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# **RESEARCH ARTICLE**

# State Estimation-Based Hybrid-Triggered Controller Design for Synchronization of Repeated Scalar Nonlinear Complex Dynamical Networks

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**ABSTRACT** The goal of this work is to examine the issue of state estimation-based synchronization of nonlinear complex dynamical networks that are prone to external disturbances, repeated scalar nonlinearities and time-varying coupling delays. In a nutshell, hybrid-triggered communication transmission nonfragile control design with respect to the estimated states and extended dissipative theory is designed, which comprises of both the time and event-triggered mechanisms, and a hybrid generator is presented between the sensor and controller. More preciously, a stochastic variable satisfying the Bernoulli random binary distribution is utilized to represent the phenomenon of random transmission between the time and event-triggered mechanism. Furthermore, delay-dependent adequate conditions for ensuring the synchronization of the addressed system are developed in the form of linear matrix inequalities. And, the requisite gain matrices and hybrid-triggered parameters are evaluated with the help of solving these procured conditions. Lastly, a numerical example with an application to memristor-based Chua's circuit model is demonstrated to ensure the effectiveness of established control technique.

**INDEX TERMS** Complex dynamical networks, repeated scalar nonlinearity, state estimation, non-fragile hybrid-triggered mechanism, extended dissipativity.

# **I. INTRODUCTION**

In control theory, complex dynamical networks (CDNs) have fascinatingly established its influence in real process and retained its ground for the last few decades. In fact, the explosive growth of the technological industry today demands the control of a wide range of networks, including wireless networks, global positioning system services, cellular networks, power grids, transportation and so on [1], [2], [3], [4]. In such a manner, these complex networks are vital and are highly correlated to our day-to-day life. In the general

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sense, CDNs are composed of several inter-linked nodes, wherein, each node signifies each dynamical system and the links meter the communication between them. Moreover, the nodes of the CDNs in this work are considered in nonlinear form. To make it more general, we ought to treat those nonlinearities as a repeated scalar nonlinearity, that encompasses distinct types of nonlinearities such as sine functions, semi-linear functions, hyperbolic tangential functions, etc. For this reason, a collection of relevant study results on repeated scalar nonlinearities is being rendered by the academicians [7], [8], [9], [10]. With this essence, the article [8] explores the question of stability of continuous-time repeated scalar nonlinear Takagi-Sugeno fuzzy systems assisted by

event-triggered  $H_{\infty}$  control protocol. Intriguingly, a design of sliding mode observer has been profoundly inspected for repeated scalar nonlinear systems by the researchers in [10]. Notably, so far in the literature, none of the works addressed the repeated scalar nonlinearity for CDNs. Thence, this CDNs with repeated scalar nonlinear is a critical framework and the acquired technical difficulties are analyzed in this study to enlarge the practical applicability.

In a related note, experts in many disciplines, including sociology, economics, biology, mathematics, etc., have oriented their focus on the synchronization problem of CDNs. This topic has indeed been relevant for years due to its plethora of exciting applications, such as image processing, harmonic oscillation generation, secure communication, heartbeat regulation, routing messages in the internet [5], [6]. Besides, this has the potential to interpret a lot of natural phenomena, notably harmony of flashing fireflies, rhythmic beating of heart cells, etc. As a consequence, many contemporary literature has intensively studied the dynamical behaviour of CDNs and other networks [13], [14], [15], [16], [17], [18], [19], [20], [21]. For instance, the literature [17] examines the synchronization transition behaviour over neuronal networks with average degree, coupling strength and information transmission delay. On another note, the existence of time delays is more common in real-world situations such as in traffic congestions, electrical signal conveyance across long distances and so forth. One cannot be much ignorant in considering time-delays in system design as it may lead to inaccurate analysis. Such time delays can sometimes potentially lead to instability and have an impact on performance of the system [11], [12]. Thus, researches on behaviour of networks in the presence of time delays would be more pertinent and perhaps other exciting works on CDNs with delays have been conducted [23], [24]. More particularly, an admirable work has been made on the Markovian CDNs with time-varying coupling delays and an incomplete transition rates addressing the issue of synchronization in [24]. The implications of aforementioned applications and their significance prompted the discussion of synchronization analysis for repeated scalar nonlinear CDNs (RSNCDNs) with coupling delays in this study, to fill the research gap.

Over and above, in this modern era of technology and communication, the inquiry on control of networks via timetriggered/sampled-data controllers drew the researchers' interest since it is more beneficial in regard to effectiveness, the ease of installation and other factors. Importantly, it assisted in ensuring the proper function of systems even in adverse circumstances like external disturbances, limited channel bandwidth, etc. However, as it periodically communicates the data to the controller regardless of whether the transfer is essential, it may result in needless consumption of communication resources. The challenge of how to tackle this flaw, led to the raise of event-triggered controllers which delivered only the selected sampled data to the controller. This choice is made with the aid of a specified event-triggering condition. As a result of these benefits, countless studies on event-triggered control for networks have been published [25], [26], [27], [28], [29], [30], [31]. The work in [27], aimed to reach the consensus for leaderfollowing multi-agent networks with effective utilization of event-triggered control. Strikingly, event-triggered approach is deployed in discrete time-varying nonlinear complex networks to prevent communication bandwidth wastage and needless executions, followed by quantization of data in [29]. Yet, in real-world scenarios, changes to the network loads may exist which must be considered. Taking these aspects into account and in addition to these kinds of advancements in this area of research, a far more suitable communication method referred to as a hybrid-triggered control scheme has been devised to strike a balance between the mitigation of communication transmission burden and system's performance assurance [32], [33], [34]. These prompted scientists to investigate on this problem for a variety of systems, including Takagi-Sugeno fuzzy systems, neural networks, multiagent systems and so on.

On the contrary, the confrontation of state estimation issue in the circumstances of inaccessibility of system's internal states have been addressed broadly due to its wide applications. Moreover, the states of the system under consideration are well reconstructed with the use of state observer and system monitoring, fault detection, realization of feedback control are only a few of its many essential applications. Extensively, in various control systems and applications, the states of a system are not always measurable. In these circumstances, stabilization is probably best served by the enforcement of observer-based controllers [22], [35], [36], [37]. Due to their propensity to approximate unavailable states from the system's available input and output, these kind of controllers are habitually used in industries to vanquish practical applications. Most of all, the design of an observer-based event-triggered controller for networks has drawn much attention among the academic researchers due to their cumulative benefits as those stated above [38], [39], [40], [41]. A remarkable work on the analysis of switched linear systems that are susceptible to disturbances and delays is made, aided by an observer-based event-triggered  $H_{\infty}$  control [38]. Besides, aging of the components, round off errors, analog-to-digital and digital-to-analog conversion in numerical computing do produce uncertainties or inaccuracies when a controller is implemented. Thus, designing a controller which ensures that it is insensitive to a certain level of flaws with its gain, commonly known as the nonfragile control problem, is extremely vital [42], [43], [44]. As a consequence of the above made discussions, this work solely contributes on the topic of designing an observer-based nonfragile hybridtriggered controller for RSNCDNs by describing the gain perturbations in a linear fractional form. Speaking of which, the linear fractional uncertain form includes the norm bounded form as its special case [45].

Motivated by the above facts, by the virtue of an extended dissipativity performance procedure, the external disturbances that are anticipated to occur in the RSNCDNs to be modelled are dampened. More pertinently, it is the incorporation of well-known performance indices which includes  $H_{\infty}$ , passivity,  $L_2 - L_{\infty}$  and dissipativity, which can be reduced to any one of them by modifying the weighting factors in design [46]. Incentivized by the foregoing discussions, the major highlights of this work are hereunder:

- This study concerns a state estimation-based synchronization issue for RSNCDNs that are exposed to external disturbances and influenced by time-varying coupling delays.
- The Luenberger observer approach has been intended in such a way as to cope up with the unmeasurable states in real course of events. These ideas are brought into application by designing a hybrid-triggered controller with respect to the observer designed with gain fluctuations in linear fractional form.
- The attempted RSNCDNs for study that are assumed to be exposed to external disturbances can readily be tackled with the aid of adopting an extended dissipativity performance scheme.
- By the virtue of Lyapunov stability theory, adequate conditions for guaranteeing the synchronization of the networks under examination are accomplished by the dint of linear matrix inequalities.

As a conclusion, a numerical example on memristor-based Chua's circuit model is rendered to exemplify the prominence of the presented results and formulated strategy. Notably, by using the MATLAB programme to solve the achieved conditions, the necessary gain matrices of the controller, observer and the matrix involved in hybrid triggering are obtained.

# **II. PROBLEM STATEMENT**

In this investigation, we take into account a class of RSNCDNs with coupling delays composed of  $\mathcal{N}$  linearly coupled nodes that are expressed in the following manner:

$$\dot{\xi}_{i}(t) = A\xi_{i}(t) + B_{\varsigma}(\xi_{i}(t)) + a \sum_{j=1}^{\mathcal{N}} c_{ij} D\xi_{j}(t) + b \sum_{j=1}^{\mathcal{N}} e_{ij} F\xi_{j}(t - d(t)) + Gu_{i}(t) + H\varpi_{i}(t), y_{i}(t) = J\xi_{i}(t), \xi_{i}(t_{0}) = \varphi_{i}(t_{0}), \ t_{0} \in [-d_{M}, 0], \ i = 1, 2, ..., \mathcal{N},$$
(1)

where  $\xi_i(t) \in \mathbb{R}^p$ ,  $u_i(t) \in \mathbb{R}^q$  and  $y_i(t) \in \mathbb{R}^r$  are the system state, control input and output vector of the *i*<sup>th</sup> node, respectively.  $\varphi_i(.)$  specifies the initial condition of RSNCDNs (1) for node *i* defined on  $[-d_M, 0]$ ; d(t) is the time-varying coupling delay with  $0 \leq d(t) \leq d_M < \infty$ ;  $\varsigma(.) \in \mathbb{R}^s \to \mathbb{R}^s$  is the repeated scalar nonlinear function;  $\varpi_i(t)$ indicates the external disturbance of the *i*<sup>th</sup> node which are assumed to be a square integrable function; a and b are positive scalars representing the coupling strengths; D and F means the inner-coupling matrices of non-delayed and delayed coupling of RSNCDNs (1), respectively. Further,  $c_{ij}$  and  $e_{ij}$  configures the elements of outer coupling matrices, wherein  $C = [c_{ij}]_{\mathcal{N} \times \mathcal{N}}$  and  $E = [e_{ij}]_{\mathcal{N} \times \mathcal{N}}$  are determined to meet the zero-row-sum property. As well, A, B, G, H and J are the appropriate dimensioned real-valued coefficient matrices. The addressed RSNCDNs (1) are further assumed to be observable and controllable.

