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METHODS

Multiobjective Tuning Technique for MPC in Grinding Circuits

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ABSTRACT We investigate the control challenges in grinding circuits—slow dynamics, long dead times, variable coupling— and the controller tuning challenge, that is, the difficulty in translating operating goals into tuning goals and closed-loop performance. A tuning algorithm for DMC (dynamic matrix control), suitable for the mineral processing industry, is proposed. The tuning problem is posed as a multiobjective optimization problem, in which the tuning goals are directly related to the desired closed-loop performance of process variables. The problem is solved using a compromise optimization, which minimizes the Euclidian distance between a feasible solution and the Utopia solution. Three case studies are presented, which validate the tuning algorithm for DMC in linear and non-linear grinding circuit models. The closed-loop performance obtained with the proposed tuning algorithm is compared to the one obtained through a benchmark tuning technique from the literature. The proposed tuning method has the following features: i) it shapes the closed-loop response according to the goal definitions for linear systems; ii) it requires tailored initial guesses and search spaces to converge to a stabilizing solution in non-linear applications; and iii) it allows the user to specify the desired closed-loop performance behavior in the tuning procedure, allowing the implementation of an adequate controller for each situation.

INDEX TERMS Grinding circuit, model predictive control, dynamic matrix control, controller tuning, multiobjective optimization, compromise optimization.

I. INTRODUCTION

Controller design of grinding processes faces several challenges such as nonlinearities, coupling and interaction between variables, time-varying parameters, long time delays, and noisy measurements [1], [2], [3], [4], [5]. Moreover, unmeasured disturbances that affect the process must be compensated and the particle size of the throughput of a grinding circuit strongly affects the degree of mineral liberation in downstream concentration processes. Therefore, feedback control is usually employed to address the operational challenges and improve the efficiency of grinding processes [6]. Model predictive control (MPC) is an attractive control methodology for grinding circuits. Long time-delays and variable coupling are easily addressed using a multiple input multiple output (MIMO) model of the process, and input and output constraints can be included in the controller formulation. The work in [7] has estimated that a well-designed MPC framework can yield 1-2% throughput increase— or USD 1 million in revenue per

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year— when compared to proportional-integral-derivative (PID) controllers in copper grinding mines.

Many industries and companies overlook the importance of MPC tuning. In a depressed economy, decision makers cannot justify investments in new hardware for the implementation or expansion of advanced control strategies [8]. Therefore, properly tuning of existing control frameworks, aiming towards better utilization of resources and higher profit, is an interesting and cheaper alternative to improve a mining plant's bottom line [9]. According to [10], a properly tuned decentralized control strategy based on PID controllers performs better than advanced control strategies (e.g. MPC), given that the effects of model mismatch and disturbances are mild. Therefore, from the financial point of view, the task of tuning an MPC should yield even better operating performance at a low investment cost.

In this paper, a tuning technique for MPC based on multiobjective optimization, originally developed and applied in the petrochemical industry [11], is extended for grinding circuits. The main contributions are:

- An offline tuning method for the weighting matrices of MPC based on multiobjective optimization is presented. The Euclidian distance between feasible solutions and the Utopia solution is minimized to obtain the optimal tuning parameters.
- The goals of the multiobjective optimization method are defined in terms of how fast or slow the closed-loop response should be, compared to an open-loop response of key input and output variables, represented as first order plus dead time transfer functions. This approach is intuitive for plant operators.
- The proposed tuning method has been assessed in simulations of increased complexity when it comes to the number of inputs and outputs of the simulated system. The method has also been tested in a validated model of a regrinding circuit in the Andritz® IDEAS® dynamic simulator.
- The proposed method is compared to a similar tuning strategy from the literature, and the simulations indicate that it outperforms a established tuning strategy in terms output reference tracking and disturbance rejection.
- Finally, the proposed technique is a strong candidate for industrial applications because: (i) it is an offline tuning method with (ii) intuitive definition of goals and (iii) can outperform an established tuning technique from the literature.

This paper is organized as follows: Section II presents the state of the art of tuning strategies for grinding circuit control; Sections III and IV summarize a typical grinding circuit and the compromise tuning method [11], respectively. In Section V, three application examples of the tuning technique are presented, in increasing order of complexity: *i*) a 2×2 grinding circuit from the literature; *ii*) a 4×4 grinding circuit from the literature; and *iii*) a 4×4 non-linear regrinding plant. The paper closes with final remarks in Section VI.

II. LITERATURE REVIEW

MPC is commonly classified as a category of advanced process control (APC), automation technologies that typically operate as a supervisory system above the regulatory-level control, which are designed to improve the performance of the process [12]. A survey conducted by [13] has gathered data from academics and process engineers regarding the level of automation present in the mineral processing industry. The survey has reported that MPC is the most common APC in the mining and mineral processing industry. However, MPC research applies mainly to grinding/milling, and flotation circuits [12]. For example, [6] has implemented a non-linear MPC (NMPC) in a simulated grinding and milling circuit. Despite the nonlinearities and disturbances, the proposed controller has successfully managed output tracking with the selected prediction horizon, which was large enough to contemplate the process dynamics but not overly large as to hinder the NMPC calculation step. Reference [7] has developed an MPC to control a 5×10 transfer function matrix model of a semi-autogenous grinding (SAG) grinding circuit, identified by inferential measurements. The process matrix is mostly comprised of first order plus dead time (FOPDT) models, except for the integrating processes, which have been modeled using rate-of-change models. Simulation results have indicated that the performance of the MPC is as good as a nominal PID. The authors in [3] have investigated the industrial application of constrained MPC in a 4×4 grinding circuit. The MPC has been compared to decentralized PIDs and manual control in industrial settings. Results have showed that MPC yields smoother output responses, satisfying all quality constraints, and smoother control moves. However, none of the works above has provided tuning guidelines, even though this is a critical aspect of the controller's formulation and implementation. Compared to PID control, MPC is more flexible but also more complex, in particular regarding the number of tuning variables. Depending on the MPC formulation, the list of tuning variables include: prediction horizon, control horizon, weights on the output error, weights on the rates of change of manipulated variables, weights on the magnitudes of manipulated variables, reference trajectory parameters, and soft constraint weights [14]. In the following, we summarize applications of tuning strategies for MPC in grinding circuits reported in the literature: trial and error, expert knowledge, heuristics, and multiobjective optimization.

In industrial practice, MPC tuning aims to balance the trade-off between performance and robustness and is often done by trial and error, in which the tuning parameters are arbitrarily adjusted until the closed-loop performance meets some desired criteria. The authors in [2] have studied the application of constrained and unconstrained MPC in a laboratory-scale grinding circuit. Tuning has been performed by trial and error, seeking adequate closed-loop performance. The performance of MPC and decentralized PI controllers has been compared for setpoint changes in a grinding circuit simulation. Results have showed that multivariable controllers yield smoother and faster output responses with little to no overshoot. A constrained MPC formulation has been suggested to account for overly large input moves. In the model mismatch scenario, the trial and error tuning compensated for mild plant-model mismatch, when compared to decentralized PI. In [4], the MPC control has been studied for a 3×3 grinding circuit model implemented in a simulation platform. Tuning has been also performed by trial and error. The results have revealed that the multiobjective control approach successfully tracked setpoint changes on all three outputs. The authors have emphasized that unconstrained MPC leads to aggressive input moves, which may be outside the feasible range of real equipment. Thus, it might not be feasible for practical implementations.

