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RESEARCH ARTICLE

A Single-Vendor Multi-Retailer VMI Partnership Under Individual Carbon-Cap Constraints

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ABSTRACT Vendor Managed Inventory (VMI) is increasingly being adopted as a mean of enhancing the efficiency of supply chain (SC) operations. In this paper, we consider a scenario where a vendor is responsible for managing the inventory of several retailers following a VMI partnership and subject to restrictions on their maximum inventory levels. To better reflect reality, we first propose a mixed integer nonlinear programming model where the vendor and the retailers operate under different environmental legislations with distinct carbon caps. Additionally, to further reduce chain-wide costs, the developed model is extended to allow for carbon exchange among SC members. We derive structural properties for both models to devise efficient exact solution algorithms. The resulting cost savings can be shared with SC members to ensure they are not worse off under the newly proposed "carbon cap and exchange" policy. The benefits of this enhanced collaborative scheme are illustrated through a small example involving a single vendor and five retailers. Furthermore, a one-way sensitivity analysis is conducted to evaluate the impact of the number of retailers and the restrictiveness of the SC members' carbon caps on the performance of the two proposed models. The obtained results indicate that carbon exchange among SC members is more advantageous for a larger number of retailers. Moreover, the implementation of the new carbon policy yields higher cost reductions when there are strict constraints on the SC members' carbon emissions.

INDEX TERMS VMI, single-vendor multi-buyers, sustainability, individual carbon caps, exact solution algorithms.

I. INTRODUCTION

It is universally agreed that supply chain (SC) collaboration is paramount for driving meaningful and everlasting change. Collaborative efforts can provide greater transparency, promotes higher ethical and environmental standards, and exert better leverage over resistant SC members. Consequently, several collaborative initiatives have been put in place and continue to be successfully implemented by many firms, such as Vendor Managed Inventory (VMI), Continuous Replenishment (CR), and Consignment Stock (CS) among others. For VMI in specific, the early partnership between Procter & Gamble and Walmart back in 1985 has initiated a widespread popularity of this strategy among industry partners [1]. Many leading companies such as HP and Shell [2] as well as Barilla the largest pasta vendor in the world [3] later adopted VMI. Currently, around 60% of the consumer processed goods retail stores in the United States operate through VMI [4]. Several researchers exploited the many benefits of VMI implementation including improved service levels and reduced stock-outs, increased inventory turnovers and reduced lead times, improved control of the bullwhip effect and chain-wide cost savings, among others (e.g. Disney and Towill [5]; Angulo et al. [6]; Hariga et al. [7], Kim and Shin [8]). For readers interested in a more in-depth discussion of the benefits and limitations of a VMI-based collaborative scheme, Anand et al. [9] provide a thorough analysis.

In a VMI arrangement, the vendor is responsible for initiating orders on behalf of the buyers and managing their inventory. The transparency and sharing of real-time information between such partners provide an elevated level

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of visibility that enhances the operations of both of them, allowing the vendor to better plan production, schedule deliveries, and manage stock levels at the buyers' facilities. To guard against the vendor's opportunistic practice of pushing more inventory to buyers and to avoid any potential conflicts between SC members, the VMI contract typically stipulates a restriction on the maximum inventory level at the buyers' premises. The vendor is then penalized on a per-unit basis upon exceeding this limit (see Fry et al. [10]; Darwish and Odah [11]; Hariga et al. [7]; Verma and Chetterjee [12], Hariga et al. [13]). It is important to point out that while focusing exclusively on the potential operational and financial gains associated with VMI partnership may have made sense in a pure commercially-driven business framework, such an approach could have unintended negative consequences for the environment. In fact, an average of 75% of all industrial sector carbon emissions can be attributed to SC activities encompassing procurement, production, distribution and warehousing [14]. As a result, companies are under increasing pressure to adopt sustainable practices and continuously rethink their day-to-day operations to ensure they comply with environmental regulations.

To address concerns about climate change and combat the global rise in carbon emissions, several regulatory policies have been established over the years. These policies include carbon tax, carbon cap, carbon offset, cap-and-trade, and eco-friendly technology standards. For detailed discussion of these policies aimed at curbing carbon emissions, along with their efficacy and benefits, readers can refer to the works of Chelly et al. [15], Hariga et al. [16] and Mohammed et al. [17]. In this paper, we adopt a quantity-based carbon cap policy that imposes a strict limit on the carbon footprint generated by each SC member, rather than a price-based approach. This cap may be imposed externally by governmental or legislative entities, or set internally by organizations that proactively seek to adopt environmentally-conscious practices as part of their response to increasing customer awareness and sustainability obligations. According to a poll conducted by Yale University, 64% of the participants support the imposition of strict carbon caps on existing power plants [18]. Environmentalists consider carbon cap to be the most effective policy for reducing carbon emissions, with many countries seriously considering its adoption across various industries, especially those with high levels of carbon dioxide (CO2) emissions [19]. While carbon tax policy is more prevalent and has been implemented across 21 countries or regions, carbon cap policy is gaining momentum due to its proven efficiency with eight countries having already adopted it [14].

In this paper, we consider integrated economic and environmental VMI-based models for a two-stage single-vendor multi-buyer (SV-MB) SC with finite storage space at the retailers' facilities. The vendor delivers an equal number of shipments to all retailers at common, equally-spaced replenishment cycles. Such synchronized policy is simple to implement and continues to be adopted in many practical situations since it is appealing from both economic and environmental standpoints, as detailed in Section III. Recently, Hariga et al. [13] tackled this problem in the literature by considering a carbon cap constraint on the total emissions, meaning that the total carbon emitted by all SC members should not exceed a stipulated predetermined level. However, in reality, the vendor and the retailers are independent entities that may operate under different environmental legislations with distinct carbon caps. Moreover, in order to more accurately assign responsibility for carbon footprint, or as part of an individual organization's own initiative to act responsibly and adopt environmentally-friendly practices, emission caps could be internally set at the organizational level. These individual caps may be set based on historical carbon generation by SC members, which they aim not to exceed after joining the VMI collaboration. In such situation, the carbon cap policy should impose a carbon cap constraint for each party involved (i.e. individual carbon caps) instead of an overall carbon cap. Hereafter, we call such modified policy as the ''individual carbon cap'' policy.

To enhance the cost efficiency of this policy, it may be beneficial to allow SC members to exchange some of their carbon permits among each other. Indeed, when operating within a strategic alliance or consortium, SC members may exchange carbon credits in case some members have depleted their allowable credits while others have surpluses. In another scenario, some members can reduce their carbon emissions by a specific amount and transfer the resulting emission reduction to other members, allowing the latter to increase their carbon footprint. Such internal carbon free carbon trading among SC members leads to a new carbon regulatory policy, hereafter referred to as ''carbon cap and exchange'' policy. The objective of this exchange mechanism is to explore the possibility of altering the lot-sizing policy towards attaining further reduction in the chain-wide total cost whilst ensuring that none of the partners is worse off cost-wise.

In this work, we develop a mathematical model for the SV-MB SC operating under a VMI contractual agreement and individual carbon caps, followed by another model addressing the extended case that employs the proposed carbon exchange policy. We exploit the structure of both models, identify optimality properties, and devise efficient solution algorithms that guarantee the attainment of the optimal solution.

The remainder of this paper is organized as follows. In Section II, we provide a survey of the literature pertaining to the problem at hand along with a highlight of the contributions of this work. Section III presents the derivation of the VMI based model under the individual carbon cap policy as well as the solution algorithm and an illustrative example. Section IV extends the model in Section III to address carbon exchange among SC members and reflects on the needed operational adjustments and the potential savings via a numerical example. In Section V, we conduct a one-way sensitivity analysis experiment seeking to evaluate the impact of the number of retailers and the restrictiveness of the individual carbon caps on the costs and carbon emission performance of the two developed models. Finally, managerial insights as well as concluding remarks, limitations and future research avenues are included in Section VI.

II. RELEVANT LITERATURE

The cross-organizational collaborative schemes, such as VMI and CS, were made possible via centralized SC systems inherently assuming a vertically integrated SC which allows for the joint optimization of production and inventory related decisions. The origin of this idea roots back to the integrated single-vendor single-buyer (SV-SB) system presented in the seminal work of Goyal [20] and bringing rise to the so called joint economic lot sizing (JELS) problem. Goyal's work has triggered a stream of research addressing different demand and lot-sizing settings and for various SC configurations. While the extended works provide the building block for the problem tackled in this paper, they do not necessarily address VMI collaboration. Interested readers are referred to the works of Ben-Daya et al. [21], Glock [22] and Utama et al. [23] for comprehensive reviews on the JELS problem. Moreover, for the vast majority of these works, the optimization criteria is solely driven by economic measures where the typical objective is to minimize (maximize) chain-wide cost (profit) while establishing benefit sharing mechanisms among participating entities to ensure everyone's interest to partake in such an integrated system.

