IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 17 March 2023, accepted 24 April 2023, date of publication 27 April 2023, date of current version 17 May 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3271141

RESEARCH ARTICLE

r, s, t-Spherical Fuzzy VIKOR Method and Its Application in Multiple Criteria Group Decision Making

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This work was supported by the Deputyship for Research and Innovation, Ministry of Education, Saudi Arabia, under Project IFP22UQU4310396DSR35.

ABSTRACT This study intends to significantly enhance the capacity of decision experts (DEs) to capture their judgment in a larger area. In order to accomplish this, we propound the r, s, t-spherical fuzzy set (r, s, t-SFS), an expansion of the t-spherical fuzzy set. In r, s, t-SFS the sum of the rth power of membership grade, sth power of neutral grade and the tth power of non-membership grade is less than or equal to 1, where r, s and t are natural numbers. Due to the inclusion of the extra parameters r and s, the r, s, t-SFS is able to describe assessment information in a more flexible and comprehensive manner. This work begins by defining r, s, t-SFS and demonstrating that it is an extension of various existing fuzzy sets. The fundamental operations, score, and accuracy functions of r, s, t-SFS are then introduced, and their mathematical features are examined. Also, we study some distance measures between r, s, t-SFSs and their required properties. Next, to aggregate r, s, t-spherical fuzzy data, r, s, t-spherical fuzzy weighted averaging (r, s, t-SFWA) and r, s, t-spherical fuzzy weighted geometric (r, s, t-SFWG) operators are bring forward along with some of their essential properties. Based on the proposed distance measure, maximizing deviation method is combined with r, s, t-spherical fuzzy information to establish the criteria weight determination method. Following this, we present r, s, t-spherical fuzzy VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method using the grounds of classical VIKOR method depending upon two focal properties, namely, group utility and individual regret of opponent. To demonstrate the use of the framed approach and exhibit its validity, we present a case study of arc welding robot selection. Besides, the effectiveness and accuracy of the proposed VIKOR are proved by parameter analysis and comparison analysis findings.

INDEX TERMS r, s, t-spherical fuzzy set. aggregation operators, maximizing deviation method, VIKOR, MCGDM.

I. INTRODUCTION

The multi-criteria group decision making (MCGDM) is intended to select the optimal option from restricted possibilities by integrating evaluation data for each option. As a valuable evaluation tool throughout the past few decades, it has been widely utilized in numerous applications, including site selection, medical diagnostics, granular computing approaches, pattern recognition, etc. How to signify evaluation information for different attributes is the initial step of MCDM. Decision experts (DEs) frequently use crisp numbers to evaluate alternatives. However, it is sometimes inappropriate to use crisp numbers since the decision environment is extremely complicated, and DEs fail to understand the opinion target completely. On the basis of these requirements, Zadeh [1] proposed the definition of fuzzy set (FS) in 1965, which utilized the membership grade to evaluate alternatives and has been the subject of several issues [2], [3], [4], [5]

The associate editor coordinating the review of this manuscript and approving it for publication was Aasia Khanum.

and in-depth investigations, particularly in MCGDM [6], [7]. Atanassov [8] subsequently added the non-membership grade to FS and produced the definition of intuitionistic FS (IFS). As a valuable tool for complex fuzzy messages, IFS has been extensively studied by a large number of scholars. However, IFSs are constrained by the fact that the total of membership grade and non-membership grade resides within the interval [0, 1], which is a very small range for representing fuzzy information. To this end, Yager [9] initially made this observation and produced Pythagorean FS (PyFS), which broadened the restrictive criteria of IFSs such that the sum of the squares of the membership grade and the non-membership grade fits inside [0, 1]. As a comprehensive type of IFS, PyFS has since piqued the attention of several researchers, who have carried out a number of noteworthy publications [10], [11], [12]. With the passage of time, however, it became apparent that PyFS cannot simulate DEs' views when we get information in the form of a pair, where the sum of the squares of membership and non-membership grades exceeds 1. To address such problems, Yager [13] upgraded PyFS and presented a q-rung orthopair fuzzy set (q-ROFS) as a new approach for addressing uncertainty. In q-ROFS, the sum of the qth power of the membership grade and the non-membership grade is less than or equal to one. Since it's introduction, numerous scholars have investigated the characteristics of q-ROFSs [14], [15], [16].

Structures such as IFS, PyFS, and q-ROFS fail when there are more than three possibilities, such as when electing between two parties: some individuals vote for party $\mathscr{X}$, some vote for party $\mathscr{Y}$, some sabotage their vote by stamping both, and some refuse to vote. To manage this type of information, Cuong and Kreinovich [17] introduced the picture fuzzy set (PFS), which has the requirement that the sum of membership grade, neutral grade, and non-membership grade must be between [0, 1]. Although PFS generalizes FS and IFS, it fails when the total of membership grade, neutral grade, and non-membership grade does not lie in [0, 1], and DE is hesitant to employ PFS to solve their problems as a result. To address this issue, Mahmood et al. [18] presented the concept of the spherical fuzzy set (SFS), in which they eased the requirement that the square sum of the aforesaid grades be in the range [0, 1]. DEs in SFS have more leeway in making decisions than in PFS. The most generalized form of all the discussed fuzzy structures is the t-spherical fuzzy set (t-SFS), which was presented by Mahmood et al. [18]. The DE in t-SFSs has to find a positive integer t such that the sum of the tth power of membership grade, neutral grade, and non-membership grade is from [0, 1]. t-SFS is an appropriate structure for handling uncertain environments. Ullah et al. [19] defined certain t-spherical fuzzy similarity measures and explained their applicability in pattern recognition. Garg et al. [20] studied improved interactive operational laws of t-SFSs and their corresponding aggregation operators along with their application. The authors in [21] put forward the axiomatic entropy measure and formulated some frank aggregation operators with their desired characteristics. Recently, Huang et al. [22] established a divergence-based maximizing deviation technique for determining expert and risk factor weights. Subsequently, they devised a t-spherical fuzzy combined compromise solution approach to stably rank failure modes.

However, in t-SFSs, the term level of membership grade σ, neutral grade ϑ, and non-membership grade ϱ is taken the same, i.e., $0 \leq \sigma^t+\vartheta^t+\varrho^t \leq 1$. But in practice, the term level of σ, ϑ, and ϱ may be different. For example, a DE define the σ as 0.7, ϑ as 0.6 and ϱ as 0.5. Clearly, $0.7^2+0.6^2+0.5^2 = 1.1 > 1$. Therefore, we should next check $0.7^3+0.6^3+0.5^3 = 0.684 < 1$. But since $0.7^3 + 0.6^2 + 0.5^2 = 0.953 < 1$. So the situation can be more successfully captured if the term level of σ, ϑ, and ϱ is allowed to be different. Thus, there is a need to initiate a novel fuzzy tool with the constrain $0 \leq \sigma^{\mathbf{r}} + \vartheta^{\mathbf{s}} + \varrho^{\mathbf{t}} \leq 1$ with $\mathbf{r}, \mathbf{s}, \mathbf{t} \geq 1$.

MCGDM is the most well-known branch of decision-support mechanisms, which includes a number of techniques. For instance, technique for order preference by similarity to ideal solutions (TOPSIS) method studied by Hwang and Yoon [23], preference ranking organization method for enrichment evaluations (PROMETHEE) proposed by Brans and Mareschel [24], vlsekriterijumska optimizacija i kompromisno resenje (VIKOR)method studied by opricovic [25], elimination et choice translating reality (ELECTRE) introduced by Benayoun, Roy, and Sussman [26], and tomada de decisao interativa e multicrit'erio (TODIM) dispatched by Gomes and Lima [27]. Highlighting the shortcomings of TOPSIS, ELECTRE, and PROMOTHEE approaches, Opricovic and Tzeng [28] expounded an extended VIKOR method. Among the different MCGDM approaches published by numerous writers in the literature, the VIKOR [28] has attained widespread acceptance. Considering the criteria, this strategy aims to find a compromise solution for ranking the options. A compromise solution is a workable solution that comes closest to the ideal solution. The conventional VIKOR approach has been expanded to its fuzzy counterparts by supplying FSs such as type-2 fuzzy VIKOR [29], hesitant fuzzy VIKOR [30], fermatean hesitant fuzzy VIKOR [31]. To the best of our information, t-spherical VIKOR approach has not yet been reported in the literature.

The prime reasons for conducting this research are as follows:

1). The existing t-SFS is based on the rule that the sum of the tth power of membership, neutral, and non-membership grades should be bounded by 1. To get reasonable results, DEs must pick the lowest integer t meeting the inequality $0 \leq \sigma^t + \vartheta^t + \varrho^t \leq 1$ since, in most problems, the higher value of t affects the results [32], [33]. In practice, however, we may face situations where the aforesaid inequality may satisfy for different powers of σ, ϑ and ϱ. For instance, if we take $\sigma = 0.6$, $\vartheta = 0.7$, and $\varrho = 0.5$, then in the light of t-SFS, $t = 3$ is the lowest possible integer that satisfies the condition $0 \leq \sigma^t + \vartheta^t + \varrho^t \leq 1$, but it also satisfies if we fix the power of σ, ϑ and ϱ to 2, 3 and 2, respectively. Since t-SFS only has a single parameter, t, it is not possible to specify various powers for membership, neutral, and non-membership grades. To overcome this issue, there is a need to add two more parameters to the existing t-SFS.

2). To use the full potential of the multi-parameter spherical fuzzy set, there is a need to extend the known literature on t-SFS and investigate a number of novel conceptions.

3). VIKOR is an important method for modeling decision ranking problems; however, in the literature, there is no research related to VIKOR regarding t-SFS. To fill this gap in the literature, it is necessary to explore the VIKOR to the environment of t-SFS or its extension.

This study mainly aims to devise a novel fuzzy tool from the viewpoint of capturing uncertainties in a better way. The following are some of this study's contributions:

1). The key goal is to introduce a more effective conception, namely r, s, t-spherical fuzzy set (r, s, t-SFS) by expanding the range of t-SFS parameters to aid DEs in developing their viewpoints in an authentic manner when dealing with decision-making challenges.

2). To lays a solid foundation for r, s, t-spherical fuzzy setting, some basic theory, including operational laws, the score and accuracy functions, and distance measures are studied.

3). We design some arithmetic and geometric aggregation operators for aggregating r, s, t-spherical fuzzy data and to verify several valuable properties associated with them.

4). Using the maximum deviation model and the Hamming distance measure on r, s, t-SFSs, we create a weighting approach for determining criteria weights.

5). An integrated assessment framework combining the VIKOR method and maximum deviation model is brought forward on the basis of the proposed r, s, t-SFS.

6). A thorough application example is presented to illustrate the application impact of the suggested approach. Compared to the previous MAGDM approaches, our method offers much larger constraints, more stability, a broader application range, and greater adaptability.

The subject contents of this paper are ordered as follows: Section II describes the primary purpose of presenting the article, the source of inspiration, and the work described by previous writers in the area. Section III initiated the notion of r, s, t-SFS, along with its basic theory, including operation rules, ranking rule, and distance measures. Section IV comprises some fundamental weighted aggregation operators along with their pertinent results. Section V summarizes the stepwise procedure of r, s, t-spherical fuzzy maximizing deviation and VIKOR to work out practical MCGDM problems with unknown weight information. Section VI discuss an application of the developed VIKOR methodology by means of an explanatory numerical example. In addition, it also conducts a detailed sensitivity analysis. In Section VII, we highlight the validity and potentiality of the frame approach by dint of comparison study. In the last section, some concluding remarks are drawn.

II. PRELIMINARIES

The notion of t-SFS is propounded by Mahmood et al. [18] as a synthesis of SFS to offer a broader range of preferences for DEs and enable them to express their hesitation about an alternative. Some basic definitions of t-SFS and terms relevant to planned work are delineated as follows.

Definition 1 ([18]): Let Z be a given nonempty set. A t-spherical fuzzy set (t-SFS) $\mathcal{S}$ on Z is given by

$$\mathcal{S} = \{\langle \mathrm{z}, \sigma(\mathrm{z}), \vartheta(\mathrm{z}), \varrho(\mathrm{z})\rangle \,|\mathrm{z} \in \mathrm{Z}\}, \tag{1}$$

where $\sigma(\mathrm{z}), \vartheta(\mathrm{z}), \varrho(\mathrm{z}) \in [0, 1]$ denote the membership, neutral and non-membership grades of $\mathrm{z} \in \mathrm{Z}$ to the set $\mathcal{S}$, respectively, with the restriction that $0 \leq \sigma^t(\mathrm{z}) + \vartheta^t(\mathrm{z}) + \varrho^t(\mathrm{z}) \leq 1$. The degree of refusal is $\pi(\mathrm{z}) = \sqrt[t]{1 - \sigma^t(\mathrm{z}) - \vartheta^t(\mathrm{z}) - \varrho^t(\mathrm{z})}$. For convince, $\langle \sigma(\mathrm{z}), \vartheta(\mathrm{z}), \varrho(\mathrm{z})\rangle$ is called a t-spherical fuzzy number (t-SFN), labeled by $\mathcal{S} = \langle \sigma, \vartheta, \varrho\rangle$.

Remark 1:
- The Definition 1 reduced to SFS if we set $t = 2$.
- The Definition 1 reduced to PFS if we set $t = 1$.
- The Definition 1 reduced to q-ROFS if we set $\vartheta = 0$.
- The Definition 1 reduced to PyFS if we set $t = 2$ and $\vartheta = 0$.
- The Definition 1 reduced to IFS if we set $t = 1$ and $\vartheta = 0$.

Definition 2 ([19]): Let $\mathcal{S}_1 = \langle \sigma_1, \vartheta_1, \varrho_1\rangle$ and $\mathcal{S}_2 = \langle \sigma_2, \vartheta_2, \varrho_2\rangle$ be two t-SFNs and $\eta > 0$, then

1) $\mathcal{S}_1 \oplus \mathcal{S}_2 = \left\langle \sqrt[t]{\sigma_1^t + \sigma_2^t - \sigma_1^t\sigma_2^t}, \vartheta_1\vartheta_2, \varrho_1\varrho_2 \right\rangle$;

2) $\mathcal{S}_1 \otimes \mathcal{S}_2 = \left\langle \begin{array}{c} \sigma_1\sigma_2, \sqrt[t]{\vartheta_1^t + \vartheta_2^t - \vartheta_1^t\vartheta_2^t}, \\ \sqrt[t]{\varrho_1^t + \varrho_2^t - \varrho_1^t\varrho_2^t} \end{array} \right\rangle$;

3) $\mathcal{S}_1^\eta = \left\langle \sigma_1^\eta, \sqrt[t]{1 - (1 - \vartheta_1^t)^\eta}, \sqrt[t]{1 - (1 - \varrho_1^t)^\eta} \right\rangle$;

4) $\eta\mathcal{S}_1 = \left\langle \sqrt[t]{1 - (1 - \sigma_1^t)^\eta}, \vartheta_1^\eta, \varrho_1^\eta \right\rangle$;

5) $\mathcal{S}_1^c = \langle \varrho_1, \vartheta_1, \sigma_1\rangle$.

Definition 3 ([18], [34]): $\mathcal{S}_1 = \langle \sigma_1, \vartheta_1, \varrho_1\rangle$ and $\mathcal{S}_2 = \langle \sigma_2, \vartheta_2, \varrho_2\rangle$ be any two t-SFNs, let $S(\mathcal{S}_1) = \sigma_1^t - \vartheta_1^t - \varrho_1^t + \left(\frac{\exp^{\sigma_1^t - \vartheta_1^t - \varrho_1^t}}{\exp^{\sigma_1^t - \vartheta_1^t - \varrho_1^t} + 1} - \frac{1}{2}\right)\pi^t$ and $S(\mathcal{S}_2) = \sigma_2^t - \vartheta_2^t - \varrho_2^t + \left(\frac{\exp^{\sigma_2^t - \vartheta_2^t - \varrho_2^t}}{\exp^{\sigma_2^t - \vartheta_2^t - \varrho_2^t} + 1} - \frac{1}{2}\right)\pi^t$ be the score values of $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively, and let $A(\mathcal{S}_1) = \sigma_1^t + \vartheta_1^t + \varrho_1^t$ and $A(\mathcal{S}_2) = \sigma_2^t + \vartheta_2^t + \varrho_2^t$ be the accuracy values of $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively. Then,

1) If $S(\mathcal{S}_1) < S(\mathcal{S}_1)$, then $\mathcal{S}_1 < \mathcal{S}_2$;

2) If $S(\mathcal{S}_1) = S(\mathcal{S}_1)$, then

  a. If $A(\mathcal{S}_1) < A(\mathcal{S}_1)$, then $\mathcal{S}_1 < \mathcal{S}_2$;

  b. If $S(\mathcal{S}_1) = S(\mathcal{S}_1)$, then $\mathcal{S}_1 = \mathcal{S}_2$.

As an important tool in information fusion, t-spherical fuzzy aggregation operator has received much attention, Mahmood et al. [35] propound the t-spherical fuzzy weighted averaging (t-SFWA) operator and the T-spherical fuzzy weighted geometric (t-SFWG) operator as follows:

Definition 4 ([35]): Let $\mathcal{S}_i\,(i = 1, 2, \ldots, n)$ be a collection of t-SFNs, then the t-spherical fuzzy weighted averaging (t-SFWA) operator is a mapping $\mathcal{S}^n \longrightarrow \mathcal{S}$ such that, as in (2), shown at the bottom of the next page, where $w = \{w_1, w_2, \ldots, w_n\}^T$ is the weight vector of $(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_n)$ such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$.

Definition 5 ([35]): Let $\mathcal{S}_i\,(i = 1, 2, \ldots, n)$ be a collection of t-SFNs, then the t-spherical fuzzy weighted geometric (t-SFWG) operator is a mapping $\mathcal{S}^n \longrightarrow \mathcal{S}$ such that, as in (3), shown at the bottom of the next page, where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_n)$ such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$.

III. r, s, t-SPHERICAL FUZZY SET

This section is devoted to propound a novel fuzzy tool and its fundamental concepts.

A. NOTION OF r, s, t-SFSs

In what follows, we constitute the concept of $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFS and investigate its idiosyncrasies to a larger extent.

