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RESEARCH ARTICLE

# Geodesic Affinity Propagation Clustering Based on Angle-Based Outlier Factor

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**ABSTRACT** The affinity propagation (AP) clustering algorithm has received a lot of attention over the past few years. AP is efficient and insensitive to initialization, and generates clustering results with lower error and in less time. However, there are still two key limitations: AP-related algorithms cannot identify outliers in clusters. And they are usually not ideal for processing nonlinear data. To address the above issues, we propose a geodesic affinity propagation clustering algorithm based on angle-based outlier factor (ABOF-GAP). First, outliers are identified according to the value of angle-based outlier factor. Besides, Euclidean distance is replaced with geodesic distance to measure similarity. Experiments on synthetic data and real data illustrate the effectiveness of the ABOF-GAP algorithm.

**INDEX TERMS** Affinity propagation (AP), geodesic distances, outlier identification, angle-based outlier factor (ABOF).

## I. INTRODUCTION

Cluster analysis is a critical step in data science [1], [2], [3]. As a classical clustering algorithm based on similarity measure, K-means algorithm starts with a random set of initial cluster centers and minimize iterately the sum of squared errors between the data point and the corresponding examplars [4], [5], [6]. However, this algorithm is very sensitive to the setting of initial cluster centers. And it usually reruns many times for different initializations. In 2007, Frey and Dueck proposed Affinity Propagation (AP) [7], a clustering method which simultaneously considers all data as potential exemplars. It takes as input the similarities between data points. Then the messages between data points are continuously passed until a set of high-quality exemplars appear. By contrast, AP does not need to initialize the cluster centers and the number of clusters. AP is shown to perform more efficiency (with lower error and in less time) than k-means algorithm [8], [9], [10].

Many improved versions of AP have been proposed in recent years. In 2013, Wang et al presented a multi-exemplar affinity propagation (MEAP) algorithm to extend the single-exemplar model [11]. In 2017, Li et al proposed an adjustable preference affinity propagation algorithm (APAP). It computes the value of each element preference based on the data distribution in the initial stage, and the preference will adjust automatically during iteration process [12]. Besides, many other similarity measurement method were also developed to improve the Euclidean distance used in AP. In 2017, Liu et al used the negative sine-squared value of the acute angle between discontinuity unit normal vectors as similarity measurement [13]. In 2020, Wang proposed an IMFSE-AP method, which measures similarities between signals based on the sample entropy of IMFs [14].

Although many achievements have been made in AP-related researches, the performance of AP or AP-based algorithms is vulnerable to outliers. The existence of outliers will affect the accuracy of clustering results. In some practical problems, the identification of outliers is even critical. As for this issue, some improved methods have been developed. In 2012, Sushravya et al addressed this limitation through the use of an asymmetric dissimilarity metric and a density-based outlier detection technique [15]. In 2018, Lei and Li proposed a Semi-supervised Affinity Propagation Clustering Algorithm based on outlier pruning (LOF-SAP), which used the local outlier factor algorithm (LOF) to identify

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outliers [16]. But there still is a problem that these methods are hard to apply to other kinds of data.

The identification of cluster outliers is a big challenge for the application and development of AP algorithms. In addition, the clustering performance of AP-related algorithms may get worse when dealing with nonlinear data. Calculating similarity between data points using Euclidean distance ignores the non-linear structure of the data, and may bring unexpected errors in clustering. The main contribution of this paper is to propose an improved AP algorithm, geodesic affinity propagation clustering based on angle-based outlier factor (ABOF-GAP). In our algorithm, outliers can be identified and eliminated more accurately based on the angle-based outlier factor. Besides, Euclidean distance is replaced with geodesic distance to measure similarity between data points. This is more consistent with the nonlinear structure of data. Experiments on both synthetic data and real data prove that the proposed ABOF-GAP algorithm can obtain better performance.

The remaining of this paper is organized as follows. In Section II, we give a brief review of AP algorithm, geodesic distance and angle-based outlier factor. In Section III, we propose the ABOF-GAP algorithm. In Section IV, we show the experimental results on simulation data and real-world data. At last, Section V states conclusions.

## II. AFFINITY PROPAGATION METHOD AND RELATED PROBLEM

### A. AP

Affinity Propagation is an exemplar-based clustering method, which considers all data points as potential exemplars simultaneously and identify clusters automatically. Compared with other clustering method, AP can avoid poor clustering results caused by unlucky initialization and hard decision, and find clusters with much lower error and in less time. AP takes as input measures of similarity between pairs of data points, then real-valued messages are transmitted between data points recursively until a high-quality set of exemplars and corresponding clusters gradually emerges. For data points $i$ and data point $k$, their similarity $s(i,k)$ is set to a negative Euclidean distance, which reflects how well data point $k$ is suited to be the exemplar for data points $i$. In particular, AP takes as input a real number $s(k,k)$ called preference for each data point $k$, and the data point with larger value of $s(k,k)$ is more likely to be chosen as an exemplar. In AP-based algorithms, all the values of preferences are commonly set as a constant, the median of the input similarities generally, which could results in a moderate number of clusters.

In the iterative process, two kinds of message, responsibility $r(i,k)$ and availability $a(i,k)$, are exchanged between data points. The responsibility $r(i,k)$ sent from $i$ to $k$, reflects the priority for data point $k$ to be chosen as the exemplar of point $i$, compared with other potential exemplars for point $i$. The availability $a(i,k)$ sent from $k$ to $i$, reflects the fitness for point $k$ to be the exemplar of point $i$, with the support from other points that point $k$ can serve as an exemplar. Messages are updated on the basis of simple formulas that search for minima of an appropriately chosen energy function. At any point in time, the magnitude of each message reflects the current affinity that one data point has for choosing another data point as its exemplar. The concrete algorithm is shown as follows:

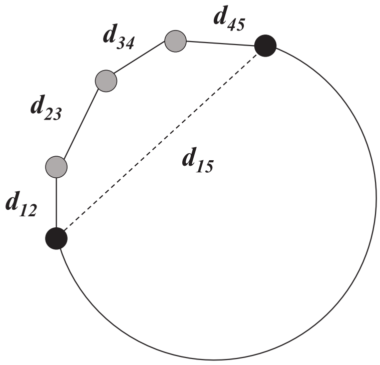


**FIGURE 1.** Geodesic distance.

1) Input the matrix of similarities, $S$, where $S(i,k)$ is the similarity $s(i,k)$ between point $i$ and point $k$. For point $x_i$ and $x_k$,
$$s(i,k) = -||x_i - x_k||^2. \tag{1}$$
and $s(k,k)$ is commonly set to be the median of the input similarities.
2) Compute responsibility $r(i,k)$, where
$$r(i,k) = s(i,k) - \max_{k' \neq k}\{a(i,k') + s(i,k')\}. \tag{2}$$
To begin with, the availabilities are set to zero in the first iteration.
3) Compute availability $a(i,k)$, where
$$a(i,k) = \min\left\{0, r(k,k) + \sum_{i' \notin \{i,k\}} \max\{0, r(i',k)\}\right\}, \tag{3}$$
$$a(k,k) = \sum_{i' \neq k} \max\{0, r(i',k)\}. \tag{4}$$
4) Combine availabilities and responsibilities to identify exemplars and corresponding clusters, and terminate the algorithm: (i) after a fixed number of iteration, (ii) after changes in the messages fall below a threshold, or (iii) after the local decisions stay constant for some number of iterations.

