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RESEARCH ARTICLE

Offloading and Pricing Strategies for Heterogeneous Mobile Networks

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ABSTRACT In this work we consider the joint problem of offloading and pricing in heterogeneous networks comprising two-tier operators, namely, a Mobile Network Operator (MNO) and a pair of competing Small-cell Service Providers (SSPs). The *offloading problem* involves the MNO deciding the amount of traffic that it wishes to offload onto the SSPs. The SSPs, in turn, interact with each other through a *pricing problem* that requires the SSPs to fix competitive prices so as to maximize their revenue by trading more offloaded data with the MNO. The nature of the pricing scheme – *flexible* or *flat* – is fixed by the regulators. In flexible-pricing the SSPs can charge the MNO differently for different amount of traffic flows offloaded onto them. In contrast, under the flat-pricing scheme the SSPs are restricted to announce a fixed price irrespective of the offloaded traffic. For both pricing schemes, the MNO's offloading problem is first formulated as a Stackelberg game with MNO as the leader, while SSPs constitute the followers. The solution to the offloading problem is characterized in terms of Stackelberg equilibria, referred to as the *optimal-offloading* strategy, which is in contrast to the *full-offloading* scenarios considered in the literature (where the entire data is naively offloaded onto the SSPs by the MNO). Next, the SSPs' pricing problem (which appears as the followers' game in the Stackelberg's formulation) is formulated as a Bayesian game and the solution is characterized in terms of Bayesian Nash equilibria (BNE). We first establish that there are no BNEs in pure strategies, and then proceed to derive the structure of a mixed strategy symmetric BNE. Finally, we conduct an extensive numerical work to compare the performances of the flexible and flat-pricing schemes under optimal-offloading and full-offloading strategies. Through our study we find that the proposed optimal-offloading strategy yields a better payoff to the MNO, while the choice of pricing scheme (flexible or flat) that is favorable for the SSPs varies with the system parameters.

INDEX TERMS Mobile data offloading, heterogeneous networks, network pricing, Stackelberg games, Bayesian games, mixed strategy Nash equilibrium.

I. INTRODUCTION

We are in the midst of a very challenging communication era that is witnessing an ever-increasing demand for mobile data. Indeed, with increasing mobile connectivity and subscriptions,[1] as well as with more-and-more data-intensive applications being introduced into the digital space, this trend (of an upward surge in the demand for mobile data) is bound to continue well into the future. In order to meet this increasing demand, although the mobile operators are proactively adopting the 5G standards, complementary solutions are also necessary that can leverage the benefit of using the services of other small-cell technologies (e.g., WiFi, femtocells, road-side units, etc) that have also been providing wireless connectivity [3], [4], [5]. One promising solution in this direction is the proposal of *mobile data offloading* [6], [7], [8], [9], [10] whereby the mobile operators are allowed to offload some of their users onto small-cell service providers

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[1] According to a latest report by Cisco [2], the number of mobile subscribers are expected to grow from 5.1 billion (66 percent of the population) in 2018 to 5.7 billion (71 percent of the population) by 2023.

(e.g., public WiFi operators, femto-cell operators, road-side unit operators, etc.), thus balancing the mobile-traffic load among the heterogeneous networks.[2]

Enabling mobile data offloading requires methods to handoff mobile users onto small-cell networks in a ubiquitous fashion. Existing standards such as the 3GPP ATSSS (Access Traffic Steering, Switching, Splitting) [13], [14], [15] can be easily adopted for this purpose. Thus, the *technological challenges* involved in realizing the proposal of mobile data offloading are relatively easy to address, although significant research and development efforts are still required in reducing the protocol and signalling overhead that may be incurred while handing-off the mobile users onto small-cell providers.

In addition to the above technological challenges, there are also some important *economic implications* that need to be addressed in parallel in order to realize the full potential of the mobile data offloading technology [16]. Such economic implications arise primarily because the heterogeneous networks, in general, are owned by competing private operators. As a result, the small-cell service providers (SSPs) will be reluctant to provide the offloading services unless the mobile network operators (MNOs) provide attractive monetary incentives to the SSPs, thus compensating the SSPs for the additional CAPEX and OPEX that they may incur.

Introducing monetary transactions into the data offloading market will yield interesting economic scenarios. For instance, since offloading incurs a cost to the MNOs, they will now exercise caution towards the amount of mobile data that they wish to offload. Simultaneously, the SSPs will seek to maximize their individual profit by fixing competitive prices for their offloading service. In this work, we study one such offloading and pricing scenario comprising a single MNO and a pair of SSPs. The offloading problem of the MNO is formulated as a Stackelberg game, while the pricing problem (involving competition among the SSPs) is captured using the framework of Bayesian games. Before proceeding to discuss in detail the main contributions of this work, we present a summary of related work in literature to highlight the novelty of the current work.

A. RELATED WORK

Some early works to have studied the economic aspects of mobile data offloading include [16], [17], [18], [19], [20], [21]. Gao et al. in [16] propose a model where the MNOs choose prices they are willing to pay, while the SSPs decide the amount of traffic they wish to offload. The resulting negotiation process between the MNOs and the SSPs is formulated as a Stackelberg game. The solution is characterized in terms of subgame perfect equilibrium (SPE), and its performance is compared against market scenarios with perfect-competition and no-competition. Authors in [17], using the framework of Nash bargaining theory, systematically study the processes of *sequential* and *concurrent* bargaining between the MNO and the SSPs. The profits-gained (or losses-incurred) by the MNO and the SSPs under the two bargaining approaches are presented. The possibility of allowing multiple SSPs to bargain in a group is also considered. Techniques from auction theory have been employed in [18] and [19] to model the interactions that arise between MNO and SSPs in data offloading markets. While the study in [18] proposes a double-auctions mechanism for pricing and rate allocation, the authors in [19] propose a reverse auction based approach to enable efficient data offloading. In contrast to auction theory, the work in [20] employ a Stackelberg game approach to conduct incentive analysis in data offloading markets. Werda et al. in [21] propose an optimization theory based pricing framework for heterogeneous networks where the users can choose to connect to either femto or macro cellular base-stations, given the respective prices. In contrast to the *user-initiated* offloading mechanism of [21], we propose a *network-initiated* mechanism where the MNO and SSPs negotiate offloading prices (thus the users remain oblivious to the offloading decisions of the MNO). More recent works employing techniques from optimization and game-theory (involving auctions, contracts, etc.,) to address the challenges involved in realizing data-offloading include [22], [23], [24], [25], and [26].

Among the recent references, studies addressing the problem of offloading in opportunistic and vehicular networks are also available [25], [26], [27], [28], [29], [30]. Zhou et al. in [27] consider the problem of selecting the initial set of mobile nodes onto which the mobile network can offload content. These initial nodes can serve requests for the offloaded content from other nodes in their vicinity, thus indirectly reducing the load on the mobile network. Authors in [25] propose to use some moving vehicles as SSPs to deliver data to other vehicles in an opportunistic fashion. Although a Stackelberg-game formulation is proposed in [25], the objective remains to identify the set of SSPs (i.e., vehicles) that can provide offloading service at the lowest price. In our context, the SSPs are fixed; the challenge lies in determining the equilibrium prices when they compete for offloading traffic that lies in the overlap of their coverage regions. For more literature on offloading in vehicular networks, see [28], [29] (and reference therein). Cellular data offloading using satellite networks has also been studied in the literature. For instance Du et al. in [30] propose a second-price auction based mechanism to facilitate a terrestrial MNO and a satellite to negotiate spectrum sharing and offloading decisions. In contrast, in our work we assume that both MNO and SSPs already possess non-overlapping spectrum – offloading is accomplished by negotiating (competitive) prices of the SSPs with the MNO.

In general, most of the above works on data-offloading assume that the coverage regions of the SSPs do not overlap,

[2]In contrast to mobile data offloading is the proposal of *5G-Unlicensed* (i.e., 5G New Radio Unlicensed [11], [12]) where the mobile operators are permitted to transmit directly on the unlicensed WiFi band (thus increasing their capacity to support more traffic). Although this approach would find merit in regions with sparse WiFi deployments, in urban settings however, transmitting on the WiFi band would significantly reduce the throughput of the users that are consciously connected to the WiFi network.

due to which the SSPs have monopoly over the traffic flows generated by the MUs in their coverage region. This is in contrast to our work where the duopoly competition, that arises due to overlapping coverage regions, is modeled. One of the few works to have considered overlapping SSPs' coverage regions include that of Li et al. in [31]. Formulating the SSPs' pricing problem as a game, the authors in [31] show that there is no Nash equilibrium in pure strategies when the SSPs coverage regions overlap. The authors then seek to derive the structure of a mixed strategy Nash equilibrium. The work in [31] however assumes that the MNO chooses to offload all traffic onto the SSPs, i.e., they do not address the MNO's offloading problem. Further, similar to all of the other work in literature, Li et al. [31] also assume that the offloaded traffic is deterministic, and is known to both the SSPs before fixing their prices. In contrast to the existing work, we model the traffic flows as random, and thus introduce uncertainty at the SSPs regarding flows offloaded onto their competitor SSPs. Thus, the SSPs' pricing problem in our case is modeled as a Bayesian game. Additionally, we also study the MNO's offloading problem (unlike that in Li et al. [31]), yielding an overall Stackelberg game. In summary, to the best of our knowledge, this is the first work to model uncertainty at SSPs regarding the traffic flow offloaded onto their competitor SSPs.

In addition to data-offloading, the framework of Stackelberg games has been extensively employed in the literature to address the problem of resource allocation in general. For instance, the problem of power allocation in heterogeneous networks and D2D networks has been considered in [32] and [33], respectively. Work in [34] and [35] propose Stackelberg game formulations to address the problem of pricing and sub-channel allocation in the context of network slicing. Similarly, [36], [37], and [38] consider adopting Stackelberg formulations to address the problem of resource allocation arising in caching networks. In the context of edge computing systems, there are works in literature that employ Stackelberg game formulations to accomplish the task of offloading computation requests to cloud servers [39], [40]. Specifically, the authors in [39] propose a Stackelberg differential game based approach for facilitating computation resource sharing between a cloud server and a collection of edge computing service providers. Similarly, interactions between multiple edge servers and Industrial IoT devices (for enabling computation offloading) are modeled using Stackelberg game in [40]. Although Stackelberg-game based resource allocation problems are commonly considered in the literature, the particular formulation that we consider (where the SSP flows are a-priori unknown) and the results that we derive (Bayesian Nash equilibrium prices in Fig. 2, etc.) are unique to the best of our knowledge.

Finally, our model comprising two SSPs falls under the realm of competition in duopoly markets [41], [42], [43], [44], [45]. Wang et al. in [41] consider the problem of designing attractive mobile subscription packages by two competing MNOs. The problem of competitive pricing for spectrum sharing in cognitive radio networks has been studied by Li et al. in [42]. Similarly, the problem of pricing in cloud markets is considered in [43], while the duopoly competition among renewable energy suppliers in energy markets is studied by Zhao et al. in [44]. With reference to the above literature, our work on competitive pricing in data-offloading markets can also be considered as a novel contribution to the studies on duopoly markets.

B. OUR CONTRIBUTIONS AND PAPER OUTLINE

Our main technical contributions are as follows:

1) We model the SSPs' pricing problem using the framework of Bayesian games, and characterize the solution in terms of Bayesian Nash equilibrium (BNE). We show that no BNE exists in pure strategies (Theorem 1), and then proceed to derive the structure of a mixed strategy symmetric BNE (Theorem 2).
2) The MNO's offloading problem is solved via a Stackelberg game formulation that involves the MNO solving a utility maximization problem by anticipating the effect of its offloading decision on the BNE prices of the SSPs.
3) Through a detailed theoretical as well as numerical study we compare and contrast the payoff's achieved by the MNO and the SSPs under *flexible and flat-pricing schemes*, when the MNO exercises (i) *optimal offloading* (i.e., Stackelberg game solution), as well as (ii) *full offloading* where the MNO naively offloads all data onto the SSPs (like in the existing work [31]).

The remainder of the paper is outlined as follows. In Section II we present the proposed flexible-pricing model, where the associated offloading and pricing problems are formulated using the frameworks of Stackelberg and Bayesian games, respectively. The solution to the pricing problem, characterized in terms of Bayesian Nash Equilibrium (BNE), is studied in Section III. Specifically, the case of BNE in pure strategies is studied in Section III-A, while in Section III-B we consider mixed strategy BNEs. In Section IV we derive the solution to the offloading problem which is characterized in terms of Stackelberg equilibrium. In Section V we present the corresponding results for the flat-pricing model where the SSPs are restricted to set a single price (irrespective of the amount of traffic offloaded onto them). In Section VI we discuss the heuristic full-offloading scheme where the MNO naively offloads all traffic onto the SSPs. Results from our numerical study are reported in Section VII, and we finally draw our conclusions in Section VIII. For convenience we have presented glossary of symbols in Table 1.