Another arbitrary approach to MPC tuning is to rely on expert knowledge. In this approach, the tuning variables (usually the weights on inputs and outputs) are defined based on the— financial, safety, operational, environmental— importance of the process variables. In [15], the authors have tuned a dynamic matrix control (DMC) based on process knowledge and trial and error. The length of the model and prediction horizon have been chosen to be large enough to accommodate the whole process dynamics, while the control horizon has been chosen to be shorter than the prediction horizon, ensuring robustness. The weights on inputs and outputs have been determined based on variable importance. The DMC has been compared to a PID implementation in a 3×3 grinding plant. The multivariate control strategy has increased the product quality criteria from 88% to 99%. The authors in [5] have compared a non-linear MPC based on static programming with a standard NMPC in a grinding circuit simulation. The weighting matrices of the MPC have been tuned by expert knowledge. Additionally, the control horizon of the static programming MPC can be set as large as the prediction horizon without compromising robustness and computational efficiency, thus allowing for smoother control actions. In [16], it has been investigated a robust multi-model DMC in which an expert system logic selects the process model based on process disturbances in a ball mill grinding circuit. The weights on outputs and inputs in the DMC cost function have been calculated based on process knowledge. High weight has been attributed to product particle size distribution (PSD), since it is the most important controller variable, and high weight has been attributed to the fresh feed rate, as it should not be allowed to vary excessively to ensure stable production. Moreover, the industrial application of the proposed algorithm has resulted in a 3% improvement in product PSD and a lower number of alarms compared to a regular DMC. Although an adaptive multivariable control strategy has been proposed, the simulation study has considered a single set of tuning variables, independent of which process model has been used by the DMC.

Although arbitrary tuning strategies may provide good performance, they do not guarantee optimal solutions. Robust results can be achieved using tuning methods based on heuristic equations, multi-objective optimization problems, or other techniques. The authors in [17] have studied the design of an MPC based on the impulse model of a grinding circuit. A time-varying gain has been introduced to mitigate overshoot and coupling in the closed-loop responses, which has been defined as the tuning variable of the proposed control strategy. According to the authors, its calculation is straightforward as long as the impulse or step response models of the plant are available. In [18], a self-tuning MPC has been developed that maximizes throughput in a grinding circuit. The self-tuning algorithm considers the weights on output deviations from the setpoint and on control moves, as well as prediction and control horizons and the sampling time. The aim is to calculate online the cost function value for different controllers (function of a particular set of tuning parameters) and pick the one that yields the lowest cost. A simulation environment has been developed in C++ to evaluate the MPC. The two-step tuning method is summarized as follows: first, an unconstrained version of the controller is used, and once the optimal tuning parameters are calculated for this case, the input constraints are added to the controller formulation. Then, a new tuning step is performed, starting from the previously calculated tuning parameters. The authors have observed a dramatic performance change between the initial guess and final solution. In a similar vein, [19] proposed an adaptive MPC strategy for paper machines under uncertainty and model degradation. The algorithm relies on exciting the process inputs to compute and minimize the variance of a Fisher information matrix defined by the process model. By recalculating the process model instead of re-tuning the controller algorithm when deviation from the nominal operation point is detected, [19] achieved adequate tracking performance under model uncertainty, but since the decision variables of the optimization problem are dependent on the prediction horizon and the number of inputs and outputs of the model, and moreover since a exhaustive search method was proposed, the method might not be feasible for real time applications in large systems. In [9], a dynamic model has been developed for a grinding system and its steady-state form was used to calculate robust H_∞ controllers that maximize throughput while maintaining the SAG mill power draw as close as possible to its upper bound. Simulation results have revealed that the selection of stability and performance requirements within the proposed framework indirectly sets how aggressive the controller is, thus providing more intuitive tuning guidelines than empirical ones available in the literature (e.g. Ziegler-Nichols). Reference [20] addressed performance requirements directly in the control design stage by proposing an economic MPC for a chemical reactor. In such formulation, a steady state vector (containing information about the inputs and states of the system at steady-state) is included as an additional decision variable to the control problem, thus reducing the conservatism of the solution. Reference [20] do not go into detail about how to select the controller gain or prediction and control horizons, but the proper definition of an economic cost function in the proposed method leads to adequate tracking and disturbance rejection performance, as well as robustness and stability of the closed-loop, even in the presence of model uncertainty. Although developing adaptive controllers with a build-in tuning mechanic is a popular strategy in the literature, researches such as [21] and [22] show that it is also worth to investigate how different optimization algorithms and different ways to define the controller tuning problem affect the calculation of optimal tuning parameters for MPC. Reference [21] studied how heuristic optimization methods (Particle Swarm Optimization, Firefly algorithm, Grey Wolf algorithm and Jaya algorithm) fare in solving the mixed integer nonlinear optimization problem resulting from minimizing a cost function based on the sum of squared errors of the output error with respect to the tuning parameters of a MPC. Results show that the analyzed optimization algorithms converge to relatively close solutions in terms of the cost function value, but widely different values for the optimal decision variables. This indicates that the posed optimization problem does not have a global solution, but rather a set of Pareto solutions. Reference [22] on the other hand focus on analyzing a Genetic Algorithm to solve the MPC tuning problem, but propose a interactive step in which the plant operator defines the relative relevance of different control objectives, which are translated into weights to calculate a single objective cost function. [22] compare how the single objective optimization fares against multiobjective optimization (i.e. all the objectives are considered at the same time resulting in a Pareto front of optimal solutions. The final solution is solved by projecting the Utopia point on the Pareto front). It is claimed that the proposed single objective approach outperforms the multiobjective approach by on average 23.74% of the tested scenarios, in which a cement kiln process in the quarrying industry was considered.

III. PROCESS DESCRIPTION

A. THE GRINDING CIRCUIT

In many references from the literature (e.g. [1], [2], [10], [17], [23]), a grinding circuit is comprised of a mill (breakage function) and a hydrocyclone (separation function), among other ancillary equipment. A mill is usually a large, cylindrical, rotating chamber with three inputs (fresh ore, a circulating stream, and water), and one output (a slurry stream of water and ore). Autogenous grinding (AG) and SAG mills use the fed ore itself as primary grinding media. SAG mills use a small number of steel spheres to help breakage. Ball and rod mills have steel spheres and steel bars, respectively, as their primary grinding media. The slurry is stored in a sump and pumped into a hydrocyclone, which separates the inflow rate into two streams. Its bottom outlet stream, namely underflow, is the circuit's circulating load mixed with fresh ore or fed directly into the mill. The top stream, namely overflow, carries the grinding circuit product to a concentration circuit to extract the final product of the concentrator plant. Ancillary equipment include pumps, conveyor belts, and pipes, in which the slurry travels from one equipment to another. Figure 1 shows a diagram of such process.

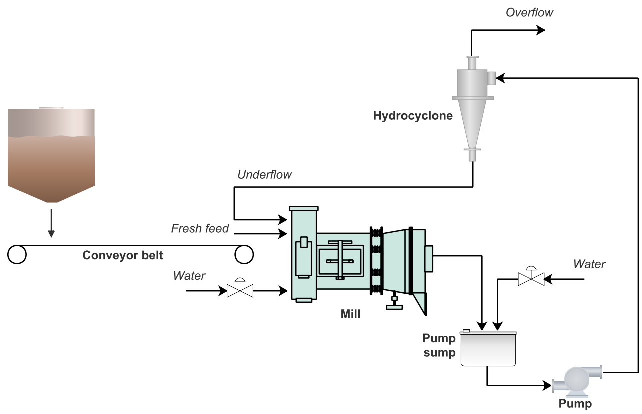

FIGURE 1. Crushing circuit diagram, adapted from [3].

The main control objective in a grinding circuit is to deliver the desired product PSD at the maximum production rate, thereby decreasing the circulating load and reducing the energy consumption of the circuit. In the event of excessive circulating load, milling and separation units might overload and cause plant stops, which is undesirable and must be avoided. Therefore, an usual control strategy considers sump water flowrate and fresh feed flowrate as manipulated variables, and the circulating load and product PSD as controlled variables. Fluctuations in the fresh feed ore hardness and flowrate are common disturbances.