### B. GEODESIC DISTANCE

In AP and AP-based algorithms, each similarity between data points is set to be a negative Euclidean distance. However, it is difficult to obtain a good clustering effect when the dimension of dataset increases gradually. Geodesic distance (GD) is an important concept in mathematical morphology. The way of calculating distance is not only determined by the spatial location of two samples, but also by the spatial distribution of the dataset. For example, in graph theory, geodesic distance is the distance of the shortest path between two points in the graph, which is different from Euclidean distance. In Figure 1, the Euclidean distance of the two black points should be the length of the line segment represented by the dashed line $d_{15}$. But geodesic distance is the shortest distance of the actual path. Its distance should be the minimum value of the sum of the distances of the solid line segments along the way, namely $d_{12} + d_{23} + d_{34} + d_{45}$.

In order to make better use of the manifold structure information in data with complex structure, this paper will adopt the negative geodesic distance as the similarity measure between data points.

### C. ABOF

The existence of outliers in original data influences the performance of AP-related methods. We use ABOF algorithm to solve this problem. ABOF algorithm is an outlier detection method based on angle [17]. As the dimensionality of the data increases, comparing distances becomes less and less meaningful. The ABOF algorithm not only uses the distance between points in a vector space, but also mainly considers the difference between the angles of the vectors. Figure 2 shows a 2D dataset with distinct in-cluster points, boundary points, and outliers. It can be found that the difference in the angle between the vectors formed by the points in the cluster and other points is very large. The difference in the angle between the vectors formed by the boundary points and other points is smaller. The difference in the angle between the vectors formed by the outliers and other points is very small.

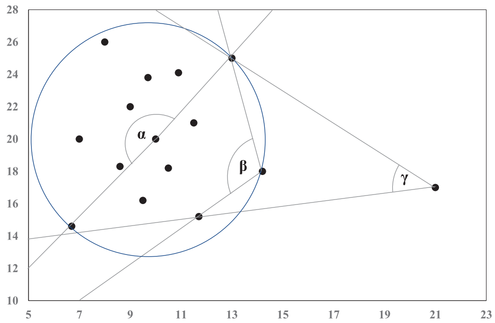


**FIGURE 2. Point distribution.**

The ABOF algorithm uses the cosine between the vectors to measure the size of the angle, and uses the variance to measure the size of the angle difference. For all data points, each data has an ABOF value, to detect outliers. For dataset $D$, the concrete descriptions of ABOF algorithm are as follows:

1) Cosine of the angle between two vectors:

$$\cos\left\langle \overrightarrow{AB}, \overrightarrow{AC} \right\rangle = \frac{\left\langle \overrightarrow{AB}, \overrightarrow{AC} \right\rangle}{\left|\overrightarrow{AB}\right| \cdot \left|\overrightarrow{AC}\right|} \tag{5}$$

2) Angle-based outlier factor of point $A$:

$$ABOF\left(\vec{A}\right) = VAR_{\vec{B},\vec{C}\in D}\left(\frac{\cos\left\langle \overrightarrow{AB}, \overrightarrow{AC} \right\rangle}{\|AB\|^2 \cdot \|AC\|^2}\right) \tag{6}$$

The essence of the ABOF algorithm is the weighted variance of the angle cosine. So the smaller the ABOF value is, the more likely the object is an outlier.

## III. THE PROPOSED CLUSTERING METHOD

In this section, we propose an improved AP algorithm.

### A. OUTLIER DETECTION

In the sample space, points inconsistent with the general behavior or characteristics of other sample points are called outliers. The outliers is generally caused by the error of calculation or operation, or the variability or elasticity of the data itself. The existence of outliers often leads to calculation errors and even some false information. But on the other hand, outliers can sometimes provide important information that researchers are concerned about. For example, outliers in medical data are usually more important than normal data. Therefore, the identification of outliers is very important in clustering problems. We give a simple example to illustrate the problem of AP algorithm on outlier detection.

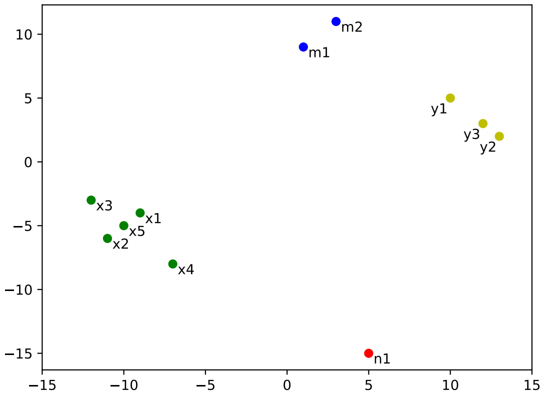


**FIGURE 3. Point distribution.**

We design a dataset with 11 data points in 2D space, which is shown in Figure 3. In this dataset, there are 1 outlier (in red), and 3 categories with 5 (in green), 3 (in gold), and 2 (in blue) data points respectively. It is visually obvious that point $n1$ is isolated and should be considered as an outlier. However, under AP algorithm, point $n1$ is assigned to the cluster with point $x5$ as its "cluster center." And point $x5$ is the closest exemplar to point $n1$. In this example, the clustering results of AP algorithm are inconsistent with the real distribution of