Definition 6: Let Z be a given nonempty set. A $\mathsf{r}, \mathsf{s}, \mathsf{t}$-spherical fuzzy set ($\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFS) $\mathscr{G}$ on Z is given by

$$\mathscr{G} = \{\langle \mathrm{z}, \sigma(\mathrm{z}), \vartheta(\mathrm{z}), \varrho(\mathrm{z})\rangle \,|\mathrm{z} \in \mathrm{Z}\}, \tag{4}$$

where $\sigma : \mathrm{Z} \longrightarrow [0, 1]$, $\vartheta : \mathrm{Z} \longrightarrow [0, 1]$, and $\varrho : \mathrm{Z} \longrightarrow [0, 1]$, characterizes membership grades, neutral grades and non-membership grades, respectively, such that for some natural numbers r, s, and t, $0 \leq \sigma^{\mathsf{r}}(\mathrm{z}) + \vartheta^{\mathsf{s}}(\mathrm{z}) + \varrho^{\mathsf{t}}(\mathrm{z}) \leq 1 \;\forall\; \mathrm{z} \in \mathrm{Z}$. The degree of indeterminacy is $\pi(\mathrm{z}) = \sqrt[\ell]{1 - \sigma^{\mathsf{r}}(\mathrm{z}) - \vartheta^{\mathsf{s}}(\mathrm{z}) - \varrho^{\mathsf{t}}(\mathrm{z})}$, where ℓ is the least common multiple (LCM) of r, s and t.

For convince, $\langle \sigma(\mathrm{z}), \vartheta(\mathrm{z}), \varrho(\mathrm{z})\rangle$ is called a generalized spherical fuzzy number (GSFN), labeled by $g = \langle \sigma, \vartheta, \varrho\rangle$. g

In the following lines, we explain the selection procedure of r, s and t in the suggested $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFS.

Remark 2: Consider the case where we have to find the minimum value of $\mathsf{r}, \mathsf{s}, \mathsf{t} \geq 1$ for a given triplet $\langle \sigma, \vartheta, \varrho\rangle$ so that $\sigma^{\mathsf{r}} + \vartheta^{\mathsf{s}} + \varrho^{\mathsf{t}} \leq 1$. It is always possible to find a unique solution to these problems using some iterative computing techniques even though they have no closed-form solution. We shall refer to the minimum values of r,s, and t satisfying $\sigma^{\mathsf{r}} + \vartheta^{\mathsf{s}} + \varrho^{\mathsf{t}} \leq 1$ as the $\mathsf{r}, \mathsf{s}, \mathsf{t}$−niche of $\langle \sigma, \vartheta, \varrho\rangle$. We note that if $\mathsf{r}_1, \mathsf{s}_1, \mathsf{t}_1$ is the $\mathsf{r}, \mathsf{s}, \mathsf{t}$−niche of $\langle \sigma, \vartheta, \varrho\rangle$, then $\langle \sigma, \vartheta, \varrho\rangle$ is valid for all $\mathsf{r} \geq \mathsf{r}_1$, $\mathsf{s} \geq \mathsf{s}_1$, and $\mathsf{t} \geq \mathsf{t}_1$.

Remark 3: Consider the case where $\sigma^{\mathsf{r}} + \vartheta^{\mathsf{s}} + \varrho^{\mathsf{t}} > 1$, and we have to find $\mathsf{r}, \mathsf{s}, \mathsf{t}$−niche. Suppose r^*, s^* and t^* are the minimum values for which $\sigma^{\mathsf{r}^*} + \vartheta^{\mathsf{s}} + \varrho^{\mathsf{t}} \leq 1$, $\sigma^{\mathsf{r}} + \vartheta^{\mathsf{s}^*} + \varrho^{\mathsf{t}} \leq 1$, and $\sigma^{\mathsf{r}} + \vartheta^{\mathsf{s}} + \varrho^{\mathsf{t}^*} \leq 1$. In such a situation, we shall refer to that minimal value that meets the condition with a larger grade.

Suppose $\mathrm{Z} = \{\mathrm{z}_1, \mathrm{z}_2, \ldots, \mathrm{z}_n\}$ be given data set and F be some fuzzy concept. Suppose a DE express his preference as a triplet $\langle \sigma_i, \vartheta_i, \varrho_i\rangle$ for each $\mathrm{z}_i \in \mathrm{Z}$. Now the challenge is how to estimate the correct values of r, s and t to represent the information appropriately. Here we can proceed as follows:

i). For each triplet $\langle \sigma_i, \vartheta_i, \varrho_i\rangle$ determine its $\mathsf{r}, \mathsf{s}, \mathsf{t}$−niche, say r_i, s_i,t_i.

ii). Ascertain the $\mathsf{r}^*, \mathsf{s}^*, \mathsf{t}^*$−niche such that $\mathsf{r}^* = \max_i \{\mathsf{r}_i\}$, $\mathsf{s}^* = \max_i \{\mathsf{s}_i\}$, and $\mathsf{t}^* = \max_i \{\mathsf{t}_i\}$.

iii) Represent $\mathscr{G}$ as a $\mathsf{r}^*, \mathsf{s}^*, \mathsf{t}^*$−SFS.

Deduction:

- The Definition 6 reduced to PFS [17] if we set $r = s = t = 1$.
- The Definition 6 reduced to SFS [18] if we set $r = s = t = 2$.
- The Definition 6 reduced to t-SFS [18] if we set $r = s = t$.
- The Definition 6 reduced to p,q-QOFS [36] if we set $\vartheta = 0$.
- The Definition 6 reduced to FFS [37] if we set $r = t = 3$ and $\vartheta = 0$.
- The Definition 6 reduced to PyFS [9] if we set $r = t = 2$ and $\vartheta = 0$.
- The Definition 6 reduced to IFS [8] if we set $r = t = 1$ and $\vartheta = 0$.

Thereby, the devised $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFS is the generalization of IFS [8], PyFS [9], FFS [37], p, q-QOFS [36], PFS [17], SFS [18], and t-SFS [18].

Now we observe that, for any natural numbers 'n_1' and 'n_2' with $n_1 > n_2$, $a^{n_1} < a^{n_2}$, $\forall\; a \in [0, 1]$. We can derive a fundamental relationship betwixt two $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFSs from this observation.

Theorem 1: Let $\mathscr{G}$ be the $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFS and $\mathsf{r}_1, \mathsf{s}_1, \mathsf{t}_1$ be the $\mathsf{r}^*, \mathsf{s}^*, \mathsf{t}^*$−niche of $\mathscr{G}$ then for $\mathsf{r}_2 \geq \mathsf{r}_1$, $\mathsf{s}_2 \geq \mathsf{s}_1$, $\mathsf{t}_2 \geq \mathsf{t}_1$, $\mathscr{G}$ is also a $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFS with $\mathsf{r}_2, \mathsf{s}_2, \mathsf{t}_2$−niche.

Proof: Since $\sigma^{\mathsf{r}_1}(\mathrm{z}) + \vartheta^{\mathsf{s}_1}(\mathrm{z}) + \varrho^{\mathsf{t}_1}(\mathrm{z}) \leq 1$, $\forall$ z. Now $\mathsf{r}_2 \geq \mathsf{r}_1$ implies that $\sigma^{\mathsf{r}_2}(\mathrm{z}) \leq \sigma^{\mathsf{r}_1}(\mathrm{z})$, $\mathsf{s}_2 \geq \mathsf{s}_1$ implies that $\vartheta^{\mathsf{r}_2}(\mathrm{z}) \leq \vartheta^{\mathsf{r}_1}(\mathrm{z})$ and $\mathsf{t}_2 \geq \mathsf{t}_1$ implies that $\sigma^{\mathsf{t}_2}(\mathrm{z}) \leq \sigma^{\mathsf{t}_1}(\mathrm{z})$ as $\sigma(\mathrm{z}), \vartheta(\mathrm{z}), \varrho(\mathrm{z}) \in [0, 1]$. So we get $\sigma^{\mathsf{r}_2}(\mathrm{z}) + \vartheta^{\mathsf{s}_2}(\mathrm{z}) + \varrho^{\mathsf{t}_2}(\mathrm{z}) \leq \sigma^{\mathsf{r}_1}(\mathrm{z}) + \vartheta^{\mathsf{s}_1}(\mathrm{z}) + \varrho^{\mathsf{t}_1}(\mathrm{z}) \leq 1$. ∎

Now, we emphasize the following fact as remark.

Remark 4: If $\mathsf{r}_2 \geq \mathsf{r}_1$, $\mathsf{s}_2 \geq \mathsf{s}_1$, $\mathsf{t}_2 \geq \mathsf{t}_1$ and $\mathscr{G}$ is a $\mathsf{r}_2, \mathsf{s}_2, \mathsf{t}_2$−SFS then $\mathscr{G}$ is not necessarily $\mathsf{r}_1, \mathsf{s}_1, \mathsf{t}_1$−SFS

In the following, we present the relationships betwixt the spaces of $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFS based on the values of r, s and t.

Theorem 2: If $\mathsf{r}_1 > \mathsf{s}_1 > \mathsf{t}_1$, then we have the following twelve cases:

i). $\mathsf{r}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS < \mathsf{s}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS < \mathsf{t}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS$;
ii). $\mathsf{r}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS < \mathsf{s}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS < \mathsf{t}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS$;
iii). $\mathsf{r}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS < \mathsf{s}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS < \mathsf{t}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS$;
iv). $\mathsf{r}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS < \mathsf{s}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS < \mathsf{t}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS$;
v). $\mathsf{s}_1, \mathsf{r}_1, \mathsf{t}_1 - SFS < \mathsf{s}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS < \mathsf{s}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS$;
vi). $\mathsf{t}_1, \mathsf{r}_1, \mathsf{s}_1 - SFS < \mathsf{t}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS < \mathsf{t}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS$;
vii). $\mathsf{s}_1, \mathsf{r}_1, \mathsf{s}_1 - SFS < \mathsf{s}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS < \mathsf{s}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS$;
viii). $\mathsf{t}_1, \mathsf{r}_1, \mathsf{t}_1 - SFS < \mathsf{t}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS < \mathsf{t}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS$;
ix). $\mathsf{s}_1, \mathsf{t}_1, \mathsf{r}_1 - SFS < \mathsf{s}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS < \mathsf{s}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS$;
x). $\mathsf{t}_1, \mathsf{s}_1, \mathsf{r}_1 - SFS < \mathsf{t}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS < \mathsf{t}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS$;

xi). $\mathsf{s}_1, \mathsf{s}_1, \mathsf{r}_1 - SFS < \mathsf{s}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS < \mathsf{s}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS$;
xii). $\mathsf{t}_1, \mathsf{t}_1, \mathsf{r}_1 - SFS < \mathsf{t}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS < \mathsf{t}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS$.

Proof: If $\mathsf{r}_1 > \mathsf{s}_1 > \mathsf{t}_1$, then we have $\sigma^{\mathsf{r}_1}(z) < \sigma^{\mathsf{s}_1}(z) < \sigma^{\mathsf{t}_1}(z)$, $\vartheta^{\mathsf{r}_1}(z) < \vartheta^{\mathsf{s}_1}(z) < \vartheta^{\mathsf{t}_1}(z)$, and $\varrho^{\mathsf{r}_1}(z) < \varrho^{\mathsf{s}_1}(z) < \varrho^{\mathsf{t}_1}(z)$.

Now

$$\begin{aligned}
&\sigma^{\mathsf{r}_1}(z) < \sigma^{\mathsf{s}_1}(z) < \sigma^{\mathsf{t}_1}(z),\\
&\Rightarrow \sigma^{\mathsf{r}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{t}_1}(z) < \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{t}_1}(z)\\
&< \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{t}_1}(z)\\
&\sigma^{\mathsf{r}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{s}_1}(z) < \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&< \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&\sigma^{\mathsf{r}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{s}_1}(z) < \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&< \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{s}_1}(z)
\end{aligned}$$

and

$$\begin{aligned}
&\sigma^{\mathsf{r}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{t}_1}(z) < \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{t}_1}(z)\\
&\quad < \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{t}_1}(z).
\end{aligned}$$

Next

$$\begin{aligned}
&\vartheta^{\mathsf{r}_1}(z) < \vartheta^{\mathsf{s}_1}(z) < \vartheta^{\mathsf{t}_1}(z),\\
&\Rightarrow \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{r}_1}(z) + \varrho^{\mathsf{t}_1}(z) < \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{t}_1}(z)\\
&< \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{t}_1}(z)\\
&\sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{r}_1}(z) + \varrho^{\mathsf{s}_1}(z) < \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&< \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&\sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{r}_1}(z) + \varrho^{\mathsf{s}_1}(z) < \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&< \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{s}_1}(z)
\end{aligned}$$

and

$$\begin{aligned}
&\sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{r}_1}(z) + \varrho^{\mathsf{t}_1}(z) < \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{t}_1}(z)\\
&\quad < \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{s}_1}(z).
\end{aligned}$$

And

$$\begin{aligned}
&\varrho^{\mathsf{r}_1}(z) < \varrho^{\mathsf{s}_1}(z) < \varrho^{\mathsf{t}_1}(z),\\
&\Rightarrow \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{r}_1}(z) < \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&< \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{t}_1}(z)\\
&\sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{r}_1}(z) < \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&< \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{t}_1}(z)\\
&\sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{r}_1}(z) < \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&< \sigma^{\mathsf{s}_1}(z) + \vartheta^{\mathsf{s}_1}(z) + \varrho^{\mathsf{t}_1}(z)
\end{aligned}$$

and

$$\begin{aligned}
&\sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{r}_1}(z) < \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{s}_1}(z)\\
&\quad < \sigma^{\mathsf{t}_1}(z) + \vartheta^{\mathsf{t}_1}(z) + \varrho^{\mathsf{t}_1}(z).
\end{aligned}$$

■

Theorem 3: If $\mathsf{r}_1 < \mathsf{s}_1 < \mathsf{t}_1$, then we have the following twelve cases:
i). $\mathsf{r}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS > \mathsf{s}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS > \mathsf{t}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS$;
ii). $\mathsf{r}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS > \mathsf{s}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS > \mathsf{t}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS$;
iii). $\mathsf{r}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS > \mathsf{s}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS > \mathsf{t}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS$;
iv). $\mathsf{r}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS > \mathsf{s}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS > \mathsf{t}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS$;
v). $\mathsf{s}_1, \mathsf{r}_1, \mathsf{t}_1 - SFS > \mathsf{s}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS > \mathsf{s}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS$;
vi). $\mathsf{t}_1, \mathsf{r}_1, \mathsf{s}_1 - SFS > \mathsf{t}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS > \mathsf{t}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS$;
vii). $\mathsf{s}_1, \mathsf{r}_1, \mathsf{s}_1 - SFS > \mathsf{s}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS > \mathsf{s}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS$;
viii). $\mathsf{t}_1, \mathsf{r}_1, \mathsf{t}_1 - SFS > \mathsf{t}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS > \mathsf{t}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS$;
ix). $\mathsf{s}_1, \mathsf{t}_1, \mathsf{r}_1 - SFS > \mathsf{s}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS > \mathsf{s}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS$;
x). $\mathsf{t}_1, \mathsf{s}_1, \mathsf{r}_1 - SFS > \mathsf{t}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS > \mathsf{t}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS$;
xi). $\mathsf{s}_1, \mathsf{s}_1, \mathsf{r}_1 - SFS > \mathsf{s}_1, \mathsf{s}_1, \mathsf{s}_1 - SFS > \mathsf{s}_1, \mathsf{s}_1, \mathsf{t}_1 - SFS$;
xii). $\mathsf{t}_1, \mathsf{t}_1, \mathsf{r}_1 - SFS > \mathsf{t}_1, \mathsf{t}_1, \mathsf{s}_1 - SFS > \mathsf{t}_1, \mathsf{t}_1, \mathsf{t}_1 - SFS$.

Proof: Based on the lines of Theorem 2, one can easily get its proof. ■

Based on Theorems 2 and 3, we may deduce that the proposed r, s, t-SFS can capture uncertainty more precisely than the current t-SFS owing to the extra parameters r and s.

B. BASIC OPERATIONS OF *r, s, t*-SFNs

We suggest the following operations for r, s, t-SFNs, based on the concept of Definition 2.

Definition 7: Let $g_1 = \langle\sigma_1, \vartheta_1, \varrho_1\rangle$ and $g_2 = \langle\sigma_2, \vartheta_2, \varrho_2\rangle$ be two r, s, t-SFNs and $\eta > 0$, then

$$1)\ \ g_1 \oplus g_2 = \left\langle \begin{array}{c} \sqrt[\mathsf{r}^*]{\sigma_1^{\mathsf{r}^*} + \sigma_2^{\mathsf{r}^*} - \sigma_1^{\mathsf{r}^*}\sigma_2^{\mathsf{r}^*}}, \vartheta_1\vartheta_2, \\ \varrho_1\varrho_2 \end{array} \right\rangle;$$

$$2)\ \ g_1 \otimes g_2 = \left\langle \begin{array}{c} \sigma_1\sigma_2, \sqrt[\mathsf{s}^*]{\vartheta_1^{\mathsf{s}^*} + \vartheta_2^{\mathsf{s}^*} - \vartheta_1^{\mathsf{s}^*}\vartheta_2^{\mathsf{s}^*}} \\ , \sqrt[\mathsf{t}^*]{\varrho_1^{\mathsf{t}^*} + \varrho_2^{\mathsf{t}^*} - \varrho_1^{\mathsf{t}^*}\varrho_2^{\mathsf{t}^*}} \end{array} \right\rangle;$$

$$\begin{aligned}
T - SFWA\left(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_n\right) &= w_1\mathcal{S}_1 \oplus w_2\mathcal{S}_2 \oplus \ldots \oplus w_n\mathcal{S}_n \\
&= \left\langle \left(1 - \prod_{k=1}^{n}\left(1 - \sigma_k^t\right)^{w_k}\right)^{1/t}, \prod_{k=1}^{n}\left(\vartheta_k\right)^{w_k}, \prod_{k=1}^{n}\left(\varrho_k\right)^{w_k} \right\rangle,
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
T - SFWG\left(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_n\right) &= w_1\mathcal{S}_1 \otimes w_2\mathcal{S}_2 \otimes \ldots \otimes w_n\mathcal{S}_n \\
&= \left\langle \prod_{k=1}^{n}\left(\sigma_k\right)^{w_k}, \left(1 - \prod_{k=1}^{n}\left(1 - \vartheta_k^t\right)^{w_k}\right)^{1/t}, \left(1 - \prod_{k=1}^{n}\left(1 - \varrho_k^t\right)^{w_k}\right)^{1/t} \right\rangle,
\end{aligned} \tag{3}$$

3) $g_1^\eta = \left\langle \begin{array}{c} \sigma_1^\eta, \sqrt[s^*]{1-(1-\vartheta_1^{s^*})^\eta}, \\ \sqrt[t^*]{1-(1-\varrho_1^{t^*})^\eta} \end{array} \right\rangle$;

4) $\eta g_1 = \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^\eta}, \vartheta_1^\eta, \varrho_1^\eta \right\rangle$;

5) $g_1^c = \langle \varrho_1, \vartheta_1, \sigma_1 \rangle$;

6) $g_1 \preceq g_2$ if and only if $\varrho_1 \leq \varrho_2$, $\vartheta_1 \geq \vartheta_2$ and $\sigma_1 \geq \sigma_2$;

7) $g_1 = g_2$ if and only if $\varrho_1 = \varrho_2$, $\vartheta_1 = \vartheta_2$ and $\sigma_1 = \sigma_2$.