The inevitable need to adopt sustainable practices has reshaped the status-quo of today's logistics operations and forced organizations around the world to rethink their day-to-day operations towards better alignment with environmental regulations. As such, an increasing number of researchers have incorporated sustainability aspects in the modeling of inventory systems and SC operations. To that end, several carbon regulatory policies have been utilized where it has been shown that better coordinated logistics and lot-sizing decisions among SC partners may yield substantial reduction in the amount of carbon footprint generated. It shall be noted, however, that the majority of these models assume single stage and single product settings [24]. In a non-VMI related context, Chen et al. [25] incorporated the four carbon policies (carbon tax, carbon cap, carbon offset, and cap-and-trade) into the classical single stage constant demand Economic Order Quantity (EOQ) model. Following the same four carbon policies, Benjaafar et al. [26] extended the previous work to address the case of a product having a time-varying demand over a finite planning horizon. The main conclusion drawn by these two works is that upon making minor operational adjustments to the lot-sizing policy, significant savings in carbon footprint can be attained at the expense of a slight increase in the operational cost. As for the strict carbon cap policy adopted in this work, several researchers have explored the effect of adopting this policy, under varying levels of the preset cap, on the operational strategy and also assessed the accompanying impact on the operational cost (e.g. Absi et al. [27], [28] and As'ad et al. [24]). For a SV-SB SC configuration, Ghosh et al. [18] explored the impact of investing in green technology under the total carbon cap policy while assuming a constant demand setting. They developed two mixed integer nonlinear programming (MINLP) models for the two cases of with or without green investment and conducted further comparative analysis. Many other works have also tackled the modeling of inventory decisions while accounting for environmental aspects as seen in the many recent and comprehensive review papers in this field (see for example Das and Jharkharia [29], Shaharudin et al. [30], Chelly et al. [15], and Zhou et al. [14]).

From a pure economic standpoint, a stream of research has focused on the optimization of integrated production and lot-sizing decisions in the context of a VMI partnership. One of the earliest such works is that of Fry et al. [10] who considered a SV-SB system in which the supplier is penalized upon failing to maintain the retailer's inventory level within a certain range. Mishra and Raghunathan [31] exploited the benefits of retailer vs. vendor managed inventory in the context of brand competition and illustrated that VMI restores the competition between suppliers and also benefits the retailers. In a related work, Lee and Chu [32] also carried out a comparative analysis seeking to identify the party that shall be in charge of controlling inventory levels and making replenishment decisions while operating in a classical newsboy setting. The authors concluded that adopting VMI collaborative mechanism has the potential to make both the vendor and the retailer better off given that some risk-sharing rule is put in place. Other works that have assessed the effectiveness of VMI coordination include Choi et al. [33], Yao et al. [34], Nagarajan and Rajagopalan [35], Lee and Ren [36], Van den Bogaert and van Jaarsveld [37] among many others. For thorough reviews of works addressing various aspects of VMI contractual agreements, interested readers are referred to Govindan et al. [38], Lee et al. [39] and Nimmy et al. [40].

When addressing SC systems comprising multiple retailers/buyers, as is the case in this work, an important issue to consider is the timing and the coordination of stock replenishments for the downstream retailers. In a general context, the SV-MB problem has been amply addressed in the literature by several researchers, including Viswanathan and Piplani [41]; Cheung and Lee [42], Lu [43], Siajadi et al. [44] among many others, where the works listed hereafter tackle the problem from an economic perspective only. Broadly speaking, there exist two approaches that have been adopted to schedule shipments to the retailers, which are equal/synchronized and unequal/unsynchronized cycle policies with the retailers receiving replenishments at the same or different points in time, respectively. For the equal cycle time and in the context of VMI partnership, Darwish and Odah [11] devised a mathematical model that assumes, similar to this work, a maximum limit on the stock levels at the retailers' premises where the vendor is penalized upon exceeding those limits. Mateen and Chatterjee [45] presented several analytical models to better coordinate lot-sizing decision following a VMI collaborative arrangement. As for the more generalized unequal cycle policy, Zhang et al. [46] developed an integrated VMI model that takes into account investment in ordering cost reduction towards improved coordination and automation among SC members. Hariga et al. [7] devised MINLP models to synchronize the vendor's cycle time with the buyers' unequal ordering cycles. In a related work, Hariga et al. [47] proposed other models allowing unequal shipment frequencies to the retailers. Verma et al. [48] also put forward an alternative ordering scheme that caters for different replenishment cycles for each retailer. The impact of retailers' heterogeneity on the effectiveness of the VMI system is explored in the works of Son and Ghosh [49] and Verma and Chatterjee [12]. For the case of two buyers, Tarhini et al. [50] showed that transshipment between buyers can prove useful towards decreasing the total cost faced by both buyers and their suppliers.

Only few works went beyond the conventional economic measure to address environmental concerns into the modeling aspects of SC problems under VMI partnership. These studies have pointed out that this collaborative strategy not only leads to cost savings but has also proven effective towards reducing carbon emissions. Mateen et al. [51] presented a VMI replenishment model for a SV-SB that incorporates both operational and emission related costs with the latter being accounted for via the carbon tax policy. Following the equal cycle time policy, Karimi and Niknamfar [52] tackled a multi-product SV-MB system following a VMI partnership under the carbon tax and the carbon cap policies. The developed model seeks to maximize the total profit along with the mean time to failure of the manufacturer's production system via a redundancy allocation problem. In a related work, Mokhtari and Rezvan [53] also considered a multi-product SV-MB system under VMI collaboration where shortages are allowed and are partially backordered. They established the production quantity and maximum shortage level for each buyer that minimize the total cost while adopting the carbon cap policy. Recently, Hariga et al. [13] developed economic, environmental and integrated economic and environmental models for the single product SV-MB system, while imposing an upper limit on the inventory levels at the buyers' premises. The authors adopted carbon tax and carbon cap policies, and further utilized the idea of the Lagrange multiplier to help with the selection of the most-suited policy. Under the carbon cap policy, they considered one carbon cap constraint on the total carbon emitted by all SC members. That is, the total carbon emission must not exceed an agreed upon level set by an environmental legislation body controlling the carbon emissions in the country (region, area, etc.) where the SC members are operating. On the other hand, it is assumed in the current paper that the vendor and the retailers are operating under different environmental legislations with distinct (individual) carbon caps, where those caps might be imposed exogenously or endogenously. In other words, the carbon footprint generated by each SC member should not exceed its own carbon permit.

As can be deduced from the above review of the relevant literature, the vast majority of the works addressing two-stage SV-MB configuration under a VMI partnership have focused solely on economic measures while overlooking environmental related concerns. This work brings the following contributions to the existing SC literature:

(1) It deviates from the few existing sustainable models in the sense that it operationalizes the environmental imperatives facing each organization in the form of individual carbon caps rather than a cap on the total emissions generated by all SC members. In reality, although a strategic alliance is assumed via a VMI collaborative environment, stand-alone caps are ought to be abide by irrespective of whether they are imposed exogenously or indigenously.
(2) Given that the vendor typically has the upper hand, a new carbon regulatory policy is proposed whereby the vendor would further orchestrate/promote an exchange mechanism of carbon allowances within SC members towards the attainment of reductions in the operational costs via adjustments to the replenishment strategy. This newly proposed exchange mechanism takes place internally at no cost. It is easily implementable in practice as it capitalizes on the already existing strategic partnership in the form of a VMI arrangement where, to the authors' best knowledge, this work stands out as being the first to put forward such a carbon regulatory policy.
(3) To optimize operational decisions under the proposed two polices "individual carbon caps" and "carbon cap and exchange", two mathematical models are developed along with computationally efficient solution algorithms that guarantee the attainment of optimal solutions. The time complexity for each solution procedure is also provided.
(4) Through the proposed carbon exchange mechanism, it turns out that substantial carbon reductions can be attained at the expense of a minor increase in the chain-wide operational cost. In particular, it has been shown numerically that the cost per ton reduction of CO2 emissions is much cheaper when carbon exchanges are allowed among SC members.

III. VMI MODELS UNDER INDIVIDUAL CARBON CAP POLICY

Consider a vendor is supplying a single product to m retailers facing deterministic demands ($D_j : j = 1, \ldots, m$). Under a VMI partnership, the vendor is responsible for managing the inventories of the retailers by initiating orders on their behalf. Accordingly, the vendor decides about the timings (order intervals, $T_j : j = 1, \ldots, m$) and ordering quantities ($q_j : j = 1, \ldots, m$) to be delivered to the retailers. In order to control its inventory, each retailer j limits its maximum on-hand stock to a level U_j that the vendor should not exceed when setting the ordering quantity, otherwise the vendor is penalized. Both the maximum on-hand limit U_j and the associated penalty cost π_j for each unit of overstock are incorporated in the VMI contract between the vendor and the retailers. Each time an order is placed, retailer j incurs a fixed cost A_j and emits $\hat{A}_j$ tons of carbon. In addition, each product unit remaining in stock by the end of the year costs h_j for retailer j and generates $\hat{h}_j$ tons of carbon. Similarly, the vendor incurs A_0 and h_0 as costs for placing an order from its supplier and holding one unit of the product in stock for one year at its premises, respectively. Moreover, ordering and storage activities of the vendor generate $\hat{A}_0$ and $\hat{h}_0$ tons of carbon emissions, respectively.

The models presented in this paper are developed under the same set of assumptions:

1. Retailers' demands are constant and known
2. Shortages are disallowed
3. The vendor orders the product from a supplier with unlimited capacity.
4. Retailers' holding cost rates are larger than that of the vendor
5. Equal number of shipments is made to each retailer.
6. The vendor is charged a penalty for each unit above the agreed on upper stock level set by the retailers.
7. A profit sharing agreement is included in the VMI contract ensuring that vendor and retailers realize cost savings as opposed to the case when they act independently. Such assumption, ensures all members willingness to be involved in the VMI partnership [54].

