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RESEARCH ARTICLE

Grey Wolf Optimizer and Whale Optimization Algorithm for Stochastic Inventory Management of Reusable Products in a Two-Level Supply Chain

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ABSTRACT Product reuse and recovery is an efficient tool that helps companies to simultaneously address economic and environmental dimensions of sustainability. This paper presents a novel problem for stock management of reusable products in a single-vendor, multi-product, multi-retailer network. Several constraints, such as the maximum budget, storage capacity, number of orders, etc., are considered in their stochastic form to establish a more realistic problem. The presented problem is formulated using a nonlinear programming mathematical model. The chance-constrained approach is suggested to deal with the constraints' uncertainty. Regarding the nonlinearity of the model, grey wolf optimizer (GWO) and whale optimization algorithm (WOA) as two novel metaheuristics are presented as solution approaches, and the sequential quadratic programming (SQP) exact algorithm validates their performance. The parameters of algorithms are calibrated using the Taguchi method for the design of experiments. Extensive analysis is established by solving several numerical results in different sizes and utilizing several comparison measures. Also, the results are compared statistically using proper parametric and non-parametric tests. The analysis of the results shows a significant difference between the algorithms, and GWO has a better performance for solving the presented problem. In addition, both algorithms perform well in searching the solution space, where the GWO and WOA differences with the optimal solution of the SQP algorithm are negligible.

INDEX TERMS Reuse and recovery, chance-constrained programming, grey wolf optimizer, whale optimization algorithm, Taguchi method.

I. INTRODUCTION

Companies' perspectives have changed from classic business principles to contemporary ideas like the supply chain in today's competitive environment. More specifically, the managers try to integrate the activities and processes of their supply chain to improve the overall performance of their companies [1]. This change is obvious in various sectors e.g., manufacturing, retail, healthcare, etc [2], [3], [4]. Integration of the supply chain necessitates collaboration across its entities, as well as coordination of information and material flows. Such coordination can play a significant role in cost-cutting and improving value for customers [5]. Inventory planning and control is one of the highly important problems in supply chains that need special coordination between supply chain entities. Unpleasant or variable inventory in a supply chain causes the Bullwhip effect and double marginalization, which ultimately degrade the performance of the supply chain and may even contribute to the demise of businesses [6]. Therefore, operations research experts investigated the problem from several years ago. The history of inventory management dates back to a century ago when Harris [7] developed the classical economic order quantity (EOQ) model. Afterward, this basic inventory model was

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extended for a wide variety of products by considering several realistic assumptions, with the details that will be discussed in the literature review section.

Despite the appearance of broad literature, the inventory management of reusable products, as an important type of items, remained unaddressed for several years. Reusable products are products that have the potential for reuse and recovery after consumption. Surgical tools, car parts, copiers, and other products are a few examples of these items. In addition to economic benefits, the recovery and reuse of reusable products help companies address the environmental dimension of sustainability appropriately. Recently, the single-product and multi-product EOQ models for inventory management of reusable items were by Mokhtari [8], and Fallahi et al. [9], respectively. In these works, one of the main assumptions is that the system is single-level, and both models ignore vendor costs in the decision-making process. In other words, there is no coordination between the upstream and downstream of the supply chain, and it is assumed that the retailers are solely responsible for decision-making. On the other hand, both works modeled and optimized their problem by considering deterministic parameters. However, several uncertainty sources may impact the parameters of inventory systems, and the deterministic formulation of the problem may negatively impact the performance [10], [11].

In this research, we try to bridge the research gap and develop a single-vendor, multi-product, multi-retailer problem for inventory management of reusable items in a two-level supply chain. Also, we try to establish a more realistic problem by considering the system's operational constraints under uncertainty in the availability of resources. A chance-constrained programming approach is suggested to handle this uncertainty. Due to the nonlinearity and the dimension of the developed constrained nonlinear model, the grey wolf optimizer (GWO) and whale optimization algorithm (WOA) are designed and implemented to solve the problem. The exact sequential quadratic programming approach is proposed (SQP) for the validation of the metaheuristics. The primary contributions of the current research are as follows:

- Developing a problem for stock management of reusable items in a single-vendor multi-product multi-retailer supply chain under uncertainty in operations constraints.
- Presenting a chance-constrained programming approach to address the stochastic constraints.
- Designing GWO and WOA metaheuristic algorithms as the solution approach.
- Presenting the SQP exact algorithm for performance validation of metaheuristics.

The rest of this research is organized as follows. Section II provides a review of the past works in the literature. Section III provides the problem definition and the model formulation. Section IV describes the presented solution methodologies to solve the developed model. Section V provides the analysis of results to show the presented model's applicability and compare the algorithms' performance. Finally, Section VI provides a conclusion of the research and suggestions for future works.

II. RELATED WORK

In this section, we will review the relevant papers in the inventory management literature. As mentioned before, Harris [7] introduced the first optimization model, EOQ, to determine the optimal ordering policy in an inventory system. This model was developed for a single-level system and included several other simple assumptions, such as infinite replenishment rate, availability of resources such as budget and space, etc. Therefore, the authors tried to extend EOQ to bring the model into a real-world setting as much as possible. A while after EOQ, Taft [12] developed the economic production quantity (EPQ) and relaxed the infinite replenishment rate assumption to determine the optimal production quantity for inventory management of manufacturing companies. After that, EOQ and EPQ models were extended by considering several assumptions and features. For example, previous authors developed EOQ and EPQ models for special types of products such as deteriorating items [13], substitutable items [14], [15], growing items [16], etc. Also, researchers considered several realistic assumptions, such as preventive maintenance [17], investment [18], trade credits [19], [20], sustainability concerns [21], [22], discount, process reliability [23], pricing decisions [24], marketing policies [25], presence of imperfect items [26], inflation and time-value of money [27], transportation policies [28], inspection errors [29], etc.

However, all these models were single-level and did not consider the vendors in the decision-making framework. Managers understood the importance of coordination in the supply chain and tried to provide integrated models for decision-making the so that the entire supply chain performance is optimized. For example, Pasandideh et al. [30] presented the formulation of EOQ for a two-level supply chain. The backorder shortage was allowed, and several operational constraints were incorporated. The complexity of the problem prompted authors to use the genetic algorithm (GA) as the solution approach. In a similar work, Pasandideh et al. [31] extended the EPQ with shortage for a constrained single-vendor single-retailer system and solved the problem via GA. Also, Pasandideh et al. [32] also worked on a stochastic inventory model for a single-vendor multi-retailer system. The authors utilized an expected value approach to handle the uncertainty. Taleizadeh et al. [33] investigated the inventory management of a multi-vendor multi-retailer supply chain with order size-dependent lead time and partial back. The demand was assumed to be uniformly distributed, and the harmony search (HS) metaheuristic was implemented to find the solutions. Chen et al. [34] considered a delay in payments as a trade credit option in a two-level supply chain and derived the optimal ordering policy of the system under this option.

In some other works, Cárdenas-Barrón & Sana [35] provided an EOQ model for a two-level supply chain in which the demand depended on the promotional effort. Also, the retailer's delay in payment was possible in the considered supply chain. Khan et al. [36] focused on developing an EOQ model for a two-level supply chain with the production of defective products. They assumed that the inspection process is subjected to error and that the production time depends on learning. Tiwari et al. [37] studied a supply chain deteriorating items with pricing and inventory decisions. It was assumed that there is a partial trade credit contract for both levels of the network. Karimian et al. [38] employed a geometric programming method for uncertainty inventory management in a single-vendor multi-retailer supply chain. The problem's applicability was shown by solving the problem for a case study in the Iranian furniture supply chain. Pourmohammad-Zia et al. [39] presented a new model for coordinating vendor and retailer inventory in a growing products supply chain using (vendor-managed inventory) VMI and a cost-sharing contract. Pourmohammad-Zia et al. [40] also aimed to determine pricing and replenishment policies of the growing items supply chain in another work. Recently, Asadkhani et al. [41] presented a sustainable supply chain under some emission reduction regulations and the VMI-consignment stock (VMI-CS) contract. They considered repair, salvage, and disposal as potential options to deal with imperfect items.

The focus of this paper is on the inventory management of reusable items. For the first time, Mokhtari [8] designed a new single-product EOQ problem for stock management of reusable products. The author assumed that the resources are infinitely available and solved the unconstrained model by determining the optimal order and recovery quantity of reusable products through an analytical derivative-based method. Recently, Fallahi et al. [9] pointed out that this model is not practical for systems that deal with multiple products and limitations of resources. Consequently, they presented a multi-product extension of the previous work, and considered the limitations on the maximum budget and the storage capacity for usable and recoverable items. They solved the model using particle swarm optimization (PSO) and differential evolution (DE) algorithms, and also developed two new versions of these algorithms using an intelligent machine learning algorithm. Table 1 compares the novelties of our research against the features of past papers in the literature.

To the best of our knowledge, no other research focuses on the inventory management of reusable products in a two-level supply chain under stochastic operational constraints. The aim of this research is to address this problem and propose a new problem that helps the supply chain managers of reusable items to coordinate the vendor with the retailer through the determination of optimal inventory decisions for the integrated systems. This problem is presented as a single-vendor multi-product multi-retailer inventory system under operational constraints. Also, several sources of uncertainty may impact the constraints of inventory systems. In this paper, we assume that the system's constraints are stochastic and handle it by the chance-constrained programming method. Additionally, GWO and WOA novel metaheuristic algorithms are designed and implemented as the solution approach. The efficiency and effectiveness of these algorithms are shown by validating the results using the SQP algorithm as a powerful exact method.

III. PROBLEM PRESENTATION AND MATHEMATICAL MODELING

In this section, we will present the new problem for inventory management of reusable products in a single-vendor multi-product multi-retailer system and formulate the mathematical model.

A. NOTATIONS

Let us consider the following notations provided in Table 2.

B. ASSUMPTIONS

The main assumption of the presented problem can be expressed as follows:

- There are one vendor, K item, and J retailer in the system.
- The demand rate for products is constated and deterministic.
- The maximum storage capacity of usable products for each retailer is less than an upper threshold with a probability equal to or greater than α.
- The maximum storage capacity of recoverable products for each retailer is less than an upper threshold with a probability equal to or greater than α.
- The maximum holding cost of usable products for each retailer is less than an upper threshold with a probability equal to or greater than α.
- The maximum holding cost of recoverable products for each retailer is less than an upper threshold with a probability equal to or greater than α.
- The maximum budget for each retailer is less than an upper threshold with a probability equal to or greater than α.
- The total number of orders in the system is less than an upper threshold with a probability equal to or greater than α.
- There is no lead time in the system.
- Backorder and lost sale shortages are not allowed.

C. PROBLEM DEFINITION

Consider a two-level supply chain of reusable items, including a vendor and $j \in \{1, \ldots, J\}$ retailers. In this system, the usable term refers to the products ready to satisfy customers' demands. In addition, recoverable products are the products that need a recovery process to become usable for demand satisfaction. Each retailer needs to place orders for $k \in \{1, \ldots, K\}$ reusable items from the vendor. The retailer j purchases the reusable item k from the vendor at the unit

TABLE 1. The features of the relevant past works.

