E_0, I_0, V_0, R_0) \in \mathbb{R}_+^5$.

Now, We have to show that, for every $(S_0, E_0, I_0, V_0, R_0) \in \mathbb{R}_+^5$, the unique solution $((S, E, I, V, R) \in \mathbb{R}_+^5$ of model (3.5) with the ICs (3.2), is nonnegative. We deny this and discuss distinguish thirty cases.

Case 1: There exist $t_S, t_E, t_I, t_V, t_R \in (0, \infty)$ such that

$$\textbf{1.1}.S(t_S) < 0 \text{ and } S(t) \ge 0 \text{ for all } t \in [0, t_s) \tag{4.2}$$
$$\textbf{1.2}.E(t_E) < 0 \text{ and } E(t) \ge 0 \text{ for all } t \in [0, t_E) \tag{4.3}$$
$$\textbf{1.3}.I(t_I) < 0 \text{ and } I(t) \ge 0 \text{ for all } t \in [0, t_I) \tag{4.4}$$
$$\textbf{1.4}.\ V(t_V) < 0 \text{ and } V(t) \ge 0 \text{ for all } t \in [0, t_V) \tag{4.5}$$
$$\textbf{1.5}.R(t_R) < 0 \text{ and } R(t) \ge 0 \text{ for all } t \in [0, t_R) \tag{4.6}$$

Case 2: $S(t)$ is non-negative on $\mathbb{R}_+$ and 4.3, 4.4, 4.5 and 4.6 holds (and other similar possible cases too)

Case 3: $S(t)$, $V(t)$ are non-negative on $\mathbb{R}_+$ and 4.3, 4.4, and 4.6 holds (and other similar possible cases too)

Case 4: $S(t)$, $E(t)$, $I(t)$ are non-negative on $\mathbb{R}_+$ 4.5 and 4.6 holds (and other similar possible cases too)

Case 5: $S(t)$, $E(t)$, $I(t)$, $V(t)$ are non-negative on $\mathbb{R}_+$ and and 4.6 holds (and other similar possible cases too)

Using contradiction approach, we will prove that none of the above all possible combination of cases can occur.

If case 1 hold, then there are five possibilities arises, which is discussed below:

Subcase 1.1. For $t_S=\min\{t_S,t_E,t_I,\ t_V,\ t_R\}$, from the first equation in (3.5), we have ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}S(t)\geq -\beta SI-(\delta_1+\gamma)S\geq -k_1S$ where, $k_1=\beta^{\alpha}max_{t\in[0,t_s]}I(t)+\delta_1+\gamma>0$; thus, after solving, ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}S(t)\geq -k_1$S we get, $S(t)\geq S(0)E_{\alpha}(-k_1t^{\alpha})$ for all $t\in[0,t_s]$. Hence, $S(t_s)\geq 0$. Which is against our assumption, $S(t_s)<0$.

Subcase 1.2. For $t_E=\min\{t_S,t_E,t_I,\ t_V,\ t_R\}$ from the second equation in (3.5), we have ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}E(t)\geq -(\delta_2+\mu)E\geq -k_2E$ where, $k_2=(\delta_2+\mu)>0$; thus, after solving, ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}E(t)\geq -k_2E$ we get, $E(t)\geq E(0)E_{\alpha}(-k_2t^{\alpha})$ for all $t\in[0,t_E]$. Hence, $E(t_E)\geq 0$. Which is against our assumption, $E(t_E)<0$.

Subcase 1.3. For $t_I=\min\{t_S,t_E,t_I,\ t_V,\ t_R\}$, from the third equation in (3.5), we have ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}I(t)\geq -(\delta_3+\eta)I\geq -k_3I$ where, $k_3=(\delta_3+\eta)>0$; thus, after solving ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}I(t)\geq -k_3I$, we get, $I(t)\geq I(0)E_{\alpha}(-k_3t^{\alpha})$ for all $t\in[0,t_I]$. Hence, $I(t_I)\geq 0$. This is again against our assumption, $I(t_I)<0$.

Subcase 1.4. For $t_V=\min\{t_S,t_E,t_I,\ t_V,\ t_R\}$, from the fourth equation in (3.5), we have ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}V(t)\geq -(\rho\beta I+\delta_4)V\geq -k_4V$ where, $k_4=(\rho\beta max_{t\in[0,t_V]}I(t)+\delta_4)>0$; thus, after solving ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}V(t)\geq -k_4V$, we get, $V(t)\geq V(0)E_{\alpha}(-k_4t^{\alpha})$ for all $t\in[0,t_V]$. Hence, $V(t_V)\geq 0$. This is also against our assumption, $V(t_4)<0$.

Subcase 1.5. For $t_R=\min\{t_S,t_E,t_I,\ t_V,\ t_R\}$, from the fifth equation in (3.5), we have ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}R(t)\geq \eta I-\delta_5R\geq -k_5R$ where, $k_5=(\eta\ max_{t\in[0,t_V]}I(t)+\delta_5)>0$; thus, after solving ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}R(t)\geq -k_5$R, we get, $R(t)\geq R(0)E_{\alpha}(-k_{54\text{ and }5\ (4.5)}(t^{\alpha})$ for all $t\in[0,t_R]$. Hence, R(t_R)≥ 0. This is also against our assumption, R(t_R) <0.

Hence, combining all the subcases (1.1) – (1.5) we reach the conclusion that case (1) does not possible. Now assuming case (2) holds. Proceeding in the similar way as in case we can also shows that case (2) cannot occur. Similar arguments can be used to show that none of the cases occur i.e. case (2) to case (5) and other all possible combinations of cases cannot occur. On the basis of the above analysis, we reached on the conclusion that the unique solution (S,E,I,V,R) is non-negative. This conclusion establishes the non-negativity of our solution. Now, we have to prove that set Y defined by (4.1) is positively invariant. Adding the equations in (3.5), we get:

$$\begin{aligned}&{}^{ABC}_{\ \ 0}D^{\alpha}_{t}\ (S+E+I+V+R)\\&\quad=b-\delta_1S-\delta_2E-\delta_3I-\delta_4V-\delta_5R\end{aligned}$$

Let $\delta=\text{Max}\{\delta_1,\delta_2,\delta_3,\delta_4,\delta_5\}$ and also assume that $\delta=\delta_1=\delta_2=\delta_3=\delta_4=\delta_5$ for the sake of our conveniencei.e. ${}^{ABC}_{\ \ 0}D^{\alpha}_{t}M=b-\delta M$ (where, $M=S+E+I+R+V$ solving above equation [24], remark 7.1]), we get:

$$\begin{aligned}M(t)&=M(0)E_{\alpha}\left(-\delta t^{\alpha}\right)+b\int_0^t p^{\alpha-1}E'_{\alpha}\left(-\delta p^{\alpha}\right)dp\\&=\left(-\frac{b}{\delta}+M(0)\right)E_{\alpha}\left(-\delta t^{\alpha}\right)+\frac{b}{\delta}\end{aligned}$$

Hence, we note that $E_{\alpha}\left(-\delta t^{\alpha}\right)\geq 0$. If $M(0)\leq\frac{b}{\delta}$, we have S+$E+I+V+R=M(t)\leq\frac{b}{\delta}$, Hence, Y is positively invariant. In the end, we successfully reached the conclusion that Y contains all solutions of the model (3.5) initially in $\mathbb{R}^5_+$ as $\lim_{t\to\infty}E_{\alpha}\left(-\delta^{\alpha}t^{\alpha}\right)=0$ (by Lemma 2.) we get $\lim_{t\to\infty}M(t)=\frac{b}{\delta}$ This completes the proof.