The fifth assumption states that the retailers share a common ordering cycle, T, as they receive the same number of shipments from the vendor, n. This ordering policy is frequently used in practice and it is called complete aggregation policy [55] or common replenishment epochs [41]. From an economic standpoint, it allows the vendor to reduce his order processing costs through the aggregation of retailers' orders and having them delivered in one shipment. From an environmental perspective, it significantly lowers carbon emissions as the order shipments to the retailers require one delivery with multiple stops at the different retailers' premises in lieu of a single round trip for each retailer's delivery. For instance, a common practice for the delivery of dairy products, soft drinks and water bottles is to replenish all retailers within the same district on the same day. Moreover, this assumption of equal replenishment cycles is quite common in the SV-MB literature (e.g. Banerjee and Banerjee [56], Viswanathan and Piplani [41], Woo et al. [57], Mishra [58], Darwish and Odah [11], Ben-Daya et al. [59], Karimi and Niknamfar [52], and Pramudyo and Luong [60] among many others).

In their recent work, Hariga et al. [13] developed economic, environmental, and integrated economic and environmental models under the above assumptions. The latter model was formulated under the carbon-cap regulation policy, whereby the overall carbon footprint generated by the ordering and storage activities of the vendor and the retailers does not surpass the allowable overall carbon cap, C. Their integrated mathematical model under overall carbon cap (OCC) restriction is as follows:

VMI_OCC:

$$\begin{aligned} &Min\ TOC\left(n, q, z_j\right) \\ &\quad = \frac{A_0}{nq} D_1 + 0.5 h_0 \frac{D}{D_1}(n-1) q \\ &\quad + \sum_{j=1}^{m}\left[\frac{A_j}{q} D_1 + 0.5 h_j \frac{D_j}{D_1} q\right] + 0.5 \frac{D_1}{q} \sum_{j=1}^{m} \frac{\pi_j}{D_j} z_j^2 \end{aligned} \tag{1}$$

$s.t.$

$$\begin{aligned} &E(n, q) = \frac{\hat{A}_0}{nq} D_1 + 0.5 \hat{h}_0 \frac{D}{D_1}(n-1) q \\ &\quad + \sum_{j=1}^{m}\left[\frac{\hat{A}_j}{q} D_1 + 0.5 \hat{h}_j \frac{D_j}{D_1} q\right] \leq C \end{aligned} \tag{2}$$

$$\frac{D_j}{D_1} q - U_j - z_j \leq 0 \text{ for } j = 1, 2, \ldots, m$$

$$q \geq 0, \quad n \text{ integer } z_j \geq 0 \text{ for } j = 1, 2, \ldots, m \tag{3}$$

where

$E(n, q)$ is the overall carbon emitted by the vendor and the retailers,

$q = q_1$ is the ordering quantity of the first retailer,

z_j is the overstock quantity at the j th retailer, where $z_j = Max\left(0, \frac{D_j}{D_1} q - U_j\right)$, and $D = \sum_{j=1}^{m} D_j$.

The first two terms in the objective function are the vendor's ordering and holding costs, respectively. The third term is the sum of the retailers' ordering and holding costs and the last term is total overstock penalty cost charged to the vendor. Constraint (2) ensures that the overall carbon emitted by all members of the SC does not exceed the overall carbon cap. The first two terms in this constraint provide the carbon generated by the vendor, while each term included in the summation represents the carbon footprint of a retailer. Constraint set (3) along with the non-negativity constraints of the overstock quantities guarantees that $z_j = Max\left(0, \frac{D_j}{D_1} q - U_j\right)$. Note that the number of decision variables related to the retailers' ordering quantities is reduced to only one variable $q = q_1$ due to the assumption of equal number of shipments made to the retailers. In this case, the retailers' ordering cycles are set to be equal. Therefore,

$$T_i = T_j \text{ and } q_i / D_i = q_j / D_j \text{ for all } i \neq j, \tag{4}$$

and consequently the ordering quantity to any retailer $j \neq 1$ can be determined by

$$q_j = q_1 \frac{D_1}{D_j}. \tag{5}$$

Moreover, the vendor's reorder cycle is given by

$$T = nT_j = \frac{nq}{D_1}. \tag{6}$$

The following 5-retailer example will be used as an illustrative example to the mathematical models presented in this paper. In addition, the solution to VMI_OCC problem of Hariga et al. [13] for this example will serve as a benchmark to assess the performance of the mathematical models developed hereafter. The problem parameters are shown in Table 1.

TABLE 1. Example data.

Retailer	D_j	A_j	h_j	π_j	U_j	$\hat{A}_j$	$\hat{h}_j$
1	1200	3	0.85	0.45	60	1.8	5
2	800	2.5	0.9	0.35	50	1.6	5
3	2300	4.5	0.75	0.4	170	2.5	4.5
4	1800	3.5	0.8	0.4	140	2.0	5
5	3000	6	0.7	0.25	240	3.0	4.5
Vendor	$C = 6000, A_0 = 300, h_0 = 0.5, \hat{A}_0 = 50$ and $\hat{h}_0 = 4$						

TABLE 2. Unconstrained and VMI_OCC optimal solutions.

Retailer	Unconstrained optimal solution			VMI_OCC optimal solution		
	q_j^{unc}	z_j	E_j^{unc}	q_j^{occ}	z_j	E_j^{occ}
1	111.4	51.4	297.7	118.4	58.8	315.0
2	74.26	24.3	202.8	79.2	29.2	214.1
3	213.5	43.5	507.3	227.6	57.6	537.3
4	167.1	27.1	439.3	178.1	38.1	465.5
5	278.5	38.5	658.9	296.8	56.8	698.2
Vendor	$E_0^{unc} = 5202.8$			$E_0^{occ} = 3769.9$		
	$n = 4$, $TOC = 1986.7$, $E^{unc} = 7308.9$			$n = 3$, $TOC = 2018.8$, $E^{occ} = 6000$		

Table 2 shows the carbon-cap unconstrained and VMI_OCC optimal solutions where:

q_j^{unc} is the optimal lot size for retailer j under no carbon cap constraint, $j = 1, \ldots, m$

E_j^{unc} is the carbon emitted by SC member j under no carbon cap constraint, $j = 0, 1, \ldots, m$

E^{unc} is the total carbon emitted by all SC members under no carbon cap constraint

q_j^{occ} is the optimal lot size for retailer j under overall carbon cap constraint, $j = 1, \ldots, m$

E_j^{occ} is the carbon emitted by SC member j under overall carbon cap constraint, $j = 0, \ldots, m$

E^{occ} is the total carbon emitted by all SC members under overall carbon cap constraint

As pointed out by Turken et al. [61], existing models addressing sustainability aspects in the context of two stage SC systems have typically assumed that the vendors and the buyers are subjected to the same environmental regulations. In reality, however, SC members are usually independent entities, and may thus adhere to different environmental regulatory policies with different carbon caps. In addition, different carbon caps can be set by the vendor and the retailers as internal maximum emission levels in their inevitable shift to better address rising environmental concerns. Under such situations, the optimal amount of carbon footprint reported in Table 2 may not be feasible. For example, the VMI_OCC optimal solution would not be feasible if the carbon allowance for retailer 4 is 400 tons. Therefore, individual carbon cap (ICC) constraints for vendor and retailers should be explicitly accounted for in the optimization problem instead of the overall carbon cap constraint when the SC members have to satisfy their own carbon caps. Accordingly, the VMI optimization problem under individual carbon cap constraints can be stated as

VMI_ICC:

$$Min\ TOC\left(n, q, z_j\right) = \frac{K\left(n\right)}{q} D_1 + 0.5\frac{G\left(n\right)}{D_1} q + 0.5\frac{D_1}{q}\sum_{j=1}^{m}\frac{\pi_j}{D_j} z_j^2$$

$s.t.$

$$E_0\left(n, q\right) = \frac{\hat{K}_0(n)}{q} D_1 + 0.5\hat{H}_0(n)\frac{q}{D_1} \leq C_0 \tag{7}$$

$$E_j = \frac{\hat{K}_j}{q} D_1 + 0.5\hat{H}_j\frac{q}{D_1} \leq C_j \text{ for } j = 1, \ldots, m$$

$$\frac{D_j}{D_1} q - U_j - z_j \leq 0 \text{ for } j = 1, 2, \ldots, m$$

$$q \geq 0, n \text{ integer } z_j \geq 0 \text{ for } j = 1, 2, \ldots, m \tag{8}$$

where

$$K\left(n\right) = \frac{A_0}{n} + A,\ G\left(n\right) = nDh_0 + H,\ A = \sum\nolimits_{j=1}^{m} A_j, \text{ and}$$

$$H = \sum\nolimits_{j=1}^{m}\left(h_j - h_0\right) D_j$$

$$\hat{K}_0\left(n\right) = \frac{\hat{A}_0}{n}, \hat{K}_j = \hat{A}_j, \hat{H}_0\left(n\right) = \hat{h}_0 D\left(n-1\right), \text{ and } \hat{H}_j = \hat{h}_j D_j$$

Proposition 1 below shows that the $(m + 1)$ constraints in (7) and (8) can be transformed to two simple inequality constraints.