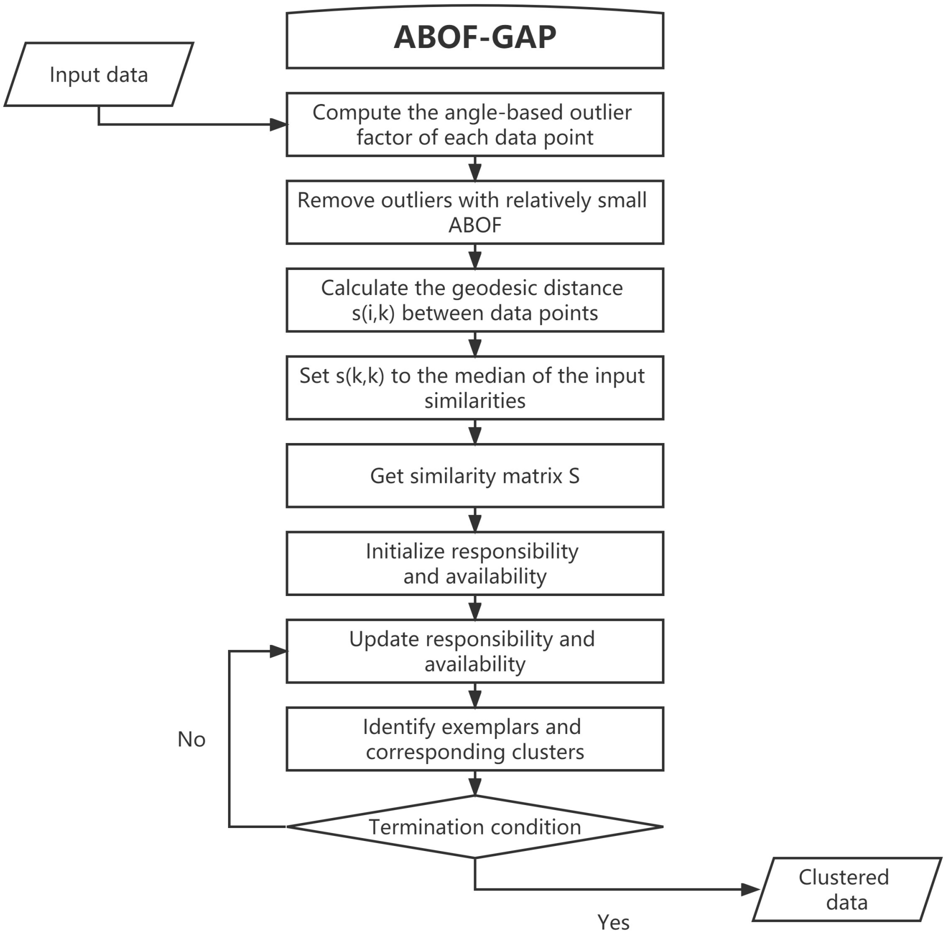

ABOF-GAP
Input data
Compute the angle-based outlier factor of each data point
Remove outliers with relatively small ABOF
Calculate the geodesic distance s(i,k) between data points
Set s(k,k) to the median of the input similarities
Get similarity matrix S
Initialize responsibility and availability
Update responsibility and availability
Identify exemplars and corresponding clusters
Termination condition
No
Yes
Clustered data

**FIGURE 4.** Flowchart of ABOF-GAP algorithm.

the data. This is because AP cannot identify outliers automatically, which would have a negative effect on the accuracy of clustering results.

### *B. ABOF-GAP*

In order to make up for the shortcoming that AP cannot identify outliers, we propose an improved AP algorithm based on geodesic distance and the ABOF method.

For each data point $x_i$, we calculate its angle-based outlier factor $ABOF(x_i)$ based on Formula (6). According to ABOF principle, the smaller the ABOF value is, the more likely the object is an outlier. So we remove outliers from original dataset with relatively small ABOF, and get a pruned dataset.

After getting rid of the outliers, we calculate geodesic distance to evaluate the similarity between data points. For data point $x_i$ and data point $x_j$, their similarity $s(i, k)$ is set to a negative geodesic distance. Then we set s(k,k) to the median of the input similarities and get the similarity matrix S.

At last we input similarity matrix S and perform AP clustering algorithm. Responsibility $r(i, k)$ and availability $a(i, k)$ are initialized and calculated iteratively via Formula (2)-(4). When meeting termination condition, we combine availabilities and responsibilities to identify exemplars and their corresponding clusters.

**Algorithm 1** ABOF-GAP

```
1) Calculate the angle-based outlier factor of each
   data point by (6) in turn to get $ABOF\ (x_i)$.
2) Remove outliers with relatively small ABOF.
3) Calculate the geodesic distance s(i,k) between data
   points.
4) Set s(k,k) to the median of the input similarities
   and get similarity matrix S.
5) Perform AP clustering algorithm.
6) Output clusters and their exemplars.
```

This algorithm is summarized in Algorithm 1. And the flowchart for the proposed ABOF-GAP algorithm is shown in Figure 4.

**TABLE 1.** The clustering results on the synthetic datasets.

| Datasets | Algorithm | RI | NMI | Precision | Recall | F-score |
|---|---|---|---|---|---|---|
| data5d | AP [7] | 0.7204 | 0.5313 | 0.1331 | 0.9777 | 0.2344 |
| | LOF-SAP [16] | 0.7224 | 0.5366 | 0.1391 | 0.9785 | 0.2436 |
| | CLAP [23] | 0.7213 | 0.5326 | 0.1383 | 0.8970 | 0.2647 |
| | SSAPEC [24] | 0.7230 | 0.5446 | 0.1463 | 0.9758 | 0.2634 |
| | DDAP [25] | 0.7211 | 0.5472 | 0.1356 | 0.9755 | 0.2856 |
| | GAP | 0.7285 | 0.5534 | 0.1593 | 0.9764 | 0.2739 |
| | **ABOF-GAP** | **0.7330** | **0.5646** | **0.1809** | **0.9810** | **0.3055** |
| data25d | AP [7] | 0.7897 | 0.6678 | 0.3503 | 0.9868 | 0.5170 |
| | LOF-SAP [16] | 0.7906 | 0.6700 | 0.3532 | 0.9928 | 0.5211 |
| | CLAP [23] | 0.7899 | 0.6845 | 0.3654 | 0.9901 | 0.5198 |
| | SSAPEC [24] | 0.7900 | 0.6783 | 0.3763 | 0.9882 | 0.5324 |
| | DDAP [25] | 0.7901 | 0.6743 | 0.3515 | 0.9879 | 0.5209 |
| | GAP | 0.7974 | 0.6847 | 0.3750 | 0.9860 | 0.5434 |
| | **ABOF-GAP** | **0.8169** | **0.7081** | **0.4419** | **0.9966** | **0.6104** |
| data50d | AP [7] | 0.8181 | 0.7090 | 0.4410 | 0.9847 | 0.6092 |
| | LOF-SAP [16] | 0.8250 | 0.7292 | 0.4691 | 0.9856 | 0.6351 |
| | CLAP [23] | 0.8194 | 0.7135 | 0.4534 | 0.9884 | 0.6235 |
| | SSAPEC [24] | 0.8198 | 0.7241 | 0.4475 | 0.9893 | 0.6135 |
| | DDAP [25] | 0.8234 | 0.8235 | 0.4575 | 0.9904 | 0.6109 |
| | GAP | 0.8351 | 0.7334 | 0.4970 | 0.9801 | 0.6595 |
| | **ABOF-GAP** | **0.8905** | **0.8158** | **0.6752** | **0.9929** | **0.8001** |