VI. EQUILIBRIUM AND STABILITY ANALYSIS OF THE MODEL

A. STABILITY ANALYSIS OF MALWARE – FREE EQUILIBRIUM

It is obvious that model (3.5) has a unique malware free equilibrium $W_0=\left(\frac{b}{\delta_1+\gamma},\ 0,\ 0,\ \frac{\gamma b}{\delta_4(\delta_1+\gamma)},\ 0\right)$ $\therefore$ Jacobian associated with the model (3.5) is:

$$J=\begin{pmatrix}-I\beta-\gamma-\delta_1 & 0 & -\beta S & 0 & 0\\ \beta I & -\mu-\delta_2 & \beta S+\beta\rho V & \beta I\rho & 0\\ 0 & \mu & -\eta-\delta_3 & 0 & 0\\ \gamma & 0 & -V\beta\rho & -\beta I\rho-\delta_4 & 0\\ 0 & 0 & \eta & 0 & -\delta_5\end{pmatrix}$$

At W_0, the Jacobian

$$J(W_0)=\begin{pmatrix}-\gamma-\delta_1 & 0 & \frac{-\beta b}{\delta_1+\gamma} & 0 & 0\\ 0 & -\mu-\delta_2 & \frac{\beta b}{\delta_1+\gamma}+\frac{\gamma b\beta\rho}{\delta_4(\delta_1+\gamma)} & 0 & 0\\ 0 & \mu & -\eta-\delta_3 & 0 & 0\\ \gamma & 0 & \frac{-\beta\rho\gamma b}{\delta_4(\delta_1+\gamma)} & -\delta_4 & 0\\ 0 & 0 & \eta & 0 & -\delta_5\end{pmatrix}$$

Its eigen values are $\lambda_1=-(\gamma+\delta_1)$, $\lambda_2=-\delta_4$, $\lambda_3=-\delta_5$,

$$\begin{aligned}\lambda_4=&\frac{1}{2(\gamma+\delta_1)\delta_4}(-\gamma\eta\delta_4-\gamma\mu\delta_4\\&-\eta\delta_1\delta_4-\mu\delta_1\delta_4-\gamma\delta_2\delta_4\\&-\delta_1\delta_2\delta_4\\&-\gamma\delta_3\delta_4-\delta_1\delta_3\delta_4\\&-\sqrt{\gamma+\delta_1}\sqrt{\delta_4}\sqrt{}(4b\beta\gamma\mu\rho+\gamma\eta^2\delta_4+4b\beta\mu\delta_4\\&-2\gamma\eta\mu\delta_4+\gamma\mu^2\delta_4+\eta^2\delta_1\delta_4\\&-2\eta\mu\delta_1\delta_4+\mu^2\delta_1\delta_4-2\gamma\eta\delta_2\delta_4+2\gamma\mu\delta_2\delta_4\\&-2\eta\delta_1\delta_2\delta_4+2\mu\delta_1\delta_2\delta_4+\gamma\delta_2^2\delta_4\end{aligned}$$

$$+\ \delta_1\delta_2^2\delta_4 + 2\gamma\eta\delta_3\delta_4 - 2\gamma\mu\delta_3\delta_4$$
$$+\ 2\eta\delta_1\delta_3\delta_4 - 2\mu\delta_1\delta_3\delta_4 - 2\gamma\delta_2\delta_3\delta_4 - 2\delta_1\delta_2\delta_3\delta_4$$
$$+\ \gamma\delta_3^2\delta_4 + \delta_1\delta_3^2\delta_4))$$

$$\begin{aligned}\lambda_5 = {} & \frac{1}{2(\gamma+\delta_1)\delta_4}(-\gamma\eta\delta_4 - \gamma\mu\delta_4 \\ & - \eta\delta_1\delta_4 - \mu\delta_1\delta_4 - \gamma\delta_2\delta_4 - \delta_1\delta_2\delta_4 \\ & - \gamma\delta_3\delta_4 - \delta_1\delta_3\delta_4 + \sqrt{\gamma+\delta_1}\sqrt{\delta_4}\sqrt{(}4b\beta\gamma\mu\rho + \gamma\eta^2\delta_4 \\ & + 4b\beta\mu\delta_4 - 2\gamma\eta\mu\delta_4 + \gamma\mu^2\delta_4 + \eta^2\delta_1\delta_4 - 2\eta\mu\delta_1\delta_4 \\ & + \mu^2\delta_1\delta_4 - 2\gamma\eta\delta_2\delta_4 + 2\gamma\mu\delta_2\delta_4 - 2\eta\delta_1\delta_2\delta_4 \\ & + 2\mu\delta_1\delta_2\delta_4 + \gamma\delta_2^2\delta_4 \\ & + \delta_1\delta_2^2\delta_4 + 2\gamma\eta\delta_3\delta_4 - 2\gamma\mu\delta_3\delta_4 \\ & + 2\eta\delta_1\delta_3\delta_4 - 2\mu\delta_1\delta_3\delta_4 \\ & - 2\gamma\delta_2\delta_3\delta_4 - 2\delta_1\delta_2\delta_3\delta_4 + \gamma\delta_3^2\delta_4 + \delta_1\delta_3^2\delta_4))\end{aligned}$$