Article	Year	Product Type	Model Type	Vendor Number		Retailer Number		Product Number		Operational Constraints	Uncertainty	Solution Approach	
				Single	Multiple	Single	Multiple	Single	Multiple			Exact	(Meta) heuristic*
Drezner et al. [15]	1995	Substitutable	EOQ			•			•			Analytical	
Salameh & Jaber [26]	2000	Imperfect	EOQ			•		•				Analytical	
Tripathy et al. [23]	2003	Imperfect	EOQ			•		•				Analytical	
Pasandideh et al. [30]	2011		SC	•		•			•	•			GA
Pasandideh et al. [32]	2011		SC	•			•		•	•			GA
Taleizadeh et al. [33]	2011		SC		•		•		•	•	•		HS,GA
Chen et al. [34]	2014		SC	•		•		•				Analytical	
Pasandideh et al. [31]	2014		SC	•		•			•	•		Solver	GA
Cárdenas-Barrón & Sana [35]	2017		SC	•		•		•				Analytical	
Khan et al. [36]	2018	Imperfect	SC	•		•		•				Analytical	
Mokhtari [8]	2018	Reusable	EOQ			•		•				Analytical	
Tiwari et al. [37]	2018	Deteriorating	SC	•		•		•		•		Analytical	
Karimian et al. [38]	2020		SC	•		•			•	•	•	Geometric Programming	
Pourmohammad-Zia et al. [39]	2021	Growing	SC	•			•	•				Game theory	
Pourmohammad-Zia et al. [40]	2021	Growing and deteriorating	SC	•		•		•				Analytical	
Mokhtari et al. [14]	2022	Substitutable	EPQ			•			•	•		Solver	
Asadkhani et al. [41]	2022	Imperfect	SC	•		•		•				Analytical	
Fallahi et al. [9]	2022	Reusable	EOQ			•			•	•		Interior point	DE,PSO, DEQL,PSOQL
This article	2023	Reusable	SC	•			•		•	•	•	Sequential Quadratic Programming	GWO,WOA

GA: Genetic algorithm, HS: Harmony Search, DE: Differential Evolution, PSO: Particle Swarm Optimization, DEQL: Differential Evolution-Q-Learning, PSOQL: Particle Swarm Optimization-Q Learning, GWO: Grey Wolf Optimizer, WOA: Whale Optimization Algorithm

TABLE 2. Mathematical notations.

Sets	
J	Retailer index; $j \in \{1, \ldots, \|J\|\}$
K	Item index; $k \in \{1, \ldots, \|K\|\}$
Parameters	
$ILU_{jk}(t)$	The inventory level of usable item k of retailer j
$ILR_{jk}(t)$	The inventory level of recoverable item k of retailer j
OCS_{jk}	The ordering cost of vendor per order of reusable item k from retailer j
OCU_{jk}	The ordering cost of retailer j per order of reusable item k
OCR_{jk}	The fixed recovery cost of retailer j per recovery of recoverable item k
PC_k	The unit purchasing cost of reusable item k with mean μ_k^{PC} and standard deviation σ_k^{PC}
RC_{jk}	The unit recovery cost of recoverable item k for retailer j
HCU_{jk}	The unit holding cost of usable item k per unit of time for retailer j with mean μ_{jk}^{HCU} and standard deviation σ_{jk}^{HCU}
HCR_{jk}	The unit holding cost of recoverable item k per unit of time for retailer j with mean μ_{jk}^{HCR} and standard deviation σ_{jk}^{HCR}
D_{jk}	The demand rate of reusable item k for retailer j with mean μ_{jk}^{D} and standard deviation σ_{jk}^{D}
m_k	The maximum number that reusable item k can be reused and recovered
f_k	The required storage capacity for storing reusable item k with mean μ_k^f and standard deviation σ_k^f
B_j	The maximum budget of retailer j with mean μ_j^B and standard deviation σ_j^B
AHU_j	The maximum holding cost for usable items of retailer j with mean μ_j^{AHU} and standard deviation σ_j^{AHU}
AHR_j	The maximum holding cost for recoverable items of retailer j with mean μ_j^{AHR} and standard deviation σ_j^{AHR}
WSU_j	The maximum storage capacity of retailer j for usable items with mean μ_j^{WSU} and standard deviation σ_j^{WSU}
WSR_j	The maximum storage capacity of retailer j for recoverable items with mean μ_j^{WSR} and standard deviation σ_j^{WSR}
WS	The total maximum storage capacity of the vendor with mean μ^{WS} and standard deviation σ^{WS}
N	The maximum number of orders for all items with mean μ^N and standard deviation σ^N
α	The probability of violating each of the constraints
Variables	
Q_{jk}	The economic order quantity for reusable item k of retailer j per cycle
q_{jk}	The economic reuse and recovery quantity for reusable item k of retailer j per cycle
p_{jk}	The economic order quantity to economic reuse and recovery quantity ratio for reusable item k of retailer j per cycle
TCB_j	The total cost of retailer j
TCS	The total cost of the vendor
TCE	The integrated total cost of the supply chain

purchasing cost PC_k. For each order, OCU_{jk} is the ordering cost of retailer j for reusable item k. In addition, OCS_{jk} is the imposed ordering cost to the vendor regarding the order of retailer j for reusable item k. Retailer j uses the purchased reusable item k to satisfy the demand of customers D_{jk}. As mentioned before, the products are reusable, and the used products can be recovered and used again for a maximum of m_k times. The recovery cost of each unit of product k for retailer j is RC_{jk}. In addition, there is a fixed recovery operational cost for the recovery of product k by retailer j, which is OCR_{jk}. The presence of usable and recoverable product k in warehouses of retailer j imposes holding costs on each retailer, which are specified by HCU_{jk} and HCR_{jk}, respectively. The stock level diagrams of usable and recoverable product k in the warehouses of retailer j are shown in Figures 1 and 2.

A set of operational constraints are considered in the system to bring it into the real-world environment. The constraints are stochastic, and it is assumed that the resources are available with a probability equal to and greater than α. The limitations on the maximum storage capacity, the holding costs, the total available purchasing budget, and the total number of orders are the stochastic constraints of the supply chain.

D. MATHEMATICAL MODELING

Regarding the explained problem, the cost components of the supply chain can be described as follows:

- The total purchasing cost of retailer j:

$$\sum_{k=1}^{K} PC_k \frac{D_{jk}}{(m_k+1)} \quad \forall j \in \mathcal{J} \tag{1}$$

- The total fixed ordering cost of the vendor:

$$\sum_{j=1}^{J}\sum_{k=1}^{K} OCS_{jk} \frac{D_{jk}}{(m_k+1)p_{jk}q_{jk}} \tag{2}$$

- The total fixed ordering cost of retailer j:

$$\sum_{k=1}^{K} OCU_{jk} \frac{D_{jk}}{(m_k+1)p_{jk}q_{jk}} \quad \forall j \in \mathcal{J} \tag{3}$$

- The total fixed recovery cost of retailer j:

$$\sum_{k=1}^{K} OCR_{jk} D_{jk} (\frac{m_k}{m_k+1}) \quad \forall j \in \mathcal{J} \tag{4}$$

- The total recovery operational cost of retailer j:

$$\sum_{k=1}^{K} RC_{jk} \frac{D_{jk}}{q_{jk}} (\frac{m_k}{m_k+1}) \quad \forall j \in \mathcal{J} \tag{5}$$

- The total holding cost of usable products of retailer j:

$$\sum_{k=1}^{K} HCU_{jk} (\frac{p_{jk}q_{jk}}{2}) \quad \forall j \in \mathcal{J} \tag{6}$$

- The total holding cost of recoverable products of retailer j:

$$\sum_{k=1}^{K} HCR_{jk} (\frac{m_k}{m_k+1}) \frac{q_{jk}}{2} \quad \forall j \in \mathcal{J} \tag{7}$$

Considering the above-described components, the total cost objective of retailer j can be expressed as follows:

$$TCB_j(p_{jk}, q_{jk}) = \sum_{k=1}^{K} PC_k \frac{D_{jk}}{(m_k+1)}$$

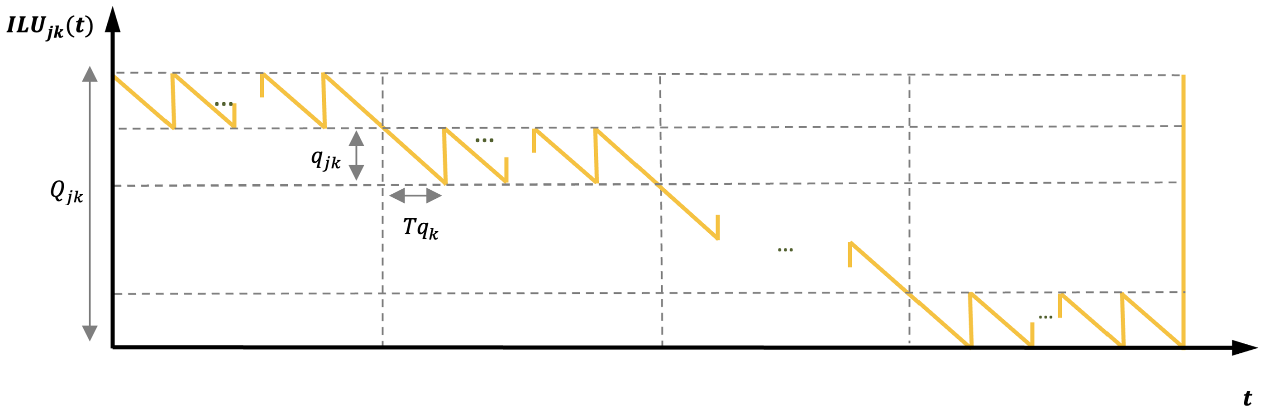


FIGURE 1. The stock level of usable product k for retailer j.

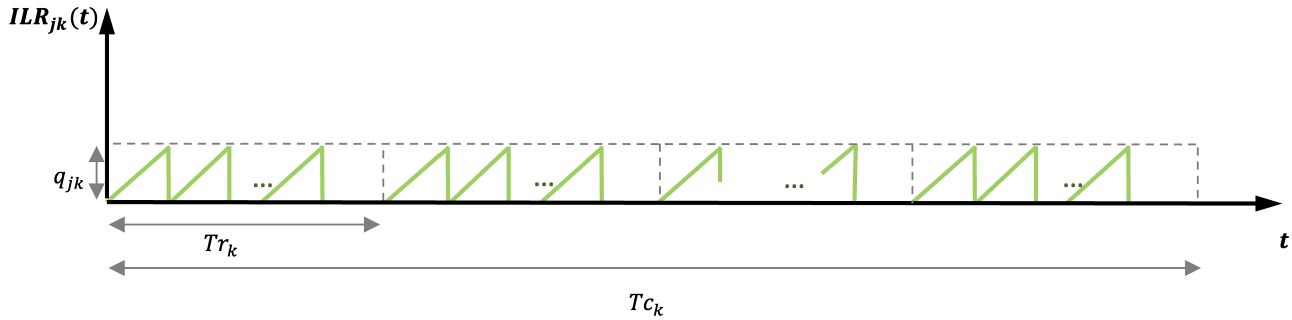


FIGURE 2. The stock level of recoverable product k for retailer j.

$$+\sum_{k=1}^{K} OCU_{jk}\frac{D_{jk}}{(m_k+1)p_{jk}q_{jk}} + \sum_{k=1}^{K} OCR_{jk}D_{jk}(\frac{m_k}{m_k+1}) + \sum_{k=1}^{K} RC_{jk}\frac{D_{jk}}{q_{jk}}(\frac{m_k}{m_k+1}) + \sum_{k=1}^{K} HCU_{jk}(\frac{p_{jk}q_{jk}}{2}) + \sum_{k=1}^{K} HCR_{jk}(\frac{m_k}{m_k+1})\frac{q_{jk}}{2} \quad (8)$$