### C. COMPUTATIONAL COMPLEXITY

This section summarizes the computational complexity of the proposed ABOF-GAP algorithm. Assuming that there are N samples in the dataset, in ABOF algorithm, each data point in the dataset is combined with any two points other than itself to form a vector. The computational complexity of this step is $O(N^2)$, which means that the computational complexity of generating vectors for all objects in the data set is $O(N^3)$. In addition, the original AP clustering takes $N \times N$ similarity matrix S as input. Therefore, the computational complexity of the original AP clustering is $O(N^2T)$, where T is the number of iterations. Overall, the computational complexity of ABOF-GAP algorithm is $O(N^3 + N^2T)$.

## IV. EXPERIMENTS

In this section, several numerical experiments are conducted to benchmark our algorithm.

### A. EVALUATION INDEX

In order to make a reasonable evaluation of the results of each clustering algorithm, we use the following five evaluation metrics to analyze and compare the clustering performance.

Suppose $U$ is the external evaluation criterion and $V$ is the clustering result. TP is the number of pairs of data points of the same class in $U$ and $V$; TN is the number of pairs of data points of the different class in $U$ and $V$; FP is the number of pairs of data points of the different class in $U$ but of the same class in $V$; FN is the number of pairs of data points of the same class in $U$ but of the different class in $V$.

The rand index (RI) can be considered as the ratio of correct clusters, which is a classic measure of the similarity between two data clusterings [18]. RI is defineded as:

$$RI = \frac{TP + TN}{TP + FP + FN + TN} \tag{7}$$

The Precision measures the proportion of sample points that are correctly assigned [19]. Precision is defined as:

$$Precision = \frac{TP}{TP + FP} \tag{8}$$

The Recall indicates the correctness of the clustering results [20]. Recall is defined as:

$$Recall = \frac{TP}{TP + FN} \tag{9}$$

The F-score counts the harmonic mean of Precision and Recall [21]. F-score is defined as:

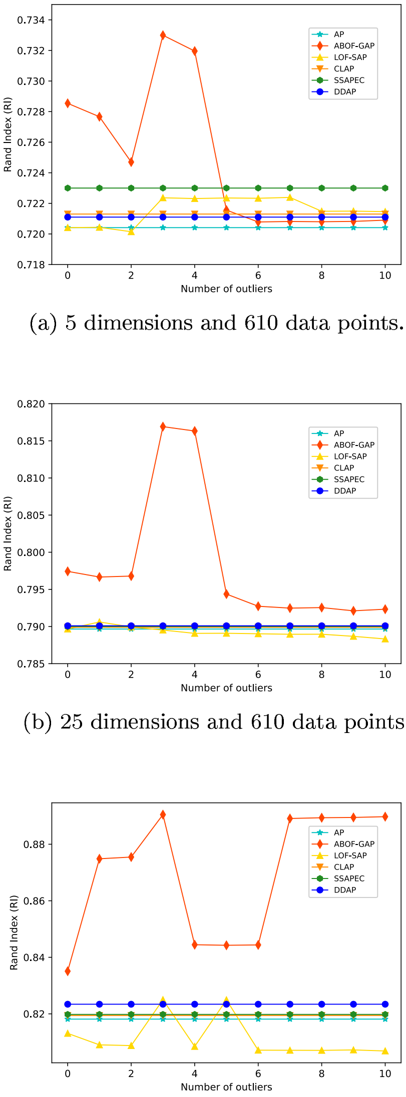
$$F - score = 2 \cdot \frac{Precision \cdot Recall}{Precision + Recall} \tag{10}$$

The normalized mutual information (NMI) evalutes the similarity between two clusters from an information theory perspective [22]. NMI is defined as:

$$NMI = \frac{\sum_{i=1}^{c} \sum_{j=1}^{c} N_{i,j} log \frac{N \cdot N_{i,j}}{N_i \cdot N_j}}{\sqrt{\sum_{i=1}^{c} N_i log \frac{N_i}{N} \cdot \sum_{j=1}^{c} N_j log \frac{N_j}{N}}} \tag{11}$$