Let s(t) and  $y_s(t)$  denote the state and output trajectory of the target node of the intended synchronization, respectively, whose dynamics is given by

$$\dot{s}(t) = As(t) + B\varsigma(s(t)),$$
  

$$y_s(t) = Js(t).$$
(2)

Let us now denote, the synchronization error for  $i^{th}$  node of RSNCDNs (1) as  $\gamma_i(t)$  defined by  $\gamma_i(t) = \xi_i(t) - s(t)$  whose dynamics is characterized as

$$\begin{split} \dot{\gamma}_i(t) &= A\gamma_i(t) + B\varsigma(\gamma_i(t)) + a \sum_{j=1}^{\mathcal{N}} c_{ij} D\gamma_j(t) \\ &+ b \sum_{j=1}^{\mathcal{N}} e_{ij} F\gamma_j(t - d(t)) + Gu_i(t) + H\varpi_i(t), \\ y_{i\gamma}(t) &= J\gamma_i(t), \end{split}$$

where  $\varsigma(\gamma_i(t)) = \varsigma(\xi_i(t)) - \varsigma(s(t)), y_{i\gamma}(t) = y_i(t) - y_s(t).$ 

Meanwhile, while we intend to design a state feedback controller, it is necessary to know about the state variables of the specified controller earlier, which is important in realworld applications. As a result, we adopt an observer-based approach in control design. In order to do this, we take into account the dynamics of the state observer as shown below:

$$\dot{\hat{\xi}}_{i}(t) = A\hat{\xi}_{i}(t) + B_{\varsigma}(\hat{\xi}_{i}(t)) + a \sum_{j=1}^{\mathcal{N}} c_{ij} D\hat{\xi}_{j}(t) + b \sum_{j=1}^{\mathcal{N}} e_{ij} F$$

$$\times \hat{\xi}_{j}(t - d(t)) + Gu_{i}(t) + L(y_{i}(t) - \hat{y}_{i}(t)),$$

$$\hat{y}_{i}(t) = J\hat{\xi}_{i}(t), \qquad (3)$$

wherein  $\hat{\xi}_i(t) \in \mathbb{R}^p$  and  $\hat{y}_i(t) \in \mathbb{R}^r$  denote the observer state and output of  $i^{th}$  node, i.e., the estimates of state  $\xi_i(t)$  and output  $y_i(t)$ , respectively. As well, L implies the observer gain matrix and let us define the state estimation error  $\xi_i(t) - \hat{\xi}_i(t)$ as  $\nabla \xi_i(t)$ .

In this paper, we ought to design a hybrid-triggered controller based on observer (3) in order to lessen the transmission of unwanted data in the network. Over and above, it should be noted that this model is constructed in a setting that allows a probabilistic switching between time and event-triggered scheme by means of a random variable which aids in enabling a balance between the system performance and data transmission. Now, in case when the time-triggered scheme is selected, then the control data denoted as  $\underline{u}_i(t)$  will be of the form

$$\underline{u}_{i}(t) = \mathbf{K}\hat{\gamma}_{i}(t - \tau(t)), \quad t \in [t_{k}h + \tau_{t_{k}}, t_{k+1}h + \tau_{t_{k+1}}), \quad (4)$$

where  $\hat{\gamma}_i(t) = \hat{\xi}_i(t) - s(t)$  and  $t_k h$  is the sequence of sampling instants with sampling period h;  $t_k$  is the sequence  $\{t_1, t_2, \ldots\} = \{1, 2, \ldots\}; \tau(t)$  is the network induced delay defined by  $\tau(t) = t - t_k h, \tau(t) \in [0, \tau_M]$ . Here,  $\tau_M$  is the upper bound of the network induced time-delay and K is the controller gain to be determined.

Now, in accordance with the event-triggered control scheme, the following condition determines whether or not the signal must be transmitted to the controller:

$$\varepsilon_i^T(t_k h)\Omega\varepsilon_i(t_k h) \le \sigma \hat{\gamma}_i^T(t_k h + lh)\Omega\hat{\gamma}_i(t_k h + lh), \quad (5)$$

wherein  $\varepsilon_i(t_k h) = \hat{\gamma}_i(t_k h) - \hat{\gamma}_i(t_k h + lh)$  which is the error between the latest transmitted data  $\hat{\gamma}_i(t_k h)$  that is sampled and the present sampling data  $\hat{\gamma}_i(t_k h + lh)$ ; l = 1, 2, ..., h;  $\sigma$  is a positive scalar in (0, 1) and  $\Omega$  is a positive definite matrix. In here, the current sampled data packets are sent to the controller, only when the above condition (5) is violated. Thus, the control signal under event-triggered control scheme, given the notation  $\overline{u}_i(t)$ , will be of the form

$$\overline{u}_i(t) = \mathbf{K}[\hat{\gamma}_i(t - \eta(t)) + \varepsilon_i(t)],$$
  
$$t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}), \quad (6)$$

herein,  $\eta(t)$  is the network induced time-delay defined by  $\eta(t) = t - t_k h - lh$  and  $\eta(t) \in [0, \eta_M]$  with  $\eta_M$  being the upper bound of  $\eta(t)$ .

Having said that, the switching between the time and event-triggered scheme in the adopted hybrid-triggered mechanism is supposed to obey Bernoulli distribution and the nonfragile control law  $u_i(t)$  with linear fractional parametric gain variations is expressed by combining (4) and (6) in the following manner:

$$u_{i}(t) = \alpha(t)(\mathbf{K} + \Delta \mathbf{K}(t))\hat{\gamma}_{i}(t - \tau(t)) + (1 - \alpha(t))$$
$$\times (\mathbf{K} + \Delta \mathbf{K}(t)) \Big[\hat{\gamma}_{i}(t - \eta(t)) + \varepsilon_{i}(t)\Big], \quad (7)$$

where  $\alpha(t)$  indicates the Bernoulli distributed random variable with expectation and variance  $\bar{\alpha}$  and  $\bar{\alpha}(1 - \bar{\alpha})$ ,  $\Delta K(t)$  signifies the variation of controller gain having the form  $\Delta K(t) = \mathscr{MF}(t)\mathscr{N}$  with  $\mathscr{F}(t) = (I - \mathscr{GF}(t))^{-1}\bar{\mathscr{F}}(t)$ ,  $I - \mathscr{G}^T\mathscr{G} > 0$ , where  $\mathscr{M}$  and  $\mathscr{N}$  are constant matrices and  $\bar{\mathscr{F}}(t)$  satisfies  $\bar{\mathscr{F}}^T(t)\bar{\mathscr{F}}(t) \leq I$ .

Remark 1: It ought to be highlighted that the switching mechanism between time and event-triggered schemes is depicted by a random variable  $\alpha(t)$ . In here, the time-triggered control scheme is activated when  $\alpha(t) = 1$  and the event-triggered controller is employed to transmit control data when  $\alpha(t) = 0$ . Also, in real-world situations, the mean value  $\bar{\alpha}$  of the random variable  $\alpha(t)$  is crucial for achieving the required performance with the available resources in line with the requirements. The value of  $\bar{\alpha}$  can be chosen to be large enough to meet the expectations when performance requirements are higher and network resources are sufficient, or in the contrary scenario to the above, the value can be fixed to be small in order to meet the demands. The designed controller (7) is now substituted in the RSNCDNs (1) and with the exploitation of Kronecker product, we obtain the following system of equations:

$$\hat{\xi}(t) = (I \otimes A)\hat{\xi}(t) + (I \otimes B)\varsigma(\hat{\xi}(t)) + a(C \otimes D)\hat{\xi}(t) + b(E \otimes F)\hat{\xi}(t - d(t)) + \alpha(t)(I \otimes G)(I \otimes (K + \Delta K(t)))[\hat{\gamma}(t - \tau(t)) - \nabla\xi(t - \tau(t))] + (1 - \alpha(t)) (I \otimes G)(I \otimes (K + \Delta K(t)))[\gamma(t - \eta(t)) - \nabla\xi(t - \eta(t)) + \varepsilon(t)] + (I \otimes LJ)\nabla\xi(t),$$
(8)

$$\nabla \dot{\xi}(t) = (I \otimes (A - LJ)) \nabla \xi(t) + (I \otimes B)_{\varsigma} (\nabla \xi(t)) + a(C \otimes D)$$
$$\nabla \xi(t) + b(E \otimes F) \nabla \xi(t - d(t)) + (I \otimes H) \varpi(t), \qquad (9)$$

$$\dot{\gamma}(t) = (I \otimes A)\gamma(t) + (I \otimes B)\varsigma(\gamma(t)) + a(C \otimes D)\gamma(t) + b(E \otimes F)\gamma(t - d(t)) + \alpha(t)(I \otimes G)(I \otimes (K + \Delta K(t)))[\hat{\gamma}(t - \tau(t)) - \nabla\xi(t - \tau(t))] + (1 - \alpha(t)) (I \otimes G)(I \otimes (K + \Delta K(t)))[\gamma(t - \eta(t)) - \nabla\xi(t - \eta(t)) + \varepsilon(t)] + (I \otimes H)\varpi(t).$$
(10)

From this spot, the following notations will be followed:

- For any matrix A, the Kronecker product  $(I \otimes A)$  will be  $\overline{A}$ .
- For any two matrices C and D, the Kronecker product  $(C \otimes D)$  will be ascertained as  $\overline{C_D}$ .