II. FLEXIBLE-PRICING MODEL

We consider a system comprising a *Mobile Network Operator (MNO)* and two *Small-cell Service Providers (SSP-1 and SSP-2)*. As illustrated in Fig. 1, the MNO and the SSPs may have installed multiple base stations (cellular and femtocell, respectively) in a geographical location with overlapping

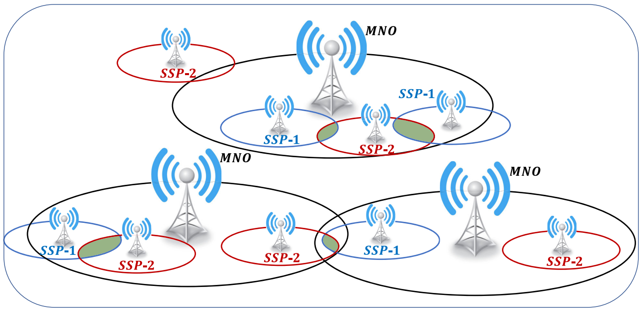


FIGURE 1. An illustration of a data offloading scenario comprising 3 cellular base-stations of MNO, and 4 and 5 femtocell base-stations of SSP-1 and SSP-2, respectively. The green-shaded portions represent the overlap regions of SSP-1 and SSP-2's coverage areas.

coverage regions. The femtocells of the SSPs could represent Road-Side Units (RSUs) of a vehicular network provider, Access Points (APs) of a public WiFi operator, etc. Thus, the SSPs could represent heterogeneous network providers, operating in the vicinity of one another. In such a setting, MNO can achieve data offloading by handing-off some of its users' requests (i.e., traffic load) onto the SSPs for service. Specifically, requests of the users that lie *exclusively* in the coverage region of (the base-stations installed by) SSP-i, $i \in \{1, 2\}$ (but not in the coverage region of SSP-j, $j \neq i$) can be offloaded to SSP-i, while the users that lie in the *overlap* regions (green-shaded regions in Fig. 1) can be offloaded to either of the SSPs.

Let $\mathbf{F}_i \in \Re_+$ ($i \in \{1, 2\}$) denote the aggregate traffic demand (or traffic *flow*) generated by the users that lie exclusively in the coverage region of SSP-i, while we use $\mathbf{F}_o \in \Re_+$ to denote the flow generated by the users that lie in the overlap region. We assume that the values of $\mathbf{F}_1$ and $\mathbf{F}_2$, being private information, are known only to the respective SSPs; whereas, the overlap flow $\mathbf{F}_o$ is commonly known to both the SSPs. We model the above scenario by assuming that $\mathbf{F}_1$ and $\mathbf{F}_2$ are random variables (and thus their realized values are known only to the respective SSPs), while the overlap flow $\mathbf{F}_o$ is considered a constant (whose value is known to both SSPs). For simplicity, we begin with a simple model where $\mathbf{F}_1$ and $\mathbf{F}_2$ are i.i.d. (independent and identically distributed) taking values $\mathbf{F}_\ell$ and $\mathbf{F}_h$ with probabilities θ_ℓ and θ_h, respectively, such that $\theta_\ell + \theta_h = 1$. We assume that $\mathbf{F}_\ell \leq \mathbf{F}_h$, so that the probability that the flow $\mathbf{F}_i$ ($i = 1, 2$) is *low* (i.e., $\mathbf{F}_\ell$) is θ_ℓ, while the flow is *high* (i.e., $\mathbf{F}_h$) with probability θ_h.

Remarks: Although we assume two flow-levels for simplicity, we note that our results can be generalized to a setup comprising more than two levels. Details of the general model will be discussed in Section III-D.

Note that we are implicitly assuming a slotted system where the values of the flows $\mathbf{F}_i$ and $\mathbf{F}_o$ (which depends on users' locations and requested traffic) do not vary within a slot (but may fluctuate across slots). The offloading and pricing strategies that we develop in this paper are for a typical slot with flow levels $\mathbf{F}_i$ and $\mathbf{F}_o$. The developed strategies can however be applied independently to each slot, yielding a continuously varying offloading and pricing decisions across slots. Thus, we are envisioning a dynamic mobile-data trading market (like the stock market) between the MNO and the SSPs in the future.

A. MNO's OFFLOADING STRATEGY

The MNO can choose to offload a portion of the above flows onto the SSPs. Specifically, when $\mathbf{F}_i = \mathbf{F}_\ell$ we assume that the MNO offloads $f_\ell := \beta_\ell \mathbf{F}_\ell$ amount of flow onto SSP-i (irrespective of the value of $\mathbf{F}_j$) where $\beta_\ell \in [0, 1]$. Similarly, when $\mathbf{F}_i = \mathbf{F}_h$ the offloaded flow is given by $f_h := \beta_h \mathbf{F}_h$ where $\beta_h \in [0, 1]$. Thus, denoting the flow offloaded onto SSP-i as F_i (referred to as the *monopoly-flow* since only SSP-i can serve this flow), we have[3]

$$F_i = \begin{cases} f_\ell = \beta_\ell \mathbf{F}_\ell & \text{w.p. } \theta_\ell \\ f_h = \beta_h \mathbf{F}_h & \text{w.p. } \theta_h. \end{cases} \tag{1}$$

Since, β_ℓ and β_h do not depend on the flow $\mathbf{F}_j$ corresponding to the other SSP (SSP-j), it follows that the monopoly flows F_i and F_j are i.i.d. The average monopoly-flow is given by $f_a := \theta_\ell f_\ell + \theta_h f_h$. Finally, the portion of the *overlap-flow* offloaded onto the SSPs is given by $f_o = \beta_o \mathbf{F}_o$, where again $\beta_o \in [0, 1]$. The offloading strategy of the MNO can thus be represented as a vector $\beta := (\beta_\ell, \beta_h, \beta_o)$ in $[0, 1]^3 \subset \Re^3$. Given an offloading strategy β, the amount of flow retained by the MNO (after offloading to SSPs) is given by

$$F_M := \begin{cases} 2(1-\beta_\ell)\mathbf{F}_\ell + (1-\beta_o)\mathbf{F}_o & \text{w.p. } \theta_\ell^2 \\ \sum_{t \in \{\ell, h\}} (1-\beta_t)\mathbf{F}_t + (1-\beta_o)\mathbf{F}_o & \text{w.p. } 2\theta_\ell\theta_h \\ 2(1-\beta_h)\mathbf{F}_h + (1-\beta_o)\mathbf{F}_o & \text{w.p. } \theta_h^2. \end{cases} \tag{2}$$

We refer to F_M as the *self-loaded flow*. The average self-loaded flow is given by

$$f_M := 2\theta_\ell(1-\beta_\ell)\mathbf{F}_\ell + 2\theta_h(1-\beta_h)\mathbf{F}_h + (1-\beta_o)\mathbf{F}_o. \tag{3}$$

where, for simplicity, we have used f_M to denote $\mathbb{E}[F_M]$.

B. SSPs' PRICING STRATEGY

The MNO directly charges the users (irrespective of whether a respective user's flow is offloaded or self-loaded) at a fixed flat-price of p_M per-unit-flow of request for data. The SSPs, on the other hand, are allowed to implement a *flexible-pricing scheme* whereby the SSPs can charge the MNO different prices for the different flow levels (low or high) they actually experience (during a slot). Formally, let $p_i = (p_{i,\ell}, p_{i,h})$ denote the *price vector* of SSP-i ($i \in \{1, 2\}$) where $p_{i,\ell}$ (resp. $p_{i,h}$) is the price per-unit-flow that SSP-i charges the MNO when the offloaded flow is $F_i = f_\ell$ (resp. f_h). Thus, given a price vector p_i, the price set by SSP-i depends on the realized value of the offloaded flow F_i (equivalently, the

[3]Notice that we are using bold-face $\mathbf{F}_i$ ($i = 1, 2$) to denote the aggregate flows while normal-case F_i are used to denote the offloaded flows.

TABLE 1. Glossary of symbols.

Notation	Description
$\mathbf{F}_i$	Traffic flow (assumed random) generated by the users that lie exclusively in the coverage region of SSP-i ($i \in \{1,2\}$)
$\mathbf{F}_o$	Traffic flow (assumed constant) generated by the users that lie in the overlap region of SSP-1 and SSP-2
$\mathbf{F}_\ell$	Value of low-level traffic flow
$\mathbf{F}_h$	Value of high-level traffic flow
θ_ℓ	Probability that the traffic flow at SSP-i is low, i.e., probability that $\mathbf{F}_i = \mathbf{F}_\ell$
θ_h	Probability that the traffic flow at SSP-i is high, i.e., probability that $\mathbf{F}_i = \mathbf{F}_h$
β_ℓ	Fraction of flow offloaded by the MNO onto SSP-i when the flow is low, i.e., when $\mathbf{F}_i = \mathbf{F}_\ell$
β_h	Fraction of flow offloaded by the MNO onto SSP-i when the flow is high, i.e., when $\mathbf{F}_i = \mathbf{F}_h$
β_o	Fraction of overlap-flow offloaded by the MNO onto the SSPs
β	$\beta := (\beta_\ell, \beta_h, \beta_o)$ represents the offloading strategy of the MNO
f_ℓ	$f_\ell = \beta_\ell \mathbf{F}_\ell$ represents the amount of flow offloaded by the MNO onto SSP-i when $\mathbf{F}_i = \mathbf{F}_\ell$
f_h	$f_h = \beta_h \mathbf{F}_h$ represents the amount of flow offloaded by the MNO onto SSP-i when $\mathbf{F}_i = \mathbf{F}_h$
f_a	$f_a = \theta_\ell f_\ell + \theta_h f_h$ denotes the expected (average) flow offloaded onto SSP-i
F_i	Random flow offloaded by the MNO onto SSP-i, i.e., $F_i = f_\ell$ w.p. θ_ℓ, while $F_i = f_h$ w.p. θ_h
F_M	Amount of flow retained by the MNO after offloading onto the SSPs; F_M is referred to as the self-loaded flow
f_M	Expected self-loaded flow, i.e., $f_M = \mathbb{E}[F_M]$
p_M	Price charged to the users by the MNO for one-unit of request for data
$p_{i,\ell}$	Price charged to the MNO by SSP-i for one-unit of offloaded traffic when the flow is low, i.e., when $F_i = f_\ell$
$p_{i,h}$	Price charged to the MNO by SSP-i for one-unit of offloaded traffic when the flow is high, i.e., when $F_i = f_h$
p_i	$p_i = (p_{i,\ell}, p_{i,h})$ represents the price vector of SSP-i
P_i	Random price set by SSP-i, i.e., $P_i = p_{i,\ell}$ w.p. θ_ℓ, while $P_i = p_{i,h}$ w.p. θ_h
$c_M(\cdot)$	Cost incurred by the MNO for serving one-unit of flow
c_S	Cost incurred by the SSPs for serving one-unit of flow
$U_M(\cdot)$	(Random) payoff accrued by the MNO
$\mathcal{U}_M(\cdot)$	Expected payoff of the MNO, i.e., $\mathcal{U}_M(\cdot) = \mathbb{E}[U_M(\cdot)]$
$U_i(\cdot)$	(Random) payoff accrued by SSP-i
$\mathcal{U}_i(\cdot)$	Expected payoff of SSP-i, i.e., $\mathcal{U}_M(\cdot) = \mathbb{E}[U_i(\cdot)]$
$\mathcal{U}_{i,\ell}(\cdot)$	Expected payoff of SSP-i conditioned on the event that $F_i = f_\ell$
$\mathcal{U}_{i,h}(\cdot)$	Expected payoff of SSP-i conditioned on the event that $F_i = f_h$
$\hat{p}_\ell$	Threshold on price such that any price $p_{i,\ell} < p_\ell$ is strictly dominated
$\hat{p}_h$	Threshold on price such that any price $p_{i,h} < p_h$ is strictly dominated
$G_{i,\ell}(p)$	Mixed pricing strategy of SSP-i when $F_i = f_\ell$, i.e., c.d.f. on the interval $[\hat{p}_\ell, p_M]$
$G_{i,h}(p)$	Mixed pricing strategy of SSP-i when $F_i = f_h$, i.e., c.d.f. on the interval $[\hat{p}_h, p_M]$
G_i	$G_i = (G_{i,\ell}, G_{i,h})$ denotes a mixed-strategy of SSP-i
$G^{(\beta)}$	$G^{(\beta)} = (G_\ell^{(\beta)}, G_h^{(\beta)})$ represents a mixed strategy such that $(G^{(\beta)}, G^{(\beta)})$ constitutes a symmetric BNE
$G_\ell^{(\beta)}$	BNE strategy when $F_i = f_\ell$
$G_h^{(\beta)}$	BNE strategy when $F_i = f_h$
q_ℓ	Threshold on price such that $G_\ell^{(\beta)}$ places all mass on $[q_\ell, q_h]$
q_h	Threshold on price such that $G_h^{(\beta)}$ places all mass on $[q_h, p_M]$
$u_\ell(\beta)$	BNE payoff achieved by SSP-i when $F_i = f_\ell$
$u_h(\beta)$	BNE payoff achieved by SSP-i when $F_i = f_h$
$u_i(\beta)$	$u_i(\beta) = \theta_\ell u_\ell(\beta) + \theta_h u_h(\beta)$ represents the average BNE payoff achieved by SSP-i
$u_M(\beta)$	Expected BNE payoff of the MNO
β^*	Optimal offloading strategy of the MNO
u_M^*	$u_M^* = u_M(\beta^*)$ denotes the optimal payoff of the MNO
u_i^*	$u_i^* = u_i(\beta^*)$ denotes the payoff of SSP-i when the MNO exercises optimal offloading strategy β^*

original aggregate flow $\mathbf{F}_i$). For convenience, we define the *price random variables* P_i as follows:

$$P_i = \begin{cases} p_{i,\ell} \text{ if } F_i = f_\ell \\ p_{i,h} \text{ if } F_i = f_h. \end{cases}$$

Note that P_1 and P_2 are independent (since F_1 and F_2 are i.i.d.) and their respective p.m.fs are given by $P_i = p_{i,\ell}$ w.p. θ_ℓ, and $P_i = p_{i,h}$ w.p. θ_h. Finally, we use $c_M(f_M)$ and c_S to denote the costs incurred by the MNO and SSPs, respectively, to serve one-unit of flow. Note that we are assuming the MNO's cost to be a (convex-increasing) function of the average self-loaded flow f_M while the SSPs' cost is considered to be simply constant.