The first four eigen values are obviously negative and last one will be negative if and only if $(-\gamma\eta\delta_4 - \gamma\mu\delta_4 - \eta\delta_1\delta_4 - \mu\delta_1\delta_4 - \gamma\delta_2\delta_4 - \delta_1\delta_2\delta_4 - \gamma\delta_3\delta_4 - \delta_1\delta_3\delta_4 + \sqrt{\gamma+\delta_1}\sqrt{\delta_4}\sqrt{(}4b\beta\gamma\mu\rho + \gamma\eta^2\delta_4 + 4b\beta\mu\delta_4 - 2\gamma\eta\mu\delta_4 + \gamma\mu^2\delta_4 + \eta^2\delta_1\delta_4 - 2\eta\mu\delta_1\delta_4 + \mu^2\delta_1\delta_4 - 2\gamma\eta\delta_2\delta_4 + 2\gamma\mu\delta_2\delta_4 - 2\eta\delta_1\delta_2\delta_4 + 2\mu\delta_1\delta_2\delta_4 + \gamma\delta_2^2\delta_4 + \delta_1\delta_2^2\delta_4 + 2\gamma\eta\delta_3\delta_4 - 2\gamma\mu\delta_3\delta_4 + 2\eta\delta_1\delta_3\delta_4 - 2\mu\delta_1\delta_3\delta_4 - 2\gamma\delta_2\delta_3\delta_4 - 2\delta_1\delta_2\delta_3\delta_4 + \gamma\delta_3^2\delta_4 + \delta_1\delta_3^2\delta_4)) < 0$ or $R_0^{th} = \frac{\delta_4 b\beta\mu + b\beta\gamma\rho\mu}{(\delta_1+\gamma)(\mu+\delta_2)(\eta+\delta_3)\delta_4} < 1$, where R_0^{th} is a dimensionless basic reproduction number.

Using Lemma 3 and above analysis, the following result is immediate:

Theorem 5.1: The malware free equilibrium W_0 is locally asymptotically stable if $R_0^{th} < 1$ and unstable if $R_0^{th} > 1$.

From Above said theorem it is clear that if the initial values $(S_0,\ E_0,\ I_0, V_0,\ R_0) \in \mathbb{R}_+^5$ are sufficiently close to W_0, the unique solution of the model (3.5) with Initial Conditions (3.2), i.e. converges to W_0. But, here we cannot depict the region of attraction, for that we discuss the next result:

Theorem 5.2: Consider $R_0^{th} < 1$. The compact set $\Upsilon = \left\{(S, E, I, V, R) \in \mathbb{R}_+^5 : 0 \le S+E+I+V+R \le \frac{b}{\delta}\right\}$, is the region of attraction of the user-free equilibrium W_0.

Proof: We have to show that, for every $((S_0,\ E_0,\ I_0, V_0,$ $R_0) \in$ Y, the unique solution (S, E, I, V, R) of model (3.5) with the Initial Conditions (ICs) (3.2) converges to W_0. Let consider the Lyapunovfunction

$$L(t) = \frac{1}{2}I^2 + \frac{1}{2}\left(s - \frac{b}{\delta} + E + I + V + R\right)^2 \tag{5.3}$$

By theorem 5.1, Υ is nonnegative invariant. Since, $(S_0, E_0,\ I_0,$ V, $R_0) \in \Upsilon$, we have $s \le \frac{b}{\delta_1+\gamma}$

Therefore, Lemma 4. and model (3.5), we get:

$$\begin{aligned}{}^{ABC}_{\ \ 0}D_t^\alpha L \le {} & I({}^{ABC}_{\ \ 0}D_t^\alpha I(t)) \\ & + \left(S - \frac{b}{\delta} + E + I + V + R\right) \\ & \times {}^{ABC}_{\ \ 0}D_t^\alpha\left(S - \frac{b}{\delta} + E + I + V + R\right) \\ = {} & I\left(\mu E - (\delta_3+\eta)I\right) \\ & + \left(S - \frac{b}{\delta} + E + I + V + R\right) \\ & \times\ (b - \delta_1 S - \delta_2 E - \delta_3 I - \delta_4 V - \delta_5 R) \\ \le {} & I\left(\mu E - (\delta_3+\eta)I\right) \\ & + \left(S - \frac{b}{\delta} + I + V + R\right)(b - \delta(S+E+I+V+R))\end{aligned}$$

(as $\delta = \max\{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}$)

$$\le I\left(\mu E - (\delta_3+\eta)I\right) - \delta\left(S - \frac{b}{\delta} + E + I + V + R\right)^2$$

$(\because S \le \frac{b}{\delta_1+\gamma})$

$$\begin{aligned}{}^{ABC}_{\ \ 0}D_t^\alpha L \le 0 \text{ if and only if } & I(\mu E - (\delta_3+\eta)I) \\ & - \delta\left(S - \frac{b}{\delta} + E + I + V + R\right)^2 \le 0\end{aligned}$$

or $I(\mu E - (\delta_3+\eta)I) \le \delta\left(S - \frac{b}{\delta} + E + I + V + R\right)^2$

Moreover, ${}^{ABC}_{\ \ 0}D_t^\alpha L \le 0$ if and only if $S+E+I+V+R = \frac{b}{\delta}$ and $I = 0$.

i.e. $W = \left\{(S, E, I, V, R);\ {}^{ABC}_{\ \ 0}D_t^\alpha L = 0\right\} = \{(S, E, I,$ $V, R);\ S+E+I+V+R = \frac{b}{\delta}\}$

By Lemma 5., every solution of model (3.5) initiating in Y tend to the largest invariant set in W. Thus, $\lim_{t\to\infty} I(t) = 0$. for I = 0, from (3.5) we get

$$\begin{aligned}{}^{ABC}_{\ \ 0}D_t^\alpha S(t) &= b - (\delta_1+\gamma)S \\ {}^{ABC}_{\ \ 0}D_t^\alpha E(t) &= -(\delta_2+\mu)E \\ {}^{ABC}_{\ \ 0}D_t^\alpha V(t) &= \gamma S - \delta_4 V \\ {}^{ABC}_{\ \ 0}D_t^\alpha R(t) &= -\delta_5 R\end{aligned} \tag{5.4}$$

In similar manner to obtaining (3.5), we can find the solution of (5.4), also thus, after solving, we get, (similar to (3.5))

$$\begin{aligned}S(t) &= \left(-\frac{b}{(\delta_1+\gamma)} + S(0)\right)E_\alpha\left(-((\delta_1+\gamma))t^\alpha\right) \\ &\quad + \frac{b}{(\delta_1+\gamma)} \\ E(t) &= E(0)E_\alpha\left(-(\delta_2+\mu)t^\alpha\right) \\ V(t) &= \left(-\frac{\gamma\ S(0)}{\delta_4} + V(0)\right)E_\alpha\left(-\delta_4 t^\alpha\right) + \frac{\gamma}{\delta_4}\frac{b}{(\delta_1+\gamma)} \\ R(t) &= r(0)E_\alpha\left(-\delta_5 t^\alpha\right)\end{aligned} \tag{5.5}$$

By Lemma 2.

$$\begin{aligned}\lim_{t\to\infty} S(t) = \frac{b}{(\delta_1+\gamma)},\ \lim_{t\to\infty} E(t) = 0,\ \lim_{t\to\infty} V(t) \\ = \frac{\gamma}{\delta_4}\frac{b}{(\delta_1+\gamma)} \text{and } \lim_{t\to\infty} R(t) = 0\end{aligned}$$

Thus, $(S, E, I, V, R) \to W_0$ as $t \to \infty$. This Thus, $(S, E, I, V, R) \to W_0$ as $t \to \infty$.

