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RESEARCH ARTICLE

Analysis and Prioritization of the Factors of the Robotic Industry With the Assistance of EDAS Technique Based on Intuitionistic Fuzzy Rough Yager Aggregation Operators

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ABSTRACT The focus of the interdisciplinary and scientific discipline of robotics is to design, maintain and use mechanical robotics. There exist many issues faced by the robotic industry but there are some factors that can cover these complexities effectively. Handling vague and imprecise data is a difficult task nowadays. So there is a need to define such kind of effective and valuable tool that can handle complex and vague data more dominantly. The evaluation based on distance from average solution (EDAS) method is a very useful tool that can handle complex data more effectively. The best alternative can be chosen based on distance from the average solution. The EDAS method is relatively simple to use and provide a quick evaluation of alternative based on multiple criteria. Yager t-norm and t-conorm are two fuzzy logic operators proposed by Yager. So based on the importance of Yager t-norm and t-conorm, initially, in this article, we have proposed the basic operative laws for intuitionistic fuzzy rough numbers. Based on these developed operational laws, we have developed some new intuitionistic fuzzy rough aggregation operators called intuitionistic fuzzy rough Yager average (weighted, ordered weighted, hybrid) aggregation operators and intuitionistic fuzzy rough Yager geometric (weighted, ordered weighted, hybrid) aggregation operators. Moreover, we have proposed the EDAS technique based on intuitionistic fuzzy rough Yager aggregation operators and used these notions for the selection of suitable factors that play a vital role in the robotic industry. Also, to show the effective use of these introduced notions, we have proposed an algorithm for the EDAS method based on intuitionistic fuzzy rough Yager aggregation operators along with a descriptive example. To show the superiority of the introduced work we have developed a comparative analysis.

INDEX TERMS Intuitionistic fuzzy rough Yager aggregation operators, EDAS method, multi-criteria group decision making.

I. INTRODUCTION

This Robotics is the synthesis of science, engineering, and technology that produces robots—machines that imitate or replace humans in activity. A robot is a programmed machine that can carry out a task, and robotics is the field of study devoted to developing robots and automation. The level of autonomy varies among different robots. These levels vary from fully autonomous robots that run on their own to robots that work under human supervision. The Robotics industry faces different challenges and these challenges include (1) New material and Fabrication Methods (2) Creating Bio-inspired robots (3) Better Power Sources (4) Brain-computer interfaces etc. To cover these complexities, there are some main factors in this regard that can cope with these complexities. In the robotics industry, some valuable factors

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play a vital role. Some of the notable factors in this regard are (1) Control software (2) Navigation (3) Battery technology (4) Platform dimension (5) Payload etc.

Multi-criteria group decision-making (MCGDM) technique is an effective method that can be used for grading the alternatives based on the evaluation of decision analysts under different criteria. Researchers have been paying close attention to decision-making based on multiple criteria to choose the best option from a variety of options. Decision-making groups have historically handled a variety of practical issues, which are made up of multiple independent experts in relevant fields. GDM has been used extensively including supplier selection [1], medical treatment [2], and supply chain risk assessment [3]. With the increasing complication of factual DM issues such as enigmatic data and human thinking problems, accurate numerical values were no longer able to take into account the accuracy of alternative information and decision-makers and potentially leading to decision-making errors. As a result, many theories have been derived for GDM problems such as fuzzy set [4], rough set [5], and so on. These theories can be used to handle imprecise data and extend the traditional MCGDM methods into joined models that can be used for MCGDM problems. The fuzzy set (FS) was an effective tool that generalize the classical set theory. Based on FS, many new tools and techniques have been developed. In 1994, Yager [6] proposed aggregation operators and fuzzy system modeling. Additionally, Ghorabaee et al. [7] invented a multi-criteria inventory classification system based on the EDAS method. As, EDAS method plays a huge part in dynamic problems, especially when more clashing criteria exist in MCGDM. This model has been extensively used in fuzzy set theory to solve MCGDM problems. Stevic et al. [8] settled the estimation of suppliers under unreliability based on the fuzzy EDAS technique. Based on FS Yang et al. [9] initiated the characterization of the minimal solution set to max-min fuzzy relation inequalities.This approach has been used by Kutlu et al. [10] in the medical field and they have applied it to the selection of hospitals. In the basic definition of fuzzy set, we have noticed that fuzzy set only uses membership grade (MG) but in many practical problems, we cannot restrict ourselves to MG. So, there was a need to develop such an effective approach that can handle MG and non-membership grade (NMG) in one structure. Atanassov [11] initiated the idea of an intuitionistic fuzzy set (IFS) in this regard to cover that issue. IFS is a more general approach to solving the MCGDM problems. It provides more space for decision-makers and more complex data to be handled through this notion. After the invention of this notion, many aggregation operators and methods have been developed. Mishra et al. [12] proposed a novel EDAS process for their evaluation of the healthcare waste disposable mechanism under the notion of IFS. Kahraman et al. [13] use EDAS methods for the selection of solid waste disposable site selection. Schitea et al. [14] introduced the WASPAS, COPRAS, and EDAS techniques based on IF information for the selection of hydrogen mobility roll-up site selection. Moreover, based on IFS some researchers have developed aggregation operators like IFWA and IFWG aggregation operators can be seen from [15] and [16]. Moreover, Seikh and Mandal [17] introduced IF Dombi aggregation operators. Also, Dong et al. [18] proposed the IF VIKOR method and IF EDAS method. Moreover, some generalized intuitionistic fuzzy Einstein hybrid aggregation operators are introduced by Rahman et al. [19]. Moreover, the idea of IF hypergraphs has been introduced by Akram et al. [20] and they have provided their applications. Also, IF graphs of the nth type with the application have been given by Davvas et al. [21]. Moreover, Jiang et al. [22] proposed entropy measures based on *IF* soft set and interval-valued IF soft set.

Based on IF cubic fuzzy operators an MCGDM system has been initiated for the selection of small hydropower plant locations given in [23]. Also, a graphical method for ranking IFS using entropy is given by Ali et al. [24].

A rough set (RS) introduced by Pawlak is a valuable mathematical apparatus to handle ambiguity and complicated data. The idea of RS has been extensively used by the researchers like Qurashi and Shabir [25] use the idea of RS in quantale modules. Moreover, Aslam et al. [26] initiated the notion of rough M-hyper systems and fuzzy M-hyper systems. Also, Shabir and Irshad [27] used roughness in ordered semigroups. Many hybrid structures are initiated by combining the RS and FS structures. The idea of a fuzzy rough set (FRS) [28] is the combination of FS and RS and the intuitionistic fuzzy rough set (IFRS) [29] is the hybrid notion for IFS and RS. Both of these notions have been widely used in different directions. Mahmood et al. [30] proposed generalized roughness in fuzzy filters and fuzzy ideals. Also, Ali et al. [31] established generalized roughness in fuzzy filters of semigroups. Based on IFRS many new ideas have been developed like IFRWA and IFRWG aggregation operators established in [32]. Also, IFR frank aggregation operators have been introduced in [33]. Many other developments have been made like Ahmmad et al. [34] proposed the notions of IFR Aczel-Alsina aggregation operators and used these notions in medical diagnosis. Moreover, Jia et al. [35] proposed the MABAC approach based on IFRNs. Also, covering-based general multi-granulation IFRSs and corresponding applications to multi-attribute group decision-making are given by Zhang et al. [36]. Furthermore, Mahmood et al. [37] established confidence level aggregation operators based on IFRNs and utilized these notions in the medical field.

As IFRS is a more generalized structure and it can provide more space to decision-makers in DM situations, we aim to use these more effective and advanced notions. Moreover, aggregation operators are basic tools for the conversion of complex data into a single value. So, firstly, we have developed the basic Yager operational laws for IFRNs, and then relying on these notions we have developed some new aggregation operators like intuitionistic fuzzy rough Yager arithmetic aggregation operators. Also, we have introduced

IFR Yager geometric aggregation operators. Furthermore, the EDAS method is an effective technique to handle complex data and it has been used in MCGDM problems in FST. So, based on this idea, we created EDAS methods using IFR Yager arithmetic and geometric aggregation operators. Also, an illustrative example that can help in the robotic industry has been introduced to support the proposed work. Additionally, a comparison of existing and introduced work demonstrates the superiority of the introduced work.

Here is the remainder of the article: We reviewed the fundamental concepts for the FS, IFS, RS, IFRS, Yager norms, and t-conorm in section II of this article. We have introduced new Yager operating laws based on IFRNs in section III. We introduced new aggregation operators, such as IFR Yager arithmetic aggregation operators, in section IV. IFR Yager geometric aggregation operators are covered in Section V. The EDAS approach based on IFR Yager aggregation operators is covered in Section VI. We offered a comparative analysis in part 7 and provided final thoughts in section VIII.

Moreover, the graphical abstract of the developed approach is given in figure 1 to show the organization of the presented work.

II. PRELIMINARIES

EDAS method is a very effective achievement for MCGDM problems. In this method, the best alternatives are chosen by using the distance from the average solution. In daily life problems, when we are facing more ambiguous and complex data. We are thinking to develop a method that can handle this complex situation. So, the EDAS method can handle more complex data effectively.

Now we will go over some fundamental definitions of FS, RS, IFRS, Yager t-norms, and t-conorm.

Definition 1 [4]: A fuzzy set is given by

$$\dot{F} = \left\{\left(\mathfrak{s}, g(\mathfrak{s})\right) | \mathfrak{s} \in \mathcal{A}\right\}, \text{where } g(\mathfrak{s}) \in [0, 1]$$

where $g(x)$ represents membership grade (MG).

Example 1: Let $\mathcal{A} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\}$ be universal set, then the fuzzy set is given by membership function $g : \mathcal{A} \to [0\ 1]$ such that

$$\dot{F} = \{(\mathfrak{s}_1, 0.5), (\mathfrak{s}_2, 0.3), (\mathfrak{s}_3, 0.4)\}.$$

Definition 2 [11]: An intuitionistic fuzzy set is given by

$$\dot{F} = \left\{\left(\mathfrak{s}, g(\mathfrak{s}), Л(\mathfrak{s})\right) | \mathfrak{s} \in \mathcal{A}\right\}$$

where $0 \leq g(\mathfrak{s}) + Л(\mathfrak{s}) \leq 1$ and $g(\mathfrak{s}), Л(\mathfrak{s})$ are MG and NMG respectively.

Example 2: Let $\mathcal{A} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\}$ be universal set then IFS is given by membership function $g : \mathcal{A} \to [0\ 1]$ and non-membership function $Л : \mathcal{A} \to [0\ 1]$ such that

$$\dot{F} = \{(\mathfrak{s}_1, 0.2, 0.3), (\mathfrak{s}_2, 0.3, 0.4), (\mathfrak{s}_3, 0.4, 0.5)\}.$$

Definition 3 [5]: Let $\mathcal{A}$ stand for general set and $'\Omega \subseteq \mathcal{A} \times \mathcal{A}$ be any crisp relation on $\mathcal{A}$. Let $'\Omega^*$ is a set-valued map (SVMP) $'\Omega^* : \mathcal{A} \to P(\mathcal{A})$ described as $'\Omega^*(\mathfrak{s}) = \left\{\mathfrak{s} \in \mathcal{A} : (\mathfrak{s}, \mathfrak{s}) \in' \Omega \text{ and } \mathfrak{s} \in \mathcal{A}\right\}$, then $\left(\mathcal{A},' \Omega\right)$ is an approximation space. Now let $\mathfrak{G} \subseteq \mathcal{A}$, then the lower approximation (LRA) and upper approximations (URA) of $\mathfrak{G}$ w.r.t $\left(\mathcal{A},' \Omega\right)$ are described by

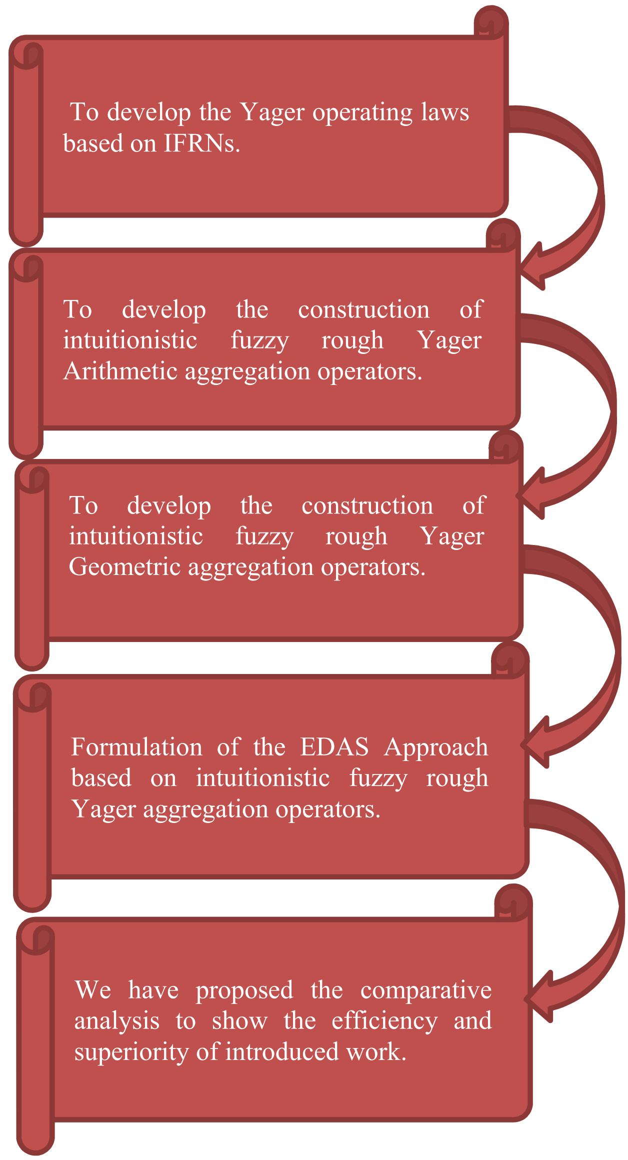

FIGURE 1. Organization of proposed work.

$$\underline{'\Omega}(\mathfrak{G}) = \left\{\left(\mathfrak{s} \in \mathcal{A} : '\Omega^*(\mathfrak{s}) \subseteq \mathfrak{G}\right)\right\}$$
$$\overline{'\Omega}(\mathfrak{G}) = \left\{\left(\mathfrak{s} \in \mathcal{A} : '\Omega *(\mathfrak{s}) \sqcap \mathfrak{G} \neq \emptyset\right)\right\}$$

The pair $\left(\underline{'\Omega}(\mathfrak{G}), \overline{\overline{'\Omega}}(\mathfrak{G})\right)$ is called a rough set (RS), where $\underline{'\Omega}(\mathfrak{G}) \neq \overline{\overline{'\Omega}}(\mathfrak{G})$. Also, $\underline{'\Omega}(\mathfrak{G}), \overline{\overline{'\Omega}}(\mathfrak{G}) : P(\mathcal{A}) \to P(\mathcal{A})$ are called crisp LR and UR approximation operators according to $\left(\mathcal{A},' \Omega\right)$.