The authors in [3] claim that, although 2×2 grinding models are common in the literature, real control applications based on such models will not operate stably for a long time. For example, the sump level must be considered; otherwise, the product particle size distribution cannot be controlled when the slurry is pumped from the sump. Then, a 4×4 model contemplating the following inputs and outputs, respectively, can be used: *i*) fresh ore feed rate, mill feed water flowrate, dilution water flowrate, and pump speed; and *ii*) product PSD, mill solids concentration, circulating load, and sump level.

B. CONTROL OBJECTIVES OF A GRINDING CIRCUIT

Similar to most industrial processes, grinding circuits benefit from stable operation. Thus, the first objective of a control framework is to deliver stable operation in the presence of disturbances and changes in operating conditions [6]. Once the first task is complete, the remaining degrees of freedom of the system are used to pursue economic goals through the optimization process. Some economic objectives in grinding circuits are [6], [18]:

- to meet product quality specifications (desired PSD) and minimize quality fluctuations;

- to maximize throughput;
- to minimize the amount of steel consumed from the grinding media for each ton of fines produced in a SAG, ball or rod mill;
- to minimize the amount of power consumed for each ton of fines produced.

Many industrial ball mill circuits aim at maximizing throughput while maintaining the product PSD within acceptable bounds. To do so, the ball mill feed rate and the water addition flowrate are the main manipulated variables, while the ore hardness of the feed rate is the main disturbance [24]. In some cases, it is also important to minimize overgrinding [25]. Table 1 summarizes input-output pairing used in the grinding literature for process control, either by decentralized or multivariable approaches.

In [7], the authors have claimed that the SAG power draw and ball charge in the SAG are high priority goals, while the SAG feed rate and density, product PSD, total water flow, cyclone feed flow, and density are lower priority goals. The author in [5] have stated that the product PSD is the most important variable to control, as it determines the economic efficiency of the circuit. In [27], the authors have advocated that the PSD of the slurry stream of grinding circuits is a proxy for the performance of the whole concentrator plant, in terms of throughput, energy efficiency, and separation efficiency. Additionally, according to [18], the power used by a mill is often an indicator of the throughput of the mill.

The most common variable pairings in milling circuits in single input single output (SISO) control strategies are: i) product PSD - sump water dilution rate; and ii) hydrocyclone feed rate - fresh solids feed rate [28]. It is noteworthy that the milling process is highly coupled, making SISO approaches insufficient. An alternative to decrease the observed coupling in the pairing above is to add the mill rotation speed (made possible by variable speed drive motors) as a manipulated variable to control the product PSD. The authors in [28] have stated that the time constant of the hydrocyclone classification is faster than any other time constant of the grinding process. Thus, precise hydrocyclone feedrate control is needed to mitigate unnecessary short-term oscillations in the overall process.

IV. METHODOLOGY

MPC is an advanced control methodology that has significantly impacted the industry, initially, mainly in the petrochemical industry but increasingly present in other process industry sectors [29]. Rather than a specific control strategy, MPC is a wide range of control methods that explicitly uses a system model to define control action by minimizing an objective function [30]. MPC is based on the moving horizon strategy, in which for every time instant t, the algorithm calculates future outputs and future optimal control actions over a prediction horizon p and a control horizon m, respectively. However, only the first control action is implemented, while the other $m-1$ control actions, calculated at time instant t, are discarded.

A. DMC FORMULATION

A DMC algorithm is used in this paper. This type of MPC uses a model of the step responses of the system to predict the outputs. According to [30], its main advantages are: *i*) easy implementation; *ii*) no prior knowledge of the process, which further simplifies implementation; and *iii*) capability to handle multivariable processes. The predicted output, based on each control increment Δu, referring to past and future actions, is obtained by

$$y(t+\mu|t) = \sum_{i=\mu-m+1}^{\mu} g_i \Delta u(t+\mu-i) + f(t+\mu), \quad (1)$$

where $\mu = 1, 2 \ldots, p$, the term $f(t+\mu)$ is the free response of the system, the notation $(t+\mu|t)$ indicates that the variable value is predicted in time instant $t+\mu$ using information available at time instant t, and g_i is the output response i to a unit step input. The free response is independent of future control actions. However, it is dependent on past values of y_i as well as on past control actions.

Equation (1) can be generalized to MIMO linear systems $n_u \times n_y$ using the superposition principle to obtain the n_y predicted outputs caused by the n_u system inputs. Therefore, the predicted output vector is defined as

$$\begin{aligned}\hat{\mathbf{y}} = [&y_1(t+1|t), \ldots, y_1(t+p|t), \ldots, y_{n_y}(t+1|t), \\ &\ldots, y_{n_y}(t+p|t)]^T. \end{aligned} \quad (2)$$

Similarly, future control signals and free response vectors are defined, respectively, as

$$\begin{aligned}\Delta\mathbf{u} = [&\Delta u_1(t), \ldots, \Delta u_1(t+m-1), \ldots, \Delta u_{n_u}(t), \\ &\ldots, u_{n_u}(t+m-1)]^T, \end{aligned} \quad (3)$$

$$\begin{aligned}\mathbf{f} = [&f_1(t+1|t), \ldots, f_1(t+p|t), \ldots, f_{n_y}(t+1|t), \\ &\ldots, f_{n_y}(t+p|t)]^T. \end{aligned} \quad (4)$$

For a multivariable process, every output y_j is affected by the inputs according to

$$y_j(t) = \sum_{k=1}^{n_u} \sum_{i=1}^{n_k} g_i^{kj} \Delta u_k(t-i), \quad (5)$$

where $g_i(t)$ is the output response j to a unit step in input k, Δu_k is the k_{th} process input, and n_k is the number of samples taken to stabilize the response to a step input at time k.

A common cost functional, in which future errors and the controller's control effort are minimized, is calculated by

$$V_{1,t} = \sum_{j=0}^{p} \left\| y(t+j|t) - y_{sp} \right\|_{Q_y}^2 + \sum_{j=0}^{m-1} \left\| \Delta u(t+j|t) \right\|_R^2 \quad (6)$$

where $Q_y \in \mathbb{R}^{n_y \times n_y}$ and $R \in \mathbb{R}^{n_u \times n_u}$ are positive definite and positive semi-definitive diagonal weighing matrices, respectively [31]. The DMC algorithm is widely used in the industry, and thus, a good candidate for validate the proposed tuning technique.

TABLE 1. Summary of input-output pairings and the control strategy from the literature.

Reference	Output	Input	Controller
[24]	Product PSD Circulating load Mill solids holdup Sump level	Feed rate and total water flowrate Constraint on feed and water addition Feed water addition Pumping rate	PID
[26]	Product PSD Cyclone feed rate	Feed rate Cyclone dilution	Robust INA[1]
[7]	SAG Power drawn	SAG Feed rate	PID
[3]	Product PSD Mill solids concentration Circulating load Sump level	Fresh ore feed rate Mill feed water flowrate Dilution water flowrate Pump speed	MPC
[16]	Product PSD Mill solids concentration Circulating load	Fresh ore feed rate Mill feed water flowrate Dilution water flowrate	Adaptive DMC
[4]	Power draw PSD reduction percentage Product Throughput	Total water flowrate Fresh ore feed rate Mill rotation speed	MPC
[27]	Product PSD > 210 Product PSD $< 74\mu$m Circulating loadμm	Ore feedrate Outlet slurry density Number of active Hydrocyclones	Fuzzy MPC
[18]	Mill load Product PSD Sump level	Flowrate to cyclone Fresh feed rate Dilution water	MPC
[25]	Mill solids concentration Circulating load Product PSD Mill discharge sump level	Pump flowrate setpoint Mill water flowrate setpoint Sump water flowrate setpoint Pump flowrate setpoint	MPC

1. Inverse Nyquist array.

B. TUNING ALGORITHM

The tuning algorithm developed in [11] for MPC with zone control and input targets is adapted for the DMC adopted in this paper. We present a brief description of the method. From the discussion presented in the previous subsection, the vector $x = \left[q_{y,1} \ \cdots \ q_{y,n_y} \ r_1 \ \cdots \ r_{n_u}\right]$ is hereafter defined as the tuning parameters of the DMC. The element $q_{y,i}$, $i = 1, \ldots, n_y$, denotes the entries of the weighing matrix Q_y, and r_i, $i = 1, \ldots, n_u$, denotes the entries of the weighing matrix R. Note that the control and prediction horizons are not considered in the tuning parameter vector. This decision is supported by the following arguments:

- There are reliable tuning methods available in the literature for the selection of these variables [11];
- The prediction and control horizons are process dependent. Therefore, well-oriented initial guesses usually provide adequate results;
- Including integer variables in the optimization problem would result in a mixed-integer problem, which is more complex.