This completes the proof of the theorem.

B. STABILITY ANALYSIS OF ENDEMIC EQUILIBRIUM

If $R_0^{th} > 1$, Firstly, we show the existence of a unique endemic equilibrium of the model (3.5).

Theorem 5.3: If $R_0^{th} > 1$. The model (3.5) has a unique endemic equilibrium $W^* = (S^*, E^*, I^*, V^*, R^*)$, where $S^* = \frac{b}{\beta I^* + (\delta_1+\gamma)}, E^* = \frac{R^*\delta_5(\delta_3+\eta)}{\mu\eta}, I^* = \frac{\delta_5 R^*}{\eta}, V^* = \frac{\gamma S^*}{\rho\beta I^* + \delta_4}$ and R^* is the unique positive solution of the cubic equation in R.

Proof.

From Definition 2.3, an endemic equilibrium point of the model (3.5) satisfies the equations

$$\begin{aligned} 0 &= b - \beta S^* I^* - (\delta_1 + \gamma) S^* \\ 0 &= \beta S^* I^* + \rho\beta V^* I^* - (\delta_2 + \mu) E^* \\ 0 &= \mu E^* - (\delta_3 + \eta) I^* \\ 0 &= \gamma S^* - \rho\beta V^* I^* - \delta_4 V^* \\ 0 &= \eta I^* - \delta_5 R^* \end{aligned} \tag{5.6}$$

After straight forward calculation, endemic equilibrium points are given by

$$S^* = \frac{\eta b}{(\beta\delta_5 R^* + (\delta_1+\gamma)\eta)}, E^* = \frac{(\delta_3+\eta)\delta_5 R^*}{\eta\mu}, \\ I^* = \frac{\delta_5 R^*}{\eta}, \quad V^* = \frac{\gamma S^*}{(\rho\beta\delta_5 R^* + \delta_4\eta)},$$

where R^* is given by the equation

$$\begin{aligned} &\frac{\beta b\eta}{\beta\delta_5 R^* + \eta(\delta_1+\gamma)} \frac{\delta_5 R^*}{\eta} - \frac{R^*\delta_5(\delta_3+\eta)}{\mu\eta} - \frac{\delta_3\delta_5 R^*}{\eta} \\ &\quad + \frac{\gamma b\eta}{\beta\delta_5 R^* + \eta(\delta_1+\gamma)} \\ &\quad - \frac{\delta_4\gamma b(\eta)^2}{[\rho\beta R^*\delta_5 + \delta_4\eta][\beta\delta_5 R^* + \eta(\delta_1+\gamma)]} - \delta_5 R^* = 0 \end{aligned} \tag{5.7}$$

From equation (5.7) it is clear that R^* have non-negative value if $R_0^{th} = \frac{\delta_4 b\beta\mu + b\beta\gamma\rho\mu}{(\delta_1+\gamma)(\mu+\delta_2)(\eta+\delta_3)\delta_4} > 1$. which proves the theorem (5.3).

Theorem 5.4: The endemic equilibrium $W^* = (S^*, E^*, I^*, V^*, R^*)$ is globally asymptotically stable.

Proof: Let $\psi(x) = x - x^* - x^* \ln\frac{x}{x^*}$

Define the LyapunovfunctionL $_1$(t) by

$$\begin{aligned} L_1(t) &= a_1\psi_1(S(t)) + a_2\psi_2(E(t)) + a_3\psi_3(I(t)) \\ &\quad + a_4\psi_4(V(t)) + a_5\psi_5(R(t)), \\ &\quad \text{where } a_j > 0,\ j = 1,2,3,4,5 \\ L_1(t) &= a_1\left(S - S^* - s^*\ln\frac{S}{S^*}\right) \\ &\quad + a_2\left(E - E^* - E^*\ln\frac{E}{E^*}\right) \\ &\quad + a_3(I - I^* - I^*\ln\frac{I}{I^*}) + a_4(V - V^* - V^*\ln\frac{V}{V^*}) \\ &\quad + a_5(R - R^* - R^*\ln\frac{R}{R^*}) \end{aligned} \tag{5.8}$$

Now, we calculate the α – order Caputo derivatives of ψ_j, $j = 1, 2, 3, 4, 5$ by Lemma 4. From the first Equation of (5.6), we have

$$b = \beta S^* I^* + (\delta_1+\gamma) S^*$$

Using this and the first equation of (3.5), we obtain

$$\begin{aligned} {}^{ABC}_{0}D_t^{\alpha}\psi_1(S) &\le \frac{S - S^*}{S}\,{}^{ABC}_{0}D_t^{\alpha}S(t) \\ &= -\frac{(S-S^*)^2}{S}\left(\beta I^* + (\delta_1+\gamma)\right) \\ &\quad + \beta(S - S^*)(I^* - I) \end{aligned} \tag{5.9}$$

From second equation of (5.6)

$$\beta S^* I^* + \rho\beta V^* I^* = (\delta_2 + \mu) E^*$$

Using this and the second equation in (3.5), we obtain

$$\begin{aligned} {}^{ABC}_{0}D_t^{\alpha}\psi_2(E(t)) &\le \frac{(E-E^*)}{E}\,{}^{ABC}_{0}D_t^{\alpha}E(t) \\ &= \frac{(E-E^*)}{E}[\beta\frac{(\text{SIE}^* - I^*S^*E)}{E^*} \\ &\quad + \rho\beta\frac{(\text{VIE}^* - I^*V^*E)}{E^*}] \end{aligned} \tag{5.10}$$

Now, from third equation of (5.6), we get

$$0 = \mu E^* - (\delta_3 + \eta) I^*$$

Using this and from third equation of (3.5), we get

$$\begin{aligned} {}^{ABC}_{0}D_t^{\alpha}\psi_3(I) &\le \frac{(I-I^*)}{I}\,{}^{ABC}_{0}D_t^{\alpha}I(t) \\ &= \frac{(I-I^*)}{I}\left[\mu(E-E^*) - \frac{\mu E^*}{I^*}(I-I^*)\right] \end{aligned} \tag{5.11}$$

From fourth equation of (5.6), we get

$$\gamma S^* = \rho\beta V^* I^* + \delta_4 V^*$$

Using this and from fourth equation of (3.5), we get

$$\begin{aligned} &{}^{ABC}_{0}D_t^{\alpha}\psi_4(V) \\ &\le \frac{(V-V^*)}{V}\,{}^{ABC}_{0}D_t^{\alpha}V(t) \\ &= \frac{(V-V^*)}{R}\left(S\frac{\rho\beta V^* I^* + \delta_4 V^*}{S^*} - \rho\beta VI - \delta_4 V\right) \end{aligned} \tag{5.12}$$