We investigate the following results using the operational laws defined in Definition 7.

Theorem 4: Let $g_i = \langle \sigma_i, \vartheta_i, \varrho_i \rangle$ $(i = 1, 2, 3)$ be any three r, s, t-SFNs, and $\eta, \eta_1, \eta_2 > 0$, then

1) $g_1 \oplus g_2 = g_2 \oplus g_1$;
2) $g_1 \otimes g_2 = g_2 \otimes g_1$;
3) $(g_1 \oplus g_2) \oplus g_3 = g_1 \oplus (g_2 \oplus g_3)$;
4) $(g_1 \otimes g_2) \otimes g_3 = g_1 \otimes (g_2 \otimes g_3)$;
5) $\eta (g_1 \oplus g_2) = \eta g_1 \oplus \eta g_2$;
6) $(g_1 \otimes g_2)^\eta = g_1^\eta \otimes g_2^\eta$;
7) $\eta_1 g_1 \oplus \eta_2 g_1 = (\eta_1 + \eta_2) g_1$;
8) $g_1^{\eta_1} \otimes g_1^{\eta_2} = g_1^{\eta_1+\eta_2}$;
9) $(\eta_1\eta_2) g_1 = \eta_1 (\eta_2 g_1)$.

Proof: We just verify components 1, 3, 5, 7 and 9, and accordingly for the remainder.

1). As

$$\begin{aligned} g_1 \oplus g_2 &= \left\langle \sqrt[r^*]{\sigma_1^{r^*} + \sigma_2^{r^*} - \sigma_1^{r^*}\sigma_2^{r^*}}, \vartheta_1\vartheta_2, \varrho_1\varrho_2 \right\rangle \\ &= \left\langle \sqrt[r^*]{\sigma_2^{r^*} + \sigma_1^{r^*} - \sigma_2^{r^*}\sigma_1^{r^*}}, \vartheta_2\vartheta_1, \varrho_2\varrho_1 \right\rangle. \end{aligned}$$

3).

$$\begin{aligned} &(g_1 \oplus g_2) \oplus g_3 \\ &= \left\langle \sqrt[r^*]{\sigma_1^{r^*} + \sigma_2^{r^*} - \sigma_1^{r^*}\sigma_2^{r^*}}, \vartheta_1\vartheta_2, \varrho_1\varrho_2 \right\rangle \\ &\quad \oplus \langle \sigma_3, \vartheta_3, \varrho_3 \rangle \\ &= \left\langle \begin{array}{c} \sqrt[r^*]{(\sigma_1^{r^*}+\sigma_2^{r^*}-\sigma_1^{r^*}\sigma_2^{r^*})+\sigma_3^{r^*}-(\sigma_1^{r^*}+\sigma_2^{r^*}-\sigma_1^{r^*}\sigma_2^{r^*})\,\sigma_3^{r^*}}, \\ (\vartheta_1\vartheta_2)\,\vartheta_3, (\varrho_1\varrho_2)\,\varrho_3 \end{array} \right\rangle. \end{aligned}$$

$$\begin{aligned} &g_1 \oplus (g_2 \oplus g_3) \\ &= \langle \sigma_1, \vartheta_1, \varrho_1 \rangle \oplus \left\langle \begin{array}{c} \sqrt[r^*]{\sigma_2^{r^*} + \sigma_3^{r^*} - \sigma_2^{r^*}\sigma_3^{r^*}}, \vartheta_2\vartheta_3, \\ \varrho_2\varrho_3 \end{array} \right\rangle \\ &= \left\langle \begin{array}{c} \sqrt[r^*]{\sigma_1^{r^*}+(\sigma_2^{r^*}+\sigma_3^{r^*}-\sigma_2^{r^*}\sigma_3^{r^*})-\sigma_1^{r^*}\,(\sigma_2^{r^*}+\sigma_3^{r^*}-\sigma_2^{r^*}\sigma_3^{r^*})} \\ , \vartheta_1\,(\vartheta_2\vartheta_3)\,, \varrho_1\,(\varrho_2\varrho_3) \end{array} \right\rangle \\ &= (g_1 \oplus g_2) \oplus g_3. \end{aligned}$$

5).

$$\begin{aligned} &\eta\,(g_1 \oplus g_2) \\ &\quad = \eta \left(\left\langle \sqrt[r^*]{\sigma_1^{r^*} + \sigma_2^{r^*} - \sigma_1^{r^*}\sigma_2^{r^*}}, \vartheta_1\vartheta_2, \varrho_1\varrho_2 \right\rangle \right) \\ &\quad = \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*}-\sigma_2^{r^*}+\sigma_1^{r^*}\sigma_2^{r^*})^\eta}, \vartheta_1^\eta\vartheta_2^\eta, \varrho_1^\eta\varrho_2^\eta \right\rangle \\ &\quad = \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^\eta\,(1-\sigma_2^{r^*})^\eta}, \vartheta_1^\eta\vartheta_2^\eta, \varrho_1^\eta\varrho_2^\eta \right\rangle. \end{aligned}$$

$$\begin{aligned} &\eta g_1 \oplus \eta g_2 \\ &\quad = \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^\eta}, \vartheta_1^\eta, \varrho_1^\eta \right\rangle \\ &\qquad \oplus \left\langle \sqrt[r^*]{1-(1-\sigma_2^{r^*})^\eta}, \vartheta_2^\eta, \varrho_2^\eta \right\rangle \\ &\quad = \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^\eta\,(1-\sigma_2^{r^*})^\eta}, \vartheta_1^\eta\vartheta_2^\eta, \varrho_1^\eta\varrho_2^\eta \right\rangle \\ &\quad = \eta\,(g_1 \oplus g_2)\,. \end{aligned}$$

7).

$$\begin{aligned} \eta_1 g_1 \oplus \eta_2 g_1 &= \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^{\eta_1}}, \vartheta_1^{\eta_1}, \varrho_1^{\eta_1} \right\rangle \\ &\quad \oplus \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^{\eta_2}}, \vartheta_1^{\eta_2}, \varrho_1^{\eta_2} \right\rangle \\ &= \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^{\eta_1+\eta_2}}, \vartheta_1^{\eta_1+\eta_2}, \varrho_1^{\eta_1+\eta_2} \right\rangle \\ &= (\eta_1+\eta_2)\,g_1. \end{aligned}$$

9).

$$(\eta_1\eta_2)\,g_1 = \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^{(\eta_1\eta_2)}}, \vartheta_1^{(\eta_1\eta_2)}, \varrho_1^{(\eta_1\eta_2)} \right\rangle.$$

$$\begin{aligned} \eta_1\,(\eta_2 g_1) &= \eta_1 \left(\left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^{\eta_2}}, \vartheta_1^{\eta_2}, \varrho_1^{\eta_2} \right\rangle \right) \\ &= \left\langle \sqrt[r^*]{1-(1-\sigma_1^{r^*})^{(\eta_2\eta_1)}}, \vartheta_1^{(\eta_2\eta_1)}, \varrho_1^{(\eta_2\eta_1)} \right\rangle \\ &= (\eta_1\eta_2)\,g_1. \end{aligned}$$

■

C. COMPARISON RULE

To compare r, s, t-SFNs, this section discusses the comparison rule based on the proposed score and accuracy function.

Definition 8: Let $g = \langle \sigma, \vartheta, \varrho \rangle$ be any r, s, t-SFNs, then its score function is denoted and defined as

$$S(g) = \frac{1}{2}\left(1 + \left(\sigma^r - \vartheta^s - \varrho^t\right)\right). \tag{5}$$

Now, we justify the suggested scoring function's subsequent characteristics.

Property 5: For any r, s, t-SFN $g = \langle \sigma, \vartheta, \varrho \rangle$, the score function lies betwixt 0 and 1.

Proof: We know for any r, s, t-SFN $g = \langle \sigma, \vartheta, \varrho \rangle$, $\sigma^r + \vartheta^s + \varrho^t \leq 1$. Now $\sigma^r - \vartheta^s - \varrho^t \leq \sigma^r \leq 1$, and $\vartheta^s + \varrho^t - \sigma^r \leq \vartheta^s + \varrho^t \leq 1 \Rightarrow \sigma^r - \vartheta^s - \varrho^t \geq -1$. Hence $-1 \leq \sigma^r - \vartheta^s - \varrho^t \leq 1 \Rightarrow 0 \leq \frac{1+\left(\sigma^r-\vartheta^s-\varrho^t\right)}{2} \leq 1$. ■

Property 6: If $g = \langle 1, 0, 0 \rangle$, then $S(g) = 1$ and if $g = \langle 0, 0, 1 \rangle$, then $S(g) = 0$.

Property 7: For any r, s, t-SFN $g = \langle \sigma, \vartheta, \varrho \rangle$, $S(g)$ monotonically increases as σ increases and monotonically decreases as ϑ and ϱ increases.

Proof: Differentiating Eq. (5) partially with respect to σ, ϑ, and ϱ, we get $\frac{\partial S(g)}{\partial \sigma} = \frac{r}{2}\sigma^{r-1}$, $\frac{\partial S(g)}{\partial \vartheta} = -\frac{s}{2}\sigma^{s-1}$, $\frac{\partial S(g)}{\partial \varrho} = -\frac{t}{2}\sigma^{t-1}$. Since $\sigma, \vartheta, \varrho \in [0, 1]$ so $\frac{\partial S(g)}{\partial \sigma} \geq 0$, $\frac{\partial S(g)}{\partial \vartheta} \leq 0$ and $\frac{\partial S(g)}{\partial \varrho} \leq 0$. Hence the result follows. ■

The following definition describes the accuracy function for r, s, t-SFNs.

Definition 9: Let $g = \langle \sigma, \vartheta, \varrho \rangle$ be any $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFN; then its accuracy function is denoted and defined as

$$A(g) = \frac{1}{2}\left(1 + \left(\sigma^{\mathsf{r}} + \vartheta^{\mathsf{s}} + \varrho^{\mathsf{t}}\right)\right). \tag{6}$$

It is evident that $0 \leq A(g) \leq 1$.

Definition 10: For any two $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs, $g_i = \langle \sigma_i, \vartheta_i, \varrho_i \rangle$ $(i = 1, 2)$, the comparison scheme is defined as follows:
i) If $S(g_1) < S(g_2)$, then $g_1 \prec g_2$;
ii) If $S(g_1) > S(g_2)$, then $g_1 \succ g_2$;
iii) If $S(g_1) = S(g_2)$, and
a) If $A(g_1) < A(g_2)$, then $g_1 \prec g_2$;
b) If $A(g_1) > A(g_2)$, then $g_1 \succ g_2$;
c) If $A(g_1) = A(g_2)$, then $g_1 \simeq g_2$.

D. DISTANCE MEASURE

The distance measure is a crucial instrument in fuzzy set analysis. It is frequently employed in decision-making issues. Generalized distance, Euclidean distance, and Hamming distance measure are the most widely utilized distance measures. In this part, we give the aforesaid distance measure between $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs that will be used later on.

Definition 11: Let $g_i = \langle \sigma_i, \vartheta_i, \varrho_i \rangle$ $(i = 1, 2, 3)$ be any three $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs, Then, a distance measure has the following characteristics:
i). $0 \leq d(g_1, g_2) \leq 1$.
ii). $d(g_1, g_2) = 0$ if and only if $g_1 = g_2$.
iii). $d(g_1, g_2) = d(g_2, g_1)$.
iv). If $g_1 \preceq g_2 \preceq g_3$, then $d(g_1, g_3) > d(g_1, g_2)$ and $d(g_1, g_3) > d(g_2, g_3)$.

Definition 12: Let $g_1 = \langle \sigma_1, \vartheta_1, \varrho_1 \rangle$ and $g_2 = \langle \sigma_2, \vartheta_2, \varrho_2 \rangle$ be any two $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs, then the Hamming distance between g_1 and g_2 is given by

$$d(g_1, g_2) = \frac{1}{3}\left(\left|\sigma_1^{\mathsf{r}} - \sigma_2^{\mathsf{r}}\right| + \left|\vartheta_1^{\mathsf{s}} - \vartheta_2^{\mathsf{s}}\right| + \left|\varrho_1^{\mathsf{t}} - \varrho_2^{\mathsf{t}}\right|\right), \tag{7}$$

the Euclidean distance between g_1 and g_2 is given by

$$d(g_1, g_2) = \sqrt{\frac{1}{3}\left(\left(\sigma_1^{\mathsf{r}} - \sigma_2^{\mathsf{r}}\right)^2 + \left(\vartheta_1^{\mathsf{s}} - \vartheta_2^{\mathsf{s}}\right)^2 + \left(\varrho_1^{\mathsf{t}} - \varrho_2^{\mathsf{t}}\right)^2\right)}, \tag{8}$$

the generalized distance between g_1 and g_2 is given by

$$d(g_1, g_2) = \sqrt[\lambda]{\frac{1}{3}\left(\left|\sigma_1^{\mathsf{r}} - \sigma_2^{\mathsf{r}}\right|^{\lambda} + \left|\vartheta_1^{\mathsf{s}} - \vartheta_2^{\mathsf{s}}\right|^{\lambda} + \left|\varrho_1^{\mathsf{t}} - \varrho_2^{\mathsf{t}}\right|^{\lambda}\right)}, \tag{9}$$

where ℓ is the LCM of $\mathsf{r}, \mathsf{s}, \mathsf{t}$, and λ is any real number.

Theorem 8: The devised generalized distance measure Eq. (9) fulfills the characteristics of Definition 11.

Proof:

i). The condition $d(g_1, g_2) \geq 0$ obviously holds. Next, consider

$$\begin{aligned} d(g_1, g_2) &= \sqrt[\lambda]{\frac{1}{3}\left(\left(\sigma_1^{\mathsf{r}} - \sigma_2^{\mathsf{r}}\right)^{\lambda} + \left(\vartheta_1^{\mathsf{s}} - \vartheta_2^{\mathsf{s}}\right)^{\lambda} + \left(\varrho_1^{\mathsf{t}} - \varrho_2^{\mathsf{t}}\right)^{\lambda}\right)} \\ &\leq \sqrt[\lambda]{\frac{1}{3}(1 + 1 + 1)} = 1. \end{aligned}$$

Therefore, $0 \leq d(g_1, g_2) \leq 1$.

Conditions ii) and iii) are straightforward.

To prove iv), grounded on Definition 8, we have $\sigma_1^{\mathsf{r}} < \sigma_2^{\mathsf{r}} < \sigma_3^{\mathsf{r}} \leq 1$, $1 \geq \vartheta_1^{\mathsf{s}} > \vartheta_2^{\mathsf{s}} > \vartheta_3^{\mathsf{s}}$, $1 \geq \varrho_1^{\mathsf{t}} > \varrho_2^{\mathsf{t}} > \varrho_3^{\mathsf{t}}$. Therefore, $\left|\sigma_1^{\mathsf{r}} - \sigma_2^{\mathsf{r}}\right|^{\lambda} < \left|\sigma_1^{\mathsf{r}} - \sigma_3^{\mathsf{r}}\right|^{\lambda}$, $\left|\vartheta_1^{\mathsf{s}} - \vartheta_2^{\mathsf{s}}\right|^{\lambda} < \left|\vartheta_1^{\mathsf{s}} - \vartheta_3^{\mathsf{s}}\right|^{\lambda}$, and $\left|\varrho_1^{\mathsf{t}} - \varrho_2^{\mathsf{t}}\right|^{\lambda} < \left|\varrho_1^{\mathsf{t}} - \varrho_3^{\mathsf{t}}\right|^{\lambda}$. Thereby, $d(g_1, g_2) \leq d(g_1, g_3)$. Analogously, $d(g_2, g_3) \leq d(g_1, g_3)$. ■

IV. AGGREGATION OPERATORS OF r, s, t-SPHERICAL FUZZY DATA

In this section, $\mathsf{r}, \mathsf{s}, \mathsf{t}$-spherical fuzzy weighted averaging, and $\mathsf{r}, \mathsf{s}, \mathsf{t}$-spherical fuzzy weighted geometric operators for aggregating the $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs are concocted to extend the fundamental aggregation operators to the $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFS environment.

A. r, s, t-SPHERICAL FUZZY AVERAGING OPERATORS

This subpart presents the novel notion of the weighted averaging operator for aggregating $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs and investigates its core characteristics.

Definition 13: Let $g_i = \langle \sigma_i, \vartheta_i, \varrho_i \rangle$ $(i = 1, 2, \ldots, n)$ be any collection of $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs, then the $\mathsf{r}, \mathsf{s}, \mathsf{t}$-spherical fuzzy weighted averaging ($\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFWA) operator is formulated as follows:

$$\mathsf{r}, \mathsf{s}, \mathsf{t} - SFWA(g_1, g_2, \ldots, g_n) = \oplus_{i=1}^{n}(\varpi_i g_i), \tag{10}$$

where $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$ is the weight vector of g_i $(i = 1, 2, \ldots, n)$, and $\varpi_i > 0$, $\sum_{i=1}^{n} \varpi_i = 1$.

Based on $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs operational rules, we deduce the following theorems.