Remarks: The justification for the above cost model are as follows:

- First, we assume that the MNO (being a big player in the mobile-data market) is operating very close to its capacity so that its service cost (being in the steeply-increasing regime) is sensitive to the average offloaded flow. Hence we assume that $c_M(\cdot)$ is convex-increasing in f_M. From a theoretical standpoint, expressing the MNO's cost $c_M(f_M)$ as a function of the self-loaded flow f_M will allow the MNO to tradeoff between the amount of offloaded vs self-loaded flows (see the remarks following expression (9) for more details).
- The SSPs on the other hand (being relatively new to the market) are assumed to be operating well below

its capacity limits, so that its service cost (being in the slowly-increasing regime) does not vary much with the amount of offloaded flows. Hence the SSPs' cost c_S is considered constant.

- Finally, we also note that the MNO's service cost $c_M(\cdot)$ is expressed as a function of the average self-loaded flow f_M rather than the instantaneous flow F_M. The justification for choosing the former comes from the consideration that the provisioning of the MNO's facility (i.e., allocation of resources such as processors, servers, bandwidth, etc., which decides the service cost of the MNO) would be based on the forecasted load (i.e., average flow) because the facility has to be setup before the commencement of the operational phase.

Now, since F_i can be served only by SSP-i, the SSPs have monopoly over their respective flows (assuming that the SSPs' prices are less than the MNO's price of p_M, otherwise the MNO has no incentive to offload). However, since the MNO would naturally offload the overlap flow f_o to the SSP that charges the lowest price,[4] the SSPs are hence expected to set competitive prices so as to optimize their profits by additionally serving the offloaded overlap-flow f_o.

Remarks: For a given offloading strategy, the above pricing model generalizes the model proposed in [31] as follows: when $\theta_\ell = 0$ or $\theta_\ell = 1$ our model degenerates to that studied in [31] where the flows are assumed to be deterministic. Alternatively, in the context of our setting, the model in [31] can be thought as simply taking the average flow f_a into account while fixing the SSPs' prices, instead of considering the detailed distribution like in our model. Thus, unlike the above flexible-pricing scheme, the authors in [31] naturally propose a *flat-pricing scheme* where the SSPs are restricted to set a single price, irrespective of the amount of flow offloaded onto them. The flat-pricing scheme of Li et al. [31] will be discussed in detail in Section V.

C. PAYOFF FUNCTIONS

Given the MNO's offloading strategy $\beta = (\beta_\ell, \beta_h, \beta_o)$, and the price vectors (p_i, p_j) of both SSPs, the revenue or payoff accrued by the MNO can be written as[5]

$$U_M\big(\beta; (p_i, p_j)\big) = F_i\big(p_M - P_i\big) + F_j\big(p_M - P_j\big) + f_o\big(p_M - \min\{P_i, P_j\}\big) + F_M\big(p_M - c_M\big) \tag{4}$$

where for simplicity we henceforth use c_M to denote $c_M(f_M)$. Thus, the average payoff of the MNO is given by

$$\mathcal{U}_M\big(\beta; (p_i, p_j)\big) = \mathbb{E}\Big[U_M\big(\beta; (p_i, p_j)\big)\Big] \tag{5}$$

where the expectation is w.r.t the joint p.m.f of (F_1, F_2, F_M).

[4]In case of a tie, the overlap flow f_o is equally split among both the SSPs (as can be seen from the $P_i = P_j$ case in (6)).

[5]For simplicity, here after we use the flexible notation (p_i, p_j) $(i \neq j)$ to denote the price-vectors of both SSPs (instead of fixing these to (p_1, p_2)).

The revenue (i.e., payoff) achieved by the SSPs can be similarly obtained. First, the payoff of SSP-i is given by

$$U_i\big((p_i, p_j); \beta\big) = \begin{cases} F_i(P_i - c_S) & \text{if } P_i > P_j \\ (F_i + f_o)(P_i - c_S) & \text{if } P_i < P_j \\ (F_i + 0.5\, f_o)(P_i - c_S) & \text{if } P_i = P_j \end{cases} \tag{6}$$

The average payoff is then given by

$$\mathcal{U}_i\big((p_i, p_j); \beta\big) = \mathbb{E}\Big[U_i\big((p_i, p_j); \beta\big)\Big]. \tag{7}$$

where the expectation is w.r.t the joint p.m.f of (F_1, F_2) (note that the self-service flow F_M does not directly affect the SSPs' payoff functions).

Further, given that SSP-i has information about its flow-level (but not that of SSP-j), we define the following conditional payoffs as follows: for $t \in \{\ell, h\}$

$$\mathcal{U}_{i,t}\big((p_{i,t}, p_j); \beta\big) = \mathbb{E}\Big[U_i\big((p_i, p_j); \beta\big)\Big| F_i = f_t\Big]. \tag{8}$$

Note that the value of the conditional payoffs, for instance say $\mathcal{U}_{i,\ell}$ (corresponding to low-flow), is not affected by the price term $p_{i,h}$ (corresponding to high-flow), and vice versa. Hence, $\mathcal{U}_{i,t}$ in (8) is shown to be a function of $p_{i,t}$ alone instead of the entire price-vector $p_i = (p_{i,\ell}, p_{i,h})$ as in the case of the unconditional payoff function $\mathcal{U}_i$ in (7). However, the unconditional payoff in (7) can be expressed using the above conditional payoffs as follows:

$$\mathcal{U}_i\big((p_i, p_j); \beta\big) = \theta_\ell\, \mathcal{U}_{i,\ell}\big((p_{i,\ell}, p_j); \beta\big) + \theta_h\, \mathcal{U}_{i,h}\big((p_{i,h}, p_j); \beta\big). \tag{9}$$

Remarks: While writing the above expressions we have assumed that $c_S \leq P_i \leq p_M$ $(i \in \{1, 2\})$. This assumption is natural because, otherwise (i) if $P_i > c_M$, the MNO will have no incentive to offload any flow onto the SSPs, or (ii) if $P_i < c_S$, then the SSPs have no benefit in accepting to serve the offloaded flow. The MNO's cost $c_M = c_M(f_M)$ on the other hand varies with the amount of (average) self-loaded flow f_M. As mentioned earlier, c_M enables the MNO to tradeoff between offloading and self-loading. For instance, increasing offloading (i.e., reducing f_M) will reduce c_M, which in-contrast entices the MNO to self-load. Alternatively, increasing self-loading (i.e., increasing f_M) will increase c_M, thus encouraging the MNO to offload. Hence, there exists an interesting trade-off between the amount of flow that the MNO can offload vs self-load. The solution to this trade-off is characterized via a Stackelberg game formulation as discussed in the following.

D. GAME FORMULATION

The above model, involving the joint problems of data-offloading and pricing, yields an interesting game formulation comprising two layers. First, (at a *higher-layer*) there exists a *data-offloading problem (or offloading game)* which can be formulated as a *Stackelberg game* (i.e., *leader-follower game*). Specifically, the game begins with the MNO choosing an offloading strategy β, following which SSP-i

and SSP-j are required to announce their respective prices p_i and p_j. Thus, the MNO acts as the **leader**, while the SSPs constitute the **set of followers** of the Stackelberg game.

Next, given the MNO's offloading decision, the SSPs themselves are involved in a (*lower-layer*) *pricing game* to resolve the problem of setting competitive prices so as to maximize their individual payoffs. More specifically, since SSP-i lacks information about the offloaded flow F_j at SSP-j, the *SSPs' pricing-game* can be formulated as a *Bayesian game* [46]. For a given β, the correspondence between the SSPs' pricing game and the Bayesian game formulation can be established as follows:

- SSP-1 and SSP-2 constitute the **set of players**.
- The **set of states** is given by the following collection of flow-pairs: $\{(f_\ell, f_\ell), (f_\ell, f_h), (f_h, f_\ell), (f_h, f_h)\}$.
- The **probability distribution** over the set of states is given by θ_ℓ^2, $\theta_\ell\theta_h$, $\theta_h\theta_\ell$ and θ_h^2, respectively.
- The **set of types** of each SSP is simply the set of possible flow-levels $\{f_\ell, f_h\}$.
- Given the type of SSP-i, the interval $[c_S, p_M]$ from which SSP-i can fix a price constitutes its **action space**.
- Finally, the payoff functions $\mathcal{U}_{i,\ell}$ and $\mathcal{U}_{i,h}$ in (8) represent the **utility functions** of SSP-i given its respective types f_ℓ and f_h.

The solution to the lower-layer pricing game can hence be characterized in terms of a Bayesian Nash equilibrium as defined in the following.

Definition 1 (Bayesian Nash Equilibrium): Given an MNO's offloading strategy β, price vectors $(p_1^{(\beta)}, p_2^{(\beta)})$, where $p_i(\beta) = (p_{i,\ell}^{(\beta)}, p_{j,h}^{(\beta)})$, are said to constitute a Bayesian Nash equilibrium (BNE) if for all $i \in \{1, 2\}$ and $t \in \{\ell, h\}$ we have

$$\mathcal{U}_{i,t}\left((p_{i,t}^{(\beta)}, p_j^{(\beta)}); \beta\right) \geq \mathcal{U}_{i,t}\left((p_{i,t}, p_j^{(\beta)}); \beta\right) \tag{10}$$

for all $p_{i,t} \in [c_S, p_M]$. Thus, unilateral deviation from a BNE is not beneficial to either of the SSPs. □

Remarks: The above definition corresponds to that of a pure strategy BNE. The mixed strategy generalization is obtained by introducing probability distributions from which the SSPs can pick their respective price-vectors. Details of mixed-strategy BNEs will be discussed in Section III-B. Meanwhile, in Section III-A we study the case of pure strategy BNEs.

The solution to the higher-layer offloading game can be characterized in terms of Stackelberg equilibrium as follows:

Definition 2 (Stackelberg Equilibrium): An offloading strategy β^* (along with its associated BNE strategy $(p_i^{(\beta^*)}, p_j^{(\beta^*)})$) is said to constitute a Stackelberg equilibrium (SE) if the following condition holds:

$$\mathcal{U}_M\left(\beta^*; (p_i^{(\beta^*)}, p_j^{(\beta^*)})\right) \geq \mathcal{U}_M\left(\beta; (p_i^{(\beta)}, p_j^{(\beta)})\right) \tag{11}$$

for all $\beta \in [0, 1]^3$. □

Stackelberg equilibrium constitutes the overall solution to the joint problem of data-offloading and pricing. We adopt the backward iteration method to solve the problem as follows: we first identify the structure of a BNE by fixing an offloading strategy β; next, using the BNE solution into the MNO's payoff function $\mathcal{U}_M$, we compute a Stackelberg equilibrium by optimizing $\mathcal{U}_M$ over all offloading strategies $\beta \in [0, 1]^3$. The details are presented in the following sections.

III. PRICING GAME: BAYESIAN NASH EQUILIBRIUM

In this section, we first establish that there are no BNEs in pure strategies. In view of this result, we then proceed to identify BNEs in mixed strategies. We finally generalize our results to a scenario comprising more than two flows.

Throughout this section we assume that the offloading strategy β of the MNO is fixed. Hence, for simplicity, we drop β from the notation of the SSPs' payoff functions; for instance, the conditional payoff function $\mathcal{U}_{i,t}\left((p_{i,t}, p_j); \beta\right)$ is simply written as $\mathcal{U}_{i,t}\left(p_{i,t}, p_j\right)$ with the understanding that the MNO's offloading strategy is β. Further, we assume that the offloaded flows satisfy $f_\ell \leq f_h$[6]; in case, $f_\ell > f_h$, then by simply interchanging the roles of f_ℓ and f_h the results of this section can be applied to identify the solution.