Example 3: Let $\mathcal{A} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4, \mathfrak{s}_5\}$ be universal set and

$$'\Omega = \left\{ \begin{array}{c} (\mathfrak{s}_1, \mathfrak{s}_2), (\mathfrak{s}_1, \mathfrak{s}_3), (\mathfrak{s}_2, \mathfrak{s}_2), \\ (\mathfrak{s}_2, \mathfrak{s}_4), (\mathfrak{s}_2, \mathfrak{s}_5), (\mathfrak{s}_3, \mathfrak{s}_1), \\ (\mathfrak{s}_3, \mathfrak{s}_2), (4, \mathfrak{s}_2), (\mathfrak{s}_4, \mathfrak{s}_3), (\mathfrak{s}_5, \mathfrak{s}_3), (\mathfrak{s}_5, \mathfrak{s}_5) \end{array} \right\} \subset \mathcal{A} \times \mathcal{A}$$

TABLE 1. IF relation.

$'\Omega$	$\mathfrak{s}_1$	$\mathfrak{s}_2$	$\mathfrak{s}_3$	$\mathfrak{s}_4$
$\mathfrak{s}_1$	$(0.3, 0.4)$	$(0.1, 0.4)$	$(0.2, 0.5)$	$(0.1, 0.6)$
$\mathfrak{s}_2$	$(0.2, 0.3)$	$(0.2, 0.4)$	$(0.3, 0.5)$	$(0.2, 0.3)$
$\mathfrak{s}_3$	$(0.1, 0.2)$	$(0.3, 0.4)$	$(0.5, 0.4)$	$(0.1, 0.7)$
$\mathfrak{s}_4$	$(0.2, 0.3)$	$(0.3, 0.5)$	$(0.2, 0.4)$	$(0.2, 0.5)$

is any crisp relation on $\mathcal{A}$. Now

$$'\Omega^*(\mathfrak{s}_1) = \{\mathfrak{s}_2, \mathfrak{s}_3\}, \quad '\Omega^*(\mathfrak{s}_2) = \{\mathfrak{s}_2, \mathfrak{s}_4, \mathfrak{s}_5\}$$
$$'\Omega^*(\mathfrak{s}_3) = \{\mathfrak{s}_1, \mathfrak{s}_2\}, \quad '\Omega^*(\mathfrak{s}_4) = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\}$$
$$'\Omega^*(\mathfrak{s}_5) = \{\mathfrak{s}_3, \mathfrak{s}_5\}.$$

Let $\mathfrak{G} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\} \subset \mathcal{A}$.

Now $\underline{'\Omega}(\mathfrak{G}) = \{\mathfrak{s}_1, \mathfrak{s}_3, \mathfrak{s}_4\}$ and $\overline{\overline{'\Omega}}((\mathfrak{G}) = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4, \mathfrak{s}_5\}$. This implies that $\underline{'\Omega}(\mathfrak{G}) \neq \overline{\overline{'\Omega}}(\mathfrak{G})$. Hence the pair $\left(\underline{'\Omega}(\mathfrak{G}), \overline{\overline{'\Omega}}(\mathfrak{G})\right)$ is called a rough set.

Definition 4 [29]: Assume that $\mathcal{A}$ stands for general set and $'\Omega \in IFS(\mathcal{A} \times \mathcal{A})$ be intuitionistic fuzzy relation. Then pair $('\Omega, \mathcal{A})$ is called IF approximation space. Now for any Now for any $\mathfrak{G} \subseteq IFS(\mathcal{A})$, then the LRA and URA of $\mathfrak{G}$ w.r.t $(\mathcal{A}, '\Omega)$ are given by

$$\underline{'\Omega}(\mathfrak{G}) = \left\{\left\langle\left(\mathfrak{s}: \underline{\mathfrak{g}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}), \underline{\text{Л}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s})\right) | \mathfrak{s} \in \mathcal{A}\right\rangle\right\}$$
$$\overline{\overline{'\Omega}}(\mathfrak{G}) = \left\{\left\langle\left(\mathfrak{s}: \overline{\overline{\mathfrak{g}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}), \overline{\overline{\text{Л}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s})\right) | \mathfrak{s} \in \mathcal{A}\right\rangle\right\}$$

where $\underline{\mathfrak{g}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}) = \wedge_{\mathfrak{c}\in\mathcal{A}}[\mathfrak{g}_{'\Omega}(\mathfrak{s}, \mathfrak{c}) \wedge \mathfrak{g}_{\mathfrak{G}}(\mathfrak{c})], \underline{\text{Л}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}) = \vee_{\mathfrak{c}\in\mathcal{A}}[\text{Л}_{'\Omega}(\mathfrak{s}, \mathfrak{c}) \vee \text{Л}_{\mathfrak{G}}(\mathfrak{c})]$ And

$$\overline{\overline{\mathfrak{g}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}) = \vee_{\mathfrak{c}\in\mathcal{A}}[\mathfrak{g}_{'\Omega}(\mathfrak{s}, \mathfrak{c}) \vee \mathfrak{g}_{\mathfrak{G}}(\mathfrak{c})], \overline{\overline{\text{Л}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}) = \wedge_{\mathfrak{c}\in\mathcal{A}}[\text{Л}_{'\Omega}(\mathfrak{s}, \mathfrak{c}) \wedge \text{Л}_{\mathfrak{G}}(\mathfrak{c})]$$

with $0 \le \overline{\overline{\mathfrak{g}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}) + \overline{\overline{\text{Л}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}) \le 1, 0 \le \underline{\mathfrak{g}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}) + \underline{\text{Л}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}) \le 1$.

As $\underline{'\Omega}(\mathfrak{G})$ *and* $\overline{\overline{'\Omega}}(\mathfrak{G})$ are IFSs, so, $\underline{'\Omega}(\mathfrak{G}), \overline{\overline{'\Omega}}(\mathfrak{G}) : IFS(\mathcal{A}) \to IFS(\mathcal{A})$ are LR and UR, approximation operators. Then pair $'\Omega(\mathfrak{G}) = \left(\underline{'\Omega}(\mathfrak{G}), \overline{\overline{'\Omega}}(\mathfrak{G})\right) = \left\{\mathfrak{s}: \left(\underline{\mathfrak{g}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}), \underline{\text{Л}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s})\right), \left(\overline{\overline{\mathfrak{g}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}), \overline{\overline{\text{Л}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s})\right) | \mathfrak{s} \in \mathcal{A}\right\}$ is called IFRS. For simplicity, $'\Omega(\mathfrak{G}) = \left(\underline{'\Omega}(\mathfrak{G}), \overline{\overline{'\Omega}}(\mathfrak{G})\right) = \left\{\mathfrak{s}: \left(\underline{\mathfrak{g}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}), \underline{\text{Л}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s})\right), \left(\overline{\overline{\mathfrak{g}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}), \overline{\overline{\text{Л}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s})\right) | \mathfrak{s} \in \mathcal{A}\right\}$ can be represented as $'\Omega(\mathfrak{G}) = \left\{\left(\underline{\mathfrak{g}}_{\check{G}}, \underline{\text{Л}}_{\check{G}}\right), \left(\overline{\overline{\mathfrak{g}}}_{\check{G}}, \overline{\overline{\text{Л}}}_{\check{G}}\right)\right\}$ known as intuitionistic fuzzy rough numbers (IFRNs).

Example 4: Let $\mathcal{A} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4\}$ be an arbitrary set and $(\mathcal{A}, '\Omega)$ be an IF approximation space with $'\Omega \in IFS(\mathcal{A} \times \mathcal{A})$ be IF relation as given in Table 1.

Now assume that experts present the optimal decision object $\mathfrak{G}$ that is an IFS given by

$$\mathfrak{G} = \{(\mathfrak{s}_1, 0.3, 0.5), (\mathfrak{s}_2, 0.4, 0.3), (\mathfrak{s}_3, 0.1, 0.4), (\mathfrak{s}_4, 0.2, 0.3)\}.$$

Now to obtain $\underline{'\Omega}(\mathfrak{G})$ and $\overline{\overline{'\Omega}}(\mathfrak{G})$, we get

$$\underline{\mathfrak{g}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}_1) = (0.3\wedge0.3) \wedge (0.1\wedge0.4) \wedge (0.2\wedge0.1) \wedge (0.1\wedge0.2) = 0.1,$$
$$\underline{\mathfrak{g}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}_2) = (0.2\wedge0.3) \wedge (0.2\wedge0.4) \wedge (0.3\wedge0.1) \wedge (0.2\wedge0.2) = 0.1,$$
$$\underline{\mathfrak{g}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}_3) = (0.1\wedge0.3) \wedge (0.3\wedge0.4) \wedge (0.5\wedge0.1) \wedge (0.1\wedge0.2) = 0.1,$$
$$\underline{\mathfrak{g}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}_4) = (0.2\wedge0.3) \wedge (0.3\wedge0.4) \wedge (0.2\wedge0.1) \wedge (0.2\wedge0.2) = 0.1$$

And

$$\overline{\overline{\mathfrak{g}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}_1) = (0.3\vee0.3) \vee (0.1\vee0.4) \vee (0.2\vee0.1) \vee (0.1\vee0.2) = 0.4,$$
$$\overline{\overline{\mathfrak{g}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}_2) = (0.2\vee0.3)\vee (0.2\vee0.4) \vee (0.3\vee0.1) \vee (0.2\vee0.2) = 0.4$$
$$\overline{\overline{\mathfrak{g}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}_3) = (0.1\vee0.3) \vee (0.3\vee0.4) \vee (0.5\vee0.1) \vee (0.1\vee0.2) = 0.5$$
$$\overline{\overline{\mathfrak{g}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}_4) = (0.2\vee0.3) \vee (0.3\vee0.4) \vee (0.2\vee0.1) \vee (0.2\vee0.2) = 0.4$$

And

$$\underline{\text{Л}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}_1) = (0.4\vee0.5)\vee(0.4\vee0.3)\vee(0.5\vee0.4)\vee(0.6\vee0.3) = 0.6$$
$$\underline{\text{Л}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}_2) =, (0.3\vee0.5)\vee(0.4\vee0.3)\vee(0.5\vee0.4)\vee(0.3\vee0.3) = 0.5$$
$$\underline{\text{Л}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}_3) =, (0.2\vee0.5)\vee(0.4\vee0.3)\vee(0.4\vee0.4)\vee(0.7\vee0.3) = 0.7$$
$$\underline{\text{Л}_{'\Omega}}_{\mathfrak{G}}(\mathfrak{s}_4) = (0.3\vee0.5)\vee(0.5\vee0.3)\vee(0.4\vee0.4)\vee(0.5\vee0.3) = 0.5.$$

And

$$\overline{\overline{\text{Л}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}_1) = (0.4\wedge0.5)\wedge(0.4\wedge0.3)\wedge(0.5\wedge0.4)\wedge(0.6\wedge0.3) = 0.3,$$
$$\overline{\overline{\text{Л}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}_2) = (0.3\wedge0.5)\wedge(0.4\wedge0.3)\wedge(0.5\wedge0.4)\wedge(0.3\wedge0.3) = 0.3,$$
$$\overline{\overline{\text{Л}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}_3) = (0.2\wedge0.5)\wedge(0.4\wedge0.3)\wedge(0.4\wedge0.4)\wedge(0.7\wedge0.3) = 0.2,$$
$$\overline{\overline{\text{Л}_{'\Omega}}}_{\mathfrak{G}}(\mathfrak{s}_4) = (0.3\wedge0.5)\wedge(0.5\wedge0.3)\wedge(0.4\wedge0.4)\wedge(0.5\wedge0.3) = 0.3$$

Thus upper and lower IF rough approximation operators are given by

$$'\Omega(\mathfrak{G}) = \{(\mathfrak{s}_1, 0.4, 0.3), (\mathfrak{s}_2, 0.4, 0.3), (\mathfrak{s}_3, 0.5, 0.2), (\mathfrak{s}_4, 0.4, 0.3)\}$$
$$\underline{'\Omega}(\mathfrak{G}) = \{(\mathfrak{s}_1, 0.1, 0.6), (\mathfrak{s}_2, 0.1, 0.5), (\mathfrak{s}_3, 0.1, 0.7), (\mathfrak{s}_4, 0.1, 0.5)\}$$

Therefore,

$'\Omega(\mathfrak{G})$

$$= \left\{ '\underline{\Omega}(\mathfrak{G}), '\overline{\overline{\Omega}}(\mathfrak{G}) \right\}$$
$$= \begin{pmatrix} (\mathfrak{s}_1, (0.1, 0.6), (0.4, 0.3)), (\mathfrak{s}_2, (0.1, 0.5), (0.4, 0.3)), \\ (\mathfrak{s}_3, (0.1, 0.7), (0.5, 0.2)), (\mathfrak{s}_4, (0.1, 0.5), (0.4, 0.3)) \end{pmatrix}.$$

Definition 5 [6]: For arbitrary real numbers ð *and* ∂, Yager t-norm '$\mathfrak{H}$' and t-conorm '$\mathfrak{H}^\star$' are given as

$$\mathfrak{H}(ð, \partial) = 1 - \min \begin{pmatrix} 1, \\ \left((1-ð)^{\text{ɼ}} + (1-\partial)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}} \end{pmatrix} \quad (1)$$

$$\mathfrak{H}^\star(ð, \partial) = \min \begin{pmatrix} 1, \\ \left((ð)^{\text{ɼ}} + (\partial)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}} \end{pmatrix} \quad (2)$$

where $\text{ɼ} \in (0, \infty)$.