Thus, with the above acknowledged details, the augmented closed-loop form of the system of equations (8)-(10) is acquired as

$$\dot{\Phi}(t) = \hat{A}\Phi(t) + \hat{B}\mathbf{f}(\Phi(t)) + \hat{C}\Phi(t - d(t)) + \alpha(t)\hat{D}\Phi(t - \tau(t)) + (1 - \alpha(t))\hat{E}\Phi(t - \eta(t)) + (1 - \alpha(t))\hat{F}\varepsilon(t) + \hat{H}\varpi(t), \mathbf{y}(t) = \hat{J}\Phi(t),$$
(11)

where  $\Phi^{T}(t) = [\hat{\xi}^{T}(t) \nabla \xi^{T}(t) \gamma^{T}(t)], \quad \varsigma^{T}(\Phi(t)) = [\varsigma^{T}(\hat{\xi}(t)) \varsigma^{T}(\nabla \xi(t)) \varsigma^{T}(\gamma(t))],$ 

$$\begin{split} \hat{A} &= \begin{bmatrix} \overline{\mathbf{A}} + \mathbf{a}\overline{\mathbf{C}_{\mathbf{D}}} & \overline{\mathbf{L}}\overline{\mathbf{J}} & 0 \\ 0 & \overline{\mathbf{A}} + \mathbf{a}\overline{\mathbf{C}_{\mathbf{D}}} - \overline{\mathbf{L}}\overline{\mathbf{J}} & 0 \\ 0 & 0 & \overline{\mathbf{A}} + \mathbf{a}\overline{\mathbf{C}_{\mathbf{D}}} \end{bmatrix}, \\ \hat{B} &= \begin{bmatrix} \overline{\mathbf{B}} & 0 & 0 \\ 0 & \overline{\mathbf{B}} & 0 \\ 0 & 0 & \overline{\mathbf{B}} \end{bmatrix}, \hat{C} &= \begin{bmatrix} \mathbf{b}\overline{\mathbf{E}_{\mathbf{F}}} & 0 & 0 \\ 0 & \mathbf{b}\overline{\mathbf{E}_{\mathbf{F}}} & 0 \\ 0 & 0 & \mathbf{b}\overline{\mathbf{E}_{\mathbf{F}}} \end{bmatrix}, \\ \hat{D} &= \hat{E} &= \begin{bmatrix} 0 & -\overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} & \overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} \\ 0 & 0 & 0 & \mathbf{b}\overline{\mathbf{E}_{\mathbf{F}}} \end{bmatrix}, \\ \hat{D} &= \hat{E} = \begin{bmatrix} 0 & -\overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} & \overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} \\ 0 & 0 & \overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} \\ 0 & -\overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} & \overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} \end{bmatrix}, \\ \hat{F} &= \begin{bmatrix} \overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} \\ 0 \\ \overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} \\ \overline{\mathbf{G}(\mathbf{K} + \Delta\mathbf{K})} \end{bmatrix}, \\ \hat{H} &= \begin{bmatrix} 0 \\ \overline{\mathbf{H}} \\ \overline{\mathbf{H}} \\ \overline{\mathbf{H}} \end{bmatrix} \text{ and } \hat{J} = \begin{bmatrix} \overline{\mathbf{J}} & \overline{\mathbf{J}} & \overline{\mathbf{J}} \end{bmatrix} \end{split}$$

#### **III. PRELIMINARIES**

Additionally, the following preliminaries beneficial for the derivation of main results are granted:

Assumption 1: The function  $\varsigma(.)$  in the RSNCDNs (1) is assumed to satisfy the ensuing requirement:

$$|\varsigma(\hat{\mathfrak{a}}) + \varsigma(\hat{\mathfrak{b}})| \le |\hat{\mathfrak{a}} + \hat{\mathfrak{b}}|, \forall \hat{\mathfrak{a}}, \hat{\mathfrak{b}} \in \mathbb{R}.$$
 (12)

Assumption 2: For the provided matrices  $\hat{U}_1$ ,  $\hat{U}_2$ ,  $\hat{U}_3$  and  $\hat{U}_4$  the requirements presented herewith are met:

• 
$$\hat{U}_1 = \hat{U}_1^T \le 0, \ \hat{U}_3 = \hat{U}_3^T > 0 \text{ and } \hat{U}_4 = \hat{U}_4^T \ge 0.$$
  
•  $(\parallel \hat{U}_1 \parallel + \parallel \hat{U}_2 \parallel) \cdot \parallel \hat{U}_4 \parallel = 0.$ 

Definition 1: [46] The augmented closed-loop system (11) is referred to be extended dissipative with the afore-mentioned matrices  $\hat{U}_1$ ,  $\hat{U}_2$ ,  $\hat{U}_3$  and  $\hat{U}_4$  assenting the Assumption 2, if for every  $\varpi(t) \in \mathcal{L}_2[0, \infty)$  and any  $t_{\lambda} \ge 0$ , there exists a scalar  $\mu$  such that

$$\mathbb{E}\left[\int_{0}^{t_{\lambda}} \mathscr{J}(t)dt - \sup_{0 \le t \le t_{\lambda}} \mathbf{y}^{T}(t)\hat{U}_{4}\mathbf{y}(t)\right] \ge \mu, \qquad (13)$$

having  $\mathcal{J}(t) = \mathbf{y}^T(t)\hat{U}_1\mathbf{y}(t) + 2\mathbf{y}^T(t)\hat{U}_2\boldsymbol{\varpi}(t) + \boldsymbol{\varpi}^T(t)\hat{U}_3\boldsymbol{\varpi}(t).$ 

*Lemma 1:* [8] Any given matrix  $\overline{P}$  in  $\mathbb{R}^{\overline{s} \times \overline{s}}$  with elements  $[\overline{p}_{rs}]$  is a positive diagonally dominant if and only if  $\overline{P} > 0$  and there exists a matrix  $\overline{W}$  in  $\mathbb{R}^{\overline{s} \times \overline{s}}$  with elements  $[\overline{w}_{rs}]$  fulfilling the condition

$$\begin{aligned} \forall r \neq s, \quad \bar{w}_{rs} \geq 0, \quad \bar{p}_{rs} + \bar{w}_{rs} \geq 0, \\ \forall r, \quad \bar{p}_{rr} - \sum_{s \neq r} (\bar{p}_{rs} + 2\bar{w}_{rs}) \geq 0. \end{aligned}$$

*Lemma 2:* [8] *Every nonlinear functions of form*  $\mathbf{\tilde{f}}(.)$  *that are contigent to Assumption 1 obeys* 

$$\bar{\mathbf{f}}^T(\zeta_i(t))\bar{P}\bar{\mathbf{f}}(\zeta_i(t)) \leq \zeta_i^T(t)\bar{P}\zeta_i(t), \quad \forall \ \zeta_i(t) \in \mathbb{R}^{\bar{s}},$$

for a given positive diagonally dominant matrix  $\overline{P} > 0$  and each node i.

# **IV. MAIN RESULTS**

The purpose of this section is to determine a novel sufficient conditions for guaranteeing asymptotic stability of the augmented closed-loop system (11), which thereby ensures the state estimation-based synchronization of RSNCDNs (1) with the aid of configured non-fragile observer-based hybrid-triggered controller. Moreover, in order to demonstrate the above-said two theorems, one having known gain matrices with  $\varpi(t) = 0$ ,  $\Delta K(t) = 0$  and the later with unknown gain matrices in the presence of external disturbances and gain perturbations are offered.

Theorem 1: In consideration with Assumption 1, the closed-loop augmented system (11) with  $\overline{\omega}(t) = 0$  and  $\Delta K(t) = 0$  is asymptotically stable if for given positive scalars  $\sigma$ ,  $d_M$ ,  $\tau_M$ ,  $\eta_M$ ,  $\overline{\alpha}$ ,  $\pi$  and matrix  $\overline{\Omega} > 0$ , the controller and observer gain matrices K and L, respectively there exist matrices  $\hat{P} > 0$ ,  $\hat{Q}_y > 0$ ,  $\hat{R}_y > 0$ ,  $\hat{S}_y$ , (y = 1, 2, 3), such that subsequent relations are met

$$[\aleph]_{9\times9} < 0, \tag{14}$$

$$\begin{bmatrix} \hat{R}_y & \hat{S}_y \\ * & \hat{R}_y \end{bmatrix} > 0, (y = 1, 2, 3)$$
 (15)

with  $\aleph_{1,1} = \hat{P}\hat{A} + \hat{A}^T\hat{P} + \hat{Q}_1 + \hat{Q}_2 + \hat{Q}_3 - \hat{R}_1 - \hat{R}_2 - \hat{R}_3 + (1 - \pi)\hat{P} + \hat{A}^T\mathcal{O}\hat{A}, \ \aleph_{1,2} = \hat{P}\hat{C} + \hat{R}_1 - \hat{S}_1, \ \aleph_{1,3} = \hat{S}_1, \ \aleph_{1,4} = \hat{S}_1 + \hat{S}_1 + \hat{S}_2 + \hat{S}_2 + \hat{S}_3 + \hat{S}$ 

$$\begin{split} \bar{\alpha}\hat{P}\hat{D} + \hat{R}_2 - \hat{S}_2, \ &\aleph_{1,5} = \hat{S}_2, \ &\aleph_{1,6} = (1 - \bar{\alpha})\hat{P}\hat{E} + \hat{R}_3 - \hat{S}_3, \ &\aleph_{1,7} = \hat{S}_3, \ &\aleph_{1,8} = (1 - \bar{\alpha})\hat{P}\hat{F}, \ &\aleph_{1,9} = \hat{P}\hat{B}, \ &\aleph_{2,2} = -2\hat{R}_1 + \hat{S}_1^T + \hat{C}^T\mathcal{O}\hat{C}, \ &\aleph_{2,3} = \hat{R}_1 - \hat{S}_1, \ &\aleph_{3,3} = -\hat{R}_1 - \hat{Q}_1, \ &\aleph_{4,4} = -2\hat{R}_2 + \hat{S}_2 + \hat{S}_2^T + \nu_1\hat{D}^T\mathcal{O}\hat{D}, \ &\aleph_{4,5} = \hat{R}_2 - \hat{S}_2, \ &\aleph_{5,5} = -\hat{R}_2 - \hat{Q}_2, \ &\aleph_{6,6} = -2\hat{R}_3 + \hat{S}_3 + \hat{S}_3^T + \sigma\hat{\Omega} + \nu_2\hat{E}^T\mathcal{O}\hat{E}, \ &\aleph_{6,7} = \hat{R}_3 - \hat{S}_3, \ &\aleph_{7,7} = -\hat{R}_3 - \hat{Q}_3, \ &\aleph_{8,8} = -\overline{\Omega} + \nu_2\hat{F}^T\mathcal{O}\hat{F}, \ &\aleph_{9,9} = -(1 - \pi)\hat{P} + \hat{B}^T\mathcal{O}\hat{B} \ where \ \mathcal{O} = d_M^2\hat{R}_1 + \tau_M^2\hat{R}_2 + \eta_M^2\hat{R}_3, \ \hat{\Omega} = diag\{0, -\sigma\,\overline{\Omega}, \sigma\,\overline{\Omega}\} \ and \ other \ terms \ zero \ and \ &\nu_1 = \bar{\alpha}^2 + \bar{\alpha}(1 - \bar{\alpha}), \ &\nu_2 = (1 - \bar{\alpha})^2 + \bar{\alpha}(1 - \bar{\alpha}). \end{split}$$