**TABLE 2.** The clustering results on the synthetic datasets.

| Datasets | Algorithm | RI | NMI | Precision | Recall | F-score |
|---|---|---|---|---|---|---|
| D=10 | AP [7] | 0.7435 | 0.5768 | 0.2055 | 0.9807 | 0.3286 |
| | LOF-SAP [16] | 0.7451 | 0.5811 | 0.2105 | 0.9833 | 0.3361 |
| | CLAP [23] | 0.7442 | 0.5832 | 0.2140 | 0.9280 | 0.3497 |
| | SSAPEC [24] | 0.7453 | 0.5892 | 0.2230 | 0.9799 | 0.3531 |
| | DDAP [25] | 0.7441 | 0.5896 | 0.2076 | 0.9796 | 0.3640 |
| | **ABOF-GAP** | **0.7610** | **0.6124** | **0.2679** | **0.9862** | **0.4071** |
| D=20 | AP [7] | 0.7782 | 0.6451 | 0.3141 | 0.9853 | 0.4699 |
| | LOF-SAP [16] | 0.7792 | 0.6478 | 0.3175 | 0.9904 | 0.4749 |
| | CLAP [23] | 0.7785 | 0.6592 | 0.3276 | 0.9746 | 0.4773 |
| | SSAPEC [24] | 0.7788 | 0.6560 | 0.3380 | 0.9861 | 0.4876 |
| | DDAP [25] | 0.7786 | 0.6531 | 0.3155 | 0.9858 | 0.4817 |
| | **ABOF-GAP** | **0.8029** | **0.6842** | **0.3984** | **0.9940** | **0.5596** |
| D=30 | AP [7] | 0.8011 | 0.6843 | 0.3866 | 0.9860 | 0.5539 |
| | LOF-SAP [16] | 0.8044 | 0.6937 | 0.3996 | 0.9899 | 0.5667 |
| | CLAP [23] | 0.8017 | 0.6961 | 0.4006 | 0.9894 | 0.5613 |
| | SSAPEC [24] | 0.8019 | 0.6966 | 0.4048 | 0.9886 | 0.5648 |
| | DDAP [25] | 0.8034 | 0.7340 | 0.3939 | 0.9889 | 0.5569 |
| | **ABOF-GAP** | **0.8463** | **0.7512** | **0.5352** | **0.9951** | **0.6863** |
| D=40 | AP [7] | 0.8124 | 0.7008 | 0.4229 | 0.9851 | 0.5908 |
| | LOF-SAP [16] | 0.8181 | 0.7174 | 0.4459 | 0.9870 | 0.6123 |
| | CLAP [23] | 0.8135 | 0.7077 | 0.4358 | 0.9887 | 0.6028 |
| | SSAPEC [24] | 0.8138 | 0.7149 | 0.4333 | 0.9891 | 0.5973 |
| | DDAP [25] | 0.8167 | 0.7937 | 0.4363 | 0.9899 | 0.5929 |
| | **ABOF-GAP** | **0.8758** | **0.7943** | **0.6285** | **0.9936** | **0.7622** |
| D=50 | AP [7] | 0.8181 | 0.7090 | 0.4410 | 0.9847 | 0.6092 |
| | LOF-SAP [16] | 0.8250 | 0.7292 | 0.4691 | 0.9856 | 0.6351 |
| | CLAP [23] | 0.8194 | 0.7135 | 0.4534 | 0.9884 | 0.6235 |
| | SSAPEC [24] | 0.8198 | 0.7241 | 0.4475 | 0.9893 | 0.6135 |
| | DDAP [25] | 0.8234 | 0.8235 | 0.4575 | 0.9904 | 0.6109 |
| | **ABOF-GAP** | **0.8905** | **0.8158** | **0.6752** | **0.9929** | **0.8001** |
| D=60 | AP [7] | 0.8217 | 0.7149 | 0.4523 | 0.9847 | 0.6220 |
| | LOF-SAP [16] | 0.8322 | 0.7422 | 0.4928 | 0.9852 | 0.6609 |
| | CLAP [23] | 0.8238 | 0.7199 | 0.4670 | 0.9906 | 0.6392 |
| | SSAPEC [24] | 0.8292 | 0.7403 | 0.4738 | 0.9902 | 0.6439 |
| | DDAP [25] | 0.8328 | 0.8567 | 0.4874 | 0.9915 | 0.6389 |
| | **ABOF-GAP** | **0.9109** | **0.8468** | **0.7394** | **0.9930** | **0.8577** |
| D=70 | AP [7] | 0.8231 | 0.7173 | 0.4569 | 0.9847 | 0.6271 |
| | LOF-SAP [16] | 0.8346 | 0.7465 | 0.5007 | 0.9850 | 0.6695 |
| | CLAP [23] | 0.8252 | 0.7220 | 0.4715 | 0.9913 | 0.6444 |
| | SSAPEC [24] | 0.8365 | 0.7527 | 0.4941 | 0.9909 | 0.6673 |
| | DDAP [25] | 0.8376 | 0.8733 | 0.5023 | 0.9920 | 0.6528 |
| | **ABOF-GAP** | **0.9268** | **0.8711** | **0.7899** | **0.9931** | **0.9030** |
| D=80 | AP [7] | 0.8160 | 0.7054 | 0.4342 | 0.9847 | 0.6015 |
| | LOF-SAP [16] | 0.8314 | 0.7408 | 0.4901 | 0.9852 | 0.6580 |
| | CLAP [23] | 0.8194 | 0.7135 | 0.4534 | 0.9884 | 0.6235 |
| | SSAPEC [24] | 0.8387 | 0.7564 | 0.5002 | 0.9911 | 0.6743 |
| | DDAP [25] | 0.8384 | 0.8761 | 0.5048 | 0.9921 | 0.6552 |
| | **ABOF-GAP** | **0.9327** | **0.8800** | **0.8082** | **0.9932** | **0.9195** |
| D=90 | AP [7] | 0.8059 | 0.6888 | 0.4025 | 0.9846 | 0.5657 |
| | LOF-SAP [16] | 0.8218 | 0.7234 | 0.4586 | 0.9858 | 0.6236 |
| | CLAP [23] | 0.8085 | 0.6975 | 0.4194 | 0.9830 | 0.5842 |
| | SSAPEC [24] | 0.8380 | 0.7552 | 0.4982 | 0.9910 | 0.6719 |
| | DDAP [25] | 0.8344 | 0.8622 | 0.4924 | 0.9916 | 0.6435 |
| | **ABOF-GAP** | **0.9370** | **0.8866** | **0.8220** | **0.9932** | **0.9318** |
| D=100 | AP [7] | 0.7909 | 0.6639 | 0.3549 | 0.9845 | 0.5119 |
| | LOF-SAP [16] | 0.8074 | 0.6974 | 0.4112 | 0.9867 | 0.5720 |
| | CLAP [23] | 0.7925 | 0.6740 | 0.3695 | 0.9750 | 0.5266 |
| | SSAPEC [24] | 0.8336 | 0.7477 | 0.4860 | 0.9906 | 0.6579 |
| | DDAP [25] | 0.8281 | 0.8401 | 0.4724 | 0.9909 | 0.6249 |
| | **ABOF-GAP** | **0.9385** | **0.8888** | **0.8266** | **0.9932** | **0.9359** |



(a) 5 dimensions and 610 data points.

(b) 25 dimensions and 610 data points.

(c) 50 dimensions and 610 data points.

**FIGURE 5.** **Rand Index(RI) of clustering result on synthetic data sets for 5, 25 and 50 dimensions.**

where, $N$ is the number of data, $N_i$ and $N_j$ denote the number of data in category $i$ and cluster $j$ respectively, $N_{i,j}$ denotes the number of data in category $i$ as well as in cluster $j$.

The above five methods have a range of value [0,1]. As the value is higher, the clustering quality is better. The value is close to 1 if the clustering results perfectly match the category labels. And the value is close to 0 if data are randomly partitioned.

### B. SYNTHETIC DATA EXPERIMENTS

The synthetic dataset for simulation evaluation consists of 600 normal data points and 10 outliers. Normal data points are from three different Gaussian distributions. Outliers are randomly generated for each data set. To exactly evaluate the behavior of our new method for different dimensionalities, we generated multiple data sets in 5, 25 and 50 dimensions.