Example 5: Let ð = 0.5 and ∂ = 0.3 and let ɼ = 2 then

$$\mathfrak{H}(0.5, 0.3) = 1 - \min \begin{pmatrix} 1, \\ \left((1-0.5)^2 + (1-0.3)^2\right)^{\frac{1}{2}} \end{pmatrix} = 0.0100$$

And

$$\mathfrak{H}^\star(0.5, 0.3) = \min \begin{pmatrix} 1, \\ \left((1-0.5)^2 + (1-0.3)^2\right)^{\frac{1}{2}} \end{pmatrix} = 0.5830.$$

Definition 6 [32]: Let $\dot{F} = \left\{ \left(\underline{g}_{\dot{F}}, \underline{Л}_{\dot{F}}\right), \left(\overline{\overline{g}}_{\dot{F}}, \overline{\overline{Л}}_{\dot{F}}\right) \right\}$ be an IFRN then the score and accuracy function are given by

$$Sc(\dot{F}) = \frac{1}{4}\left(2 + \underline{g}_{\dot{F}} + \overline{\overline{g}}_{\dot{F}} - \underline{Л}_{\dot{F}} - \overline{\overline{Л}}_{\dot{F}}\right), \quad S(\dot{F}) \in [0, 1]$$
$$Ac(\dot{F}) = \frac{1}{4}\left(2 + \underline{g}_{\dot{F}} + \overline{\overline{g}}_{\dot{F}} + \underline{Л}_{\dot{F}} + \overline{\overline{Л}}_{\dot{F}}\right), \quad A(\dot{F}) \in [0, 1].$$

Example 6: Let $\dot{F} = \{(0.2, 0.3), (0.5, 0.4)\}$ be an IFRN then the score and accuracy function are given by

$$Sc(\dot{F}) = = \frac{1}{4}(2 + 0.2 + 0.5 - 0.3 - 0.4) = 0.5$$
$$Ac(\dot{F}) = \frac{1}{4}(2 + 0.2 + 0.5 + 0.3 + 0.4) = 0.85.$$

Definition 7 [32]: For two IFRNs, $\dot{F}_1 = \left\{ \left(\underline{g}_{\dot{F}_1}, \underline{Л}_{\dot{F}_1}\right), \left(\overline{\overline{g}}_{\dot{F}_1}, \overline{\overline{Л}}_{\dot{F}_1}\right) \right\}$ and $\dot{F}_2 = \left\{ \left(\underline{g}_{\dot{F}_2}, \underline{Л}_{\dot{F}_2}\right), \left(\overline{\overline{g}}_{\dot{F}_2}, \overline{\overline{Л}}_{\dot{F}_2}\right) \right\}$ we get

1) If $S(\dot{F}_1) > S(\dot{F}_2)$ then $\dot{F}_1 > \dot{F}_2$,
2) If $S(\dot{F}_1) < S(\dot{F}_2)$ then $\dot{F}_1 < \dot{F}_2$,
3) If $S(\dot{F}_1) = S(\dot{F}_2)$ then
   i. If $A(\dot{F}_1) > A(\dot{F}_2)$ then $\dot{F}_1 > \dot{F}_2$,
   ii. If $A(\dot{F}_1) < A(\dot{F}_2)$ then $\dot{F}_1 < \dot{F}_2$,
   iii. If $A(\dot{F}_1) = A(\dot{F}_2)$ then $\dot{F}_1 = \dot{F}_2$.

III. YAGER OPERATIONAL LAWS FOR INTUITIONISTIC FUZZY ROUGH NUMBERS

In this section based on Yager t-norm and t-conorm, we have established the basic Yager operational laws based on IFR numbers.

Definition 8: For two IFRNs, $\dot{F}_1 = \left\{ \left(\underline{g}_{\dot{F}_1}, \underline{Л}_{\dot{F}_1}\right), \left(\overline{\overline{g}}_{\dot{F}_1}, \overline{\overline{Л}}_{\dot{F}_1}\right) \right\}$ and $\dot{F}_2 = \left\{ \left(\underline{g}_{\dot{F}_2}, \underline{Л}_{\dot{F}_2}\right), \left(\overline{\overline{g}}_{\dot{F}_2}, \overline{\overline{Л}}_{\dot{F}_2}\right) \right\}$, $\text{ɼ} > 0$ and $\text{ɇ} > 0$, then Yager t-norm and t-conorm operations for IFRNs can be defined as

1. $\dot{F}_1 \oplus \dot{F}_2 =$

$$\left[\begin{Bmatrix} \begin{pmatrix} \min\left(1, \left(\left(\underline{g}_{\dot{F}_1}\right)^{\text{ɼ}} + \left(\underline{g}_{\dot{F}_2}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right), \\ 1 - \min\left(1, \left(\left(1 - \underline{Л}_{\dot{F}_1}\right)^{\text{ɼ}} + \left(1 - \underline{Л}_{\dot{F}_2}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right) \end{pmatrix}, \\ \begin{pmatrix} \min\left(1, \left(\left(\overline{\overline{g}}_{\dot{F}_1}\right)^{\text{ɼ}} + \left(\overline{\overline{g}}_{\dot{F}_2}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right), \\ 1 - \min\left(1, \left(\left(1 - \overline{\overline{Л}}_{\dot{F}_1}\right)^{\text{ɼ}} + \left(1 - \overline{\overline{Л}}_{\dot{F}_2}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right) \end{pmatrix} \end{Bmatrix}\right],$$

2. $\dot{F}_1 \otimes \dot{F}_2 =$

$$\begin{bmatrix} 1 - \min\left(1, \left(\left(1 - \underline{g}_{\dot{F}_1}\right)^{\text{ɼ}} + \left(1 - \underline{g}_{\dot{F}_2}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right), \\ \left(\min\left(1, \left(\left(\underline{Л}_{\dot{F}_1}\right)^{\text{ɼ}} + \left(\underline{Л}_{\dot{F}_2}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right)\right), \\ 1 - \min\left(1, \left(\left(1 - \overline{\overline{g}}_{\dot{F}_1}\right)^{\text{ɼ}} + \left(1 - \overline{\overline{g}}_{\dot{F}_2}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right), \\ \left(\min\left(1, \left(\left(\overline{\overline{Л}}_{\dot{F}_1}\right)^{\text{ɼ}} + \left(\overline{\overline{Л}}_{\dot{F}_2}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right)\right) \end{bmatrix},$$

3. $\text{ɇ}\dot{F}_1 =$

$$\left[\begin{Bmatrix} \begin{pmatrix} \min\left(1, \left(\left(\text{ɇ}\underline{g}_{\dot{F}_1}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right), \\ 1 - \min\left(1, \left(\text{ɇ}\left(1 - \underline{Л}_{\dot{F}_1}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right) \end{pmatrix}, \\ \begin{pmatrix} \min\left(1, \left(\left(\text{ɇ}\overline{\overline{g}}_{\dot{F}_1}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right), \\ 1 - \min\left(1, \left(\text{ɇ}\left(1 - \overline{\overline{Л}}_{\dot{F}_1}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right) \end{pmatrix} \end{Bmatrix}\right],$$

4. $\dot{F}_1^{\text{ɇ}} =$

$$\begin{bmatrix} 1 - \min\left(1, \left(\text{ɇ}\left(1 - \underline{g}_{\dot{F}_1}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right), \\ \left(\min\left(1, \left(\left(\text{ɇ}\underline{Л}_{\dot{F}_1}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right)\right), \\ 1 - \min\left(1, \left(\text{ɇ}\left(1 - \overline{\overline{g}}_{\dot{F}_1}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right), \\ \left(\min\left(1, \left(\left(\text{ɇ}\overline{\overline{Л}}_{\dot{F}_1}\right)^{\text{ɼ}}\right)^{\frac{1}{\text{ɼ}}}\right)\right) \end{bmatrix}.$$

Example 7: Let $\dot{F} = \{(0.3, 0.1), (0.4, 0.3)\}, \dot{F}_1 = \{(0.6, 0.3), (0.2, 0.7)\}$ and $\dot{F}_2 = \{(0.1, 0.8), (0.4, 0.4)\}$ be three IFRSs and $\text{ɍ} = 4$ and $\text{ꞙ} = 2$, then we get

1) $\dot{F}_1 \oplus \dot{F}_2 =$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\min\left(1, \left((0.6)^3 + (0.1)^3\right)^{\frac{1}{3}}\right), \\ 1 - \min\left(1, \left((1-0.2)^3 + (1-0.8)^3\right)^{\frac{1}{3}}\right)\end{array}\right), \\ \left(\begin{array}{c}\min\left(1, \left((0.2)^3 + (0.4)^3\right)^{\frac{1}{3}}\right), \\ 1 - \min\left(1, \left((1-0.7)^3 + (1-0.4)^3\right)^{\frac{1}{3}}\right)\end{array}\right)\end{array}\right\};$$
$$= ((0.6001, 0.2988), (0.4061, 0.3908))$$

2) $\dot{F}_1 \otimes \dot{F}_2 =$

$$\left\{\begin{array}{c}1 - \min\left(1, \left((1-0.6)^3 + (1-0.1)^3\right)^{\frac{1}{3}}\right), \\ \left(\min\left(1, \left((0.2)^3 + (0.8)^3\right)^{\frac{1}{3}}\right)\right), \\ 1 - \min\left(1, \left((1-0.2)^3 + (1-0.4)^3\right)^{\frac{1}{3}}\right), \\ \left(\min\left(1, \left((0.7)^3 + (0.4)^3\right)^{\frac{1}{3}}\right)\right)\end{array}\right\};$$
$$= ((0.0913, 0.8039), (0.1430, 0.7179))$$

3)

$$2\dot{F}_1 = \left\{\begin{array}{c}\left(\begin{array}{c}\min\left(1, \left(2(0.6)^3\right)^{\frac{1}{3}}\right), \\ 1 - \min\left(1, \left(2(1-0.2)^3\right)^{\frac{1}{3}}\right)\end{array}\right), \\ \left(\begin{array}{c}\min\left(1, \left(2(0.2)^3\right)^{\frac{1}{3}}\right), \\ 1 - \min\left(1, \left(2(1-0.7)^3\right)^{\frac{1}{3}}\right)\end{array}\right)\end{array}\right\};$$
$$= ((0.7135, 0.1675), (0.2378, 0.6432))$$

4)

$$\dot{F}_1^2 = \left\{\begin{array}{c}1 - \min\left(1, \left(2(1-0.6)^3\right)^{\frac{1}{3}}\right), \\ \left(\min\left(1, \left(2(0.3)^3\right)^{\frac{1}{3}}\right)\right), \\ 1 - \min\left(1, \left(2(1-0.2)^3\right)^{\frac{1}{3}}\right), \\ \left(\min\left(1, \left(2(0.7)^3\right)^{\frac{1}{3}}\right)\right)\end{array}\right\}$$
$$= ((0.5243, 0.3567), (0.04863, 0.8324)).$$

Theorem 1: For three IFRNs $\dot{F} = \left\{\left(\underline{g}_{\dot{F}}, \underline{\text{Л}}_{\dot{F}}\right), \left(\overline{\overline{g}}_{\dot{F}_1}, \overline{\overline{\text{Л}}}_{\dot{F}_1}\right)\right\}$, $\dot{F}_1 = \left\{\left(\underline{g}_{\dot{F}_1}, \underline{\text{Л}}_{\dot{F}_1}\right), \left(\overline{\overline{g}}_{\dot{F}_1}, \overline{\overline{\text{Л}}}_{\dot{F}_1}\right)\right\}$ and $\dot{F}_2 = \left\{\left(\underline{g}_{\dot{F}_2}, \underline{\text{Л}}_{\dot{F}_2}\right), \left(\overline{\overline{g}}_{\dot{F}_2}, \overline{\overline{\text{Л}}}_{\dot{F}_2}\right)\right\}$, we get

1) $\dot{F}_1 \oplus \dot{F}_2 = \dot{F}_2 \oplus \dot{F}_1$
2) $\dot{F}_1 \otimes \dot{F}_2 = \dot{F}_2 \otimes \dot{F}_1$
3) $\text{ꞙ}\left(\dot{F}_1 \oplus \dot{F}_2\right) = \text{ꞙ}\left(\dot{F}_1\right) \oplus \text{ꞙ}\left(\dot{F}_2\right)$
4) $\left(\text{ꞙ}_1 + \text{ꞙ}_2\right)\dot{F} = \text{ꞙ}_1\dot{F} \oplus \text{ꞙ}_2\dot{F}$
5) $\left(\dot{F}_1 \otimes \dot{F}_2\right)^{\text{ꞙ}} = \dot{F}_1^{\text{ꞙ}} \otimes \dot{F}_2^{\text{ꞙ}}$ *for* $\text{ꞙ} > 0$
6) $\dot{F}^{\text{ꞙ}_1} \otimes \dot{F}^{\text{ꞙ}_2} = \dot{F}^{(\text{ꞙ}_1 + \text{ꞙ}_2)}$ *for* $\text{ꞙ}_1, \text{ꞙ}_2 > 0$.