Proposition 1: Constraints (7) and (8) can be replaced by the following two inequalities

$$q^{l,max}(n) = Max[q_0^l\left(n\right), q_{ret}^{l,max}] \leq q \leq q^{u,min}(n) = Min[q_0^u\left(n\right), q_{ret}^{u,min}] \tag{9}$$

where $q_{ret}^{l,max} = Max\left[q_j^l j = 1, \ldots, m\right]$ and $q_{ret}^{u,min} = Min\left[q_j^u j = 1, \ldots, m\right]$,

$q_0^l\ (n)$ and $q_0^u\ (n)$ are given in (12),

q_j^l and q_j^u are given in (13)

Proof:

Given the quadratic form of the carbon cap constraints (7) and (8), they can be easilytransformed to simple inequalities given by:

$$q_0^l(n) \le q \le q_0^u(n) \tag{10}$$
$$q_j^l \le q \le q_j^u \text{ for } j = 1, \ldots, m \tag{11}$$

respectively, where

$$q_0^l(n) = \frac{C_0 - \sqrt{C_0^2 - 2\hat{K}_0(n)\hat{H}_0(n)}}{\hat{H}_0(n)} D_1 \text{ and } q_0^u(n) = \frac{C_0 + \sqrt{C_0^2 - 2\hat{K}_0(n)\hat{H}_0(n)}}{\hat{H}_0(n)} D_1 \tag{12}$$

$$q_j^l = \frac{C_j - \sqrt{C_j^2 - 2\hat{K}_j\hat{H}_j}}{\hat{H}_j} D_1 \text{ and } q_j^u = \frac{C_j + \sqrt{C_j^2 - 2\hat{K}_j\hat{H}_j}}{\hat{H}_j} D_1 \text{ for } j = 1, \ldots, m. \tag{13}$$

It is then obvious that the inequalities in (9) to (11) can be replaced by

$$Max[q_0^l(n), (q_j^l j = 1, \ldots, m)] \le q \le Min[q_0^u(n), (q_j^u j = 1, \ldots, m)] \quad \blacksquare$$

Therefore, the mathematical form of VMI_ICC optimization problem can be rewritten as

VMI_ICC:

$$Min\ TOC(n, q, z_j) = \frac{K(n)}{q} D_1 + 0.5\frac{G(n)}{D_1} q + 0.5\frac{D_1}{q}\sum_{j=1}^{m}\frac{\pi_j}{D_j} z_j^2$$

$s.t.$

$$q^{l,max}(n) \le q \le q^{u,min}(n)$$
$$\frac{D_j}{D_1} q - U_j - z_j \le 0 \text{ for } j = 1, 2, \ldots, m$$
$$q \ge 0, n \text{ integer } z_j \ge 0 \text{ for } j = 1, 2, \ldots, m \tag{14}$$

The next proposition states the feasibility conditions for the optimization problem VMI_ICC.

Proposition 2: VMI_ICC is feasible under the following conditions

1-

$$C_j \ge \sqrt{2\hat{K}_j\hat{H}_j} = \sqrt{2\hat{A}_j\hat{h}_j D_j} = E_j^{min} \text{ for } j = 1, \ldots, m. \tag{15}$$

2-

$$C_0 \ge \sqrt{2\hat{A}_0\hat{h}_0 D} = E_0^{min} \tag{16}$$

3-

$$q^{l,max} \le q^{u,min}$$

Proof:

1- The least amount of carbon emitted by the jth retailer is obtained by minimizing its carbon footprint E_j in (8), which is $E_j^{min} = \sqrt{2\hat{K}_j\hat{H}_j} = \sqrt{2\hat{A}_j\hat{h}_j D_j}$. Therefore, VMI_ICC would not be feasible if the carbon cap set for any retailer is smaller than the minimum amount of carbon it can generate. In addition, the lower and upper bounds for the ordering quantity in (13) are not defined for $C_j < E_j^{min}$.

2- It clear from (12) that $q_0^l(n)$ and $q_0^u(n)$ are not defined when $C_0^2 > 2\hat{K}_0(n)\hat{H}_0(n)$. Substituting the expressions of $\hat{K}_0(n)$ and $\hat{H}_0(n)$, the inequality becomes $C_0^2 > 2\frac{\hat{A}_0}{n}\hat{h}_0(n-1)D$ or $C_0^2 > 2\hat{A}_0\hat{h}_0 D\left(1 - \frac{1}{n}\right) > 2\hat{A}_0\hat{h}_0 D$.

3- It is clear that VMI_ICC would not be feasible because of constraints (14) if the condition $q^{l,max} \le q^{u,min}$ is not satisfied. $\blacksquare$

It should also be noted that when the carbon caps for the SC members satisfy $C_j > E_j^{unc}$ for $j = 0, \ldots, m$, then the unconstrained carbon problem solution $(q_j^{unc}, j = 1, \ldots, m)$ is also the optimal solution to the VMI_ICC problem. In the following, it is assumed that the problem parameters satisfy all conditions in Proposition 2 to ensure the feasibility of VMI_ICC and $C_j < E_j^{unc}$ for $j = 0, \ldots, m$. Therefore, the carbon caps for the SC members should satisfy:

$$E_j^{min} \le C_j \le E_j^{max} = E_j^{unc}\ for\ j = 0, 1, \ldots, m \tag{17}$$

We next outline a simple algorithm to solve VMI_ICC problem optimally. Given the convexity of the objective function of VMI_ICC with respect to n,the algorithm starts with $n = 1$ and then increments its value by one until the first time the cost function increases. In addition, given that $q_0^u(n)$ in (12) is decreasing with respect to n, the algorithm also stops when it reaches the infeasibility $q_0^u(n) < Max[q_j^l j = 1, \ldots, m]$. Therefore, the largest number of iterations needed before the infeasibility condition takes place is the largest integer n such that $q_0^u(n) \ge q_{ret}^{l,max}$. In order to find such n,we need to solve a third-degree polynomial equation $q_0^u(n) = q_{ret}^{l,max}$, which can only be solved numerically. Alternatively, we provide an upper bound value, n^{max}, on the largest number of iterations. The equation $q_0^u(n) = q_r^{l,max}$ can be rewritten as:

$$\left[C_0 + \sqrt{C_0^2 - 2\hat{K}_0(n)\hat{H}_0(n)}\right] D_1 = \hat{H}_0(n)\, q_{ret}^{l,max}.$$

Given that the function in the left-hand (right-hand) side of the above equation is decreasing (increasing) in n, then an upper bound value for n is the solution to

$$\left[C_0 + \sqrt{C_0^2 - 2\hat{K}_0(1)\hat{H}_0(1)}\right] D_1 = \hat{H}_0(n)\, q_{ret}^{l,max},$$

which is

$$n_1^{max} = \left\lfloor \frac{2D_1C_0}{\hat{h}_0 D q_{ret}^{l,max}} + 1 \right\rfloor \tag{18}$$

where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x.

A. VMI_ICC ALGORITHM

0- Compute n_1^{max} using (18).

1- Set $n = 1$ and $TOC^* = \infty$.

2- Find $q^{unc}(n)$, the optimal ordering quantity to the unconstrained carbon cap optimization problem, which can be found using the algorithm in Hariga et al. [13].

3- Compute $q^{l,max}(n)$ and $q^{u,min}(n)$ using (9).

4- The optimal ordering quantity $q^*(n)$ is given by:

$$q^*(n) = \begin{cases} q^{unc}(n) \text{ if } & q^{l,max}(n) \leq q^{unc}(n) \leq q^{u,min}(n) \\ q^{l,max}(n) \text{ if } & q^{unc}(n) \leq q^{l,max}(n) \\ q^{u,min}(n) \text{ if } & q^{unc}(n) \geq q^{u,min}(n) \end{cases}$$

Compute $TOC\left(n, q^*(n), z_j\right)$ using (1), where $z_j = Max\left[0, \frac{D_j}{D_1} q^*(n) - U_j\right]$ for $j = 1, \ldots, m$.

5- If $TOC\left(n, q^*(n), z_j\right) < TOC^*$ then set $TOC^* = TOC\left(n, q^*(n), z_j\right), n^* = n, q^* = q^*(n)$, and $n \leftarrow n + 1$.

6- $n \leq n_1^{max}$ go to step 3. Otherwise, set $n \leftarrow n - 1$ and stop.

7- If $TOC\left(n, q^*(n), z_j\right) \geq TOC^*$, stop.

It should also be noted that at least one SC member (one of the retailers or the vendor) will fully utilize its allowable carbon quota when $q^*(n) = q^{l,max}(n)$ or $q^*(n) = q^{u,min}(n)$ (i.e., at least one carbon cap constraint will be binding).

It is easy to see from the different steps of the algorithm that its time complexity depends on step 2 to find $q^{unc}(n)$ and the number of iterations needed before convergence. Given that step 2 makes use of the algorithm in Hariga et al. [13], then this step runs in the order of O($mlog(m)$). Therefore, the time complexity of the above algorithm is of the order O($mlog(m) + n_1^{max}$).

The following theorem shows that the above algorithm indeed yields the optimal solution to VMI_ICC problem.

Theorem 1: The solution generated by the VMI_ICC algorithm is the optimal solution to VMI_ICC problem.