A. PURE STRATEGY BNE

The results in this sub-section are along the lines of the results in [31]. However, caution is required as the offloaded flows F_i ($i = 1, 2$) are random which is unlike the case in [31]. Further, in our case the actions are represented as price-vectors $p_i = (p_{i,\ell}, p_h)$ instead of a single real-valued price like in [31]. As a result, the process of establishing the non-existence of a pure strategy BNE equilibrium (in Theorem 1) requires careful investigation of all of the several exhaustive cases. The details are as follows.

Suppose $F_i = f_t$ ($t \in \{\ell, h\}$) then SSP-i can accrue a guaranteed payoff of at least $u_t := f_t(p_M - c_S)$ by setting the price to maximum (i.e., $p_{i,t} = p_M$). However, by reducing the price (i.e., $p_{i,t} < p_M$), it is possible to achieve a higher payoff of $v_t(p_{i,t}) := (f_t + f_o)(p_{i,t} - c_S)$ (by acquiring the overlap flow f_o as well). Since $v_t(p_{i,t})$ reduces as we decrease $p_{i,t}$, it follows that there exists a *threshold price* $\hat{p}_t$ such that for prices lower than $\hat{p}_t$ there is no incentive in acquiring the overlap flow; SSP-i is instead better-off by simply serving the monopoly flow at the maximum price p_M. The value of $\hat{p}_t$ is obtained by solving for $p_{i,t}$ from the equation $v_t(p_{i,t}) = u_t$, and is given by

$$\hat{p}_t = \frac{p_M f_t + c_S f_o}{f_t + f_o}. \tag{12}$$

Note that, since $\hat{p}_t$ in the above expression is a weighted sum of c_S and p_M, it follows that $c_S < \hat{p}_t < p_M$. Further, since the RHS of (12) is increasing in f_t we have $\hat{p}_\ell \leq \hat{p}_h$ (because $f_\ell \leq f_h$).

In the following lemma we formally show that the prices lower than $\hat{p}_t$ are indeed dominated strategies.

Lemma 1: For any $t \in \{\ell, h\}$ and $i \in \{1, 2\}$, the pricing strategy $p_{i,t} < \hat{p}_t$ is strictly dominated.

[6]Note that, although the original flows satisfy $\mathbf{F}_\ell \leq \mathbf{F}_h$, since $f_t = \beta_t \mathbf{F}_t$ for $t \in \{\ell, h\}$, it is not necessary for the offloaded flows to satisfy the same.

Proof: For any price-vector p_j of SSP-j, we can write

$$\begin{aligned}\mathcal{U}_{i,t}(p_{i,t}, p_j) &\overset{(a)}{\leq} (f_t + f_o)(p_{i,t} - c_S)\\ &\overset{(b)}{<} (f_t + f_o)(\hat{p}_t - c_S)\\ &\overset{(c)}{=} f_t(p_M - c_S)\\ &\overset{(d)}{\leq} \mathcal{U}_{i,t}(p_M, p_j)\end{aligned}$$

In the above expression, (a) holds because the RHS represents the maximum payoff that SSP-i can possibly accrue at price $p_{i,t}$, (b) is due to the hypothesis $p_{i,t} < \hat{p}_t$, (c) simply follows from the expression of $\hat{p}_t$ in (12), and finally (d) holds because the actual payoff received for playing the pricing strategy p_M may include contribution from the overlap flow (in case there is a tie with any of the SSP-j's pricing strategy, i.e., if $p_{j,s} = p_M$ for some $s \in \{\ell, h\}$; otherwise equality holds). Thus, we see that the pricing strategy $p_{i,t}$ is strictly dominated by the strategy of choosing the maximum price p_M. ■

Since the SSPs have no incentive to choose a dominated strategy, we can hence reduce the set of feasible actions to $[\hat{p}_t, p_M] \subset [c_S, p_M]$. With the assistance of the above result, we now state the following main theorem.

Theorem 1: There exists no BNE in pure strategies for the SSPs' pricing game (recall Definition 1).

Proof: Let (p_1, p_2) represent a generic pure strategy price-vector pair where $p_i = (p_{i,\ell}, p_{i,h})$ for $i \in \{1, 2\}$. For each $t \in \{\ell, h\}$ and $i \in \{1, 2\}$, partitioning the price range into $p_{i,t} \in (\hat{p}_t, p_M]$ (referred to as *range*-1) and $p_{i,t} = \hat{p}_t$ (*range*-2), a total of 16 regions are possible for the values of (p_1, p_2) to lie into. We group these 16 regions into 3 exhaustive cases. For each case, we show that there is always incentive for one of the SSPs to deviate, thus implying that (p_1, p_2) cannot constitute a BNE. The details under each case are discussed in the following.

Case-1: For some SSP, say SSP-j, both prices $p_{j,\ell}$ and $p_{j,h}$ are in range-1. Without loss of generality, we assume that $p_{j,\ell} \leq p_{j,h}$. Then, SSP-i can benefit by deviating from its price $p_{i,\ell}$ under different scenarios as follows:

(a) If $p_{i,\ell} \in (p_{j,h}, p_M]$ $(i \neq j)$ then SSP-i can acquire the overlap flow, and thus increase its payoff $\mathcal{U}_{i,\ell}$, by choosing a price $\tilde{p}_{i,\ell} \in (\hat{p}_\ell, p_{j,\ell})$.

(b) If $p_{i,\ell} = p_{j,h}$ then choosing a price $\tilde{p}_{i,\ell}$ less than, but arbitrarily close to $p_{j,h}$, SSP-i can benefit by acquiring the complete overlap flow f_o whenever $F_j = f_h$ (instead of only $0.5 f_o$ if it continues to use $p_{i,\ell}$).

(c) If $p_{i,\ell} \in (p_{j,\ell}, p_{j,h})$ then SSP-i can increase its payoff by choosing a price $\tilde{p}_{i,\ell} \in (p_{i,\ell}, p_{j,h})$.

(d) If $p_{i,\ell} = p_{j,\ell}$ then, as in case-(b), choosing a price $\tilde{p}_{i,\ell}$ less than, but arbitrarily close to $p_{j,\ell}$, SSP-i can benefit by acquiring the complete overlap flow f_o whenever $F_j = f_\ell$ (instead of only $0.5 f_o$ if it continues to use $p_{i,\ell}$).

(e) Finally, if $p_{i,\ell} \in [\hat{p}_\ell, p_{j,\ell})$ then SSP-i is better-off by deviating to a price $\tilde{p}_{i,\ell} \in (p_{i,\ell}, p_{j,\ell})$.

Case-2: For both SSPs, one price is in range-1 while the other price is in range-2. Let $p_{i,t} \in (\hat{p}_t, p_M]$ and $p_{j,s} \in (\hat{p}_s, p_M]$ denote the range-1 prices of SSP-i and SSP-j, respectively. Then one SSP can benefit under different scenarios as follows:

(a) If $p_{i,t} < p_{j,s}$ then SSP-i can benefit by choosing a price $\tilde{p}_{i,t} \in (p_{i,t}, p_{j,s})$.

(b) If $p_{i,t} = p_{j,s}$ then choosing a price $\tilde{p}_{i,t}$ less than, but arbitrarily close to $p_{j,s}$, SSP-i can benefit by acquiring the complete overlap flow f_o whenever $F_j = f_s$ (instead of only $0.5 f_o$ if it continues to use $p_{i,t}$).

(c) Finally, if $p_{i,t} > p_{j,s}$ then SSP-j can benefit by choosing a price $\tilde{p}_{j,s} \in (p_{j,s}, p_{i,t})$.

Case-3: Suppose both prices of SSP-j are in range-2, while one or both prices of SSP-i are in range-2. Let $p_{i,t} = \hat{p}_t$ denote the price of SSP-i in range-2. Then, SSP-i can benefit by deviating to $\tilde{p}_{i,t} = p_M$ as shown below:

$$\begin{aligned}\mathcal{U}_{i,t}(\tilde{p}_{i,t}, p_j) &= f_t(p_M - c_S)\\ &= (f_t + f_o)(\hat{p}_t - c_S)\\ &> \mathcal{U}_{j,t}(p_{i,t}, p_j).\end{aligned}$$

where the inequality simply follows because, using $p_{i,t} = \hat{p}_t$, SSP-i would only acquire partial overlap flow (i.e., $0.5\ f_o$) whenever $F_j = f_t$. ■

With the above non-existence result in place, it is now natural to ask questions about mixed-strategy BNEs. Specifically, we are interested in determining *probability distributions* over the range of price-vectors (instead of individual price-vectors) that can constitute a solution to the SSPs' pricing game. The details are presented in the following sub-section.

B. MIXED STRATEGY BNE

The definition of mixed-strategies involves allowing the SSPs to choose prices $p_{i,t}$ in a random fashion. Specifically, for $i \in \{1, 2\}$ and $t \in \{\ell, h\}$, let $G_{i,t}$ denote the c.d.f of $p_{i,t}$, i.e., $G_{i,t}(p) = \mathbb{P}(p_{i,t} \leq p)$. We assume that $G_{i,t}$ is a distribution on the set of undominated prices $[\hat{p}_t, p_M]$ (since there is no rational in choosing any price less than $\hat{p}_t$; recall Lemma 1). Finally, we use $G_i = (G_{i,\ell}, G_{i,h})$ to denote a mixed-strategy of SSP-i.

Given a mixed strategy G_j of SSP-j, the payoff received by SSP-i for playing price p, given that $F_i = f_t$ $(t \in \{\ell, h\})$, can be written as in (13), shown at the bottom of the next page.[7] The payoff term associated with θ_ℓ in (13) can be understood as follows. With probability θ_ℓ the flow at SSP-j is f_ℓ. Then, further, with probability $G_{j,\ell}(p)$ SSP-j chooses a price less than p in which case SSP-i gets to serve only the monopoly flow f_t accruing a payoff of $f_t(p - c)$; on the other hand, with probability $(1 - G_{j,\ell}(p))$ the price set by SSP-j is greater than p in which case SSP-j receives a payoff of $(f_t + f_o)(p - c)$ by serving both monopoly as well as the overlap flows. The term associated with θ_h can be similarly understood. Simplifying (13) yields the simpler form in (14), as shown at the bottom of the next page, for the expression of the payoff $\mathcal{U}_{i,t}$.

[7]For simplicity, in (13) we overload the notation $\mathcal{U}_{i,t}$ from (8) to also denote the payoff received by SSP-i in response to a mixed strategy of SSP-j. Also, for simplicity, β is dropped from the notation as mentioned earlier.

The expected payoff received by SSP-i for using the mixed-strategy $G_{i,t}$ whenever $F_i = f_t$ is given by

$$\mathcal{U}_{i,t}(G_{i,t}, G_j) := \int_{\hat{p}_t}^{p_M} \mathcal{U}_{i,t}(p, G_j)\, dG_{i,t}(p) \tag{15}$$

where the integral in the above expression is understood as the *Riemann-Stieltjes integral* with respect to the distribution function $G_{i,t}$ [47].

We can now define mixed-strategy BNEs as follows.

Definition 3 (Mixed-Strategy BNE): For a given β, a mixed-strategy pair $(G_1^{(\beta)}, G_2^{(\beta)})$ is said to constitute a mixed-strategy BNE for the flexible-pricing game if the following holds: for all $i \in \{1, 2\}$ and $t \in \{\ell, h\}$ we have $\mathcal{U}_{i,t}(G_{i,t}^{(\beta)}, G_j^{(\beta)}) \geq \mathcal{U}_{i,t}(G_{i,t}, G_j^{(\beta)})$ for all mixed-strategies $G_{i,t}$ of SSP-i. A mixed-strategy BNE $(G_1^{(\beta)}, G_2^{(\beta)})$ is said to be *symmetric* if $G_i^{(\beta)} = G_j^{(\beta)} = G^{(\beta)}$. □

C. STRUCTURE OF A MIXED STRATEGY BNE

Obtaining mixed-strategy BNEs directly from the definition is difficult in general. However, there is an equivalent representation that can be used to compute mixed-strategy BNEs [46, Section 4.11, Proposition 140.1]. We state this result without proof (however, adapted to our notation) in the following.

Proposition 1 (Osborne 2004): Consider a mixed-strategy $(G_i^{(\beta)}, G_j^{(\beta)})$. For simplicity, denote $u_{i,t}(\beta) := \mathcal{U}_{i,t}(G_{i,t}^{(\beta)}, G_j^{(\beta)})$. Then, $(G_i^{(\beta)}, G_j^{(\beta)})$ is a BNE if and only if, for each player $i \in \{1, 2\}$ and state $t \in \{\ell, h\}$

- $G_{i,t}^{(\beta)}$ does not place any probability distribution on any p such that $\mathcal{U}_{i,t}(p, G_j^{(\beta)}) < u_{i,t}(\beta)$.
- There exists no p such that $\mathcal{U}_{i,t}(p, G_j^{(\beta)}) > u_{i,t}(\beta)$.

The above two conditions imply that $\mathcal{U}_{i,t}(p, G_j^{(\beta)}) = u_{i,t}(\beta)$ for all p that lies in the domain of $G_{i,t}^{(\beta)}$, while $\mathcal{U}_{i,t}(p, G_j^{(\beta)}) < u_{i,t}(\beta)$ for p that are outside its domain.