1) TUNING GOALS

The tuning goals are solely defined in terms of a desired dynamic of the process outputs. The following procedure is used:

1) List process outputs in order of importance. Use operational, financial, environmental criteria;
2) Select the most representative process input for each output, following the order defined in item 1. Observe that it is not prohibited to use duplicate inputs if the process demands it.
3) Define unitary first-order plus dead time (FOPDT) transfer functions, $G_{des,i}(s) = \frac{e^{-\theta_i s}}{1+\tau_i s}$, $i = 1, \cdots, n'_y$ i.e. a time constant, unitary gain, and dead-time when applicable, for each input-output pair defined in item 2. The term n'_y denotes the number of outputs that are considered for the tuning method, $n'_y \leq n_y$.

Now, it is possible to calculate reference trajectories based on $G_{des,i}(s)$. The tuning goals are defined as

$$F_i(t) = \sum_{t=1}^{\Theta} (y_{ref,i}(t) - y_i(t))^2, \ i = 1, \cdots, n'_y, \qquad (7)$$

where Θ denotes the tuning horizon, which should be chosen to be large enough to contemplate the entire process dynamics; $y_{ref,i}(t)$ is the desired value of output i calculated at time instant t, using the appropriate step response of $G_{des,i}(s)$, and $y_i(t)$ is the closed-loop value of output i. Observe that $y_i(t)$ is a function of the vector of decision variables since the tuning parameters determine the closed-loop response of the system.

2) MULTIOBJECTIVE OPTIMIZATION

Consider the following multiobjective optimization problem:

a: PROBLEM 2

$$\min_x F(x) = \left[F_1(x) \ \cdots \ F_w(x)\right]^T \qquad (8)$$

subject to:

$$g(x)_j \leq 0,\ j = 1, \ldots, z,$$
$$h(x)_l = 0,\ l = 1, \ldots, e, \tag{9}$$

where $g(x)_j$ and $h(x)_l$ denote inequality and equality constraints, respectively. The feasible design space is defined as $\mathbf{X} = \{x \in \mathbb{R}^n | g_j(x) \leq 0,\ j = 1, \cdots, z$ and $h(x)_l = 0,\ l = 1, \cdots, e\}$, in which n is the number of decision variables of the problem, and the feasible criterion space is defined as $\mathbf{Z} = \{z \in \mathbb{R}^w | z = F(x),\ x \in \mathbf{X}\}$.

Definition 1: A solution $x^* \in \mathbf{X}$ is a Pareto optimum, or a non-dominated solution iff there does not exist another solution $x \in \mathbf{X}$, such that $F_i(x) \leq F_i(x^*)$, for at least one i.

Definition 2: A solution $F^\circ(x) \in \mathbf{Z}$ is an Utopia point iff for each $i = 1, \cdots, w$, $F_i^{\deg} = min_x\{F_i(x)|x \in \mathbf{X}\}$.

A compromise solution approach is proposed, in which the Pareto optimum closest to the Utopia point is selected as the solution of Problem 2.

3) COMPROMISE TUNING ALGORITHM

The compromise tuning algorithm is based on the following procedure. First, the Utopia solution is calculated. Observe that the Utopia solution is often unfeasible due to conflicting constraints.

a: PROBLEM 3a

$$F_i^\circ = \min_x F_i(x),\ i = 1, \ldots, w \tag{10}$$

subject to (7) and

$$LB \leq x \leq UB. \tag{11}$$

Consider that $w = n_y'$ is the number of tuning objectives, $x \in \mathbb{R}^{n_y+n_u}$ is the vector of decision variables, and LB and UB are its upper and lower bounds, respectively. Observe that w optimization problems are defined, each one taking into account one objective at a time.

Once the Utopia solution is available, the following compromise optimization problem is solved:

b: PROBLEM 3b

$$\min_x \left\| F^\circ - F(x) \right\| \tag{12}$$

subject to (7) and (11), where $x \in \mathbf{X}$ is calculated as the closest point to the Utopia solution, in terms of Euclidean distance.

Remark: The proposed methodology assumes that a process model is available for control design, but the designer has no freedom to modify the process model, or to re-identify the process.

V. APPLICATION RESULTS

In this section, we present applications of the proposed tuning technique for DMC in grinding circuits. The applications are in ascending order of model complexity. First, an unconstrained DMC is tuned for a linear 2 × 2 system; after, an unconstrained DMC is synthesized for a linear 4 × 4 system; and lastly, the tuning algorithm is used to adjust a DMC for a non-linear 4 × 4 grinding circuit. In all applications, the proposed tuning technique is compared to a benchmark tuning technique.

TABLE 2. Input and output variables, units and tags of Application I.

Variable name	Units	Tag
Fresh ore feed rate and dilution	*	u_1
Cyclone dilution	*	u_2
Product particle size	*	y_1
Flow rate of feed to the cyclone	*	y_2

A. BRIEF DESCRIPTION OF THE BENCHMARK TUNING TECHNIQUE

In [32], the authors have proposed a simple, yet robust, tuning technique for predictive controllers. This technique makes use of the following penalty function:

$$PM_i = 3\left(1 + \frac{6DT_i}{p} + \frac{3G_iDT_i}{p}\right), \tag{13}$$

to automatically calculate the penalty on controller moves, in which PM is equivalent to the diagonal elements of R. It is noteworthy that the penalty on errors, PE (equivalent to the diagonal elements of Q_y), is kept in a default value of 1. A trial and error approach should be used to adjust PE for better performance if necessary. Also, the term G_i is the gain for each pair, DT_i is the dead time for each loop pair, and p is the prediction horizon. The pairing decision is made based on the coupling level between variables.

B. APPLICATION I - LINEAR 2 × 2 SYSTEM

The goal of Application I is to corroborate that the proposed tuning technique for DMCs is capable to calculate tuning parameters, in which the desired closed-loop performance is specified as reference trajectories for the actual closed-loop control performance.

1) PROCESS MODEL

In the first application, the nominal system obtained from [26] is represented by the following transfer functions:

$$G(s)_1 = \begin{bmatrix} \frac{-0.9362e^{-350s}}{1164s+1} & \frac{(10.252s+2.819\times10^{-3})e^{-200s}}{(80218s^2+652s+1)} \\ \frac{36.49}{792s+1} & \frac{1.1405}{179s+1} \end{bmatrix}. \tag{14}$$

Table 2 summarizes the inputs and outputs of the process.

2) TUNING GOALS

In [26], the authors have designed a robust Inverse Nyquist Array controller to promote variable decoupling, thus reducing the steady-state offset and overshoot of the non-dominant outputs. In order to translate it as a tuning goal for the compromise tuning technique, we selected the pairing $y_1 - u_1$, and $y_2 - u_2$. Since transfer functions $G(s)_1(i, i)$, $i = 1, \ldots, n_y$, are already FOPDT, no approximation or fitting is

TABLE 3. Tuning results in Application I.