Also, the vendor bears the following cost:

$$TCS(p_{jk}, q_{jk}) = \sum_{j=1}^{J}\sum_{k=1}^{K} OCS_{jk}\frac{D_{jk}}{(m_k+1)p_{jk}q_{jk}} \quad (9)$$

In the decentralized systems, each retailer places the order regarding her cost function. Here, the total cost of other retailers and the vendor do not play any role in the determination of the replenishment decisions. As a consequence of such a policy, a huge cost may impact the overall performance of the system. In centralized decision-making, managers try to determine the optimal decisions regarding the costs of all entities. In this situation, a coordination mechanism should be used to establish the integrated total cost function of the supply chain. The presented mechanism by Hill [42] is one of the well-known mechanisms managers tend to adapt to coordinate such supply chains [41]. Therefore, we use this coordination approach to integrate the cost components of the vendor and retailer. The centralized total cost of the two-layer network under the coordination mechanism of Hill [42] can be expressed as follows:

$$TCE = \sum_{j=1}^{J}\sum_{k=1}^{K} OCS_{jk}\frac{D_{jk}}{(m_k+1)p_{jk}q_{jk}} + \sum_{k=1}^{K} PC_k\frac{D_{jk}}{(m_k+1)} + \sum_{k=1}^{K} OCU_{jk}\frac{D_{jk}}{(m_k+1)p_{jk}q_{jk}} + \sum_{k=1}^{K} OCR_{jk}D_{jk}(\frac{m_k}{m_k+1}) + \sum_{k=1}^{K} RC_{jk}\frac{D_{jk}}{q_{jk}}(\frac{m_k}{m_k+1}) + \sum_{k=1}^{K} HCU_{jk}(\frac{p_{jk}q_{jk}}{2}) + \sum_{k=1}^{K} HCR_{jk}(\frac{m_k}{m_k+1})\frac{q_{jk}}{2} \quad (10)$$

The integrated objective function is subjected to the following stochastic operational constraints:

$$Min\ TCE \quad (11)$$

$$\text{s.t. } P\left(\sum_{k=1}^{K} PC_k.p_{jk}.q_{jk} \le B_j\right) \ge 1-\alpha \quad \forall j \in \mathcal{J} \quad (12)$$

$$P\left(\sum_{j=1}^{J}\sum_{k=1}^{K} f_k.p_{jk}.q_{jk} \le WS\right) \ge 1-\alpha \tag{13}$$

$$P\left(\sum_{k=1}^{K} f_k.p_{jk}.q_{jk} \le WSU_j\right) \ge 1-\alpha \quad \forall j \in \mathcal{J} \tag{14}$$

$$P\left(\sum_{k=1}^{K} f_k.q_{jk} \le WSR_j\right) \ge 1-\alpha \quad \forall j \in \mathcal{J} \tag{15}$$

$$P\left(\sum_{k=1}^{K} HCU_{jk}.(\frac{p_{jk}.q_{jk}}{2}) \le AHU_j\right) \ge 1-\alpha \quad \forall j \in \mathcal{J} \tag{16}$$

$$P\left(\sum_{k=1}^{K} HCR_{jk}.(\frac{m_k}{m_k+1}).\frac{q_{jk}}{2} \le AHR_j\right) \ge 1-\alpha \quad \forall j \in \mathcal{J} \tag{17}$$

$$P\left(\sum_{j=1}^{J}\sum_{k=1}^{K} \frac{D_{jk}}{(m_k+1).p_{jk}.q_{jk}} \le N\right) \ge 1-\alpha \tag{18}$$

$$p_{jk}, q_{jk} \ge 0 \quad \forall j \in \mathcal{J},\ k \in \mathcal{K} \tag{19}$$

Constraints (12) to (18) are the chance constraints of the system. Stochastic constraints (12) ensure that the total purchasing cost of the products for each retailer does not exceed the retailer's maximum budget. Constraint (13) limits the total available storage capacity for the vendor. Stochastic constraints (14) and (15) specify the storage capacity of each retailer for usable and recoverable products. The constraints of the maximum holding cost of usable and recoverable for each retailer is shown via stochastic constraints (16) and (17). Stochastic constraints (18) limit the system's total number of orders. Constraints (19) determine the type of decision variables.

We utilize the chance constraint programming approach to deal with the uncertainty of the constraints. Considering a normal probability distribution with mean μ and standard deviation σ for the upper bound of each stochastic constraint, the constraints can be rewritten as (20)-(28) [10]. In the below equations, the upper α-percentile point of the normal probability distribution (standard form) is shown by Z_α.

$$Min\ TCE \tag{20}$$

$$\text{s.t.} \sum_{k=1}^{K} \mu_k^{PC}.p_{jk}.q_{jk} + Z_\alpha.\sqrt{\sum_{k=1}^{K}(\sigma_k^{PC}.p_{jk}.q_{jk})^2 + (\sigma_j^B)^2} \le \mu_j^B \quad \forall j \in \mathcal{J} \tag{21}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} \mu_k^f.p_{jk}.q_{jk} + Z_\alpha.\sqrt{\sum_{j=1}^{J}\sum_{k=1}^{K}(\sigma_k^f.p_{jk}.q_{jk})^2 + (\sigma^{WS})^2} \le \mu^{WS} \tag{22}$$

$$\sum_{k=1}^{K} \mu_k^f.p_{jk}.q_{jk} + Z_\alpha.\sqrt{\sum_{k=1}^{K}(\sigma_k^f.p_{jk}.q_{jk})^2 + (\sigma_j^{WSU})^2} \le \mu_j^{WSU} \quad \forall j \in \mathcal{J} \tag{23}$$

$$\sum_{k=1}^{K} \mu_k^f.q_{jk} + Z_\alpha.\sqrt{\sum_{k=1}^{K}(\sigma_k^f.q_{jk})^2 + (\sigma_j^{WSR})^2} \le \mu_j^{WSR} \quad \forall j \in \mathcal{J} \tag{24}$$

$$\sum_{k=1}^{K} \mu_{jk}^{HCU}.(\frac{p_{jk}.q_{jk}}{2}) + Z_\alpha.\sqrt{\sum_{k=1}^{K}\left(\sigma_{jk}^{HCU}.(\frac{p_{jk}.q_{jk}}{2})\right)^2 + (\sigma_j^{AHU})^2} \le \mu_j^{AHU} \quad \forall j \in \mathcal{J} \tag{25}$$

$$\sum_{k=1}^{K} \mu_{jk}^{HCR}.(\frac{m_k}{m_k+1}).\frac{q_{jk}}{2} + Z_\alpha.\sqrt{\sum_{k=1}^{K}\left(\sigma_{jk}^{HCR}.(\frac{m_k}{m_k+1}).\frac{q_{jk}}{2}\right)^2 + (\sigma_j^{AHR})^2} \le \mu_j^{AHR} \quad \forall j \in \mathcal{J} \tag{26}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} \frac{\mu_{jk}^D}{(m_k+1).p_{jk}.q_{jk}} + Z_\alpha.\sqrt{\sum_{j=1}^{J}\sum_{k=1}^{K}\left(\frac{\sigma_{jk}^D)}{(m_k+1).p_{jk}.q_{jk}}\right)^2 + (\sigma^N)^2} \le \mu^N \tag{27}$$

$$p_{jk}, q_{jk} \ge 0 \quad \forall j \in \mathcal{J},\ \forall k \in \mathcal{K} \tag{28}$$

The presented model is a constrained nonlinear programming model, which can not easily be solved via exact classical methods or commercial solvers. Therefore, we utilize the metaheuristic algorithms as the solution methodology. The algorithms will be presented in the next section.

IV. SOLUTION APPROACH

This section will discuss the proposed solution methodologies to solve the problem. The presented model is a constrained nonlinear programming mathematical model. Previous research pointed out that such inventory management problems are challenging to solve with classical methods due to the nonlinearity of the model and several local optimum solutions [9], [43]. Therefore, metaheuristic algorithms are widely used as a powerful solution for multi-product inventory management in supply chains [9], [30], [33]. This paper uses GWO and WAO as two recently developed metaheuristics to solve the problem. In addition, we use SQP as an exact approach to show the efficiency of the metaheuristics.

A. GREY WOLF OPTIMIZER METAHEURISTIC ALGORITHM

GWO is a nature-inspired population-based metaheuristic that was introduced by Mirjalili et al. [44], and extensively used as the solution approach to optimization problems in several fields [45], [46]. The GWO metaheuristic is designed to find the near global optimum of a given function for the solution spaces with continuous variables. The algorithm is inspired from the hunting behavior of grey wolves in a pack, where each wolf plays a specific role in cooperating and competing with each other to find the best prey. GWO is started with a population of random solutions, and then it iteratively updates the solutions by mimicking the interactions among the wolves in the pack. The steps of the GWO metaheuristic can be summarized are as below:

1) INITIALIZE PREY AND HUNTERS

The first step of the GWO metaheuristic is to initialize the population of solutions $X = \{x_1, x_2, \ldots, x_n\}$ where x_i is a solution vector in the search space. The population is initialized with a set of random solutions. The solutions number, also known as the population size, is a user-defined parameter that can vary depending on the problem at hand. Larger population size can increase the diversity of solutions but also increases the computational cost. GWO also starts with initializing the position and fitness of the alpha, beta, and delta wolves, denoted by x_α, x_β and x_δ, respectively. These wolves are used as reference points in the next steps of the algorithm. The x_α wolf is considered the leader of the pack and has the best objective function in the population. The x_β wolf is the second-best solution, and the x_δ wolf is the solution with the third-best objective function. After the initialization of the population in the first iteration, there is a set of common steps in the next iterations of GWO, which is the core of the algorithm. These steps iteration process are executed until a stopping criterion is met, such as the maximum iterations or achieving a satisfactory solution. In each iteration, the algorithm updates the position of the x_α, x_β, and x_δ wolves, as well as the position of the other wolves in the population, based on the details that are as follows.

2) HUNTING

GWO updates the position of the solutions with respect to the hunting behavior of grey wolves, where the wolves cooperate and compete to find the best prey. The position of the x_α, x_β, and x_δ wolves is updated using the following equations:

$$x_{t+1}^{\alpha} = x_t^{\alpha} + \alpha_t(x_t^{\beta} - x_t^{\alpha}) + \beta_t(x_t^{\delta} - x_t^{\alpha}) \tag{29}$$

$$x_{t+1}^{\beta} = x_t^{\beta} + \alpha_t(x_t^{\alpha} - x_t^{\beta}) + \beta_t(x_t^{\delta} - x_t^{\beta}) \tag{30}$$

$$x_{t+1}^{\delta} = x_t^{\delta} + \alpha_t(x_t^{\alpha} - x_t^{\delta}) + \beta_t(x_t^{\beta} - x_t^{\delta}) \tag{31}$$

where α_t and β_t are linearly decreasing functions of the iteration t, and are used to control the step size of the search. In addition, the position of the other wolves is updated using the following equation:

$$x_{t+1}^{(i)} = x_t^{(i)} + r.(x_{t+1}^{\alpha} - x_t^{(i)}) + r.(x_{t+1}^{\beta} - x_t^{(i)}) + r.(x_{t+1}^{\delta} - x_t^{(i)}) \tag{32}$$

where $r \in (0, 1)$ is a random number generator. After updating the position of the solutions, the fitness of the x_α, x_β, and x_δ wolves is re-evaluated, and the new x_α, x_β, and x_δ wolves are selected from the population. The search process of the algorithm is shown in Figure 3.