Using the above proposition we proceed to identify mixed-strategy BNEs. Specifically, we compute a symmetric BNE whose structure is as reported in the following theorem.

Theorem 2: The mixed-strategy pair $(G^{(\beta)}, G^{(\beta)})$ constitutes a symmetric BNE for the flexible-pricing game where $G^{(\beta)} = (G_\ell^{(\beta)}, G_h^{(\beta)})$ is given by

$$G_h^{(\beta)}(p) = \frac{(f_h + \theta_h f_o)}{\theta_h f_o} \frac{(p - q_h)}{(p - c_S)} \quad \text{for } q_h \leq p \leq p_M \tag{16}$$

$$G_\ell^{(\beta)}(p) = \frac{(f_\ell + f_o)}{\theta_\ell f_o} \frac{(p - q_\ell)}{(p - c_S)} \quad \text{for } q_\ell \leq p \leq q_h. \tag{17}$$

The thresholds q_ℓ and q_h are computed as follows:

$$q_h = \frac{p_M f_h + c_S \theta_h f_o}{f_h + \theta_h f_o} \tag{18}$$

$$q_\ell = \frac{q_h(f_\ell + \theta_h f_o) + c_S \theta_\ell f_o}{f_\ell + f_o}. \tag{19}$$

Proof: Before proceeding to the details of the proof, it is useful to note that $G_h^{(\beta)}$ and $G_\ell^{(\beta)}$ are indeed valid probability distributions on their respective domains. For instance, we see that $G_h^{(\beta)}$ is increasing in p; also it can be verified (by direct substitution and simplification) that $G_h^{(\beta)}(q_h) = 0$ and $G_h^{(\beta)}(p_M) = 1$. Similarly, $G_\ell^{(\beta)}$ is also increasing in p, satisfying $G_\ell^{(\beta)}(q_\ell) = 0$ and $G_\ell^{(\beta)}(q_h) = 1$.

Now, the proof is essentially based on verifying the sufficient conditions in Proposition 1. Equivalently, we show that the payoff function $\mathcal{U}_{i,t}(p, G^{(\beta)})$ is constant on the domain of $G_t^{(\beta)}$, and lower elsewhere. The details are as follows.

We begin by recalling the expression for $\mathcal{U}_{i,t}$ in (14) (adapted to the current case where $(G_i, G_j) = (G^{(\beta)}, G^{(\beta)})$):

$$\mathcal{U}_{i,t}(p, G^{(\beta)}) = (f_t + f_o)(p - c_S) - \Big(\theta_\ell G_\ell^{(\beta)}(p) + \theta_h G_h^{(\beta)}(p)\Big) f_o (p - c_S). \tag{20}$$

For $t = \ell$, evaluating the above expression for $q_\ell \leq p \leq q_h$ we obtain (by noting that $G_h^{(\beta)}(p) = 0$ for p in the above range)

$$\mathcal{U}_{i,\ell}(p, G^{(\beta)}) = (f_\ell + f_o)(q_\ell - c_S) =: u_\ell(\beta). \tag{21}$$

Thus, the payoff stays constant in the domain of $G_\ell^{(\beta)}$. However, for $p < q_\ell$, since both $G_\ell^{(\beta)}(p) = G_h^{(\beta)}(p) = 0$, we obtain

$$\mathcal{U}_{i,\ell}(p, G^{(\beta)}) = (f_\ell + f_o)(p - c_S) < u_\ell(\beta).$$

Finally, for $p \geq q_h$, substituting $G_\ell^{(\beta)}(p) = 1$ and $G_h^{(\beta)}(p)$ from (16), and simplifying yields

$$\mathcal{U}_{i,\ell}(p, G^{(\beta)}) = (f_\ell - f_h)p + \kappa_\ell \tag{22}$$

$$\mathcal{U}_{i,t}(p, G_j) := \theta_\ell\Big(G_{j,\ell}(p) f_t (p - c_S) + \big(1 - G_{j,\ell}(p)\big)(f_t + f_o)(p - c_S)\Big) + \theta_h\Big(G_{j,h}(p) f_t (p - c_S) + \big(1 - G_{j,h}(p)\big)(f_t + f_o)(p - c_S)\Big) \tag{13}$$

$$= (f_t + f_o)(p - c_S) - \Big(\theta_\ell G_{j,\ell}(p) + \theta_h G_{j,h}(p)\Big) f_o (p - c_S). \tag{14}$$

where $\kappa_\ell := \left(f_h q_h - f_\ell c_S + \theta_h f_o(q_h - c_S)\right)$ is a constant in p. Since $f_\ell < f_h$, it follows that the above expression is decreasing in p. Thus, for $p > q_h$ we have $\mathcal{U}_{i,\ell}(p, G^{(\beta)}) < \mathcal{U}_{i,\ell}(q_h, G^{(\beta)}) = u_\ell(\beta)$.

Similarly, for $t = h$, evaluating the payoff expression in (20) for $q_h \le p \le p_M$ we obtain

$$\mathcal{U}_{i,h}(p, G^{(\beta)}) = (f_h + \theta_h f_o)(q_h - c_S) =: u_h(\beta). \tag{23}$$

On the other hand, for $p \le q_h$ we obtain

$$\mathcal{U}_{i,h}(p, G^{(\beta)}) = (f_h - f_\ell)p + \kappa_h \tag{24}$$

where $\kappa_h = \left(f_\ell q_\ell - f_h c_S + f_o(q_\ell - c_S)\right)$. Since $f_h > f_\ell$ the above expression is increasing in p. Thus, for $p < q_h$ we obtain $\mathcal{U}_{i,h}(p, G^{(\beta)}) < \mathcal{U}_{i,h}(q_h, G^{(\beta)}) = u_h(\beta)$. ■

Using the conditional payoffs $u_\ell(\beta)$ and $u_h(\beta)$ in (21) and (23), respectively, the average payoff of SSP-i at the BNE $(G^{(\beta)}, G^{(\beta)})$ can be written as

$$\begin{aligned} u_i(\beta) &= \theta_\ell u_\ell(\beta) + \theta_h u_h(\beta) \\ &= f_a(p_M - c_S) + \frac{\theta_\ell \theta_h (p_M - c_S)(f_h - f_\ell) f_o}{f_h + \theta_h f_o}. \end{aligned}$$

Substituting for q_h and q_ℓ from (18) and (19) in the above expression, and simplifying finally yields (which has been verified using Mathematica)

$$u_i(\beta) = f_a(p_M - c_S) + \frac{\theta_\ell \theta_h (p_M - c_S)(f_h - f_\ell) f_o}{f_h + \theta_h f_o} \tag{25}$$

where recall that $f_a = \theta_\ell f_\ell + \theta_h f_h$ denotes the average flow.

D. ILLUSTRATION AND DISCUSSION

In Fig. 2 we illustrate the structure of the BNE distributions $G_h^{(\beta)}$ and $G_\ell^{(\beta)}$ for a numerical example (where $f_\ell = 5, f_h = 20, f_o = 10, p_M = 10, c_S = 1$, and $\theta_\ell = 0.6$). Also depicted in the figure are the respective payoff functions $\mathcal{U}_{i,\ell}$ and $\mathcal{U}_{i,h}$ (see the plots corresponding to the right-sided y-axis). The q_h and q_ℓ values (computed using (18) and (19)) are 8.5 and 5.5, respectively. These thresholds (marked on the x-axis) determine the respective domains of the BNE distributions $G_h^{(\beta)}$ and $G_\ell^{(\beta)}$. The payoff values $u_h(\beta) = 180$ and $u_\ell(\beta) = 67.5$ (computed using (23) and (21), respectively) are similarly marked on the plot's RHS y-axis. From Fig. 2 we make the following observations:

- We first note that the *domains are non-overlapping*. This condition is necessary as otherwise (due to the form of the payoff function in (20)) it would not be possible to satisfy the condition $\mathcal{U}_{i,t}(p, G^{(\beta)}) = u_t(\beta)$ (i.e., a constant) for all p in the domain of $G_t^{(\beta)}$.
- Further, we also notice that *the distribution corresponding to the state of larger flow has a "greater" domain, and vice versa*. Specifically, the domain of $G_h^{(\beta)}$ is $[q_h, p_M]$ which is greater than that of $G_\ell^{(\beta)}$ (which is $[q_\ell, q_h]$). The above requirement is necessary in the proof to show that the value of the payoff $\mathcal{U}_{i,t}(p, G_t^{(\beta)})$ is lower outside the domain of $G_t^{(\beta)}$. For instance, from (22) we notice that the condition $f_\ell < f_h$ is critical to ensure that $\mathcal{U}_{i,\ell}(p, G^{(\beta)}) < u_\ell^{(\beta)}$ for $p \in (q_h, p_M]$. The above condition (i.e., $f_\ell < f_h$) is similarly required in (24) to argue that $\mathcal{U}_{i,h}(p, G^{(\beta)}) < u_h(\beta)$ for p outside the domain of $G_h^{(\beta)}$.

The above two observations, in fact, enabled us to derive the result in Theorem 2 as follows:

1) We first recognize that the domains of $G_h^{(\beta)}$ and $G_\ell^{(\beta)}$ are of the form $[q_h, p_M]$ and $[q_\ell, q_h]$, respectively.
2) We then identify the forms of the distributions $G_t^{(\beta)}(p)$ $(t \in \{\ell, h\})$ that is necessary to ensure that the payoff $\mathcal{U}_{i,t}(p, G^{(\beta)})$ remains constant over the respective domains. Expressions (16) and (17) are results of this process.
3) We finally calculate q_h and q_ℓ in an iterative fashion: we first obtain q_h by solving $G_h^{(\beta)}(p_M) = 1$; we then derive q_ℓ by solving $G_\ell^{(\beta)}(q_h) = 1$. The resultant expressions are as reported in (18) and (19).

The above procedure can be extended to scenarios comprising more than two flow levels as presented in the following.

Consider a general model where $f_1 < f_2 < \cdots < f_n$ $(n \ge 2)$ denote the different offloaded-flow levels (with $\beta = (\beta_1, \beta_2, \cdots, \beta_n) \in [0,1]^n$ denoting the offloading decision). The probability that F_i takes value f_t is given by θ_t (for $t \in [n]$ where $[n] := \{1, 2, \cdots, n\}$). The other aspects of the model remain unchanged. As a result, we have the following analog of expression (20):

$$\begin{aligned} \mathcal{U}_{i,t}(p, G^{(\beta)}) &= (f_t + f_o)(p - c_S) \\ &\quad - \sum_{s=1}^{n} \theta_s G_s^{(\beta)}(p) f_o(p - c_S) \end{aligned}$$

where $G^{(\beta)} = (G_1^{(\beta)}, G_2^{(\beta)}, \cdots, G_n^{(\beta)})$ represents a mixed-strategy BNE. Using the above payoff function into the 3-step procedure discussed earlier (and extending the procedure to $n \ge 2$ flow-levels), we obtain the following generalization of the result in Theorem 2.

Theorem 3: The mixed-strategy pair $(G^{(\beta)}, G^{(\beta)})$ constitutes a symmetric BNE where $G^{(\beta)} = (G_t^{(\beta)} : t \in [n])$ is given by

$$G_t^{(\beta)}(p) = \frac{\left(f_t + \sum_{s=t}^{n} \theta_s f_o\right)}{\theta_t f_o} \frac{(p - q_t)}{(p - c_S)} \text{ for } q_t \le p \le q_{t+1} \tag{26}$$

for $t \in [n]$. The thresholds q_t can be computed via. backward induction as follows: $q_{n+1} := p_M$ and

$$q_t = \frac{q_{t+1}\left(f_t + \left(1 - \sum_{s=1}^{t} \theta_s\right) f_o\right) + c_S \theta_t f_o}{f_t + \sum_{s=t}^{n} \theta_s f_o} \tag{27}$$

for $t = n, (n-1), \cdots, 1$.

Proof: The proof is exactly along the lines of the proof of Theorem 2. We do not repeat the proof for brevity. ■

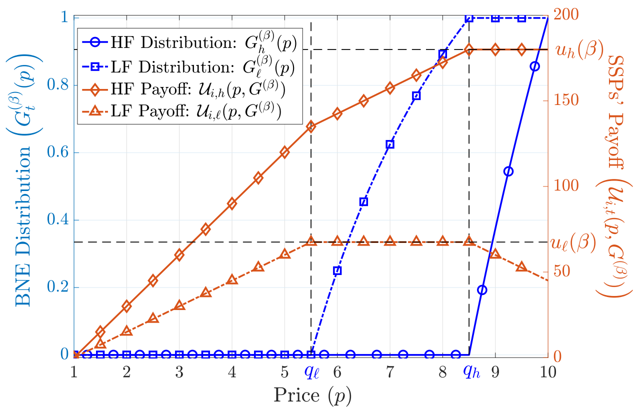


FIGURE 2. **Illustration of BNE distribution and SSPs' payoff functions. The distribution functions are depicted using blue coloured curves, while the payoff functions are shown in red colour. Solid lines represent quantities corresponding to high-flow (HF), while that corresponding to low-flow (LF) are shown using dash lines.**

Note that, for $n = 2$ the above result readily simplifies to the expressions in Theorem 2. Also, we note that the payoffs functions remain constant over the domain of the respective distributions. The value of the constant payoff $u_t(\beta)$ that is achieved by any p in the domain of $G_t^{(\beta)}$ is given by

$$u_t(\beta) := \left(f_t + \sum_{s=t}^{n} \theta_s f_o\right)(q_t - c_S). \tag{28}$$

We can again easily verify that the above expression is consistent with the expressions of $u_\ell(\beta)$ and $u_h(\beta)$ in (21) and(23).