Proof: Given the convexity of its objective function and the feasible region, VMI_ICC has a unique global minimum for a given n. Accordingly, to show that the solution $q^*(n)$ is optimal, we need to show that it satisfies the Karush-Kuhn-Tucker (KKT) equations (Bazaraa et al. [62]). Let $\theta^l(n), \theta^u(n)$, and λ_j $(j = 1, 2, \ldots, m)$ be the Lagrangian multipliers corresponding to the constraints ($q^{l,max}(n) \leq q$), ($q \leq q^{u,mn}(n)$) and the constraints ($\frac{D_j}{D_1} q - U_j - z_j \leq 0 : j = 1, 2, \ldots, m$) in VMI_ICC. Therefore, ($q, z_j : j = 1, 2, \ldots, m$) is an optimal solution if and only if there exists non-negative $\theta^l(n), \theta^u(n)$, and λ_j $(j = 1, 2, ..m)$ satisfying the following KKT conditions, which are obtained from the initial KKT equations after some algebraic manipulations:

$$q^2\left(n, \theta^l(n), \theta^u(n)\right) = \frac{2K(n) + \sum_{j \in J^+(n)} \frac{\pi_j U_j^2}{D_j}}{G(n) + \theta^u(n) - \theta^l(n) + \sum_{j \in J^+(n)} \pi_j D_j} D_1^2 \tag{19}$$

where $J^+(n)$ is the set of retailers with over-stock satisfying:

$$q^2\left(n, \theta^l(n), \theta^u(n)\right) \geq U_j^2 \big/ D_j^2 \text{e} \tag{20}$$

$$\theta^l(n)\left(q^{l,max}(n) - q\left(n, \theta^l(n), \theta^u(n)\right)\right) = 0 \tag{21}$$

$$\theta^u(n)\left(q\left(n, \theta^l(n), \theta^u(n)\right) - q^{u,min}(n)\right) = 0 \tag{22}$$

$$z_j\left(n, \theta^l(n), \theta^u(n)\right) = D_j \frac{q\left(n, \theta^l(n), \theta^u(n)\right)}{D_1} - U_j \text{ for } j \in J^+(n) \tag{23}$$

$$z_j\left(n, \theta^l(n), \theta^u(n)\right) = 0 \text{ for } j \notin J^+(n) \tag{24}$$

$$\lambda_j = \frac{D_1 \pi_j}{D_j q\left(n, \theta^l(n), \theta^u(n)\right)} z_j \text{ for } j = 1, \ldots, m. \tag{25}$$

First, when $q^{l,max}(n) \leq q_u(n) \leq q^{u,min}(n)$, the Lagrangian multipliers $\theta^l(n)$ and $\theta^u(n)$ are set equal to 0, $q\left(n, \theta^l(n), \theta^u(n)\right) = q_u(n)$, and λ_j $(j = 1, 2, ..m)$ can be found using (25) after computing z_j $(j = 1, 2, ..m)$ from (23) and (24). Therefore, the unconstrained carbon cap solution, $q_u(n)$, is optimal as it satisfies KKT conditions. Next, if $q_u(n) \leq q^{l,max}(n)$, set $q\left(n, \theta^l(n), \theta^u(n)\right) = q^{l,max}(n)$, $\theta^u(n) = 0$, and determine $\theta^l(n)$ from (19) after identifying the set $J^+(n)$ using (23) and (24). The Lagrangian multipliers λ_j $(j = 1, 2, ..m)$ can be calculated using (25). Therefore, $q^{l,max}(n)$ is optimal as it satisfies KKT conditions. In a similar way, all Lagrangian multipliers can be determined for the case $q_u(n) \geq q^{u,min}(n)$. In this case, $q^{u,min}(n)$ is the optimal solution. As mentioned above, given the convexity of the objective function with respect to n,then performing the different steps of VMI_ICC algorithm will result in the optimal solution to the VMI_ICC optimization problem. ■

Illustrative example:

Consider the same illustrative example above with the carbon caps for the retailers and vendor given in Table 3. The same table shows the minimum and maximum carbon emissions for all SC members.

TABLE 3. Carbon allowances of the SC members.

Retailer	E_j^{min}	E_j^{max}	C_j
1	146.97	297.86	200
2	113.14	202.88	160
3	227.49	507.29	440
4	189.74	439.25	200
5	284.6	658.88	500
Vendor	1907.88	5202.82	5000

We define the tightness or the restrictiveness of the problem constraints as how close are the individual carbon allowances to the minimum allowed carbon emissions. Mathematically, it is given by

$$T_g = \frac{\sum_{j=0}^{m} (C_j - E_j^{min}) / \sum_{j=0}^{m} (E_j^{max} - E_j^{min})}{m+1}$$

For the illustrative example at hand, the tightness value is equal to 0.53. Therefore, it cannot be considered as a restrictive problem since the tightness value is not low.

TABLE 4. Results of VMI_ICC algorithm.

n	q^{unc}	$q^{l,max}$	$q^{u,min}$	q^*	TOC^*	E^{icc}
1	304.0	18.23	35.1	35.1	11026	2709
2	186.2	18.23	35.1	35.1	5963.9	2386
3	138.0	18.23	35.1	35.1	4321.1	2634
4	111.4	18.23	35.1	35.1	3532.9	3024
5	94.3	18.23	35.1	35.1	3086.6	3471
6	81.6	18.23	35.1	35.1	2811.3	396
7	72.2	18.23	35.1	35.1	2633.6	4438
8	64.9	18.23	35.1	35.1	2517	4939
9	59.2	18.23	35.1	35.1	2441.1	5448
10	54.5	18.23	35.1	35.1	2393.7	5961

The results of the different iterations when running VMI_ICC algorithm are reported in Table 4, where E^{icc} is the total carbon emitted by all SC members under individual carbon cap constraint. The maximum number of iterations before reaching the infeasibility condition is $n^{max} = 19$. However, the optimal solution was obtained after 11 iterations.

As shown in Table 4, the optimal solution is to make 10 deliveries to the retailers with the first retailer receiving a quantity equal to 35.1 units at a minimum total cost of 2393.7. Compared to the cost of the constrained overall carbon cap problem (see Table 2), the constrained individual carbon cap resulted in an increase in the total operational costs of 374.9. Such result is expected as the solution to the VMI_OCC problem is not a feasible solution to VMI_ICC problem since $E_j^{occ} > C_j$ for all retailers. This can be observed by comparing the right-most column in Table 2 to the right-most column in Table 3. In addition, VMI_ICC optimal solution leads to 1347.9 tons reduction in carbon emissions with a SC total operational costs increase of 407 when compared to the solution of the unconstrained carbon cap problem. Therefore, the price per ton reduction in CO2 emissions when implementing the VMI_ICC solution is 0.302.

Given that the retailers are subject to tight individual carbon cap constraints, the vendor can exchange part of its carbon allowance with the retailers in order to further reduce the SC total operational costs. For instance, when the vendor allocates 50, 30, 20, 200 and 300 tons of his/her carbon allowance to retailers 1 to 5, respectively, the SC total operational costs will decrease to 2086, corresponding to a cost saving of 308. In the following section, we formulate the problem of finding the optimal carbon allowances exchange levels between SC members towards the attainment of further reduction in the total operational costs.

IV. VENDOR MANAGED INVENTORY AND CARBON EXCHANGE MODEL

As it was observed in the illustrative example of the last section, the exchange of carbon allowances between SC members can result in a reduction of the chain-wide total operational costs. Given that the vendor determines the order quantity for each member of the SC which affects their inventory level and the number of placed orders, he/she can build on this strategic partnership to promote a collaborative environment whereby he/she would also be responsible for deciding on the carbon emitted by all SC members. That is, he/she should also manage the carbon emitted by the SC members in addition to their inventories. From a practical standpoint, this approach is appealing, as it would further incentivize SC members (particularly retailers) to engage in such partnership due to three main reasons. Firstly, given that the vendor is in charge of managing inventories as well as carbon emissions for all members, each retailer would thus be relieved from the task of managing those emissions and acquiring extra credit in case of exceeding his/her carbon quota. Secondly, as the VMI partnership essentially presents a sort of consortium between SC members, such exchange of carbon would take place at no charge rendering this approach very attractive in practice. Finally, this internal exchange of carbon credits has the potential of yielding cost savings that are shared among members in such a way that no member is worse-off from a cost standpoint.

Thereafter, we call this problem the vendor managed inventory and carbon (VMIC) problem and the new regulation policy the "carbon cap and exchange" policy. Under such policy, SC members are encouraged to exchange portion of their carbon allowances with other members. In essence, this newly proposed policy is similar to the well-established carbon cap-and-trade policy with the difference that the carbon is being traded internally between the SC members instead of the carbon market at no costs and revenues. As noted earlier, the resulting chain-wide cost savings can be shared with members who exchanged portion of their carbon credits to ensure they are not worse off from the cost perspective.

The following additional decision variables are introduced to formulate the VMIC model.

E_j^+ : Amount of carbon credit acquired by member j from other SC members, $j = 0, 1, \ldots, m$.

E_j^- : Amount of carbon credit handed out by member j to other SC members, $j = 0, 1, \ldots, m$.

The problem can then be stated mathematically as:

VMIC:

$$Min\ TOC\left(n, q, z_j\right) = \frac{K(n)}{q} D_1 + 0.5 \frac{G(n)}{D_1} q + 0.5 \frac{D_1}{q} \sum_{j=1}^{m} \frac{\pi_j}{D_j} z_j^2$$

$$s.t.$$
$$E_0(n,q) = \frac{\hat{K}_0(n)}{q}D_1 + 0.5\hat{H}_0(n)\frac{q}{D_1} \leq C_0 + E_0^+ - E_0^- \tag{26}$$
$$E_j(q) = \frac{\hat{K}_j}{q}D_1 + 0.5\hat{H}_j\frac{q}{D_1} \leq C_j + E_j^+ - E_j^- \text{ for } j = 1,\ldots,m \tag{27}$$
$$E_j^- \leq C_j - E_j^{min} \text{ for } j = 0,1,\ldots,m \tag{28}$$
$$\sum_{j=0}^{m} E_j^+ = \sum_{j=0}^{m} E_j^-$$
$$\frac{D_j}{D_1}q - U_j - z_j \leq 0 \text{ for } j = 1,2,\ldots,m$$
$$E_j^+ \geq 0 \text{ and } E_j^- \geq 0 \text{ for } j = 0,1,\ldots,m$$
$$q \geq 0, n \text{ integer}, z_j \geq 0 \text{ for } j = 1,2,\ldots,m \tag{29}$$

Constraint (26) states that the carbon emitted by the vendor should not exceed the new carbon limit after the carbon exchange mechanism. Similarly, constraint set (27) ensures the carbon generated by each retailer should be limited to its new carbon allowance after the exchange. Constraints (28) guarantee that the amount of carbon traded by SC member j is not exceeding its maximum amount of carbon that can be exchanged, $C_j - E_j^{min}$. Given that carbon exchange is carried out only within the SC, then constraint (29) warrants that the total amount of carbon received by some of the SC members is equal to the total amount of carbon handed out by the remaining members. It should also be noted that the optimal solution to VMI_ICC problem is feasible to the above optimization problem VMIC. Therefore, from an economic standpoint, VMI_ICC problem will provide an upper bound to the VMIC problem. The next lemma states an important property of the VMIC optimal solution whereby the amounts of carbon credits acquired and handed out by the jth SC member cannot be both positive.