Theorem 9: Let $g_i = \langle \sigma_i, \vartheta_i, \varrho_i \rangle$ $(i = 1, 2, \ldots, n)$ be any collection of $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs, then the result of $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFWA operator is still a $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFN, shown as follows:

$$\begin{aligned} \mathsf{r}, \mathsf{s}, \mathsf{t} - SFWA(g_1, g_2, \ldots, g_n) &= \oplus_{i=1}^{n}(\varpi_i g_i) \\ &= \left\langle \sqrt[\mathsf{r}]{1 - \prod_{i=1}^{n}\left(1 - \sigma_i^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{n} \vartheta_i^{\varpi_i}, \prod_{i=1}^{n} \varrho_i^{\varpi_i} \right\rangle, \end{aligned} \tag{11}$$

Proof: To verify Eq. (11), we use the mathematical induction principle.

For $n = 2$,

$$\begin{aligned} &\mathsf{r}, \mathsf{s}, \mathsf{t} - SFWA(g_1, g_2) \\ &= \varpi_1 g_1 \oplus \varpi_2 g_2 \\ &= \left\langle \sqrt[\mathsf{r}]{1 - \left(1 - \sigma_1^{\mathsf{r}}\right)^{\varpi_1}\left(1 - \sigma_2^{\mathsf{r}}\right)^{\varpi_2}}, \vartheta_1^{\varpi_1}\vartheta_2^{\varpi_2}, \varrho_1^{\varpi_1}\varrho_2^{\varpi_2} \right\rangle \\ &= \left\langle \sqrt[\mathsf{r}]{1 - \prod_{i=1}^{2}\left(1 - \sigma_i^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{2} \vartheta_i^{\varpi_i}, \prod_{i=1}^{2} \varrho_i^{\varpi_i} \right\rangle. \end{aligned}$$

Therefore, the result is true for $i = 2$.

Suppose the result is true for $i = k$. Thus, we have

$$\begin{aligned}&\mathsf{r,s,t}-SFWA\left(g_1, g_2, \ldots, g_k\right)\\&=\oplus_{i=1}^{k}\left(\varpi_i g_i\right)\\&=\left\langle\sqrt[\mathsf{r}]{1-\prod_{i=1}^{k}\left(1-\sigma_i^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{k}\vartheta_i^{\varpi_i}, \prod_{i=1}^{k}\varrho_i^{\varpi_i}\right\rangle.\end{aligned}$$

Now, for $i = k+1$

$$\begin{aligned}&\mathsf{r,s,t}-SFWA\left(g_1, g_2, \ldots, g_{k+1}\right)\\&=\oplus_{i=1}^{k+1}\left(\varpi_i g_i\right)=\oplus_{i=1}^{k}\left(\varpi_i g_i\right)\oplus\varpi_{k+1}g_{k+1}\\&=\left\langle\sqrt[\mathsf{r}]{1-\prod_{i=1}^{k}\left(1-\sigma_i^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{k}\vartheta_i^{\varpi_i}, \prod_{i=1}^{k}\varrho_i^{\varpi_i}\right\rangle\\&\quad\oplus\left\langle\sqrt[\mathsf{r}^*]{1-\left(1-\sigma_{k+1}^{\mathsf{r}*}\right)^{\varpi_{k+1}}}, \vartheta_{k+1}^{\varpi_{k+1}}, \varrho_{k+1}^{\varpi_{k+1}}\right\rangle\\&=\left\langle\sqrt[\mathsf{r}]{1-\prod_{i=1}^{k+1}\left(1-\sigma_i^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{k+1}\vartheta_i^{\varpi_i}, \prod_{i=1}^{k+1}\varrho_i^{\varpi_i}\right\rangle.\end{aligned}$$

Hence, the result is true for $n = k+1$. Therefore, the mathematical induction process ensures that the stated result holds true for all natural numbers. ∎

Theorem 10: If all $\mathsf{r,s,t}$-SFNs $g_i = \langle\sigma_i, \vartheta_i, \varrho_i\rangle$ $(i = 1, 2, \ldots, n)$ are equal, i.e., $g_i = g = \langle\sigma, \vartheta, \varrho\rangle$ for all i, then

$$\mathsf{r,s,t}-SFWA\left(g_1, g_2, \ldots, g_n\right) = g. \tag{12}$$

Proof: Since $g_i = g = \langle\sigma, \vartheta, \varrho\rangle$ for all $i = 1, 2, \ldots, n$, then

$$\begin{aligned}&\mathsf{r,s,t}-SFWA\left(g_1, g_2, \ldots, g_n\right)\\&=\left\langle\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\sigma_i^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{n}\vartheta_i^{\varpi_i}, \prod_{i=1}^{n}\varrho_i^{\varpi_i}\right\rangle\\&=\left\langle\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\sigma^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{n}\vartheta^{\varpi_i}, \prod_{i=1}^{n}\varrho^{\varpi_i}\right\rangle\\&=\left\langle\sqrt[\mathsf{r}]{1-\left(1-\sigma^{\mathsf{r}}\right)^{\sum_{i=1}^{n}\varpi_i}}, \vartheta^{\sum_{i=1}^{n}\varpi_i}, \varrho^{\sum_{i=1}^{n}\varpi_i}\right\rangle\\&=\langle\sigma, \vartheta, \varrho\rangle = g.\end{aligned}$$

∎

Theorem 11: Let $g_i = \langle\sigma_i, \vartheta_i, \varrho_i\rangle$ and $g_i' = \langle\sigma_i', \vartheta_i', \varrho_i'\rangle (i = 1, 2, \ldots, n)$ be two collections of $\mathsf{r,s,t}$-SFNs, if $g_i \leq g_i'$, for all i then

$$\begin{aligned}&\mathsf{r,s,t}-SFWA\left(g_1, g_2, \ldots, g_n\right)\\&\quad\leq\mathsf{r,s,t}-SFWA\left(g_1', g_2', \ldots, g_n'\right).\end{aligned} \tag{13}$$

Proof: Since $g_i \leq g_i'$, for all $i = 1, 2, \ldots, n$, then we have

$$\prod_{i=1}^{n}\left(1-\sigma_i^{\mathsf{r}}\right)^{\varpi_i}\geq\prod_{i=1}^{n}\left(1-\sigma_i'^{\mathsf{r}}\right)^{\varpi_i}\Rightarrow\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\sigma_i^{\mathsf{r}}\right)^{\varpi_i}}$$

$$\leq\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\sigma_i'^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{n}\vartheta_i^{\varpi_i}\geq\prod_{i=1}^{n}\vartheta_i'^{\varpi_i}$$

and $\prod_{i=1}^{n}\varrho_i^{\varpi_i}\geq\prod_{i=1}^{n}\varrho_i'^{\varpi_i}$. Therefore,

$$\begin{aligned}&\left\langle\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\sigma_i^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{n}\vartheta_i^{\varpi_i}, \prod_{i=1}^{n}\varrho_i^{\varpi_i}\right\rangle\\&\leq\left\langle\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\sigma_i'^{\mathsf{r}}\right)^{\varpi_i}}, \prod_{i=1}^{n}\vartheta_i'^{\varpi_i}, \prod_{i=1}^{n}\varrho_i'^{\varpi_i}\right\rangle.\end{aligned}$$

Hence $\mathsf{r,s,t}-SFWA\left(g_1, g_2, \ldots, g_n\right) \leq \mathsf{r,s,t}-SFWA\left(g_1', g_2', \ldots, g_n'\right)$. ∎

Theorem 12: Let $g_i = \langle\sigma_i, \vartheta_i, \varrho_i\rangle (i = 1, 2, \ldots, n)$ be a collection of $\mathsf{r,s,t}$-SFNs, and let $g^- = \min\left\{g_1, g_2, \ldots, g_n\right\}$ and $g^+ = \max\left\{g_1, g_2, \ldots, g_n\right\}$; then

$$g^- \leq \mathsf{r,s,t}-SFWA\left(g_1, g_2, \ldots, g_n\right) \leq g^+. \tag{14}$$

Proof: According to Theorem 11,

$$\begin{aligned}&\mathsf{r,s,t}-SFWA\left(g^-, g^-, \ldots, g^-\right)\\&\quad\leq\mathsf{r,s,t}-SFWA\left(g_1, g_2, \ldots, g_n\right)\\&\quad\quad\leq\mathsf{r,s,t}-SFWA\left(g^+, g^+, \ldots, g^+\right).\end{aligned}$$

Next, in line with Theorem 10, $g^- \leq \mathsf{r,s,t}-SFWA\left(g_1, g_2, \ldots, g_n\right) \leq g^+$. ∎

B. r, s, t-SPHERICAL FUZZY GEOMETRIC OPERATORS

The present subsection introduces the novel notion of the weighted geometric operator for aggregating $\mathsf{r,s,t}$-SFNs and investigates its core features.

Definition 14: Let $g_i = \langle\sigma_i, \vartheta_i, \varrho_i\rangle\ (i = 1, 2, \ldots, n)$ be any collection of $\mathsf{r,s,t}$-SFNs, then the $\mathsf{r,s,t}$-spherical fuzzy weighted geometric ($\mathsf{r,s,t}$-SFWG) operator is formulated as follows:

$$\mathsf{r,s,t}-SFWG\left(g_1, g_2, \ldots, g_n\right) = \otimes_{i=1}^{n}\left(g_i^{\varpi_i}\right), \tag{15}$$

where $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$ is the weight vector of $g_i\ (i = 1, 2, \ldots, n)$, and $\varpi_i > 0$, $\sum_{i=1}^{n}\varpi_i = 1$.

Based on $\mathsf{r,s,t}$-SFNs operational rules, we derive the following results.

Theorem 13: Let $g_i = \langle\sigma_i, \vartheta_i, \varrho_i\rangle\ (i = 1, 2, \ldots, n)$ be any collection of $\mathsf{r,s,t}$-SFNs, then the result of $\mathsf{r,s,t}$-SFWG operator is still a $\mathsf{r,s,t}$-SFN, shown as follows:

$$\begin{aligned}&\mathsf{r,s,t}-SFWG\left(g_1, g_2, \ldots, g_n\right) = \otimes_{i=1}^{n}\left(g_i^{\varpi_i}\right)\\&=\left\langle\prod_{i=1}^{n}\sigma_i^{\varpi_i}, \sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\vartheta_i^{\mathsf{r}}\right)^{\varpi_i}}, \sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\varrho_i^{\mathsf{r}}\right)^{\varpi_i}}\right\rangle,\end{aligned} \tag{16}$$

Proof: To verify Eq. (16), we use the mathematical induction principle.

For $n=2$,

$$\begin{aligned}
&\mathsf{r,s,t}-SFWG\left(g_1,g_2\right)\\
&=g_1^{\varpi_1}\otimes g_2^{\varpi_2}\\
&=\left\langle\begin{array}{c}\sigma_1^{\varpi_1}\sigma_2^{\varpi_2},\sqrt[\mathsf{r}]{1-\left(1-\vartheta_1^{\mathsf{r}}\right)^{\varpi_1}\left(1-\vartheta_2^{\mathsf{r}}\right)^{\varpi_2}},\\ \sqrt[\mathsf{r}]{1-\left(1-\varrho_1^{\mathsf{r}}\right)^{\varpi_1}\left(1-\varrho_2^{\mathsf{r}}\right)^{\varpi_2}}\end{array}\right\rangle\\
&=\left\langle\prod_{i=1}^{2}\sigma_i^{\varpi_i},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{2}\left(1-\vartheta_i^{\mathsf{r}}\right)^{\varpi_i}},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{2}\left(1-\varrho_i^{\mathsf{r}}\right)^{\varpi_i}}\right\rangle.
\end{aligned}$$

Therefore, the result is true for $i=2$.

Suppose the result is true for $i=k$. Thus, we have

$$\begin{aligned}
&\mathsf{r,s,t}-SFWG\left(g_1,g_2,\ldots,g_k\right)\\
&=\otimes_{i=1}^{k}\left(g_i^{\varpi_i}\right)\\
&=\left\langle\prod_{i=1}^{k}\sigma_i^{\varpi_i},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{k}\left(1-\vartheta_i^{\mathsf{r}}\right)^{\varpi_i}},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{k}\left(1-\varrho_i^{\mathsf{r}}\right)^{\varpi_i}}\right\rangle.
\end{aligned}$$

Now, for $i=k+1$

$$\begin{aligned}
&\mathsf{r,s,t}-SFWG\left(g_1,g_2,\ldots,g_{k+1}\right)\\
&=\otimes_{i=1}^{k+1}\left(g_i^{\varpi_i}\right)\\
&=\otimes_{i=1}^{k}\left(g_i^{\varpi_i}\right)\otimes g_{k+1}^{\varpi_{k+1}}\\
&=\left\langle\begin{array}{c}\prod_{i=1}^{k}\sigma_i^{\varpi_i},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{k}\left(1-\vartheta_i^{\mathsf{r}}\right)^{\varpi_i}},\\ \sqrt[\mathsf{r}]{1-\prod_{i=1}^{k}\left(1-\varrho_i^{\mathsf{r}}\right)^{\varpi_i}}\end{array}\right\rangle\\
&\quad\otimes\left\langle\begin{array}{c}\sigma_{k+1}^{\varpi_{k+1}},\sqrt[\mathsf{r}]{1-\left(1-\vartheta_{k+1}^{\mathsf{r}}\right)^{\varpi_{k+1}}},\\ \sqrt[\mathsf{r}]{1-\left(1-\varrho_{k+1}^{\mathsf{r}}\right)^{\varpi_{k+1}}}\end{array}\right\rangle\\
&=\left\langle\begin{array}{c}\prod_{i=1}^{k+1}\sigma_i^{\varpi_i},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{k+1}\left(1-\vartheta_i^{\mathsf{r}}\right)^{\varpi_i}},\\ \sqrt[\mathsf{r}]{1-\prod_{i=1}^{k+1}\left(1-\varrho_i^{\mathsf{r}}\right)^{\varpi_i}}\end{array}\right\rangle.
\end{aligned}$$

Hence, the result is true for $n=k+1$. Therefore, the mathematical induction process ensures that the stated result holds true for all natural numbers. ■

Theorem 14: If all $\mathsf{r,s,t}$-SFNs $g_i=\langle\sigma_i,\vartheta_i,\varrho_i\rangle$ $(i=1,2,\ldots,n)$ are equal, i.e., $g_i=g=\langle\sigma,\vartheta,\varrho\rangle$ for all i, then

$$\mathsf{r,s,t}-SFWG\left(g_1,g_2,\ldots,g_n\right)=g. \tag{17}$$

Proof: Since $g_i=g=\langle\sigma,\vartheta,\varrho\rangle$ for all $i=1,2,\ldots,n$, then

$$\begin{aligned}
&\mathsf{r,s,t}-SFWG\left(g_1,g_2,\ldots,g_n\right)\\
&=\left\langle\begin{array}{c}\prod_{i=1}^{n}\varrho_i^{\sigma_i},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\vartheta_i^{\mathsf{r}}\right)^{\varpi_i}},\\ \sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\varrho_i^{\mathsf{r}}\right)^{\varpi_i}}\end{array}\right\rangle\\
&=\left\langle\begin{array}{c}\prod_{i=1}^{n}\sigma^{\varpi_i},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\vartheta^{\mathsf{r}}\right)^{\varpi_i}},\\ \sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\varrho^{\mathsf{r}}\right)^{\varpi_i}}\end{array}\right\rangle\\
&=\left\langle\begin{array}{c}\sigma^{\sum_{i=1}^{n}\varpi_i},\sqrt[\mathsf{r}]{1-\left(1-\vartheta^{\mathsf{r}}\right)^{\sum_{i=1}^{n}\varpi_i}},\\ \sqrt[\mathsf{r}]{1-\left(1-\varrho^{\mathsf{r}}\right)^{\sum_{i=1}^{n}\varpi_i}}\end{array}\right\rangle\\
&=\langle\sigma,\vartheta,\varrho\rangle=g.
\end{aligned}$$

■

Theorem 15: Let $g_i=\langle\sigma_i,\vartheta_i,\varrho_i\rangle$ and $g_i'=\langle\sigma_i',\vartheta_i',\varrho_i'\rangle(i=1,2,\ldots,n)$ be two collections of $\mathsf{r,s,t}$-SFNs, if $g_i\leq g_i'$, for all i then

$$\begin{aligned}
&\mathsf{r,s,t}-SFWG\left(g_1,g_2,\ldots,g_n\right)\\
&\quad\leq\mathsf{r,s,t}-SFWG\left(g_1',g_2',\ldots,g_n'\right).
\end{aligned} \tag{18}$$

Proof: Since $g_i\leq g_i'$, for all $i=1,2,\ldots,n$, then we have

$$\begin{aligned}
&\prod_{i=1}^{n}\sigma_i^{\varpi_i}\geq\prod_{i=1}^{n}\sigma_i^{\prime\varpi_i},\prod_{i=1}^{n}\left(1-\vartheta_i^{\mathsf{r}}\right)^{\varpi_i}\geq\prod_{i=1}^{n}\left(1-\vartheta_i^{\prime\mathsf{r}}\right)^{\varpi_i}\\
&\quad\Rightarrow\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\vartheta_i^{\mathsf{r}}\right)^{\varpi_i}}\leq\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\vartheta_i^{\prime\mathsf{r}}\right)^{\varpi_i}},
\end{aligned}$$

and

$$\begin{aligned}
&\prod_{i=1}^{n}\left(1-\varrho_i^{\mathsf{r}}\right)^{\varpi_i}\geq\prod_{i=1}^{n}\left(1-\varrho_i^{\prime\mathsf{r}}\right)^{\varpi_i}\Rightarrow\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\varrho_i^{\mathsf{r}}\right)^{\varpi_i}}\\
&\leq\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\varrho_i^{\prime\mathsf{r}}\right)^{\varpi_i}}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\left\langle\prod_{i=1}^{n}\sigma_i^{\varpi_i},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\vartheta_i^{\mathsf{r}}\right)^{\varpi_i}},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\varrho_i^{\mathsf{r}}\right)^{\varpi_i}}\right\rangle\\
&\leq\left\langle\prod_{i=1}^{n}\sigma_i^{\prime\varpi_i},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\vartheta_i^{\prime\mathsf{r}}\right)^{\varpi_i}},\sqrt[\mathsf{r}]{1-\prod_{i=1}^{n}\left(1-\varrho_i^{\prime\mathsf{r}}\right)^{\varpi_i}}\right\rangle.
\end{aligned}$$

Hence $\mathsf{r,s,t}-SFWG\left(g_1,g_2,\ldots,g_n\right)\leq\mathsf{r,s,t}-SFWG\left(g_1',g_2',\ldots,g_n'\right)$. ■

Theorem 16: Let $g_i=\langle\sigma_i,\vartheta_i,\varrho_i\rangle(i=1,2,\ldots,n)$ be a collection of $\mathsf{r,s,t}$-SFNs, and let $g^-=\min\left\{g_1,g_2,\ldots,g_n\right\}$ and $g^+=\max\left\{g_1,g_2,\ldots,g_n\right\}$; then

$$g^-\leq\mathsf{r,s,t}-SFWG\left(g_1,g_2,\ldots,g_n\right)\leq g^+. \tag{19}$$

Proof: According to Theorem 15,

$$\begin{aligned}
&\mathsf{r,s,t}-SFWG\left(g^-,g^-,\ldots,g^-\right)\\
&\quad\leq\mathsf{r,s,t}-SFWG\left(g_1,g_2,\ldots,g_n\right)\\
&\quad\leq\mathsf{r,s,t}-SFWG\left(g^+,g^+,\ldots,g^+\right).
\end{aligned}$$

Next, in line with Theorem 14, $g^-\leq\mathsf{r,s,t}-SFWG\left(g_1,g_2,\ldots,g_n\right)\leq g^+$. ■

V. MCGDM BASED ON THE MAXIMIZING DEVIATION AND VIKOR METHOD

This segment introduces a novel way for determining the weight of criteria based on the deviation measure and extends the VIKOR method to r, s, t-spherical fuzzy settings in order to tackle MCGDM issues.