3) STOP CRITERIA

The stopping criterion is a pre-defined condition that is used to determine when the search process should stop. The stopping criterion is usually based on the number of iterations or the quality. A common stopping criterion is to stop the algorithm after a certain number of iterations, also known as the maximum number of iterations. This criterion is used to prevent the algorithm from running indefinitely, and it is typically set based on the computational resources available and the complexity of the problem. Another stopping criterion is to stop the algorithm when a satisfactory solution is found. This criterion is used to stop the optimization process when the solution quality reaches a certain level. We set the first criterion as the stop criterion of GWO. Algorithm 1 presents the pseudocode of the GWO.

B. WHALE OPTIMIZATION ALGORITHM

WOA is another population-based metaheuristic algorithm inspired by humpback whales foraging behavior. The algorithm was proposed by Mirjalili & Lewis [47], and it is used to solve problems in several research areas [48], [49]. It is known for its ability to find high-quality solutions, as well as its ability to avoid getting stuck in local optima [50]. As pointed out, WOA is a population-based optimization algorithm, where a group of candidate solutions, called a population, is iteratively improved to converge towards a near-optimal solution. The algorithm simulates the foraging behavior of humpback whales, where each whale represents a candidate solution, and the search space is divided into subproblems. The main steps of WOA are:

1) INITIALIZE PREY

The first step of the WOA algorithm is to randomly initialize a population of solutions. The population is typically a set of n solutions, where each solution is represented by a vector of d variables, $x_i^T = \{x_i^{(1)}, x_i^{(2)}, \ldots, x_i^{(d)}\}$. The values of the variables should be chosen within the specified bounds.

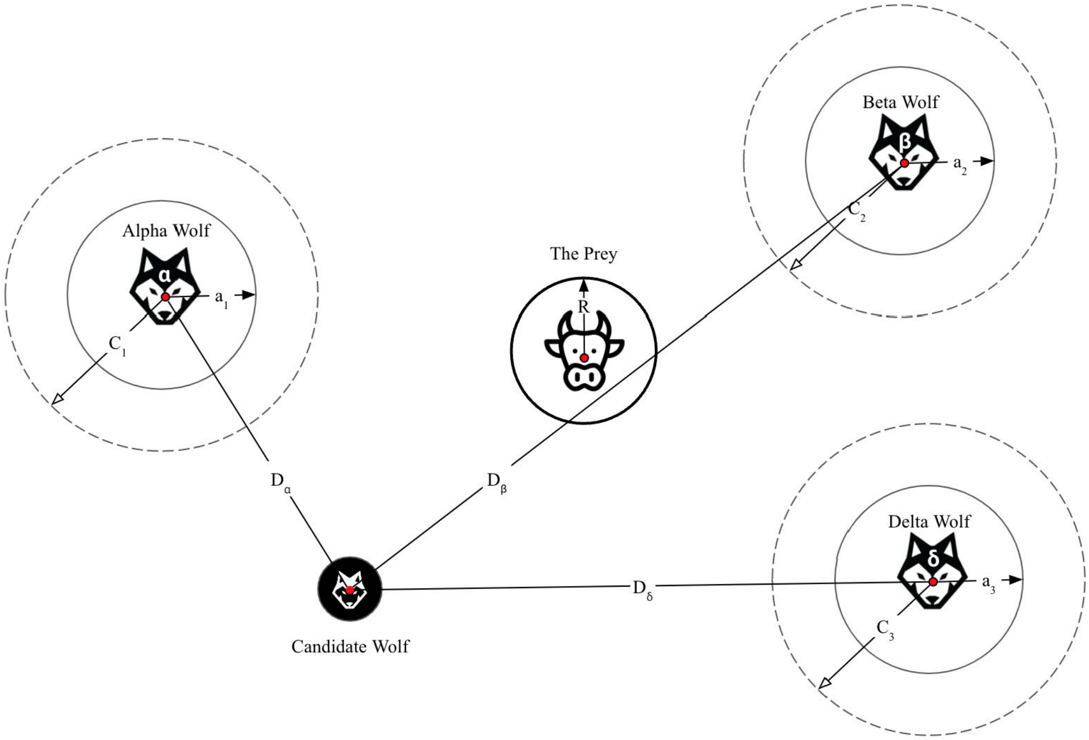


FIGURE 3. The search process of the GWO metaheuristic algorithm.

2) HUNTING

In the WOA, the current best solution, also known as the leader, is assumed to be close to the optimum, and the other solutions in the population are guided towards it, similar to how humpback whales encircle their prey. In other words, the leader solution is used as a point of reference for the other solutions to follow, guiding the search of the other solutions toward better regions of the space. The other solutions are updated simultaneously, based on their distance and fitness difference from the leader solution. This behavior is represented by the mathematical equations (33) and (34) that are used to update the positions of the solutions in the population. These equations are designed to mimic the foraging behavior of humpback whales.

$$\vec{D} = |\vec{C}.\vec{X}_t^* - \vec{X}_t| \tag{33}$$

$$\vec{X}_{t+1} = \vec{X}_t^* - \vec{A}.\vec{D} \tag{34}$$

where $\vec{X}_t^*$ is the best solution obtained in t^{th} iteration of WOA. The parameters $\vec{A}$ and $\vec{C}$ are as follows:

$$\vec{A} = 2.\vec{a}.\vec{r}_1 - \vec{a} \tag{35}$$

$$\vec{C} = 2.\vec{r} \tag{36}$$

where $\vec{a}$ decreases linearly from 2 to 0 over the algorithm. Also, $\vec{r}$ is a random number generated uniformly between 0 and 1. Two models are used to represent the attacking behavior of humpback whales:

a: SHRINKING UPDATING POSITION

This behavior is accomplished by decreasing the value of $\vec{a}$ in equation (35). The range of fluctuation is also decreased by $\vec{A}$. Figure 4 illustrates the potential positions that can be reached from (X, Y) to (X^*, Y^*) when $0 \leq A \leq 1$ in a 2-dimensional space.

b: SPIRAL UPDATING POSITION

Another observation of Humpback whales' hunting is swimming in a helical path toward their prey [47]. To replicate this behavior, a spiral function is defined to modify the position of search as:

$$\vec{X}_{t+1} = \vec{D}'.e^{bt}.\cos(2\pi l) + \vec{X}_t^* \tag{37}$$

where the distance of the i^{th} whale to the prey (current ideal solution) is represented by $|\vec{X}_t^* - \vec{X}_t|$, b is a constant for shaping the logarithmic spiral, $l \in (0, 1)$ is a random number. The procedure is shown in Figure 5.

To simulate how humpback whales move around their prey by swimming in a shrinking circle and along a spiral-shaped path, the algorithm utilizes a probability of 50% to either use the first or second method.

3) STOP CRITERIA

WOA is to repeat the process of evaluating the fitness, selecting the leader, and updating the positions of the solutions until a stopping condition is met. We set the maximum number of

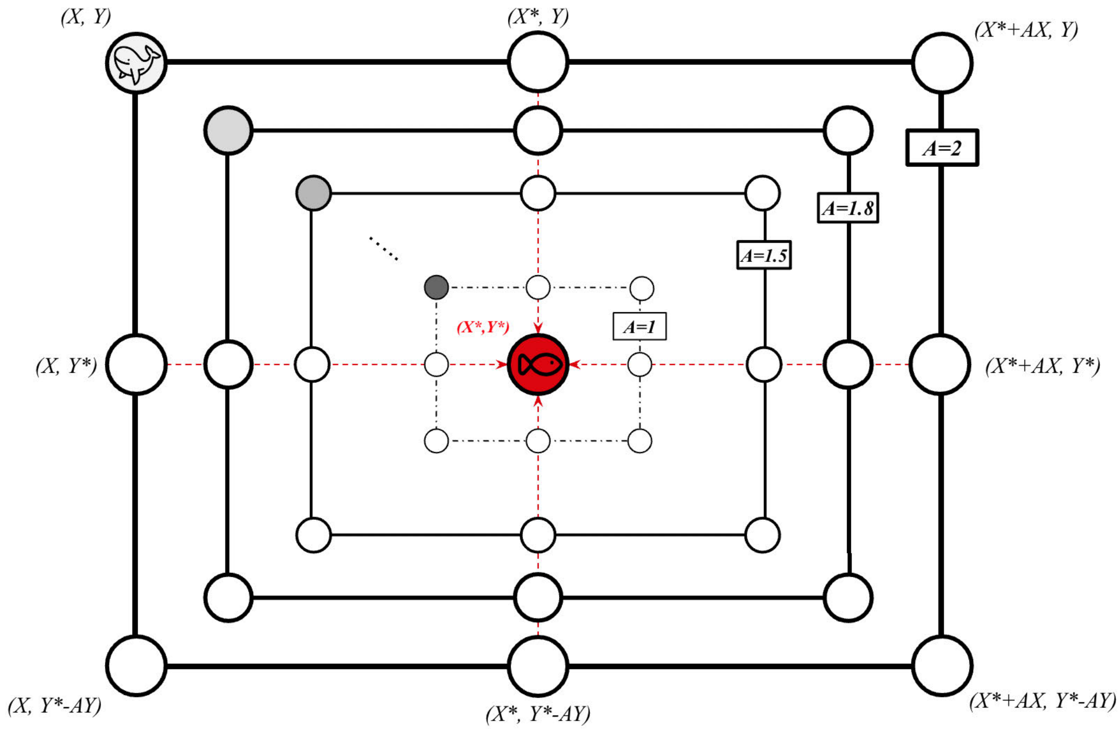


FIGURE 4. The shrinking updating process of the WOA metaheuristic algorithm.

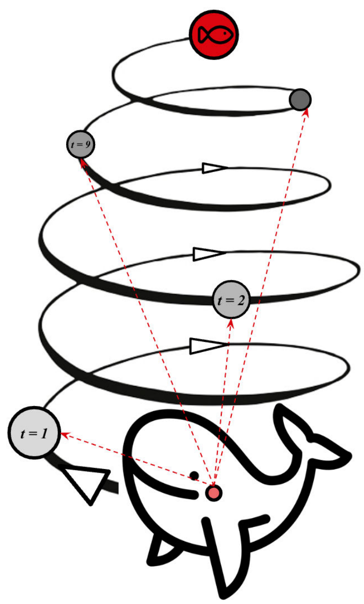


FIGURE 5. The spiral updating process of the WOA metaheuristic algorithm.

iterations as the stopping criterion. Algorithm 2 shows the pseudocode of the WOA.

C. SEQUENTIAL QUADRATIC PROGRAMMING

The SQP algorithm is a technique for solving nonlinear optimization problems involving smooth and nonsmooth functions. It is an iterative method that uses a combination of gradient and Hessian information to determine the next iterate. SQP is particularly well-suited for solving large-scale NLP problems and has been shown to be effective in many applications.