Lemma 1: The VMIC optimal solution is characterized by
$$E_j^+E_j^- = 0 \text{ for } j = 0,1,\ldots,m \tag{30}$$

Proof: Let $(\tilde{E}_j^+, \tilde{E}_j^-)$ be a feasible solution to VMIC problem that does not satisfy (30). Then, let

$\bar{E}_j = Min(\tilde{E}_j^+, \tilde{E}_j^-)$, $E_j^+ = \tilde{E}_j^+ - \bar{E}_j$ and $E_j^- = \tilde{E}_j^- - \bar{E}_j$ for all j. It is clear that (E_j^+, E_j^-) is also a feasible solution to VMIC problem and satisfies (30). Therefore, there is an optimal solution satisfying (30). ■

Next, note that by summing the inequalities in (26) and (27) we get
$$\frac{\hat{K}_0(n)}{q}D_1 + 0.5\hat{H}_0(n)\frac{q}{D_1} + \sum\nolimits_{j=1}^{m}\left[\frac{\hat{K}_j}{q}D_1 + 0.5\hat{H}_j\frac{q}{D_1}\right] \leq \sum\nolimits_{j=0}^{m}\left[C_j + E_j^+ - E_j^-\right]$$
which after using (29) can be rewritten as
$$\frac{\hat{K}_0(n)}{q}D_1 + 0.5\hat{H}_0(n)\frac{q}{D_1} + \sum\nolimits_{j=1}^{m}\left[\frac{\hat{K}_j}{q}D_1 + 0.5\hat{H}_j\frac{q}{D_1}\right] \leq \sum\nolimits_{j=0}^{m} C_j$$

The last inequality can be further simplified and written as
$$\frac{\hat{K}(n)}{q}D_1 + 0.5\frac{\hat{G}(n)}{D_1}q \leq \sum\nolimits_{j=0}^{m} C_j \tag{31}$$
where
$$\hat{K}(n) = \sum_{j=0}^{m}\hat{K}_j \text{and } \hat{G}(n) = \sum\nolimits_{j=0}^{m}\hat{H}_j \tag{32}$$

It is clear that any VMIC feasible solution would also satisfy (31). In the following lemma, we show that any feasible solution, q, to (31) would also be a VMIC feasible solution.

Lemma 2: A feasible solution, q, to (31) is also feasible to VMIC problem.

Proof: Let q be an ordering quantity satisfying (31). Then the amount of carbon emitted by the vendor and the retailers are:
$$E_0(n,q) = \frac{\hat{K}_0(n)}{q}D_1 + 0.5\hat{H}_0(n)\frac{q}{D_1}, \text{ and}$$
$$E_j(q) = \frac{\hat{K}_j}{q}D_1 + 0.5\hat{H}_j\frac{q}{D_1} \text{ for } j = 1,\ldots,m,$$
respectively.

For q to satisfy constraints (26) and (27), we set
$$E_0^+ = E_0(n,q) - C_0 \text{ if } E_0(n,q) > C_0 \text{ and } E_0^+ = 0 \text{ otherwise,} \tag{33}$$
$$E_0^- = C_0 - E_0(n,q) \text{ if } E_0(n,q) < C_0 \text{ and } E_0^- = 0 \text{ otherwise,} \tag{34}$$
$$E_j^+ = E_j(q) - C_j \text{ if } E_j(q) > C_j \text{ and } E_j^+ = 0 \text{ otherwise for } j = 1,\ldots,m, \tag{35}$$
$$E_j^- = C_j - E_j(q) \text{ if } E_j(q) < C_j \text{ and } E_j^- = 0 \text{ otherwise for } j = 1,\ldots,m. \tag{36}$$

We also define $J^{>0}(n)$ and $J^{<0}(n)$ to denote the set of SC members, including the vendor, with $E_j^+ > 0$ and $E_j^- > 0$, respectively. By the result of Lemma 1, we have $E_j^- = 0$ for all $j \in J^{>0}(n)$ and $E_j^+ = 0$ for all $j \in J^{<0}(n)$. We need now to show that constraints (28) and (29) are also satisfied.

Given that by definition $E_0^{min} \leq E_0(n,q)$ and $E_j^{min} \leq E_j(q)$ for $j = 1,\ldots,m$, then $C_0 - E_0^{min} \geq C_0 - E_0(n,q) = E_0^-$ and $C_j - E_j^{min} \geq C_j - E_j(q) = E_j^-$ for $j = 1,\ldots,m$. Therefore, constraints (28) are satisfied.

To show the feasibility of constraint (29), first suppose that $\sum_{j\in J^{>0}(n)} E_j^+ < \sum_{j\in J^{<0}(n)} E_j^-$. Then, $(E_j^-, j \in J^{<0}(n))$ can be reduced to make equation (29) feasible without affecting the feasibility of constraints (28). Next, we show that $\sum\limits_{j\in J^{>0}(n)} E_j^+$ cannot be strictly larger than $\sum_{j\in J^{<0}(n)} E_j^-$. For this purpose, assume that $\sum_{j\in J^{>0}(n)} E_j^+ > \sum_{j\in J^{<0}(n)} E_j^-$ for a feasible q satisfying (31). Then, we have $\sum_{j\in J^{>0}(n)} \left[E_j(q) - C_j\right] >$

$\sum_{j\in J^{<0}(n)}\left[C_j - E_j(q)\right]$, or after arranging terms $\sum_{j=0}^{m} E_j(q) > \sum_{j=0}^{m} C_j$, which contradicts the feasibility of q.

Based on the results of Lemmas 1 and 2, we state the next theorem providing the optimal solution to VMIC problem.

Theorem 2: The optimal ordering quantity for the first retailer in the VMIC problem is given by:

$$q^*(n) = \begin{cases} q^{unc}(n) \text{ if } Q^l(n) \le q^{unc}(n) \le Q^u(n) \\ Q^l(n) \text{ if } q^{unc}(n) \le Q^l(n) \\ Q^u(n) \text{ if } q^{unm}(n) \ge Q^u(n) \end{cases} \tag{37}$$

where

$q^{unc}(n)$ is the optimal solution to the unconstrained carbon problem,

$$Q^l(n) = \frac{\sum_{j=0}^{m} C_j - \sqrt{\left(\sum_{j=0}^{m} C_j\right)^2 - 2\hat{K}(n)\hat{G}(n)}}{\hat{G}(n)} D_1 \text{ and}$$
$$Q^u(n) = \frac{\sum_{j=0}^{m} C_j + \sqrt{\left(\sum_{j=0}^{m} C_j\right)^2 - 2\hat{K}(n)\hat{G}(n)}}{\hat{G}(n)} D_1 \tag{38}$$

Proof:

First $Q^l(n)$ and $Q^u(n)$, given in (38), are the two roots of the quadratic inequality (31). In addition, both roots are also feasible to VMIC problem by Lemma 2. Suppose that one of the last two conditions in (37) is satisfied and assume that there exists another optimal solution $q'(n)$, different to $q^{unc}(n)$ as the latter is outside the range $[Q^l(n), Q^u(n)]$, satisfying (31) but not binding. This contradicts that either $Q^l(n)$ or $Q^u(n)$ is optimal to VMI_OCC problem under the condition regarding $q^{unc}(n)$. Therefore, the optimal solution to VMIC problem is one of the roots of (31) depending on which of the last two conditions in (37) holds. Suppose now that the first condition is satisfied. Then, $q^{unc}(n)$ is feasible by Lemma 2 and it is optimal as it provides a lower bound on the total operational costs, TOC. ■

It can be noted that $Q^l(n)$ and $Q^u(n)$ in (38) are defined only when $\left(\sum_{j=0}^{m} C_j\right)^2 > 2\hat{K}(n)\hat{G}(n)$. Therefore, the largest number of deliveries that can be made to the retailers for every vendor cycle, n_2^{max}, is the largest integer, n, such that $\left(\sum_{j=0}^{m} C_j\right)^2 > 2\hat{K}(n)\hat{G}(n)$. In the following, we provide two properties of the optimal solution to VMIC problem.

*Property 1:*If $q^*(n) = Q^l(n)$ or $q^*(n) = Q^u(n)$, then equation (31) is binding and we have $\sum_{j\in J^{>0}(n)} E_j^+ = \sum_{j\in J^{<0}(n)} E_j^-$.