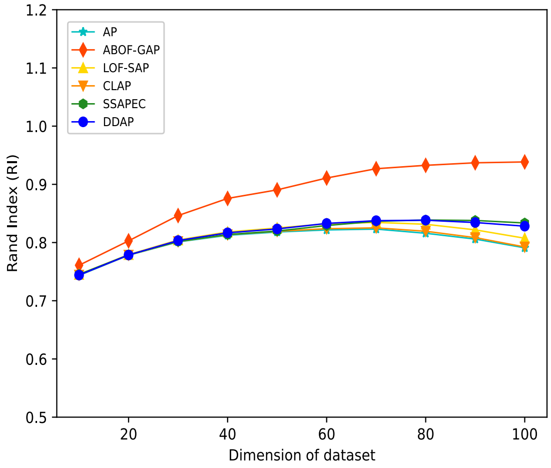


**FIGURE 6.** **Rand Index(RI) of clustering result on synthetic data sets for 10-100 dimensions.**

**TABLE 3.** **Characteristics of datebases.**

| Dateset | Size | Class | Dimensions |
|---|---|---|---|
| Wine | 178 | 3 | 13 |
| Iris | 150 | 4 | 4 |
| Glass | 178 | 7 | 9 |
| Segmnt | 210 | 7 | 19 |
| PenDigits | 10992 | 10 | 16 |
| Semeion | 1593 | 10 | 256 |

Besides, we also constructed an "intermediate" algorithm, geodesic affinity propagation (GAP) for comparison. The GAP method uses geodesic distance instead of Euclidean distance to measure similarity between data points. The GAP method can be taken as a reference for the proposed ABOF-GAP algorithm. This is in a view that the GAP does not include the outlier elimination process based on ABOF. For comparison, AP [7], LOF-SAP [16], CLAP [23], SSAPEC [24], DDAP [25] and GAP algorithm are all performed on these three datasets for comparison. The results are shown in Table 1.

As shown in Table 1, all five indices of ABOF-GAP are significantly higher than the other six algorithms. To better show the difference between the new algorithm and other algorithms in different dimensions, we make RI for each clustering result, which are shown in Figure 5. As the dimension increases, the gap between ABOF-GAP and other algorithms in indexes also increases. The above results also indicate that, as the dimension increases, distance gradually loses its value as a measure of outlier degree. However, this does not mean that the more outliers are eliminated, the better effect of the clustering could be got. Two indexes of ABOF-GAP reach the highest values when 3 outliers are removed, instead of all outliers being pruned. This may because the relative positions of all data points will change after an outlier is removed.

**TABLE 4.** The clustering results on the UCI datasets.

| Datasets | Algorithm | RI | NMI | Precision | Recall | F-score |
|---|---|---|---|---|---|---|
| Wine | AP [7] | 0.7141 | 0.4805 | 0.1690 | 0.1998 | 0.1832 |
| | LOF-SAP [16] | 0.7228 | 0.5077 | 0.2006 | 0.2486 | 0.2221 |
| | CLAP [23] | 0.7265 | 0.5194 | 0.1843 | 0.2683 | 0.2597 |
| | SSAPEC [24] | 0.7354 | 0.5423 | 0.2298 | 0.3743 | 0.2790 |
| | DDAP [25] | 0.7234 | 0.5254 | 0.2156 | 0.2943 | 0.2609 |
| | GAP | 0.7537 | 0.5717 | 0.3022 | 0.4147 | 0.3496 |
| | **ABOF-GAP** | **0.7555** | **0.5725** | **0.3103** | **0.4265** | **0.3593** |
| Iris | AP [7] | 0.7528 | 0.5638 | 0.2713 | 0.3610 | 0.3098 |
| | LOF-SAP [16] | 0.7589 | 0.5782 | 0.2745 | 0.3792 | 0.3294 |
| | CLAP [23] | 0.7578 | 0.5723 | 0.2853 | 0.3764 | 0.3268 |
| | SSAPEC [24] | 0.7612 | 0.5843 | 0.2963 | 0.3901 | 0.3367 |
| | DDAP [25] | 0.7583 | 0.5798 | 0.2784 | 0.3854 | 0.3298 |
| | GAP | 0.7641 | 0.5989 | 0.3127 | 0.4359 | 0.3641 |
| | **ABOF-GAP** | **0.7698** | **0.6295** | **0.3235** | **0.4629** | **0.3808** |
| Glass | AP [7] | 0.7521 | 0.4072 | 0.1571 | 0.1646 | 0.1607 |
| | LOF-SAP [16] | 0.7523 | 0.4089 | 0.1571 | 0.1650 | 0.1626 |
| | CLAP [23] | 0.7522 | 0.4087 | 0.1579 | 0.1661 | 0.1630 |
| | SSAPEC [24] | 0.7527 | 0.4103 | 0.1610 | 0.1690 | 0.1639 |
| | DDAP [25] | 0.7530 | 0.4091 | 0.1598 | 0.1654 | 0.1629 |
| | GAP | 0.7534 | **0.4158** | 0.1567 | 0.1651 | 0.1608 |
| | **ABOF-GAP** | **0.7567** | 0.4126 | **0.1620** | **0.1718** | **0.1668** |
| Segment | AP [7] | 0.7909 | 0.4531 | 0.4234 | 0.3423 | 0.3786 |
| | LOF-SAP [16] | 0.8523 | 0.5523 | 0.4398 | 0.3490 | 0.3892 |
| | CLAP [23] | 0.8432 | 0.5743 | 0.4313 | 0.3487 | 0.3856 |
| | SSAPEC [24] | 0.8619 | 0.5762 | 0.4367 | 0.3502 | 0.3887 |
| | DDAP [25] | 0.8532 | 0.5773 | 0.4298 | 0.3498 | 0.3857 |
| | GAP | 0.8623 | 0.5872 | 0.4412 | 0.3505 | 0.3907 |
| | **ABOF-GAP** | **0.8712** | **0.5902** | **0.4443** | **0.3512** | **0.3923** |
| Pendigits | AP [7] | 0.8022 | 0.6733 | 0.3870 | 0.4589 | 0.4199 |
| | LOF-SAP [16] | 0.8032 | 0.6873 | 0.3994 | 0.4679 | 0.4309 |
| | CLAP [23] | 0.8040 | 0.6856 | 0.3954 | 0.4634 | 0.4267 |
| | SSAPEC [24] | 0.8052 | 0.6878 | 0.4034 | 0.4680 | 0.4333 |
| | DDAP [25] | 0.8024 | 0.6867 | 0.3921 | 0.4656 | 0.4257 |
| | GAP | 0.8043 | 0.6879 | 0.4038 | 0.4678 | 0.4335 |
| | **ABOF-GAP** | **0.8102** | **0.6890** | **0.4189** | **0.4786** | **0.4468** |
| Semeion | AP [7] | 0.7021 | 0.3819 | 0.2198 | 0.3843 | 0.2797 |
| | LOF-SAP [16] | 0.7235 | 0.4280 | 0.2219 | 0.3927 | 0.2836 |
| | CLAP [23] | 0.7124 | 0.4156 | 0.2226 | 0.3910 | 0.2837 |
| | SSAPEC [24] | 0.7467 | 0.4298 | 0.2210 | 0.3925 | 0.2828 |
| | DDAP [25] | 0.7398 | 0.4198 | 0.2204 | 0.3819 | 0.2795 |
| | GAP | 0.7465 | 0.4296 | 0.2209 | 0.3920 | 0.2826 |
| | **ABOF-GAP** | **0.7562** | **0.4376** | **0.2234** | **0.3935** | **0.2850** |