*Proof:* In order to assert the necessary result, the Lyapunov-Krasovskii functional candidate is assigned as follows:

$$\mathbf{V}(t) = \sum_{k=1}^{3} \mathbf{V}_{k}(t),$$
(16)

where

$$V_{1}(t) = \Phi^{T}(t)\hat{P}\Phi(t),$$
(17)  

$$V_{2}(t) = \int_{t-d_{M}}^{t} \Phi^{T}(s)\hat{Q}_{1}\Phi(s)ds + \int_{t-\tau_{M}}^{t} \Phi^{T}(s)\hat{Q}_{2}\Phi(s)ds + \int_{t-\eta_{M}}^{t} \Phi^{T}(s)\hat{Q}_{3}\Phi(s)ds,$$
(18)  

$$0 \quad t \qquad 0 \quad t$$

$$V_{3}(t) = d_{M} \int_{-d_{M}}^{0} \int_{t+\theta}^{T} \Phi^{T}(s)\hat{R}_{1}\Phi(s)dsd\theta + \tau_{M} \int_{-\tau_{M}}^{0} \int_{t+\theta}^{T} \Phi^{T}(s)$$
$$\times \hat{R}_{2}\Phi(s)dsd\theta + \eta_{M} \int_{-\eta_{M}}^{0} \int_{t+\theta}^{t} \Phi^{T}(s)\hat{R}_{3}\Phi(s)dsd\theta,$$
(19)

with  $\hat{P} = \text{diag}\{\overline{\mathbf{P}_1}, \overline{\mathbf{P}_2}, \overline{\mathbf{P}_2}\}, \hat{Q}_z = \text{diag}\{\overline{\mathbf{Q}_{z1}}, \overline{\mathbf{Q}_{z2}}, \overline{\mathbf{Q}_{z3}}\}, \hat{R}_z = \text{diag}\{\overline{\mathbf{R}_{z1}}, \overline{\mathbf{R}_{z2}}, \overline{\mathbf{R}_{z3}}\}, \text{where } \overline{\mathbf{P}_1} > 0, \overline{\mathbf{P}_2} > 0, \overline{\mathbf{Q}_{z1}} > 0, \overline{\mathbf{Q}_{z2}} > 0, \overline{\mathbf{Q}_{z3}} > 0, \overline{\mathbf{Q}_{z1}} > 0, \overline{\mathbf{Q}_{z2}} > 0, \overline{\mathbf{Q}_{z3}} > 0, \overline{\mathbf{R}_{z1}} > 0, \overline{\mathbf{R}_{z2}} > 0, \overline{\mathbf{R}_{z3}} > 0, (z = 1, 2, 3).$ 

It is to be mentioned that, in the further, the mathematical expectation will be denoted by  $\mathbb{E}\{.\}$ . The outcomes listed below are obtained by evaluating the expectation of first derivative of the above-mentioned Lyapunov-Krasovskii functional (16) along the trajectories of the augmented closed-loop system (11):

$$\mathbb{E}\{\dot{\mathbf{V}}_{1}(t)\} = \mathbb{E}\{\Phi^{T}(t)\hat{P}\dot{\Phi}(t) + \dot{\Phi}^{T}(t)\hat{P}\Phi(t)\}, \qquad (20)$$

$$\mathbb{E}\{\dot{\mathbf{V}}_{2}(t)\} = \mathbb{E}\{\Phi^{T}(t)(\hat{Q}_{1} + \hat{Q}_{2} + \hat{Q}_{3})\Phi(t) - \Phi^{T}(t - d_{M})$$

$$\hat{Q}_{1}\Phi(t - d_{M}) - \Phi^{T}(t - \tau_{M})\hat{Q}_{2}\Phi(t - \tau_{M})$$

$$- \Phi^{T}(t - \eta_{M})\hat{Q}_{3}\Phi(t - \eta_{M})\}, \qquad (21)$$

$$\mathbb{E}\{\dot{\mathbf{V}}_{3}(t)\} = \mathbb{E}\{\dot{\Phi}^{T}(t)(d_{M}^{2}\hat{R}_{1} + \tau_{M}^{2}\hat{R}_{2} + \eta_{M}^{2}\hat{R}_{3})\dot{\Phi}(t)$$

$$- d_{M}\int_{t - d_{M}}^{t}\Phi^{T}(s)\hat{R}_{1}\Phi(s)ds - \tau_{M}\int_{t - \tau_{M}}^{t}\Phi^{T}(s)$$

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$$\times \hat{R}_2 \Phi(s) ds - \eta_M \int_{t-\eta_M}^t \Phi^T(s) \hat{R}_3 \Phi(s) ds \}.$$
 (22)

With an aim to acquire the desired results in terms of linear matrix inequalities the single integral terms in the preceding equation can indeed be reconfigured as follows with the aid of Lemma 1 in [30]:

$$-\varrho_{M} \int_{t-\varrho_{M}}^{t} \Phi^{T}(s)\hat{R}_{l}\Phi(s)ds \leq \begin{bmatrix} \Phi(t) \\ \Phi(t-\varrho(t)) \\ \Phi(t-\varrho_{M}) \end{bmatrix}^{T} \\ \begin{bmatrix} \hat{R}_{l} & \hat{R}_{l} - \hat{S}_{l} & \hat{S}_{l} \\ * & -2\hat{R}_{l} + \hat{S}_{l} + \hat{S}_{l}^{T} & \hat{R}_{l} - \hat{S}_{l} \\ * & * & -\hat{R}_{l} \end{bmatrix} \begin{bmatrix} \Phi(t) \\ \Phi(t-\varrho(t)) \\ \Phi(t-\varrho_{M}) \end{bmatrix}, \quad (23)$$

where for l = 1,  $\rho$  implies d, for l = 2,  $\rho$  implies  $\tau$  and for l = 3,  $\rho$  implies  $\eta$ .

As well, using the Lemma 2, we can now articulate the below inequality;

$$\varsigma^{T}(\Phi(t))\hat{P}\varsigma(\Phi(t)) \le \Phi^{T}(t)\hat{P}\Phi(t)$$
(24)

which for any  $\pi = [0, 1)$  becomes

$$-(1-\pi)\Phi^{T}(t)\hat{P}\Phi(t) \le -(1-\pi)\varsigma^{T}(\Phi(t))\hat{P}\varsigma(\Phi(t)).$$
(25)

Besides, from the inequality (5), we arrive at

$$\sigma \hat{\gamma}^{T}(t-\eta(t))\overline{\mathbf{\Omega}}\hat{\gamma}(t-\eta(t)) - \varepsilon^{T}(t)\overline{\mathbf{\Omega}}\varepsilon(t) \ge 0.$$
 (26)

Putting together the equations (20)-(26), we yield

$$\mathbb{E}\{\dot{\mathbf{V}}(t)\} \le F^{T}(t) \boldsymbol{\aleph} F(t), \tag{27}$$

where  $F^{T}(t) = \left[ \Phi^{T}(t) \Phi^{T}(t - d(t)) \Phi^{T}(t - d_{M}) \Phi^{T}(t - \tau(t)) \Phi^{T}(t - \tau_{M}) \Phi^{T}(t - \eta(t)) \Phi^{T}(t - \eta_{M}) \varepsilon^{T}(t) \varsigma^{T}(\Phi(t)) \right]$ having the elements as posted in the statement of Theorem 1.

Ultimately, we come to the conclusion that  $\mathbb{E}\{\dot{V}(t)\} < 0$  holds true as long as the condition (14) in the statement of Theorem 1 does. Thus, when  $\varpi(t) = 0$  and  $\Delta K(t) = 0$ , i.e., in the absence of external disturbances and gain perturbations, it is inferred that the augmented closed-loop system (11) exhibits asymptotic stability and is the conclusion of the proof.

The augmented closed-loop system (11) with non-zero disturbances, gain perturbations and unknown gain matrices is now integrated into the inference of Theorem 1.