Proof: As $Q^l(n)$ and $Q^u(n)$ are the roots of inequality (31), then it is binding when $q^*(n) = Q^l(n)$ or $q^*(n) = Q^u(n)$. Therefore, the values for (E_j^+ and $E_j^-, j = 0, 1, \ldots, m$) have to be set such that $E_j(q^*(n)) = C_j + E_j^+ - E_j^-$ for $j = 0, 1, \ldots, m$ and $\sum_{j\in J^{>0}(n)} E_j^+ = \sum_{j\in J^{<0}(n))} E_j^-$, otherwise inequality (31) will not be binding. ■

*Property 2:*If $q^*(n) = q^{unc}(n)$, then constraint (31) is not binding and we have $\sum_{j\in J^{>0}(n)} E_j^+ < \sum_{j\in J^{<0}(n)} E_j^-$, where ($E_j^+, j \in J^{>0}(n)$) and ($E_j^-, j \in J^{<0}(n)$) are given by (33) to (36). In this case, ($E_j^-, j \in J^{<0}(n)$) are adjusted to reach the equality in (29).

Proof: First, it is clear that constraint (31) is not binding since $q^{unc}(n)$ is not one of its roots. Next, given that it is assumed that the carbon caps, ($C_j, j = 0, 1, \ldots, m$) do not satisfy the conditions for the optimality of the unconstrained solution, then for some SC members $C_j < E_j(q^*(n)) = E_j^{unc}$. For these SC members, $j \in J^{>0}(n)$, $E_j^+ = E_j(q^{unc}(n)) - C_j$ and $E_j^- = 0$. As shown in Lemma 2, we have $\sum_{j\in J^{>0}(n)} E_j^+ < \sum_{j\in J^{<0}(n)} E_j^-$ if E_j^- for $j \in J^{<0}(n)$ are determined using (34) and (36). Therefore, ($E_j^-, j \in J^{<0}(n)$) has to be adjusted to satisfy the equality in (29). ■

Based on the above results, we propose the following simple algorithm to generate the optimal solution to VMIC model.

A. VMIC ALGORITHM

1) Find n_2^{max}, the largest integer, n,such that $\left(\sum_{j=0}^{m} C_j\right)^2 > 2\hat{K}(n)\hat{G}(n)$.
2) Set $n = 1$ and $TOC^* = \infty$.
3) Find $q^{unc}(n)$, the optimal ordering quantity to the unconstrained carbon cap optimization problem.
4) Compute $Q^l(n)$ and $Q^u(n)$ using (38).
5) The optimal ordering quantity $q^*(n)$ is given by:

$$q^*(n) = \begin{cases} q^{unc}(n) \text{ if } Q^l(n) \le q^{unc}(n) \le Q^u(n) \\ Q^l(n) \text{ if } q^{unc}(n) \le Q^l(n) \\ Q^u(n) \text{ if } q^{unc}(n) \ge Q^u(n) \end{cases}$$

Compute $TOC\left(n, q^*(n), z_j\right)$ using (1), where $z_j = Max\left[0, \frac{D_j}{D_1}q^*(n) - U_j\right]$ for $j = 1, \ldots, m$.

Compute $E_0(n, q^*(n))$ and $E_j(q)$ for $j = 1, \ldots, m$ using (26) and (27), respectively.

If $q^*(n)$ is equal to $Q^l(n)$ or $Q^u(n)$, compute (E_j^+ and $E_j^-, j = 0, 1, \ldots, m$) using (33) to (36).

If $q^*(n) = q^{unc}(n)$, compute ($E_j^+, j = 0, 1, \ldots, m$) using (33) and (35). For all $j \in J^{<0}(n)$), compute E_j^- such that $\sum_{j\in J^{>0}(n)} E_j^+ = \sum_{j\in J^{<0}(n))} E_j^-$.

6) If $TOC\left(n, q^*(n), z_j\right) < TOC^*$ then set $TOC^* = TOC\left(n, q^*(n), z_j\right)$, $n^* = n, q^* = q^*(n)q^*, n \leftarrow n+1$.
7) If $n \le n_2^{max}$ go to step 2. Otherwise, set $n \leftarrow n - 1$ and stop.
8) If $TOC\left(n, q^*(n), z_j\right) \ge TOC^*$, stop.

As shown for the algorithm in Section III, the time complexity of the proposed algorithm is also of the order $O(m\log(m) + n_2^{max})$.

In order to determine the amount of carbon handed out by SC member $j \in J^{<0}(n)$) to member $j' \in J^{>0}(n)$, $E_{jj'}^-$, one needs to solve the following equations:

$$E_j^- = \sum_{j' \in J^{>0}} E_{jj'}^- \text{ for } j \in J^{<0} \quad (39)$$

$$E_{j'}^+ = \sum_{j \in J^{<0}} E_{jj'}^- \text{ for } j' \in J^{>0} \quad (40)$$

TABLE 5. An example of a feasible solution to Equations (39) and (40).

j	0	1	2	3	4	5	E_j^+
0	-	0	0	0	0	0	0
1	20	-	0	20	0	0	40
2	0	0	-	0	0	0	0
3	0	0	0	-	0	0	0
4	35	0	0	0	-	0	35
5	15	0	30	0	0	-	45
E_j^-	70	0	30	20	0	0	

Table 5 reports one of the multiple solutions to the above equations for the case of 5 retailers with $E_j^+ = (0, 40, 0, 0, 35, 45)$ and $E_j^- = (70, 0, 30, 20, 0, 0)$. For example, the vendor gave 20, 35, and 15 tons to retailers 1, 4, and 5, respectively, and retailer 2 gave 30 tons to retailer 5.

Illustrative Example

Continuing with the same example of the previous section, we now illustrate the mechanism of the VMIC algorithm. The results of the different iterations of the VMIC algorithm are exhibited in Table 6. The maximum number of iterations needed to solve the VMIC problem, n_2^{max}, is 48. However, the algorithm generated the optimal solution after four iterations.

The minimum total operational cost of VMIC problem is 1997.97 when three orders are delivered to each retailer with the first retailer receiving 138.05 units. This VMIC ordering policy resulted in a cost saving of 395.7 when compared to the VMI_ICC solution. However, due to the relaxation of the retailers' carbon allowances, the carbon generated by all SC members increased by 538.67 tons. However, when compared to the solution of the unconstrained carbon cap problem, the VMIC ordering policy leads to 808.9 tons reduction in carbon footprint at a mere increase in the total operational cost of 11.27. Consequently, the price of a one-ton reduction of CO2 is reduced from 0.302 to 0.014 when implementing VMIC instead of VMI_ICC.

Table 7 shows the amount of carbon exchanged between SC members as well as the operational costs and carbon emissions for all SC members under VMI_ICC and VMIC ordering policies. It can be observed from the table that all SC members benefited from VMIC partnership (i.e. they are all better off cost-wise) when comparing their operational costs under VMI_ICC and VMIC ordering policies, as a result of the vendor transferring 384.48 tons of his/her carbon allowance (5000 tons) to the retailers. This transfer increased the carbon allowances of the retailers and relaxed their constraints as it can be noticed in the carbon emissions of the SC members (E_j values) under both ordering policies. In general, the cost savings resulting from the implementation of the "carbon cap and exchange" policy can be used to ensure that no SC member becomes worse off in the worst-case scenario due to his/her engagement in the VMIC partnership.

V. SENSITIVITY ANALYSIS

In this section, we conduct a sensitivity analysis to assess the benefits of exchanging carbon allowances between the SC members. To that end, three measures of performance are adopted throughout the analysis. The first one measures the cost improvement (reduction) of VMIC policy over the case where no carbon exchanges are allowed (VMI_ICC policy) and it is given by:

$$\%CR = 100 * \frac{VMI\_ICCcost - VMICcost}{VMI\_ICCcost}$$

The second one measures the price (cost) associated with reducing carbon emissions of the whole SC by one ton when implementing VMI policy, which is expressed as:

$$PR_{VMIC} = \frac{VMICcost - VMI\_unconstrainedcost}{VMI\_unconstrainedCO2 - VMICCO2}$$

The third one, PR_{VMI_ICC}, is the price of one ton reduction of CO2 emissions under VMI_ICC policy:

$$PR_{\mathrm{VMI\_ICC}} = \frac{\mathrm{VMI\_ICC}cost - VMI\_unconstrainedcost}{VMI\_unconstrainedCO2 - \mathrm{VMI\_ICC}CO2}$$

The only parameter that was varied in the analysis is the number of retailers $m \in \{5, 6, 7, 8, 9, 10\}$. We use the same data of the illustrative example when $m = 5$. The problem data for the other cases of m are shown in Table 8.

Moreover, the carbon allowance, C_j for $j = 0, 1, \ldots, m$, is randomly generated from a uniform distribution. For each m,a total of 20 random test instances are generated for the carbon allowances. In the first 10 random instances, (C_j for $j = 0, 1, \ldots, m$) were uniformly generated in the range $[E_j^{min}, 0.5 * (E_j^{min} + E_j^{max})]$. On the other hand, the carbon allowances were randomly generated from the uniform distribution $[0.5 * (E_j^{min} + E_j^{max}), E_j^{max}]$ for the last 10 random cases. The reason for generating two different sets of carbon allowances is to avoid having most of the test instances either having restricted or relaxed individual carbon constraints. As a result, a total of 120 instances were solved. For each solved problem, the above three performance measures are computed as well as the constraints tightness, T_g. The results of the sensitivity study are analyzed next.

It is noted that the two solution algorithms for VMI_ICC and VMIC problems, respectively, were coded using Visual Basic and executed on a personal computer with a 3.60 GHz processor and 16 GB RAM. Solving each problem instance in the sensitivity analysis experiments took only few iterations and less than a second of CPU time.

A. IMPACT OF THE NUMBER OF RETAILERS

The first analysis relates to the impact of the number of retailers involved in the VMI partnership. For each of the

TABLE 6. Results of VMIC algorithm.