Parameters	Benchmark	Proposed
Q_y	$\begin{bmatrix}1.0000 & 1.0000\end{bmatrix}$	$\begin{bmatrix}11.6324 & 0.3145\end{bmatrix}$
R	$\begin{bmatrix}33.8301 & 3.0000\end{bmatrix}$	$\begin{bmatrix}100.0000 & 11.3830\end{bmatrix}$
$[f_1\ f_2]$	*	$\begin{bmatrix}0.5 & 1\end{bmatrix}$

required. The transfer functions of the reference trajectories are selected as

$$G(s)_{1,des} = \begin{bmatrix} \frac{1e^{-350s}}{1164s\times f_1+1} & 0 \\ 0 & \frac{1}{179s\times f_2+1} \end{bmatrix}, \quad (15)$$

where f_1 and f_2 are tuning factors that define (based on the process requirements) whether tuning should be done to prioritize faster or slower transient responses to output set point changes. It is worth highlighting that the steady-state gains from the original transfer functions in (14) have been replaced by 1, since in the tuning step the gain value of the reference trajectories is not relevant, as arbitrary values can be chosen as long as input and output bounds are respected.

The decision variables of the tuning optimization problem are $x = \left[Q_y\ R\right]^T$, the initial guesses are $x_0 = \left[10\ 10\ 0.1\ 0.1\right]^T$, and the lower and upper bounds of the tuning problem are $x_{min} = \left[0.01\ 0.01\ 0.01\ 0.01\right]^T$ and $x_{max} = \left[100\ 100\ 100\ 100\right]^T$, respectively. The tuning problem is solved using *fmincon* (MATLAB$^{\text{TM}}$, *sqp* algorithm, 10^{-4} TolX, 10^{-4} TolFun, 4×10^4 MaxFunEvals, 10^6 MaxIter). In this example, we consider an unconstrained DMC (there are no constraints on inputs, outputs, and input increments).

3) RESULTS

The prediction horizon, control horizon, and sample time for both tuning techniques are $p = 300$, $m = 3$, and 1 sec, respectively. Table 3 summarizes the optimal tuning parameters (Q_y and R), and the performance factors used. For the benchmark technique, the following pairing was selected: $y1 - u1$, $y2 - u2$.

Figure 2 illustrates the output responses in the tuning step for the proposed technique. The trajectory reference changes at 0 and 60 minutes for y_2 and y_1, respectively. A correlation between the values of Q_y and the aggressiveness of the MPC is observed. By comparing the response of y_1 and y_2, it is worth mentioning that y_2 was much closer to its reference after a setpoint change, while the desired performance of y_1 was somewhat neglected.

This example illustrated that by varying the response factors f_1 and f_2, it is possible to obtain, in an automated fashion, DMC tuning parameters that reflect whether the desired response is fast or slow. Here, the reference trajectories were defined as FOPDT transfer functions, since the system model are already described by FOPDT transfer functions, making the tuning procedure.

A simulation is proposed to illustrate how the tuned controllers fare against each other in a setpoint tracking example. The simulation scenario considers that the systems start from the origin, and the setpoints of y_2 and y_1 change from 0 to 1 at 0 and 100 minutes, respectively.

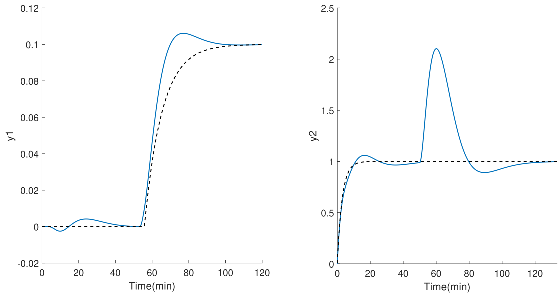

FIGURE 2. Tuning results of Application I, process outputs (solid line, blue) and reference trajectories (dashed line).

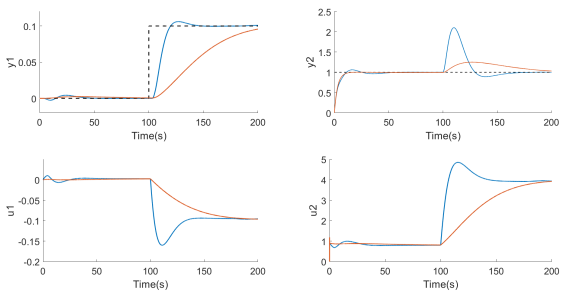

FIGURE 3. Output and input responses of the system in Application I in closed-loop with MPC tuned by the proposed method (solid line, blue), benchmark technique (solid line, orange), and the output setpoints (dashed line).

Figure 3 shows the output and input responses of the system. The proposed tuning technique yields a more aggressive controller, and therefore, the outputs follow the setpoint closely. The control signal calculated by the benchmark technique are sluggish compared to the proposed one.

Usually in the industry, DMCs are tuned by trial and error. Therefore, expert knowledge is required to translate desired economic goals and process constraints into closed-loop responses. An experienced control engineer knows the relationship between the process variables, and by association, can select DMC tuning parameters (weights on input increments and output deviations from setpoints) to somehow translate the process knowledge into control performance. Such a task may sound straightforward for a 2×2 system, such as in this example; however, it becomes harder for larger systems.

To summarize, by imposing that the tracking speed of the objective related to output y_1 is higher than that of output y_2, the proposed tuning method calculated a set of tuning parameters that implements the intent of the control designer, without the need to select the tuning parameters directly based on interpreting their effect in the closed-loop performance of the controller. Thus far, we have validated the proposed tuning method when applied to the linearized grinding process, despite long time delays and variable

TABLE 4. Input and output variables, units and tags of Application II.

Variable name	Units	Tag
Fresh ore feed rate	ton/h	u_1
Ball mill feed water flowrate	m^3/h	u_2
Dilution water flowrate	m^3/h	u_3
Pump speed	Hz	u_4
Particle size	%-200 mesh	y_1
Mill solids concentration	% solids	y_2
Circulating load	ton/h	y_3
Sump level	m	y_4

coupling. It allows for the calculation of tuning parameters of an MPC controller based on desired output tracking trajectories, in terms of how fast or slow the output tracking dynamics should be.

C. APPLICATION II - LINEAR 4 × 4 SYSTEM

The goal of Application II is to illustrate how the proposed tuning technique performs in more complex settings than in the previous example. Here, an unconstrained DMC for a 4 × 4 grinding circuit is tuned.

1) PROCESS MODEL

Table 4 presents input and output variables of the system, which is represented by the following transfer function matrix:

$$G(s)_2 = \begin{bmatrix} \frac{-0.58e^{-41s}}{83s+1} & \frac{0.97e^{-40s}(1-1.08e^{-232s})}{(125s+1)(195s+1)} & \frac{0.67e^{-8s}(1-1.07e^{-214s})}{(20s+1)(92s+1)} & \frac{0.50e^{-2s}}{18s+1} \\ \frac{0.62}{123s+1} & \frac{-1.75}{118s+1} & \frac{0.51e^{-87s}}{(81s+1)(182s+1)} & \frac{0.64e^{-9s}}{137s+1} \\ \frac{2.61e^{-45s}}{110s+1} & \frac{9.52e^{-93s}}{(98s+1)(137s+1)} & \frac{2.83e^{-8s}}{128s+1} & \frac{2.81e^{-5s}}{(108s+1)} \\ \frac{0.001e^{-30s}}{s(150s+1)} & \frac{0.011e^{-30s}}{s(100s+1)} & \frac{0.032}{s} & \frac{-0.031}{s} \end{bmatrix}. \quad (16)$$

For more information see [3].

2) TUNING GOALS

The authors in [3] have mentioned that previously to their study, a decentralized PID control has been implemented in the grinding circuit. The most concerning problems observed in the operation has been the inability of PID controllers to attenuate the effects of disturbances such as ore hardness, feed rate and feed particle size. Therefore, in order to prevent mill overload, the feedrate has been shut down, leading to unwanted fluctuations in the product particle size distribution. Based on this information, we consider that the most important output variables are the PSD of the product (y_1), the solids concentration (y_2), and the circulating load (y_3). The following variable pairs $y_1 - u_1$, $y_2 - u_2$, and $y_3 - u_3$ were selected for the definition of the reference trajectories. Finally, the resulting reference dynamics are given by

TABLE 5. Tuning results in Application 2.