A. r, s, t-SPHERICAL FUZZY MAXIMIZING DEVIATION METHOD

Due to the complexity and intricacy of the decision-making environment, it is often impossible to gather complete information on criteria and their respective weights. Therefore, identifying and distributing the criterion weights is a crucial problem [38], [39]. The maximizing deviation approach, proffered by Yingming [40], has been used to such MCGDM issues in order to identify and describe each criterion's suitable value [41].

Suppose that $\mathcal{O}l = \{\mathcal{O}l_1, \mathcal{O}l_2, \ldots, \mathcal{O}l_m\}$ is the set of 'm' alternatives and $\mathcal{C}r = \{\mathcal{C}r_1, \mathcal{C}r_2, \ldots, \mathcal{C}r_n\}$ is the set of criteria 'n' so that the weights of criteria are entirely unrecognized. Let $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$, be the weight vector of criteria, so that $\Sigma_{j=1}^{n}\varpi_i = 1,\ \varpi_j \geq 0$. Thereby, for the criterion $\mathcal{C}r_j \in \mathcal{C}r$, the deviation of alternative $\mathcal{O}l_i$ to all the other alternatives under the criteria $\mathcal{C}r_j$ can be described as shown:

$$D_{ij}\left(\varpi_j\right) = \sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)\varpi_j, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n, \tag{20}$$

where g_{ij} and g_{kj} are the evaluation values stated by r, s, t-SFNs, and $d\left(g_{ij}, g_{kj}\right)$ is the Hamming distance between them ascertained by Eq. (7). The total deviation among all the alternatives under attribute $\mathcal{C}r_j$ is represented and defined as

$$D_j\left(\varpi_j\right) = \sum_{i=1}^{m} D_{ij}\left(\varpi_j\right) = \sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)\varpi_j. \tag{21}$$

Next the total deviation among all the alternatives in respect of all the criteria is symbolized and formulated as

$$D\left(\varpi_j\right) = \sum_{j=1}^{n} D_j\left(\varpi_j\right) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)\varpi_j. \tag{22}$$

Then, the following optimal model (named as M) is constructed:

$$M = \begin{cases} \max D\left(\varpi_j\right) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)\varpi_j, \\ \varpi_j \geq 0,\ j = 1, 2, \ldots, n,\ \sum_{j=1}^{n}\varpi_j^2 = 1. \end{cases}$$

For the solution of the preceding model M, we utilize the Lagrange multiplier function:

$$L\left(\varpi_j, \daleth\right) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)\varpi_j + \daleth\left(\sum_{j=1}^{n}\varpi_j^2 - 1\right). \tag{23}$$

Then, the partial derivatives of the Lagrange function with respect to ϖ_j and $\daleth$ are computed and set to zero:

$$\begin{cases} \dfrac{\partial L\left(\varpi_j, \daleth\right)}{\partial \varpi_j} = \sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right) + 2\daleth\varpi_j = 0, \\ \dfrac{\partial L\left(\varpi_j, \daleth\right)}{\partial \daleth} = \sum_{j=1}^{n}\varpi_j^2 - 1 = 0, \end{cases} \tag{24}$$

Solving Eq. (24), we get

$$\begin{cases} 2\daleth = \sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)\right)^2}, \\ \varpi_j = \dfrac{\sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)\right)^2}}, \end{cases} \tag{25}$$

After normalizing the vector $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$, one obtains the weight vector.

$$\varpi_j = \frac{\varpi_j}{\sum_{j=1}^{n}\varpi_j} = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d\left(g_{ij}, g_{kj}\right)}. \tag{26}$$

B. VIKOR APPROACH FOR MCGDM UNDER r, s, t-SPHERICAL ENVIRONMENT

This section purposes to devise the VIKOR approach for the r, s, t-spherical fuzzy environment to take into consideration the MCGDM issues. Our primary objective is to provide a compromise solution that optimizes group utility while minimizing individual regret.

Consider an MCGDM issue with m alternatives as depicted by $\mathcal{O}l_i\ (i = 1, 2, \ldots, m)$. Suppose there are l number of DEs $D_k\ (k = 1, 2, \ldots, l)$ who need to choose the best choice based on n number of criteria $\mathcal{C}r_j\ (j = 1, 2, \ldots, n)$. Let each of the l DEs be given a weight $w_k\ (k = 1, 2, \ldots, l)$ so that $w_k > 0$ and $\sum_{k=1}^{l} w_k = 1$.

The mechanism of the r, s, t-spherical fuzzy VIKOR approach is described in detail below:

Step 1: Creation of individual r, s, t-spherical fuzzy decision matrices:

The DEs examine the capabilities of alternatives in relation to the chosen criteria and record their opinions in the form of r, s, t-SFNs. The judgments of the DE

D_k $(k = 1, 2, \ldots, l)$ are systematically organised as a decision matrix $M^{(k)} = \left(g_{ij}^{k}\right)_{m\times n}$ (see Table 1). wherein the subscript $i\,(i = 1, 2, \ldots, m)$ corresponds to the alternative $\mathcal{O}l_i$, subscript $j(j = 1, 2, \ldots, n)$ corresponds to the criteria $\mathcal{C}r_j$, and superscript k symbolize the decision made by the DE D_k. Further, an entry $g_{mn}^{k} = \langle \sigma_{mn}^{k}, \vartheta_{mn}^{k}, \varrho_{mn}^{k}\rangle$ presents $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFN allocated to the alternative $\mathcal{O}l_i$ relative to the criteria $\mathcal{C}r_j$ by the DE D_k. Analogously, l independent decision matrices $M^{(1)}, M^{(2)}, \ldots, M^{(l)}$ are created by placing $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFNs, assigned by DEs $D^{(1)}, D^{(2)}, \ldots, D^{(l)}$, respectively, into decision matrices.

Step 2: Formation of the aggregated $\mathsf{r}, \mathsf{s}, \mathsf{t}$-spherical fuzzy decision matrix:
Aggregate the individual decision matrices $M^{(k)} = \left(g_{ij}^{k}\right)_{m\times n}$ $(k = 1, 2, \ldots, l)$ into collective decision matrix $M = \left(g_{ij}\right)_{m\times n}$ by the aid of $\mathsf{r}, \mathsf{s}, \mathsf{t}$-SFWA operator given in Eq. (11). Thus, the aggregated $\mathsf{r}, \mathsf{s}, \mathsf{t}$-spherical fuzzy rating g_{ij} of each alternative with respect to different criteria can be obtained as given in Eq. (27).

$$\begin{aligned} g_{ij} &= \mathsf{r}, \mathsf{s}, \mathsf{t} - SFWA\left(g_{ij}^{1}, g_{ij}^{2}, \ldots, g_{ij}^{l}\right) \\ &= \oplus_{k=1}^{l} w_k g_{ij}^{k} \\ &= \left\langle \begin{array}{c} \sqrt[\mathsf{r}]{1 - \prod_{k=1}^{l}\left(1 - \sigma_{ij}^{k\,\mathsf{r}}\right)^{w_k}}, \\ \prod_{k=1}^{l}\left(\vartheta_{ij}^{k}\right)^{w_k}, \prod_{k=1}^{l}\left(\varrho_{ij}^{k}\right)^{w_k} \end{array} \right\rangle; \\ & \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n. \end{aligned} \tag{27}$$

Then, the aggregated $\mathsf{r}, \mathsf{s}, \mathsf{t}$-spherical fuzzy decision matrix corresponding to given MCGDM problem is designed as Table 2:

Step 3: Criteria weights determination:
Ascertain the weight vector of criteria $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ by the method outlined in Section V-A.

Step 4: Determination of relative ideal solutions:
The objective of an MCGDM issue is to choose the optimal solution that meets the specified criteria. In accordance with the VIKOR technique, this objective is attained by picking the alternative closest to the positive ideal solution (or ideal solution) and furthest from the negative ideal solution. In actuality, however, there are no optimal answers. To analyze the alternatives, we formulate the following relative ideal solutions:
Define the $\mathsf{r}, \mathsf{s}, \mathsf{t}$-spherical fuzzy positive ideal solution $\mathcal{O}l^{+} = \left(\mathcal{O}l_1^{+}, \mathcal{O}l_2^{+}, \ldots, \mathcal{O}l_n^{+}\right)$ and the $\mathsf{r}, \mathsf{s}, \mathsf{t}$-spherical fuzzy negative ideal solution $\mathcal{O}l^{-} = \left(\mathcal{O}l_1^{-}, \mathcal{O}l_2^{-}, \ldots, \mathcal{O}l_n^{-}\right)$ as follows:

$$\mathcal{O}l_j^{+} = \begin{cases} \max\limits_i g_{ij}, & \text{for benefit criteria} \\ \min\limits_i g_{ij}, & \text{for cost criteria} \end{cases}; \quad j = 1, 2, \ldots, n, \tag{28}$$

and

$$\mathcal{O}l_j^{-} = \begin{cases} \min\limits_i g_{ij}, & \text{for benefit criteria} \\ \max\limits_i g_{ij}, & \text{for cost criteria} \end{cases}; \quad j = 1, 2, \ldots, n. \tag{29}$$

Step 5: Ascertainment of group utility measure and individual regret measure:
This steps aims to calculate the group utility measure and individual regret measure for each alternative. Group utility measure $\mathscr{S}_i$ and individual regret measure $\mathscr{R}_i$ for the alternative $\mathcal{O}l_i$ can be found using Euclidean distance and criterion weights as follows:

$$\mathscr{S}_i = \sum_{j=1}^{n} \varpi_j \frac{d\left(\mathcal{O}l_j^{+}, \mathcal{O}l_{ij}\right)}{d\left(\mathcal{O}l_j^{+}, \mathcal{O}l_j^{-}\right)}, \tag{30}$$

$$\mathscr{R}_i = \max_j \varpi_j \frac{d\left(\mathcal{O}l_j^{+}, \mathcal{O}l_{ij}\right)}{d\left(\mathcal{O}l_j^{+}, \mathcal{O}l_j^{-}\right)}. \tag{31}$$

Step 6: Determination of VIKOR index:
Determine the VIKOR index $\mathscr{V}_i$ for the alternative $\mathcal{O}l_i$ as follows:

$$\mathscr{V}_i = ¶\left(\frac{\mathscr{S}_i - \mathscr{S}_*}{\mathscr{S}^* - \mathscr{S}_*}\right) + (1 - ¶)\left(\frac{\mathscr{R}_i - \mathscr{R}_*}{\mathscr{R}^* - \mathscr{R}_*}\right), \tag{32}$$

wherein $\mathscr{S}^* = \max\limits_i \mathscr{S}_i$, $\mathscr{S}_* = \min\limits_i \mathscr{S}_i$, $\mathscr{R}^* = \max\limits_i \mathscr{R}_i$, $\mathscr{R}_* = \min\limits_i \mathscr{R}_i$. The coefficients ¶ and $(1-¶)$ indicate the weight-age assigned to group utility $\mathscr{S}_i$ and individual regret $\mathscr{R}_i$, and play a key role in the appraisal of the compromise solution. It takes a value from the closed unit interval $[0, 1]$, but often 0.5, such that the compromise solution simultaneously has both properties, namely maximum group utility and minimum individual regret of opponent. However, it may be used by DEs in accordance with the MCGDM issue. When $¶ = 1$, the compromise solution focuses on maximising the group utility, whereas when $¶ = 0$, the compromise solution focuses on minimising individual regret.

Step 7: Compromise solution and ranking of alternatives:
In this stage, the alternatives are arranged in increasing order based on the group utility measure, the individual regret measure, and the VIKOR index. Here, we acquire three rating lists that can be used to assess the compromise solution.
The alternatives listed at the first and second position with regard to $\mathscr{V}$ are indicated by $\mathcal{O}l^{(1)}$ and $\mathcal{O}l^{(2)}$, respectively, for further assessments. The compromise solution includes alternative $\mathcal{O}l^{(1)}$ if the following requirements are met:

Requirement 1 Acceptable advantage: $\mathscr{V}\left(\mathcal{O}l^{(2)}\right) - \mathscr{V}\left(\mathcal{O}l^{(1)}\right) \geq \frac{1}{m-1}$, where m is the number of alternatives.

Requirement 2 Acceptable stability in decision making: The alternative $\mathcal{O}l^{(1)}$ is also at the top of

TABLE 1. Assessment information provided by D_k.

	$\mathcal{C}r_1$	$\mathcal{C}r_2$	$\cdots$	$\mathcal{C}r_n$
$\mathcal{O}l_1$	$\langle\sigma_{11}^k, \vartheta_{11}^k, \varrho_{11}^k\rangle$	$\langle\sigma_{12}^k, \vartheta_{12}^k, \varrho_{12}^k\rangle$	$\cdots$	$\langle\sigma_{1n}^k, \vartheta_{1n}^k, \varrho_{1n}^k\rangle$
$\mathcal{O}l_2$	$\langle\sigma_{21}^k, \vartheta_{21}^k, \varrho_{21}^k\rangle$	$\langle\sigma_{22}^k, \vartheta_{22}^k, \varrho_{22}^k\rangle$	$\cdots$	$\langle\sigma_{2n}^k, \vartheta_{2n}^k, \varrho_{2n}^k\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\mathcal{O}l_m$	$\langle\sigma_{m1}^k, \vartheta_{m1}^k, \varrho_{m1}^k\rangle$	$\langle\sigma_{m2}^k, \vartheta_{m2}^k, \varrho_{m2}^k\rangle$	$\cdots$	$\langle\sigma_{mn}^k, \vartheta_{mn}^k, \varrho_{mn}^k\rangle$

TABLE 2. Aggregated r, s, t-spherical fuzzy decision matrix: $M = \left(g_{ij}\right)_{m\times n}$.

	$\mathcal{C}r_1$	$\mathcal{C}r_2$	$\cdots$	$\mathcal{C}r_n$
$\mathcal{O}l_1$	$\langle\sigma_{11}, \vartheta_{11}, \varrho_{11}\rangle$	$\langle\sigma_{12}, \vartheta_{12}, \varrho_{12}\rangle$	$\cdots$	$\langle\sigma_{1n}, \vartheta_{1n}, \varrho_{1n}\rangle$
$\mathcal{O}l_2$	$\langle\sigma_{21}, \vartheta_{21}, \varrho_{21}\rangle$	$\langle\sigma_{22}, \vartheta_{22}, \varrho_{22}\rangle$	$\cdots$	$\langle\sigma_{2n}, \vartheta_{2n}, \varrho_{2n}\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\mathcal{O}l_m$	$\langle\sigma_{m1}, \vartheta_{m1}, \varrho_{m1}\rangle$	$\langle\sigma_{m2}, \vartheta_{m2}, \varrho_{m2}\rangle$	$\cdots$	$\langle\sigma_{mn}, \vartheta_{mn}, \varrho_{mn}\rangle$

the list with regard to either $\mathscr{S}$ or $\mathscr{R}(\mathscr{S}$ and $\mathscr{R})$. This compromise solution is stable within a decision-making procedure: "voting by majority rule" (if $\P > 0.5$), "by consensus" (if $\P = 0.5$), or "with veto" (if $\P < 0.5$).

If any of the requirements are not met, the following compromise solutions are proposed:

- The compromise solution set consists of both $\mathcal{O}l^{(1)}$ and $\mathcal{O}l^{(2)}$ if only **Requirement 2** is not met.
- If the **Requirement 1** is not satisfied, the set of compromise solutions consists of $\mathcal{O}l^{(1)}$, $\mathcal{O}l^{(2)}$,…,$\mathcal{O}l^{(v)}$, where $\mathcal{O}l^{(v)}$ is determined by the inequality $\mathscr{V}\left(\mathcal{O}l^{(v)}\right) - \mathscr{V}\left(\mathcal{O}l^{(1)}\right) < \frac{1}{m-1}$, for maximum v.

VI. CASE STUDY

In this part, a case study regarding the robot selection problem (adopted from [42]) is presented to demonstrate the decision procedure of the provided **r**, **s**, **t**-spherical fuzzy VIKOR approach.