IV. OFFLOADING GAME: STACKELBERG EQUILIBRIUM

For a given offloading strategy β of the MNO, we first compute the expected payoff received by the MNO when the SSPs set prices using the symmetric BNE $G^{(\beta)} = (G_\ell^{(\beta)}, G_h^{(\beta)})$ derived in Theorem 2. We then optimize the MNO's payoff function to eventually derive a Stackelberg equilibrium. We proceed as follows.

Let $p_i = (p_{i,\ell}, p_{i,h})$ denote the random price-vector of SSP-i where the distribution of $p_{i,t}$ is given by $G_t^{(\beta)}$ (for $t \in \{\ell, h\}$); equivalently, the distribution of p_i is given by $G^{(\beta)}$. Define $p_j = (p_{j,\ell}, p_{j,h})$ similarly, and assume that p_i and p_j are independent. Then, recalling (4) and (5), the average payoff of the MNO can be expressed as

$$u_M(\beta) := \mathbb{E}\Big[\mathcal{U}_M\big(\beta; (p_i, p_j)\big)\Big]$$

where the expectation is w.r.t the joint distribution $(G^{(\beta)}, G^{(\beta)})$ of the random vector (p_i, p_j). Towards computing the above expectation, we define the conditional payoffs of the MNO as

$$u_M\big(\beta|t_i, t_j\big) := \mathbb{E}\Big[\mathcal{U}_M\big(p_i, p_j\big)\Big| F_i = t_i, F_j = t_j\Big]$$

for $t_i, t_j \in \{\ell, h\}$. Now, noting that $u_M\big(\beta|\ell, h\big) = u_M\big(\beta|h, \ell\big)$ (since the statistics associated with both SSPs are identical), we can write

$$\begin{aligned} u_M(\beta) \\ = \theta_\ell^2 u_M(\beta|\ell, \ell)) + 2\theta_\ell\theta_h u_M(\beta|\ell, h) + \theta_h^2 u_M(\beta|h, h). \end{aligned} \tag{29}$$

Recalling (4) and (5) again, the individual payoff terms in the above expression can be computed as follows: for $t \in \{\ell, h\}$

$$\begin{aligned} u_M(\beta|t, t) = 2f_t\Big(p_M - \mathbb{E}[p_{i,t}]\Big) \\ + f_o\Big(p_M - \mathbb{E}\Big[\min\{p_{i,t}, p_{j,t}\}\Big]\Big) \\ + f_M(t, t)\big(p_M - c_M(f_M)\big) \end{aligned} \tag{30}$$

$$\begin{aligned} u_M(\beta|\ell, h) = f_\ell\Big(p_M - \mathbb{E}[p_{i,\ell}]\Big) + f_h\Big(p_M - \mathbb{E}[p_{i,h}]\Big) \\ + f_o\Big(p_M - \mathbb{E}[p_{i,\ell}]\Big) + f_M(\ell, h)\big(p_M - c_M(f_M)\big). \end{aligned} \tag{31}$$

where (recall (2))

$$\begin{aligned} f_M(t, t) &:= \mathbb{E}[F_M | F_i = f_t, F_j = f_t] \\ &= 2(1 - \beta_\ell)\mathbf{F}_\ell + (1 - \beta_o)\mathbf{F}_o \\ f_M(\ell, h) &:= \mathbb{E}[F_M | F_i = f_\ell, F_j = f_h] \\ &= \sum_{t \in \{\ell, h\}} (1 - \beta_t)\mathbf{F}_t + (1 - \beta_o)\mathbf{F}_o. \end{aligned}$$

Note that the expressions (30) and (31) have been simplified by replacing $\mathbb{E}[p_{j,t}]$ by $\mathbb{E}[p_{i,t}]$ for $t \in \{\ell, h\}$ (since $p_{i,t}$ and $p_{j,t}$ are i.i.d with $G_t^{(\beta)}$ being their common distribution). Also, in order to simplify the min-term in the second expression we have made use of the observation that $p_{i,\ell} \in [q_\ell, q_h]$ and $p_{j,h} \in [q_h, p_M]$ so that $p_{i,\ell} \leq p_{j,h}$ always (recall Theorem 2).

Expectation of $p_{i,t}$ can be computed using the identity

$$\mathbb{E}[p_{i,t}] = \int_0^\infty \mathbb{P}(p_{i,t} > p)dp = \int_0^\infty (1 - G_t^{(\beta)}(p))dp. \tag{32}$$

Similarly, since $p_{i,t}$ and $p_{j,t}$ are i.i.d it follows that

$$\mathbb{E}\Big[\min\{p_{i,t}, p_{j,t}\}\Big] = \int_0^\infty \Big(1 - G_t^{(\beta)}(p)\Big)^2 dp. \tag{33}$$

Using the above development in (29) and simplifying yields[8]

$$\begin{aligned} u_M(\beta) = f_o(p_M - c_S) - \frac{2\theta_\ell\theta_h(p_M - c_S)(f_h - f_\ell)f_o}{f_h + \theta_h f_o} \\ + f_M\big(p_M - c_M(f_M)\big). \end{aligned} \tag{34}$$

However, note that the above expression is applicable whenever $f_\ell \leq f_h$, i.e., for $\beta = (\beta_\ell, \beta_h, \beta_o)$ such that $\beta_\ell\mathbf{F}_\ell \leq \beta_h\mathbf{F}_h$. Suppose β satisfies $\beta_\ell\mathbf{F}_\ell > \beta_h\mathbf{F}_h$ (i.e., $f_\ell > f_h$), then interchanging the roles of f_ℓ and f_h we obtain

$$\begin{aligned} u_M(\beta) = f_o(p_M - c_S) - \frac{2\theta_h\theta_\ell(p_M - c_S)(f_\ell - f_h)f_o}{f_\ell + \theta_\ell f_o} \\ + f_M\big(p_M - c_M(f_M)\big). \end{aligned} \tag{35}$$

Expressions (34) and (35) together constitute of the payoff of the MNO for any given offloading strategy $\beta \in [0, 1]^3$.

[8] We have used Mathematica for simplifying the expressions; for brevity the intermediate steps of simplification have been omitted.

Finally, stackelberg equilibrium $\beta^* = (\beta_\ell^*, \beta_h^*, \beta_o^*)$ can be computed as follows:

$$\beta^* = \arg\max_{\beta\in[0,1]^3} u_M(\beta) \tag{36}$$

while the MNO's payoff at the equilibrium is given by

$$u_M^* := u_M(\beta^*) = \max_{\beta\in[0,1]^3} u_M(\beta). \tag{37}$$

Given β^*, the average payoff achieved by SSP-i ($i \in \{1, 2\}$) at equilibrium can be computed using (25) as follows:

$$\begin{aligned} u_i^* &:= u_i(\beta^*) \\ &= f_a^*(p_M - c_S) + \frac{\theta_\ell\theta_h(p_M - c_S)(f_h^* - f_\ell^*)f_o^*}{f_h^* + \theta_h f_o^*} \end{aligned} \tag{38}$$

where $f_\ell^* = \beta_\ell^*\mathbf{F}_\ell$, $f_h^* = \beta_h^*\mathbf{F}_h$, and $f_o^* = \beta_o^*\mathbf{F}_o$ denote the offloaded flows at β^*, while $f_a^* = \theta_\ell f_\ell^* + \theta_h f_h^*$ represents the average offloaded flow. Again note that the above payoff expression is valid provided $f_\ell^* \le f_h^*$. In case $f_\ell^* > f_h^*$, then

$$u_i^* = f_a^*(p_M - c_S) + \frac{\theta_h\theta_\ell(p_M - c_S)(f_\ell^* - f_h^*)f_o^*}{f_\ell^* + \theta_\ell f_o^*} \tag{39}$$

which is simply obtained by interchanging the roles of f_ℓ and f_h in (25).

In Section VII we will conduct a detailed numerical study to determine the structure of the Stackelberg equilibrium β^*, along with the payoffs u_M^* and u^* received by the MNO and the SSPs at the equilibrium, respectively. Before proceeding to Section VII, we will discuss the case of *flat-pricing scheme* where the SSPs are restricted to announce a single price irrespective of the flow-level that the SSPs may experience. This is in contrast to the flexible-pricing scheme studied thus far where the SSPs are allowed to set different prices for different flow-levels they could experience. Details are presented in the following section.

V. FLAT PRICING MODEL

Like in the flexible-pricing scheme (studied in Sections II to IV) we begin by fixing the offloading strategy $\beta = (\beta_\ell, \beta_h, \beta_o)$ of the MNO so that the offloaded flows are given by $f_\ell = \beta_\ell\mathbf{F}_\ell$, $f_h = \beta_h\mathbf{F}_h$, and $f_o = \beta_o\mathbf{F}_o$ as before. However, unlike the flexible-pricing scheme, here the SSPs are restricted to announce flat-prices. Let $\overline{p}_i \in [c_S, p_M]$ denote[9] the price announced by SSP-i ($i \in \{1, 2\}$). Note that $\overline{p}_i$ and $\overline{p}_j$ are scalar prices, which is unlike the case in the flexible-pricing scheme where these are vectors (e.g., $p_i = (p_{i,\ell}, p_{i,h})$).

Now, given the *flat-prices* $(\overline{p}_i, \overline{p}_j)$ of the SSPs, the payoff received by SSP-i is given by

$$\overline{\mathcal{U}}_i\left((\overline{p}_i, \overline{p}_j)\,;\beta\right) = \begin{cases} f_a(\overline{p}_i - c_S) & \text{if } \overline{p}_i > \overline{p}_j \\ (f_a + f_o)(\overline{p}_i - c_S) & \text{if } \overline{p}_i < \overline{p}_j \\ (f_a + 0.5\,f_o)(\overline{p}_i - c_S) & \text{if } \overline{p}_i = \overline{p}_j \end{cases}$$

[9]We use overline for the notations corresponding to the flat-pricing scheme.

where $f_a = \theta_\ell f_\ell + \theta_h f_h$ denotes the average flow as before. The above expression is identical to the payoff expression considered in [31]. Thus, leveraging the results from [31], we immediately identify the structure of the mixed-strategy BNE in the flat-pricing scheme as follows.

Theorem 4 (Li et al. 2019): The mixed strategy $(\overline{G}^*, \overline{G}^*)$ constitutes a symmetric BNE for the flat-pricing scheme where

$$\overline{G}^*(p) = \frac{(f_a + f_o)}{f_o}\frac{(p - q_a)}{p - c_S} \quad \text{for } q_a \le p \le p_M. \tag{40}$$

The threshold q_a is given by

$$q_a = \frac{p_M f_a + c_S f_o}{f_a + f_o}. \tag{41}$$

Discussion: It is interesting to compare the above result with the form of the BNE for the flexible-pricing scheme in Theorem 2. For this, we first note that for $\theta = 0$ or $\theta = 1$, since only one of the flow-levels occur with probability 1, our model reduces to the scenario studied in [31]. Suppose $\theta = 0$ then, noting that $f_a = f_h$, and simplifying (16) and (18) we obtain $G_h^* = \overline{G}^*$ and $q_h = q_a$ (while the distribution G_ℓ^* degenerates). The case $\theta = 1$ similarly yields $G_\ell^* = \overline{G}^*$ and $q_\ell = q_a$. Thus, our result in Theorem 2 is a generalization of the above result by Li et al. in [31].