Theorem 2: In concern with Assumptions 1 and 2, for given positive scalars  $\kappa$ ,  $\sigma$ ,  $\pi$ ,  $\mu$ ,  $\bar{\alpha}$ ,  $d_M$ ,  $\tau_M$ ,  $\eta_M$ , there exists positive scalar  $\epsilon$ , positive definite matrices  $\hat{X} = diag\{\overline{\mathbf{X_1}}, \overline{\mathbf{X_2}}, \overline{\mathbf{X_2}}\}, \tilde{X}, \hat{Q}_y, \hat{R}_y, \hat{S}_y, (y = 1, 2, 3),$  $\hat{P} = diag\{\overline{\mathbf{P_1}}, \overline{\mathbf{P_2}}, \overline{\mathbf{P_2}}\}, \hat{W} = diag\{\overline{\mathbf{W_1}}, \overline{\mathbf{W_2}}, \overline{\mathbf{W_2}}\}, with \overline{\mathbf{P_x}} = [\mathfrak{p}_x^{\mathfrak{uv}}] \in \mathbb{R}^{p \times p}, \overline{\mathbf{W_x}} = [\mathfrak{w}_x^{\mathfrak{uv}}] \in \mathbb{R}^{p \times p}$  and appropriate dimensioned matrices  $\mathcal{Y}$ ,  $\mathcal{M}$ , the dynamics of closed-loop system (11) is asymptotically stable, if the following holds:

$$\tilde{\tilde{\aleph}} = \begin{bmatrix} \tilde{\tilde{\aleph}} & \Xi_1 & \bar{\Xi}_1^T & \bar{\Xi}_2^T & \Xi_2 \\ * -\epsilon I & \epsilon \mathcal{G}^T & 0 & 0 \\ * & * & -\epsilon I & 0 & 0 \\ * & * & * & -\epsilon I & \epsilon \mathcal{G}^T \\ * & * & * & \epsilon & \epsilon & \epsilon \end{bmatrix} < 0,$$
(28)

$$\begin{bmatrix} -\kappa I \ JX_2 - \check{X}J \\ * & -I \end{bmatrix} < 0,$$
(29)

$$\begin{bmatrix} -\omega \hat{X} \ \hat{X} \mathbf{J}^T \hat{U}_4^{1/2} \\ * \ -I \end{bmatrix} < 0, \tag{30}$$

$$\begin{bmatrix} \hat{\hat{R}}_{y} & \hat{\hat{S}}_{y} \\ * & \hat{\hat{R}}_{y} \end{bmatrix} > 0, (y = 1, 2, 3),$$
(31)

$$\forall \mathfrak{u} \neq \mathfrak{v}, \quad \mathfrak{w}_x^{\mathfrak{u}\mathfrak{v}} \ge 0, \quad \mathfrak{p}_x^{\mathfrak{u}\mathfrak{v}} + \mathfrak{w}_x^{\mathfrak{u}\mathfrak{v}} \ge 0, \tag{32}$$

$$\forall \mathfrak{u}, \quad \mathfrak{p}_x^{\mathfrak{u}\mathfrak{v}} - \sum_{\mathfrak{u}\neq\mathfrak{v}} \mathfrak{p}_x^{\mathfrak{u}\mathfrak{v}} + 2\mathfrak{w}_x^{\mathfrak{u}\mathfrak{v}} \ge 0, \hat{P}\hat{X} = I \qquad (33)$$

with  $\tilde{\aleph}_{1,1} = \Psi_{\hat{A}} + \Psi_{\hat{A}}^T + \hat{Q}_1 + \hat{Q}_2 + \hat{Q}_3 - \hat{R}_1 - \hat{R}_2 - \hat{R}_3 +$  $(1 - \pi)\hat{X}, \ \tilde{\aleph}_{1,2} = \hat{C}\hat{X} + \hat{R}_1 - \hat{S}_1, \ \tilde{\aleph}_{1,3} = \hat{S}_1, \ \tilde{\aleph}_{1,4} =$  $\bar{\alpha}\Psi_{\hat{D}} + \hat{R}_2 - \hat{S}_2, \ \tilde{\aleph}_{1.5} = \hat{S}_2, \ \tilde{\aleph}_{1.6} = (1 - \bar{\alpha})\Psi_{\hat{F}} +$  $\hat{\hat{R}}_3 - \hat{\hat{S}}_3, \ \tilde{\aleph}_{1,7} = \hat{\hat{S}}_3, \ \tilde{\aleph}_{1,8} = (1 - \bar{\alpha})\Psi_{\hat{r}}, \ \tilde{\aleph}_{1,9} =$  $\hat{B}, \ \tilde{\aleph}_{1,10} = \hat{H} - 2\hat{X}\hat{J}^T\hat{U}_2, \ \tilde{\aleph}_{1,11} = d_M\Psi_2^T, \ \tilde{\aleph}_{1,12} =$  $\tau_M \Psi_{\hat{\lambda}}^T$ ,  $\tilde{\aleph}_{1,13} = \eta_M \Psi_{\hat{\lambda}}^T$ ,  $\tilde{\aleph}_{1,14} = \hat{X} \hat{J}^T \sqrt{\hat{U}_1}$ ,  $\tilde{\aleph}_{2,2} =$  $\begin{array}{rcl} -2\hat{\hat{R}}_{1}^{T} &+& \hat{\hat{S}}_{1}^{T} &+& \hat{\hat{S}}_{1}^{T}, & \tilde{\aleph}_{2,3} &=& \hat{\hat{R}}_{1} &+& \hat{\hat{S}}_{1}, & \tilde{\aleph}_{2,11} &=\\ d_{M}\hat{X}\hat{C}^{T} &,& \tilde{\aleph}_{2,12} &=& \tau_{M}\hat{X}\hat{C}^{T}, & \tilde{\aleph}_{2,13} &=& \eta_{M}\hat{X}\hat{C}^{T}, & \tilde{\aleph}_{3,3} &= \end{array}$  $-\hat{\hat{R}}_1 - \hat{\hat{Q}}_1, \ \tilde{\aleph}_{4,4} = -2\hat{\hat{R}}_2 + \hat{\hat{S}}_2 + \hat{\hat{S}}_2^T, \ \tilde{\aleph}_{4,5} = \hat{\hat{R}}_2 - \hat{\hat{R}}$  $\hat{\hat{S}}_2$ ,  $\tilde{\aleph}_{4,11} = d_M \sqrt{\nu_1} \Psi_{\hat{D}}^T$ ,  $\tilde{\aleph}_{4,12} = \tau_M \sqrt{\nu_1} \Psi_{\hat{D}}^T$ ,  $\tilde{\aleph}_{4,13} =$  $\eta_M \sqrt{\nu_1} \Psi_{\hat{n}}^T$ ,  $\tilde{\aleph}_{5,5} = -\hat{\hat{R}}_2 - \hat{\hat{Q}}_2$ ,  $\tilde{\aleph}_{6,6} = -2\hat{\hat{R}}_3 + \hat{\hat{S}}_3 +$  $\hat{\hat{S}}_{3}^{T} + \sigma \hat{\hat{\Omega}}, \quad \tilde{\aleph}_{6,7} = \hat{\hat{R}}_{3} - \hat{\hat{S}}_{3}, \quad \tilde{\aleph}_{6,11} = d_{M} \sqrt{\nu_{2}} \Psi_{\hat{\pi}}^{T}, \quad \tilde{\aleph}_{6,12} =$  $\tau_M \sqrt{\nu_2} \Psi_{\hat{r}}^T$ ,  $\tilde{\aleph}_{6,13} = \eta_M \sqrt{\nu_2} \Psi_{\hat{r}}^T$ ,  $\tilde{\aleph}_{7,7} = -\hat{R}_3 - \hat{R}_3$  $\hat{\hat{Q}}_{3}, \tilde{\aleph}_{8,8} = -\overline{\Omega}, \tilde{\aleph}_{8,11} = d_M \sqrt{\nu_2} \Psi_{\hat{\pi}}^T, \tilde{\aleph}_{8,12} =$  $\tau_M \sqrt{\nu_2} \Psi_{\hat{F}}^T, \quad \tilde{\aleph}_{8,13} = \eta_M \sqrt{\nu_2} \Psi_{\hat{F}}^T, \quad \tilde{\aleph}_{9,9} \stackrel{r}{=} -(1 - 1)$  $\pi)\hat{X}, \ \tilde{\aleph}_{9,11}^{T} = d_M\hat{B}^T, \ \tilde{\aleph}_{9,12}^{T} = \tau_M\hat{B}^T, \ \tilde{\aleph}_{9,13} = \eta_M\hat{B}^T, \ \tilde{\aleph}_{10,10} = \hat{U}_3, \ \tilde{\aleph}_{10,11} = d_M\hat{H}^T, \ \tilde{\aleph}_{10,12} =$  $\tau_M \hat{H}^T$ ,  $\tilde{\aleph}_{10,13} = \eta_M \hat{H}^T$ ,  $\tilde{\aleph}_{11,11} = -2\hat{X} + \hat{\hat{R}}_1$ ,  $\tilde{\aleph}_{12,12} =$  $-2\hat{X} + \hat{R}_2, \quad \tilde{\aleph}_{13,13} = -2\hat{X} + \hat{R}_3, \quad \tilde{\aleph}_{14,14} = -I, \text{ and all other}$ terms zero, where  $v_1 = \bar{\alpha}^2 + \bar{\alpha}(1-\bar{\alpha}), v_2 = (1-\bar{\alpha})^2 + \bar{\alpha}(1-\bar{\alpha}).$ Moreover,

$$\begin{aligned} \boldsymbol{\Xi}_{1}^{T} &= \begin{bmatrix} \boldsymbol{\epsilon} \overline{\mathbf{G}} \overline{\mathcal{M}} & \boldsymbol{0} & \boldsymbol{\epsilon} \overline{\mathbf{G}} \overline{\mathcal{M}} & \underline{\boldsymbol{0}} & \dots & \underline{\boldsymbol{0}} \end{bmatrix}, \\ \bar{\boldsymbol{\Xi}}_{1} &= \begin{bmatrix} \underline{\boldsymbol{0}} & \dots & \underline{\boldsymbol{0}} & -\bar{\boldsymbol{\alpha}} \overline{\mathcal{N}} \mathbf{X}_{2} \bar{\boldsymbol{\alpha}} \overline{\mathcal{N}} \mathbf{X}_{2} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\bar{\boldsymbol{\alpha}} \overline{\mathcal{N}} \mathbf{X}_{2} \\ & \hat{\boldsymbol{\alpha}} \overline{\mathcal{N}} \mathbf{X}_{2} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\alpha} \overline{\mathcal{N}} \mathbf{X}_{2} & \underline{\boldsymbol{0}} & \dots & \underline{\boldsymbol{0}} \end{bmatrix}, \\ \boldsymbol{\Xi}_{2}^{T} &= \begin{bmatrix} \underline{\boldsymbol{0}} & \dots & \underline{\boldsymbol{0}} & -\sqrt{\nu_{1}} \overline{\mathcal{N}} \mathbf{X}_{2} & \sqrt{\nu_{1}} \overline{\mathcal{N}} \mathbf{X}_{2} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \end{aligned}$$



FIGURE 1. Memristor-based Chua's circuit [10].