The algorithm is based on the theory of Quadratic Programming (QP), and it's a combination of gradient and Hessian information to determine the next iterate [51], [52]. It uses the Karush-Kuhn-Tucker (KKT) conditions to manage equality constraints in the same way that Newton's technique does when solving an unconstrained NLP optimization problem. [53]. The KKT conditions are a set of necessary and sufficient conditions that a solution to a constrained optimization problem must satisfy. In this algorithm, the solution of QP sub-problem is typically utilized to establish a line search direction. SQP is similar to the active-set algorithm and has some advantages over other exact methodologies. One advantage is that the SQP method guarantees exact feasibility with respect to bounds. This means that the algorithm will always find a feasible solution that satisfies all the bounds constraints. Another advantage of SQP is that it is more robust to problems with complex values [53], [54]. This is because the SQP algorithm approximates the objective function and constraints, which can help avoid getting stuck in poor local solutions and help the algorithm converge to a global optimum.

SQP is also used in the literature to determine the economic order (production) quantity in constrained multi-product inventory problems [10], [43], [55]. The SQP method is

Algorithm 1 GWO Algorithm

Input: Maximum iteration (t_{max}), Population size (N_{pop}), a, A, and C
Output: Best solution ($\vec{X}_\alpha$)
for $i = 1 : N_{pop}$ **do**
  Initialize GWO solution $\vec{X}_i^0$
  Calculate the fitness $f(\vec{X}_i^0)$
end for
$\vec{X}_\alpha \leftarrow$ The first best wolve
$\vec{X}_\beta \leftarrow$ The second best wolve
$\vec{X}_\delta \leftarrow$ The third best wolve
for $t = 1 : t_{max}$ **do**
  $\vec{A} \leftarrow 2\vec{a}.\vec{r}_1 - \vec{a}$
  $\vec{C} \leftarrow 2\vec{r}_2$
  $\vec{D}_\alpha \leftarrow |\vec{C}_1.\vec{X}_\alpha - \vec{X}|, \vec{D}_\beta \leftarrow |\vec{C}_2.\vec{X}_\beta - \vec{X}|, \vec{D}_\delta \leftarrow |\vec{C}_3.\vec{X}_\delta - \vec{X}|$
  $\vec{X}_1^t \leftarrow \vec{X}_\alpha - \vec{A}_1.(\vec{D}_\alpha), \vec{X}_2^t \leftarrow \vec{X}_\beta - \vec{A}_2.(\vec{D}_\beta), \vec{X}_3^t \leftarrow \vec{X}_\delta - \vec{A}_3.(\vec{D}_\delta)$
  $\vec{A} \leftarrow 2.\vec{a}.\vec{r}_1 - \vec{a}$
  $\vec{X}^{t+1} \leftarrow \frac{\vec{X}_1^t + \vec{X}_2^t + \vec{X}_3^t}{3}$ {Update the position of the wolve}
  Update a, A, and C multipliers
  Calculate the fitness $f(\vec{X}^{t+1})$
  Update x_α, x_β, and x_δ using equations (29) - (31)
  $t \leftarrow t + 1$
end for
Return: $\vec{X}_\alpha$

particularly well-suited to solving this type of problem because it can handle the nonlinear and nonconvex nature of the objective function and constraints that arise in the EPQ problem with stochastic constraints. In addition, studies have shown that the SQP method can perform significantly better than other approaches, such as the interior-point method. This is because the SQP method can often converge to a global optimum, whereas the interior-point method can get stuck in poor local solutions. Also, the SQP method can handle the nonlinear and nonconvex nature of the objective function and constraints that arise in the EPQ problem with stochastic constraints more effectively than the interior-point exact approach [10], [43].

D. CONSTRAINT HANDLING

Metaheuristic algorithms are mainly designed to solve unconstrained optimization problems. Thus, when these algorithms are applied to real-world optimization problems, one of the most difficult challenges is handling the problem's constraints. There are several methods for constraint handling in the literature, including penalty-based techniques, separation of objective and constraints techniques, repair algorithm-based techniques, and boundary-based techniques. Among these methods, penalty-based techniques have proven to be particularly effective in increasing the feasibility of the solutions obtained by metaheuristic algorithms.

Algorithm 2 WOA Algorithm

Input: Maximum iteration (t_{max}), Population size (N_{pop}), a, b, A, C, p, and l
Output: Best solution ($\vec{X}^*$)
for $i = 1 : N_{pop}$ **do**
  Initialize whale position $\vec{X}_i^0$
  Calculate the fitness $f(\vec{X}_i^0)$
end for
for $i = 1 : t_{max}$ **do**
  for *Eachsearchagent* **do**
    Update a, A, C, p, and l
    if $p < 0.5$ **then**
      $\vec{A} \leftarrow 2.\vec{a}.\vec{r}_1 - \vec{a}$
      $\vec{C} \leftarrow 2.\vec{r}$
      if $|\vec{A}| < 1$ **then**
        $\vec{D} \leftarrow |\vec{C}.\vec{X}_t^* - \vec{X}_t|$
        $\vec{X}_{t+1} \leftarrow \vec{X}_t^* - \vec{A}.\vec{D}$
      else if $|\vec{A}| \geq 1$ **then**
        Select a random search agent ($\vec{X}_{rand}$)
        $\vec{D} \leftarrow |\vec{C}.\vec{X}_{rand} - \vec{X}_t|$
        $\vec{X}_{t+1} \leftarrow \vec{X}_{rand} - \vec{A}.\vec{D}$
      end if
    else if $p \geq 0.5$ **then**
      $\vec{D}' \leftarrow |\vec{X}^* - \vec{X}|$
      $\vec{X}_{t+1} \leftarrow \vec{D}'.e^{bt}.\cos(2\pi l) + \vec{X}_t^*$
    end if
  end for
  Check and amend if any agent goes beyond the space
  Calculate the fitness $f(\vec{X}^{t+1})$
  Update $\vec{X}_t^*$ if there is a better solution
  $t \leftarrow t + 1$
end for
Return: $\vec{X}^*$

We choose the static penalty approach as a penalty-based technique to handle the model's constraints. Static constraint handling is a straightforward and effective approach that utilizes the knowledge gained from unfeasible solutions to steer the solutions toward feasibility. The static penalty has proven highly efficient in handling nonlinear constraints in non-linear programming problems. Fallahi et al. [9] also used this approach for constraint handling of the developed metaheuristics in solving their problem for inventory management of reusable products. In this method, the user specifies distinct degrees of violation for each constraint, along with a related penalty for each degree. As the degree of constraint violation becomes more severe, the associated penalty coefficient also increases.

V. COMPUTATIONAL EXPERIMENTS

In this section, we will evaluate the model's and metaheuristic algorithms' performance by solving the numerical examples. We use the data from the works by Mokhtari [8], and

TABLE 3. The data of numerical examples.

Parameter	Range	Parameter	Range
OCS	UN(1300,1900)	f	UN(1,2)
OCU	UN(1300,1900)	B	UN(290000000,310000000)
OCR	UN(80,100)	AHU	UN(380000,420000)
PC	UN(40,60)	AHR	UN(1900000,2100000)
RC	UN(16,24)	WSR	UN(18000,22000)
HCU	UN(1,2)	WSU	UN(18000,22000)
HCR	UN(6,10)	WS	25000*J
D	UN(10000,14000)	N	10000*K
m	DUN(2,5)		

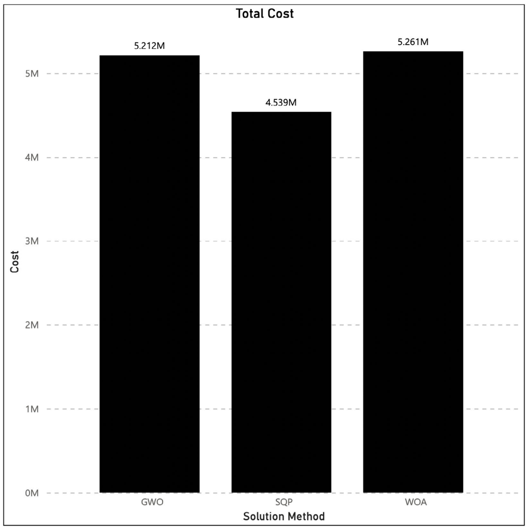


FIGURE 6. Comparison between the value of objective functions using different solution methods.

Fallahi et al. [9] to generate numerical examples. The details of the data are presented in Table 3.

The algorithms are run on a personal laptop with 16 GB Ram and an Intel Core i7 4.7 GHz CPU. We also provide the SAS code for SQP solver in Appendix A.

First, we are going to validate the performance of metaheuristic algorithms by comparing the results of a small-size numerical example. For this goal, we consider a numerical example with two retailers and one product in the system. Figure 6 shows the calculated results of algorithms for the numerical example. As obvious, there is no significant difference between the performance of the metaheuristics. The total costs of GWO and WOA are more than SQP by about 14.78 and 15.90, respectively. Such difference confirms that the algorithms perform well in searching the solution space. As obvious, GWO has a better performance than WOA.

To provide better insight, the cost component by each algorithm is also provided in Figure 7. As can be seen, a great portion of the total cost is due to the fixed recovery cost. In addition, the holding cost of recoverable products is less than the other cost components of the system.

A. PARAMETER TUNING

The input parameters highly impact the performance of metaheuristic algorithms [56]. Various methods are employed in the literature to determine the input parameters of metaheuristics. The trial-and-error methods are very time-consuming and do not guarantee the quality of solutions. Therefore, using a systematic approach for parameter tuning seems necessary. Taguchi's design [57] of experiments is one of the widely used statistical methods for this goal. The Taguchi method utilizes the concept of the orthogonal array to manage the number of experiments. In this statistical approach, the affecting factors are grouped into two categories of signal (S) and noise (N) factors. There is no direct control over the noise factors, and they can not easily be changed or removed. Therefore, Taguchi tries to find the optimum level of signal factors in such as way that the effect of noise factors is minimized. Taguchi defines the relative importance of individual components in terms of their primary effects on the objective function in order to determine the best parameter levels. The repeated data is transformed by Taguchi into a different value, which is the variation measure. This transformation is signal-to-noise (S/N) ratio, which is calculated as below for a minimization problem:

$$S/N = -10 \log \frac{1}{m} \sum_{j=1}^{m} z_j^2 \tag{38}$$

where m is the total replications and z_j is the response in j^{th} replications. Three considered levels for each parameter of algorithms are presented in Table 4. The L9 orthogonal arrays are utilized for the parameter calibration of GWO and WOA. Three levels are defined for each parameter based on the details in Table 4. In addition, each orthogonal array is run in five replications.

TABLE 4. The considered levels for parameters of metaheuristics.

Parameter	GWO		WOA		
	A	B	A	B	C
	Max_it	N_pop	Max_it	N_pop	b
Level 1	100	100	100	100	-0.9
Level 2	150	150	150	150	-1
Level 3	200	200	200	200	-1.1

TABLE 5. The optimal input parameters of metaheuristic algorithms.

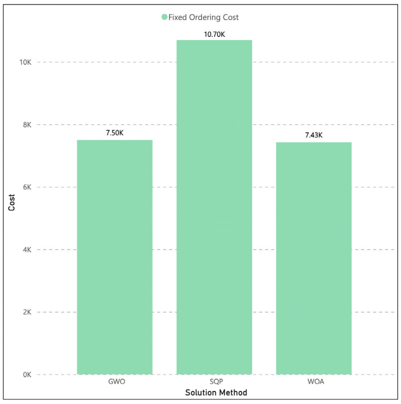
Parameter	GWO		WOA		
	A	B	A	B	C
	Max_it	N_pop	Max_it	N_pop	b
Optimal level	200	200	200	200	-1.1

The optimal level of parameters are presented in Table 5.