Proof: For three IFRNs $\dot{F}, \dot{F}_1, \dot{F}_2$ and $\text{ꞙ}, \text{ꞙ}_1, \text{ꞙ}_2 > 0$. Then by using definition (8), we get

1) $\dot{F}_1 \oplus \dot{F}_2$

$$= \left\{\begin{array}{c}\left(\begin{array}{c}\min\left(1, \left(\left(\underline{g}_{\dot{F}_1}\right)^{\text{ɍ}} + \left(\underline{g}_{\dot{F}_2}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right), \\ 1 - \min\left(1, \left(\left(1 - \underline{\text{Л}}_{\dot{F}_1}\right)^{\text{ɍ}} + \left(1 - \underline{\text{Л}}_{\dot{F}_2}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right)\end{array}\right), \\ \left(\begin{array}{c}\min\left(1, \left(\left(\overline{\overline{g}}_{\dot{F}_1}\right)^{\text{ɍ}} + \left(\overline{\overline{g}}_{\dot{F}_2}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right), \\ 1 - \min\left(1, \left(\left(1 - \overline{\overline{\text{Л}}}_{\dot{F}_1}\right)^{\text{ɍ}} + \left(1 - \overline{\overline{\text{Л}}}_{\dot{F}_2}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right)\end{array}\right)\end{array}\right\}$$

$$= \left\{\begin{array}{c}\left(\begin{array}{c}\min\left(1, \left(\left(\underline{g}_{\dot{F}_2}\right)^{\text{ɍ}} + \left(\underline{g}_{\dot{F}_1}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right), \\ 1 - \min\left(1, \left(\left(1 - \underline{\text{Л}}_{\dot{F}_2}\right)^{\text{ɍ}} + \left(1 - \underline{\text{Л}}_{\dot{F}_1}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right)\end{array}\right), \\ \left(\begin{array}{c}\min\left(1, \left(\left(\overline{\overline{g}}_{\dot{F}_2}\right)^{\text{ɍ}} + \left(\overline{\overline{g}}_{\dot{F}_1}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right), \\ 1 - \min\left(1, \left(\left(1 - \overline{\overline{\text{Л}}}_{\dot{F}_2}\right)^{\text{ɍ}} + \left(1 - \overline{\overline{\text{Л}}}_{\dot{F}_1}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right)\end{array}\right)\end{array}\right\}$$

$= \dot{F}_2 \oplus \dot{F}_1$.

2) $\dot{F}_1 \otimes \dot{F}_2$

$$= \left\{\begin{array}{c}1 - \min\left(1, \left(\left(1 - \underline{g}_{\dot{F}_1}\right)^{\text{ɍ}} + \left(1 - \underline{g}_{\dot{F}_2}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right), \\ \left(\min\left(1, \left(\left(\underline{\text{Л}}_{\dot{F}_1}\right)^{\text{ɍ}} + \left(\underline{\text{Л}}_{\dot{F}_2}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right)\right), \\ 1 - \min\left(1, \left(\left(1 - \overline{\overline{g}}_{\dot{F}_1}\right)^{\text{ɍ}} + \left(1 - \overline{\overline{g}}_{\dot{F}_2}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right), \\ \left(\min\left(1, \left(\left(\overline{\overline{\text{Л}}}_{\dot{F}_1}\right)^{\text{ɍ}} + \left(\overline{\overline{\text{Л}}}_{\dot{F}_2}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right)\right)\end{array}\right\}$$

$$= \left\{\begin{array}{c}1 - \min\left(1, \left(\left(1 - \underline{g}_{\dot{F}_2}\right)^{\text{ɍ}} + \left(1 - \underline{g}_{\dot{F}_1}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right), \\ \left(\min\left(1, \left(\left(\underline{\text{Л}}_{\dot{F}_2}\right)^{\text{ɍ}} + \left(\underline{\text{Л}}_{\dot{F}_1}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right)\right), \\ 1 - \min\left(1, \left(\left(1 - \overline{\overline{g}}_{\dot{F}_2}\right)^{\text{ɍ}} + \left(1 - \overline{\overline{g}}_{\dot{F}_1}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right), \\ \left(\min\left(1, \left(\left(\overline{\overline{\text{Л}}}_{\dot{F}_2}\right)^{\text{ɍ}} + \left(\overline{\overline{\text{Л}}}_{\dot{F}_1}\right)^{\text{ɍ}}\right)^{\frac{1}{\text{ɍ}}}\right)\right)\end{array}\right\}$$

$= \dot{F}_2 \otimes \dot{F}_1$.

3) $\text{ꞙ}(\dot{F}_1 \oplus \dot{F}_2)$

$$= \text{ꞙ}\left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left(\left(\underline{g}_{\dot{F}_1}\right)^{\mathrm{r}} + \left(\underline{g}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\left(1-\underline{Л}_{\dot{F}_1}\right)^{\mathrm{r}} + \left(1-\underline{Л}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left(\left(\overline{\overline{g}}_{\dot{F}_1}\right)^{\mathrm{r}} + \left(\overline{\overline{g}}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\left(1-\overline{\overline{Л}}_{\dot{F}_1}\right)^{\mathrm{r}} + \left(1-\overline{\overline{Л}}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right)\end{array}\right\}$$

$$= \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left(\text{ꞙ}\left(\underline{g}_{\dot{F}_1}\right)^{\mathrm{r}} + \text{ꞙ}\left(\underline{g}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}\left(1-\underline{Л}_{\dot{F}_1}\right)^{\mathrm{r}} + \text{ꞙ}\left(1-\underline{Л}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left(\text{ꞙ}\left(\overline{\overline{g}}_{\dot{F}_1}\right)^{\mathrm{r}} + \text{ꞙ}\left(\overline{\overline{g}}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}\left(1-\overline{\overline{Л}}_{\dot{F}_1}\right)^{\mathrm{r}} + \text{ꞙ}\left(1-\overline{\overline{Л}}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right)\end{array}\right\}$$

$\text{ꞙ}(\dot{F}_1) \oplus \text{ꞙ}(\dot{F}_2)$

$$= \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left(\left(\text{ꞙ}\underline{g}_{\dot{F}_1}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}\left(1-\underline{Л}_{\dot{F}_1}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left(\left(\text{ꞙ}\overline{\overline{g}}_{\dot{F}_1}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}\left(1-\overline{\overline{Л}}_{\dot{F}_1}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right)\end{array}\right\}$$

$$\oplus \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left(\left(\text{ꞙ}\underline{g}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}\left(1-\underline{Л}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left(\left(\text{ꞙ}\overline{\overline{g}}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}\left(1-\overline{\overline{Л}}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right)\end{array}\right\}$$

$$= \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left(\text{ꞙ}\left(\underline{g}_{\dot{F}_1}\right)^{\mathrm{r}} + \text{ꞙ}\left(\underline{g}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}\left(1-\underline{Л}_{\dot{F}_1}\right)^{\mathrm{r}} + \text{ꞙ}\left(1-\underline{Л}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left(\text{ꞙ}\left(\overline{\overline{g}}_{\dot{F}_1}\right)^{\mathrm{r}} + \text{ꞙ}\left(\overline{\overline{g}}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}\left(1-\overline{\overline{Л}}_{\dot{F}_1}\right)^{\mathrm{r}} + \text{ꞙ}\left(1-\overline{\overline{Л}}_{\dot{F}_2}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right)\end{array}\right\}$$

Hence $\text{ꞙ}(\dot{F}_1 \oplus \dot{F}_2) = \text{ꞙ}(\dot{F}_1) \oplus \text{ꞙ}(\dot{F}_2)$.

4) $\text{ꞙ}_1\dot{F} \oplus \text{ꞙ}_2\dot{F}$

$$= \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left(\text{ꞙ}_1\left(\underline{g}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}_1\left(1-\underline{Л}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left(\text{ꞙ}_1\left(\overline{\overline{g}}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}_1\left(1-\overline{\overline{Л}}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right)\end{array}\right\}$$

$$\oplus \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left(\text{ꞙ}_2\left(\underline{g}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}_2\left(1-\underline{Л}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left(\text{ꞙ}_2\left(\overline{\overline{g}}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left(\text{ꞙ}_2\left(1-\overline{\overline{Л}}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right)\end{array}\right\}$$

$$= \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left((\text{ꞙ}_1+\text{ꞙ}_2)\left(\underline{g}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left((\text{ꞙ}_1+\text{ꞙ}_2)(1-\underline{Л}_{\dot{F}})^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left((\text{ꞙ}_1+\text{ꞙ}_2)\left(\overline{\overline{g}}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right),\\ 1-\min\left(1, \left((\text{ꞙ}_1+\text{ꞙ}_2)\left(1-\overline{\overline{Л}}_{\dot{F}}\right)^{\mathrm{r}}\right)^{\frac{1}{\mathrm{r}}}\right)\end{array}\right)\end{array}\right\}$$

$$= (\text{ꞙ}_1+\text{ꞙ}_2)\,\dot{F}.$$

Properties 5) and 6) are similar. So, we omit their proofs.

IV. INTUITIONISTIC FUZZY YAGER AGGREGATION OPERATORS

In this section, we aim to develop some aggregation operators like IFRYWA, IFRYOWA, and IFRYHA operators.

A. INTUITIONISTIC FUZZY ROUGH YAGER WEIGHTED ARITHMETIC OPERATOR

As IFRYWA aggregation operators weigh the IFR values so based on this observation, in this subsection, we aim to discuss the basic definition of IFRYWA operators and discover their properties.

Definition 9: For the family of IFRNs $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{Л}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{Л}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$. Then IFR Yager weighted arithmetic (IFRYWA) operator is defined by a mapping $\mathfrak{F} : \mathcal{G}^m \rightarrow \mathcal{G}$ such that

$$IFRYWA_{\theta}\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \oplus_{i=1}^{m}\left(\theta_i \dot{F}_i\right)$$

where $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_m)^{T}$ is the weight vector (WV) of $\dot{F}_i$ with condition that $\sum_{i=1}^{m} \theta_i = 1$ and $\theta_i > 0$.

Theorem 2: Let $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{J}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{J}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$ be the family of IFRNs. Then aggregated result obtained from the IFRYWA operator is again IFRN given by

$$\begin{aligned} &IFRYWA_{\theta}\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) \\ &= \oplus_{i=1}^{m}\left(\theta_i \dot{F}_i\right) \\ &= \left\{ \begin{array}{c} \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\underline{g}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\underline{J}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right) \end{array} \right), \\ \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\overline{\overline{J}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right) \end{array} \right) \end{array} \right\} \end{aligned} \tag{3}$$

Proof: We employ the mathematical induction procedure to prove this result

Step 1: When $m = 2$, As

$$\theta_1 \dot{F}_1 = \left\{ \begin{array}{c} \left(\begin{array}{c} \min\left(1, \left(\theta_1\left(\underline{g}_{\dot{F}_1}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_1\left(1-\underline{J}_{\dot{F}_1}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right), \\ \left(\begin{array}{c} \min\left(1, \left(\theta_1\left(\overline{\overline{g}}_{\dot{F}_1}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_1\left(1-\overline{\overline{J}}_{\dot{F}_1}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right) \end{array} \right\}$$

and

$$\theta_2 \dot{F}_2 = \left\{ \begin{array}{c} \left(\begin{array}{c} \min\left(1, \left(\theta_2\left(\underline{g}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_2\left(1-\underline{J}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right), \\ \left(\begin{array}{c} \min\left(1, \left(\theta_2\left(\overline{\overline{g}}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_2\left(1-\overline{\overline{J}}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right) \end{array} \right\};$$

Therefore, we get

$$\begin{aligned} &\theta_1 \dot{F}_1 \oplus \theta_2 \dot{F}_2 \\ &= \left\{ \begin{array}{c} \left(\begin{array}{c} \min\left(1, \left(\left(\theta_1 \underline{g}_{\dot{F}_1}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_1\left(1-\underline{J}_{\dot{F}_1}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right), \\ \left(\begin{array}{c} \min\left(1, \left(\left(\theta_1 \overline{\overline{g}}_{\dot{F}_1}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_1\left(1-\overline{\overline{J}}_{\dot{F}_1}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right) \end{array} \right\} \\ &\oplus \left\{ \begin{array}{c} \left(\begin{array}{c} \min\left(1, \left(\left(\theta_2 \underline{g}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_2\left(1-\underline{J}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right), \\ \left(\begin{array}{c} \min\left(1, \left(\left(\theta_2 \overline{\overline{g}}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_2\left(1-\overline{\overline{J}}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right) \end{array} \right\} \\ &= \left\{ \begin{array}{c} \left(\begin{array}{c} \min\left(1, \left(\theta_1\left(\underline{g}_{\dot{F}_1}\right)^{r} + \theta_2\left(\underline{g}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_1\left(1-\underline{J}_{\dot{F}_1}\right)^{r} + \theta_2\left(1-\underline{J}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right), \\ \left(\begin{array}{c} \min\left(1, \left(\theta_1\left(\overline{\overline{g}}_{\dot{F}_1}\right)^{r} + \theta_2\left(\overline{\overline{g}}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\theta_1\left(1-\overline{\overline{J}}_{\dot{F}_1}\right)^{r} + \theta_2\left(1-\overline{\overline{J}}_{\dot{F}_2}\right)^{r}\right)^{\frac{1}{r}}\right) \end{array} \right) \end{array} \right\} \\ &= \left\{ \begin{array}{c} \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{2}\left(\theta_i\left(\underline{g}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{2}\left(\theta_i\left(1-\underline{J}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right) \end{array} \right), \\ \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{2}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{2}\left(\theta_i\left(1-\overline{\overline{J}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right) \end{array} \right) \end{array} \right\} \end{aligned}$$