Additionally, the relation to determine the controller and observer gain matrices is proffered by

 $\mathbf{K} = \mathcal{Y} \mathbf{X}_2^{-1} \text{ and } \mathbf{L} = \mathbf{M} \breve{X}^{-1}.$ 

*Proof:* The same Lyapunov-Krasovskii functional (16) is fabricated and similar procedures as in proof of Theorem 1 is performed by considering the circumstances in the presence of external disturbances and control gain perturbations, i.e.,  $\varpi(t) \neq 0$  and  $\Delta K(t) \neq 0$ . Besides, the further advancement is carried on by implementing the extended dissipativity performance condition along with. In the vein of above discussions, we reach

$$\mathbb{E}\{\dot{\mathbf{V}}(t)\} - \mathscr{J}(t) \le \bar{F}^{T}(t)\bar{\aleph}\bar{F}(t), \qquad (34)$$

where  $\bar{F}^T(t) = [F^T(t) \ \varpi^T(t)]$  and  $\bar{\aleph} = \begin{bmatrix} \aleph & \hat{P}\hat{H} - 2\hat{J}^T\hat{U}_2 \\ \ast & -\hat{U}_3 + \hat{H}^T\mathcal{O}\hat{H} \end{bmatrix}$ , here the elements of  $\aleph$  are same as in (14) except the term  $\aleph_{1,1}$ is replaced with  $\hat{P}\hat{A} + \hat{A}^T\hat{P} + \hat{Q}_1 + \hat{Q}_2 + \hat{Q}_3 - \hat{R}_1 - \hat{R}_2 - \hat{R}_3 - \hat{P} + \hat{A}^T\mathcal{O}\hat{A} - \hat{J}^T\hat{U}_1\hat{J}.$ 

Let us allow,  $\hat{X} = \hat{P}^{-1}$ , where  $\hat{X} = \text{diag}\{\overline{\mathbf{X}_1}, \overline{\mathbf{X}_2}, \overline{\mathbf{X}_2}\}$ . Following this, pre and post multiplying the matrix  $\bar{\aleph}$  with diag{ $\hat{X}$ ,..., $\hat{X}$ ,  $\overline{\mathbf{X}_2}$ , I, I} and treating the gain perturbations 7 times

 $\Delta K(t)$  in the resulting design to be in a linear fractional form split in accordance with Lemma 3 of [45] followed by

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**FIGURE 2.** Synchronization of three trajectories of RSNCDNs  $\xi_i(t)$ , (i = 1, 2, ..., 5) in (39) to their isolated node s(t).

the application of Schur complement lemma, we obtain the matrix  $\tilde{\aleph}$  as clearly proclaimed in the statement of Theorem 2. Moreover, on the condition that the linear matrix inequality  $\tilde{\aleph}$  in (28) holds it would be evident that

$$\mathbb{E}\{\dot{\mathbf{V}}(t)\} - \mathcal{J}(t) < 0. \tag{35}$$

So as to proceed further, the integration of above accomplished inequality is carried out from 0 to t, yielding

$$\int_{0}^{T} \mathscr{J}(s)ds > \mathbb{E}\{\mathbf{V}(t) - \mathbf{V}(0)\} > \mathbb{E}\{\Phi^{T}(t_{\lambda})\hat{P}\Phi(t_{\lambda})\} + \mu.$$
(36)

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**FIGURE 3.** Trajectories of RSNCDNs  $\xi_g(t)$  in (39) and their estimates  $\hat{\xi}_g(t)$  as in (3), (g = 1, 2, 3).

Now, with an objective to prove that the relation (13) in Definition 1 is true for the two possible cases of  $\hat{U}_4$ , i.e.  $\| \hat{U}_4 \| = 0$  and  $\| \hat{U}_4 \| > 0$ , the discussion is proceeded. We initiate with the case when  $\| \hat{U}_4 \| = 0$ . Then, from (36), we get  $\int_{0}^{t_{\lambda}} \mathscr{J}(s) ds \ge \mathbb{E}\{\Phi^T(t_{\lambda})\hat{P}\Phi(t_{\lambda})\} + \mu \ge \mu, \quad \forall 0 \le t_{\lambda}.$ 

Thus, it is obvious that when  $\| \hat{U}_4 \| = 0$ , the relation (13) is fulfilled. Progressing forward to the next case, it is



**FIGURE 4.** Trajectories of RSNCDNs  $\xi_g(t)$  in (39) and their estimates  $\hat{\xi}_g(t)$  as in (3), (g = 4, 5).

apparent from the Assumption 2 that, other matrices involved in extended dissipativity will be  $\hat{U}_1 = \hat{U}_2 = 0$  and  $\hat{U}_3 > 0$ . Let  $\hat{J}^T \hat{U}_4 \hat{J} < \omega \hat{P}$  for some  $\omega \in (0, 1)$  and  $0 \le t \le t_{\lambda}$ , then in compliance with above findings, we acquire

$$\int_{0}^{t_{\lambda}} \mathscr{J}(s)ds \geq \mathbb{E}\{\omega\Phi^{T}(t)\hat{P}\Phi(t)\} + \mu \geq \mathbb{E}\{\Phi^{T}(t)\hat{J}^{T}\hat{U}_{4}\hat{J}\Phi(t)\} + \mu = \mathbb{E}\{\mathbf{y}^{T}(t)\hat{U}_{4}\mathbf{y}(t)\} + \mu$$
(37)

which verifies that equation (13) is possessed for any nonnegative  $t_{\lambda}$ . In accordance with the above evaluation, it is proved that the closed-loop augmented system (11) achieves stability in the extended dissipative sense. In addition, it should be pointed out that, in tandem with the proof of Theorem 2 of [8], it is also clear that  $\hat{P}$  is diagonally dominant. Ultimately, the needed proof is obtained.

# **V. SIMULATION VERIFICATION**

This part exemplifies the effectiveness of the designed controller and correctness of the compiled results by taking the memristor-based Chua's circuit (as in Fig. 1) into consideration, whose parameters are borrowed from [10]. The



**FIGURE 5.** Control data release instants and intervals of nodes 1, 2, 3 of RSNCDNs (39).

memristor-based Chua's circuit to be examined is of the following form:

$$\mathscr{C}_{1}\dot{\mathcal{V}}_{\mathscr{C}1}(t) = \frac{1}{\mathscr{R}_{p}} \left( \mathcal{V}_{\mathscr{C}2}(t) - \mathcal{V}_{\mathscr{C}1}(t) \right) - \mathfrak{g}(\mathcal{V}_{\mathscr{C}1}(t)),$$
  
$$\mathscr{C}_{2}\dot{\mathcal{V}}_{\mathscr{C}2}(t) = -\frac{1}{\mathscr{R}_{p}} \left( \mathcal{V}_{\mathscr{C}2}(t) - \mathcal{V}_{\mathscr{C}1}(t) \right) + \mathbf{i}(t),$$
  
$$\mathscr{L}\mathbf{i}(t) = -\mathscr{R}_{q}\mathbf{i}(t) - \mathcal{V}_{\mathscr{C}2}(t), \qquad (38)$$



FIGURE 6. Control data release instants and intervals of nodes 4, 5 of RSNCDNs (39).

having components,  $\mathscr{C}_1$ ,  $\mathscr{C}_2$  are the two capacitors with voltages  $\mathcal{V}_{\mathscr{C}_1}$ ,  $\mathcal{V}_{\mathscr{C}_2}$  across them,  $\mathscr{L}$  being the inductor with current  $\mathbf{i}(t)$  through them,  $\mathscr{R}_p$  and  $\mathscr{R}_q$  are the two linear resistances and g(.) is the characteristic of nonlinear resistance defined by  $\mathfrak{g}(\mathcal{V}_{\mathscr{C}_1}(t)) = \mathfrak{g}_1 \mathcal{V}_{\mathscr{C}_1} + \frac{1}{2}(\mathfrak{g}_2 - \mathfrak{g}_1)(|\mathcal{V}_{\mathscr{C}_1}(t) + 1| - |\mathcal{V}_{\mathscr{C}_1}(t) - 1|)$ . Note that the notations  $\mathcal{C}$ ,  $\mathcal{R}$  and  $\mathcal{L}$  in Fig. 1 represents  $\mathscr{C}$ ,  $\mathscr{R}$  and  $\mathscr{L}$  in (38).

Based on the displayed circuit model, the RSNCDNs (1) with 5 nodes, assuming that there exists external disturbances  $\overline{\omega}_i(t)$  along with the control input  $u_i(t)$  is described as

$$\dot{\xi}_{i}(t) = A\xi_{i}(t) + B\zeta(\xi_{i}(t)) + a \sum_{j=1}^{5} c_{ij}D\xi_{j}(t) + b \sum_{j=1}^{5} e_{ij}F$$
  
$$\xi_{j}(t - d(t)) + Gu_{i}(t) + H\varpi_{i}(t), (i = 1, 2, ..., 5),$$
  
(39)

with the borrowed parameters A =  $\begin{bmatrix} -z_1z_2 & z_1z_2 & 0\\ z_2 & -z_2 & z_2\\ 0 & z_3z_2 & z_4z_2 \end{bmatrix},$ 

$$\mathbf{B} = \begin{bmatrix} -z_1 z_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and the nonlinear function } \varsigma(\xi_i(t)) \text{ to}$$



FIGURE 7. Event occuring instants of nodes 1, 2, 3 of RSNCDNs (39).

be  $\mathfrak{g}(.)$  in (38), wherein  $z_1 = \frac{\mathscr{C}2}{\mathscr{C}1}$ ,  $z_2 = \frac{1}{\mathscr{R}_p \mathscr{C}2}$ ,  $z_3 = -\frac{\mathscr{C}2\mathscr{R}_p}{\mathscr{L}}$ and  $z_4 = -\frac{\mathscr{C}2\mathscr{R}_p \mathscr{R}_q}{\mathscr{L}}$ .