Parameters	*Benchmark*	*Proposed*
Q_y	[1.0000 1.0000 1.0000 1.0000]	[0.0163 0.0057 0.0001 0.0696]
R	[4.1250 52.1333 244.0560 16.5068]	[0.0001 24.7334 26.8158 14.7946]
$[f_1\ f_2\ f_3\ *]$	*	[1.0000 1.0000 1.0000 *]

$$G(s)_{2,des} = \begin{bmatrix} \frac{1e^{-41s}}{83s\times f_1+1} & 0 & 0 & 0 \\ 0 & \frac{1}{118s\times f_1+1} & 0 & 0 \\ 0 & 0 & \frac{1e^{-8s}}{128s\times f_1+1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (17)$$

The initial guesses of the decision variables $x = [Q_y\ R]^T$ are defined as $x_0 = [0.001\ 0.001\ 0.001\ 0.001\ 10\ 10\ 10\ 10]^T$, and the lower and upper bounds of the tuning problem are $x_{min} = [10^3\ 10^3\ 10^3\ 10^3\ 10^3\ 10^3\ 10^3\ 10^3]^T$ and $x_{max} = [10^{-4}\ 10^{-4}\ 10^{-4}\ 10^{-4}\ 10^{-4}\ 10^{-4}\ 10^{-4}\ 10^{-4}]^T$.The tuning problem is solved using *fmincon* (MATLAB™, *sqp* algorithm, 10^{-4} TolX, 10^{-4} TolFun, 4×10^4 MaxFunEvals, 10^4 MaxIter).

3) RESULTS

The tuning results are summarized in Table 5. For both techniques, the prediction horizon, control horizon, and sample time are $p = 40$, $m = 3$, and 10 sec, respectively. For the benchmark technique, the following pairing was selected: $y1 - u4$, $y2 - u3$, $y3 - u2$, and $y4 - u1$.

Figure 4 illustrates the output responses of the proposed tuned MPC against the reference trajectories during the tuning design. The tuning simulation considers setpoint changes on y_1, y_2, and y_3, with magnitude of 0.1 at 0, 25, and 50 minutes, respectively. The tuning simulations run for 80 minutes. It is worth noting that there is no reference trajectory for y_4, the sump level. In industrial practice, the tank levels do not require strict control; variations around the setpoint are allowed as long as the upper and lower bounds are respected. Hence, to simplify the tuning method, no tuning goal is imposed on y_4. In the event of reference change, y_3 keeps close track of reference; y_1 responds fast but overshoots, and the response of y_2 is sluggish.

A simulation is proposed to compare the proposed and benchmark tuning techniques. The system starts from the origin. Figure 5 presents the closed-loop response of the system outputs for each tuning method for setpoint changes in the output variables (0 to 1 for y_1 at t = 0 min, 0 to 1 for y_2 at t = 50 min, and 0 to 1 for y_3 at t = 100 min) and Figure 6 illustrates the behavior of the input variables.

Note that outputs y_1 and y_4 yield more contrasting responses between the techniques. The proposed tuning technique is much faster in both cases since R is smaller as seen in Table 5. Hence, less penalty is applied over the control actions, allowing the DMC to act more freely. However, y_2 and y_3 have similar responses throughout the simulation, except for a moment in minute 50, when the proposed tuning technique further mitigated coupling effects in y_3.

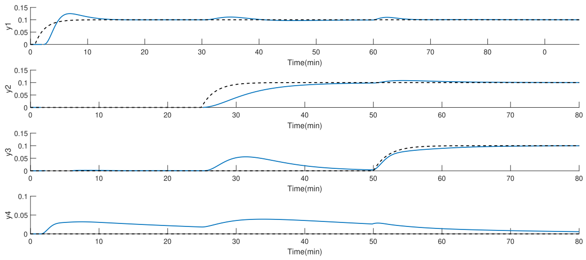

FIGURE 4. Tuning results of Application II, process outputs (solid line, blue) and reference trajectories (dashed line).

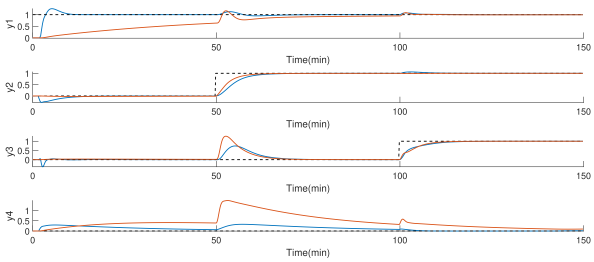

FIGURE 5. Output responses of the system in Application II in a closed-loop with MPC tuned by the proposed method (solid line, blue), benchmark technique (solid line, orange), and the output setpoints (dashed line).

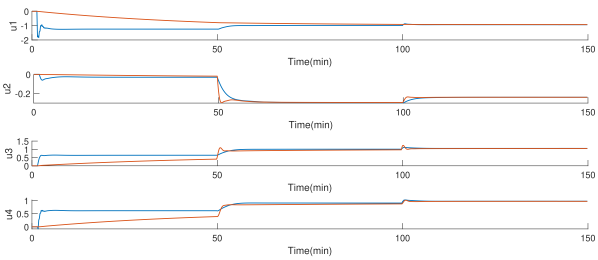

FIGURE 6. Input responses of the system in Application II in a closed-loop with MPC tuned by the proposed method (solid line, blue) and the benchmark technique (solid line, orange).

To summarize, then compared to the benchmark tuning technique, it was observed that by applying the proposed tuning method, faster setpoint tracking was achieved for outputs y_1 and y_4, and the coupling effect between output y_3 and input u_2, as illustrated by the transfer function $G(s)_2(3,2) = \frac{0.011e30s}{s(100s+1)}$, is mitigated when the setpoint change of y_2 is implemented.This application example showed that the proposed tuning technique is able to yield adequate tuning parameters, even in more complex settings and for larger systems.

D. APPLICATION III - NON-LINEAR 4 × 4 SYSTEM

The non-linear application presented here consists of the process control of a regrinding circuit in the Concentrator 3 plant, from Samarco's iron ore mine, located in the city of Mariana, Minas Gerais, Brazil.

1) REGRINDING CIRCUIT

The regrinding circuit consists of two parallel lines of hydrocyclones and ball mills. The feedrate to each line is the product of the primary flotation circuit. Hydrocyclones installed at the beginning of the circuit pre-classify the inflow rate into underflow and overflow. The underflow of the first classification stage feeds the ball mill; the overflow bypasses the milling circuit, which is mixed with the ball mill throughput and water, producing a slurry. After, it is stored in a sump pump, from which it is pumped to a second set of hydrocyclones. For this second stage of classification, the underflow of the hydrocyclone (circulating load) also feeds the ball mill, and the overflow feeds another flotation stage. Table 6 summarizes the input and output variables of the regrinding circuit.

TABLE 6. Input and output variables of Application III.

Variable	Units	Tag
Mill solids concentration	$\%$	y_1
Circulating load	t/h	y_2
Product granulometry	$\%325mesh$	y_3
Mill discharge sump level	$\%$	y_4
Pump flowrate setpoint	t/h	u_1
Mill water flowrate setpoint	m^3/h	u_2
Sump water flowrate setpoint	m^3/h	u_3
Pump flowrate setpoint	t/h	u_4

2) PROCESS MODEL

The transfer function matrix of the process is given by

$$G(s) = \begin{bmatrix} \frac{0.0015}{(478.93s+1)} & \frac{(-20.35\ s-0.015)e^{-12.7s}}{(8101s^2+363.2\ s+1)} & \frac{(1.364e-05s-4.75e-09)}{(s^2+0.00927s+1.315e-05)} & \frac{8.192e^{-19.5s}}{(24041s+1)} \\ \frac{(-0.003774s-1.075e-05)}{(s^2+0.1526s+0.0002904)} & \frac{(42.24\ s-0.07543)e^{-9.19s}}{(3342s^2+281.9s+1)} & \frac{-0.002034e^{-30s}}{(24.6s+1)} & \frac{1.009e^{-2.34s}}{(1363s+1)} \\ \frac{-0.008363}{(139.3s+1)} & \frac{-0.2201}{(92.77s+1)} & \frac{0.004587}{(505.6s+1)} & \frac{0.9465}{(2244s+1)} \\ \frac{(0.0001724\ s+8.383e-07)}{(s^2+0.05738s+0.0001375)} & \frac{0.5578e^{-4.16s}}{(13.49\ s+1)} & \frac{(0.0002044s+1.799e-07)}{(s^2+0.05934s+0.0001905)} & \frac{-0.616}{(2749s+1)} \end{bmatrix}. \quad (18)$$

It was generated using the System Identification Toolbox from MATLAB^{TM} and historical process data.