Step 2: Suppose equation (3) is true for $m = k$ that is

$$\begin{aligned} &IFRYWA_{\theta}\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) \\ &= \left\{ \begin{array}{c} \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{k}\left(\theta_i\left(\underline{g}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{k}\left(\theta_i\left(1-\underline{J}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right) \end{array} \right), \\ \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{k}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{k}\left(\theta_i\left(1-\overline{\overline{J}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right) \end{array} \right) \end{array} \right\} \end{aligned}$$

Now for $m = k + 1$, we get

$$IFRYWA_{\theta}\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_k, \dot{F}_{k+1}\right)$$

$$= \left\{\begin{array}{l} \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{k}\left(\theta_i\left(\underline{g}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{k}\left(\theta_i\left(1-\underline{Л}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right), \\ \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{k}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{k}\left(\theta_i\left(1-\overline{\overline{Л}}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right) \end{array}\right\}$$

$$\oplus \left\{\begin{array}{l} \left(\begin{array}{c} \min\left(1, \left(\theta_{k+1}\left(\underline{g}_{\dot{F}_{k+1}}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\theta_{k+1}\left(1-\underline{Л}_{\dot{F}_{k+1}}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right), \\ \left(\begin{array}{c} \min\left(1, \left(\left(\theta_{k+1}\left(\overline{\overline{g}}_{\dot{F}_{k+1}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\left(\theta_{k+1}\left(1-\overline{\overline{Л}}_{\dot{F}_{k+1}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right) \end{array}\right\}$$

$$= \left\{\begin{array}{l} \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{k+1}\left(\theta_i\left(\underline{g}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{k+1}\left(\theta_i\left(1-\underline{Л}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right), \\ \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{k+1}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{k+1}\left(\theta_i\left(1-\overline{\overline{Л}}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right) \end{array}\right\}$$

Hence equation (3) is true for $m = k + 1$. So the result is valid for all m.

Example 8: Let $\dot{F}_1 = \{(0.4, 0.5), (0.3, 0.4)\}, \dot{F}_2 = \{(0.6, 0.2), (0.4, 0.2)\}, \dot{F}_3 = \{(0.6, 0.3), (0.5, 0.3)\}$ and $\dot{F}_4 = \{(0.3, 0.5), (0.2, 0.7)\}$ be four IFRNs and $\mathfrak{r} = 4$. Also, $\theta = (0.31, 0.14, 0.34, 0.21)^{T}$. Now we use equation 3 to get the aggregated result as follows

$$IFRYWA_{\theta}\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \dot{F}_4\right)$$

$$= \left\{\begin{array}{l} \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(\underline{g}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(1-\underline{Л}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right), \\ \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(1-\overline{\overline{Л}}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right) \end{array}\right\}$$

$$= \left\{\begin{array}{l} \left(\begin{array}{c} \min\left(1, \left(\left(\begin{array}{c} 0.31\times(0.4)^4 + 0.14\times(0.6)^4 \\ +0.34\times(0.6)^4 + 0.21\times(0.3)^4 \end{array}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\begin{array}{c} 0.31\times(1-0.5)^4 \\ +0.14\times(1-0.2)^4 \\ +0.34\times(1-0.3)^4 \\ +0.21\times(1-0.5)^4 \end{array}\right)^{\frac{1}{4}}\right) \end{array}\right), \\ \left(\begin{array}{c} \min\left(1, \left(\left(\begin{array}{c} 0.31\times(0.3)^4 \\ +0.14\times(0.4)^4 \\ +0.34\times(0.5)^4 \\ +0.21\times(0.2)^4 \end{array}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\begin{array}{c} 0.31\times(1-0.4)^4 \\ +0.14\times(1-0.2)^4 \\ +0.34\times(1-0.3)^4 \\ +0.21\times(1-0.7)^4 \end{array}\right)^{\frac{1}{4}}\right) \end{array}\right) \end{array}\right\}$$

$$= ((0.5177, 0.3564), (0.4078, 0.3478)).$$

Theorem 3 (Idempotency): If $\dot{F}_i = \dot{F}$ for all i where $(i = 1, 2, 3, \ldots, m)$, then

$$IFRYWA\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_i\right) = \dot{F}.$$

Proof: As $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{Л}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{Л}}_{\dot{F}_i}\right)\right\} = \dot{F} = \left\{\left(\underline{g}_{\dot{F}}, \underline{Л}_{\dot{F}}\right), \left(\overline{\overline{g}}_{\dot{F}}, \overline{\overline{Л}}_{\dot{F}}\right)\right\}$ for all $(i = 1, 2, 3, \ldots, m)$. Then by using equation (3), we get

$$IFRYWA_{\theta}\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right)$$

$$= \left\{\begin{array}{l} \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\underline{g}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\underline{Л}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right), \\ \left(\begin{array}{c} \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\overline{\overline{Л}}_{\dot{F}_i}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right) \end{array}\right\}$$

$$= \left\{\begin{array}{l} \left(\begin{array}{c} \min\left(1, \left(\left(\underline{g}_{\dot{F}}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\left(1-\underline{Л}_{\dot{F}}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right), \\ \left(\begin{array}{c} \min\left(1, \left(\left(\overline{\overline{g}}_{\dot{F}}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right), \\ 1-\min\left(1, \left(\left(1-\overline{\overline{Л}}_{\dot{F}}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right) \end{array}\right\}$$

$$= \left\{\begin{array}{l} \left(\min\left(1, \left(\underline{g}_{\dot{F}}\right)\right), 1-\min\left(1, \left(1-\underline{Л}_{\dot{F}}\right)\right)\right), \\ \left(\min\left(1, \left(\overline{\overline{g}}_{\dot{F}}\right)\right), 1-\min\left(1, \left(1-\overline{\overline{Л}}_{\dot{F}}\right)\right)\right) \end{array}\right\}$$

$$= \left\{\left(\underline{g}_{\dot{F}}, \underline{Л}_{\dot{F}}\right), \left(\overline{\overline{g}}_{\dot{F}}, \overline{\overline{Л}}_{\dot{F}}\right)\right\} = \dot{F}$$

Theorem 4 (Boundedness): Suppose $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{Л}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{Л}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$ be the family of IFRNs and let $\dot{F}^{-} = \min\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right)$ *and* $\dot{F}^{+} = \max\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right)$. Then

$$\dot{F}^{-} \leq IFRYWA_{\theta}\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) \leq \dot{F}^{+}.$$

Proof: As $\dot{F}^{-} = \min\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \left\{\left(\underline{g}^{-}{}_{\dot{F}}, \underline{\text{Л}}^{-}{}_{\dot{F}}\right), \left(\overline{\overline{g}}^{-}{}_{\dot{F}}, \overline{\overline{\text{Л}}}^{-}{}_{\dot{F}}\right)\right\}$ *and* $\dot{F}^{+} = \max(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m) = \left\{\left(\underline{g}^{+}{}_{\dot{F}}, \underline{\text{Л}}^{+}{}_{\dot{F}}\right), \left(\overline{\overline{g}}^{+}{}_{\dot{F}}, \overline{\overline{\text{Л}}}^{+}{}_{\dot{F}}\right)\right\}$, where $\underline{g}^{-}{}_{\dot{F}} = \min\left(\underline{g}_{\dot{F}_\mathrm{i}}\right), \overline{\overline{g}}^{-}{}_{\dot{F}} = \min\left(\overline{\overline{g}}_{\dot{F}_\mathrm{i}}\right)$ and $\underline{\text{Л}}^{-}{}_{\dot{F}} = \max\left(\underline{\text{Л}}_{\dot{F}_\mathrm{i}}\right), \overline{\overline{\text{Л}}}^{-}{}_{\dot{F}} = \max\left(\overline{\overline{\text{Л}}}_{\dot{F}_\mathrm{i}}\right)$. Also, $\underline{g}^{+}{}_{\dot{F}} = \max\left(\underline{g}_{\dot{F}_\mathrm{i}}\right)$, $\overline{\overline{g}}^{+}{}_{\dot{F}} = \max\left(\overline{\overline{g}}_{\dot{F}_\mathrm{i}}\right)$ and $\underline{\text{Л}}^{+}{}_{\dot{F}} = \min\left(\underline{\text{Л}}_{\dot{F}_\mathrm{i}}\right), \overline{\overline{\text{Л}}}^{+}{}_{\dot{F}} = \min\left(\overline{\overline{\text{Л}}}_{\dot{F}_\mathrm{i}}\right)$. So, inequalities for lower values of MG are

$$\min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\underline{g}^{-}{}_{\dot{F}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\underline{g}_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\underline{g}^{+}{}_{\dot{F}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \textit{ and}$$

$$\min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\overline{\overline{g}}^{-}{}_{\dot{F}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(g_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\overline{\overline{g}}^{+}{}_{\dot{F}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right)$$

Similarly, for lower values of NMG, we get

$$1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \underline{\text{Л}}^{+}{}_{\dot{F}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq 1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \text{Л}_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq 1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \underline{\text{Л}}^{-}{}_{\dot{F}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right)$$

And

$$1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \overline{\overline{\text{Л}}}^{+}{}_{\dot{F}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq 1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \text{Л}_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq 1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \overline{\overline{\text{Л}}}^{-}{}_{\dot{F}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right).$$

Therefore

$$\dot{F}^{-} \leq IFRYWA\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) \leq \dot{F}^{+}.$$

Theorem 5 (Monotonicity): Suppose $\dot{F}^{*}_{\mathrm{i}} = \left\{\dot{F}^{*}_1, \dot{F}^{*}_2, \dot{F}^{*}_3, \ldots, \dot{F}^{*}_m\right\}$ *and* $\dot{F}_\mathrm{i} = \left\{\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right\}$ be the two families IFRNs. If $\underline{g}^{*}{}_{\dot{F}^{*}_\mathrm{i}} \leq \underline{g}_{\dot{F}_\mathrm{i}}, \overline{\overline{g}}^{*}{}_{\dot{F}^{*}_\mathrm{i}} \leq \overline{\overline{g}}^{*}{}_{\dot{F}_\mathrm{i}}$ and $\underline{\text{Л}}^{*}{}_{\dot{F}^{*}_\mathrm{i}} \geq \underline{\text{Л}}_{\dot{F}_\mathrm{i}}, \overline{\overline{\text{Л}}}^{*}{}_{\dot{F}^{*}_\mathrm{i}} \geq \overline{\overline{\text{Л}}}^{*}{}_{\dot{F}_\mathrm{i}}$ for all i, then

$$IFRYWA\left(\dot{F}^{*}_1, \dot{F}^{*}_2, \dot{F}^{*}_3, \ldots, \dot{F}^{*}_m\right) \leq IFRYWA\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right).$$

Proof: As $\underline{g}^{*}{}_{\dot{F}^{*}_\mathrm{i}} \leq \underline{g}_{\dot{F}_\mathrm{i}}$ then

$$\left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\underline{g}^{*}{}_{\dot{F}^{*}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}} \leq \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\underline{g}_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}$$

$$\min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\underline{g}^{*}{}_{\dot{F}^{*}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\underline{g}_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right),$$

Similarly

$$\left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\overline{\overline{g}}^{*}{}_{\dot{F}^{*}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}} \leq \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\overline{\overline{g}}_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}$$

$$\min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\overline{\overline{g}}^{*}{}_{\dot{F}^{*}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \leq \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(\overline{\overline{g}}_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right)$$

As

$$\underline{\text{Л}}^{*}{}_{\dot{F}^{*}_\mathrm{i}} \geq \underline{\text{Л}}_{\dot{F}_\mathrm{i}}$$

$$1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \underline{\text{Л}}^{*}{}_{\dot{F}^{*}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \geq 1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \underline{\text{Л}}_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right)$$

Similarly

$$1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \overline{\overline{\text{Л}}}^{*}{}_{\dot{F}^{*}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right) \geq 1 - \min\left(1, \left(\sum_{\mathrm{i}=1}^{m}\left(\theta_\mathrm{i}\left(1 - \overline{\overline{\text{Л}}}_{\dot{F}_\mathrm{i}}\right)^{\mathrm{r}}\right)\right)^{\frac{1}{\mathrm{r}}}\right).$$

From the above inequalities, we can conclude that

$$IFRYWA\left(\dot{F}^{*}_1, \dot{F}^{*}_2, \dot{F}^{*}_3, \ldots, \dot{F}^{*}_m\right) \leq IFRYWA\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right).$$

Theorem 6 (Reducibility): For the family of IFRNs $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{Л}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{Л}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$ and for $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_m)^T = \left(\frac{1}{m}, \frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}\right)^T$ is the WV of $\dot{F}_i$. Then

$$IFRYWA_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \frac{1}{m}\left(\sum_{i=1}^{m}\left(\left(\underline{g}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right),\\ 1-\min\left(1, \frac{1}{m}\left(\sum_{i=1}^{m}\left(\left(1-\underline{Л}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \frac{1}{m}\left(\sum_{i=1}^{m}\left(\left(\overline{\overline{g}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right),\\ 1-\min\left(1, \frac{1}{m}\left(\sum_{i=1}^{m}\left(\left(1-\overline{\overline{Л}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\end{array}\right)\end{array}\right\}.$$

Proof: Straightforward

Theorem 7 (Commutativity): Consider the family $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{Л}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{Л}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$ of IFRNs. If $\dot{F}_i^*$ is the permutation of $\dot{F}_i$ then

$$IFRYWA_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = IFRYWA_\theta\left(\dot{F}_1^*, \dot{F}_2^*, \dot{F}_3^*, \ldots, \dot{F}_m^*\right).$$

Proof: Straightforward

B. INTUITIONISTIC FUZZY ROUGH YAGER ORDERED WEIGHTED ARITHMETIC OPERATORS

As IFRYWA aggregation operators only weigh the IFR values and it does not weigh the ordered position, so to cover this issue, here in this subsection, we have to elaborate on the notion of IFRYOWA operators.