The parameters of memristor-based Chua's circuit to be explored is given by  $\mathscr{C}1 = 0.4$ ,  $\mathscr{C}2 = 0.48$ ,  $\mathscr{R}_p = 2$ ,  $\mathscr{R}_q = 4$ ,  $\mathscr{L} = 4$ . In addition, the other parameters associated with RSNCDNs (39) taken as  $G = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ ,  $H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ ,  $J = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}$ .

The inner coupling matrices of normal and delayed coupling of RSNCDNs are assigned as  $D = I_3$  and F =



FIGURE 8. Event occuring instants of nodes 4, 5 of RSNCDNs (39).

 $I_3$ , respectively. The coupling strengths are categorized as a = b = 0.1, whereas the outer coupling matrices

	1 -2 1 0 0	
are chosen to be $C =$	0 1 -2 1 0	and $E =$
	0 0 1 -2 1	
	1 0 0 1 -2	
$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \end{bmatrix}$		
1 -1 0 0 0		
$0 \ 0 \ -1 \ 1 \ 0$		
0 0 1 -2 1		

Firstly, we ought to check whether the considered system parameters meet the observability condition. For this purpose, we compute the observability matrix *Obs* with the aid of MATLAB toolbox from the system matrices A and J chosen

and is obtained as 
$$Obs = \begin{bmatrix} 1.0000 & 0 & 0 \\ -1.2500 & 1.2500 & 0 \\ 2.8646 & -2.8646 & 2.6042 \end{bmatrix}$$

This makes it clear that the rank of *Obs* is 3. As the observability matrix has full rank, ie., the rank is equal to the number of states, it is clear that the addressed RSNCDNs are observable. Moreover, the scalars employed in the feasibility assessment

![](_page_10_Figure_2.jpeg)

**FIGURE 9.** Bernoulli distributed random variable  $\alpha(t)$  with its expectation value  $\bar{\alpha} = 0.4$ .

are intended to be  $d_M = \tau_M = \eta_M = 0.1$ ,  $\sigma = 0.22$ ,  $\theta = 0.00001$ ,  $\mu = 0.01 \omega = 0.1$ ; the expectation value of the Bernoulli distributed random variable  $\alpha(t)$  engaged in hybrid triggering mechanism is opted as  $\bar{\alpha} = 0.4$ ; the parameters related to linear fractional nonfragile gain fluctuations are  $\mathcal{M} = 0.001$ ,  $\mathcal{N} = [0.001 \ 0.001 \ 0.001]$  and  $\mathcal{G} = 0.0001I$ .

The feasible collection of matrices is found under the conditions shown above. the matrix  $\Omega$  engaged in the event generator function, the requisite controller and observer gains are, respectively, earned as

$$K = \begin{bmatrix} -0.0636 & -0.0471 & -0.0160 \end{bmatrix}, L = \begin{bmatrix} -0.0636 & 0 & 0 \end{bmatrix}^{T}$$
  
and 
$$\Omega = \begin{bmatrix} 36.0269 & -25.0382 & -0.6993 \\ -25.0382 & 63.4427 & -16.9119 \\ -0.6993 & -16.9119 & 13.1793 \end{bmatrix}.$$

After which, in order to move forward with the simulation testing, we choose the necessary parameters listed herein. The time-varying coupling delay d(t) in (39) with bound [0, 0.1] is picked as  $d(t) = 0.05 + 0.05 \sin(t)$ , the external disturbance functions are taken as  $w_i(t) = 0.2(\sin(t) + 1)$  $sin(\pi t)$ , i = 1, 2, ..., 5 and function  $\mathscr{F}(t)$  concerned in nonfragilty is  $\mathscr{F}(t) = \sin(t)$  which is selected in such as way that it satisfies  $\mathscr{F}^T(t)\mathscr{F}(t) \leq 1$  as indicated in Lemma 3 of [45]. Alongside the above specified information in hand and the initial conditions  $\xi_1(t_0) = \hat{\xi}_1(t_0) = \begin{bmatrix} 8 & 6 & 8 \end{bmatrix}^T$ ,  $\xi_3(t_0) = \hat{\xi}_3(t_0) = \begin{bmatrix} 5 & 6 & 5 \end{bmatrix}^T$ ,  $s(t_0) = \xi_r(t_0) = \hat{\xi}_r(t_0) =$  $[-10 \ 2 \ -10]^T$ , r = 2, 4, 5, with sampling period 0.01, the simulation outcomes are illustrated in the Figures 2-9. The effective synchronization responses of the three trajectories of RSNCDNs (39) under the intended controller is disclosed in Fig. 2. As is shown in this figure, the trajectories of each node of RSNCDNs (39) are effectively synchronized to their respective isolated node trajectories which shows that the developed controller does its best to achieve the intended performance. Withal, it is evident from Figs. 3, 4 that, the states of the addressed RSNCDNs are estimated by the designed Luenberger observer (3) in a vivid manner for all the five nodes. The input-data release instants along with its time-intervals for 5 nodes of RSNCDNs is clearly demonstrated in Figs. 5, 6. And here, it is apparent that, the control data packets are sent only when needed and it is obvious that the unnecessary usage of resources are prevented. Meanwhile, the instants at which the event generator function (5) is violated has been marked in the Figs. 7, 8. It can be seen that, the number of triggers via the employed hybrid-triggered controller lies between 45 to 50 for all the nodes. In a further, with a traditional time-triggered controller, where data packets are exchanged on a periodic basis, there are 2000 updates to the control input data. Here, it is apparent that with the implementation of our designed controller the network burden is reduced to 2.5% which is of huge benefit in real-time scenarios. In addition, the trajectories of the random variable  $\alpha(t)$  involved in the swap process between time and event triggered control protocol is displayed in Fig. 9. Thus, from the results provided and graphs represented, it is obvious that the devised extended dissipative non-fragile hybridtriggered controller scheme served best in accomplishing the desired requirements while improving the state estimation performance under the satisfied disturbance attenuation index for the addressed RSNCDNs (1).

# **VI. CONCLUSION**

This work reports the scrutinization on the topic of state estimation-based synchronization issues for RSNCDNs liable to time-varying coupling delays and external disturbances. In particular, CDNs with a new class of repeated scalar nonlinearities is examined instead of the conventional nonlinear functions. Notably, Luenberger observer, which estimates the states of the system to a significant extent, is meant to cope with the system states that are generally immeasurable in realistic conditions. Withal, a hybridtriggered control protocol based on Luenberger observer approach has been developed that tackle problems such as limited communication bandwidth while still ensuring system's performance. Besides which, extended dissipativity a unified form of other existing performances like  $H_{\infty}$ , passivity,  $L_2 - L_\infty$  and dissipativity has been incorporated with an eye to lessen the impact generated by external disturbances on the RSNCDNs. Over and above, the adequate conditions have been fabricated by the virtue of Lyapunov stability theory in the form of linear matrix inequalities in the context of assuring the stability of resulting augmented closed-loop system. In the end, a conclusive validation on the rightness of offered theoretical results and efficacy of the attempted approach are given through a simulative representation of memristor-based Chua's circuit model. In addition, our future research will be on addressing the issues of fault estimation and security control simultaneously for a class of nonlinear complex dynamical networks with semi-Markovian jump topologies.

#### REFERENCES

[1] H. Shen, X. Hu, X. Wu, S. He, and J. Wang, "Generalized dissipative state estimation of singularly perturbed switched complex dynamic networks with persistent dwell-time mechanism," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 52, no. 3, pp. 1795–1806, Mar. 2022.