	VMIC Results					
n	$q^{unc}(n)$	$Q^l(n)$	$Q^u(n)$	$q^*(n)$	$TOC^*(n)$	$E(n)$
1	304.0	11.6	352.5	304.0	2359.4	5668
2	186.2	6.9	189.9	186.2	2062.9	6381
3	138.1	5.3	129.6	129.6	1997.9	6500

TABLE 7. Detailed carbon exchanges between the SC members and their costs.

j	E_j^+	E_j^-	Carbon Emission (E_j)		Operational Cost	
			VMI_ICC	VMIC	VMI_ICC	VMIC
0	0	384.48	4962.01	4615.52	1624.54	1439.15
1	67.22	0	149.29	267.22	117.48	82.8
2	23.04	0	113.2	183.04	96.0	62.0
3	13.55	0	236.84	453.55	179.08	134.81
4	192.27	0	200	392.27	140.72	110.16
5	88.40	0	300	588.40	235.84	168.95
Total	384.48	384.48	5961.34	6500.0	2393.67	1997.97

TABLE 8. Problem data for the cases when **m** > **5.**

Retailer	D_j	A_j	h_j	π_j	U_j	$\widehat{A}_j$	$\widehat{h}_j$
6	2000	4	0.8	0.3	180	2	4.7
7	2500	5	0.6	0.8	250	2.4	5.2
8	4000	2	0.55	0.6	420	3.2	6.4
9	1000	10	0.65	0.2	120	4.5	5.8
10	1500	8	0.7	0.3	200	4	4.7

three performance measures, we computed the average values as a function of the number of retailers where the results are presented in the two figures below.

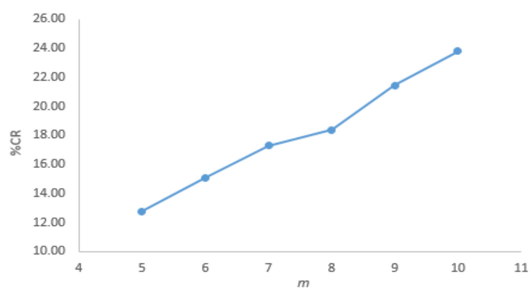

FIGURE 1. Variation of the cost reduction by VMIC policy as a function of *m*.

Figure 2 shows that the cost improvement of the VMIC policy over the VM_ICC policy is increasing with the number of retailers. This makes sense given that for larger number of retailers there exist more opportunities for carbon exchanges to take place between SC members. Moreover, the number of SC members with relaxed carbon allowances (closer to their maximum carbon emission) increases for larger m. In this case, such members can transfer some of their carbon allowances to other members with restricted carbon quotas resulting in lower total operational costs.

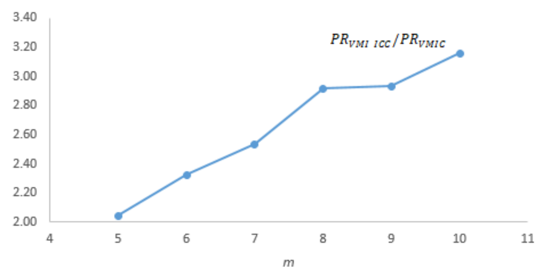

FIGURE 2. Variation of the prices of carbon reduction as function of *m*.

It should also be mentioned that the reduction of the average SC operational costs resulting from the execution of the VMIC policy was accompanied by an increase in the average SC carbon emission as observed in the illustrative example. However, as exhibited in Figure 2, it is always cheaper to reduce carbon emissions through the implementation of VMIC policy rather than the VMI_ICC policy. Indeed, Figure 2 shows that the ratio PR_{VMI_ICC}/PR_{VMIC} is always larger than one and it is increasing with the number of retailers.

B. IMPACT OF THE CONSTRAINTS TIGHTNESS LEVEL

Another relevant aspect that is assessed though this sensitivity analysis pertains to the impact of the problem constraints tightness on the cost savings following the VMIC policy. To this purpose, we first divided the test instances into four classes depending on the value of the tightness level. The first class of test instances corresponds to the very restrictive problems with their tightness levels in the interval $[E_j^{min}, 0.25 * (E_j^{min} + E_j^{max})]$. For such type of problems, most of the SC members are subject to very restraining carbon constraints as their carbon allowances are closer to E_j^{min}. The second class is related to the restrictive test problems with tightness levels in the range $[0.25 * (E_j^{min} + E_j^{max}), 0.5 * (E_j^{min} + E_j^{max})]$. It is clear that these type of problems are less restrictive than those in the first class. Test problems in the third class are bound by relaxed carbon constraints with most of the SC members having their carbon quotas larger than $0.5 * (E_j^{min} + E_j^{max})$. The tightness levels for these problems fall in the range $[0.5 * (E_j^{min} + E_j^{max}), 0.75 * (E_j^{min} + E_j^{max})]$. Finally, the fourth class deals with the most relaxed problems with high tightness level close to E_j^{max}. The tightness level for this class of problems belongs to the interval $[0.75 * (E_j^{min} + E_j^{max}), E_j^{max}]$.

TABLE 9. Impact of the constraints tightness level.

Class of problems	Number of tested cases	$\%CR$	PR_{VMI_ICC}	PR_{VMIC}
1	29	22.556	0.172	0.07
2	31	21.667	0.144	0.051
3	35	5.537	0.022	0.005
4	25	4.527	0.017	0.004

As can be noted from Table 9, the cost reduction of the VMIC policy and the prices of carbon reduction for both policies decrease with the each class of test instances. It is expected that the more relaxed the problem (higher tightness level), the less is the need for carbon exchange between the SC members since there will less number of SC members with restrictive carbon constraints. Hence, VMIC policy would not be as beneficial cost-wise for cases with relaxed individual carbon constraints.

VI. MANAGERIAL INSIGHTS AND CONCLUDING REMARKS

VMI is a collaborative supply chain (SC) initiative that continues to attract a lot of attention due to its proven ability to enhance operational efficiency for all parties involved. While the VMI framework and related models enable the vendor to better plan production, schedule deliveries to SC partners and manage inventories at the buyers' facilities, these models can also be used to manage the SC carbon footprint. In this paper, we propose integrated economic and environmental VMI based models for a two-stage single-vendor multi-buyer SC with finite storage space at the retailers' facilities.

In many realistic situations, the vendor and the retailers are independent entities that may be operating under different environmental regulations with distinct carbon caps. In such cases, individual carbon caps should be instituted in lieu of the overall carbon cap for all SC members. Rather than being imposed exogenously, these individual caps might be set internally in response to rising climate change related concerns and increased customer awareness making the shift to environmentally-conscious practices the imperative rather than a choice. This individual carbon caps policy has many benefits as it allows for better determination of each party's responsibility for the generated carbon footprint. Moreover, the vendor, who is already managing the retailers' inventories, may promote an environmental-based collaborative scheme in which he/she would manage the retailers' footprint resulting from the shipment and lot-sizing policy adopted.

In reality, SC members would typically maintain a record of the carbon footprint they have been generating over the years, based on which individual caps can be set. However, upon joining this VMI partnership, the vendor's replenishment strategy might potentially cause the carbon emissions to differ from the previously established levels. In particular, some members may exceed their carbon allowances while others may not fully utilize them. Having the upper hand in such strategic partnership, the vendor can advise some members to exchange their surplus carbon allowance for the benefit of the whole SC. This "carbon cap and exchange" policy would attain chain-wide cost savings and also entice reluctant members to participate in this collaborative environment by ensuring they would not be worse off from an economic standpoint.

The newly proposed "carbon cap and exchange" policy has many useful managerial implications. In addition to identifying responsibility for carbon footprints, it provides a rational method of carbon exchange that takes inventory and ordering factors into consideration. Essentially, since some SC members may be worse off by joining a VMI partnership while having to adhere to individual caps, the "cap and exchange" mechanism empowers the vendor by providing a tool for counterbalancing this effect and making those members better off. Therefore, the developed optimization model provides a decision-making tool for optimizing both economic and environmental measures, where the amount of exchange among the members is determined to optimize cost savings compared to a no-exchange policy.

Although this work presents a novel carbon regulatory policy that paves the way for a closer collaboration between members of SC operating under VMI partnership, it has some limitations. To prevent the vendor from opportunistically stockpiling more inventory at the retailers' premises, this work's countermeasure calls for imposing an upper limit on buyers' maximum inventory levels and penalizing the vendor on a per-unit basis for exceeding this limit. As such, a purely VMI-based collaborative framework is assumed, despite the fact that an alternative approach has been proposed in the literature to serve the same purpose. This alternative approach uses a hybrid collaborative scheme formed by combining VMI and CS initiatives, resulting in the so called vendor managed consignment inventory (VMCI), as seen in Hariga et al. [16] and many others. Another limitation of this work relates to the assumption of a deterministic and constant demand, implying stable operating conditions. This may significantly compromise the applicability of the developed models to products or industries characterized by volatile demand. Lastly, since this work adopts synchronized replenishment cycles among all retailers, it may result in suboptimal solutions in situations where retailers need to be replenished at different ordering frequencies.

One future research avenue is to incorporate the "carbon cap and exchange" policy proposed in this paper into other SC collaborative initiatives, or hybrid ones such as VMCI, and embed such policy as an inherent part of the collaboration agreement. Furthermore, adapting the proposed models to ensure their suitability for uncertain environments with stochastic and/or dynamic demand is another promising future research direction. Also, relaxing the restrictive assumption of equal replenishment cycles presents a more challenging problem to solve and might better resemble the reality of several industries.

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