Now, for a given price p, the payoff received by SSP-i when SSP-j uses the mixed strategy $\overline{G}^*$ can be written as

$$\begin{aligned} &\overline{\mathcal{U}}_i\big((p, \overline{G}^*); \beta\big) \\ &= \overline{G}^*(p)f_a(p - c_S) + \Big(1 - \overline{G}^*(p)\Big)(f_a + f_o)(p - c_S) \\ &= (f_a + f_o)(p - c_S) - \overline{G}^*(p)f_o(p - c_S) \\ &= (f_a + f_o)(q_a - c_S) \\ &= f_a(p_M - c_S) \end{aligned}$$

for $p \in [q_a, p_M]$. Thus, we see that the payoff remains constant for p in the domain of $\overline{G}^*$. As a result, the average payoff received by SSP-i under the flat-pricing scheme (for a given offloading strategy β of the MNO) is given by

$$\overline{u}_i(\beta) := \overline{\mathcal{U}}_i\big((\overline{G}^*, \overline{G}^*); \beta\big) = f_a(p_M - c_S). \tag{42}$$

The payoff received by the MNO under the flat-pricing scheme can now be computed as follows. Analogous to the conditional payoff term $u_M(\beta|t, t)$ in (30), the MNO's payoff in the flat-pricing scheme can be written as

$$\begin{aligned} \overline{u}_M(\beta) := 2f_a\Big(p_M - \mathbb{E}[\overline{p}_i]\Big) + f_M\big(p_M - c_M(f_M)\big) \\ + f_o\Big(p_M - \mathbb{E}\Big[\min\{\overline{p}_i, \overline{p}_j\}\Big]\Big) \end{aligned}$$

where $\overline{p}_i$ and $\overline{p}_j$ are i.i.d random prices with their common c.d.f given by $\overline{G}^*$ in (40), while f_M denotes the average self-loaded flow (recall (3)). Substituting the expectation terms (which can be computed using the identities in (32) and (33)) in the above expression and simplifying yields the following simple form for the payoff expression

$$\overline{u}_M(\beta) = f_o(p_M - c_S) + f_M\big(p_M - c_M(f_M)\big). \tag{43}$$

Finally, the Stackelberg equilibrium $\overline{\beta}^* = (\overline{\beta}^*_\ell, \overline{\beta}^*_h, \overline{\beta}^*_o)$ for the flat-pricing scheme can be computed as follows:

$$\overline{\beta}^* = \arg\max_{\beta\in[0,1]^3} \overline{u}_M(\beta)$$

while the MNO's payoff at the equilibrium is given by

$$\overline{u}^*_M = \overline{u}_M(\overline{\beta}^*) = \max_{\beta\in[0,1]^3} \overline{u}_M(\beta). \tag{44}$$

The SSPs payoff at equilibrium can be obtained by using the above $\overline{\beta}^*$ in (42):

$$\overline{u}^*_i = \overline{u}_i(\overline{\beta}^*) = \overline{f}^*_a(p_M - c_S) \tag{45}$$

where $\overline{f}^*_a = \theta_\ell \overline{f}^*_\ell + \theta_h \overline{f}^*_h, \overline{f}^*_\ell = \overline{\beta}^*_\ell \mathbf{F}_\ell$, and $\overline{f}^*_h = \overline{\beta}^*_h \mathbf{F}_h$.

Through our numerical study in Section VII, we will compare and contrast the performance (in terms of MNO's and SSPs' payoffs) achieved by the flat-pricing scheme against that achieved by the flexible-pricing scheme of the earlier sections.

VI. FULL-OFFLOADING STRATEGIES

Before proceeding to numerical work, in this section we will briefly discuss the *full-offloading* (flexible and flat pricing) schemes where the MNO naively offloads the entire flows onto the SSPs (i.e., without self-loading). This is in contrast to the flexible and flat pricing schemes with *optimal offloading* studied in the previous sections. In terms of prior literature, full-offloading flexible pricing scheme relates to our earlier work [1], while the case of flat-pricing with full-offloading corresponds to the work of Li et al. [31].

Full-offloading is achieved by fixing the offloading strategy of the MNO to $\beta' = [1, 1, 1]$ so that $f_M = 0$ (recall from (3) that f_M is the average self-loaded flow). Then, recalling (34), the MNO's payoff under full-offloading flexible pricing scheme is given by

$$\begin{aligned} u'_M &:= u_M(\beta') \\ &= \mathbf{F}_o(p_M - c_S) - \frac{2\theta_\ell\theta_h(p_M - c_S)(\mathbf{F}_h - \mathbf{F}_\ell)\mathbf{F}_o}{\mathbf{F}_h + \theta_h\mathbf{F}_o} \end{aligned} \tag{46}$$

Note that the offloaded flows (f_ℓ, f_h, f_o) are replaced by the overall flows $(\mathbf{F}_\ell, \mathbf{F}_h, \mathbf{F}_o)$. Similarly, for the flat-pricing scheme with full-offloading, the MNO's payoff can be easily deduced from (43):

$$\overline{u}'_M := \overline{u}_M(\beta') = \mathbf{F}_o(p_M - c_S). \tag{47}$$

Inspecting (46) and (47) we obtain the following result (stated as Lemma for easy reference):

Lemma 2: The difference in payoffs achieved by the MNO in full-offloading flexible and flat-pricing schemes is given by

$$(u'_M - \overline{u}'_M) = -\frac{2\theta_\ell\theta_h(p_M - c_S)(\mathbf{F}_h - \mathbf{F}_\ell)\mathbf{F}_o}{(\mathbf{F}_h + \theta_h\mathbf{F}_o)}. \tag{48}$$

TABLE 2. Various offloading and pricing strategies.

S No.	Offloading Strategy	Pricing Scheme	Short-form
1	Optimal Offloading	Flexible Pricing	FLEX OPT
2	Optimal Offloading	Flat Pricing	FLAT OPT
3	Full Offloading	Flexible Pricing	FLEX FULL [1]
4	Full Offloading	Flat Pricing	FLAT FULL [32]

The payoff achieved by the SSPs in full-offloading flexible and flat-pricing schemes, we recall (25) and (42), respectively, to obtain

$$\begin{aligned} u'_i &:= u_i(\beta') \\ &= \mathbf{F}_a(p_M - c_S) + \frac{\theta_\ell\theta_h(p_M - c_S)(\mathbf{F}_h - \mathbf{F}_\ell)\mathbf{F}_o}{\mathbf{F}_h + \theta_h\mathbf{F}_o} \end{aligned} \tag{49}$$

and

$$\overline{u}'_i := \overline{u}_i(\beta') = \mathbf{F}_a(p_M - c_S). \tag{50}$$

where $\mathbf{F}_a = \theta_\ell\mathbf{F}_\ell + \theta_h\mathbf{F}_h$ denotes the average overall flow (i.e., offloaded flow at β'). Analogous to Lemma 2, we thus have the following result.

Lemma 3: The difference in payoffs achieved by the SSPs in the flexible and the flat-pricing schemes with full-offloading is given by

$$(u'_i - \overline{u}'_i) = \frac{\theta_\ell\theta_h(p_M - c_S)(\mathbf{F}_h - \mathbf{F}_\ell)\mathbf{F}_o}{(\mathbf{F}_h + (1-\theta)\mathbf{F}_o)}. \tag{51}$$

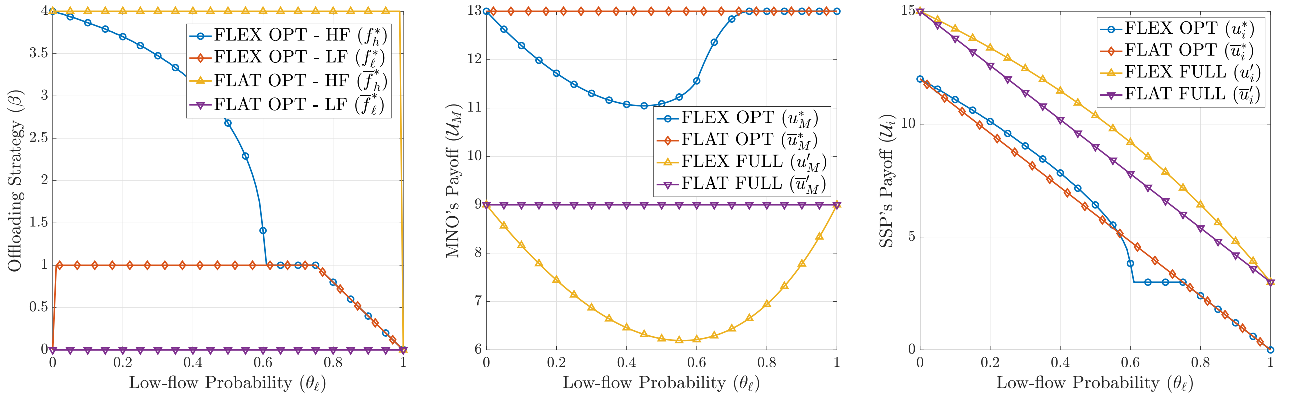
Discussion: From Lemma 3 we see that $u'_i \geq \overline{u}'_i$. Thus, under full-offloading, the SSPs can achieve a higher payoff if flexible-pricing is implemented than if flat-pricing were to be implemented. In contrast, from Lemma 2 it follows that MNO achieves a lower payoff under full-offloading flexible-pricing scheme. But interestingly we see that the amount of loss in payoff incurred by the MNO is equal to the total gain in payoff achieved by both SSPs. Thus, the net-payoff in the system is conserved when moving from flat to flexible-pricing full-offloading schemes, although there is a disparity in payoffs achieved by the SSPs and the MNO under the naive full-offloading strategy. Through our numerical work we will see that the optimal-offloading strategy can reduce this disparity in payoff, thus enabling a more favorable tradeoff between the flexible and flat pricing schemes. The details of our numerical study are reported in the next section.

VII. NUMERICAL WORK

In this section we will compare the performances of the flexible-pricing and flat-pricing schemes, under both optimal-offloading as well as full-offloading strategies. For the ease of presentation, we use the short-forms given in Table 2 to refer to each of the schemes for discussion in this section.

We first fix the values of the various parameters as follows:

- *Traffic flows*: $\mathbf{F}_\ell$ is normalized to 1, while $\mathbf{F}_h$ and $\mathbf{F}_o$ are set to 5 and 3, respectively.
- *Flow probabilities*: We set $\theta_\ell = 0.6$ so that $\theta_h = 0.4$.



(a) (b) (c)

FIGURE 3. Performance of the various schemes listed in Table 2 as a function of θ_ℓ.

- *Prices & Costs:* We normalize c_S to 1, while p_M is set to 4. We assume the MNO's cost to be a linear function of the average self-loaded flow f_M, i.e., $c_M(f_M) = f_M$.

We then vary the values of a few key parameters (namely, θ_ℓ, p_M, $\mathbf{F}_h$, and $\mathbf{F}_o$) (while keeping all other parameters' values fixed), and report the observations in the following.

We first vary θ_ℓ in the range [0, 1] and study its effect on the performance of the various strategies. The results are presented in Fig. 3(a), 3(b), and 3(c). Specifically, in Fig. 3(a) we depict the offloading strategies β^* and $\overline{\beta}^*$ of FLEX OPT and FLAT OPT, respectively, by plotting the corresponding high-flows (HF) and low-flows (LF). The overlap flows satisfy $f_o^* = \overline{f}_o^* = 3$ (i.e., the entire aggregate overlap flow $\mathbf{F}_o = 3$ is offloaded) for all $\theta_\ell \in [0, 1]$, and hence for simplicity as well as to avoid triviality, f_o^* and $\overline{f}_o^*$ are not depicted in Fig. 3(a). From Fig. 3(a) we derive the following key observations:

- For $0 \leq \theta_\ell \leq 0.6$ (approx), f_h^* decreases with θ_ℓ until $f_h^* = f_\ell^* = \mathbf{F}_\ell^*$. In this regime, under optimal offloading, the MNO maximizes its payoff by primarily reducing the gap $(f_h^* - f_\ell^*)$ between HF and LF, so that the second term of the payoff expression in (34) is optimized.
- For $0.6 < \theta_\ell \leq 0.75$, the offloading strategy remains unchanged at $f_h^* = f_\ell^* = \mathbf{F}_\ell^*$. In this regime, the third term of (34) is primarily being optimized (note that the second term vanishes once $f_h^* = f_\ell^*$).
- Finally, for $0.75 < \theta_\ell \leq 1$, both HF and LF decrease simultaneously to maintain $f_h^* = f_\ell^*$ due to which the second term of (34) remains 0, while also optimizing the self-loaded flow f_M^* so as to keep the third term optimized with increasing θ_ℓ.

The above phenomenon (of f_h^* trying to match f_ℓ^* first and then both decreasing/increasing together) will occur commonly with other parameters as well. For instance, see Fig. 4(a), 4(d) and 5(a) respectively corresponding to parameters p_M, $\mathbf{F}_h$ and $\mathbf{F}_o$. Unlike other parameters, in the case of $\mathbf{F}_h$, the offloaded HF f_h^* increases with the overall HF $\mathbf{F}_h$. This is because, for large values of $\mathbf{F}_h$, the MNO is better-off offloading a significant portion of the flow (although the second term of (34) reduces the MNO's payoff) than self-loading which will increase the MNO's service cost c_M, thus eventually reducing its payoff (via. the third term in the payoff expression (34)).

Now, returning back to the case of θ_ℓ, from Fig. 3(b), as expected, we see that the optimal-offloading schemes (FLEX OPT and FLAT OPT) yield a higher payoff to the MNO when compared with the naive full-offloading schemes (FLEX FULL and FLAT FULL). Furthermore, under optimal-offloading, the difference in MNO's payoffs with flexible and flat pricing schemes reduces when compared with the respective difference under the full-offloading strategy. In fact, for $\theta_\ell \geq 0.75$ we find that both FLEX OPT and FLAT OPT yield the same payoff to the MNO. In contrast, the SSPs' payoff reduces under optimal offloading as can be seen from Fig. 3(c). Thus, under optimal-offloading the SSPs cannot monopolize the incentive as in the case of the naive full-offloading schemes. More interestingly, we see that for $\theta_\ell \in [0, 0.55]$ FLEX OPT yields a slightly higher payoff than FLAT OPT, vice versa for $\theta_\ell \in [0.55, 0.75]$, and both strategies yield identical payoff for $\theta_\ell \in [0.75, 1]$. These results can serve as guidelines for the regulators to fix the nature of the pricing strategy (flexible or flat) so that both parties (MNO and SSPs) are fairly benefited by the system.