To further show the change of clustering effect of ABOF-GAP algorithm under different dimensions, another experiment is performed on datasets generated from three different Gaussian distributions. Each dataset consists of 600 normal data points and 10 outliers. The dimension of these datasets varies from 10 to 100. The number of outliers

**TABLE 5.** The clustering results on the BBC dataset.

| Datasets | Algorithm | RI | NMI | Precision | Recall | F-score |
|---|---|---|---|---|---|---|
| BBC10d | AP [7] | 0.7728 | 0.5146 | 0.0817 | 0.9031 | 0.1498 |
| | LOF-SAP [16] | 0.7724 | 0.5142 | 0.0815 | 0.9027 | 0.1494 |
| | CLAP [23] | 0.7731 | 0.5153 | 0.0823 | 0.9021 | 0.1616 |
| | SSAPEC [24] | 0.7767 | 0.5264 | 0.0956 | **0.9036** | 0.1792 |
| | DDAP [25] | 0.7751 | 0.5257 | 0.0876 | 0.9025 | 0.1724 |
| | GAP | 0.7787 | 0.5366 | 0.1095 | 0.8994 | 0.1952 |
| | **ABOF-GAP** | **0.7792** | **0.5369** | **0.1098** | 0.8992 | **0.1958** |
| BBC20d | AP [7] | 0.7704 | 0.4998 | 0.0701 | 0.9129 | 0.1301 |
| | LOF-SAP [16] | 0.7702 | 0.5001 | 0.0705 | 0.9133 | 0.1309 |
| | CLAP [23] | 0.7707 | 0.5010 | 0.0812 | 0.9128 | 0.1308 |
| | SSAPEC [24] | 0.7785 | 0.5491 | 0.1045 | **0.9139** | 0.2073 |
| | DDAP [25] | 0.7769 | 0.5278 | 0.1008 | 0.9087 | 0.2052 |
| | GAP | 0.7805 | 0.5482 | 0.1169 | 0.9022 | 0.2070 |
| | **ABOF-GAP** | **0.7825** | **0.5543** | **0.1199** | 0.9131 | **0.2119** |
| BBC25d | AP [7] | 0.7698 | 0.4982 | 0.0708 | 0.8781 | 0.1310 |
| | LOF-SAP [16] | 0.7692 | 0.4983 | 0.0688 | 0.8835 | 0.1277 |
| | CLAP [23] | 0.7723 | 0.5113 | 0.0892 | 0.8910 | 0.1421 |
| | SSAPEC [24] | 0.7812 | 0.5342 | 0.1072 | 0.9097 | 0.1873 |
| | DDAP [25] | 0.7793 | 0.5279 | 0.1013 | 0.8978 | 0.1762 |
| | GAP | 0.7803 | 0.5519 | 0.1138 | **0.9182** | 0.2024 |
| | **ABOF-GAP** | **0.7852** | **0.5657** | **0.1371** | 0.9079 | **0.2382** |
| BBC50d | AP [7] | 0.7653 | 0.4757 | 0.0473 | 0.9034 | 0.0898 |
| | LOF-SAP [16] | 0.7652 | 0.4783 | 0.0480 | 0.9065 | 0.0911 |
| | CLAP [23] | 0.7689 | 0.4827 | 0.0579 | 0.9024 | 0.0956 |
| | SSAPEC [24] | 0.7793 | 0.5176 | 0.0873 | **0.9102** | 0.1980 |
| | DDAP [25] | 0.7726 | 0.5092 | 0.0690 | 0.9049 | 0.1846 |
| | GAP | 0.7812 | 0.5259 | 0.1398 | 0.8104 | 0.2384 |
| | **ABOF-GAP** | **0.7832** | **0.5308** | **0.1463** | 0.8276 | **0.2487** |

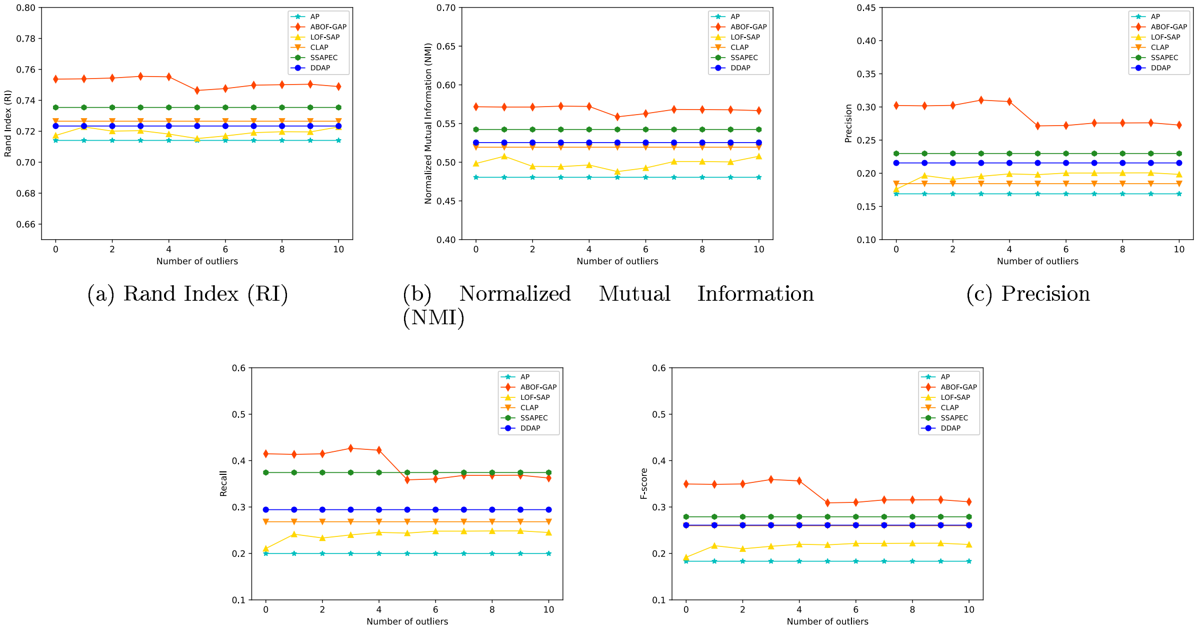
to be removed in ABOF-GAP algorithm is set to 3. The complete experimental results are shown in Table 2. ABOF-GAP achieves better clustering performance on all datasets. Figure 6 compares ABOF-GAP and AP, LOF-SAP, CLAP, SSAPEC, DDAP by RI. As the increase of dimension, the difference between other algorithms and ABOF-GAP's RI becomes more obvious. After the dimension exceeds 80, the RI of AP, CLAP and LOF-SAP clustering especially has a downward trend, which can be concluded that ABOF-GAP can better overcome the negative impact of dimension increment on clustering.