The most apparent embodiment of modern welding technology is robotic welding. The initial generation of robotic welding systems used a two-pass weld process, with the first pass devoted to learning the geometry of the seam and the second run dedicated to tracking and welding the seam. The second generation of robotic welding systems used technology breakthroughs to monitor seams in real-time while simultaneously learning and tracking seams. Third-generation robotic welding systems are the most sophisticated in robotic welding technology since they operate in real-time and comprehend the seam's rapidly changing shape while functioning in disorganized environments. According to the selection of industrial arc welding robots, higher product quality requirements should result in cheaper costs and a more trustworthy weld. Robots may be characterized by their mass density, duplicability, cargo capacity, maximum reach, average energy consumption, and motion. All of these factors must be considered while selecting robots for a certain application. Based on the workspace geometry, the most common form of robot for industrial robotic arc welding is one with a revolute (or jointed arm) configuration. This research investigates the selection of industrial robots for arc welding processes using the VIKOR method. Four separate robots with five programmable axes and different controllers from their respective manufacturers were used to collect the data for arc welding robots. These nine robots are allocated five characteristics. After reviewing several datasheets provided by robot manufacturers to define their products, the selection criteria were assessed. Additionally, the views of industry specialists are considered. The criteria for selection were determined after a debate between the research group and an industry expert. The final selection matrix was evaluated based on the consensus of both groups, with the major characteristics of each robot serving as assessment criteria. Table 3 lists the five most significant factors to consider when selecting an arc welding robot:

The set of four alternatives $\mathcal{O}l = \{\mathcal{O}l_1, \mathcal{O}l_2, \mathcal{O}l_3, \mathcal{O}l_4\}$ is reviewed by three DEs $D = \{D_1, D_2, D_3\}$, which are comprised of experienced engineers and consumers in the assessment stage and with weights $w = (0.3, 0.4, 0.3)^T$. The three DEs employ the five criteria listed in Table 3 to determine the optimal additive manufacturing choices for the linear delta robot.

A. THE DECISION-MAKING STEPS

In this part, we use the framed VIKOR approach to choose the optimal solution. The computational steps are as follows:

Step 1: The assessment information of the three DEs D_k $(k = 1, 2, 3)$ are shown in Tables 4-6.

Step 2: With the aid of Eq. (11), the original decision matrices 4-6 are aggregated using the DEs' weight $w = (0.3, 0.4, 0.3)^T$ into a single decision matrix, which is depicted in Table 7.

Step 3: The computed values of criteria weights using the maximizing deviation (outlined in Section V-A) are given as follows:
$\varpi_1 = 0.1478$, $\varpi_2 = 0.2000$, $\varpi_3 = 0.2950$, $\varpi_4 = 0.2063$, $\varpi_5 = 0.1509$.

TABLE 3. Description of evaluation criteria.

Criteria	Description
Weight density	This criteria takes into consideration the robot's physical weight. Consumers often want lightweight robots. Typically, the weight density is represented in kg ($\mathcal{C}r_1$).
Replicability	This refers to a robot's capacity to do a job repeatedly. More replication is often desired. Typically, replication is measured in millimetres ($\mathcal{C}r_2$).
Freight capacity	Freight capacity refers to the maximum total weight that a robot can lift in a single turn. Being more is often favoured. Typically, the weight density is stated in killogrammes ($\mathcal{C}r_3$).
Maximum reach	This is the mean of the greatest vertical and horizontal lengths a robot arm can extend to accomplish a job. It is typical to want to achieve more. Typically, the robot's greatest reach is measured in millimetres ($\mathcal{C}r_4$).
Motion of a robot	Robot motion refers to the movement of a robot at a reference point near the end effector's tip. Trajectory, velocity, acceleration, and acceleration derivative are often used to describe the motion of robots. Typically, the velocity of a robot is represented in meter per second or meter per second square ($\mathcal{C}r_5$).

TABLE 4. Evaluation matrix provided by D_1.

	$\mathcal{C}r_1$	$\mathcal{C}r_2$	$\mathcal{C}r_3$	$\mathcal{C}r_4$	$\mathcal{C}r_5$
$\mathcal{O}l_1$	$\langle 0.6, 0.3, 0.4\rangle$	$\langle 0.5, 0.2, 0.6\rangle$	$\langle 0.9, 0.4, 0.3\rangle$	$\langle 0.7, 0.6, 0.5\rangle$	$\langle 0.4, 0.5, 0.4\rangle$
$\mathcal{O}l_2$	$\langle 0.6, 0.3, 0.7\rangle$	$\langle 0.8, 0.5, 0.6\rangle$	$\langle 0.5, 0.4, 0.5\rangle$	$\langle 0.4, 0.2, 0.6\rangle$	$\langle 0.6, 0.4, 0.5\rangle$
$\mathcal{O}l_3$	$\langle 0.7, 0.5, 0.3\rangle$	$\langle 0.6, 0.4, 0.5\rangle$	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.4, 0.3, 0.5\rangle$	$\langle 0.5, 0.3, 0.5\rangle$
$\mathcal{O}l_4$	$\langle 0.5, 0.5, 0.6\rangle$	$\langle 0.4, 0.3, 0.7\rangle$	$\langle 0.8, 0.4, 0.5\rangle$	$\langle 0.6, 0.5, 0.3\rangle$	$\langle 0.7, 0.6, 0.5\rangle$

TABLE 5. Evaluation matrix provided by D_2.

	$\mathcal{C}r_1$	$\mathcal{C}r_2$	$\mathcal{C}r_3$	$\mathcal{C}r_4$	$\mathcal{C}r_5$
$\mathcal{O}l_1$	$\langle 0.5, 0.3, 0.4\rangle$	$\langle 0.4, 0.2, 0.5\rangle$	$\langle 0.8, 0.4, 0.3\rangle$	$\langle 0.6, 0.6, 0.4\rangle$	$\langle 0.4, 0.5, 0.5\rangle$
$\mathcal{O}l_2$	$\langle 0.5, 0.3, 0.6\rangle$	$\langle 0.7, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.5\rangle$	$\langle 0.5, 0.2, 0.6\rangle$	$\langle 0.5, 0.4, 0.6\rangle$
$\mathcal{O}l_3$	$\langle 0.6, 0.5, 0.4\rangle$	$\langle 0.5, 0.4, 0.6\rangle$	$\langle 0.3, 0.6, 0.7\rangle$	$\langle 0.4, 0.3, 0.5\rangle$	$\langle 0.4, 0.3, 0.5\rangle$
$\mathcal{O}l_4$	$\langle 0.5, 0.6, 0.5\rangle$	$\langle 0.4, 0.7, 0.3\rangle$	$\langle 0.7, 0.4, 0.6\rangle$	$\langle 0.6, 0.5, 0.4\rangle$	$\langle 0.6, 0.5, 0.6\rangle$

TABLE 6. Evaluation matrix provided by D_3.

	$\mathcal{C}r_1$	$\mathcal{C}r_2$	$\mathcal{C}r_3$	$\mathcal{C}r_4$	$\mathcal{C}r_5$
$\mathcal{O}l_1$	$\langle 0.5, 0.4, 0.3\rangle$	$\langle 0.6, 0.3, 0.6\rangle$	$\langle 0.7, 0.4, 0.3\rangle$	$\langle 0.6, 0.5, 0.5\rangle$	$\langle 0.5, 0.4, 0.4\rangle$
$\mathcal{O}l_2$	$\langle 0.7, 0.6, 0.3\rangle$	$\langle 0.8, 0.5, 0.6\rangle$	$\langle 0.5, 0.4, 0.5\rangle$	$\langle 0.6, 0.4, 0.2\rangle$	$\langle 0.6, 0.5, 0.4\rangle$
$\mathcal{O}l_3$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.4, 0.6, 0.3\rangle$	$\langle 0.4, 0.4, 0.5\rangle$	$\langle 0.3, 0.4, 0.5\rangle$
$\mathcal{O}l_4$	$\langle 0.6, 0.5, 0.5\rangle$	$\langle 0.4, 0.4, 0.8\rangle$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.6, 0.6, 0.2\rangle$	$\langle 0.7, 0.5, 0.5\rangle$

TABLE 7. Aggregated r, s, t-spherical fuzzy decision matrix.

	$\mathcal{C}r_1$	$\mathcal{C}r_2$	$\mathcal{C}r_3$	$\mathcal{C}r_4$	$\mathcal{C}r_5$
$\mathcal{O}l_1$	$\langle 0.5337, 0.3270, 0.3669\rangle$	$\langle 0.5017, 0.2259, 0.5578\rangle$	$\langle 0.8207, 0.4000, 0.3000\rangle$	$\langle 0.6354, 0.5681, 0.4573\rangle$	$\langle 0.4337, 0.4676, 0.4373\rangle$
$\mathcal{O}l_2$	$\langle 0.6025, 0.3693, 0.5104\rangle$	$\langle 0.7665, 0.5000, 0.6000\rangle$	$\langle 0.4639, 0.4373, 0.5000\rangle$	$\langle 0.5100, 0.2462, 0.4315\rangle$	$\langle 0.5640, 0.4277, 0.5030\rangle$
$\mathcal{O}l_3$	$\langle 0.6645, 0.4290, 0.3923\rangle$	$\langle 0.6025, 0.4277, 0.5030\rangle$	$\langle 0.3642, 0.6000, 0.5651\rangle$	$\langle 0.4000, 0.3270, 0.5000\rangle$	$\langle 0.4103, 0.3270, 0.5000\rangle$
$\mathcal{O}l_4$	$\langle 0.5337, 0.5378, 0.5281\rangle$	$\langle 0.4000, 0.4590, 0.5192\rangle$	$\langle 0.7362, 0.3669, 0.5378\rangle$	$\langle 0.6000, 0.5281, 0.2980\rangle$	$\langle 0.6656, 0.5281, 0.5378\rangle$

Step 4: In this step, positive ideal solution $\mathcal{O}l^+$ and negative ideal solution $\mathcal{O}l^-$ are obtained using Eqs. (28) and (29), respectively. It is worth mention that in the considered problem, only criteria $\mathcal{C}r_1$ is of the cost type, while the rest $\mathcal{C}r_i (i = 2, 3, \ldots, 5)$ are of the benefit type, as shown in the equation at the bottom of the next page.

Step 5: In the light of Eqs. (30) and (31), the values of $\mathscr{S}_i$ and $\mathscr{R}_i$ are computed for each alternative as given below:

$$\begin{aligned}\mathscr{S}_1 &= 0.2000, \mathscr{S}_2 = 0.6986,\\ \mathscr{S}_3 &= 0.9494, \mathscr{S}_4 = 0.8231,\\ \mathscr{R}_1 &= 0.2000, \mathscr{R}_2 = 0.2063,\\ \mathscr{R}_3 &= 0.2950, \mathscr{R}_4 = 0.2464.\end{aligned}$$

Step 6: According to Eq. (32), obtain the VIKOR index $\mathscr{V}_i$ (taking ¶ $= 0.5$) of each alternative is obtained as listed below:

$$\mathscr{V}_1 = 0, \mathscr{V}_2 = 0.3658, \mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.6599,$$

Step 7: Ranking the alternatives according to the values of $\mathscr{S}_i$, $\mathscr{R}_i$ and $\mathscr{V}_i$ in ascending order, the results are demonstrated as follows:

$$\begin{aligned}\mathcal{O}l^{(1)} &= \mathscr{V}_1 = 0,\\ \mathcal{O}l^{(2)} &= \mathscr{V}_2 = 0.3658,\\ &\quad \mathscr{V}_4 = 0.6599, \mathscr{V}_3 = 1.000,\\ \mathscr{V}\left(\mathcal{O}l^{(2)}\right) - \mathscr{V}\left(\mathcal{O}l^{(1)}\right) &= 0.3658 > 1/m - 1\end{aligned}$$

TABLE 8. Sensitivity analysis results w.r.t r^*.

$r^* = 4$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.7978, \mathscr{S}_2 = 1.2816,$ $\mathscr{S}_3 = 1.9409, \mathscr{S}_4 = 1.2685$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_2 > \mathcal{O}l_3$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.4334, \mathscr{R}_2 = 0.4113,$ $\mathscr{R}_3 = 0.5947, \mathscr{R}_4 = 0.5009$	$\mathcal{O}l_2 > \mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1, \mathcal{O}l_2\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.1233, \mathscr{V}_2 = 0.2116,$ $\mathscr{V}_3 = 1.523, \mathscr{V}_4 = 0.7059$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
$r^* = 6$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.2000, \mathscr{S}_2 = 0.7320,$ $\mathscr{S}_3 = 0.9965, \mathscr{S}_4 = 1.0052$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_3 > \mathcal{O}l_4$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2063,$ $\mathscr{R}_3 = 0.2950, \mathscr{R}_4 = 0.3030$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_3 > \mathcal{O}l_4$	$\{\mathcal{O}l_1\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.3609,$ $\mathscr{V}_3 = 0.9558, \mathscr{V}_4 = 1.000$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_3 > \mathcal{O}l_4$	
$r^* = 8$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.2000, \mathscr{S}_2 = 0.7476,$ $\mathscr{S}_3 = 1.0196, \mathscr{S}_4 = 1.0627$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_3 > \mathcal{O}l_4$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2063,$ $\mathscr{R}_3 = 0.2950, \mathscr{R}_4 = 0.3777$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_3 > \mathcal{O}l_4$	$\{\mathcal{O}l_1\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.3518,$ $\mathscr{V}_3 = 0.7673, \mathscr{V}_4 = 1.026$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_3 > \mathcal{O}l_4$	
$r^* = 10$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.2000, \mathscr{S}_2 = 0.8815,$ $\mathscr{S}_3 = 1.2410, \mathscr{S}_4 = 1.2175$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2950,$ $\mathscr{R}_3 = 0.5086, \mathscr{R}_4 = 0.4338$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.8273,$ $\mathscr{V}_3 = 2.124, \mathscr{V}_4 = 1.719$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
$r^* = 12$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.2000, \mathscr{S}_2 = 0.8955,$ $\mathscr{S}_3 = 1.2751, \mathscr{S}_4 = 1.2546$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2950,$ $\mathscr{R}_3 = 0.5419, \mathscr{R}_4 = 0.4676$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.4624,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.8818$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	

$\mathcal{O}l^{(1)}$ is also at the top with respect to $\mathscr{R}$ or/and $\mathscr{S}$. Thus, the compromise solution is $\left\{\mathcal{O}l^{(1)}\right\}$.

B. INFLUENCE OF THE PARAMETER VALUES ON THE MCGDM RESULTS

This section is devoted to perform a sensitivity analysis by changing r^*, s^*, t^* and ¶ in order to examine the robustness of the framed approach.

1) INFLUENCE OF THE PARAMETER r^* ON THE DECISION MAKING RESULTS

Here, we investigate how the parameter r^* impacts the decision-making process. To do this, we fix the values of s^* and t^* while varying the value of r^*. Next, we solve the numerical case described in Section VI employing the propound VIKOR approach for $\mathsf{r}^* = 4, 6, 8, 10,$ and 12. Because 4 is the smallest feasible value of r^* for which all inputs of the DEs shown in Tables 4-6 become r, s, t-SFNs. The values of the revised $\mathscr{S}_i$, $\mathscr{R}_i$, $\mathscr{V}_i$, ranking order and compromise solution of the alternatives for different values of r^* acquired by the presented method have been listed in Table 8. Table 8 demonstrates that different values of the parameter r^* yield different revised $\mathscr{S}_i$, $\mathscr{R}_i$, and $\mathscr{V}_i$, but the rank ordering is almost same. For $\mathsf{r}^* = 4, 10, 12$, the ranking order of the four available alternatives is $\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$ and for $\mathsf{r}^* = 6, 8$, the ranking order is $\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_3 > \mathcal{O}l_4$. Consequently, the sole distinction in the ranking order is between $\mathcal{O}l_3$ and $\mathcal{O}l_4$. For $\mathsf{r}^* = 4, 10, 12,$

$$\mathcal{O}l^{+} = \begin{pmatrix} \langle 0.5337, 0.3270, 0.3669\rangle, \langle 0.7665, 0.5000, 0.6000\rangle, \\ \langle 0.8207, 0.4000, 0.3000\rangle, \langle 0.6354, 0.5681, 0.4573\rangle, \langle 0.4337, 0.4676, 0.4373\rangle \end{pmatrix},$$

$$\mathcal{O}l^{-} = \begin{pmatrix} \langle 0.6645, 0.4290, 0.3923\rangle, \langle 0.5017, 0.2259, 0.5578\rangle, \\ \langle 0.3642, 0.6000, 0.5651\rangle, \langle 0.5100, 0.2462, 0.4315\rangle, \langle 0.5640, 0.4277, 0.5030\rangle \end{pmatrix}.$$

$\mathcal{O}l_4 > \mathcal{O}l_3$ and for $\mathbf{r}^* = 6, 8$, $\mathcal{O}l_3 > \mathcal{O}l_4$. Moreover, for $\mathbf{r}^* = 6, 8, 10, 12$ the only compromise solution is $\mathcal{O}l_1$ while only for $\mathbf{r}^* = 4$, $\mathcal{O}l_2$ is also the compromise solution. Thus, the devised approach is quite stable regarding different values of $\mathbf{r}^*$.

2) INFLUENCE OF THE PARAMETER s^* ON THE DECISION MAKING RESULTS

Here, we investigate the effect of parameters on the decision-making outcomes. To do this, we solve the same numerical example described in Section VI with previous values of $\mathbf{r}^*$ and $\mathbf{t}^*$ and changeable values of s. Consider $\mathbf{s}^* = 4, 6, 8, 10, 12$ and 12 the values of the redesigned $\mathscr{S}_i, \mathscr{R}_i, \mathscr{V}_i$, ranking order, and compromise solution of the alternatives for various values of $\mathbf{s}^*$ derived using the suggested methodology are summarized in 9. From Table 9, we can see that for $\mathbf{s}^* = 4$; the proposed approach is highly sensitive since $\mathcal{O}l_1$ has been moved from the first to the third place. The compromise solution is also extended from $\{\mathcal{O}l_1\}$ to $\{\mathcal{O}l_1, \mathcal{O}l_2, \mathcal{O}l_3\}$. For the remaining cases, i.e., for $\mathbf{s}^* = 6, 8, 10, 12$, we get the same ranking order and compromise solution of the alternatives as in Subsection VI-A. Thus, the parameter $\mathbf{s}^*$ does not affect the overall decision making results except for $\mathbf{s}^* = 4$.