The transfer function matrix (18) was used to implement a DMC strategy, for which a schematic representation of how it is integrated with the circuit is shown in Figure 7. The DMC acts in an upper layer connected to a regulatory layer. As mentioned in Table 6, every input of the DMC is the setpoint of PID controller in the regulatory layer, which is not described in this paper.

3) TUNING GOALS

The priority factors of the circulating load (y_2) and the mill discharge sump level (y_4) are low compared to the other variables. Hence, these two variables have been removed from the tuning process, which was also a way to reduce computational cost. The variable pairs $y_1 - u_1$, $y_3 - u_3$ were selected for the definition of the following reference

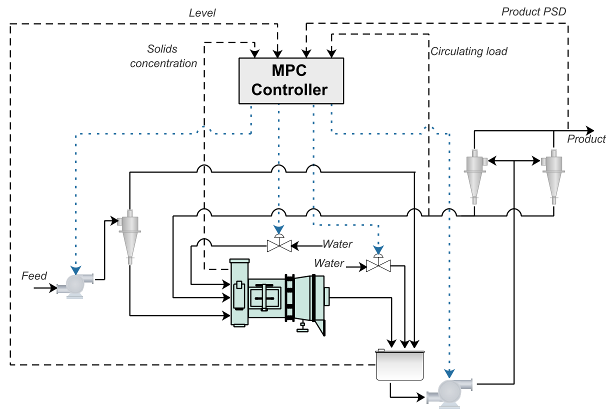

FIGURE 7. Schematic diagram of the MPC strategy for the regrinding circuit.

TABLE 7. Tuning results in Application III.

Parameters	Benchmark	Proposed
Q_y	$[0.1000 \quad 0.1000 \quad 0.1000 \quad 10.0000]$	$[1000 \quad 10^{-4} \quad 0.2211 \quad 1000]$
R	$[3 \quad 30.0275 \quad 3 \quad 3]$	$[102.2362 \quad 106.3686 \quad 81.5633 \quad 76.8041]$
$[f_1 \; * \; f_3 \; *]$	*	$[3 \quad * \quad 0.5 \quad *]$

trajectories:

$$G(s)_3 = \begin{bmatrix} \frac{1.0}{(478.9272s \times f_1 + 1)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1.0}{(505.6s \times f_2 + 1)} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{19}$$

It is worth mentioning that the tuning case studied in this example explores unmodelled dynamics and model mismatches, as the tuning technique is based on reference trajectories for linear systems and used with a DMC. However, although the controllers are applied, in simulation, to the non-linear model of the regrinding circuit, we are assuming the operation of the system around the desired set-point.

The lower and upper bounds of the tuning problem are $x_{min} = [10^3 \; 10^3 \; 10^3 \; 10^3 \; 10^3 \; 10^3 \; 10^3 \; 10^3]^T$ and $x_{max} = [10^{-4} \; 10^{-4} \; 10^{-4} \; 10^{-4} \; 10^{-4} \; 10^{-4} \; 10^{-4} \; 10^{-4}]^T$.The tuning problem is solved using *fmincon* (MATLAB^{TM}, textitsqp algorithm, 10^{-3} *tolx*, 10^{-3} *tolfun*, 4×10^3 *MaxFunEvals*, 10^3 *MaxIter*).

4) RESULTS

Considering the slow dynamics of the process, as seen in the identified transfer functions, the prediction horizon, control horizon, and sample time were selected as $p = 20$, $m = 3$, and 1 sec, respectively. The tuning results are summarized in Table 7. For the benchmark technique, the following pairing was selected: $y1 - u1$, $y2 - u3$, $y3 - u2$, and $y4 - u4$. The values of the reference trajectories are increased by 0.1 at 0 and 250 minutes for (y_1) and (y_3), respectively.

Figure 8 compares the reference trajectories and output responses in closed-loop obtained by the tuned DMC. Observe the tuning strategy failed to improve the tracking

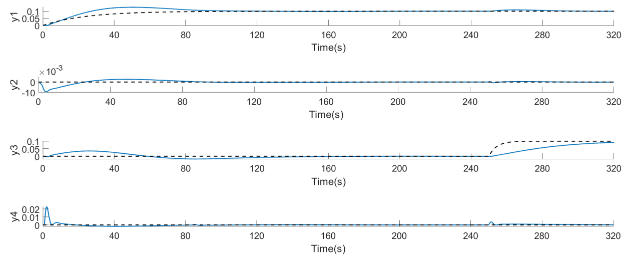

FIGURE 8. Tuning results of Application III, process outputs (solid line, blue) and reference trajectories (dashed line).

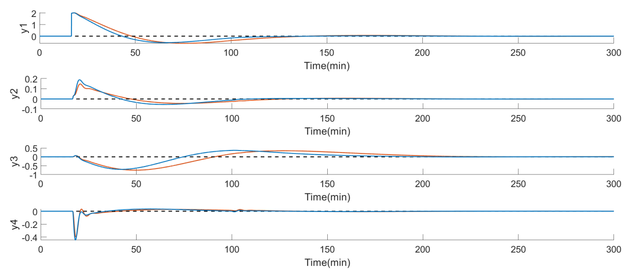

FIGURE 9. Output responses of the system in Application III (Example I) in a closed-loop with MPC tuned by the proposed method (solid line, blue), benchmark technique (solid line, orange), and the output setpoints (dashed line).

performance of outputs y_1 and y_3. This might be caused by saturation of R at the lower bound.

5) DYNAMIC SIMULATION RESULTS

Three experiments were conducted to validate the DMC implemented using the proposed tuning technique. The experiments aim to reproduce practical operational challenges in the regrinding circuit, which are summarized as follows:

- Experiment I: Increase the solids content percentage in the feed flowrate;
- Experiment II: Decrease the solids content percentage of the feed flowrate;
- Experiment III: Change the setpoint of the product PSD.

a: EXPERIMENT I

This first experiment consists of increasing the solids content percentage of the feed. An increase of 2% was applied at 15 minutes. Figure 9 presents the closed-loop response of the system. Both techniques were able to drive all controlled variables back to the setpoint after the disturbance affected the system. However, the benchmark tuning technique yielded an overall slower response, especially in y_3. Figure 10, which illustrates the control inputs for this scenario, shows the aggressiveness of the proposed tuning technique in comparison to the sluggish control action of the benchmark technique. However, the aggressiveness does not reflect in significant overshoot.

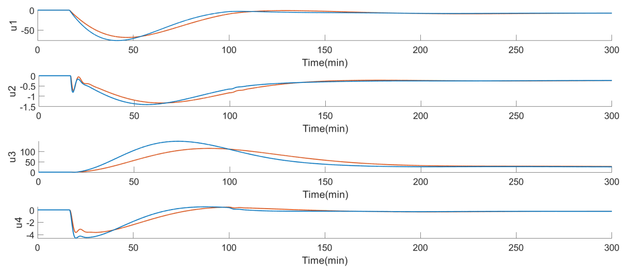

FIGURE 10. Input responses of the system in Application III (Example I) in a closed-loop with MPC tuned by the proposed method (solid line, blue), and benchmark technique (solid line, orange).