### C. REAL-WORLD DATA EXPERIMENTS

The experimental data are from the UCI reference database and text dataset. UCI database is set up by the University of California Irvine as a database for machine learning, which derived from real life [26]. In the UCI database, datasets Wine, Iris, Glass, Segmnt, PenDigits and Semeion are tested in our experiments. The data characteristics are shown in Table 3.

We use the proposed algorithm ABOF-GAP to cluster the datasets Wine, Iris, Glass, Segmnt, PenDigits and Semeion. In order to reflect the effect of the algorithm, we also test AP, LOF-SAP, CLAP, SSAPEC, DDAP and GAP for comparison. The results are shown in Table 4. As shown in Table 4, RI, Precision, Recall and F-score of ABOF-GAP on the six UCI data sets are all higher than that of AP and other algorithms. The five indexes of ABOF-GAP is also higher than that of GAP on Wine and Iris. However, the improvement effect of ABOF on clustering by GAP method on Glass is not very significant compared to performing GAP method directly. Furthermore, NMI of ABOF-GAP is slightly lower than that of GAP on Glass. This may because there are no outliers in the Glass dataset. And removing the two points with the smallest ABOF slightly leads to worse clustering results.

In addition, based on the above data experiments, we compared the clustering results of AP, LOF-SAP, CLAP,



(a) Rand Index (RI)

(b) Normalized Mutual Information (NMI)

(c) Precision

(d) Recall

(e) F-score

**FIGURE 7.** **The clustering results on wine.**

SSAPEC, DDAP and ABOF-GAP algorithm on the Wine dataset. The comparison results of the six algorithms are shown in Figure 7. The RI value of ABOF-GAP is significantly higher than that of the other algorithms. And the five indexes of ABOF-GAP reach the highest values when 3 outliers are removed.

Next we tested the effect of ABOF-GAP algorithm on text datasets. We chose BBC dataset in this work, which consists of 737 text documents, 4613 terms and 85576 words from the BBC website [27]. The data set can be divided into 5 categories: athletics, cricket, football, rugby and tennis. We use TF-IDF (Term Frequency-Inverse Document Frequency) to construct the feature vector of text [28]. In order to evaluate the influence of dimension on the clustering effect of the algorithm, the feature vector matrix of BBC text is processed into different dimensions through PCA (Principal Components Analysis) algorithm. We performed AP, LOF-SAP, CLAP, SSAPEC, DDAP, GAP and ABOF-GAP algorithm to cluster BBC dataset. As shown in Table 5, the 4 indexes of ABOF-GAP algorithm are higher than other algorithms except Recall. Figure 8 shows the RI achieved by AP, LOF-SAP, CLAP, SSAPEC, DDAP, GAP and ABOF-GAP algorithm, plotted against the dimension of text feature vector. We can find that with the increase of dimension, the clustering effect of the ABOF-GAP algorithm remains relatively stable while most of the other methods show a downtrend. And the gap of clustering effect between other algorithms and ABOF-GAP algorithm increases gradually.

The real data experimental results demonstrate that the proposed ABOF-GAP algorithm is an effective algorithm. The overall performance of the proposed ABOF-GAP algorithm is better than that of other existing AP-related methods. The results also indicate that the NMI of ABOF-GAP should be improved for such a data set like Glass, which will be studied in our future work.

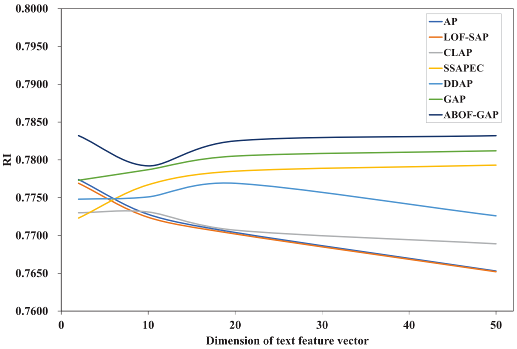


**FIGURE 8.** **RI of clustering result on BBC dataset.**

## V. CONCLUSION

Considering the problems caused by outliers, and nonlinearity of data in most AP-based algorithms, an improved AP algorithm based on angle-based outlier factor and geodesic distance (ABOF-GAP) is proposed in this work. The distinguishing features of the proposed ABOF-GAP algorithm are that outliers is firstly identified and eliminated based on ABOF method. Then Euclidean distance is replaced by geodesic distance to more realistically measure the similarity.

Experiments on six datasets in UCI and BBC dataset are carried out. Five evaluation indexes (RI, NMI, Precision, Recall and F-score) are adopted to evaluate the performance of clustering. The experimental results show that the proposed ABOF-GAP algorithm is an effective algorithm and its overall performance is better than that of other existing AP-related methods.

This work attempts to develop a new AP-based algorithm to conveniently and efficiently solve some problems in data processing, when the clustering effect is not ideal or the clustering performance needs to be improved. Since ABOF-GAP is an algorithm involving outlier problem, it is more suitable for clustering data with outliers. In addition, computational complexity and memory cost become a great bottleneck of AP when handling extremely large-scale datasets. In our future work, we will embark on solving the problem of AP clustering for extremely large-scalable and structurally complex datasets.

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