3) INFLUENCE OF THE PARAMETER t^* ON THE DECISION MAKING RESULTS

In this subsection, we study how the parameter $\mathbf{r}^*$ affects the decision-making process. To do this, we fix the previous values of $\mathbf{r}^*$ and $\mathbf{s}^*$ while varying the value of $\mathbf{t}^*$. Next, we work out the numerical case described in Section VI employing the propound VIKOR approach (from Step 4) for $\mathbf{t}^* = 4, 6, 8, 10$, and 12. Because 4 is the smallest feasible value of $\mathbf{t}^*$ for which all inputs of the DEs displayed in Tables 4-6 become $\mathbf{r}, \mathbf{s}, \mathbf{t}$-SFNs. The values of the revised $\mathscr{S}_i$, $\mathscr{R}_i$, $\mathscr{V}_i$, ranking order and compromise solution of the four alternatives for various values of $\mathbf{t}^*$ acquired by the provided method have been depicted in Table 10. Table 10 illustrates that different values of the parameter $\mathbf{t}^*$ yield different revised $\mathscr{S}_i, \mathscr{R}_i$, and $\mathscr{V}_i$. For $\mathbf{t}^* = 4, 6, 8$, the ranking order of the four available alternatives is $\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$ and for $\mathbf{t}^* = 10, 12$, the ranking order is $\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$. For $\mathbf{t}^* = 8$, the only compromise solution is $\mathcal{O}l_1$ while for $\mathbf{t}^* = 4, 10, 12$, the compromise solution consist of $\mathcal{O}l_1$ and $\mathcal{O}l_2$ and for $\mathbf{t}^* = 6$ the compromise solution is $\{\mathcal{O}l_1, \mathcal{O}l_4\}$. Thus for the parameter $\mathbf{t}^*$, the proposed $\mathbf{r}, \mathbf{s}, \mathbf{t}$-spherical fuzzy VIKOR approach is quite sensible.

4) INFLUENCE OF THE PARAMETER ¶ ON THE DECISION MAKING RESULTS

This section discusses in depth the effect of parameter ¶ on model outcomes. The parameter ¶ reflects the degree of preference of DEs regarding utility measure and individual regret measure. When ¶ $= 0.5$, DEs consider the difference in utility measure and the difference in individual regret measure to be of equal importance. Here, we analyze this situation for various possible values of ¶.

From Table 11, we can see that ¶ $= 0.1, 0.2, 0.3, 0.4$, alternative $\mathcal{O}l_1$ is the optimal ranked by $\mathscr{V}$ and satisfies the **Requirement 2**, but it does not meet the **Requirement 1** i.e., $\mathscr{V}\left(\mathcal{O}l^{(2)}\right) - \mathscr{V}\left(\mathcal{O}l^{(1)}\right) < \frac{1}{m-1}$. Here, because of $\mathscr{V}\left(\mathcal{O}l^{(3)}\right) - \mathscr{V}\left(\mathcal{O}l^{(1)}\right) \geq \frac{1}{m-1}$, set of compromise solution consist of $\mathcal{O}l_1$ and $\mathcal{O}l_2$. The reason is that in these cases, DEs pay enough attention to the difference in individual regret measures and are less concerned about utility measure.

Table 11 further illustrates that when $0.5 \leq$ ¶ ≤ 1, the raking result attained by the proposed VIKOR approach is $\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$ which is the same as that obtained for $0 \leq$ ¶ < 0.5. Hoverer, the compromise consist of just $\mathcal{O}l_1$. This is because, when ¶ $= 0.5, 0.6, \ldots, 1.0$, alternative $\mathcal{O}l_1$ is the top-ranked by $\mathscr{V}$ and also the top-ranked by $\mathscr{S}$ and/or $\mathscr{R}$. In addition, $\mathcal{O}l_1$ meets the following requirements:

$$\begin{aligned}
\mathscr{V}_{\P=0.5}(\mathcal{O}l_2) - \mathscr{V}_{\P=0.5}(\mathcal{O}l_1) &= 0.3658 - 0.000 \\
&= 0.3658 > \tfrac{1}{3-1}, \\
\mathscr{V}_{\P=0.6}(\mathcal{O}l_2) - \mathscr{V}_{\P=0.6}(\mathcal{O}l_1) &= 0.4257 - 0.000 \\
&= 0.4257 > \tfrac{1}{3-1}, \\
\mathscr{V}_{\P=0.7}(\mathcal{O}l_2) - \mathscr{V}_{\P=0.7}(\mathcal{O}l_1) &= 0.4856 - 0.000 \\
&= 0.4856 > \tfrac{1}{3-1}, \\
\mathscr{V}_{\P=0.8}(\mathcal{O}l_2) - \mathscr{V}_{\P=0.8}(\mathcal{O}l_1) &= 0.5455 - 0.000 \\
&= 0.5455 > \tfrac{1}{3-1}, \\
\mathscr{V}_{\P=0.9}(\mathcal{O}l_2) - \mathscr{V}_{\P=0.9}(\mathcal{O}l_1) &= 0.6054 - 0.000 \\
&= 0.6054 > \tfrac{1}{3-1}, \\
\mathscr{V}_{\P=1.0}(\mathcal{O}l_2) - \mathscr{V}_{\P=1.0}(\mathcal{O}l_1)\, 0.6653 - 0.000 & \\
&= 0.6653 > \tfrac{1}{3-1}.
\end{aligned}$$

Thereby, $\mathcal{O}l_1$ is the compromise solution for these six cases.

The results listed in Table 11 show that when the parameter ¶ changes, the ranking results remain the same. Thus the framed approach has good stability regarding the parameter ¶. Though for ¶ < 0.5, the compromise solution is changed. This change reflects the influence of different preferences of DEs on the decision outcomes. Therefore, the parameter ¶ is necessary, and the developed VIKOR approach is reasonable.

VII. COMPARATIVE STUDY

To validate the predictability and efficacy of the proposed MCGDM method, we compare it to previous decision-making approaches [43], [44]. We apply the t-spherical fuzzy TOPSIS technique [43] and the t-spherical fuzzy MULTI-MOORA technique [44] to the problem presented in Section VI and compare the results obtained to those of the framed method.

A. COMPARISON WITH TOPSIS METHOD

This part employs the prevailing technique [43], namely TOPSIS, on the data provided in Table 7.

The t-spherical fuzzy TOPSIS technique comprises the following steps:

TABLE 9. Sensitivity analysis results w.r.t s*.

$s^* = 4$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.6531, \mathscr{S}_2 = 0.65734, \mathscr{S}_3 = 0.65342, \mathscr{S}_4 = 1.1491$	$\mathcal{O}l_1 > \mathcal{O}l_3 > \mathcal{O}l_2 > \mathcal{O}l_4$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.4531, \mathscr{R}_2 = 0.2124, \mathscr{R}_3 = 0.2950, \mathscr{R}_4 = 0.5137$	$\mathcal{O}l_2 > \mathcal{O}l_3 > \mathcal{O}l_1 > \mathcal{O}l_4$	$\{\mathcal{O}l_1, \mathcal{O}l_2, \mathcal{O}l_3\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.2731, \mathscr{V}_2 = 0.004274, \mathscr{V}_3 = 0.09404, \mathscr{V}_4 = 0.8418$	$\mathcal{O}l_2 > \mathcal{O}l_3 > \mathcal{O}l_1 > \mathcal{O}l_4$	
$s^* = 6$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.2000, \mathscr{S}_2 = 0.68074, \mathscr{S}_3 = 0.98057, \mathscr{S}_4 = 0.8177$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2255, \mathscr{R}_3 = 0.2950, \mathscr{R}_4 = 0.2464$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.4422, \mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.6399$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
$s^* = 8$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.2000, \mathscr{S}_2 = 0.69072, \mathscr{S}_3 = 0.99658, \mathscr{S}_4 = 0.8311$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2322, \mathscr{R}_3 = 0.3435, \mathscr{R}_4 = 0.2509$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.4202, \mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.5735$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
$s^* = 10$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.2000, \mathscr{S}_2 = .69447, \mathscr{S}_3 = 1.00233, \mathscr{S}_4 = 0.8353$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2351, \mathscr{R}_3 = 0.3495, \mathscr{R}_4 = 0.2523$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.4255, \mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.5708$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
$s^* = 12$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.2000, \mathscr{S}_2 = 0.69581, \mathscr{S}_3 = 1.00424, \mathscr{S}_4 = 0.8362$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2362, \mathscr{R}_3 = 0.3515, \mathscr{R}_4 = 0.2524$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.4277, \mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.5685$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	

Step 1. Normalized the decision matrix based on the nature of each criteria as follows:

$$\mathcal{S}_{ij} = \langle \sigma_{ij}, \vartheta_{ij}, \varrho_{ij} \rangle = \begin{cases} \langle \sigma_{ij}, \vartheta_{ij}, \varrho_{ij} \rangle, & \text{for benefit criteria} \\ \langle \varrho_{ij}, \vartheta_{ij}, \sigma_{ij} \rangle, & \text{for cost criteria} \end{cases}; \quad j = 1, 2, \ldots, n, \tag{33}$$

Based on Eq. (33), the data of Table 7 is normalized and is arranged in Table 12.

Step 2: Determine the criteria weights according to Eq. (34)

$$\varpi_j = \frac{1 - \mathbb{H}_j}{n - \sum_{j=1}^{n} \mathbb{H}_j}, \tag{34}$$

where $\mathbb{H}_j \in [0, 1], j = 1, 2, \ldots, n$ is defined as

$$\mathbb{H}_j = \frac{1}{m} \sum_{i=1}^{m} \left\{ \begin{array}{l} \sin\left(\frac{\pi \times \left(2 + \sigma_{ij}^t - \vartheta_{ij}^t - \varrho_{ij}^t\right)}{8}\right) + \\ \sin\left(\frac{\pi \times \left(2 - \sigma_{ij}^t + \vartheta_{ij}^t + \varrho_{ij}^t\right)}{8}\right) - 1 \end{array} \right\} \times \frac{1}{2^{\frac{1}{t}} - 1}. \tag{35}$$

According to formulation (34), we work out the weights of five criteria as given below.

$\varpi_1 = 0.1998, \varpi_2 = 0.2004, \varpi_3 = 0.1988, \varpi_4 = 0.2005, \varpi_5 = 0.2005$.

Step 3: Find positive ideal solution (PIS) and negative ideal solution (NIS), according to Eqs. (36) and (37), respectively.

$$S^+ = \left(s_{i1}^+, s_{i2}^+, \ldots, s_{in}^+\right), \tag{36}$$

where $S^+ = \left(\max\left(\sigma_{ij}\right), \min\left(\vartheta_{ij}\right), \min\left(\varrho_{ij}\right)\right)$.

$$S^- = \left(s_{i1}^-, s_{i2}^-, \ldots, s_{in}^-\right), \tag{37}$$

where $S^- = \left(\min\left(\sigma_{ij}\right), \min\left(\vartheta_{ij}\right), \max\left(\varrho_{ij}\right)\right)$.

Thus, we have, as shown in the equation at the bottom of the next page.

Step 4: Determine the alternatives measure from PIS by the following formula, as in (38), shown at the bottom of the next page.

Also, determine the alternatives measure from NIS by the following formula, as in (39), shown at the bottom of the next page.

TABLE 10. Sensitivity analysis results w.r.t t*.

$t^* = 4$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.27382, \mathscr{S}_2 = 1.1015,$ $\mathscr{S}_3 = 1.0560, \mathscr{S}_4 = .55996$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.3922,$ $\mathscr{R}_3 = 0.2950, \mathscr{R}_4 = 0.2505$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$	$\{\mathcal{O}l_1, \mathcal{O}l_2\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 1.000,$ $\mathscr{V}_3 = 0.7197, \mathscr{V}_4 = 0.3042$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$	
$t^* = 6$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.26116, \mathscr{S}_2 = 1.02456,$ $\mathscr{S}_3 = 0.9731, \mathscr{S}_4 = 0.44648$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.3894,$ $\mathscr{R}_3 = 0.2950, \mathscr{R}_4 = 0.2508$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$	$\{\mathcal{O}l_1, \mathcal{O}l_4\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 1.000,$ $\mathscr{V}_3 = 0.7171, \mathscr{V}_4 = 0.2555$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$	
$t^* = 8$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.21367, \mathscr{S}_2 = 0.91090,$ $\mathscr{S}_3 = 0.8934, \mathscr{S}_4 = 0.43327$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.1683, \mathscr{R}_2 = 0.3254,$ $\mathscr{R}_3 = 0.2950, \mathscr{R}_4 = 0.2501$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$	$\{\mathcal{O}l_1\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 1.000,$ $\mathscr{V}_3 = 0.8907, \mathscr{V}_4 = 0.4178$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_3 > \mathcal{O}l_2$	
$t^* = 10$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.25698, \mathscr{S}_2 = 0.62850,$ $\mathscr{S}_3 = 0.9229, \mathscr{S}_4 = 0.59242$	$\mathcal{O}l_1 > \mathcal{O}l_4 > \mathcal{O}l_2 > \mathcal{O}l_3$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2063,$ $\mathscr{R}_3 = 0.2950, \mathscr{R}_4 = 0.2495$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1, \mathcal{O}l_2\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.3121,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.5124$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
$t^* = 12$	Indexes	Computed values	Preferential sequences	Compromise solution
	$\mathscr{S}_i$	$\mathscr{S}_1 = 0.25683, \mathscr{S}_2 = 0.62826,$ $\mathscr{S}_3 = 0.9218, \mathscr{S}_4 = 0.59052$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	
	$\mathscr{R}_i$	$\mathscr{R}_1 = 0.2000, \mathscr{R}_2 = 0.2063,$ $\mathscr{R}_3 = 0.2950, \mathscr{R}_4 = 0.2493$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1, \mathcal{O}l_2\}$
	$\mathscr{V}_i$	$\mathscr{V}_1 = 0.0000, \mathscr{V}_2 = 0.3124,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.5104$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	

Using Eq. (38), the measure between the alternatives and PIS is calculated as follows:

$$K_1\left(g_{1j}, S^+\right) = 0.3307,\ K_2\left(g_{2j}, S^+\right) = 0.3837,$$
$$K_3\left(g_{3j}, S^+\right) = 0.2609,\ K_4\left(g_{4j}, S^+\right) = 0.4012.$$

And the measure between the alternatives and NIS based on Eq. (39), is calculated as follows:

$$K_1\left(g_{1j}, S^-\right) = 0.2174,\ K_2\left(g_{2j}, S^-\right) = 0.2942,$$
$$K_3\left(g_{3j}, S^-\right) = 0.3259,\ K_4\left(g_{4j}, S^-\right) = 0.2569.$$

$$S^+ = \begin{pmatrix} \langle 0.5281, 0.3270, 0.5337\rangle, \langle 0.7665, 0.2259, 0.5030\rangle, \\ \langle 0.8207, 0.3669, 0.3000\rangle, \langle 0.6354, 0.2462, 0.2980\rangle, \langle 0.6656, 0.3270, 0.4373\rangle \end{pmatrix}$$

and

$$S^- = \begin{pmatrix} \langle 0.3669, 0.3270, 0.6645\rangle, \langle 0.4000, 0.2259, 0.6000\rangle, \\ \langle 0.3642, 0.3669, 0.5651\rangle, \langle 0.4000, 0.2462, 0.5000\rangle, \langle 0.4103, 0.3270, 0.5378\rangle \end{pmatrix}$$

$$K_i\left(g_{ij}, S^+\right) = \frac{\sum_{j=1}^{n} \varpi_j \left(\sigma_{ij}^t \sigma_{ij}^{+t}, \vartheta_{ij}^t \vartheta_{ij}^t, \varrho_{ij}^t \varrho_{ij}^{+t}\right)}{\left(\left(\sum_{j=1}^{n} \varpi_j \left(\sigma_{ij}^{2t}, \vartheta_{ij}^{2t}, \varrho_{ij}^t\right)\right)\left(\sum_{j=1}^{n} \varpi_j \left(\sigma_{ij}^{+2t}, \vartheta_{ij}^{+2t}, \varrho_{ij}^{+2t}\right)\right)\right)^{\frac{1}{n}}} \tag{38}$$

$$K_i\left(g_{ij}, S^-\right) = \frac{\sum_{j=1}^{n} \varpi_j \left(\sigma_{ij}^t \sigma_{ij}^{-t}, \vartheta_{ij}^t \vartheta_{ij}^t, \varrho_{ij}^t \varrho_{ij}^{-t}\right)}{\left(\left(\sum_{j=1}^{n} \varpi_j \left(\sigma_{ij}^{2t}, \vartheta_{ij}^{2t}, \varrho_{ij}^t\right)\right)\left(\sum_{j=1}^{n} \varpi_j \left(\sigma_{ij}^{-2t}, \vartheta_{ij}^{-2t}, \varrho_{ij}^{-2t}\right)\right)\right)^{\frac{1}{n}}} \tag{39}$$

Step 5: Determine the closeness of each alternative utilizing Eq. (40).

$$\mathcal{Q}_i = \frac{K_i\left(g_{ij}, S^{+}\right)}{K_i\left(g_{ij}, S^{+}\right) + K_i\left(g_{ij}, S^{-}\right)} \tag{40}$$

Using Eq. (40), we find the closeness coefficient of each alternative as given below:
$\mathcal{Q}_1 = 0.6034$, $\mathcal{Q}_2 = 0.5660$,
$\mathcal{Q}_3 = 0.4446$, $\mathcal{Q}_4 = 0.6096$.

Step 6: Based on the closeness coefficients, rank the alternatives in decreasing order:

$$\mathcal{O}l_4 > \mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_3.$$

B. COMPARISON WITH MULTIMOORA METHOD

This section aims to address the considered problem via t-spherical fuzzy MULTIMOORA [44] described in the below phase.

Step 1: Normalized the decision matrix based on the nature of each criteria as follows:

$$\mathcal{S}_{ij} = \left\langle \sigma_{ij}, \vartheta_{ij}, \varrho_{ij} \right\rangle = \begin{cases} \left\langle \sigma_{ij}, \vartheta_{ij}, \varrho_{ij} \right\rangle, & \text{for benefit criteria} \\ \left\langle \varrho_{ij}, \vartheta_{ij}, \sigma_{ij} \right\rangle, & \text{for cost criteria} \end{cases}; \quad j = 1, 2, \ldots, n. \tag{41}$$

Based on Eq. (41), the data of Table 7 is normalized and is arranged in Table 12.