From fifth equation of (5.6), we get

$$\eta I^* = \delta_5 R^*$$

Using this and from fifth equation of (3.5), we get

$$\begin{aligned} {}^{ABC}_{0}D_t^{\alpha}\psi_5(R) &\le \frac{(R-R^*)}{R}\,{}^{ABC}_{0}D_t^{\alpha}R(t) \\ &= \frac{(R-R^*)}{R}\left(\frac{\delta_5 R^* I - \delta_5 R I^*}{I^*}\right) \end{aligned} \tag{5.13}$$

From equation (5.3) to (5.13), it follows that ${}^{ABC}_{0}D_t^{\alpha}L_1(t) \le 0$ *and* ${}^{ABC}_{0}D_t^{\alpha}L_1(t) = 0$ if and only if $S = S^*$, $E = E^*, I = I^*$, $V = V^*$ and $R = R^*$, Hence, the largest invariant set $\left\{(S, E, I, V, R);\, {}^{ABC}_{0}D_t^{\alpha} L = 0\right\}$ is the singleton set W^*. From Theorem 4.1, we found that the set Y attracts all solutions of the model (3.5) initiating in $\mathbb{R}^5_+$. By Lemma 5, W^* is global asymptotically stable. This proofs the theorem.

VII. NUMERICAL SOLUTION AND ITS ANALYSIS

Here, we find the numerical solutions of the considered model (3.1). Then the numerical results are gained through the proposed scheme. To achieve this purpose, we apply the fractional Adams Bashforth method [60] to approximate the AB fractional integral.

The equation (3.1) can be converted to a fractional integral equation by applying the fundamental theorem of fractional calculus [[24], [42], [47]] as follows:

$$
\begin{aligned}
:S_n(t) &= S_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{b - \beta S_n I_n - (\delta_1+\gamma) S_n\} \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} \\
&\quad \times \{b - \beta S_n I_n - (\delta_1+\gamma) S_n\}\, dl \\
E_n(t) &= E_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{\beta S_n I_n + \rho\beta V_n I_n - \mu E_n - \delta E_n\} \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} \\
&\quad \times \{\beta S_n I_n + \rho\beta V_n I_n - \mu E_n - \delta_2 E_n\}\, dl \\
I_n(t) &= I_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{\mu E_n - (\delta_3+\eta) I_n\} \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} \{\mu E_n - (\delta_3+\eta) I_n\}\, dl \\
V_n(t) &= V_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{\gamma S_n - \rho\beta V_n I_n - \delta_4 V_n\} \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} \\
&\quad \times \{\gamma S_n - \rho\beta V_n I_n - \delta_4 V_n\}\, dl \\
R_n(t) &= R_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{\eta I_n - \delta_5 R_n\} \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} \{\eta I_n - \delta_5 R_n\}\, dl
\end{aligned}
\tag{6.1}
$$

For the sake of convenience, let us assume

$$
\begin{aligned}
&H_{n,1}(t) = b - \beta S_n(t) I_n(t) - (\delta_1+\gamma) S_n(t), \\
&H_{n,2}(t) = \beta S_n(t) I_n(t) + \rho\beta V_n(t) I_n(t) - \mu E_n(t) - \delta E_n(t), \\
&H_{n,3}(t) = \mu E_n(t) - (\delta_3+\eta) I_n(t), \\
&H_{n,4}(t) = \gamma S_n - \rho\beta V_n(t) I_n(t) - \delta_4 V_n(t), \\
&H_{n,5}(t) = \eta I_n(t) - \delta_5 R_n(t)
\end{aligned}
\tag{6.2}
$$

The expressions $H_{n,1}(t),\ H_{n,2}(t),\ H_{n,3}(t),\ H_{n,4}(t),\ and\ H_{n,5}(t)$ are said to satisfy the Lipschitz condition if and only if $S_n(t),\ E_n(t), I_n(t), V_n(t)$ and $R_n(t)$ have an upper bound. Let $S_n(t)$ and $S_m(t)$ be two functions, then we get

$$
\begin{aligned}
&\left\|H_{n,1}(t) - H_{m,1}(t)\right\| \\
&= \|-\beta(S_n - S_m)\, I_n - (\delta_1+\gamma)(S_n - S_m)\| \\
&= \|-(\beta I_n + (\delta_1+\gamma))(S_n - S_m)\| \\
&\le \|-(\beta I_n + (\delta_1+\gamma))\|\, \|(S_n - S_m)\| \\
&\le \varepsilon_1\, \|(S_n - S_m)\|
\end{aligned}
\tag{6.3}
$$

Where $\varepsilon_1 = \left\|-(\beta max_{t\in[0,t_S]} I_n(t) + (\delta_1+\gamma))\right\|.$

Hence, we have

$$
\left\|H_{n,1}(t) - H_{m,1}(t)\right\| \le \varepsilon_1\, \|(S_n - S_m)\| \tag{6.4}
$$

Similarly, we can obtain

$$
\begin{aligned}
\left\|H_{n,2}(t) - H_{m,2}(t)\right\| &\le \varepsilon_2\, \|(E_n - E_m)\| \\
\left\|H_{n,3}(t) - H_{m,3}(t)\right\| &\le \varepsilon_3\, \|(I_n - I_m)\| \\
\left\|H_{n,4}(t) - H_{m,4}(t)\right\| &\le \varepsilon_4\, \|(V_n - V_m)\| \\
\left\|H_{n,5}(t) - H_{m,5}(t)\right\| &\le \varepsilon_5\, \|(R_n - R_m)\|
\end{aligned}
\tag{6.5}
$$

Thus, the Lipschitz condition is satisfied for all the five functions $H_{n,1}(t),\ H_{n,2}(t),\ H_{n,3}(t),\ H_{n,4}(t),\ and\ H_{n,5}(t)$ where $\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3,\ \varepsilon_4$ and ε_5 are the corresponding Lipschitz constant

Now, using equation (6.2) in equation (6.1) we get:

$$
\begin{aligned}
S_n(t) &= S_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{b - \beta S_n I_n - (\delta_1+\gamma) S_n\} \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} H_{n,1}(l) dl \\
E_n(t) &= E_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{\beta S_n I_n + \rho\beta V_n I_n - \mu E_n - \delta E_n\} \\
&\quad \times \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} H_{n,2}(l) dl \\
I_n(t) &= I_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{\mu E_n - (\delta_3+\eta) I_n\} \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} H_{n,3}(l) dl \\
V_n(t) &= V_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{\gamma S_n - \rho\beta V_n I_n - \delta_4 V_n\} \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} H_{n,4}(l) dl \\
R_n(t) &= R_n(0) + \frac{(1-\alpha)}{B(\alpha)} \{\eta I_n - \delta_5 R_n\} \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-l)^{\alpha-1} H_{n,5}(l) dl
\end{aligned}
\tag{6.6}
$$