Definition 10: For the family of IFRNs $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{Л}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{Л}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$. Then IFR Yager ordered weighted arithmetic (IFRYOWA) operator is defined by a mapping $\mathfrak{F} : \mathcal{G}^m \rightarrow \mathcal{G}$ such that

$$IFRYOWA_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \oplus_{i=1}^{m}\left(\theta_i \dot{F}_{\propto(i)}\right)$$

where $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_m)^T$ is WV of $\dot{F}_i$ with condition that $\sum_{i=1}^{m}\theta_i = 1$ and $\theta_i > 0$ and $(\propto(1), \propto(2), \propto(3), \ldots, \propto(m))$ is the permutation of $(i = 1, 2, 3, \ldots, m)$ such that $\dot{F}_{\propto(i-1)} \geq \dot{F}_{\propto(i)}$ for all $i = 1, 2, 3, \ldots, m$.

Theorem 8: Let $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{Л}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{Л}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$ be the family of IFRNs with $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_m)^T$ is WV of $\dot{F}_i$ with condition that $\sum_{i=1}^{m}\theta_i = 1$. Then aggregated result obtained from the IFRYOWA operator is again IFRN given by

$$IFRYOWA_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \oplus_{i=1}^{m}\left(\theta_i \dot{F}_{\propto(i)}\right)$$

$$= \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\underline{g}_{\dot{F}_{\propto(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right),\\ 1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\underline{Л}_{\dot{F}_{\propto(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_{\propto(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right),\\ 1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\overline{\overline{Л}}_{\dot{F}_{\propto(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\end{array}\right)\end{array}\right\} \quad (4)$$

Example 9: Let $\dot{F}_1 = \{(0.3, 0.2), (0.1, 0.8)\}$, $\dot{F}_2 = \{(0.5, 0.4), (0.2, 0.6)\}$, $\dot{F}_3 = \{(0.6, 0.2), (0.7, 0.1)\}$ and $\dot{F}_4 = \{(0.2, 0.4), (0.3, 0.6)\}$ be four IFRNs and $r = 4$. Also, $\theta = (0.20, 0.22, 0.34, 0.24)^T$.

Note that

$$S\left(\dot{F}_1\right) = \frac{1}{4}(2 + 0.3 + 0.1 - 0.2 - 0.8) = 0.35$$
$$S\left(\dot{F}_2\right) = \frac{1}{4}(2 + 0.5 + 0.2 - 0.4 - 0.6) = 0.425$$
$$S\left(\dot{F}_3\right) = \frac{1}{4}(2 + 0.6 + 0.7 - 0.2 - 0.1) = 0.75$$
$$S\left(\dot{F}_4\right) = \frac{1}{4}(2 + 0.2 + 0.3 - 0.4 - 0.6) = 0.375$$

It means that $S\left(\dot{F}_3\right) > S\left(\dot{F}_2\right) > S\left(\dot{F}_4\right) > S\left(\dot{F}_1\right)$, therefore

$$\dot{F}_{\propto(1)} = \dot{F}_1 = \{(0.6, 0.2), (0.7, 0.1)\},$$
$$\dot{F}_{\propto(2)} = \dot{F}_2 = \{(0.5, 0.4), (0.2, 0.6)\},$$
$$\dot{F}_{\propto(3)} = \dot{F}_3 = \{(0.2, 0.4), (0.3, 0.6)\},$$
$$\dot{F}_{\propto(4)} = \dot{F}_4 = \{(0.3, 0.2), (0.1, 0.8)\}$$

Now we use equation 4 to get the aggregated result as follows

$$IFRYWA_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \dot{F}_4\right) = \left\{\begin{array}{l}\left(\begin{array}{c}\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(\underline{g}_{\dot{F}_{\propto(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right),\\ 1-\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(1-\underline{Л}_{\dot{F}_{\propto(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\end{array}\right),\\ \left(\begin{array}{c}\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_{\propto(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right),\\ 1-\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(1-\overline{\overline{Л}}_{\dot{F}_{\propto(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\end{array}\right)\end{array}\right\}$$
$$= ((0.4531, 0.2909), (0.4755, 0.3819)).$$

Remark 1: Intuitionistic fuzzy rough Yager-ordered weighted arithmetic operators satisfy all properties given in theorems 3, 4, 5, 6, and 7.

C. INTUITIONISTIC FUZZY ROUGH YAGER HYBRID WEIGHTED ARITHEMATIC OPERATOR

From definitions 9 and 10, it is clear that IFRYWA aggregation operators only weights the IFR values and IFRYOWA aggregation operators weigh the ordered position of IFR values, but to discuss both characteristics in one frame, in this subsection, we aim is to define the notion for IFRYHWA operators.

Definition 11: For the family of IFRNs $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{J}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{J}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$. Then IFR Yager hybrid weighted arithmetic (IFRYHWA) operator is defined by a mapping $\mathfrak{F}: \mathcal{G}^m \rightarrow \mathcal{G}$ such that

$$IFRYHWA_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \oplus_{i=1}^{m}\left(\theta_i \check{\dot{F}}_{\alpha(i)}\right) = \left\{\begin{array}{c}\left(\begin{array}{c}\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\underline{\check{g}}_{\dot{F}_{\alpha(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\underline{\check{J}}_{\dot{F}_{\alpha(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\end{array}\right), \\ \left(\begin{array}{c}\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\overline{\overline{\check{g}}}_{\dot{F}_{\alpha(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\overline{\overline{\check{J}}}_{\dot{F}_{\alpha(i)}}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\end{array}\right)\end{array}\right\} \tag{5}$$

where $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_m)^T$ is WV of $\dot{F}_i$ with condition that $\sum_{i=1}^{m}\theta_i = 1$ and $\theta_i > 0$ and $\check{\dot{F}}_{\alpha(i)}$ is the *ith* biggest weighted IFR values $\check{\dot{F}}_i$ $\left(\check{\dot{F}}_i = m\theta_i\dot{F}_i \; i = 1, 2, 3, \ldots, m\right)$ and m is the balancing coefficient.

Remark 2: For $\theta = \left(\frac{1}{m}, \frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}\right)^T$, IFRYWA and IFRYOWA operators are special cases for IFRYHWA operators. Thus IFRYHWA operator is the more generalized operator.

V. INTUITIONISTIC FUZZY ROUGH YAGER WEIGHTED GEOMETRIC AGGREGATION OPERATORS

As IFRYWG aggregation operators weigh the IFR values so based on this observation, we aim to discuss the basic definition of IFRYWG operators and discover their properties.

Definition 12: For the family of IFRNs $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{J}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{J}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$. Then IFRYWG operators are defined by a mapping $\mathfrak{F}: \mathcal{G}^m \rightarrow \mathcal{G}$ such that

$$IFRYWG_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \otimes_{i=1}^{m}\left(\dot{F}_i^{\theta_i}\right)$$

where $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_m)^T$ is the WVs of $\dot{F}_i$ with condition that $\sum_{i=1}^{m}\theta_i = 1$ and $\theta_i > 0$.

Theorem 9: Let $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{J}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{J}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$ be the family of IFRNs. Then the result obtained from the IFRYWG operator is again IFRN given by

$$IFRYWG_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \otimes_{i=1}^{m}\left(\dot{F}_i^{\theta_i}\right) = \left\{\begin{array}{c}1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\underline{g}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ \left(\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\underline{J}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1-\overline{\overline{g}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ \left(\min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\overline{\overline{J}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\right)\end{array}\right\} \tag{6}$$

Example 10: Consider the data of example 8 and theorem 9 we get

$$IFRYWG_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \left\{\begin{array}{c}1-\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(1-\underline{g}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ \left(\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(\underline{J}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\right), \\ 1-\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(1-\overline{\overline{g}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ \left(\min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(\overline{\overline{J}}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right)\right)\end{array}\right\}$$

$$= \left\{\begin{array}{c}\left(\begin{array}{c}1-\min\left(1, \left(\begin{array}{c}0.31\times(1-0.4)^4 \\ +0.14\times(1-0.6)^4 \\ +0.34\times(1-0.6)^4 \\ +0.21\times(1-0.3)^4\end{array}\right)^{\frac{1}{4}}\right), \\ \min\left(1, \left(\left(\begin{array}{c}0.31\times(0.5)^4 \\ +0.14\times(0.2)^4 \\ +0.34\times(0.3)^4 \\ +0.21\times(0.5)^4\end{array}\right)\right)^{\frac{1}{r}}\right)\end{array}\right), \\ \left(\begin{array}{c}1-\min\left(1, \left(\begin{array}{c}0.31\times(1-0.3)^4 \\ +0.14\times(1-0.4)^4 \\ +0.34\times(1-0.5)^4 \\ +0.21\times(1-0.2)^4\end{array}\right)^{\frac{1}{4}}\right), \\ \min\left(1, \left(\left(\begin{array}{c}0.31\times(0.4)^4 \\ +0.14\times(0.2)^4 \\ +0.34\times(0.3)^4 \\ +0.21\times(0.7)^4\end{array}\right)\right)^{\frac{1}{r}}\right)\end{array}\right)\end{array}\right\}$$

$$= ((0.4336, 0.4340), (0.3313, 0.4976)).$$

Remark 3: IFRYWG operators satisfy all properties given in theorem 3, 4, 5, 6, and 7.

Now we propose the basic definition for IFRYOWG operator.

Definition 13: For the family of IFRNs $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{J}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{J}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$. Then IFRYOWG operators are defined by a mapping $\mathfrak{F}: \mathcal{G}^m \rightarrow \mathcal{G}$ such that

$$IFRYOWG_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \otimes_{i=1}^{m}\left(\dot{F}_{\alpha(i)}\right)^{\theta_i}$$

where $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_m)^T$ is WV of $\dot{F}_i$ with condition that $\sum_{i=1}^{m} \theta_i = 1$ and $\theta_i > 0$ and $(\propto(1), \propto(2), \propto(3), \ldots, \propto(m))$ is the permutation of $(i = 1, 2, 3, \ldots, m)$ such that $\dot{F}_{\propto(i-1)} \geq \dot{F}_{\propto(i)}$ for all $i = 1, 2, 3, \ldots, m$.

As IFRYWG aggregation operators only weigh the IFR values and it does not weigh the ordered position, to cover this issue, in the next theorem, we have to elaborate on the notion of IFRYOWG operators.

Theorem 10: Let $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{J}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{J}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$ be the family of IFRNs with $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_m)^T$ is WV of $\dot{F}_i$ with condition that $\sum_{i=1}^{m} \theta_i = 1$. Then aggregated result obtained from the IFRYOWG operator is again IFRN given by

$$IFRYOWG_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \otimes_{i=1}^{m}\left(\theta_i \dot{F}_{\propto(i)}\right) = \left\{\begin{array}{l} \left(\begin{array}{c} 1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1 - \underline{J}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\underline{g}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right), \\ \left(\begin{array}{c} 1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1 - \overline{\overline{J}}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right) \end{array}\right\} \quad (7)$$

Proof: Similar to Theorem 2.

Example 11: Using the data of example 9 and theorem 10, we get

$$IFRYOWG_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \otimes_{i=1}^{m}\left(\theta_i \dot{F}_{\propto(i)}\right) = \left\{\begin{array}{l} \left(\begin{array}{c} 1 - \min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(1 - \underline{J}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ \min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(\underline{g}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right), \\ \left(\begin{array}{c} 1 - \min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(1 - \overline{\overline{J}}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ \min\left(1, \left(\sum_{i=1}^{4}\left(\theta_i\left(\overline{\overline{g}}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right) \end{array}\right\} = ((0.3184, 0.3501), (0.2415, 0.6429)).$$

Remark 4: IFRYOWG operators satisfy all properties given in theorem 3, 4, 5, 6, and 7.

From definitions 12 and 13, it is clear that IFRYWG aggregation operators only weights the IFR values and IFRYOWG aggregation operators weights the ordered position, but to discuss both characteristics in one frame, we aim is to define the notion for IFRYHWG operators.