- [2] H. Shen, D. Wang, Z. Wang, J. H. Park, and K. Shi, " $H_{\infty}$  load frequency control for power systems under communication delays: An event-triggered dynamic output feedback scheme," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 69, no. 8, pp. 3495–3499, Aug. 2022.
- [3] H. Ren, P. Shi, F. Deng, and Y. Peng, "Fixed-time synchronization of delayed complex dynamical systems with stochastic perturbation via impulsive pinning control," *J. Franklin Inst.*, vol. 357, no. 17, pp. 12308–12325, Nov. 2020.
- [4] H. Shen, Y. Xia, J. Wang, and J. H. Park, "Fault-tolerant event-triggered H<sub>∞</sub> load frequency control for multiarea power systems with communication delay," *IEEE Syst. J.*, vol. 16, pp. 6624–6634, Dec. 2022.
- [5] P. Shi, X. Li, Y. Zhang, and J. Yan, "Event-triggered quantized input-output finite-time synchronization of Markovian neural networks," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 70, no. 3, pp. 1381–1391, Mar. 2023.
- [6] X. Wang, H. R. Karimi, M. Shen, D. Liu, L.-W. Li, and J. Shi, "Neural network-based event-triggered data-driven control of disturbed nonlinear systems with quantized input," *Neural Netw.*, vol. 156, pp. 152–159, Dec. 2022.
- [7] J. Cheng and Y. Zhan, "Nonstationary ℓ<sub>2</sub>−ℓ<sub>∞</sub> filtering for Markov switching repeated scalar nonlinear systems with randomly occurring nonlinearities," *Appl. Math. Comput.*, vol. 365, Jan. 2020, Art. no. 124714.
- [8] Y. Wen, H. Chang, X. Su, and W. Assawinchaichote, "Event-triggered fuzzy control of repeated scalar nonlinear systems and its application to Chua's circuit system," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 67, no. 12, pp. 5347–5357, Dec. 2020.
- [9] L. Wu, X. Su, and P. Shi, "Output feedback control of Markovian jump repeated scalar nonlinear systems," *IEEE Trans. Autom. Control*, vol. 59, no. 1, pp. 199–204, Jan. 2014.
- [10] X. Liu, X. Su, P. Shi, C. Shen, and Y. Peng, "Event-triggered sliding mode control of nonlinear dynamic systems," *Automatica*, vol. 112, Feb. 2020, Art. no. 108738.
- [11] B. Ren, H. R. Karimi, T. Yin, and S. Fu, "Asynchronous H<sub>∞</sub> filtering for semi-Markov jump T–S fuzzy systems within partial state delay and deception attack: Applied to aircraft-pilot state estimation," J. Franklin Inst., Oct. 2022, doi: 10.1016/j.jfranklin.2022.10.026.
- [12] R. Sakthivel, R. Sakthivel, P. Selvaraj, and H. R. Karimi, "Delaydependent fault-tolerant controller for time-delay systems with randomly occurring uncertainties," *Int. J. Robust Nonlinear Control*, vol. 27, no. 18, pp. 5044–5060, Dec. 2017.
- [13] R. Sakthivel, R. Sakthivel, O. M. Kwon, and B. Kaviarasan, "Fault estimation and synchronization control for complex dynamical networks with time-varying coupling delay," *Int. J. Robust Nonlinear Control*, vol. 31, no. 6, pp. 2205–2221, Apr. 2021.
- [14] R. Sakthivel, R. Sakthivel, O. M. Kwon, and P. Selvaraj, "Observerbased synchronization of fractional-order Markovian jump multi-weighted complex dynamical networks subject to actuator faults," *J. Franklin Inst.*, vol. 358, no. 9, pp. 4602–4625, Jun. 2021.
- [15] Z. Hu, H. Ren, and P. Shi, "Synchronization of complex dynamical networks subject to noisy sampling interval and packet loss," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 33, no. 8, pp. 3216–3226, Aug. 2022.
- [16] Y. Zhao, F. Fu, J. Wang, J. Feng, and H. Zhang, "Synchronization of hybrid-coupled delayed dynamical networks with noises by partial mixed impulsive control strategy," *Phys. A, Stat. Mech. Appl.*, vol. 492, pp. 1181–1193, Feb. 2018.
- [17] Q. Wang, M. Perc, Z. Duan, and G. Chen, "Synchronization transitions on scale-free neuronal networks due to finite information transmission delays," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 80, no. 2, Aug. 2009, Art. no. 026206.
- [18] Q. Wang, M. Perc, Z. Duan, and G. Chen, "Impact of delays and rewiring on the dynamics of small-world neuronal networks with two types of coupling," *Phys. A, Statist. Mech. Appl.*, vol. 389, no. 16, pp. 3299–3306, Aug. 2010.
- [19] F. Parastesh, K. Rajagopal, S. Jafari, M. Perc, and E. Schöll, "Blinking coupling enhances network synchronization," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 105, no. 5, May 2022, Art. no. 054304.
- [20] F. Parastesh, H. Azarnoush, S. Jafari, B. Hatef, M. Perc, and R. Repnik, "Synchronizability of two neurons with switching in the coupling," *Appl. Math. Comput.*, vol. 350, pp. 217–223, Jun. 2019.
- [21] S. Majhi, M. Perc, and D. Ghosh, "Dynamics on higher-order networks: A review," J. Roy. Soc. Interface, vol. 19, no. 188, Mar. 2022, Art. no. 20220043.

- [22] Y. Lu, H. R. Karimi, and N. Zhang, "Observer-based H<sub>∞</sub> consensus for linear multi-agent systems subject to measurement outliers," J. Smart Environ. Green Comput., vol. 1, pp. 60–75, Mar. 2021.
- [23] X. Wang, X. Liu, K. She, S. Zhong, and L. Shi, "Delay-dependent impulsive distributed synchronization of stochastic complex dynamical networks with time-varying delays," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 49, no. 7, pp. 1496–1504, Jul. 2019.
- [24] J. Feng, K. Cheng, J. Wang, J. Deng, and Y. Zhao, "Pinning synchronization for delayed coupling complex dynamical networks with incomplete transition rates Markovian jump," *Neurocomputing*, vol. 434, pp. 239–248, Apr. 2021.
- [25] N. Wang and F. Hao, "Event-triggered sliding mode control with adaptive neural networks for uncertain nonlinear systems," *Neurocomputing*, vol. 436, pp. 184–197, May 2021.
- [26] J. Liu, T. Yin, D. Yue, H. R. Karimi, and J. Cao, "Event-based secure leader-following consensus control for multiagent systems with multiple cyber attacks," *IEEE Trans. Cybern.*, vol. 51, no. 1, pp. 162–173, Jan. 2021.
- [27] G. Song, P. Shi, and R. K. Agarwal, "Fixed-time sliding mode cooperative control for multiagent networks via event-triggered strategy," *Int. J. Robust Nonlinear Control*, vol. 31, no. 1, pp. 21–36, Jan. 2021.
- [28] J. Liu, Y. Wang, L. Zha, X. Xie, and E. Tian, "An event-triggered approach to security control for networked systems using hybrid attack model," *Int. J. Robust Nonlinear Control*, vol. 31, no. 12, pp. 5796–5812, Aug. 2021.
- [29] F. Rahimi and H. Rezaei, "An event-triggered recursive state estimation approach for time-varying nonlinear complex networks with quantization effects," *Neurocomputing*, vol. 426, pp. 104–113, Feb. 2021.
- [30] L. Zha, E. Tian, X. Xie, Z. Gu, and J. Cao, "Decentralized event-triggered  $H_{\infty}$  control for neural networks subject to cyber-attacks," *Inf. Sci.*, vol. 457, pp. 141–155, Aug. 2018.
- [31] Y.-A. Liu, S. Tang, Y. Liu, Q. Kong, and J. Wang, "Extended dissipative sliding mode control for nonlinear networked control systems via eventtriggered mechanism with random uncertain measurement," *Appl. Math. Comput.*, vol. 396, May 2021, Art. no. 125901.
- [32] J. Qi and Y. Li, "Hybrid-triggered fault detection filter design for networked Takagi–Sugeno fuzzy systems subject to persistent heavy noise disturbance," *Int. J. Adapt. Control Signal Process.*, vol. 35, no. 6, pp. 1062–1082, Jun. 2021.
- [33] J. Liu, J. Xia, J. Cao, and E. Tian, "Quantized state estimation for neural networks with cyber attacks and hybrid triggered communication scheme," *Neurocomputing*, vol. 291, pp. 35–49, May 2018.
- [34] X. Chen, L.-Y. Yin, Y.-T. Liu, and H. Liu, "Hybrid-triggered consensus for multi-agent systems with time-delays, uncertain switching topologies, and stochastic cyber-attacks," *Chin. Phys. B*, vol. 28, no. 9, Sep. 2019, Art. no. 090701.
- [35] D. Liu and D. Ye, "Pinning-observer-based secure synchronization control for complex dynamical networks subject to DoS attacks," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 67, no. 12, pp. 5394–5404, Dec. 2020.
- [36] Y. Li, K. Li, and S. Tong, "An observer-based fuzzy adaptive consensus control method for nonlinear multiagent systems," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 11, pp. 4667–4678, Nov. 2022.
- [37] J. Wang, H. Wang, J. Xia, and H. Shen, "H<sub>∞</sub> synchronization of persistent dwell-time switched neural networks based on an observer-based sliding mode scheme," *Nonlinear Anal., Hybrid Syst.*, vol. 41, Aug. 2021, Art. no. 101046.
- [38] T. Li, W. Wang, and W. Chen, "Event-triggered observer-based H<sub>∞</sub> control of switched linear systems with time-varying delay," *Int. J. Syst. Sci.*, vol. 52, no. 8, pp. 1618–1632, Jan. 2021.
- [39] B. Jiang, D. Liu, H. R. Karimi, and B. Li, "Observer-based event-triggered H-infinity sliding control of Markovian jump system suffer from actuator attacks," *Asian J. Control*, Nov. 2022, doi: 10.1002/asjc.2998.
- [40] X. Li, H. Zhu, and S. Song, "Input-to-state stability of nonlinear systems using observer-based event-triggered impulsive control," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 51, no. 11, pp. 6892–6900, Nov. 2021.
- [41] J. Zhang, H. Zhang, K. Zhang, and Y. Cai, "Observer-based output feedback event-triggered adaptive control for linear multiagent systems under switching topologies," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 33, no. 12, pp. 7161–7171, Dec. 2022.
- [42] C. Zhao, S. Zhong, X. Zhang, Q. Zhong, and K. Shi, "Novel results on nonfragile sampled-data exponential synchronization for delayed complex dynamical networks," *Int. J. Robust Nonlinear Control*, vol. 30, no. 10, pp. 4022–4042, Jul. 2020.

- [43] X. Zhang, C. Li, H. Li, and Z. Cao, "Mean-square stabilization of impulsive neural networks with mixed delays by non-fragile feedback involving random uncertainties," *Neural Netw.*, vol. 154, pp. 469–480, Oct. 2022.
- [44] C. Ge, Y. Zhang, L. Wang, and Y. Liu, "Non-fragile control with eventtriggered scheme for networked control systems under probabilistic timevarying delays," *J. Franklin Inst.*, vol. 359, no. 10, pp. 4856–4873, Jul. 2022.
- [45] G. Liu, J. H. Park, S. Xu, and G. Zhuang, "Robust non-fragile H<sub>∞</sub> fault detection filter design for delayed singular Markovian jump systems with linear fractional parametric uncertainties," *Nonlinear Anal., Hybrid Syst.*, vol. 32, pp. 65–78, May 2019.
- [46] Y.-A. Liu, J. Xia, B. Meng, X. Song, and H. Shen, "Extended dissipative synchronization for semi-Markov jump complex dynamic networks via memory sampled-data control scheme," *J. Franklin Inst.*, vol. 357, pp. 10900–10920, Oct. 2020.

![](_page_12_Picture_6.jpeg)

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![](_page_12_Picture_9.jpeg)

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![](_page_12_Picture_11.jpeg)

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![](_page_12_Picture_13.jpeg)

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