To start an iterative scheme, at given point $t = t_{k+1},$ for $k = 0,\ 1,\ 2, \ldots\ldots$ the above equation is reformulated as:

$$\begin{aligned}
S_n(t_{k+1}) &= S_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,1}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \int_{t_j}^{t_{j+1}} (t_{k+1} - l)^{\alpha-1} H_{n,1}(l) dl \\
E_n(t_{k+1}) &= E_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,2}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \int_{t_j}^{t_{j+1}} (t_{k+1} - l)^{\alpha-1} H_{n,2}(l) dl \\
I_n(t_{k+1}) &= I_n(0) \\
&\quad + \frac{(1-\alpha)}{B(\alpha)} H_{n,3}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \int_{t_j}^{t_{j+1}} (t_{k+1} - l)^{\alpha-1} H_{n,3}(l) dl \\
V_n(t_{k+1}) &= V_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,4}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \int_{t_j}^{t_{j+1}} (t_{k+1} - l)^{\alpha-1} H_{n,4}(l) dl \\
R_n(t_{k+1}) &= R_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,5}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \int_{t_j}^{t_{j+1}} (t_{k+1} - l)^{\alpha-1} H_{n,5}(l) dl
\end{aligned} \tag{6.7}$$

Within the interval $[t_j,\ t_{j+1}]$, we can utilize the two points interpolation for approximate the functions $H_{n,1}(l),\ H_{n,2}(l)$, $H_{n,3}(l)$, $H_{n,4}(l)$and $H_{n,5}(l)$ inside the above integral.

$$\begin{aligned}
H_{n,1}(l) &\cong \frac{H_{n,1}(t_j)}{h}(l - t_{j-1}) - \frac{H_{n,1}(t_{j-1})}{h}(l - t_j) \\
H_{n,2}(l) &\cong \frac{H_{n,2}(t_j)}{h}(l - t_{j-1}) - \frac{H_{n,2}(t_{j-1})}{h}(l - t_j) \\
H_{n,3}(l) &\cong \frac{H_{n,3}(t_j)}{h}(l - t_{j-1}) - \frac{H_{n,3}(t_{j-1})}{h}(l - t_j) \\
H_{n,4}(l) &\cong \frac{H_{n,4}(t_j)}{h}(l - t_{j-1}) - \frac{H_{n,4}(t_{j-1})}{h}(l - t_j) \\
H_{n,5}(l) &\cong \frac{H_{n,5}(t_j)}{h}(l - t_{j-1}) - \frac{H_{n,5}(t_{j-1})}{h}(l - t_j)
\end{aligned} \tag{6.8}$$

Using equation (6.4) and (6.5) in (6.3) and on simplification we get:

$$\begin{aligned}
&S_n(t_{k+1}) \\
&= S_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,1}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \left(\frac{H_{n,1}(t_j)}{h} I_{j-1}^{\alpha} - \frac{H_{n,1}(t_{j-1})}{h} I_j^{\alpha} \right) \\
&E_n(t_{k+1}) \\
&= E_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,2}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \left(\frac{H_{n,2}(t_j)}{h} I_{j-1}^{\alpha} - \frac{H_{n,2}(t_{j-1})}{h} I_j^{\alpha} \right)
\end{aligned} \tag{6.9}$$

$$\begin{aligned}
&I_n(t_{k+1}) \\
&= I_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,3}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \left(\frac{H_{n,3}(t_j)}{h} I_{j-1}^{\alpha} - \frac{H_{n,3}(t_{j-1})}{h} I_j^{\alpha} \right) \\
&V_n(t_{k+1}) \\
&= V_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,4}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \left(\frac{H_{n,4}(t_j)}{h} I_{j-1}^{\alpha} - \frac{H_{n,4}(t_{j-1})}{h} I_j^{\alpha} \right) \\
&R_n(t_{k+1}) \\
&= R_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,5}(t_k) \\
&\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{j=k} \left(\frac{H_{n,5}(t_j)}{h} I_{j-1}^{\alpha} - \frac{H_{n,5}(t_{j-1})}{h} I_j^{\alpha} \right)
\end{aligned}$$

where,

$$I_{j-1}^{\alpha} = \int_{t_j}^{t_{j+1}} (l - t_{j-1})(t_{k+1} - l)^{\alpha-1} dl$$

$$I_j^{\alpha} = \int_{t_j}^{t_{j+1}} (l - t_j)(t_{k+1} - l)^{\alpha-1} dl$$

On further simplification of the integrals I_{j-1}^{α} and I_j^{α}, and assuming, $t_j = jh$, we can easily find that

$$\begin{aligned}
I_{j-1}^{\alpha} &= \frac{h^{\alpha+1}}{\alpha(\alpha+1)} \left[(k+1-j)^{\alpha}(k-j+2+\alpha) \right. \\
&\quad \left. -(k-j)^{\alpha}(k-j+2+2\alpha) \right]
\end{aligned} \tag{6.10}$$

$$\begin{aligned}
I_j^{\alpha} &= \frac{h^{\alpha+1}}{\alpha(\alpha+1)} \left[(k+1-j)^{\alpha+1} \right. \\
&\quad \left. -(k-j)^{\alpha}(k-j+1+\alpha) \right]
\end{aligned} \tag{6.11}$$

Using values from equations (6.10) and (6.11) in equation (6.9) and after simplification, we get

$$\begin{aligned}
&S_n(t_{k+1}) \\
&= S_n(0) + \frac{(1-\alpha)}{B(\alpha)} H_{n,1}(t_k) + \frac{\alpha}{B(\alpha)} \sum_{j=0}^{j=k} \\
&\quad \times \left(\frac{H_{n,1}(t_j)}{\Gamma(\alpha+2)} h^{\alpha} \left[(k+1-j)^{\alpha}(k-j+2+\alpha) \right. \right. \\
&\quad \left. -(k-j)^{\alpha}(k-j+2+2\alpha) \right] - \frac{H_{n,1}(t_{j-1})}{\Gamma(\alpha+2)} h^{\alpha}
\end{aligned}$$

$$\left[(k+1-j)^{\alpha+1}-(k-j)^{\alpha}(k-j+1+\alpha)\right]\Big) \tag{6.12}$$

$$\begin{aligned}
&E_n(t_{k+1})\\
&=E_n(0)+\frac{(1-\alpha)}{B(\alpha)}H_{n,2}(t_k)+\frac{\alpha}{B(\alpha)}\sum\nolimits_{j=0}^{j=k}\\
&\quad\times\Bigg(\frac{H_{n,2}(t_j)}{\Gamma(\alpha+2)}h^{\alpha}\left[(k+1-j)^{\alpha}(k-j+2+\alpha)\right.\\
&\quad\left.-(k-j)^{\alpha}(k-j+2+2\alpha)\right]-\frac{H_{n,2}(t_{j-1})}{\Gamma(\alpha+2)}h^{\alpha}\\
&\quad\left[(k+1-j)^{\alpha+1}-(k-j)^{\alpha}(k-j+1+\alpha)\right]\Bigg)
\end{aligned} \tag{6.13}$$