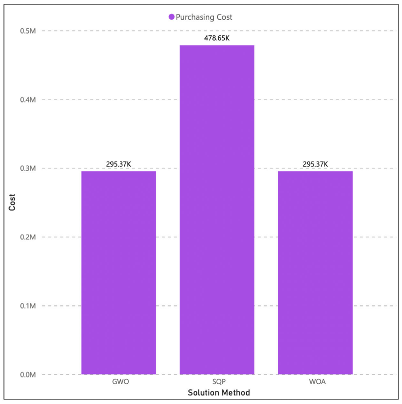
In addition, Figures 8 and 9 are the main effects plots of S/N ratios.

B. PERFORMANCE ANALYSIS

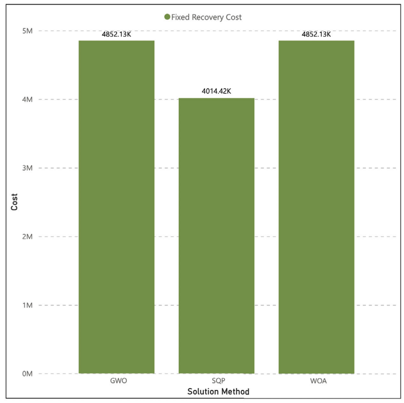
A detailed analysis of algorithms is provided in this section. 15 numerical examples with different dimensions (number of products and retailers) are solved by the algorithms. Each algorithm is run in 10 replications for each generated numerical example. We consider four measures, including average RPD, average RDI, average CPU time, and average standard



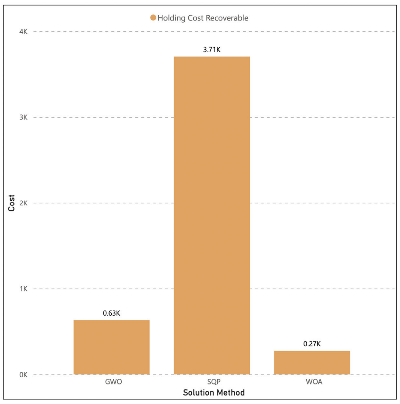
(a) Fixed Ordering Cost



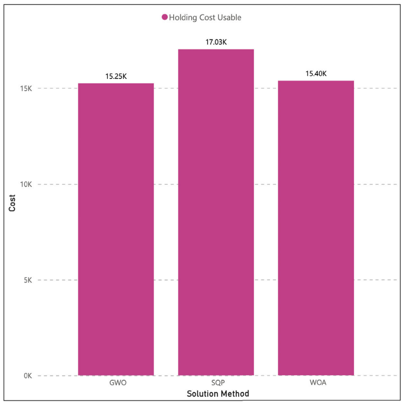
(b) Purchasing Cost



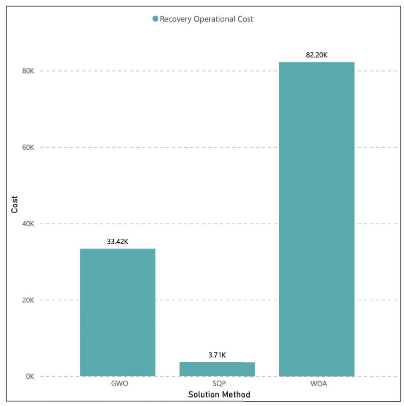
(c) Recovery Cost (Fixed)



(d) Holding Cost (Recoverable)



(e) Holding Cost (Usable)



(f) Recovery Cost (Operational)

FIGURE 7. The calculated cost components of GWO, WOA, and SQP algorithms.

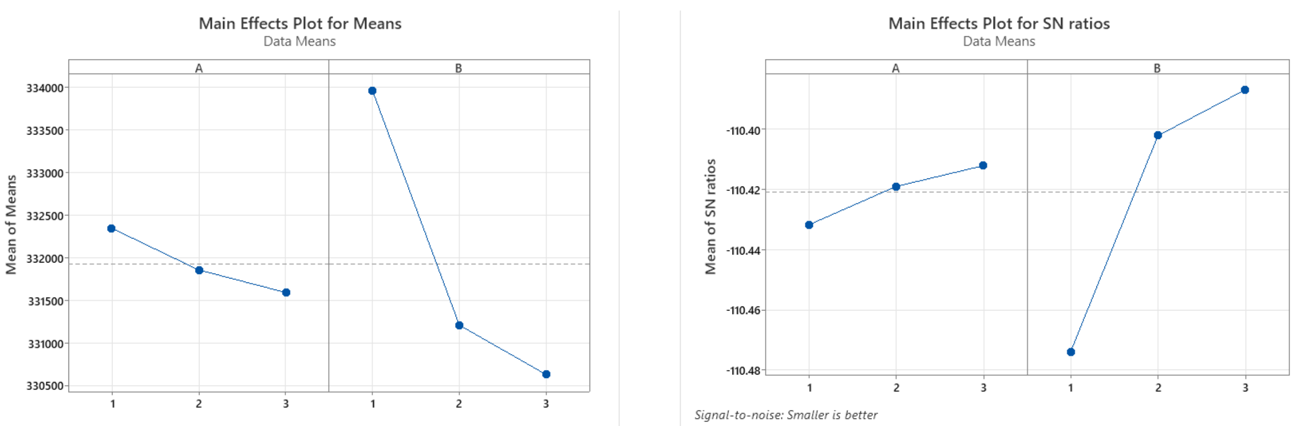


FIGURE 8. The main effects plot for S/N of GWO.

deviation, to asses and compare the efficiency of metaheuristics. The average RPD and RDI as two relative error indicators are used to investigate the solution's quality. These measures are calculated based on the following equations:

$$RPD = \frac{Cur_{sol} - Best_{sol}}{Best_{sol}} \tag{39}$$

$$RDI = \frac{Cur_{sol} - Best_{sol}}{Worst_{sol} - Best_{sol}} \tag{40}$$

where Cur_{sol} is the current obtained solution, $Best_{sol}$ and $Worst_{sol}$ are the best and worst calculated solutions, respectively. Note that the lower values of these measures demonstrate better performance. We utilized average CPU time to demonstrate the required time of algorithms to solve the

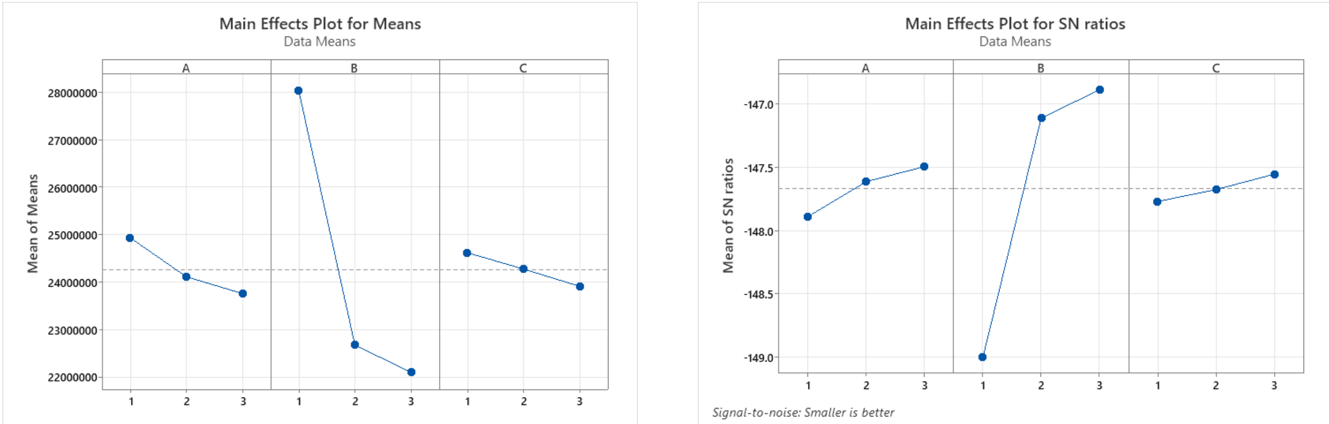


FIGURE 9. The main effects plot for S/N of WOA.

TABLE 6. The computational results of the metaheuristic algorithms.

Problem size		Average RPD		Average RDI		Average standard deviation		Average CPU time	
Vendor	Retailer	GWO	WOA	GWO	WOA	GWO	WOA	GWO	WOA
1	2	7.119E-07	1.930E-09	4.218E-01	4.275E-01	0.921	0.003	14.569	16.158
2	3	5.470E-06	2.118E-07	6.211E-01	3.869E-01	14.125	1.059	16.919	18.362
2	6	3.410E-05	1.892E-04	3.529E-01	4.506E-01	319.281	1626.265	18.117	22.751
3	3	7.666E-06	3.495E-06	4.170E-01	3.970E-01	50.032	25.751	18.228	19.260
3	5	3.215E-05	2.121E-04	3.977E-01	4.572E-01	440.535	1941.583	18.992	20.257
3	6	4.348E-05	5.121E-04	4.886E-01	5.082E-01	620.624	6005.441	19.902	21.817
4	3	6.805E-06	3.325E-06	3.537E-01	3.307E-01	67.446	35.478	19.843	19.148
4	4	3.170E-05	2.659E-04	4.893E-01	3.385E-01	356.253	4552.478	21.917	23.415
4	5	5.143E-04	1.316E-02	4.667E-01	4.230E-01	7752.310	280268.084	26.476	26.376
5	5	7.856E-05	1.055E-03	3.881E-01	3.097E-01	1868.231	23823.369	22.619	23.665
5	7	3.475E-04	2.442E-03	5.998E-01	3.487E-01	5200.742	68809.121	24.730	24.539
6	4	3.436E-05	6.543E-04	4.863E-01	4.669E-01	643.454	11541.881	21.909	22.957
6	5	2.168E-04	1.335E-03	4.572E-01	3.394E-01	4176.421	31895.015	23.000	24.088
6	6	1.950E-04	2.059E-03	6.193E-01	4.524E-01	3689.708	48374.604	25.398	24.530
7	3	1.819E-05	2.274E-04	3.379E-01	3.774E-01	325.777	4136.163	21.729	22.145
Average		1.045E-04	1.475E-03	4.598E-01	4.009E-01	1701.724	32202.420	20.957	21.965

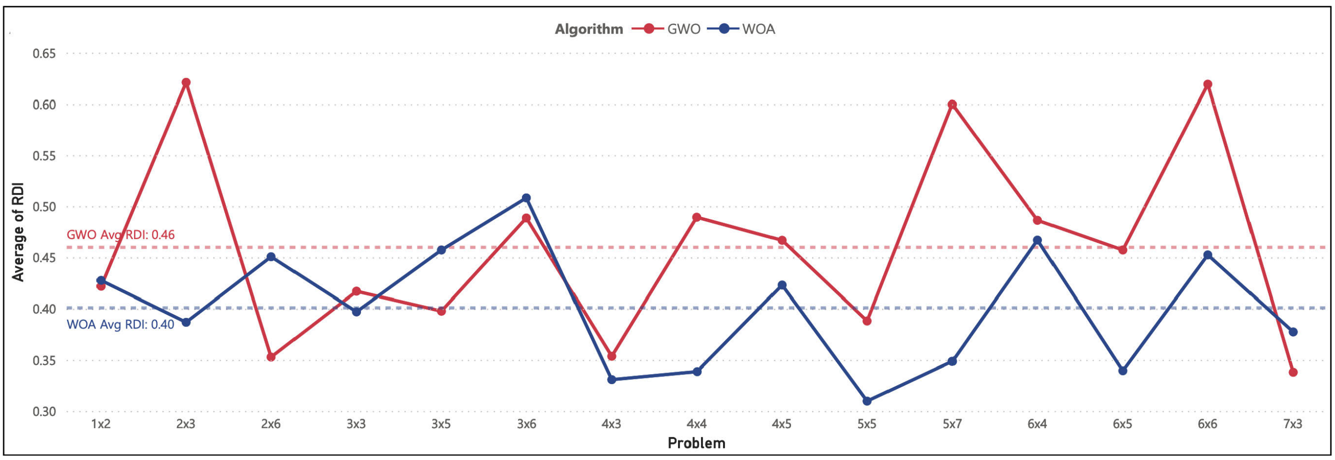


FIGURE 10. The average RDI of metaheuristics for the test examples.

problem. In addition, the standard deviation measure is used to evaluate the robustness of algorithms in different runs. Table 6 summarizes the obtained results of the metaheuristic algorithm.