Step 2: Aggregate the normalized data by employing t-spherical fuzzy Dombi prioritized weighted arithmetic (t-SFDPWA) operator given in Eq. (42).

$$\widetilde{\mathcal{S}}_i = t - SFDPWA\left(\mathcal{S}_{i1}, \mathcal{S}_{i2}, \ldots, \mathcal{S}_{in}\right) = \left\langle \begin{array}{c} \sqrt[t]{1 - \dfrac{1}{1 + \left\{ \sum\limits_{j=1}^{n} \left(\dfrac{\varpi_j w_{ij}}{\sum\limits_{j=1}^{n} w_{ij}} \right) \left(\dfrac{\sigma_{ij}^t}{1 - \sigma_{ij}^t} \right)^{\Xi} \right\}^{\frac{1}{\Xi}}}}, \\ \sqrt[t]{1 - \dfrac{1}{1 + \left\{ \sum\limits_{j=1}^{n} \left(\dfrac{\varpi_j w_{ij}}{\sum\limits_{j=1}^{n} w_{ij}} \right) \left(\dfrac{1 - \vartheta_{ij}^t}{\vartheta_{ij}^t} \right)^{\Xi} \right\}^{\frac{1}{\Xi}}}}, \\ \sqrt[t]{1 - \dfrac{1}{1 + \left\{ \sum\limits_{j=1}^{n} \left(\dfrac{\varrho_j w_{ij}}{\sum\limits_{j=1}^{n} w_{ij}} \right) \left(\dfrac{1 - \varrho_{ij}^t}{\varrho_{ij}^t} \right)^{\Xi} \right\}^{\frac{1}{\Xi}}}} \end{array} \right\rangle \tag{42}$$

where Ξ is prioritized parameter, $w_{ij} = \frac{\daleth_{ij}}{\sum_{j=1}^{n} \daleth_{ij}}$, $\daleth_{ij} = \prod_{k=1}^{j-1} \mathcal{C}r_k\left(\mathcal{O}l_i\right)$, $(j = 1, 2, \ldots, n)$, $\daleth_{i1} = 1$, and $\mathcal{C}r_j\left(\mathcal{O}l_i\right)$ is the performance of alternative $\mathcal{O}l_i$ under criteria $\mathcal{C}r_j$.

Using Eq. (42) by taking $\Xi = 2$, the aggregated values are obtained as:

$$\widetilde{\mathcal{S}}_1 = \langle 0.5712, 0.3610, 0.5021 \rangle,$$
$$\widetilde{\mathcal{S}}_2 = \langle 0.5390, 0.4580, 0.6712 \rangle,$$
$$\widetilde{\mathcal{S}}_3 = \langle 0.3885, 0.5462, 0.6994 \rangle,$$
$$\widetilde{\mathcal{S}}_4 = \langle 0.4884, 0.5753, 0.5602 \rangle.$$

Step 3: Apply the score function Eq. (43), on the above aggregated values. $\mathcal{S}_{ij} = \left\langle \sigma_{ij}, \vartheta_{ij}, \varrho_{ij} \right\rangle$

$$S\left(\widetilde{\mathcal{S}}_i\right) = \frac{1 + \widetilde{\sigma}_{ij}^t - \widetilde{\vartheta}_{ij}^t - \widetilde{\varrho}_{ij}^t}{2}; \quad i = 1, 2, \ldots, m, \tag{43}$$

we have $S\left(\widetilde{\mathcal{S}}_1\right) = 0.5064$, $S\left(\widetilde{\mathcal{S}}_2\right) = 0.3791$, $S\left(\widetilde{\mathcal{S}}_3\right) = 0.2768$, $S\left(\widetilde{\mathcal{S}}_4\right) = 0.3751$.

Next, we normalized the above score values via Eq. (44)

$$\widehat{S\left(\mathcal{S}_i\right)} = \frac{S\left(\widetilde{\mathcal{S}}_i\right)}{\max\limits_i S\left(\widetilde{\mathcal{S}}_i\right)}; \quad i = 1, 2, \ldots, m. \tag{44}$$

Using Eq. (44), the normalized score valued are computes as follows:
$\widehat{S\left(g_1\right)} = 1.000$, $\widehat{S\left(g_2\right)} = 0.7486$, $\widehat{S\left(g_3\right)} = 0.5466$, $\widehat{S\left(g_4\right)} = 0.7407$.

Step 4: List the ranking alternatives according to their normalized score values in increasing order.
Based on the normalized score values, the ranking order of alternatives is given as:

$$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3.$$

Step 5: Calculate the weighted distance between t-SFNs and the ideal solution using Eq. (45).

$$d_{ij} = d_H\left(\varpi_j \mathcal{S}_{ij}, \varpi_j s_{ij}^{+}\right); \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n \tag{45}$$

here d_H represent the t-spherical fuzzy Hamming distance. Moreover, for each alternative, the maximum hamming distance from the ideal solution is detected following Eq. (46)

$$d_i = \max_i d_{ij}. \tag{46}$$

According to Eqs. (45) and (46), we get
$d_1 = 0.4764$, $d_2 = 0.3710$, $d_3 = 0.4806$, $d_4 = 0.2420$.

Step 6: Normalize each d_i via Eq. (47)

$$\widehat{d_i} = \frac{\min\limits_i d_i}{d_i} \quad i = 1, 2, \ldots, m. \tag{47}$$

In the light of Eq. (47), we have $\widehat{d_1} = 0.5080$, $\widehat{d_2} = 0.6523$, $\widehat{d_3} = 0.5035$, $\widehat{d_4} = 1.000$.

Step 7: Based on the computed $\widehat{d_i}(i = 1, 2, \ldots, m$, rank the alternatives in decreasing order.
Hence, the ranking order of alternatives is obtained as: $\mathcal{O}l_4 > \mathcal{O}l_2 > \mathcal{O}l_1 > \mathcal{O}l_3$.

Step 8: Aggregate the normalized data by employing t-spherical fuzzy Dombi prioritized weighted geometric (t-SFDPWG) operator given in Eq. (48).

$$\widetilde{\mathcal{S}}_i = t - SFDPWG\left(\mathcal{S}_{i1}, \mathcal{S}_{i2}, \ldots, \mathcal{S}_{in}\right)$$

TABLE 11. Sensitivity analysis results w.r.t ¶.

¶	VIKOR indices	Ranking	Compromise solution
0.1	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.1262,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.5227$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1, \mathcal{O}l_2\}$
0.2	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.1861,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.5570$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1, \mathcal{O}l_2\}$
0.3	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.1262,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.5227$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1, \mathcal{O}l_2\}$
0.4	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.3059,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.6256$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1, \mathcal{O}l_2\}$
0.5	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.3658,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.6599$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
0.6	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.4257,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.6942$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
0.7	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.4856,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.7286$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
0.8	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.5455,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.7629$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
0.9	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.6054,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.7972$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$
1.0	$\mathscr{V}_1 = 0.000, \mathscr{V}_2 = 0.6653,$ $\mathscr{V}_3 = 1.000, \mathscr{V}_4 = 0.8315$	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$	$\{\mathcal{O}l_1\}$

TABLE 12. Normalized r, s, t-spherical fuzzy decision matrix.

	$\mathcal{C}r_1$	$\mathcal{C}r_2$	$\mathcal{C}r_3$	$\mathcal{C}r_4$	$\mathcal{C}r_5$
$\mathcal{O}l_1$	$\langle 0.3669, 0.3270, 0.5337\rangle$	$\langle 0.5017, 0.2259, 0.5578\rangle$	$\langle 0.8207, 0.4000, 0.3000\rangle$	$\langle 0.6354, 0.5681, 0.4573\rangle$	$\langle 0.4337, 0.4676, 0.4373\rangle$
$\mathcal{O}l_2$	$\langle 0.5104, 0.3693, 0.6025\rangle$	$\langle 0.7665, 0.5000, 0.6000\rangle$	$\langle 0.4639, 0.4373, 0.5000\rangle$	$\langle 0.5100, 0.2462, 0.4315\rangle$	$\langle 0.5640, 0.4277, 0.5030\rangle$
$\mathcal{O}l_3$	$\langle 0.3923, 0.4290, 0.6645\rangle$	$\langle 0.6025, 0.4277, 0.5030\rangle$	$\langle 0.3642, 0.6000, 0.5651\rangle$	$\langle 0.4000, 0.3270, 0.5000\rangle$	$\langle 0.4103, 0.3270, 0.5000\rangle$
$\mathcal{O}l_4$	$\langle 0.5281, 0.5378, 0.5337\rangle$	$\langle 0.4000, 0.4590, 0.5192\rangle$	$\langle 0.7362, 0.3669, 0.5378\rangle$	$\langle 0.6000, 0.5281, 0.2980\rangle$	$\langle 0.6656, 0.5281, 0.5378\rangle$

$$
= \left\langle
\begin{array}{l}
\sqrt[t]{1 - \dfrac{1}{1 + \left\{ \sum\limits_{j=1}^{n} \left(\dfrac{\varpi_j w_{ij}}{\sum\limits_{j=1}^{n} w_{ij}} \right) \left(\dfrac{1-\sigma_{ij}^t}{\sigma_{ij}^t} \right)^{\Xi} \right\}^{\frac{1}{\Xi}}}}, \\
\sqrt[t]{1 - \dfrac{1}{1 + \left\{ \sum\limits_{j=1}^{n} \left(\dfrac{\varpi_j w_{ij}}{\sum\limits_{j=1}^{n} w_{ij}} \right) \left(\dfrac{\vartheta_{ij}^t}{1-\vartheta_{ij}^t} \right)^{\Xi} \right\}^{\frac{1}{\Xi}}}}, \\
\sqrt[t]{1 - \dfrac{1}{1 + \left\{ \sum\limits_{j=1}^{n} \left(\dfrac{\varrho_j w_{ij}}{\sum\limits_{j=1}^{n} w_{ij}} \right) \left(\dfrac{\varrho_{ij}^t}{1-\varrho_{ij}^t} \right)^{\Xi} \right\}^{\frac{1}{\Xi}}}}
\end{array}
\right\rangle \tag{48}
$$

where Ξ is prioritized parameter, $w_{ij} = \frac{\urcorner_{ij}}{\sum_{j=1}^{n} \urcorner_{ij}}$, $\urcorner_{ij} = \prod_{k=1}^{j-1} \mathcal{C}r_k(\mathcal{O}l_i)$, $(j = 1, 2, \ldots, n)$, $\urcorner_{i1} = 1$, and $\mathcal{C}r_j(\mathcal{O}l_i)$ is the performance of alternative $\mathcal{O}l_i$ under criteria $\mathcal{C}r_j$.

Using Eq. (48) by taking $\Xi = 2$, the aggregated values are obtained as:

$$
\begin{aligned}
\widetilde{\mathcal{S}}_1 &= \langle 0.5301, 0.3210, 0.4025\rangle, \\
\widetilde{\mathcal{S}}_2 &= \langle 0.6471, 0.3356, 0.4553\rangle, \\
\widetilde{\mathcal{S}}_3 &= \langle 0.5200, 0.3812, 0.4932\rangle, \\
\widetilde{\mathcal{S}}_4 &= \langle 0.5990, 0.3903, 0.4060\rangle.
\end{aligned}
$$

Step 9: Apply the score function Eq. (49), on the above aggregated values. $\mathcal{S}_{ij} = \langle \sigma_{ij}, \vartheta_{ij}, \varrho_{ij} \rangle$

$$
S\left(\widetilde{\mathcal{S}}_i\right) = \frac{1 + \widetilde{\sigma}_{ij}^t - \widetilde{\vartheta}_{ij}^t - \widetilde{\varrho}_{ij}^t}{2}; \quad i = 1, 2, \ldots, m, \tag{49}
$$

we have $S\left(\widetilde{\mathcal{S}}_1\right) = 0.5253$, $S\left(\widetilde{\mathcal{S}}_2\right) = 0.5694$, $S\left(\widetilde{\mathcal{S}}_3\right) = 0.4826$, $S\left(\widetilde{\mathcal{S}}_4\right) = 0.5443$.

Next, we normalized the above score values via Eq. (50)

$$
\widehat{S(\mathcal{S}_i)} = \frac{S\left(\widetilde{\mathcal{S}}_i\right)}{\max\limits_{i} S\left(\widetilde{\mathcal{S}}_i\right)}; \quad i = 1, 2, \ldots, m. \tag{50}
$$

Using Eq. (50), the normalized score values are computed as follows:

$\widehat{S(\mathcal{S}_1)} = 0.9226$, $\widehat{S(\mathcal{S}_2)} = 1.000$, $\widehat{S(\mathcal{S}_3)} = 0.8476$, $\widehat{S(\mathcal{S}_4)} = 0.9559$.

Step 10: List the ranking alternatives according to their normalized score values in increasing order.

Based on the normalized score values, the ranking order of alternatives is given as: $\mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_1 > \mathcal{O}l_3$.

TABLE 13. Comparative analysis.

Method	Criteria's weight	multi-parameters	Compromise solution	Ranking
TOPSIS method [44]	Determined by entropy -based method	No	No	$\mathcal{O}l_4 > \mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_3$
MULTIMOORA method [45]	Should be known in advance	No	No	$\mathcal{O}l_4 > \mathcal{O}l_2 > \mathcal{O}l_1 > \mathcal{O}l_3$
Proposed VIKOR method	Determined by maximizing deviation method	Yes	Yes	$\mathcal{O}l_1 > \mathcal{O}l_2 > \mathcal{O}l_4 > \mathcal{O}l_3$

Step 11: Based on the ranking order of alternatives, obtained in Steps 4, 7, and 10, obtain the final ranking order of alternatives by applying dominance theory [45].
Thus, by applying the dominance theory [45], the final ranking is obtained as:

$$\mathcal{O}l_4 > \mathcal{O}l_2 > \mathcal{O}l_1 > \mathcal{O}l_3.$$

C. RESULTS DISCUSSION

In this section, we compare the results of the propound VIKOR approach with the existing ones [43], [44] to further elaborate on the benefits of the established MCGDM technique, as presented in Table 13.

From Table 13, we can observe that the ranking result derived by the framed approach is different from the existing ones. According to the proposed approach, the alternative $\mathcal{O}l_1$ is the optimal alternative, but the ranking results given by [43] and [44] reveal that $\mathcal{O}l_4$ is the most preferable alternative. Further, the difference between the TOPSIS [43], and MULTIMOORA [44] on the ranking result is reflected in the position of alternative $\mathcal{O}l_2$ where the ranking position of $\mathcal{O}l_2$ changes from the second by the TOPSIS method to the third position. These differences may be caused due to the following aspects:

- The existing methods utilize t-spherical fuzzy data in which the term level of σ, ϑ, and ϱ are taken the same. But in practice, the term level of σ, ϑ, and ϱ may be different. For example, the decision information provided by DE D_1 regarding $\mathcal{O}l_4$ with respect to criteria $\mathcal{C}r_5$ is $\langle 0.7, 0.6, 0.5\rangle$. Clearly, $0.7^2 + 0.6^2 + 0.5^2 = 1.1 > 1$. Therefore, we should next check $0.7^3 + 0.6^3 + 0.5^3 = 0.684 < 1$. But since $0.7^3 + 0.6^2 + 0.5^2 = 0.953 < 1$. Thus the situation can be more successfully captured if the term level of σ, ϑ, and ϱ are allowed to be different, as is the proposed method. The proposed method's ability to use various term levels may be the key factor of the ranking difference between it and preexisting t-spherical fuzzy methods.
- The weight vector derived by the proposed maximizing deviation method is $(0.1478, 0.2000, 0.2950, 0.2063, 0.1509)$, whereas the weight vector derived by the Entropy-based method [43] is $(0.1998, 0.2004, 0.1988, 0.2005, 0.2005)$. The weight vector of criteria $\mathcal{C}r_1$, $\mathcal{C}r_3$, and $\mathcal{C}r_5$ obtained by the maximizing deviation method differs significantly from the weight vector computed using an entropy-based method. This is due to the fact that the entropy measure worked in the existing entropy-based model is invalid. It does not take into account the degree of indeterminacy, has various counterintuitive instances, and may produce misleading findings [15]. The decision-making process largely relies on these assigned weights of criteria. Thus, the ranking variation may also be attributable to the varying weights.
- In addition to differences in the obtained weights, the provided method takes into account the weighted distance between decision components. The comparison approach [43] uses unweighted distance measures, which may not effectively depict the differences between decision components and have a negative impact on the final decision findings.

Based on the preceding comparison, the provided method has the following superiorities.

1) The developed approach is based on r, s, t-spherical fuzzy setting. Due to multi-parameters the usage of r, s, t-SFS makes the information more flexible and has the capability to capture the uncertainty more accurately. Many existing structures of FS become special cases of proposed set after certain conditions are added (see III-A). The presence of the parameters enable DEs to provide their assessments regarding any object in a broadened way.
2) Existing methods are based on a single opinion matrix, however the suggested method works with opinion matrices from multiple experts. Moreover, in the previous method [44], the weight of each criterion must be known in advance, but the suggested method uses maximizing deviation model to determine objective criteria weights that improve the reliability of decision results.
3) The proposed methodology not only delivers a solution that is closer to the ideal, but it also achieves a balance between the minimum individual regret for the "opponent" and the maximum group utility of the "majority." Thus, the r, s, t-spherical fuzzy VIKOR gives more practicable results than the existing approaches.

VIII. CONCLUSION

This study initiated the notion of r, s, t-SFSs as an expansion to several preexisting FSs. In r, s, t-SFS, the sum of the rth power of membership grade, the sum of the sth power of neutral grade and the sum of the tth power of nonmembership grade is bounded by 1. The r, s, t-SFS enables DEs to articulate their evaluations more extensively in order to handle ambiguous data more effectively. Keeping these advantages in mind, we came up with basic operational laws and discussed their properties. Based on the devised score and accuracy function, ranking order between two r, s, t-SNs was defined for comparison purposes. Besides r, s, t-spherical fuzzy distance measure was defined, and its required axioms were proved mathematically. In addition, r, s, t-SFWA and r, s, t-SFWG operators were introduced along with their characteristics. Based on the devised fundamental concepts, VIKOR method is extended to r, s, t-spherical fuzzy setting in which criteria weights are determined objectively via developed maximizing deviation model. To illustrate the practicality, a case study related to robot selection using r, s, t-spherical fuzzy data was solved by the developed approach. Meanwhile, detailed sensitive analysis and comparison with previous methods were carried out to elaborate on the robustness and feasibility of the propound approach.

AVAILABILITY OF DATA AND MATERIALS

All data generated or analysed during this study are included in this published article.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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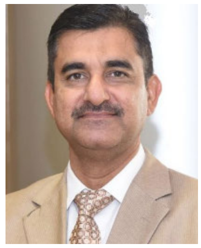

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