$$\begin{aligned}
&I_n(t_{k+1})\\
&=I_n(0)+\frac{(1-\alpha)}{B(\alpha)}H_{n,3}(t_k)+\frac{\alpha}{B(\alpha)}\sum\nolimits_{j=0}^{j=k}\\
&\quad\times\Bigg(\frac{H_{n,3}(t_j)}{\Gamma(\alpha+2)}h^{\alpha}\left[(k+1-j)^{\alpha}(k-j+2+\alpha)\right.\\
&\quad\left.-(k-j)^{\alpha}(k-j+2+2\alpha)\right]-\frac{H_{n,3}(t_{j-1})}{\Gamma(\alpha+2)}h^{\alpha}\\
&\quad\left[(k+1-j)^{\alpha+1}-(k-j)^{\alpha}(k-j+1+\alpha)\right]\Bigg)
\end{aligned} \tag{6.14}$$

$$\begin{aligned}
&V_n(t_{k+1})\\
&=V_n(0)+\frac{(1-\alpha)}{B(\alpha)}H_{n,4}(t_k)+\frac{\alpha}{B(\alpha)}\sum\nolimits_{j=0}^{j=k}\\
&\quad\times\Bigg(\frac{H_{n,4}(t_j)}{\Gamma(\alpha+2)}h^{\alpha}\left[(k+1-j)^{\alpha}(k-j+2+\alpha)\right.\\
&\quad\left.-(k-j)^{\alpha}(k-j+2+2\alpha)\right]-\frac{H_{n,4}(t_{j-1})}{\Gamma(\alpha+2)}h^{\alpha}\\
&\quad\left[(k+1-j)^{\alpha+1}-(k-j)^{\alpha}(k-j+1+\alpha)\right]\Bigg)
\end{aligned} \tag{6.15}$$

$$\begin{aligned}
&R_n(t_{k+1})\\
&=R_n(0)+\frac{(1-\alpha)}{B(\alpha)}H_{n,5}(t_k)+\frac{\alpha}{B(\alpha)}\sum\nolimits_{j=0}^{j=k}\\
&\quad\times\Bigg(\frac{H_{n,5}(t_j)}{\Gamma(\alpha+2)}h^{\alpha}\left[(k+1-j)^{\alpha}(k-j+2+\alpha)\right.\\
&\quad\left.-(k-j)^{\alpha}(k-j+2+2\alpha)\right]-\frac{H_{n,5}(t_{j-1})}{\Gamma(\alpha+2)}h^{\alpha}\\
&\quad\times\left[(k+1-j)^{\alpha+1}-(k-j)^{\alpha}(k-j+1+\alpha)\right]\Bigg)
\end{aligned} \tag{6.16}$$

A. FOR THE STABILITY OF THE NUMERICAL SOLUTION OBTAINED ABM METHOD

This is clear from (6.4) and (6.5) that the functions $H_{n,1}(t)$, $H_{n,2}(t)$, $H_{n,3}(t)$, $H_{n,4}(t)$, *and* $H_{n,5}(t)$ Satisfies the Lipschitz condition,

Now,

$$\begin{aligned}
&\left\|H_{n,1}(t_k)-H_{n,1}(t_{k-1})\right\|_{\infty}\\
&=\|-\beta S_n(t_k)\ I_n(t_k)-(\delta_1+\gamma)S_n(t_k)\\
&\quad+\beta S_n(t_{k-1})\ I_n(t_{k-1})+(\delta_1+\gamma)S_n(t_{k-1})\|_{\infty}
\end{aligned} \tag{6.17}$$

using (6.12) and (6.13) in (6.17) and from ref. (**60**)

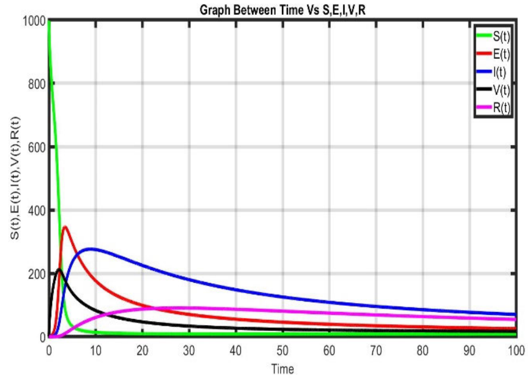

FIGURE 2. **Basic Reproduction number less than one (R_0^{th} = 0.316235).**

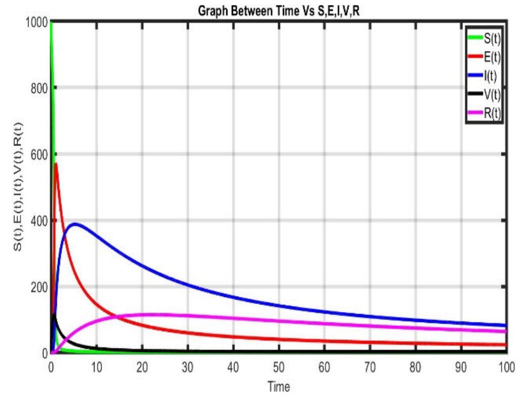

FIGURE 3. **Basic Reproduction number greater than one (R_0^{th}= 1.192941).**

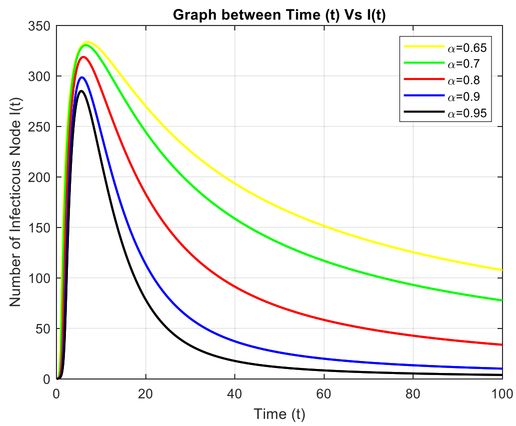

FIGURE 4. **Analyse the effect of fractional derivative (α) on infectious nodes.**

for $k\to\infty$ *as* $\left\|H_{n,1}(t_k)-H_{n,1}(t_{k-1})\right\|_{\infty}\to 0$, therefore, from Theorem 3.3, the solution $S_n(t_k)$ is stable.