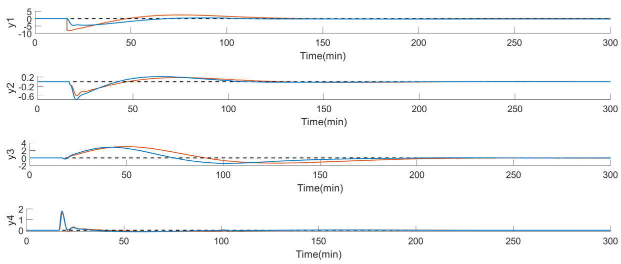

FIGURE 11. Output responses of the system in Application III (Example II) in a closed-loop with MPC tuned by the proposed method (solid line, blue), benchmark technique (solid line, orange), and the output setpoints (dashed line).

b: EXPERIMENT II

Abrupt drops in the feed flowrate of the regrinding circuit are usual in real operation. In this situation, water in the feed slurry increases to compensate for the lesser amount of solids. The percentage of solids in the feed flowrate of the circuit was reduced by 8% to simulate this condition on the simulated plant.

Figures 11 and 12 show the output and input responses of the system, respectively. The output responses y_1 and y_3 expose a considerable difference between the robustness of both techniques. The proposed tuning technique rejects the disturbance with less error, avoiding large and long overshoots around the setpoint, especially in y_1. In addition, the output performance is achieved without forcing overly aggressive control moves, except in u_3. Regarding y_2 and y_4, the output responses for both techniques are similar.

c: EXPERIMENT III

In the third experiment, the setpoint of y_3 is decreased by 0.5%. This variation represents a demand for finer products and seems reasonable since variations in product PSD are hard to achieve. Figures 13 and 14 show the output and input responses of the system, respectively. The proposed tuning technique is more aggressive, moving y_3 to the setpoint faster but at the cost of larger fluctuations in the other variables. Additionally, the control moves of the benchmark controller are slower compared to the moves of the proposed tuned

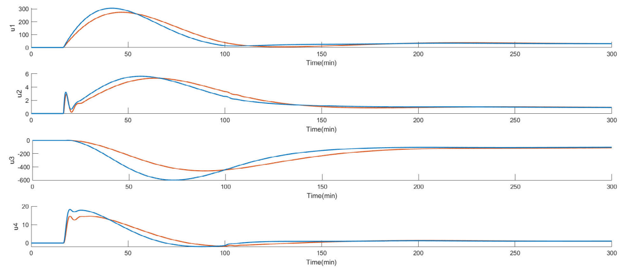

FIGURE 12. Input responses of the system in Application III (Example II) in a closed-loop with MPC tuned by the proposed method (solid line, blue), and benchmark technique (solid line, orange).

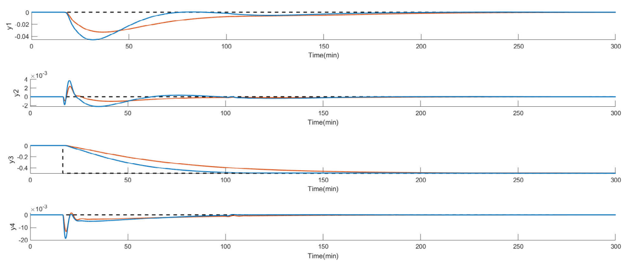

FIGURE 13. Output responses of the system in Application III (Example III) in a closed-loop with MPC tuned by the proposed method (solid line, blue), benchmark technique (solid line, orange), and the output setpoints (dashed line).

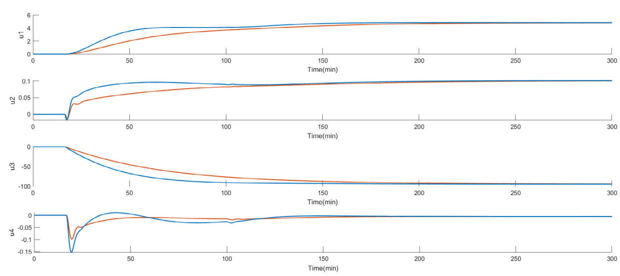

FIGURE 14. Input responses of the system in Application III (Example III) in a closed-loop with MPC tuned by the proposed method (solid line, blue), and benchmark technique (solid line, orange).

controller. Thus, y_3 takes twice as much time to accommodate around the new setpoint.

To summarize, the setpoint tracking and disturbance rejection capabilities of a DMC implemented in a dynamic simulation of validated regrinding circuit was analyzed, regarding the closed-loop performance obtained by DMCs tuned by the proposed technique and a benchmark technique. The results presented in the Figures above indicate that the proposed tuning method succeeded in translating the desired performance criteria into appropriate values of tuning parameters for the DMC. This is demonstrated in Experiment I and II for disturbance rejection, as the proposed tuning strategy results in quick rejection without significant large overshoot, even in lower priority outputs. This is also demonstrated in Experiment III for reference tracking, as the new steady state is reached quickly, but at the expense of some overshoot in lower priority outputs. Finally, when compared to a benchmark tuning technique, the proposed technique delivers faster stabilization under disturbance rejection and reference tracking, without compromising the transient performance.

VI. CONCLUSION

We explored the adaptation of a tuning method for MPC controllers applied to the mineral processing industry, which was originally developed for the petrochemical industry. In the mineral processing industry, the settling times and dead times are in general larger, which posed an additional challenge to the tuning procedure. Three case studies were used to investigate the suitability of the methodology. Applications I and II illustrated the tuning method on linear models when they were closed-loop with a DMC. The prediction horizon and control horizon were not included in the tuning method, but rather selected following well-established guidelines from the literature. Application I demonstrated that the tuning method works for processes with longer time delays and time constants. In addition, it was demonstrated that it is still possible to define the closed-loop response of the system by selecting a fast or slow response for each output. Such a choice is simpler and more straightforward than selecting the values of tuning parameters directly. Application II illustrated the tuning of a DMC in closed-loop with a 4×4 grinding circuit. In this type of process, one of the outputs is usually a level measurement, and the system does not benefit of the setpoint control of this variable. As long as the level is within acceptable ranges, the performance of the process remains unchanged. With this in mind, the level variable was not included in the tuning goals. Nonetheless, the complexity of the tuning problem increased significantly compared to Application I. Finally, Application III explored the tuning of a linear DMC in closed-loop with a non-linear model of a regrinding plant. In the tuning stage, a linearized model of the plant was considered. In order to achieve acceptable tuning parameters in this scenario, the initial guess and the search space were tailored around a known stabilizing set of parameters (obtained by trial and error). Comparison tests between the proposed tuning technique and a benchmark tuning technique from literature corroborated the superior performance of the proposed technique. We would like to point out that the superiority of the proposed tuning method has been demonstrated for the specific conditions analyzed in the presented experiments. Disturbance rejection and zone control of level variables are important characteristics of the mining and mineral processing industries and could thus be explored as additional tuning goals in future work. Tuning of more complex control algorithms, for example robust MPC with zone control and input targets, can also be addressed. Similarly to [21], optimization methods other than the gradient-based method implemented in *fmincon*, in particular heuristic optimization methods can be tested with respect to efficiency and convergence and finally, it is worth exploring innovative ways to design the control problem and consider adaptive control frameworks build on the proposed tuning method, i.e. a intuitive method in with plant operators can specify the desired performance of a closed-loop process, rather than the tuning parameters of the controller implemented in the process.

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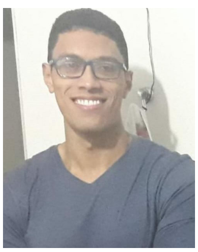

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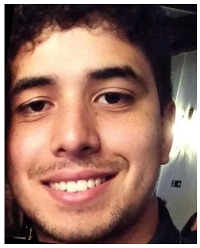

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