Definition 14: For the family of IFRNs $\dot{F}_i = \left\{\left(\underline{g}_{\dot{F}_i}, \underline{J}_{\dot{F}_i}\right), \left(\overline{\overline{g}}_{\dot{F}_i}, \overline{\overline{J}}_{\dot{F}_i}\right)\right\}$ $(i = 1, 2, \ldots, m)$. Then IFRYHWG operators are defined by a mapping $\mathfrak{F}: \mathcal{G}^m \rightarrow \mathcal{G}$ such that

$$IFRYHWG_\theta\left(\dot{F}_1, \dot{F}_2, \dot{F}_3, \ldots, \dot{F}_m\right) = \otimes_{i=1}^{m}\left(\check{\dot{F}}_{\propto(i)}\right)^{\theta_i} = \left\{\begin{array}{l} \left(\begin{array}{c} 1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1 - \check{\underline{J}}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\check{\underline{g}}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right), \\ \left(\begin{array}{c} 1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(1 - \check{\overline{\overline{J}}}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right), \\ \min\left(1, \left(\sum_{i=1}^{m}\left(\theta_i\left(\check{\overline{\overline{g}}}_{\dot{F}_{\propto(i)}}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right) \end{array}\right) \end{array}\right\} \quad (8)$$

where $\boldsymbol{\theta} = (\boldsymbol{\theta_1}, \boldsymbol{\theta_2}, \boldsymbol{\theta_3}, \ldots, \boldsymbol{\theta_m})^{\mathbf{T}}$ is WV of $\dot{F}_i$ with condition that $\sum_{i=1}^{m} \boldsymbol{\theta}_i = \mathbf{1}$ and $\boldsymbol{\theta}_i > \mathbf{0}$ and $\check{\dot{F}}_{\propto(i)}$ is the ***ith*** biggest weighted IFR values $\check{\dot{F}}_i$ $\left(\check{\dot{F}}_i = \boldsymbol{m\theta}_i\dot{F}_i\, i = \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots, \boldsymbol{m}\right)$ and $\boldsymbol{m}$ is the balancing coefficient.

VI. EDAS METHOD BASED ON INTUITIONISTIC FUZZY ROUGH YAGER AGGREGATION OPERATORS

In this section based on the proposed approach, we will discuss the EDAS method. The EDAS method was established by Ghorabaeee et al. [7].

Suppose $\{\mathscr{B}_1, \mathscr{B}_2, \mathscr{B}_3, \ldots, \mathscr{B}_m\}$ be set of $'m'$ alternatives and $\{\mathscr{d}_1, \mathscr{d}_2, \mathscr{d}_3, \ldots, \mathscr{d}_n\}$ denote the set of $'n'$ attributes. Also, suppose $\{\eth_1, \eth_2, \eth_3, \ldots, \eth_\mathfrak{h}\}$ be the set of $\mathfrak{h}$ experts for each alternative $\mathscr{B}_i$ $(i = 1, 2, 3, \ldots, m)$ against attributes $\mathscr{d}_j$ $(j = 1, 2, 3, \ldots, n)$. Let $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_n)^T$ be the WVs for attributes $\mathscr{d}_j$ and $\mathfrak{L} = (\mathfrak{L}_1, \mathfrak{L}_2, \mathfrak{L}_3, \ldots, \mathfrak{L}_\mathfrak{h})^T$ be the WVs for decision-makers $\eth_l$ $(l = 1, 2, 3, \ldots, \mathfrak{h})$ such that $\sum_{j=1}^{n} \theta_j = 1$ and $\sum_{l=1}^{\mathfrak{h}} \mathfrak{L}_j = 1$. Now algorithm for the EDAS method based on initiated work is given by

Step 1: Collect the assessment values given by experts $\eth_l$ for each alternative $\mathscr{B}_i$ against attributes $\mathscr{d}_j$ in the form of a matrix

$$M = \dot{F}_{ij}^{l} = \left[\left(\underline{g}_{\dot{F}_{ij}}, \underline{J}_{\dot{F}_{ij}}\right), \left(\overline{\overline{g}}_{\dot{F}_{ij}}, \overline{\overline{J}}_{\dot{F}_{ij}}\right)\right]_{m \times n}$$

where $\dot{F}_{ij}^{l}$ represent IFRNs for each alternative $\mathscr{B}_i$ against attributes $\mathscr{d}_j$.

Step 2: Utilize the suggested method to aggregate the collective decision matrix and obtain the aggregated decision matrix as

$$M = \dot{F}_{ij} = \left[\left(\underline{g}_{\dot{F}_{ij}}, \underline{J}_{\dot{F}_{ij}}\right), \left(\overline{\overline{g}}_{\dot{F}_{ij}}, \overline{\overline{J}}_{\dot{F}_{ij}}\right)\right]_{m \times n}$$

Step 3: Normalize the aggregated matrix by using the formula given below

Normalized the information given in step 1 by using the formula given below

$$M^n = \dot{F}^n_{ij} = \begin{cases} \dot{F}_{ij} = \left\{ \left(\underline{g}_{\dot{F}_{ij}}, \underline{Л}_{\dot{F}_{ij}} \right), \left(\overline{\overline{g}}_{\dot{F}_{ij}}, \overline{\overline{Л}}_{\dot{F}_{ij}} \right) \right\}; \\ \quad \textit{For benefit type attributes} \\ (\dot{F}_{ij})^c = \left\{ \left(\underline{Л}_{\dot{F}_{ij}}, \underline{g}_{\dot{F}_{ij}} \right), \left(\overline{\overline{Л}}_{\dot{F}_{ij}}, \overline{\overline{g}}_{\dot{F}_{ij}} \right) \right\}; \\ \quad \textit{For cost type attributes} \end{cases}$$

where $(\dot{F}_{ij})^c$ is the complement of $\dot{F}_{ij} = \left(\underline{g}_{\dot{F}_{ij}}, \underline{Л}_{\dot{F}_{ij}} \right), \left(\overline{\overline{g}}_{\dot{F}_{ij}}, \overline{\overline{Л}}_{\dot{F}_{ij}} \right)$.

Step 4: Calculate the values of Avs by using the established approach for all alternatives under each attribute

$$Avs = [Avs]_{1\times n} = \left[\frac{1}{m} \sum_{i=1}^{m} \dot{F}^n_{ij} \right]_{1\times n}$$

$$Avs = [Avs]_{1\times n} = \left[\frac{1}{m} \sum_{i=1}^{m} \dot{F}^n_{ij} \right]_{1\times n}$$

$$= \left\{ \begin{pmatrix} \min\left(1, \left(\sum_{i=1}^{m} \left(\frac{1}{m} \left(\underline{g}_{\dot{F}_i}{}^{n}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1 - \min\left(1, \left(\sum_{i=1}^{m} \left(\frac{1}{m} \left(1 - \underline{Л}_{\dot{F}_i}{}^{n}\right)^{r}\right)\right)^{\frac{1}{r}}\right) \end{pmatrix}, \begin{pmatrix} \min\left(1, \left(\sum_{i=1}^{m} \left(\frac{1}{m} \left(\overline{\overline{g}}^{n}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right), \\ 1 - \min\left(1, \left(\sum_{i=1}^{m} \left(\frac{1}{m} \left(1 - \overline{\overline{Л}}^{n}_{\dot{F}_i}\right)^{r}\right)\right)^{\frac{1}{r}}\right) \end{pmatrix} \right\}$$

Step 5: Based on obtained Avs obtained from step 4, we will calculate $PDAS$ and $NDAS$ by using the formula given below

$$PDAS_{ij} = \left[PDAS_{ij}\right]_{m\times n} = \frac{max\left(0, S\left(\dot{F}^n_{ij}\right) - S\left(Avs_j\right)\right)}{S\left(Avs_j\right)}$$

$$NDAS_{ij} = \left[NDAS_{ij}\right]_{m\times n} = \frac{max\left(0, S\left(Avs_j\right) - S\left(\dot{F}^n_{ij}\right)\right)}{S\left(Avs_j\right)}$$

Step 6: Calculate the positive weight distance (SP_i) and negative weight distance SN_i by

$$SP_i = \sum_{j=1}^{n} \theta_j PDAS_{ij}, SN_i = \sum_{j=1}^{n} \theta_j NDAS_{ij}$$

Step 7: Normalize SP_i *and* SN_i by using formula

$$NSP_i = \frac{SP_i}{\max_i (SP_i)}, \quad NSN_i = 1 - \frac{SN_i}{\max_i (SN_i)}$$

Step 8: Based on NSP_i *and* NSN_i determine the score values by utilizing the formula

$$AS_i = \frac{1}{2}(NSP_i + NSN_i)$$

Step 9: Rank all values based on $\boldsymbol{AS_i}$ and choose the best result.

A. ILLUSTRATIVE EXAMPLE

To guarantee a successful robotics project in the robotic industry, you need to develop a strong specification, regardless of whether you are new to the robotics process or have a team of professionals on staff. You and your robotics designer will be better able to select the appropriate platform and technological update if you consider the fundamental robot requirements and capabilities. There are many challenges to decide the best factor in robotics projects. The robotics industry faces many issues. These challenges include (1) better power sources (2) Navigation unmapped environment (3) Brain-computer interfaces (4) Social robots for long-term engagement (5) Ethics etc.

Here we have four main factors that play a vital role in any robot industry and try to cope with all the challenges faced by any robot project.

1) PLATFORM DIMENSION

Your robot must have a small enough footprint to fit in the area where you plan to utilize it. The size and weight of all the hardware mounted on your robot are determined by the platform's dimensions and construction.

2) PAYLOAD

Your robotics project's payload is an important component. Other elements are impacted by the amount of weight that your platform must support. Speed, size, and platform weight are all strongly connected to the payload as well. Make sure your robotics integrator is aware of your speed and payload needs.

3) BATTERY TECHNOLOGY

The majority of robotics solutions provide lead-acid or lithium-ion batteries. Based on aspects including safety, environmental impact, capacity, charge time, efficiency, size/weight, and longer life cycle, lithium-ion is regarded as the most cutting-edge technology for transportation systems. Advanced sensors and lithium-ion batteries enable even more efficient energy use and longer battery life.

4) CONTROL SOFTWARE

You could require access to the primary software of your robotics platform depending on the application of your project and the level of customization you intend to perform. The software may be open source or proprietary, depending on the platform. Make sure to choose an open-source solution or an integrator with a robotics expert who has access to the proprietary software if you need access to the program.

Example 12: Assume that we have four main factors for any robot project and we are going to select the best factor that plays the main role in any robotic project. Let these four factors be given by

$\mathscr{b}_1$ = Platform dimension
$\mathscr{b}_2$ = Payload
$\mathscr{b}_3$ = Battery Technology

TABLE 2. IFR data given by $ð_1$.

	d_1	d_2	d_3	d_4	d_5
$\mathscr{b}_1$	$\{(0.21, 0.31), (0.43, 0.41)\}$	$\{(0.16, 0.36), (0.17, 0.37)\}$	$\{(0.26, 0.44), (0.27, 0.45)\}$	$\{(0.34, 0.54), (0.35, 0.55)\}$	$\{(0.23, 0.63), (0.24, 0.64)\}$
$\mathscr{b}_2$	$\{(0.31, 0.33), (0.11, 0.31)\}$	$\{(0.18, 0.38), (0.19, 0.39)\}$	$\{(0.28, 0.46), (0.29, 0.47)\}$	$\{(0.36, 0.56), (0.37, 0.57)\}$	$\{(0.25, 0.65), (0.26, 0.66)\}$
$\mathscr{b}_3$	$\{(0.12, 0.32), (0.13, 0.33)\}$	$\{(0.20, 0.41), (0.21, 0.42)\}$	$\{(0.30, 0.49), (0.31, 0.51)\}$	$\{(0.38, 0.58), (0.39, 0.59)\}$	$\{(0.27, 0.67), (0.11, 0.71)\}$
$\mathscr{b}_4$	$\{(0.14, 0.34), (0.15, 0.35)\}$	$\{(0.22, 0.43), (0.23, 0.45)\}$	$\{(0.32, 0.52), (0.33, 0.53)\}$	$\{(0.21, 0.61), (0.22, 0.62)\}$	$\{(0.15, 0.81), (0.14, 0.22)\}$

TABLE 3. IFR data given by $ð_2$.

	d_1	d_2	d_3	d_4	d_5
$\mathscr{b}_1$	$\{(0.22, 0.33), (0.23, 0.42)\}$	$\{(0.26, 0.46), (0.57, 0.36)\}$	$\{(0.46, 0.49), (0.57, 0.15)\}$	$\{(0.31, 0.50), (0.30, 0.51)\}$	$\{(0.13, 0.23), (0.34, 0.54)\}$
$\mathscr{b}_2$	$\{(0.41, 0.23), (0.41, 0.21)\}$	$\{(0.68, 0.18), (0.59, 0.29)\}$	$\{(0.21, 0.26), (0.27, 0.43)\}$	$\{(0.26, 0.46), (0.35, 0.47)\}$	$\{(0.15, 0.19), (0.46, 0.16)\}$
$\mathscr{b}_3$	$\{(0.52, 0.22), (0.63, 0.30)\}$	$\{(0.10, 0.11), (0.19, 0.52)\}$	$\{(0.20, 0.41), (0.21, 0.11)\}$	$\{(0.48, 0.28), (0.60, 0.19)\}$	$\{(0.47, 0.17), (0.19, 0.21)\}$
$\mathscr{b}_4$	$\{(0.74, 0.14), (0.85, 0.11)\}$	$\{(0.26, 0.53), (0.63, 0.18)\}$	$\{(0.12, 0.18), (0.13, 0.23)\}$	$\{(0.61, 0.33), (0.12, 0.52)\}$	$\{(0.18, 0.29), (0.24, 0.39)\}$

$\mathscr{b}_4 =$ Control Software

To assess these factors there are three experts $\{ð_1, ð_2, ð_3\}(ð_i = 1, 2, 3)$ under WVs $\mathfrak{L} = \{0.38, 0.28, 0.34\}^T$. Suppose the experts assess these four factors that play their role in any robotic projects under the five attributes as $\{d_1 = Intelligence\ d_2 = Sence\ perception, d_3 = Power, d_4 = Independence, d_5 = Functionality\}$. Suppose the WVs for attributes are $\theta = \{0.20, 0.18, 0.24, 0.22, 0.16\}^T$. Now we use the stepwise algorithm for EDAS techniques as given below

Step 1: Collect the information proposed by each expert in the form IFRNs given in Tables 2-4.