Considering the RPD and RDI measures, the algorithms are competitive. As can be seen, GWO reaches lower RPD in most cases. On the other hand, the RDI values of WOA are lower than GWO. The results express that GWO is more

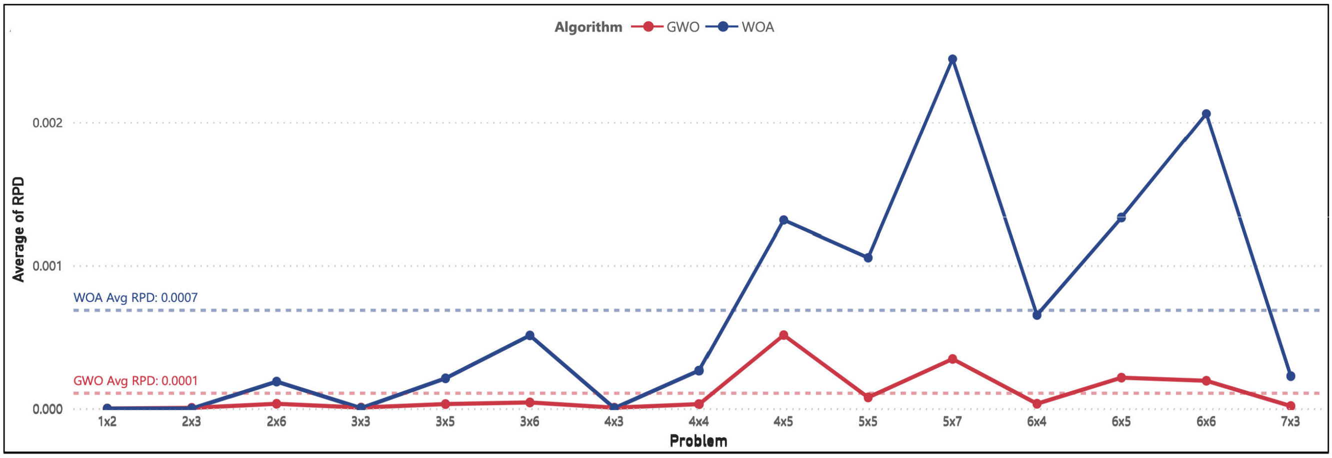


FIGURE 11. The average RPD of metaheuristics for the test examples.

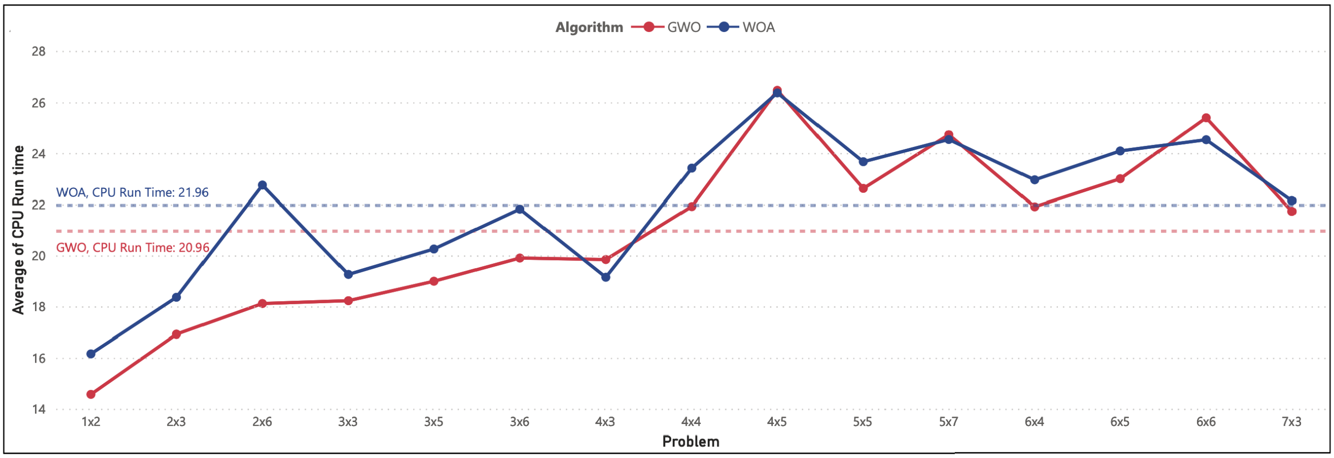


FIGURE 12. The average CPU time of metaheuristics for the test examples.

TABLE 7. The results of statistical test for algorithms comparison.

Measure	Test	P-value	Superior algorithm
Average RPD	Wilcoxon signed-rank test	0.002	GWO
Average RDI	Paired sample	0.048	WOA
Average CPU time	Paired sample	0.010	GWO
Average Standard deviation	Wilcoxon signed-rank test	0.002	GWO

robust than WOA. As can be seen, the variation of the calculated solutions by GWO is less than WOA, and GWO has less standard deviation for different numerical examples. Also, the CPU time of the algorithms is very competitive. However, GWO solves most problems in less amount of time. The schematic comparisons of results are presented in Figures 10 to 13 to provide better insight.

In continuing, the performance of algorithms is compared statistically. Here, we use statistical hypothesis testing to see whether there is a significant difference between the performance of metaheuristics. All tests and comparisons are performed in $\alpha = 0.05$ significance level. The paired-sample t-test and Wilcoxon signed-rank test are parametric and non-parametric tests for the statistical comparison of two populations. To select the proper test, we need to evaluate the normality distribution of considered measures for the solution of algorithms. Therefore, the normal probability plots are provided and presented in Figure 14.

Based on the results, we use Wilcoxon signed-rank test for average RPD and average standard deviation and test the difference between average RDI and average CPU time by paired-sample t-test. The results of these tests are provided in Table 7.

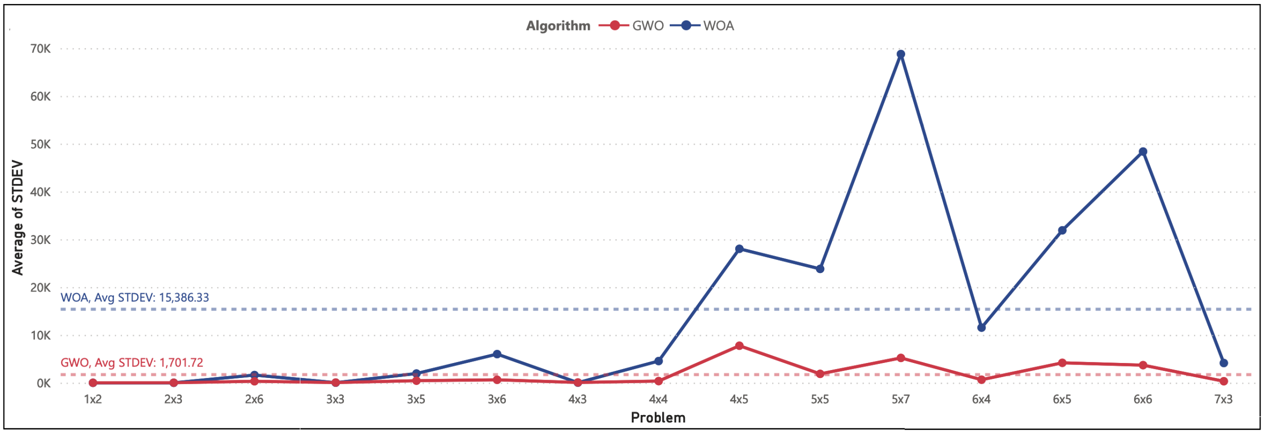
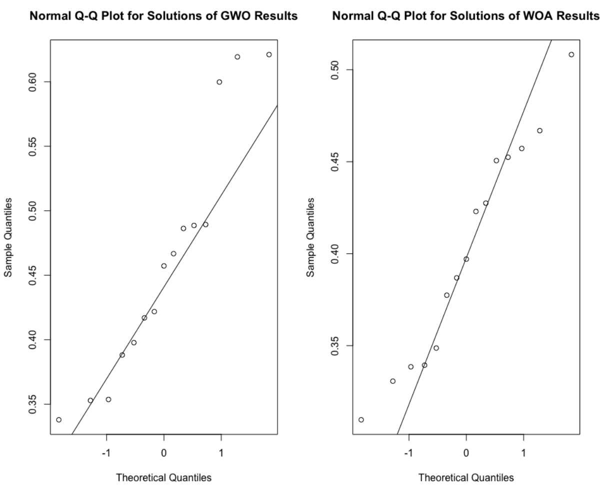
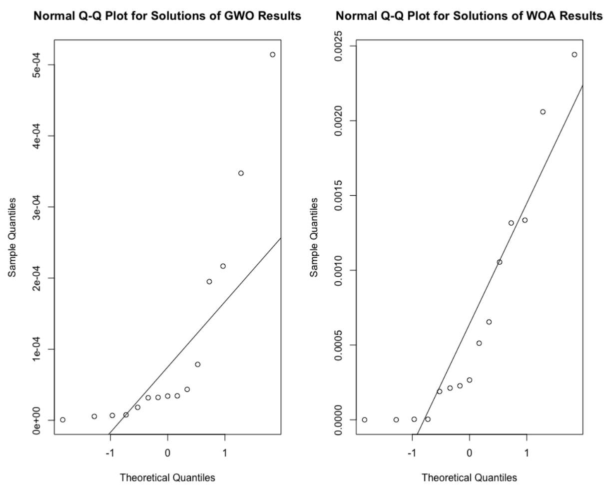


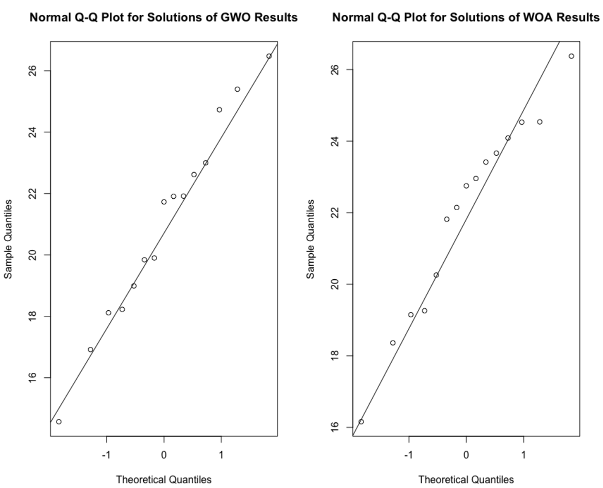
FIGURE 13. The average standard deviation of metaheuristics for the test examples.