Similar conclusions can be drawn from the results in other plots. For instance from Fig. 4(b), where we depict MNO's payoff as a function of the MNO's price p_M, we again see that the gap in the payoffs of flexible and flat pricing schemes reduces under optimal offloading. From Fig. 4(c), where the SSPs' payoff are shown, we can again identify different range of prices p_M where FLEX OPT is better than FLAT OPT, and vice versa. The performances of the various strategies w.r.t the variation in the parameter $\mathbf{F}_h$ are shown in Fig. 4(d), 4(e), and 4(f). Similar conclusion can be drawn by investigating the MNO's and the SSPs' payoffs in Fig 4(e), and 4(f), respectively.

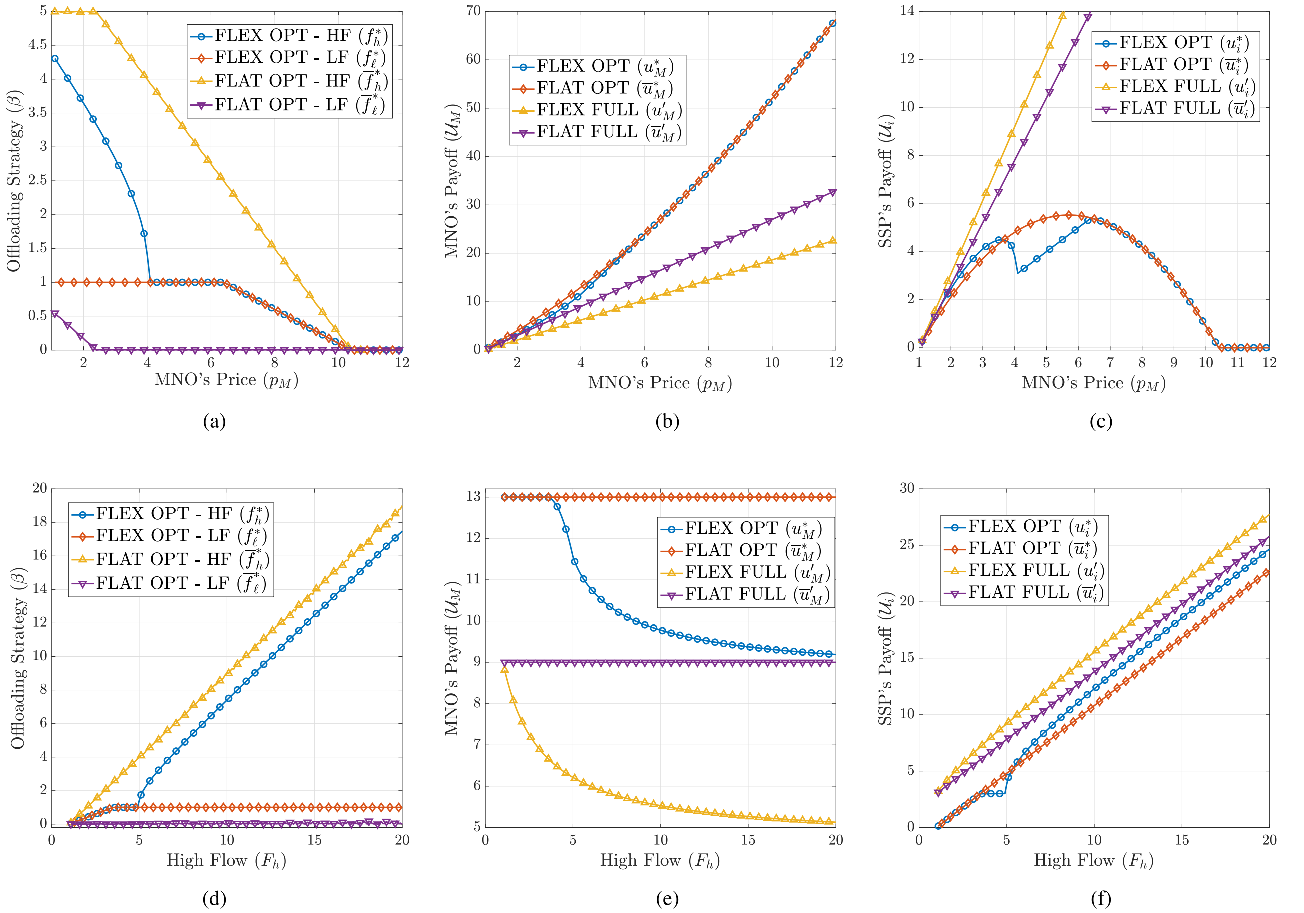


FIGURE 4. Performance of the various schemes listed in Table 2 as functions of p_M and $\mathbf{F}_h$.

Results corresponding to parameter $\mathbf{F}_o$ are illustrated in Fig. 5(a), 5(b) and 5(c). From the results we see that FLAT OPT yields a better performance than FLEX OPT, both in terms of achieving a higher payoff to MNO as well as the SSPs (Fig. 5(b) and 5(c)). However, note that this observation is for the case where the cost function c_M is linear, i.e., $c_M(f_M) = f_M$. For general polynomial cost functions of the form $c_M(f_M) = f_M^2$ (for $n = 2$ the cost function becomes quadratic), from Fig. 5(e) and 5(f) we find that the FLEX OPT scheme is more favorable than FLAT OPT in terms of yielding a higher payoff to the SSPs (while the MNOs' payoff remains marginally lower than that of FLAT OPT). The corresponding optimal strategies are depicted in Fig. 5(d).

Based on our above results and discussions, we summarize the following:

- Optimal-offloading strategies (FLEX OPT and FLAT OPT) prevents the SSPs from monopolizing the payoff, unlike the full-offloading strategies in the literature (FLEX FULL and FLAT FULL which represent prior work [1] and [31], respectively).
- Given optimal-offloading, the choice of the pricing strategy (FLEX OPT or FLAT OPT) depends heavily on the operating point (i.e., the values of the various parameters of the model). Specifically, some observations regarding the SSPs pricing strategy are as follows:
  - o Flexible-pricing yields better payoff to SSPs if the value of θ_ℓ is low (i.e., approximately $\theta_\ell \leq 0.55$), while flat-pricing suffices otherwise (see Fig. 3(c)).
  - o Similarly, for low values of MNO's price (i.e., approximately $p_M \leq 3.8$) flexible-pricing is a better choice (see Fig. 4(c)).
  - o In contrast, flexible pricing is a suitable choice for higher values of high flow (i.e., $\mathbf{F}_h > 5$); see Fig. 4(f).
  - o With respect to varying overlap flow, flat-pricing is optimal whenever the MNO's cost function c_M is linear, while flexible-pricing should be adopted otherwise (i.e., whenever c_M is super-linear); see Fig. 5(c) and Fig. 5(f), respectively.

In summary, given the estimates of the system parameters, our above observations can serve as a guideline for the regulators to decide upon the pricing scheme to use (FLEX OPT or FLAT OPT) that will mutually benefit a local network of privately owned MNO and SSPs, thus enticing them to participate in mobile-data offloading.

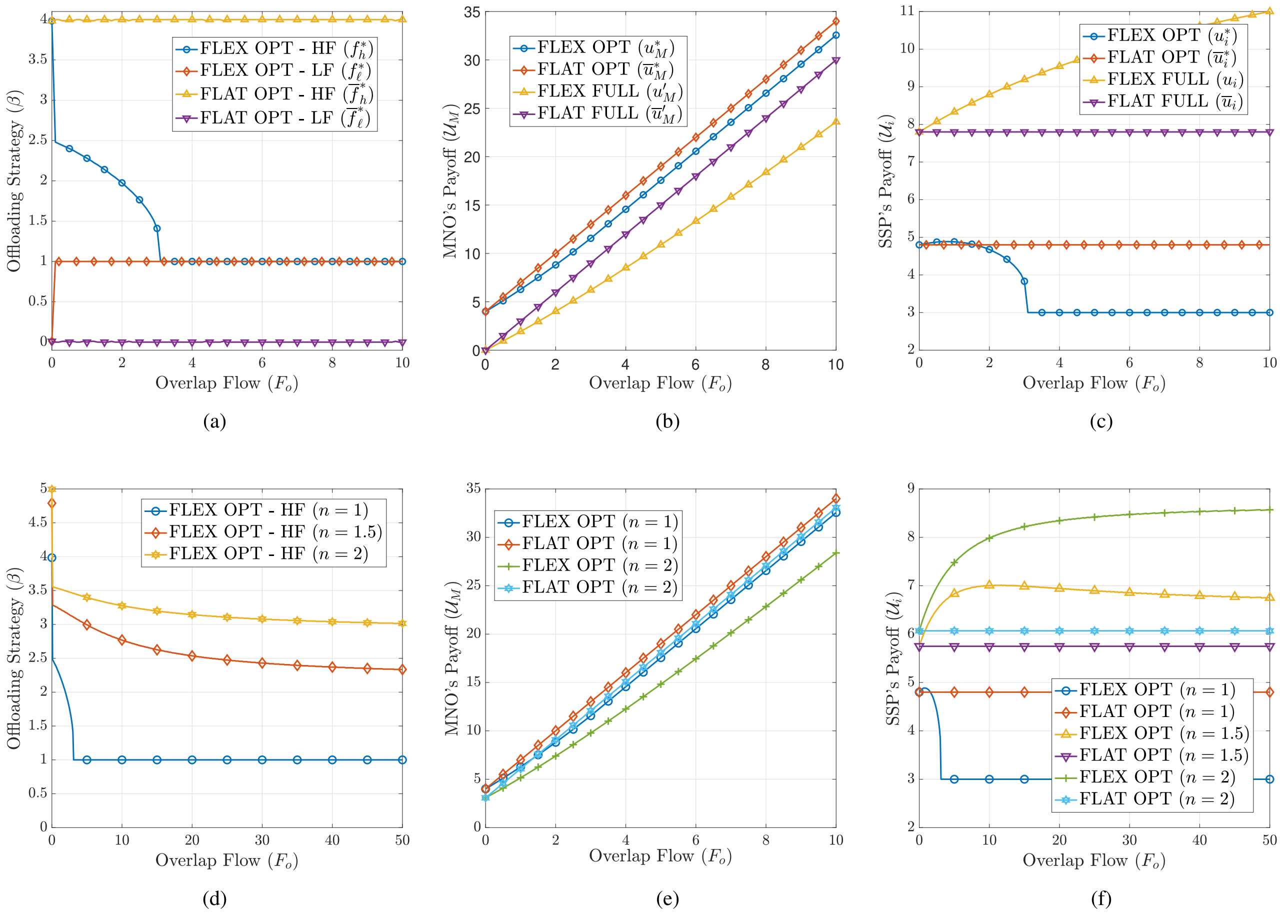


FIGURE 5. Performance of the various schemes listed in Table 2 as functions of F_o and $c_M(\cdot)$.

VIII. CONCLUSION AND FUTURE SCOPE

For a model where the SSPs are allowed to set flexible prices, we proposed a Stackelberg game formulation to study the joint problem of offloading and pricing in heterogeneous networks. The pricing problem appears in the Stackelberg formulation as a Bayesian-game played between the SSPs, the solution to which is characterized in terms of Bayesian Nash equilibrium (BNE). We first showed that no BNE exists in pure strategies, and then proceeded to derive results illustrating the structure of a symmetric mixed-strategy BNE (Theorem 2). The BNE solution of the pricing problem is used to obtain the Stackelberg equilibrium of the MNO's offloading problem (see (36)). All the above results are specialized to the flat-pricing scenario considered in the literature, where the SSPs are restricted to announce a single price irrespective of the amount of data that may be offloaded onto them. We conducted an extensive numerical study to compare the performances of the flexible and flat-pricing schemes under optimal-offloading and full-offloading strategies of the MNO. Through our study we find that the optimal-offloading strategy prevents the SSPs from monopolizing the market payoff (unlike the full-offloading strategy), while the pricing scheme (flexible or flat) that yields a favorable payoff to the SSPs depends on the system parameters. Thus, given the estimates of the system parameters, our work can simultaneously recommend the offloading strategy for the MNO (i.e., amount of traffic to offload) and the pricing scheme (flexible or flat) that can enable SSPs to accrue a higher payoff.

Extending our study to a scenario comprising more than two SSPs would constitute an interesting direction for future work. We note that when multiple SSPs are involved, the overlap flow amongst different pairs of SSPs may be different, which would render the problem more challenging. Along similar lines, it will be interesting to consider a setup comprising multiple MNOs, where the offloading decisions of the MNOs and the pricing strategies of the SSPs are intricately coupled with one another. In this context, determining the Stackelberg equilibrium offloading strategy would be an interesting contribution.

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