Step 2: A collective decision matrix given by experts against their WVs is aggregated by the proposed IFRYWA operator to get the aggregated decisions matrix as given in Table 5.

Step 3: As all criteria are of benefit type so no need to normalize this matrix.

Step 4: Calculate the values of *Avs* by using the established approach for all alternatives under each attribute and the results are given in Table 6.

Step 5: Based on obtained *Avs* obtained from step 4, we can find the score value for each Avs_i (i = 1, 2, ..., 5) and then calculate *PDAS* and *NDAS* given in Table 7-8.

$$S(Avs_1) = 0.5230, \quad S(Avs_2) = 0.5426,$$

TABLE 4. IFR data given by $ð_3$.

	d_1	d_2	d_3	d_4	d_5
$\mathscr{b}_1$	$\{(0.26, 0.46), (0.57, 0.36)\}$	$\{(0.41, 0.23), (0.41, 0.21)\}$	$\{(0.61, 0.33), (0.12, 0.52)\}$	$\{(0.12, 0.18), (0.13, 0.23)\}$	$\{(0.18, 0.29), (0.24, 0.39)\}$
$\mathscr{b}_2$	$\{(0.26, 0.53), (0.63, 0.18)\}$	$\{(0.74, 0.14), (0.85, 0.11)\}$	$\{(0.18, 0.38), (0.19, 0.39)\}$	$\{(0.38, 0.58), (0.39, 0.59)\}$	$\{(0.23, 0.63), (0.24, 0.64)\}$
$\mathscr{b}_3$	$\{(0.12, 0.18), (0.13, 0.23)\}$	$\{(0.18, 0.29), (0.24, 0.39)\}$	$\{(0.52, 0.22), (0.63, 0.30)\}$	$\{(0.18, 0.18), (0.20, 0.29)\}$	$\{(0.17, 0.47), (0.59, 0.14)\}$
$\mathscr{b}_4$	$\{(0.15, 0.17), (0.31, 0.24)\}$	$\{(0.19, 0.23), (0.73, 0.10)\}$	$\{(0.57, 0.17), (0.63, 0.24)\}$	$\{(0.21, 0.49), (0.22, 0.29)\}$	$\{(0.19, 0.39), (0.38, 0.18)\}$

TABLE 5. Collective aggregated decision matrix by using IFRYWA operator.

	d_1	d_2	d_3	d_4	d_5
$\mathscr{b}_1$	$\{(0.2330, 0.3565), (0.4720, 0.3940)\}$	$\{(0.3248, 0.3254), (0.4457, 0.3006)\}$	$\{(0.4976, 0.4056), (0.4217, 0.3321)\}$	$\{(0.2965, 0.3413), (0.2996, 0.3763)\}$	$\{(0.1968, 0.3380), (0.2798, 0.4931)\}$
$\mathscr{b}_2$	$\{(0.3375, 0.3384), (0.4980, 0.2314)\}$	$\{(0.6346, 0.2207), (0.6784, 0.2380)\}$	$\{(0.2385, 0.3612), (0.2606, 0.4285)\}$	$\{(0.3485, 0.5304), (0.3721, 0.5402)\}$	$\{(0.2253, 0.3965), (0.3522, 0.3774)\}$
$\mathscr{b}_3$	$\{(0.3779, 0.2371), (0.4588, 0.2836)\}$	$\{(0.1775, 0.2549), (0.2173, 0.4308)\}$	$\{(0.4104, 0.3437), (0.4899, 0.2730)\}$	$\{(0.3900, 0.2994), (0.4621, 0.3169)\}$	$\{(0.3554, 0.3634), (0.4516, 0.2613)\}$
$\mathscr{b}_4$	$\{(0.5388, 0.2114), (0.6218, 0.2270)\}$	$\{(0.2260, 0.3547), (0.6135, 0.2125)\}$	$\{(0.4470, 0.2549), (0.4910, 0.3071)\}$	$\{(0.4476, 0.4550), (0.2043, 0.4251)\}$	$\{(0.1746, 0.4130), (0.3005, 0.2402)\}$

TABLE 6. The value of the average solution *Avs*.

d_1	$\{(0.03211, 0.0028), (0.0660, 0.0031)\}$
d_2	$\{(0.07279, 0.0027), (0.1049, 0.0045)\}$
d_3	$\{(0.0222, 0.0057), (0.0240, 0.0056)\}$
d_4	$\{(0.0129, 0.0167), (0.0149, 0.0172)\}$
d_5	$\{(0.0040, 0.0081), (0.0133, 0.0095)\}$

TABLE 7. The evaluation of $PDAS_{ij}$.

	d_1	d_2	d_3	d_4	d_5
$\mathscr{b}_1$	0.0000	0.0000	0.07214	0.0000	0.0000
$\mathscr{b}_2$	0.08294	0.3151	0.0000	0.0000	0.0000
$\mathscr{b}_3$	0.1074	0.0000	0.1221	0.1212	0.09137
$\mathscr{b}_4$	0.3010	0.0469	0.1657	0.0000	0.0000

$$S(Avs_3) = 0.5087, \quad S(Avs_4) = 0.4984,$$
$$S(Avs_5) = 0.4999$$

Step 6: Calculate the positive weight distance (SP_i) and negative weight distance SN_i by

$$SP_i = \sum\nolimits_{j=1}^{n} \theta_j PDAS_{ij}, SN_i = \sum\nolimits_{j=1}^{n} \theta_j NDAS_{ij}$$

as given in Table 9.

TABLE 8. The evaluation of $NDAS_{ij}$.

	d_1	d_2	d_3	d_4	d_5
$\mathscr{b}_1$	0.0658	0.0119	0.0000	0.0578	0.1771
$\mathscr{b}_2$	0.0000	0.0000	0.1599	0.1724	0.0980
$\mathscr{b}_3$	0.000	0.2125	0.0000	0.0000	0.0000
$\mathscr{b}_4$	0.0000	0.0000	0.0000	0.1113	0.0888

TABLE 9. Values of SP_i and SN_i.

SP_i $(i = 1, 2, 3, 4)$	SN_i $(i = 1, 2, 3, 4)$
0.01731	0.05638
0.07330	0.0920
0.09211	0.0382
0.10888	0.0387

TABLE 10. Values of NSP_i and N SN_i.

NSP_i $(i = 1, 2, 3, 4)$	NSN_i $(i = 1, 2, 3, 4)$
0.15902	0.38727
0.67327	0.00000
0.84599	0.58418
1.0000	0.57919

TABLE 11. Ranking order for proposed approaches.

Proposed operators based on the EDAS method	AS_i values for alternatives $\mathscr{b}_1$ $\mathscr{b}_2$ $\mathscr{b}_3$ $\mathscr{b}_4$	Ranking
IFRYWA	0.2731, 0.3336, 0.7150, 0.7895	$\mathscr{b}_4 > \mathscr{b}_3 > \mathscr{b}_2 > \mathscr{b}_1$
IFRYWG	0.0092, 0.0000, 0.04476, 0.5767	$\mathscr{b}_4 > \mathscr{b}_3 > \mathscr{b}_1 > \mathscr{b}_2$
IFRYOWA	1.0000, 0.6855, 0.3633, 0.0000	$\mathscr{b}_1 > \mathscr{b}_2 > \mathscr{b}_3 > \mathscr{b}_4$
IFRYOWG	0.8902, 0.2544, 0.1506, 0.0000	$\mathscr{b}_1 > \mathscr{b}_2 > \mathscr{b}_3 > \mathscr{b}_4$

Step 7: Normalize SP_i and SN_i by using formula

$$NSP_i = \frac{SP_i}{\max_i (SP_i)}, \quad NSN_i = 1 - \frac{SN_i}{\max_i (SN_i)}$$

as given in Table 10.

Step 8: Based on NSP_i and NSN_i calculate the score values by using the formula

$$AS_i = \frac{1}{2}(NSP_i + NSN_i)$$

$$AS_1 = 0.2731, AS_2 = 0.3366,$$

$$AS_3 = 0.7150, AS_4 = 0.7895.$$

Step 9: As $AS_4 > AS_3 > AS_2 > AS_1$. So, the experts should select $\mathscr{b}_4$ as the best factor that is valuable in any robotics project.

Now in Table 11, we present the ranking orders of other proposed aggregation operators like IFRYOWA operators, IFRYWG operators, and IFRYOWG operators.

TABLE 12. Comparative study.

Methods	AS_i values for alternatives $\mathscr{b}_1$ $\mathscr{b}_2$ $\mathscr{b}_3$ $\mathscr{b}_4$	Ranking
IFWA [15]	Cannot be calculated	×
IFWG [16]	Cannot be calculated	×
IFDWA [17]	Cannot be calculated	×
IFWDG [17]	Cannot be calculated	×
IF EDAS Method [32]	Cannot be calculated	×
IF VIKOR method [18]	Cannot be calculated	×
IFRYWA proposed	0.2731, 0.3336, 0.7150, 0.7895	$\mathscr{b}_4 > \mathscr{b}_3 > \mathscr{b}_2 > \mathscr{b}_1$
IFRYOWA proposed	1.0000, 0.6855, 0.3633, 0.0000	$\mathscr{b}_1 > \mathscr{b}_2 > \mathscr{b}_3 > \mathscr{b}_4$
IFRYWG Proposed	0.0092, 0.0000, 0.04476, 0.5767	$\mathscr{b}_4 > \mathscr{b}_3 > \mathscr{b}_1 > \mathscr{b}_2$
IFRYOWD proposed	0.8902, 0.2544, 0.1506, 0.0000	$\mathscr{b}_1 > \mathscr{b}_2 > \mathscr{b}_3 > \mathscr{b}_4$

VII. COMPARATIVE ANALYSIS

In this part, we will discuss the comparative assessment of the introduced work. For this, we will take the data from Table 6 and WVs $\boldsymbol{\theta} = \{\mathbf{0.20}, \mathbf{0.18}, \mathbf{0.24}, \mathbf{0.22}, \mathbf{0.16}\}^{\mathbf{T}}$. The results are listed in Table 12. From the analysis of Table 12, we can say the IF VIKOR method [18] method, IF EDAS method [32], and some aggregation operators based on IF data selected from [15], [16], and [17] are inaccessible to cover the data given in Table 12. Because all these theories are based on IFNs while the initiated approach is based on intuitionistic fuzzy rough numbers. It means that all existing theories fail due to a lack of rough information. The main reason is that the developed theory is based on intuitionistic fuzzy rough numbers that consist of upper and lower approximation operators, while the existing theories cannot discuss the information that consists of upper and lower approximation operators. That's why all the prevailing theories cannot handle the data given in Table 12. On the other hand, all proposed work can handle the rough data. We can observe that established work can provide more space for decision-makers and it can cover the deficiency of rough information.

VIII. CONCLUSION

These days we have to deal with ambiguous and complex data. In MCGDM, we can process the multiple criteria in all areas of DM for valuable results. In fuzzy set theory, IFS and RS are effective tools to handle complex data. EDAS method is one of the efficient and fruitful methods to handle DM problems that depend upon ***PDAS*** and ***NDAS*** from ***Avs***. The superior value of ***PDAS*** and inferior of ***NDAS*** is considered the optimal choice. We have started with IFR operative laws based on Yager t-norms and t-conorm. To use these operational laws we have introduced intuitionistic fuzzy rough Yager weighted athematic aggregation operators like IFRYWA and IFRYOWA aggregation operators. Also, we have established IFRYWG and IFRYOWG operators. Moreover, the properties of these operatives have been elaborated on in detail. To study the combined structure of the EDAS method with IFR Yager aggregation operators, in the application section, we have developed EDAS methods based on IFRY aggregation operators. An illustrative example shows the effective use of these notions in MCGDM problems. Also, a comparative analysis of the introduction work shows the effectiveness of established notions.

These notions are also limited because if someone comes up with Pythagorean fuzzy rough information like $\{(\mathbf{0.5}, \mathbf{0.6}), (\mathbf{0.4}, \mathbf{0.7})\}$, then the developed notion cannot handle this data because the necessary condition for the intuitionistic fuzzy rough set is violated which is $\mathbf{0.5} + \mathbf{0.6} \notin [\mathbf{0}, \mathbf{1}]$ ***and*** $\mathbf{0.4} + \mathbf{0.7} \notin [\mathbf{0}, \mathbf{1}]$. So the basic condition is violated and IFRS failed to cover that data. So proposed notions are also limited.

Some new theories can be examined like similarities measures based on these notions as given in [38]. Additionally, as stated in [39], this technique can be exactly generalized to a stronger structure. The given approach can be extended to m-polar fuzzy soft rough set [40], IF N-soft rough set [41], and some new decision-making algorithms can be developed as given in [42]. Moreover, we can define some new aggregation operators like interactive Hamacher power aggregation operators as given in [43]. We can also define the TOPSIS method and COPRAS method based on the developed approach as given in [44] and [45].

Data availability:

Not applicable

Conflict of Interest:

The authors declare no conflict of interest.

Author Contribution:

All the authors contributed equally.

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