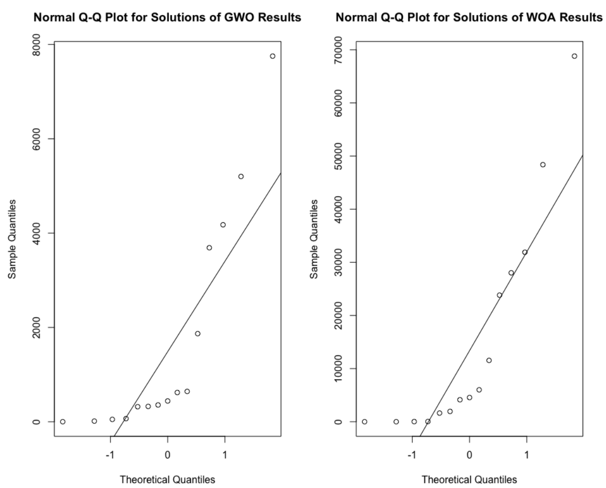
(a) RDI



(b) RPD



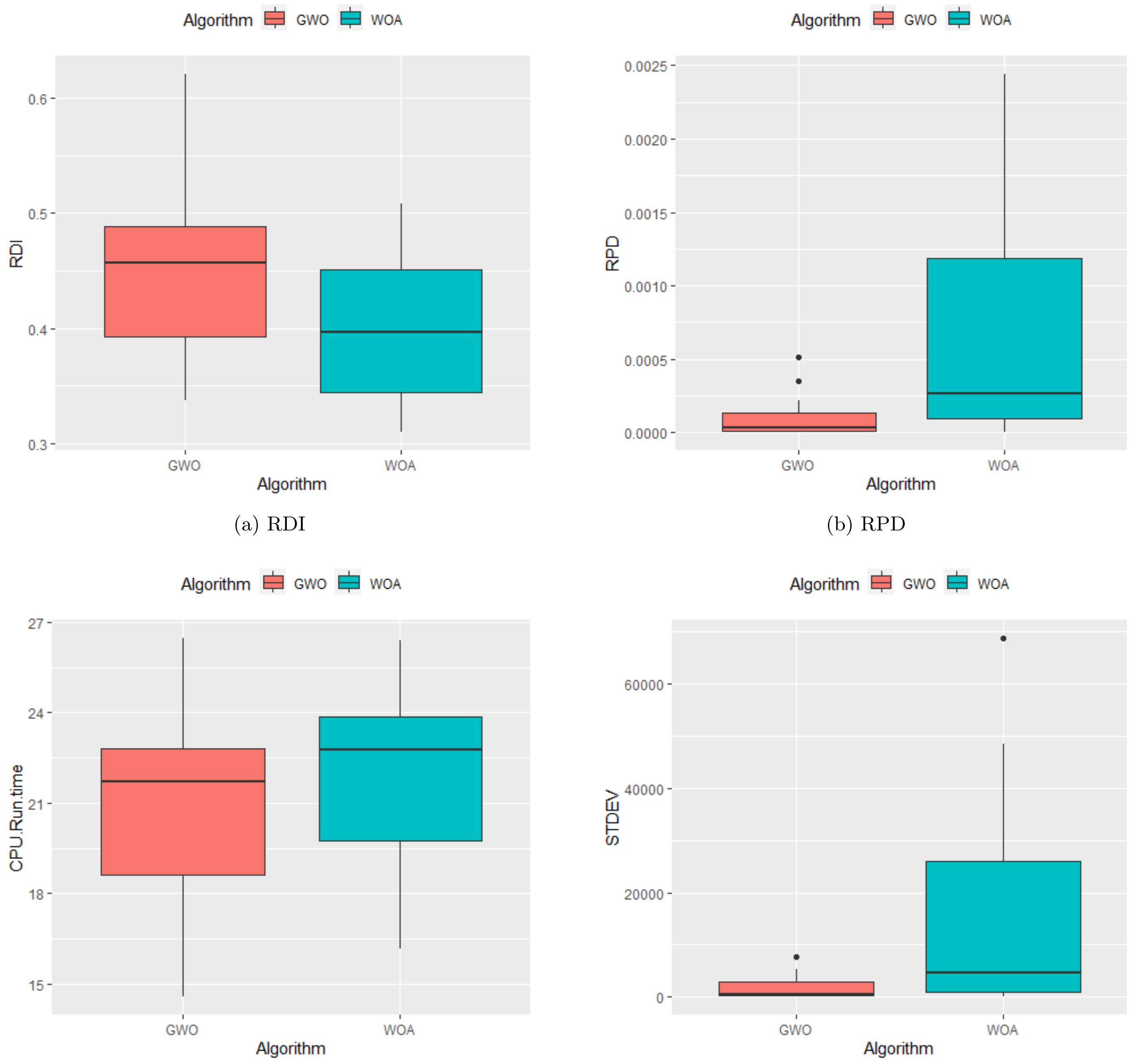
(c) CPU run time



(d) Standard deviation

FIGURE 14. The normal Q-Q plots for performance indicators.

In this table, the p-value of tests is less than 0.05. Consequently, we can infer the significant difference of metaheuristics at this significance level. Considering the results, GWO is the superior algorithm regarding average RPD, CPU time, and standard deviation. However, the WOA algorithm has a significantly better performance in terms of average RDI.



(a) RDI (b) RPD (c) CPU run time (d) Standard deviation

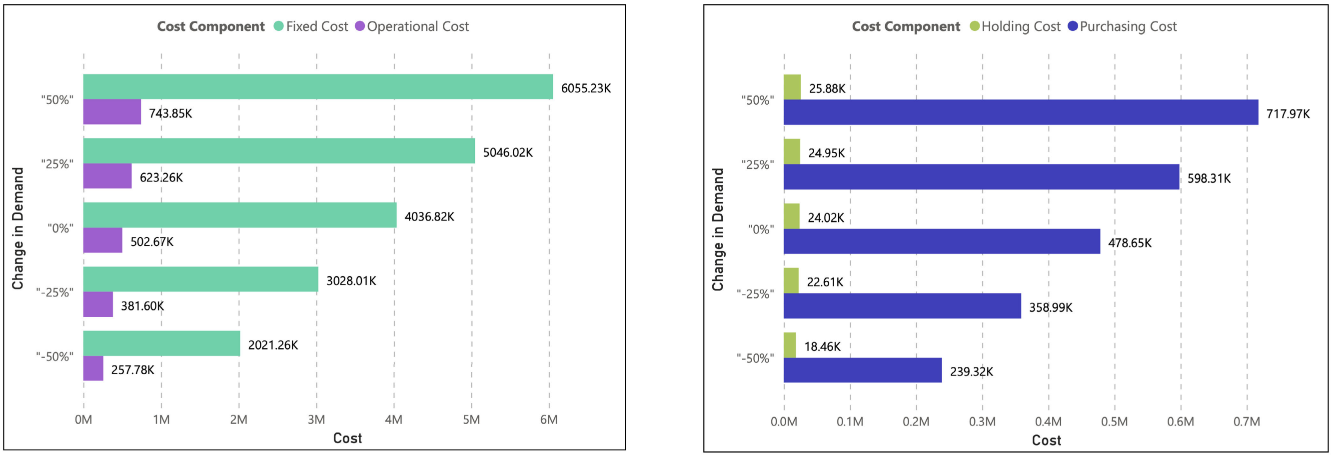
FIGURE 15. **The boxplots for performance indicators.**

In addition, the boxplot of each performance measure is also presented in Figure 15.

As can be seen, the boxplots of GWO are lower than the boxplots of WOA for all measures except the average RDI.

C. SENSITIVITY ANALYSIS

Sensitivity analysis is a systematic approach that aims to provide more insights for managers into the system's considering the variability of parameters [58]. As the last step, sensitivity analysis is carried out to investigate the impact of change in demand as one of the main parameters on the cost components of the inventory system. Since the SQP can calculate the optimal results, this is used to perform the sensitivity analysis. For this goal, we consider the change in parameters at −50% to +50% rates. In addition, we categorize the cost components to provide a better insight. The fixed costs include the fixed ordering cost of the vendor and retailers and the fixed recovery cost of retailers. Also, the purchasing cost and holding cost of retailers are the operational costs. Figure 16 shows the details of obtained results.



(a) Fixed and operational cost components (b) Operational cost components

FIGURE 16. Sensitivity analysis for cost components by changing in demand rate.

As evident, the increasing demand negatively impacts the total cost of the system. In fact, more demand satisfaction requires more ordering, holding, and recovery of products. Based on the results, the fixed cost parameters are more sensitive than the operational cost parameters to changes in demand. In addition, the purchasing cost is more impacted than the holding cost, when the demand is violated. This means managers should focus more on controlling and reducing fixed components to improve the system's performance. Various approaches, such as investment in the infrastructures of the supply chain, can help the managers with this goal.

VI. CONCLUSION

The proposed multi-product model for inventory management addresses the complex dynamics of reusable items in a single-vendor multi-retailer two-level supply chain, which involves significant uncertainties such as stochastic resource constraints. The chance-constrained programming approach was employed to handle these uncertainties and establish the optimal ordering and recovery policies for each product at each retailer. The objective of the model was to minimize the overall cost of the supply chain while ensuring the inventory level and service level requirements of each retailer were met. The use of metaheuristic algorithms has gained significant attention in recent years due to their ability to effectively search the solution space of complex problems. Given the nonlinearity of the constrained model, two novel metaheuristic algorithms, GWO and WOA, were proposed as solution approaches. These algorithms were chosen for their ability to handle complex and nonlinear optimization problems. In addition, the SQP exact algorithm was employed to evaluate the performance of GWO and WOA, and further analysis was conducted. To ensure the optimal performance of the metaheuristic algorithms, the parameters of GWO and WOA were tuned using the Taguchi statistical method. This method is known for its efficiency in optimizing parameters for complex systems.

The numerical example mentioned in the statement highlights the potential of GWO and WOA algorithms in finding solutions that are comparable to those obtained using exact optimization techniques such as SQP. The study extended the analysis to include 15 different sizes of numerical examples. This allowed for a more comprehensive evaluation of the algorithms under different problem sizes. The results showed that the GWO algorithm outperformed the WOA algorithm in terms of solution quality and computational time. Moreover, the study evaluated the robustness of the GWO and WOA algorithms by analyzing their performance variation in multiple runs. The results showed that the GWO algorithm is more robust and produces solutions with lower variation compared to the WOA algorithm. Finally, the study also analyzed the sensitivity of the problem to changes in the demand parameter. The results revealed that the fixed cost components of the problem are more sensitive to changes in demand than the operational cost components. This information can be valuable to decision-makers in understanding the impact of demand fluctuations on the cost structure of the problem.

For future research, it would be beneficial to explore the potential impact of incorporating uncertainty in additional parameters beyond those previously examined. This could be accomplished through the application of stochastic or fuzzy programming methods, which would allow for a more comprehensive understanding of the problem at hand. Additionally, the use of multi-criteria decision-making techniques, including ABC analysis, could help to classify products and optimize system performance. To further enhance the efficacy of the study, researchers may wish to employ other heuristic and metaheuristic algorithms as potential solution approaches, and conduct comparative analyses with both GWO and WOA to assess their relative strengths and weaknesses in addressing the research question.

APPENDIX A
SAS CODE FOR SQP SOLVER

The SAS code for the SQP solver used for this study may be viewed at https://github.com/amir-sadeghi-kh/Reusable-Supply-Chain

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