Similarly, we can obtain the stability condition for solutions $E_n(t_k)$, $I_n(t_k)$, $V_n(t_k)$ and $R_n(t_k)$.

VIII. RESULTS AND DISCUSSION

Conduct a series of numerical simulation experiments to study the impact of different factors on the spread of malware in WSN.

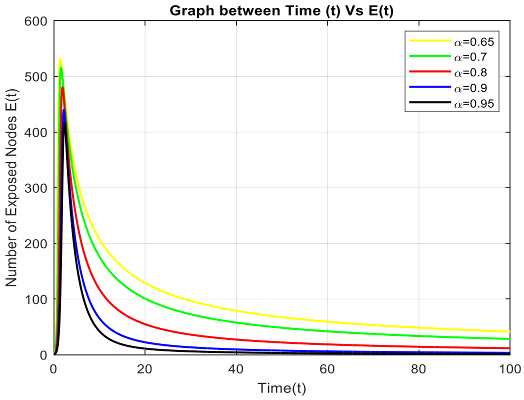

FIGURE 5. Analyse the effect of fractional derivative (α) on exposed nodes.

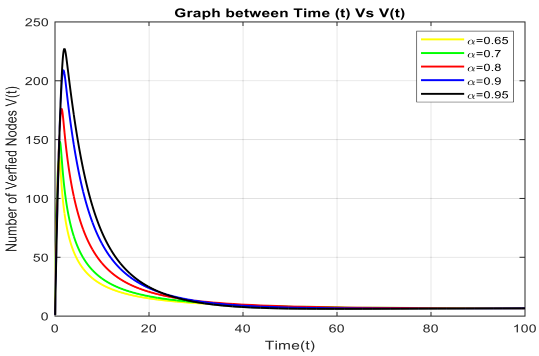

FIGURE 6. Analyse the effect of fractional derivative (α) on vaccinated nodes.

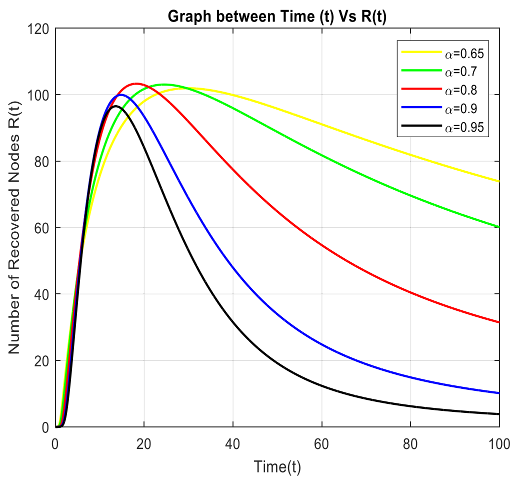

FIGURE 7. Analyse the effect of fractional derivative (α) on recovered nodes.

The proposed work is simulated using MATLAB on a 3-GHzIntel Xeon system running UBUNTU 19.2 LTS with 16-GBRAM. The results of simulation are illustrated in Figure 2 to Figure 7, for susceptible (S), Exposed(E), infectious (I), vaccinated and recovered (R) class with respect to time (t).The values of the various parameters considered for simulation are $b = 1, \beta = .02, \Upsilon = .2, \rho = .004, \eta = .07, \delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = .1$ and taking the fractional derivative value $\alpha = .7$ In this analysis, it is ascertainedthat initially infected nodes increase with time and achievethe maximum value. After that, it begins to disappear with timeand become zero. This shows that the system is asymptoticallystable in these conditions. The basic reproduction number incise 1 depicted in Figure1, is0.316235 which is less than one, so malware disappears from the wireless sensor network. This result validates Theorems 5.1 and 5.2. When change some parametric value such as $\beta = .07, \rho = .007, \mu = .6$, and and other remain same. Then the value of $R_0^{th} = 1.192941$. Fig. 2 demonstrates that malware persists in the system continuously and in this case, the value of basic reproduction number (1.192941) is greater than 1. In this situation, malware will spreading the wireless sensor network., so malware continuously persist in the wireless sensor network. This result validates Theorems 5.3 and (5.4).

The impact of fractional derivative (α) on infectious nodes and exposed nodes are shown in Figure 3 and Figure 4. Shows that relationship between time and infectious nodes and exposed class nodes. It is evident from graph that when initially infectious class nodes and exposed class nodes achieve its maximum value for different value of α and after some time decline. For higher value of α the number of infectious nodes and exposed class nodes is less and for lower value of α the number of infectious nodes and exposed class nodes is more as compared to higher value of α.

Figure 5 shows the dynamic relationship between time and vaccinated class nodes for different value of α. It is clear from figure that when $\alpha = .095$ vaccination is very fast as compared to lesser value of α but after some time there is no impact of α on the vaccination. The link between time and recovered class nodes is shown in Figure 7, and it can be seen that as rises, recovery becomes quicker.

IX. CONCLUSION AND FUTURE WORK

The fractional-order derivatives are normally well suited for modelling since the determination about the derivative order offers one more degree of freedom, which results in a better fit to the real-time data with less inaccuracy than the integer-order model. Therefore, for controlling malware propagation and extending the lifespan of WSN, a fractional order SEIVR epidemic model is proposed. The proposed model offers a technique to quickly identify sensor node transitions from susceptible to infected state. Such detection provides a chance to further incorporate techniques for maintaining the sensor nodes and stabilizing WSN under various circumstances. The drawbacks of the earlier discussed model are overcome by an amended SEIRV model through the mechanism of vaccination. The computation of the basic reproduction number is one of the critical parameters that plays an important role in controlling malware transmission in the WSN. This value is computed. Additionally, it is found that endemic equilibrium and malware free equilibrium are locally and globally asymptotically stable, if the values of $\boldsymbol{R_0^{th}}>\mathbf{1}$ and $\boldsymbol{R_0^{th}}<\mathbf{1}$, respectively and this is supported by simulation results. These results are presented in Figure 2 and 3. Additionally, MATLAB(R2018a) simulations were run to test the validity of the proposed model. Numerical solutions for the system have been found using the Adam-Bashforth-Moulton predictor-corrector method. The impact of various factors on malware propagation in WSN under various circumstances has been extensively analysed. Numerous results show that the proposed model offers a better defence mechanism against malware attacks. Further, fractional derivative models can be used to develop more effective control strategies for epidemics, including quarantine measures, and social distancing like policies depends on the type of network. There is a need for more research on how to use these models to optimize control strategies and minimize the impact of epidemics on WSN as well as online social network. In upcoming research work, we proposed our research in the field of variable order fractional derivative model through artificial intelligence-based approach.

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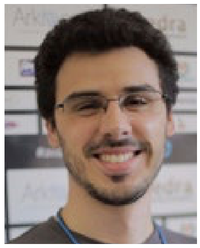

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